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Enhanced Worst-case Timing Analysis of Ring-based Networks with Cyclic Dependencies using Network Calculus

A. AMARI, A. MIFDAOUI, F. FRANCES, J. LACAN
University of Toulouse-ISAE, France

Abstract—The recent research effort towards defining new communication solutions for cyber-physical systems (CPS), to guarantee high availability level with limited cabling costs and complexity, has renewed the interest in ring-based networks. This topology has been recently used for various networked cyber-physical systems (Net-CPS), e.g., avionics and automotive, with the implementation of many Real Time Ethernet (RTE) profiles. A relevant issue for such networks is to prove timing predictability, a key requirement for safety-critical systems. For the most common ring-based Real Time Ethernet (RTE) profiles, conducting such performance analyses has been greatly simplified due to their implemented time-triggered communication scheme, e.g. Master/slave or TDMA. Unlike these existing approaches, we are interested in this paper in event-triggered ring-based networks, which guarantee high resource utilization efficiency and (re)configuration flexibility, at the cost of increasing the timing analysis complexity. The implementation of such a communication scheme on top of a ring topology actually induces cyclic dependencies, in comparison to time-triggered solutions. To cope with this arising issue of cyclic dependencies, only few techniques have been proposed in the literature, mainly based on Network Calculus framework, and consist in analyzing locally the delay upper bound in each crossed node, resulting in pessimistic end-to-end delay bounds. Hence, the main contribution in this paper is enhancing the delay bounds tightness of such networks, through an innovative global analysis based on Network Calculus, considering the flow serialization phenomena along the flow path. An extensive analysis of such a proposal is conducted herein regarding the accuracy of delay bounds and its impact on the system performance, i.e., scalability and resource-efficiency; and the results highlight its outperformance, in comparison to conventional methods.

Index Terms—Network Calculus, PMOO, Performance analysis, Ring, Cyclic dependencies, Delay bounds, Non-feedforward.

I. INTRODUCTION

The recent research effort towards defining new communication solutions for cyber-physical systems (CPS), to guarantee high availability level with limited cabling costs and complexity, has renewed the interest in ring-based networks, which provide an implicit redundant path by introducing only one additional connection between the two end nodes, compared to line or star topologies [13]. The ring-based networks have been prominently used for industrial networked cyber-physical systems (Net-CPS) with the implementation of many Real Time Ethernet (RTE) profiles cited in IEC 61784-2 [4], e.g., EtherCAT [1], SERCOSIII [2] and Profinet-IRT

[15], and recently in other application fields like automotive, e.g. RACE [19], and avionics, e.g. AeroRing [5]. A relevant issue for such networks is proving time predictability, a key requirement for safety-critical applications. Hence, to deal with the performance evaluation of such networks, accurate timing analysis to compute worst-case delays or at least upper bounds has to be considered.

For the most common ring-based Real Time Ethernet (RTE) profiles, conducting such performance analyses has been greatly simplified due to their implemented time-triggered communication scheme, e.g. Master/slave or TDMA. Unlike these existing approaches, we are interested in this paper in event-triggered ring-based networks, which guarantee high resource utilization efficiency and (re)configuration flexibility, at the cost of increasing timing analysis complexity. The implementation of such a communication scheme on top of a ring topology actually induces cyclic dependencies, i.e., there exist interdependent flows with paths forming cycles, in comparison to time-triggered solutions.

To cope with this arising issue of cyclic dependencies, only few techniques have been proposed in the literature, mainly based on Network Calculus framework [14]. This framework is considered as one of the most efficient methodologies for the worst-case performance analysis and has been recently used to certify the avionics standard AFDX [3] [11]. These existing approaches are based on local analyses of delay upper bounds, e.g., [9] [8] [12], or backlog upper bounds, e.g., [21] [14], in each crossed node; thus resulting in end-to-end delay bounds computation. However, these main conventional analysis methods lead to overly pessimistic upper bounds, which limits the system's scalability and resource efficiency, as it will be illustrated in Section VI.

To handle these limitations, an innovative global analysis based on Network Calculus¹, considering the flow serialization phenomena along the flow path, is proposed in this paper to enhance the delay bounds tightness of such networks. The main idea is based on one of the most recent results in Network Calculus framework, denoted as "Pay Multiplex Only Once" (PMOO) proposed in [17]. This principle consists in paying the bursts of interfering flows only once, to compute tight end-to-end delay bounds. This latter has been proposed in [17]

¹This is an updated version at July 2016

for networks with acyclic graph under arbitrary multiplexing; thus it is extended in this paper to ring-based networks with cyclic dependencies under Fixed Priority (FP) multiplexing. The main contributions of this work are twofold:

- First, a new closed-form formula of the guaranteed end-to-end service curve of a flow of interest, crossing a ring-based network with cyclic dependencies under FP policy, is defined and the formal proof of its correctness is detailed. This introduced service curve will infer the computation of tight end-to-end delay upper bounds, accounting for cyclic dependencies impact;
- Then, an extensive analysis of the approach is conducted, regarding the delay bound tightness and its impact on the system performance, e.g., scalability and resource-efficiency. Results highlight its outperformance, in comparison to conventional timing analysis and an achievable worst-case delay lower bound.

In the next section, we present the main principles of the Network Calculus framework, necessary to conduct the worst-case timing analysis of ring-based networks. Then, we give an overview of the most relevant timing analysis approaches in this specific area, and relate them to our work in Section III. Afterwards, we detail the main assumptions, notations and system model in Section IV. Our proposed approach and the formal proofs of its correctness are then detailed in Section V. Finally, Section VI presents the performance evaluation of our proposal, in comparison with conventional approaches.

II. NETWORK CALCULUS BACKGROUND

Network Calculus formalism provides deterministic upper bounds on delays and backlogs (queue sizes), through modeling the maximum input traffic arrival and the minimum availability of the crossed nodes, described by the so called *maximum arrival curve* and *minimum service curve*, respectively. The definitions of these curves are explained in following.

Definition 1. (*Arrival Curve*) a function $\alpha(t)$ is an arrival curve for a data flow with an input cumulative traffic function $F(t)$, i.e., the number of bits received until time t , iff:

$$\forall t, F(t) \leq F \otimes^2 \alpha(t)$$

Definition 2. (*Service curve*) The function $\beta(t)$ is the minimum simple service curve for a data flow with an input and output cumulative traffic functions $F(t)$ and $F^*(t)$, respectively, iff:

$$F^*(t) \geq F \otimes \beta(t)$$

These definitions allow us to compute performance characteristics of flows, according to the following theorem.

Theorem 1 (Performance Bounds). *Consider a flow i constrained by an arrival curve α crossing a system \mathcal{S} , offering a service curve β . The performance bounds obtained at any time t are given by:*

$${}^2 f \otimes g(t) = \inf_{0 \leq s \leq t} \{f(t-s) + g(s)\}$$

Output arrival curve: $\alpha^(t) = \alpha \otimes^3 \beta(t)$*

Backlog: $\forall t : q(t) \leq (\alpha \otimes \beta)(0) =: v(\alpha, \beta)$ ⁴

Delay: $\forall t : d(t) \leq \inf\{t \geq 0 : (\alpha \otimes \beta)(-t) \leq 0\}$ ⁵

The computation of these bounds is greatly simplified in the case of a leaky bucket arrival curve $\alpha(t) = \sigma + \rho \cdot t$, with σ the maximal burst and ρ the maximum rate, i.e., the flow is (σ, ρ) -constrained; and the Rate-Latency service curve $\beta_{R,T}(t) = [R \cdot (t - T)]^+$ ⁶ with latency T and rate R . In this case, the delay is upper bounded by $\frac{\sigma}{R} + T$, the backlog bound is $\sigma + \rho \cdot T$, and the output arrival curve is $\sigma + \rho(T + t)$. This service curve is easy to define in the case of one input/output node serving one or many traffic flows coming from the same source and going to the same destination. However, to handle more realistic scenario with a network of nodes, implementing aggregate scheduling to multiplex the crossing flows at the input and demultiplex them at the output, one needs to define the left over service curve offered to each traffic flow within each crossed node, accounting for the impact of interfering flows. The computation of such a left over service curve depends on the implemented scheduling policy within each crossed node, and the most common ones are Arbitrary Multiplexing, FIFO and Fixed Priority (FP). It is worth noting that this derivation needs strict service curve property in the general case, except for FIFO and Constant bit rate nodes. A minimum strict service curve is defined as follows.

