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Nonlinear Dynamic Inversion for Redundant Systems using the EKF Formalism

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Abstract—This paper presents an allocator for over-actuated systems based on the Extended Kalman Filter (EKF). The main advantages of the proposed approach are the greater flexibility in handling the constraints and its real-time capabilities. Based on the literature, theoretical convergence results, which ensure the convergence towards the local optimal values looked for, are presented. Another formulation of the kinematic equations of redundant systems that meet some constraints is also proposed in order to go through and/or avoid singularities. The two formulations are combined and applied to an academic example (a planar redundant manipulator arm).

Keywords: redundant actuators, allocation, non-linear dynamic inversion, EKF, singularities.

I. INTRODUCTION

When a system is over-actuated, control allocation methods are needed to divide the work over the actuators, in order to generate the required forces or torques on the system. Moreover, depending on the configuration of the actuators, internal singularities can occur (configurations where it becomes impossible to create a force in a direction even if the maximum capacity of the actuators is not reached). Other issues control allocation must face are the saturations of the actuators and the suitability for real-time implementation. These control allocation problems include the control of multi-DOFs (degrees of freedom) manipulator arms, cable robots and submarines. We propose to focus only on the allocation problem (see Fig. 1), and not on the closed-loop regulation. In this paper, vectors are written in bold small

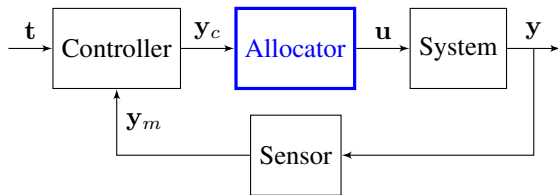


Fig. 1. Block diagram of a typical control loop

letters and the matrices are in bold capital letters.

Let's first write a general controllable nonlinear system (1).

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{G}(\mathbf{x})\mathbf{u} \\ \mathbf{y} = \mathbf{h}(\mathbf{x}) \end{cases} \quad (1)$$

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where $\mathbf{x} \in \mathbb{R}^n$ is the state, $\mathbf{u} \in \mathbb{R}^m$ the control input, $\mathbf{y} \in \mathbb{R}^l$ the controlled output, $\mathbf{f} \in \mathbb{R}^n$, $\mathbf{h} \in \mathbb{R}^l$ differentiable functions, and $\mathbf{G} \in \mathbb{R}^{n \times m}$ a matrix function. $m \geq l$ for an over-actuated (redundant) system.

In a classic nonlinear dynamic inversion, extensively studied for the past thirty by researchers like A. Isidori in [1] for instance, the successive Lie derivatives of \mathbf{y} are calculated so that the input appears explicitly, assuming the nonlinear system has a relative degree.

For a system of relative degree one, one can write:

$$\dot{\mathbf{y}} = \mathbf{J}(\mathbf{x})\dot{\mathbf{x}} \quad (2)$$

with $\mathbf{J} \in \mathbb{R}^{l \times n}$ the jacobian matrix of \mathbf{h} . It then derives :

$$\mathbf{J}(\mathbf{x})\mathbf{G}(\mathbf{x})\mathbf{u} = \dot{\mathbf{y}} - \mathbf{J}(\mathbf{x})\mathbf{f}(\mathbf{x}) \quad (3)$$

If $\mathbf{J}(\mathbf{x})\mathbf{G}(\mathbf{x})$ is invertible, then the solution is:

$$\mathbf{u} = (\mathbf{J}(\mathbf{x})\mathbf{G}(\mathbf{x}))^{-1}(-\mathbf{J}(\mathbf{x})\mathbf{f}(\mathbf{x}) + \dot{\mathbf{y}}) \quad (4)$$

where $\dot{\mathbf{y}}$ is imposed to follow specifications. In a general case, $\mathbf{J}(\mathbf{x})\mathbf{G}(\mathbf{x})$ is a $l \times m$ matrix and can be rank-deficient for some values of \mathbf{x} . Then its inversion can be carried out using the approach developed in this paper. For simplicity, this paper focuses on the case where \mathbf{f} is the null function, \mathbf{G} is the identity matrix and $m = n \geq l$. It comes down to the pure allocation problem where $\dot{\mathbf{x}} = \mathbf{u}$.