Definition 3. (*Strict service curve*) The function β is the strict service curve for a data flow with an input and output cumulative functions F and F^* , if for any backlogged period (s, t) , $F^*(t) - F^*(s) \geq \beta(t - s)$.

The main results concerning the left over service curves computation are in following.

Theorem 2 (Left-over service curve - Arbitrary Multiplex). [7] *let f_1 and f_2 be two flows crossing a server that offers a strict service curve β such that f_1 is α_1 -constrained, then the residual service curve offered to f_2 is:*

$$\beta_2 = (\beta - \alpha_1)_{\uparrow}$$

where $f_{\uparrow}(t) = \max\{0, \sup_{0 \leq s \leq t} f(s)\}$

Corollary 1 (Left-over service curve - FP Multiplex). [7] *Consider a system with the strict service β and m flows crossing it, f_1, f_2, \dots, f_m . The maximum packet length of f_i is $l_{i, \max}$ and is upper-constrained by the arrival curve α_i . The flows are scheduled under a non-preemptive fixed priority (NP-FP) multiplexing, where f_i has higher priority than f_j if $i < j$. For each $i \in \{1, \dots, m\}$, the **strict** service curve of f_i is given by:*

$$\left(\beta - \sum_{j < i} \alpha_j - \max_{k > i} l_{k, \max}\right)_{\uparrow} \quad (1)$$

³ $f \otimes g(t) = \sup_{s \geq 0} \{f(t+s) - g(s)\}$

⁴ $v(f, g)$: the maximum vertical distance between f and g

⁵ $h(f, g)$: the maximum horizontal distance between f and g

⁶ $[x]^+$ is the maximum between x and 0

For $i = 1$, it is $(\beta - \max_{k>1} l_{k,max}) \uparrow$

Afterwards, one of the strongest result in the Network Calculus framework is the computation of an end-to-end service curve for a tandem of nodes crossed by the same flows. This curve is computed as the convolution of left-over service curves in each node, and is used to infer end-to-end performance bounds according to Th. 1. This result is described in the following theorem.

Theorem 3 (Concatenation-Pay Bursts Only Once). *Assume a flow crossing n servers with respective service curves β_1, \dots, β_n . The system composed of the concatenation of the n servers offers a minimal service curve $\otimes_{i \in [1,n]} \beta_i$ to the flow. In the case of rate-latency service curves β_{R^i, T^i} for server i , the end-to-end service curve offered to a flow crossing n servers from 1 to n can be simplified as follow:*

$$\beta_{1,2,\dots,n}(t) = \min_{i \in [1,n]} [R^i] \cdot [t - \sum_{i \in [1,n]} T^i]$$

This result infers an interesting property known as "Pay bursts Only Once" (PBOO) phenomena. The end-to-end delay bound for a data flow, computed using the end-to-end service curve obtained with Th. 3, actually outperforms the sum of delay bound per node, computed iteratively using Th. 1 and denoted as additive delay bound. The computation of these two bounds shows the appearance of the burst term many times in the additive delay bound, and only once for the other. This property has been recently extended in [17] to account the bursts of interfering flows only once within the end-to-end delay bound, which is known as "Pay Multiplex Only Once" (PMOO). The idea of this principle will be detailed in Section V, to prove our proposed closed-form formula of the end-to-end service curve, guaranteed to a flow of interest crossing a ring-based network under FP multiplexing.

III. RELATED WORK: TIMING ANALYSIS OF RING-BASED NETWORKS

A large body of work, based on Network Calculus [14], exists for timing analysis of networks with acyclic network graph, called also feedforward networks, and an interesting overview of the most relevant approaches in this area is detailed in [10]. However, these approaches could not be directly applicable for ring-based networks with cyclic dependencies. The fundamental problem to handle such dependencies consists in defining the input traffic upstream the node of interest, depending on the output of the node downstream, which in turn depends on its input.

To handle such cyclic dependencies, a first class of interesting approaches has been proposed to break the potential cycles through prohibiting the use of some links or sub-paths to ensure the feed-forward property [18] [20]. Although these approaches simplify the timing analysis of such networks, they imply at the same time a reliability level deterioration, since the ring topology is transformed into line.

The second class of approaches introduces computation methods to support cycles using an iterative approach by

successively analyzing the delay bound in each crossed node in the network, resulting in end-to-end delay bounds computation. The most relevant approaches are focusing on, either each crossed node delay bound, e.g., [9] [8] [12], or each crossed node backlog bound, e.g., [21] [14].

For the particular case of ring-based network, Cruz [9] defines an interesting approach, called **Time Stopping Method**, which consists in two steps. First, a finite burstiness bound for the transmitted flows is assumed to compute the delay bounds. Then, the feasibility conditions to solve these equations are defined. This method presents some limitations in terms of resource utilization, since the utilization rate decreases dramatically when the network size increases. Another interesting approach in this area has been proposed in [21] and then generalized in [14] to prove the ring stability through the existence of a backlog bound; thus called **Backlog-based Method**. The maximum bound on the delay within a node is the processing time of the maximum backlogged traffic and the end-to-end delay communication bound is the sum of the crossed nodes delays. The inferred delay bound increases polynomially with the number of nodes, which limits inherently the network scalability.

To overcome these limitations, our main proposal in this paper consists in introducing an enhanced worst-case timing analysis of ring-based networks with cyclic dependencies, based on a global method, when considering the flow serialization phenomena along the flow path. First, a closed-form formula of the guaranteed end-to-end service curve for a flow of interest is defined and proved. Then, a computational resolution method to solve the cycle issue is detailed. Finally, the performance evaluation process of such a proposal shows its outperformance to enhance bound tightness and system performance, in comparison to the conventional analysis methods, i.e., Time Stopping and Backlog-based methods.