Therefore, this paper focuses on computing $\dot{\mathbf{x}}$ which has to respect (2) with a given $\dot{\mathbf{y}}$, as well as saturation constraints and avoiding/passing singularities. The literature on the topic is rather extensive, but only methods that can be implemented in real-time are mentioned here. For a comprehensive survey of usual allocation methods (also called kinematic control), see the work of B. Siciliano and L. Sciavicco [2] [3] for example. The most common method is the Moore-Penrose (MP) pseudo-inversion:

$$\dot{\mathbf{x}} = \mathbf{J}^T(\mathbf{J}\mathbf{J}^T)^{-1}\dot{\mathbf{y}} = \mathbf{J}^\dagger\dot{\mathbf{y}} \quad (5)$$

However, when the matrix $\mathbf{J}\mathbf{J}^T$ becomes non-invertible, the system is in a singularity and (5) fails to compute $\dot{\mathbf{x}}$. To avoid this issue, L. Sciavicco and B. Siciliano [4] propose to use the Jacobian transpose which always exists instead of the MP pseudo-inverse. In addition, stability results are given with this formulation for closed-loop systems. Another possibility is to add a term in the MP formulation so that the inversion becomes always possible (6): it is the Singular Robust Inverse method (SRI) (or Damped Least-Square solution), first introduced by [5] [6].

$$\dot{\mathbf{x}} = \mathbf{J}^T(\mathbf{J}\mathbf{J}^T + \mathbf{W})^{-1}\dot{\mathbf{y}} \quad (6)$$

with \mathbf{W} a matrix to tune. The main difficulty is to choose \mathbf{W} . Different possibilities exist, including the use of the Singular Value Decomposition [7], or methods to take constraints into account [8]. These methods are therefore more robust to singularities, but create errors and do not ensure the passing of the singularities.

A more general solution of (2) is:

$$\dot{\mathbf{x}} = \mathbf{J}^\dagger \dot{\mathbf{y}} + (\mathbf{I} - \mathbf{J}^\dagger \mathbf{J}) \mathbf{x}_v \quad (7)$$

with \mathbf{x}_v an arbitrary vector. $(\mathbf{I} - \mathbf{J}^\dagger \mathbf{J})$ is a projection into the null-space of \mathbf{J} . It is called the gradient method since \mathbf{x}_v is often calculated by optimizing different cost functions, as in [9] which maximizes the dynamic manipulability.

Other methods that do not create errors include the ‘‘Extended Jacobian’’ where constraints are added in the \mathbf{J} matrix. J. Bailleuil in [10] for instance worked on possible constraint formulations. However, the main issues are that constraints have to be well modelled because they need to be always verified, and numerical singularities can occur [2]. C.W. Wampler [11] studied in depth the inverse kinematics functions, in particular the reduction of the workspace in domains where no singularities exist and the problem can be inverted (see also [12]). [13] propose a task-priority based method. However, the main issue with these methods is that they reduce the capabilities of the system by restricting the workspace and imposing hard constraints.

Optimization solutions are also provided in the literature. In particular, T. Fossen and T. Johansen [14] give a survey of possible optimizations for controlling underwater vehicles. Quadratic optimization with constraints is presented, with multi-parametric quadratic resolutions methods for instance [14]. Nonlinear optimization also exists, which can be transformed into linear programming problems [14]. Another optimization method is the use of the Karush-Kuhn-Tucker conditions [15] after defining an optimization problem. An iterative algorithm is then proposed. These methods give optimal solutions, but require a high load of calculation.

Finally, different methods are nowadays proposed, like the Cyclic Co-ordinate Descent where each actuator is moved independently from the others and the Triangulation [16]. They are fast but cannot handle global constraints. Alternatives methods based on Neural Networks are also proposed, as well as hybrid methods that mix numerical and analytical calculations [17].

In this paper, the Extended Kalman Filter (EKF) formalism is used to do nonlinear dynamic inversions for redundant systems. It can take constraints into account in a new way and can be easily implemented in real-time. A comparison with respect to the methods listed before is outlined, an example of theoretical implementation and convergence proofs are presented. Then, a new formulation of the kinematic equations is proposed, which can better handle the singularities. It is well adapted for mechanical systems with some assumptions, and is compatible with the EKF. Finally, the EKF and the singularities-bypassing method are applied on an academic redundant manipulator arm example.

II. PROPOSED ALLOCATION METHOD : THE EXTENDED KALMAN FILTER

A. Advantages of the method and comparisons

To the best knowledge of the authors, using the EKF for allocation has not been done before. This method minimizes the same criterion as a SRI, of the type $r = \frac{1}{2}(\mathbf{m} - \mathbf{C}\mathbf{v})^T \mathbf{R}^{-1}(\mathbf{m} - \mathbf{C}\mathbf{v}) + \frac{1}{2}(\mathbf{v} - \mathbf{v}_d)^T \mathbf{P}^{-1}(\mathbf{v} - \mathbf{v}_d)$ with: (i) \mathbf{m} the measurements ($\dot{\mathbf{y}}$ in the SRI case), (ii) \mathbf{v} the estimation of the state variables ($\dot{\mathbf{x}}$ in the SRI case), (iii) \mathbf{v}_d the predicted value of \mathbf{v} , (iv) \mathbf{C} the jacobian matrix of the measurement equations (\mathbf{J} in the SRI case) and (v) \mathbf{P} and \mathbf{R} weight matrices. In the EKF, the gain matrix \mathbf{P} is updated with a weight matrix \mathbf{Q} and a prediction model, so as to minimize the covariance of the estimation errors.

The EKF also provides more flexibility than methods where constraints are added in the jacobian matrix [10]. Indeed, as some equations may not be exactly verified, they can be more or less weighted through matrices \mathbf{R} and \mathbf{Q} . It can be useful if the modelling is not perfect, especially when taking constraints into account, or simplified models of dynamics. Also, the possibility of time-varying these matrices gives a flexibility and can act as a task-priority controller since it enables the system to emphasize on different equations depending on the current configuration of the system.

The formulation is not very far from the optimization problems, nevertheless the solution proposed (the usual algorithm to solve the EKF) can be carried out in real-time, it is therefore a powerful tool.

Finally, another interesting point is the convergence results that already exist for EKFs, and which can be applied to help tuning the gains so as to remain optimal and convergent.

B. The Extended Kalman Filter

This new allocation method uses the Extended Kalman Filter (EKF) that was initially introduced by [18]. It is commonly used to estimate state variables based on the evaluation of different equations. It is a local optimal observer for nonlinear systems. The discrete formulation is chosen here to remain consistent with the next section, however continuous implementations can be carried out. Let’s take the first-order approximation of (2):

$$\Delta \mathbf{y} = \mathbf{J}(\mathbf{x}) \Delta \mathbf{x} \quad \text{with} \quad \begin{cases} \Delta \mathbf{y} = \mathbf{y}(t) - \mathbf{y}(t - T_s) \\ \Delta \mathbf{x} = \mathbf{x}(t) - \mathbf{x}(t - T_s) \end{cases} \quad (8)$$

with T_s a chosen sampling period. The goal of EKF is to estimate an optimal $\Delta \mathbf{x}$ for a desired $\Delta \mathbf{y}$.

The estimated variables in the EKF can be chosen simply as (9):

$$\mathbf{v} = \begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \end{bmatrix} \quad \text{with} \quad \begin{cases} \mathbf{v}_1 = \Delta \mathbf{x} \\ \mathbf{v}_2 = \mathbf{y} \end{cases} \quad (9)$$

\mathbf{v}_1 is a vector of dimension n and \mathbf{v}_2 of dimension l .

Considering a reference model \mathbf{f}_1 on the internal states dynamics as well as equation (8), the Kalman model becomes:

$$\begin{cases} \mathbf{v}_1(t + T_s) = \mathbf{f}_1(\mathbf{v}_1(t), \mathbf{v}_2(t)) + \boldsymbol{\epsilon}_{v_1} \\ \mathbf{v}_2(t + T_s) = \mathbf{J}(\mathbf{x})\mathbf{v}_1(t) + \mathbf{v}_2(t) + \boldsymbol{\epsilon}_{v_2} \end{cases} \quad (10)$$

where the state evolution errors $\boldsymbol{\epsilon}_{v_1}$ and $\boldsymbol{\epsilon}_{v_2}$ are neglected in the prediction model and sized by the covariance matrix \mathbf{Q} . \mathbf{f}_1 can be any function associated with the studied system. In the general case, the prediction equations read : $\hat{\mathbf{v}}(t + T_s) = \mathbf{f}_k(\hat{\mathbf{v}}(t))$. The Jacobian matrix associated with this model is denoted \mathbf{F} and is of size $(n + l) \times (n + l)$:

$$\mathbf{F} = \frac{\partial \mathbf{f}_k}{\partial \mathbf{v}} \quad (11)$$

The measurement equation (\mathbf{m} is the measurement vector) is:

$$\mathbf{m} = \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \end{bmatrix} = \mathbf{r}(\mathbf{v}) + \boldsymbol{\epsilon}_m \quad \text{with} \quad \begin{cases} \mathbf{m}_1(t) = \mathbf{v}_2(t) + \boldsymbol{\epsilon}_{m_1} \\ \mathbf{m}_2(t) = \mathbf{f}_m(\mathbf{v}_1(t)) + \boldsymbol{\epsilon}_{m_2} \end{cases} \quad (12)$$

\mathbf{m}_1 is the reference input \mathbf{y}_c sent by the controller for instance (Fig. 1). To ensure the observability of the EKF, the second equation of (12) is added, and can be used to model additional constraints (e.g. saturations). Perturbations $\boldsymbol{\epsilon}_{m_1}$ and $\boldsymbol{\epsilon}_{m_2}$ are sized by the covariance matrix \mathbf{R} . Let us denote $\mathbf{C} = \frac{\partial \mathbf{r}}{\partial \mathbf{v}}$.