IV. SYSTEM MODEL

We consider the following assumptions and notations to compute the worst-case end-to-end delay bounds for a flow of interest f crossing the network:

- We consider a unidirectional ring topology, as shown in Fig. 1, connecting M nodes, labelled from 1 to M , and serving a fixed number of flows I . The notations $l \oplus k$ and $l \ominus k$ designate $(l+k) \bmod M$ and $(l-k) \bmod M$ for the k -*eth* successor and k -*eth* predecessor of node l , respectively;
- Each flow $i \in I$ follows a fixed path from its initial source until the final sink, defined as $path_i = (0, i.first, i.first \oplus 1, \dots, i.first \oplus (h_i - 1))$, where 0 is a virtual node representing the source, $i.first$ the first hop and h_i the number of hops of flow i with $h_i \leq M$. Moreover, we define for each flow i , its subpath with $n \in [1, h_i]$ hops as $subpath_i(n) = (0, i.first, \dots, i.first \oplus (n - 1))$;
- We denote $i \ni k$ the set of flows crossing the node k ;
- For each flow f , consider $\mathbb{K}_f(n)$ the set of interfering flows with flow f along its subpath $subpath_f(n)$; thus

$\mathbb{K}_f(n) = \{i \neq f / \exists k \in \text{subpath}_f(n) / i \ni k\}$. Moreover, for any flow $i \in \mathbb{K}_f(n)$, consider its first (last) multiplexing node label with flow f along the subpath $\text{subpath}_f(n)$ as $M\text{first}(i, f, n)$ ($M\text{last}(i, f, n)$);

- Within the network, flows are treated according to an aggregate scheduling, i.e. flows are classified within aggregates according to a common parameter, such as priority. Within an aggregate, flows are served under arbitrary multiplexing in each crossed node;
- Each node k serves the traffic of an aggregate according to a strict service curve having a rate-latency form, with a rate R^k and a latency T^k , $\beta^k(t) = R^k(t - T^k)^+$;
- Each flow i is constrained by one leaky bucket of rate ρ_i and an initial burst σ_i^0 at its input source 0, thus admits an initial input arrival curve $\alpha_i^0(t) = \sigma_i^0 + \rho_i \cdot t$. Moreover, we define its input arrival curve at each crossed node k along its path path_i as $\alpha_i^{k \ominus 1}(t) = \sigma_i^{k \ominus 1} + \rho_i \cdot t$;
- We consider the case of a stable network, i.e., for any node $k \in [1, M]$, $\frac{\sum_{i \ni k} \rho_i}{R^k} \leq 1$.
- The general assumption for notations consists in considering upper indices to indicate nodes or a set of nodes, and lower indices to indicate flows.

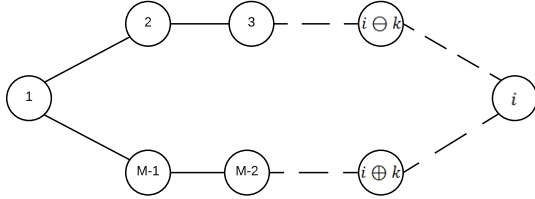


Fig. 1: Ring-based Network Example

V. ENHANCED TIMING ANALYSIS OF RING-BASED NETWORKS

The aim of this section is to conduct the worst-case timing analysis of ring-based networks with cyclic dependencies. First, a closed-form service curve, guaranteed to any flow of interest f along its subpath $\text{subpath}_f(n)$ in such a network under arbitrary multiplexing, is presented and proved. Then, this formula is extended to the particular case of ring-based network under Fixed Priority (FP) multiplexing. It is worth noting that the worst-case behavior under FP multiplexing is covered under Arbitrary multiplexing, but this latter may infer overly-pessimistic bounds since it do not take into account the priority impact, i.e., any flow may be delayed by all the other flows independently from their priorities. Finally, the analysis approach of maximum end-to-end delay bounds, accounting for cyclic dependencies, is detailed.

A. End-to-end Service Curve under Arbitrary Multiplexing

The main interesting work in the literature on the end-to-end performance analysis of feedforward networks under arbitrary multiplexing are dealing with the trade off between bounds accuracy and computational effort complexity. The seminal work in this area is proposed in [17] and consists in

computing a tight closed-form formula of end-to-end service curve, considering the PMOO phenomena. However, the same authors have showed later in [16] that this approach could be outperformed in some particular cases by the classic PBOO approach, and proposed an optimization-based method to handle this issue. Afterwards, this idea has been extended in [6] to general feed-forward networks using Integer-Liner Programming approach to infer exact worst-case end-to-end delay. However, this latter has been proved as NP-hard problem under general assumptions. Since our objective in this paper is enhancing delay bound tightness while keeping a reasonable computation effort, our main idea is based on extending the closed-form formula of service curve proved in [17], to ring-based networks with cyclic dependencies. Hence, for any flow of interest f along its subpath $\text{subpath}_f(n)$ under the assumptions detailed in Section IV, its guaranteed service curve in ring-based network under arbitrary multiplexing is detailed in Theorem 4.

Theorem 4 (End-to-End Service Curve in Ring Network under Arbitrary Multiplexing). *The service curve offered to a flow of interest f along its subpath, $\text{subpath}_f(n)$, in ring-based network under arbitrary multiplexing with strict service curve nodes of the rate-latency type $\beta_{R,T}$ and leaky bucket constrained arrival curves $\alpha_{\sigma,\rho}$, is a rate-latency curve defined as follows:*

$$\beta_f^{\text{subpath}_f(n)}(t) = R^{\text{subpath}_f(n)}(t - T^{\text{subpath}_f(n)})^+ \quad (2)$$

where,

$$\left\{ \begin{array}{l} R^{\text{subpath}_f(n)} = \min_{k \in \text{subpath}_f(n)} [R^k - \sum_{j \ni k, j \neq f} \rho_j] \\ T^{\text{subpath}_f(n)} = \sum_{k \in \text{subpath}_f(n)} T^k \\ + \sum_{i \in \mathbb{K}_f(n)} \frac{\sigma_i^{f, \text{first} \ominus 1} \cdot \mathbf{1}_{\{i, \text{first} \neq f, \text{first}\}}}{R^{\text{subpath}_f(n)}} \\ + \sum_{i \in \mathbb{K}_f(n)} \frac{\sigma_i^0 \cdot \mathbf{1}_{\{f \ni i, \text{first}\}} + \rho_i \cdot \sum_{j \in \text{subpath}_f(n) \cap \text{path}_i} T^j}{R^{\text{subpath}_f(n)}} \end{array} \right.$$

As we can see from Eq. (2), some flow bursts are payed twice. These particular flows have actually two convergence points⁷ with the path of flow of interest: one at their own source and one at the flow of interest source. For instance, in Fig 2(a), there are two convergence points between the interfering flow 3 and the flow of interest 1, which are nodes 1 (the source of flow 1) and 3 (the source of flow 3). This fact is due to the cyclic dependency impact and does not violate the PMOO principle, since the interfering flow can be considered as a new flow at each convergence point with the flow of interest. Hence, as illustrated in Fig. 2(b), for instance, the interfering flows 3 is splitter in two subflows: $3'$ crossing node 3, and $3''$ from node 1 to node 2.

⁷A convergence point between two flows is the first multiplexing point after their paths divergence.

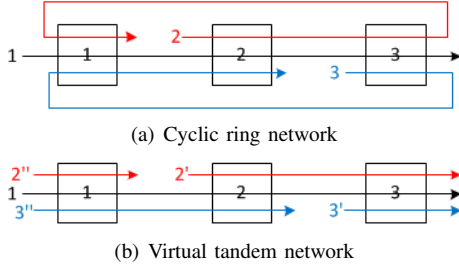


Fig. 2: From a cyclic ring to a virtual tandem network

Proof. To prove this theorem, we need first to prove the following lemma, which extends the closed-form formula of the end-to-end service curve guaranteed to a flow of interest f given in [17], when crossing a tandem of nodes with an arbitrary overlapping interfering flows scenario.