The EKF theory provides a local optimal estimate $\hat{\mathbf{v}}$. The parameters that need to be set in this filter are \mathbf{P}_0 the initial covariance matrix on the estimate errors, \mathbf{R} and \mathbf{Q} . The equations of evolution of the discrete EKF are not given here, but can be easily found in the literature [18].

The method gives the best least-square solution of the problem posed. Since the equations are deterministic, the solution is a compromise between the different equations that can include constraints on the system. The different weights give the importance of each equation.

C. Convergence results

1) *Existing results:* The convergence of the allocator is an important point to verify. Some results already exist for allocation methods, as the Jacobian transposed [4], already cited. The convergence of the error dynamics was proven in a closed-loop. Another result by T. Johansen [19] shows the exponential global convergence of a closed-loop which contains an exponentially stable controller and an asymptotically optimal allocator. [20] gives a generic form of an input allocation with a first-order dynamics that is internally stable, with different characteristics and with saturation constraints.

Therefore, some methods have a stability proof, but not all. Moreover, except for the last one, they are closed-loop results. The method from [20] seems to be less friendly for adding constraints (except saturations). This paper focuses only on the open-loop local convergence of the allocator. Therefore, it gives an interesting result whatever the controller used, as well as when there is no controller. The Lyapunov function for the proof is from [21] and can be used to show the global stability of the closed-loop system.

2) *Theory:* [21] gives the conditions for the EKF to be a local uniform asymptotic observer, and thus for $\hat{\mathbf{v}}$ (resp. $\hat{\mathbf{v}}_2$) to tend to \mathbf{v} (resp. \mathbf{y}). In our case, these conditions will guarantee the proposed allocation method to converge. These conditions are :

- \mathbf{F} is invertible at each time step.
- The following norms are bounded : $\|\mathbf{F}\|, \|\mathbf{F}^{-1}\|, \|\mathbf{C}\|, \|\mathbf{R}\|, \|\mathbf{Q}\|, \|\partial \mathbf{F} / \partial \mathbf{v}\|, \|\partial \mathbf{C} / \partial \mathbf{v}\|$.
- Let's call $\mathbf{k}(\mathbf{p}, \mathbf{q}) = \mathbf{r}(\mathbf{p}) - \mathbf{r}(\mathbf{q}) - \mathbf{C}(\mathbf{p})(\mathbf{p} - \mathbf{q})$. There exists a scalar m such that

$$\|\mathbf{k}(\mathbf{p}, \mathbf{q})\| \leq m \|\|\partial \mathbf{C} / \partial \mathbf{v}\|\| \|\mathbf{p} - \mathbf{q}\|^2$$

- The system is uniformly observable, or equivalently (\mathbf{F} , \mathbf{C}) verify the uniform observability condition given in [21], taken from the article of J.J. Deyst and C.F. Price [22], as long as the estimation errors are small enough. That gives boundaries on the estimation covariance error matrix.
- The last condition ensuring the Lyapunov function $V = (\hat{\mathbf{v}} - \mathbf{v})^T \mathbf{P}^{-1}(\hat{\mathbf{v}} - \mathbf{v})$ to be decreasing is omitted for brevity and is detailed in [21].

That gives conditions and a proof of the convergence of the nonlinear dynamic inversion proposed. With this proof, it is possible to size the covariance matrices of the EKF and to ensure convergence when additional constraints (that can depend on the configuration of the system) are added. Nevertheless, this formulation does not ensure that the singularities will be passed. As in the SRI formulations, a solution always exists, but for the convergence proof, well-conditioned matrices are needed, that is not always the case with the matrix \mathbf{J} . Moreover, there may be a lack of stability in the vicinity of a singularity because our variational formulation of (10) may not be representative enough: if a singularity-avoidance is needed and is not well-modelled, the system cannot search in the null-space of \mathbf{J} and then the errors created make the system go outside the stability region. That is why it is proposed to improve the model implemented in the EKF, so as the optimal value towards which the system converges always exists. The second-order Taylor expansion, which will help passing the singularities, is now studied.