Lemma 5 (End-to-End Service Curve in Tandem Network under Arbitrary Multiplexing). *The service curve offered to a flow of interest f along its subpath, $subpath_f(n)$, in tandem network under arbitrary multiplexing with strict service curve nodes of the rate-latency type $\beta_{R,T}$ and leaky bucket constrained arrival curves $\alpha_{\sigma,\rho}$, is a rate-latency curve defined as follows:*

$$\beta_f^{subpath_f(n)}(t) = R^{subpath_f(n)}(t - T^{subpath_f(n)})^+ \quad (3)$$

where,

$$\left\{ \begin{array}{l} R^{subpath_f(n)} = \min_{k \in subpath_f(n)} [R^k - \sum_{j \ni k, j \neq f} \rho_j] \\ T^{subpath_f(n)} = \sum_{k \in subpath_f(n)} T^k \\ + \sum_{i \in \mathbb{K}_f(n)} \frac{\sigma_i^0 + \rho_i \cdot \sum_{j \in subpath_f(n) \cap path_i} T^j}{R^{subpath_f(n)}} \end{array} \right.$$

Proof. Let's consider a flow of interest f with a subpath of length n , $subpath_f(n)$. Any crossed node $l \in subpath_f(n)$ admits a strict service curve, hence according to Def. 3, for any instant $t_l \geq 0$, there exists $t_{l \ominus 1} \leq t_l$ the start of the backlogged period such that:

$$\begin{aligned} & F_f^{l,*}(t_l) - F_f^{l,*}(t_{l \ominus 1}) \\ & + \sum_{i \ni l, i \neq f} (F_i^{l,*}(t_l) - F_i^{l,*}(t_{l \ominus 1})) \\ & \geq \beta^l(\Delta_l) \end{aligned} \quad (4)$$

where $F_k^{l,*}(t_l)$ and $F_k^l(t_l)$ are the output and input cumulative function of flow k at node l , respectively; and $\Delta_l = t_l - t_{l \ominus 1}$. The time indices are chosen to match the node indices. Then, we sum up the expression in Eq. (4) when

varying $l \in subpath_f(n)$, which infers the following:

$$\begin{aligned} & \sum_{l \in subpath_f(n)} F_f^{l,*}(t_l) - F_f^{l,*}(t_{l \ominus 1}) \\ & \geq \sum_{l \in subpath_f(n)} \beta^l(\Delta_l) \\ & - \sum_{l \in subpath_f(n)} \sum_{i \ni l, i \neq f} (F_i^{l,*}(t_l) - F_i^{l,*}(t_{l \ominus 1})) \end{aligned} \quad (5)$$

Eq. (5) can be simplified due to the following:

$$\begin{aligned} & \sum_{l \in subpath_f(n)} F_f^{l,*}(t_l) - F_f^{l,*}(t_{l \ominus 1}) \\ & = F_f^{f.first,*}(t_{f.first}) - F_f^{f.first,*}(t_{f.first \ominus 1}) \\ & + F_f^{f.first \oplus 1,*}(t_{f.first \oplus 1}) - F_f^{f.first \oplus 1,*}(t_{f.first}) \\ & + F_f^{f.first \oplus 2,*}(t_{f.first \oplus 2}) - F_f^{f.first \oplus 2,*}(t_{f.first \oplus 1}) \\ & \dots \\ & + F_f^{f.first \oplus (n-1),*}(t_{f.first \oplus (n-1)}) - F_f^{f.first \oplus (n-1),*}(t_{f.first \oplus (n-2)}) \\ & = F_f^{f.first \oplus (n-1),*}(t_{f.first \oplus (n-1)}) - F_f^{f.first,*}(t_{f.first \ominus 1}) \end{aligned} \quad (6)$$

Moreover, giving the definition of $\mathbb{K}_f(n) = \{i \neq f / \exists k \in subpath_f(n) / i \ni k\}$, we have an equivalence between $\sum_{l \in subpath_f(n)} \sum_{i \ni l, i \neq f}$ and $\sum_{i \in \mathbb{K}_f(n)} \sum_{l \in subpath_f(n) \cap path_i}$. Therefore, using Eq. (6) and this latter equivalence with the definitions of $Mfirst(i, f, n)$ and $Mlast(i, f, n)$, Eq. (5) can be rewritten as follows:

$$\begin{aligned} & F_f^{f.first \oplus (n-1),*}(t_{f.first \oplus (n-1)}) - F_f^{f.first,*}(t_{f.first \ominus 1}) \\ & \geq \sum_{l \in subpath_f(n)} \beta^l(\Delta_l) \\ & - \sum_{i \in \mathbb{K}_f(n)} \sum_{l \in subpath_f(n) \cap path_i} (F_i^{l,*}(t_l) - F_i^{l,*}(t_{l \ominus 1})) \\ & \geq \sum_{l \in subpath_f(n)} \beta^l(\Delta_l) \\ & - \sum_{i \in \mathbb{K}_f(n)} (F_i^{Mlast(i,f,n),*}(t_{Mlast(i,f,n)}) - F_i^{Mfirst(i,f,n),*}(t_{Mfirst(i,f,n) \ominus 1})) \end{aligned} \quad (7)$$

To substitute the cumulative traffic functions of flows in $\mathbb{K}_f(n)$ in Eq. (7) by their arrival curves, we use the causality constraint of cumulative traffic functions, i.e., $\forall t, F(t) \geq F^*(t)$ and the property of the start of backlogged period at $t_{Mfirst(i,f,n) \ominus 1}$, i.e., $F_i^{Mfirst(i,f,n),*}(t_{Mfirst(i,f,n) \ominus 1}) = F_i^{Mfirst(i,f,n)}(t_{Mfirst(i,f,n) \ominus 1})$. Moreover, since the network topology is tandem, for a fixed instant t and a flow i , we have $F_i^k(t) \leq F_i^{k \ominus 1}(t)$. Knowing that the input arrival curve of a flow i at each crossed node k is $\alpha_i^{k \ominus 1}(t)$ and $\Delta_l = t_l - t_{l \ominus 1}$,

we infer the following:

$$\begin{aligned}
& \sum_{i \in \mathbb{K}_f(n)} (F_i^{Mlast(i,f,n),*}(t_{Mlast(i,f,n)}) - F_i^{Mfirst(i,f,n),*}(t_{Mfirst(i,f,n) \ominus 1})) \geq \\
& \sum_{i \in \mathbb{K}_f(n)} (F_i^{Mlast(i,f,n)}(t_{Mlast(i,f,n)}) - F_i^{Mfirst(i,f,n)}(t_{Mfirst(i,f,n) \ominus 1})) \quad (8) \\
& \leq \sum_{i \in \mathbb{K}_f(n)} (F_i^{Mfirst(i,f,n)}(t_{Mlast(i,f,n)}) - F_i^{Mfirst(i,f,n)}(t_{Mfirst(i,f,n) \ominus 1})) \geq \\
& \leq \sum_{i \in \mathbb{K}_f(n)} \alpha_i^{Mfirst(i,f,n) \ominus 1} (t_{Mlast(i,f,n)} - t_{Mfirst(i,f,n) \ominus 1}) \\
& \leq \sum_{i \in \mathbb{K}_f(n)} \alpha_i^{Mfirst(i,f,n) \ominus 1} \left(\sum_{l=Mfirst(i,f,n)}^{Mlast(i,f,n)} \Delta_l \right)
\end{aligned}$$