III. A SINGULARITY-HANDLING FORMULATION

Let's first express differently (2), by doing a Taylor expansion to the second-order approximation:

$$\begin{aligned} \Delta \mathbf{y} &= \mathbf{h}(\mathbf{x} + \Delta \mathbf{x}) - \mathbf{h}(\mathbf{x}) \\ &= \mathbf{J}(\mathbf{x})\Delta \mathbf{x} + \frac{1}{2} \sum_{i=1}^n (\Delta x_i) \frac{\partial^2 \mathbf{h}(\mathbf{x})}{\partial x_i \partial \mathbf{x}} \Delta \mathbf{x} + o(\Delta \mathbf{x})^2 \end{aligned} \quad (13)$$

Let's denote $\mathbf{H}_i = \frac{\partial^2 \mathbf{h}}{\partial x_i \partial \mathbf{x}} = \frac{\partial \mathbf{J}}{\partial x_i}$ and define the matrix \mathbf{H} as the concatenation of the columns number i in the matrices \mathbf{H}_i (14). Its size is $l \times n$.

$$\mathbf{H} = [\mathbf{H}_1(:, 1) \quad \mathbf{H}_2(:, 2) \quad \dots \quad \mathbf{H}_i(:, i) \quad \dots \quad \mathbf{H}_n(:, n)] \quad (14)$$

with $\mathbf{H}_i(:, i)$ referring to the i^{th} column of \mathbf{H}_i .

Then, (13) can be written as:

$$\Delta \mathbf{y} = \mathbf{J}(\mathbf{x})\Delta \mathbf{x} + \frac{1}{2}\mathbf{H}(\mathbf{x})\Delta^2 \mathbf{x} + \frac{1}{2}\sum_{j=1}^n (\Delta x_j)\mathbf{H}'_j \Delta \mathbf{x} + o(\Delta^2 \mathbf{x}) \quad (15)$$

with \mathbf{H}'_j the matrix \mathbf{H}_j whose column j is replaced by a null vector and $\Delta^2 \mathbf{x}$ the vector composed of the components Δx_i^2 .

Let's call \mathbf{M} the $(l \times 2n)$ matrix defined in (16).

$$\mathbf{M} = [\mathbf{J}(\mathbf{x}) \quad \mathbf{H}(\mathbf{x})] \quad (16)$$

In the case of manipulator arms, since the direct kinematic model is composed of sinus and cosinus of the x_i , then each column vector of \mathbf{J} is rotated by $\pi/2$ in \mathbf{H} , and so the vectors $\mathbf{J}(:, i)$ and $\mathbf{H}(:, i)$ are perpendicular to each other for any i . A vector \mathbf{s} is then a singular direction to \mathbf{M} if and only if it is perpendicular to all vectors of \mathbf{M} (equivalently $\mathbf{s}^T \mathbf{M} = \mathbf{0}_{1 \times 2n}$). It means that all vectors of \mathbf{M} are in the same hyperplane, hence that we have a planar manipulator arm. If the system is intrinsically 3-D, that guarantees that \mathbf{M} is full rank for any \mathbf{x} .

The main assumption in this paper is that the matrix \mathbf{M} defined in (16) is always of rank l whatever the values of \mathbf{x} . From the robotic example given above, this assumption is verified in many mechanical systems.

Another advantage of using \mathbf{M} is that the search of the best solution also takes into account the null-space of \mathbf{J} through \mathbf{H} and therefore will help pass and/or avoid the singularities. See the work of K. Yamada and I. Jikuya [23] for theoretical research and proofs on the usefulness of this formulation.

This matrix \mathbf{M} supposed regular is then used to invert the relation (15) rewritten as (17).

$$\Delta \mathbf{y} - \frac{1}{2}\sum_{j=1}^n (\Delta x_j)\mathbf{H}'_j \Delta \mathbf{x} = \mathbf{M}(\mathbf{x}) \begin{bmatrix} \Delta \mathbf{x} \\ \frac{1}{2}\Delta^2 \mathbf{x} \end{bmatrix} + o(\Delta^2 \mathbf{x}) \quad (17)$$

$o(\Delta^2 \mathbf{x})$ is assumed to be negligible compared to the other terms (assumption true if the time step is small enough).

Then, our goal is to find the vector $\begin{bmatrix} \Delta \mathbf{x} \\ \frac{1}{2}\Delta^2 \mathbf{x} \end{bmatrix}$ that verifies (17) and that also verifies the obvious internal relationship.

It should be noticed that the input of the equation (17) is $\Delta \mathbf{y} - \frac{1}{2}\sum_{j=1}^n (\Delta x_j)\mathbf{H}'_j \Delta \mathbf{x}$, the first term being the reference input assumed to be available from the controller, and the second term is considered to be a feed-forward from the previously calculated \mathbf{x} and $\Delta \mathbf{x}$.

\mathbf{H}'_i are in general not null for manipulator arms. The theoretical study of the null-space of $\mathbf{J}\mathbf{J}^T$ and the associated singularities is then more complicated, however it can be noticed that this formulation enables us to avoid numerical singularities, and still search into the null-space of $\mathbf{J}\mathbf{J}^T$ to bypass the kinematic singularities.