Rewrite the input arrival curve of a flow i at node k , $\alpha_i^{k \ominus 1}$, as follows:

$$\begin{aligned}
\alpha_i^{k \ominus 1} \left(\sum_{l=1}^m \Delta_l \right) &= \sigma_i^{k \ominus 1} + \rho_i \cdot \sum_{l=1}^m \Delta_l \\
&= \sigma_i^{k \ominus 1} + \rho_i \cdot \Delta_1 + \rho_i \cdot \sum_{l=2}^m \Delta_l \\
&= \alpha_i^{k \ominus 1}(\Delta_1) + \sum_{l=2}^m \bar{\alpha}_i(\Delta_l) \quad (9)
\end{aligned}$$

where $\bar{\alpha}_i(\Delta_l) = \rho_i \cdot \Delta_l$. Furthermore, giving the tandem topology, the first multiplexing point of an interfering flow i with a flow of interest f is necessarily the first hop of flow i , i.e., $Mfirst(i, f, n) = i.first$. Therefore, using Eq. (8) and Eq. (9), and the definitions of $Mfirst(i, f, n)$ and $\mathbb{K}_f(n)$, Eq. (7) becomes as following:

$$\begin{aligned}
& F_f^{f.first \oplus (n-1),*}(t_{f.first \oplus (n-1)}) - F_f^{f.first,*}(t_{f.first \ominus 1}) \\
& \geq \sum_{l \in subpath_f(n)} \beta^l(\Delta_l) \\
& - \sum_{i \in \mathbb{K}_f(n)} \alpha_i^{Mfirst(i,f,n) \ominus 1} \left(\sum_{l=Mfirst(i,f,n)}^{Mlast(i,f,n)} \Delta_l \right) \\
& \geq \sum_{l \in subpath_f(n)} \beta^l(\Delta_l) \\
& - \sum_{i \in \mathbb{K}_f(n)} (\alpha_i^{i.first \ominus 1}(\Delta_{i.first}) + \sum_{l=i.first \oplus 1}^{Mlast(i,f,n)} \bar{\alpha}_i(\Delta_l)) \\
& \geq \sum_{l \in subpath_f(n)} \beta^l(\Delta_l) \\
& - \sum_{l \in subpath_f(n)} \sum_{i \ni l, i \neq f} (\alpha_i^{i \ominus 1}(\Delta_l) \cdot 1_{\{l=i.first\}} + \bar{\alpha}_i(\Delta_l) \cdot 1_{\{l \neq i.first\}}) \\
& \geq \sum_{l \in subpath_f(n)} [\beta^l(\Delta_l) \\
& - \sum_{i \ni l, i \neq f} \alpha_i^{i \ominus 1}(\Delta_l) \cdot 1_{\{l=i.first\}} + \bar{\alpha}_i(\Delta_l) \cdot 1_{\{l \neq i.first\}}]
\end{aligned}$$

Considering the expressions of $\beta^l(\Delta_l) = R^l(\Delta_l - T^l)^+$, $\alpha_i^{l \ominus 1}(\Delta_l) \cdot 1_{\{l=i.first\}} = \sigma_i^0 + \rho_i \cdot \Delta_l$ and $\bar{\alpha}_i(\Delta_l) = \rho_i \cdot \Delta_l$, we

obtain the following:

$$\begin{aligned}
& F_f^{f.first \oplus (n-1),*}(t_{f.first \oplus (n-1)}) - F_f^{f.first,*}(t_{f.first \ominus 1}) \\
& \geq \sum_{l \in subpath_f(n)} [(R^l - \sum_{i \ni l, i \neq f} \rho_i) \cdot \\
& (\Delta_l - T^l - \frac{\sum_{i \ni l, i \neq f} \sigma_i^0 + T^l \cdot \sum_{i \ni l, i \neq f} \rho_i}{R^l - \sum_{i \ni l, i \neq f} \rho_i})] \\
& \geq \min_{l \in subpath_f(n)} (R^l - \sum_{i \ni l, i \neq f} \rho_i) \cdot \\
& [\sum_{l \in subpath_f(n)} \Delta_l - \sum_{l \in subpath_f(n)} T^l \\
& - \sum_{l \in subpath_f(n)} \frac{\sum_{i \ni l, i \neq f} \sigma_i^0 + T^l \cdot \sum_{i \ni l, i \neq f} \rho_i}{R^l - \sum_{i \ni l, i \neq f} \rho_i}] \quad (11)
\end{aligned}$$

Giving the definition of $\mathbb{K}_f(n)$, we can easily verify the following equivalence :

$$\begin{aligned}
& \sum_{l \in subpath_f(n)} T^l \cdot \sum_{i \ni l, i \neq f} \rho_i \quad \Leftrightarrow \\
& \sum_{i \in \mathbb{K}_f(n)} \rho_i \cdot \sum_{j \in subpath_f(n) \cap path_i} T^j \\
& \text{Hence, Eq. (11) becomes:}
\end{aligned}$$

$$\begin{aligned}
& F_f^{f.first \oplus (n-1),*}(t_{f.first \oplus (n-1)}) - F_f^{f.first,*}(t_{f.first \ominus 1}) \\
& \geq \min_{l \in subpath_f(n)} (R^l - \sum_{i \ni l, i \neq f} \rho_i) \cdot \\
& [t_{f.first \oplus (n-1)} - t_{f.first \ominus 1} - \sum_{l \in subpath_f(n)} T^l \quad (12) \\
& - \sum_{i \in \mathbb{K}_f(n)} \frac{\sigma_i^0 + \rho_i \cdot \sum_{j \in subpath_f(n) \cap path_i} T^j}{\min_{l \in subpath_f(n)} (R^l - \sum_{i \ni l, i \neq f} \rho_i)}]
\end{aligned}$$

This latter represents the definition of the end-to-end service curve of the flow of interest f along its subpath $subpath_f(n)$, which finishes the proof of Lemma 5. \square

The ring-based network can be seen as a particular case of a tandem network with cyclic dependencies, where we can identify three categories of interfering flows with the flow of interest f : (i) the first one includes the flows having only one convergence point with the flow of interest f , which is their first hop, i.e., for a flow i in this category, it is $i.first$; (ii) the second one includes the flows having only one convergence point with the flow of interest f , which is the first hop of flow f , i.e., $f.first$; (iii) the third one includes the flows having two distinct convergence points with the flow of interest f , i.e., for a flow i in this category, there are two convergence points $i.first$ and $f.first$, if $i.first \neq f.first$. For both first categories, we consider the arrival curve of each interfering flow i at the point of convergence, i.e., (σ_i^0, ρ_i) -constrained or $(\sigma_i^{f.first \ominus 1}, \rho_i)$ -constrained. For the third category, to model these particular interfering flows, we split each one of them in two subflows to cut virtually the cyclic dependency, as illustrated in Fig. 2: (i) $i1$: the subflow of i along its subpath $path_{i1} = (0, i.first, i.first \oplus 1, \dots, f.first \ominus 1)$ and it is (σ_i^0, ρ_i) -constrained; (ii) $i2$: the subflow of i along its subpath $path_{i2} = (f.first \ominus 1, f.first, \dots, i.first \oplus (h_i - 1))$, and it is $(\sigma_i^{f.first \ominus 1}, \rho_i)$ -constrained. Therefore, each interfering flow i in the third category is split in two subflows ($i1, i2$), where

$i1$ fulfills the conditions of the first category of interfering flows, whereas $i2$ fulfills the ones of the second category. Hence, when splitting the flows in the third category within $\mathbb{K}_f(n)$ in two subflows, we obtain a transformed set $\overline{\mathbb{K}_f(n)}$. This latter can be rewritten considering the conditions of both first categories as: $\overline{\mathbb{K}_f(n)} = \{\mathbb{K}_f(n), f \ni i.first\} \cup \{\mathbb{K}_f(n), i \ni f.first, i.first \neq f.first\}$. For instance, for the example in Fig. 2, $\overline{\mathbb{K}_f(n)} = \{2', 3'\} \cup \{2'', 3''\}$. Moreover, for each interfering flow i in the third category, we have the following:

$$\begin{aligned} & \sigma_{i1}^0 + \rho_i \cdot \sum_{j \in \text{subpath}_f(n) \cap \text{path}_{i1}} T^j \\ + & \sigma_{i2}^0 + \rho_i \cdot \sum_{j \in \text{subpath}_f(n) \cap \text{path}_{i2}} T^j \\ = & \sigma_i^0 + \sigma_i^{f.first \ominus 1} + \rho_i \cdot \sum_{j \in \text{subpath}_f(n) \cap (\text{path}_{i1} \cup \text{path}_{i2})} T^j \\ = & \sigma_i^0 + \sigma_i^{f.first \ominus 1} + \rho_i \cdot \sum_{j \in \text{subpath}_f(n) \cap \text{path}_i} T^j \end{aligned} \quad (13)$$

Therefore, when applying the Lemma 5 to this particular case of tandem network, i.e., ring-based network, and using Eq. (13), we deduce the following latency for the end-to-end service curve:

$$\begin{aligned} T^{\text{subpath}_f(n)} &= \sum_{k \in \text{subpath}_f(n)} T^k \\ + & \sum_{i \in \overline{\mathbb{K}_f(n)}, f \ni i.first} \frac{\sigma_i^0 + \rho_i \cdot \sum_{j \in \text{subpath}_f(n) \cap \text{path}_i} T^j}{R^{\text{subpath}_f(n)}} \\ + & \sum_{\substack{i \in \mathbb{K}_f(n) \\ i \ni f.first \\ i.first \neq f.first}} \frac{\sigma_i^{f.first \ominus 1} + \rho_i \cdot \sum_{j \in \text{subpath}_f(n) \cap \text{path}_i} T^j}{R^{\text{subpath}_f(n)}} \\ \Leftrightarrow T^{\text{subpath}_f(n)} &= \sum_{k \in \text{subpath}_f(n)} T^k \\ + & \sum_{i \in \mathbb{K}_f(n)} \frac{\sigma_i^{f.first \ominus 1} \cdot \mathbb{1}_{\{i \ni f.first, i.first \neq f.first\}} + \rho_i \cdot \sum_{j \in \text{subpath}_f(n) \cap \text{path}_i} T^j}{R^{\text{subpath}_f(n)}} \\ + & \sum_{i \in \mathbb{K}_f(n)} \frac{\sigma_i^0 \cdot \mathbb{1}_{\{f \ni i.first\}} + \rho_i \cdot \sum_{j \in \text{subpath}_f(n) \cap \text{path}_i} T^j}{R^{\text{subpath}_f(n)}} \end{aligned} \quad (14)$$

This latency corresponds to the latency of Th. 4, which finishes the proof of the theorem. \square

B. End-to-end Service Curve under FP Multiplexing

The closed-form formula expressed in Th. 4 under Arbitrary multiplexing is extended herein to FP multiplexing. To achieve this aim, we need to define the following terms:

- $P(i)$ for the priority level of flow i , where each crossed node has at maximum NP priority levels and 0 denotes the highest one;

- $L_{max}(i)$ for the maximum packet length of flow i , accounting for the communication protocol overhead;
- $hp_f^k = \{i \neq f / i \ni k, P(i) \leq P(f)\}$ for the set of flows crossing the node k excluding the flow f , with priority higher or equal to the f one;
- $lp_f^k = \{i \ni k, P(i) > P(f)\}$ for the set of flows crossing the node k with priority lower than the f one;
- $\mathbb{K}_{\leq f}(n) = \{i \neq f / \exists k \in \text{subpath}_f(n) / i \ni k, P(i) \leq P(f)\}$ for the set of flows interfering with the flow f along its subpath, $\text{subpath}_f(n)$, with a priority higher or equal to f one.

For any flow of interest f along its subpath $\text{subpath}_f(n)$, its guaranteed service curve in ring-based network under FP multiplexing is detailed in Corollary 2.

Corollary 2 (End-to-End Service Curve in Ring Network under FP Multiplexing). *The service curve offered to a flow of interest f along its subpath, $\text{subpath}_f(n)$, in ring-based network under FP multiplexing with strict service curve nodes of the rate-latency type $\beta_{R,T}$ and leaky bucket constrained arrival curves $\alpha_{\sigma,\rho}$, is a rate-latency curve defined as follows:*

$$\beta_f^{\text{subpath}_f(n)}(t) = R^{\text{subpath}_f(n)}(t - T^{\text{subpath}_f(n)})^+ \quad (15)$$

where,

$$\left\{ \begin{aligned} R^{\text{subpath}_f(n)} &= \min_{k \in \text{subpath}_f(n)} \left[R^k - \sum_{j \ni hp_f^k} \rho_j \right] \\ T^{\text{subpath}_f(n)} &= \sum_{k \in \text{subpath}_f(n)} T^k + \frac{\max_{j \in lp_f^k} L_{max}(j)}{R^k} \\ + & \sum_{i \in \mathbb{K}_{\leq f}(n)} \frac{\sigma_i^{f.first \ominus 1} \cdot \mathbb{1}_{\{i \ni f.first, i.first \neq f.first\}}}{R^{\text{subpath}_f(n)}} \\ + & \sum_{i \in \mathbb{K}_{\leq f}(n)} \frac{\sigma_i^0 \cdot \mathbb{1}_{\{f \ni i.first\}} + \rho_i \cdot \sum_{j \in \text{subpath}_f(n) \cap \text{path}_i} T^j + \frac{\max_{p \in lp_f^j} L_{max}(p)}{R^j}}{R^{\text{subpath}_f(n)}} \end{aligned} \right.$$

Proof. The proof is straightforward following the Theorem 4 one. First, the left-over service curve of each crossed node, considering the impact of lower priority flows due to the non-preemptive transmission, is computed through the application of Cor. 1. The obtained service curve is a strict service curve and still is a rate-latency service curve for each crossed node k , with a rate R^k and a latency $\frac{\max_{j \in lp_f^k} L_{max}(j)}{R^k} + T^k$. Afterwards, we need to apply Th. 4 to infer the end-to-end service curve of a flow of interest f , when considering the flow set $\mathbb{K}_{\leq f}(n)$ instead of $\mathbb{K}_f(n)$, to account only the impact of flows with higher or equal priority. \square

C. Computation of End-to-end Delay Bounds

To compute the end-to-end service curve offered to a flow of interest f , we need to compute the bursts of flows arriving upstream its source node. When considering the latency $T^{\text{subpath}_f(n)}$ expressed in Th. 4, the following relationship

between these bursts and the latency is obtained as follows:

$$\begin{aligned}
T^{\text{subpath}_f(n)} &= \overbrace{\sum_{k \in \text{subpath}_f(n)} T^k}^{\text{Constant}} \\
&+ \sum_{i \in \mathbb{K}_f(n)} \frac{\overbrace{\sigma_i^0 \cdot 1_{\{f \ni i, \text{first}\}} + \rho_i \cdot \sum_{j \in \text{subpath}_f(n) \cap \text{path}_i} T^j}^{\text{Constant}}}{R^{\text{subpath}_f(n)}} \\
&+ \underbrace{\sum_{i \in \mathbb{K}_f(n)} \frac{\sigma_i^{f, \text{first} \oplus 1} \cdot 1_{\{i \ni f, \text{first}\}}}{R^{\text{subpath}_f(n)}}}_{\text{Variable}}
\end{aligned} \tag{16}$$

On the other hand, the arrival curve of the flow of interest f at the output of the last node of its subpath, $\text{subpath}_f(n)$, is obtained throughout the application of Theorem 1, as follows:

$$\begin{aligned}
\alpha_f^{f, \text{first} \oplus (n-1)}(t) &= \alpha_f^0 \otimes \beta_f^{\text{subpath}_f(n)}(t) \\
\Rightarrow \sigma_f^{f, \text{first} \oplus (n-1)} &= \sigma_f^0 + \rho_f \times T^{\text{subpath}_f(n)} \tag{17}
\end{aligned}$$

Hence, we can see the cyclic dependency between the latency (Eq. (16)), which depends on the propagated bursts, and the propagated bursts (Eq. 17), which depends in its turn on the latency.