To our best knowledge, only the article [23] uses the second-order Taylor expansion for control. However, the method proposed in article [23] to find $\begin{bmatrix} \Delta \mathbf{x} \\ \frac{1}{2}\Delta^2 \mathbf{x} \end{bmatrix}$ is rather costly since it is not analytically solvable and requires an

iterative procedure at each sample time. It is hardly generalizable to greater numbers of actuators since the resolution of the equations in the Gröbner bases can become of very high order. Using the EKF formalism as presented before enables us to estimate $\Delta \mathbf{x}$ and $\Delta^2 \mathbf{x}$ that verify their intrinsic relationship and verifies equation (17) in only one iteration at each time step. For instance, it is possible to modify the EKF presented before by adding $\Delta^2 \mathbf{x}$ as new estimated variables. The term $\frac{1}{2}\sum_{j=1}^n (\Delta x_j)\mathbf{H}'_j \Delta \mathbf{x}$ can be considered as a known input perturbation in the model.

The combination of the EKF and this formulation gives a new tool that can efficiently invert the dynamic equation (1), take constraints into account with flexibility, handle the singularities, with convergence results that help tuning the parameters of the EKF.

IV. APPLICATION

A. Modelling

To apply the proposed method, let's consider the simple case of a 2-DOFs manipulator arm moving in a reference plane supposed fixed $(\mathbf{u}_r, \mathbf{v}_r)$ (see Fig. 2). The manipulator has 3 arms linked by 3 revolute joints. The objective is to control the end-effector position (y_1, y_2) . The parametrization is chosen relative : the angles between the arms are the controlled parameters, called q_1, q_2 and q_3 . The length of the arms are noted l_1, l_2 and l_3 . In the $(\mathbf{u}_r, \mathbf{v}_r)$ plane, the

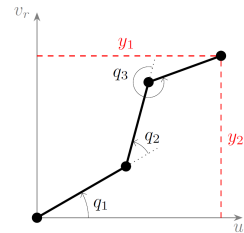


Fig. 2. Parametrization of the system studied

dynamics of the system (assumed rigid) has the form (18), taken from the book of W. Khalil and E. Dombre [24].

$$\ddot{\mathbf{q}} = -\mathbf{A}^{-1}(\mathbf{q})\mathbf{C}_q(\dot{\mathbf{q}}, \mathbf{q}) + \mathbf{A}^{-1}(\mathbf{q})\mathbf{u} \quad (18)$$

with \mathbf{A} the inertia matrix and \mathbf{C}_q the quadratic velocity vector including the Coriolis and the centrifugal forces. The joints are controlled by imposing a torque vector \mathbf{u} .

To identify to (1), let us denote $\mathbf{x} = \begin{bmatrix} \mathbf{q} \\ \dot{\mathbf{q}} \end{bmatrix}$.

The kinematic equations read:

$$\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \mathbf{h}(\mathbf{q}), \quad \dot{\mathbf{y}} = \begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} = \mathbf{J}(\mathbf{q})\dot{\mathbf{q}} \quad (19)$$

$\Delta \mathbf{q}$ is computed by using the EKF and the singularity-handling formulation. Indeed, the estimated variables of the EKF were chosen as (20). The prediction equations and the

measurement equations are (20) and (21).

$$\begin{cases} \mathbf{v}_1 = \Delta \mathbf{q} \\ \mathbf{v}_2 = \Delta^2 \mathbf{q} \\ \mathbf{v}_3 = \Delta \mathbf{y} \end{cases} \quad \text{and} \quad \begin{cases} \hat{\mathbf{v}}_1(t + T_s) = \frac{2}{3} \hat{\mathbf{v}}_1(t) \\ \hat{\mathbf{v}}_2(t + T_s) = \frac{4}{9} \hat{\mathbf{v}}_2(t) \\ \hat{\mathbf{v}}_3(t + T_s) = \frac{1}{2} \hat{\mathbf{v}}_3(t) + \frac{1}{2} \mathbf{w}(t) \end{cases} \quad (20)$$

with \mathbf{w} the reference input $\Delta \mathbf{y}_c$.

In (20), the internal state dynamics of reference model \mathbf{f}_1 is $(2/3, 4/9, 1/2)$ and is chosen in the following way: the first two equations of (20) are used to limit the maximal accelerations of the joints. The third equation of (20) was chosen to keep the \mathbf{F} matrix (11) invertible as required for the convergence proof, and take into account the reference input $\mathbf{w}(t)$.

$$\begin{cases} \mathbf{m}_1 = \sum_{j=1}^n (\Delta q_j) \frac{\mathbf{H}_j'}{2} \Delta \mathbf{q} = -\mathbf{J} \mathbf{v}_1 - \frac{\mathbf{H}}{2} \mathbf{v}_2 + \mathbf{v}_3(t) + \epsilon_{m1} \\ \mathbf{m}_2 = \mathbf{0}_{3 \times 1} = -\mathbf{v}_1^2 + \mathbf{v}_2 + \epsilon_{m2} \\ \mathbf{m}_3 = \mathbf{k} = \mathbf{v}_1 + \epsilon_{m3} \end{cases} \quad (21)$$

The first two equations of (21) are imposed by the constraints on the problem. Let's develop the choice of \mathbf{k} :

$$\mathbf{k} = k_1 \begin{bmatrix} \mathbf{I}_{3 \times 3} & \mathbf{0}_{3 \times 3} \end{bmatrix} \mathbf{N} \begin{bmatrix} \mathbf{1}_{1 \times 3} & \mathbf{0}_{1 \times 3} \end{bmatrix}^T \quad (22)$$

with $\mathbf{N} = \mathbf{I} - \mathbf{M}^\dagger \mathbf{M}$, the projection on the null-space of $\mathbf{M} = [\mathbf{J}(\mathbf{q}) \quad \mathbf{H}(\mathbf{q})]$. This operation is always defined because of the regularity of \mathbf{M} . k_1 is defined as:

$$k_1 = \frac{\|\mathbf{w}\|}{w_{max}} \dot{q}_{sat} T_s \quad (23)$$

with w_{max} the maximal norm of \mathbf{w} , \dot{q}_{sat} the maximal allowable speed of the joint angles and T_s the sampling period. (22) is relevant for two aspects : it takes into account the saturation constraint, which depends on the distance to the reference value (via \mathbf{w}), thus not creating movement when the system has reached its goal. In addition, projecting the obtained value on the null-space of \mathbf{M} avoids creating errors (because these components are associated with the null-space of \mathbf{J}) and it helps steering the system out of initial singularities, when the system has no speed. Adding a term in the null-space of \mathbf{M} is also interesting because \mathbf{v}_2 gives components along \mathbf{H} which comprises a part of the null-space of \mathbf{J} complementary to the null-space of \mathbf{M} , therefore expanding the search in the null-space of \mathbf{J} .

The convergence results of the EKF were used to tune the parameters \mathbf{R} , \mathbf{Q} and \mathbf{P}_0 : $\mathbf{Q} = 4.0 \cdot 10^{-6} \mathbf{I}$ and $\mathbf{P}_0 = 5.10^{-5} \mathbf{I}$. As for \mathbf{R} , a valid interval for its singular values has been computed. $\mathbf{R} = \text{diag}([10^{-5}, 10^{-5}, 1.23 \cdot 10^{-5}])$. Indeed, a lower confidence appears on the last measurement since the last equation does not model perfectly the saturation constraint. The estimates \mathbf{v} are all initialized at 0.

The block diagram summarizing the implementation of the application is given in Fig. 3.

B. Simulations

In the simulation, $\mathbf{u} = \mathbf{K}_w (\Delta \mathbf{q} / T_s - \dot{\mathbf{q}}_m)$ (with \mathbf{K}_w a gain matrix and $\dot{\mathbf{q}}_m$ the measured joint velocities). It is assumed that \mathbf{q} and $\dot{\mathbf{q}}$ are measured.

Numerical application : The 3 beams are uniform with the same mass per unit of length (3.39 kg/m). $l_1 = l_2 = l_3 = 0.4 \text{ m}$, and for each joint $K_w = 3 \text{ Nm s/rad}$, $\dot{q}_{sat} = 10 \text{ rad/s}$ and $T_s = 0.001 \text{ s}$.

\mathbf{w} is taken as the distance between the required position and the current position of the end-effector, with a saturation calculated from the saturation of \dot{q}_{sat} and the maximal singular value of \mathbf{J} . The value of the saturation obtained is $w_{sat} = 0.015 \text{ m}$.

The system is initially in a singular folded configuration: $q_1 = 0, q_2 = \pi, q_3 = \pi \text{ rad}$. The final state is another singular state $(y_{1i}, y_{2i}) = (-(l_1 + l_2 + l_3), 0)$ which is in the initial singular direction. The motion created by the proposed allocator is given in Fig. 4. After 31s, a new singular state is

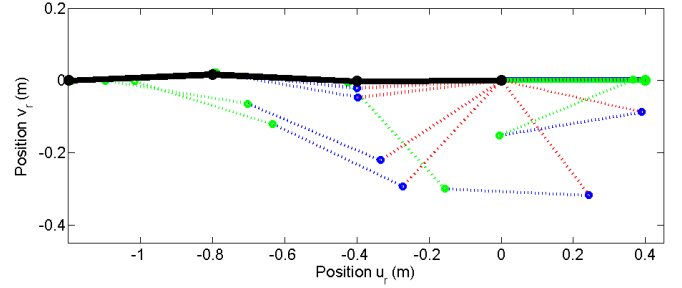


Fig. 4. Trajectory of the manipulator arm for the first movement. Solid green : initial state. Solid black : final state. Dashed lines : intermediate states (red, blue and green for arm (1), (2) and (3) respectively).