The main issue is to find the different latencies and bursts of any flow $f \in I$ along any of its subpaths with a length $n \in [1, M]$. To cope with this issue, we construct the following matrix system, from formula (16) and (17):

$$\begin{cases} T = C_1 + A_1 \times \sigma \\ \sigma = C_2 + A_2 \times T \end{cases} \tag{18}$$

where,

- T a vector that holds all the $T^{\text{subpath}_f(n)}$ variables, for $f \in I$ and $n \in [1, M]$;
- σ a vector that holds all the $\sigma_f^{f, \text{first} \oplus (n-1)}$ variables, for $f \in I$ and $n \in [1, M]$;
- A_1 a matrix that holds all the coefficients of the unknown bursts and C_1 the constants of formula (16);
- A_2 a matrix that holds all the coefficients of the unknown latencies and C_2 the constants of formula (17).

Afterwards, we obtain the following relation through the constraints propagation:

$$T = (Id - A_1 \times A_2)^{-1} \times C_3 \tag{19}$$

where $C_3 = C_1 + A_1 \times C_2$

The system admits a solution if the matrix $(Id - A_1 \times A_2)$ is invertible, i.e., its determinant is not null. If this condition is verified, then we can compute the vectors T and σ . Finally, the maximum delay bound of any flow i after crossing $n \in [1, M]$ nodes, $D_i^{\text{subpath}_i(n)}$, can be computed as following after applying Theorem 1:

$$D_i^{\text{subpath}_i(n)} = \frac{\sigma_f^0}{R^{\text{subpath}_i(n)}} + T^{\text{subpath}_f(n)}$$

Consider the simple case of broadcast communication with one transmitted flow per node, (σ, ρ) -constrained, the determinant of the matrix $(Id - A_1 \times A_2)$ is a polynomial function of the variable $x = \frac{\rho}{R - (M-1)\rho}$ with a degree M :

$$(1 - M) \times (x + 1)^{(M-1)} \times \left(x - \frac{1}{M-1}\right)$$

This matrix system resolution is feasible for $x \leq \frac{1}{M-1}$, which induces the following network stability condition, i.e., bounded delays: $\rho \leq \frac{R}{2(M-1)}$. The consequences of this condition will be deeply discussed in Section VI.

VI. PERFORMANCE EVALUATION

In this section, we conduct performance analysis of the proposed approach to measure the obtained delay bound tightness and its impact on the system performance, in reference with conventional approaches based on Network Calculus, e.g. Backlog-based and Time stopping methods. Moreover, we consider an achievable worst-case delay bound, to have a more precise idea on the pessimism ratio of the computed upper bound delay, in comparison with the exact worst-case delay, i.e., if the gap between the upper and lower bounds is small, then the exact worst-case delay is not far away from the computed maximum delay bound. First, we describe the case of study and the considered scenarios. Then, we detail the tightness of bounds and its impact on the system performance, under the different analysis approaches.

A. Case of study

We consider the case study with the following assumptions:

- The topology is a unidirectional ring topology, connecting M nodes;
- The links speed is 1Gbit/s ;
- All equipments are similar, and send the same traffic in broadcast mode;
- Technological latency within each node is 600ns ;
- Each equipment can generate 3 types of traffic classes (TC), as shown in the table I, where HRT (for Hard Real Time), SRT (for Soft Real Time) and NRT (for Non Real Time).

	TC	Payload (byte)	rate (Kbps)
I/O data	HRT	64	80
Audio streaming	SRT	128	128
File transfer	NRT	1024	1000

TABLE I: Traffic Characteristics

To conduct the performance analysis of our proposed timing analysis approach. We first compare our approach with the conventional ones under the following three scenarios, where each node transmits only SRT traffic:

- Scenario 1: to analyse the impact of the traffic bursts, we increase the burst size from 166 bytes until 1500 bytes for a network of 35 nodes;
- Scenario 2: to analyse the impact of increasing the network congestion, the upper bounds on end-to-end

delays are computed when the number of nodes is fixed, $M = 10$, and the network load is increasing by a step of 10%;

- Scenario 3: to analyse the impact of enlarging the network scalability, i.e. network size, the upper bounds on end-to-end delays are computed under the variation of the node number, from 10 to 100 nodes by a step of 10 nodes.

Finally, to analyse the delay bound of each traffic class, we consider the scenario 4 where each node sends the 3 types of traffic classes, when varying the node number from 10 to 100 nodes, by steps of 10 nodes.

B. Tightness of Bounds

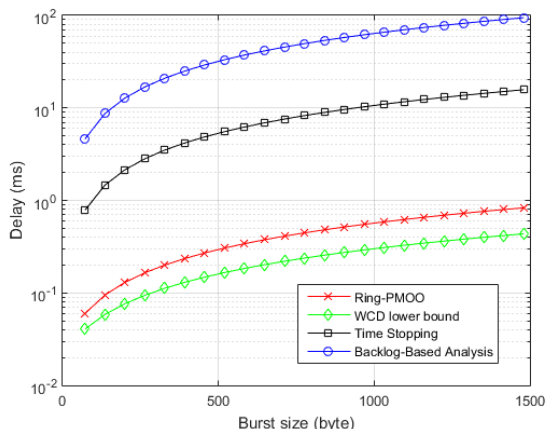


Fig. 3: Upper bounds on end-to-end delays vs size of burst

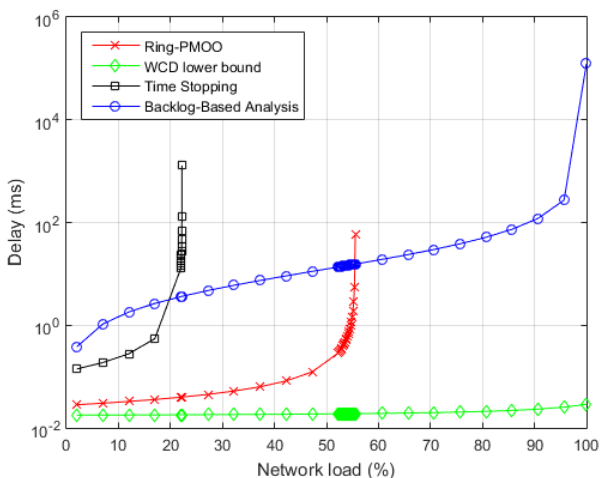


Fig. 4: Upper bounds on end-to-end delay bounds vs network load

To investigate the delay bound tightness computed with the different approaches, we benchmark the delay bounds obtained with our proposed method, denoted as Ring-PMOO in the figures, against the two conventional analyses, i.e., Time Stopping and the Backlog-based methods, and in reference

with the achievable worst-case delay, denoted as WCD lower bound. This latter is computed when considering an intuitive worst-case scenario, which consists in considering for each flow of interest only the impact of direct interferences within each crossed node, and ignoring the impact of the upstream flows at its source node, i.e. it is considered as null. The gap between the computed upper and WCD lower bounds will give us an idea about the delay bound tightness. In fact, this interval includes necessarily the exact worst-case delay; thus if this interval duration is small, then the upper bound delay is tight.