required: $(y_{1f}, y_{2f}) = (0, 0)$. This part of the motion is given in Fig. 5. The joints motion and end-effector positions versus time are in Fig. 6. Even if beginning in a folded singular

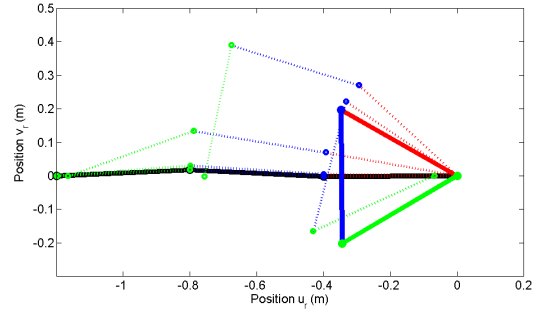


Fig. 5. Trajectory of the manipulator arm for the second movement. Solid lines : final state.

configuration, the limits of the workspace has been reached with an error of less than 0.0003 m and the system has gone back to a singular position, hence the allocation method manages the worst cases. The maximal error of the end-effector along the \mathbf{v}_r -axis is less than 1 cm . The time spent to escape from a singular position is due to the reconfiguration in the null-space of \mathbf{J} that does not create movement for the end-effector. In addition, other singularities were avoided along the trajectory (the $(0,0)$ point for instance).

V. CONCLUSION

The method proposed to allocate over-actuated systems is real-time and has stability proofs. The different parameters

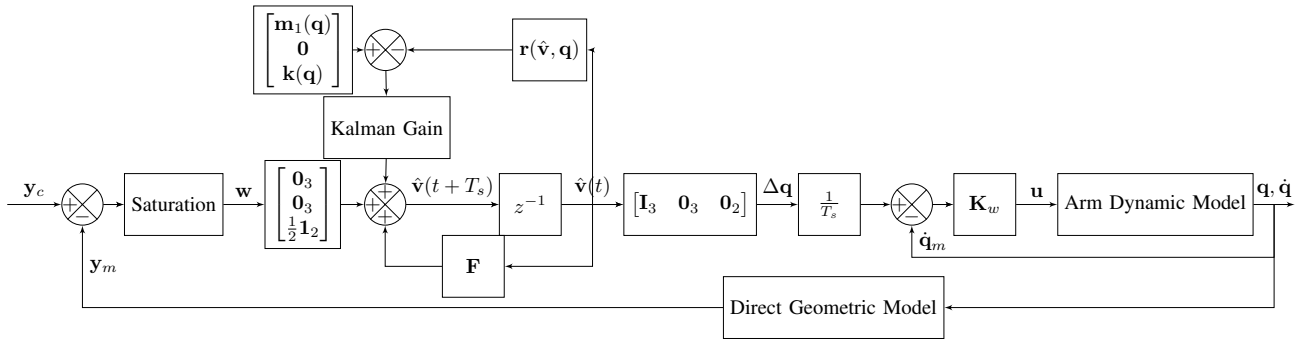


Fig. 3. Block diagram of the whole system

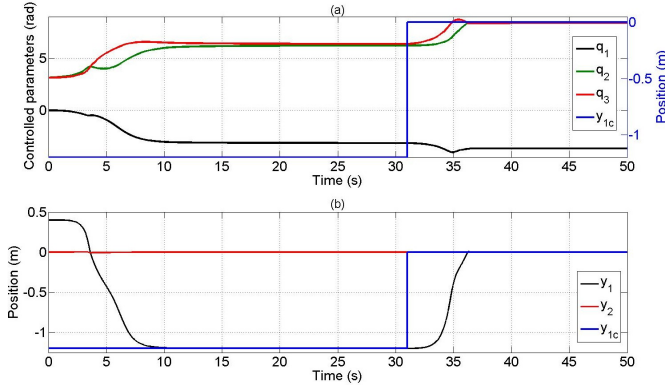


Fig. 6. (a) Angles \mathbf{q} (b) End-effector position with respect to time.

can be tuned to ensure the convergence, the time response as well as the priority among the different equations. Another formulation for singularity-passing and avoiding has been proposed, which has proven efficient on the given example. The forthcoming work will now focus on a better integration of the constraints in the formulation in particular cases, as well as a study on the sensitivity of the method on parameter uncertainties. In addition, a comparison with other existing techniques will be carried out.

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