First, we consider Scenario 1 to analyse the impact of the traffic bursts on the tightness of bounds. As illustrated in Fig 3, the delay bound increases when increasing the burst size, since the multiplexing delay within each crossed node increases. As we can notice, the conventional approaches lead to overly pessimistic bounds, in comparison with the proposed one. For example, for a burst of 1500 bytes, the upper bound on the end-to-end delay is almost equal to 1ms, 16ms and 95ms with Ring-PMOO, Time stopping and the Backlog-based approaches, respectively. Moreover, the WCD lower bound is about 0.5ms, which yields to a low pessimism ratio of the computed upper bound with Ring-PMOO approach, i.e., ≤ 0.5 ms. This fact proves the delay bound tightness obtained with the proposed approach under high bursty traffic, in comparison with the conventional methods.

Then, we consider Scenario 2 to analyse the impact of increasing congestion on the tightness of bounds. As shown in Fig. 4, the proposed approach outperforms the conventional methods, when the network load is less than 55.55%, i.e. this network load corresponds to the stability condition explained in Section V-C. However, the Time Stopping method leads to infinite upper bounds when the utilization rate is higher than 22.22%, whereas the Backlog-based method still is stable under full utilization rate. Nevertheless, this latter yields loose upper bounds (100ms), in comparison with the WCD lower bound (0.1ms).

In Scenario 3, we analyse the impact of enlarging the network scalability on the tightness of bounds. As shown in Fig. 5, the proposed approach still outperforms the conventional approaches in terms of delay tightness, when the network scale increases. For instance, for a network size of 100 nodes, the upper bounds are more than 1s and 30ms with the Backlog-based and Time Stopping methods, respectively; whereas it is less than 0.5ms with the proposed approach. Moreover, the gap between the WCD lower bound and the computed upper bound is less than 0.2ms, which proves the bound tightness for large-scale networks.

Discussion: *These results under various scenarios show the tightness of the end-to-end delay upper bounds computed with the proposed approach (Ring-PMOO), in comparison with the conventional ones and in reference with the WCD lower bound. It is worth noting that the network condition stability is required to infer bounded delays.*

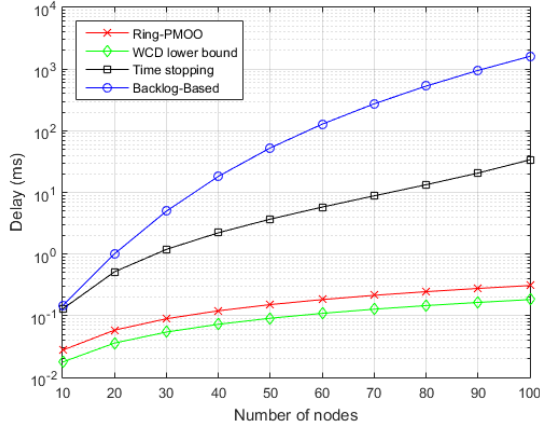


Fig. 5: Upper bounds on end-to-end delays vs number of nodes

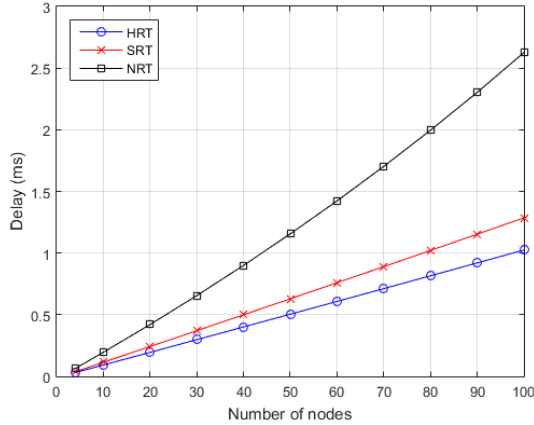


Fig. 6: Upper bounds on end-to-end delays of three traffic classes vs number of nodes

C. Sensitivity Analysis

We discuss herein the impact of the timing analysis method on the system performance, in terms of resource efficiency, i.e., the maximum achievable network load guaranteeing the network stability condition, and system scalability, i.e. the network size guaranteeing the system schedulability. Hence, we reconsider the different scenarios to show their impact on both metrics.

Impact of increasing congestion: as illustrated in Figure 4, when considering Scenario 2, the time stopping method diverges for a global utilization rate around 22.22%, which corresponds to a feasibility condition of $\frac{2}{M-1}$; whereas it achieves 55.55% with our proposed approach, which corresponds to the feasibility condition of $\frac{M}{2 \times (M-1)} = \frac{M \rho_{max}}{R}$ where $\rho_{max} = \frac{R}{2(M-1)}$. It is worth noting that this feasibility condition is computed for broadcast communication, which is the worst-case of contentions since all the flows cross all the nodes. However, a full utilization rate still is achievable under the Backlog-based even if the delay bounds are overly pessimistic.

Impact of enlarging network scalability: Figure 5 illustrates the impact of the network size on the end-to-end delay bounds using the different conventional methods and the proposed one, when considering Scenario 3. Obviously, the delay bounds increase with the network size, since the number of generated messages and crossed nodes increase. As we can notice, the proposed approach leads to tighter delay bounds for large-scale networks, in comparison with the conventional methods. This fact enhances definitely the system scalability.

Impact of QoS Mangemet: Fig 6 illustrates the upper bounds on end-to-end delays of the different traffic classes, described in Table I, when increasing the number of nodes. Obviously, the delay bounds increase with the network size, but the large-scale network of 100 nodes still is stable, i.e., bounded delays. For example, the delay bound of HRT traffic for a network of 100 nodes is 1ms.

Discussion: this detailed sensitivity analysis shows that using the proposed timing analysis approach yields to enhance the guaranteed system schedulability for large-scale network, when the stability condition is verified. The Time Stopping method actually limits the network performance in terms of resource efficiency, i.e. the utilization rate decreases dramatically when the network size increases; whereas the Backlog-based method limits the system scalability, i.e. the nodes number is hardly constrained to guarantee tight temporal deadlines. Hence, the proposed approach allows to bridge the gap between these two conventional methods, by enhancing the resource efficiency and system scalability.

VII. CONCLUSIONS

In this paper, an enhanced worst-case timing analysis of ring-based networks with cyclic dependencies, based on Network Calculus framework, has been proposed. Unlike conventional approaches based on local analysis of the delay bound in each crossed node in the network, our proposed approach is based on a global analysis method, considering the flow serialization phenomena along the flow path, to allow the computation of tighter end-to-end delay bounds. Hence, we have defined and proved the closed-form formula of the end-to-end service curve of any flow of interest crossing such a network, under arbitrary and FP multiplexings. Afterwards, the performance evaluation of such a proposal under various scenarios has been conducted, and results highlight the tightness of computed delay bounds, in contrast to conventional methods, i.e. Time Stopping and Backlog-based methods, and in reference with a lower bound of the exact worst case delay. Furthermore, the proposed method yields to guarantee enhanced system performance, in terms of resource efficiency and network scalability.

As a next step, we envision to extend this result to FIFO service policy and any network topology with cyclic dependencies.

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