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Preliminary Study on the Feasibility of a Plasma-Based Electrically Small ENG Antenna

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***Abstract*—The use of low pressure non-equilibrium plasma as epsilon negative medium for a zero-order spherical antenna resonator is considered. Static behaviour of the resonance for a non-homogeneous plasma is investigated under quasi-static assumption and discrete radial description of the plasma. Analytical and numerical results are consistent and show that the implementation of such a plasma-based antenna is possible.**

***Index Terms*—miniaturization, low pressure non-equilibrium plasma, ENG resonator.**

I. Introduction

For many applications, antenna miniaturization is of particular interest, especially in the VHF-band where the wavelength is about one meter. Many different miniaturization techniques have been investigated [1]. Amongst them, the use of a negative permittivity medium has been proposed in order to create a so-called Epsilon NeGative (ENG) spherical resonator [2], [3]. Small ENG spheres can indeed reveal a resonant behaviour when disturbed by incident field.

This scattering behaviour can be found using Mie theory and can be reduced in our case to a Laplace problem by assuming a quasi-static response of the sphere. Let us consider the general scattering problem shown in Figure 1. It consists of a homogeneous sphere of radius a, filled with an isotropic dielectric with relative permittivity ε_p and surrounded by vacuum. If we assume that this sphere is small compared with the wavelength ($ka \ll 1$), one can determine under quasi-static assumption the normalized polarizability α of such a sphere immersed in a constant vertically polarized E-field by solving Laplace equation for the scalar potential [2]:

$$\alpha = 3\frac{\varepsilon_p - 1}{\varepsilon_p + 2} \tag{1}$$

This expression exhibits the well-known static resonance of a negative permittivity sphere when $\varepsilon_p = -2$. Note that although (1) is valid for $ka = 0$ only, this approach still provide relevant information on the resonance for $ka \ll 1$ [4].

Practical implementations of this static resonance based on metamaterials confirmed the previous analytical result [5], [6]. In this work, we suggest to realize this ENG resonator using a plasma discharge. The plasma indeed exhibits this property and its parameter settings let us foresee a possible tunability.

Fig. 1. Schema of the scattering problem of a homogeneous sphere in a constant vertically E-field.

However, the use of a low pressure non-equilibrium plasma involves many practical downsides that need to be considered. This paper will address the following problems: plasma confinement, non homogeneous permittivity, dispersion, and losses.

II. Low pressure non-equilibrium plasma properties

A plasma is an ionized gas whose global charge remains macroscopically neutral. In a low pressure non-equilibrium plasma, only electrons have enough energy to ionize the gas and the degree of ionization δ (ratio of free-moving electrons to the total amount of particles) is usually low: $\delta < 10^{-3}$.

Non-magnetized low pressure non-equilibrium plasma medium can be represented by its complex relative permittivity ε_p which obeys the well-known Drude model [7]:

$$\varepsilon_p(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + i\nu)} \tag{2}$$

where ν is the electron collision frequency which represents the losses (hereafter ν is set to 40 MHz) and ω_p is the plasma angular frequency defined by:

$$\omega_p = \sqrt{\frac{e^2 n_e}{m_e \epsilon_0}} \tag{3}$$

with e the Coulomb charge, n_e the electron density, m_e the mass of electron and ϵ_0 the vacuum permittivity.

Considering the problem of a plasma sphere, one must note that the electron density n_e will not be uniform in the sphere and neither will the relative permittivity ε_p. To determine the

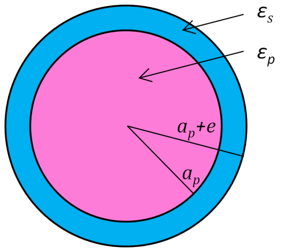

Fig. 2. Schema of the scattering problem of a homogeneous sphere in a constant vertically E-field surrounded by a glass shell ($\varepsilon_s = 5.5$)

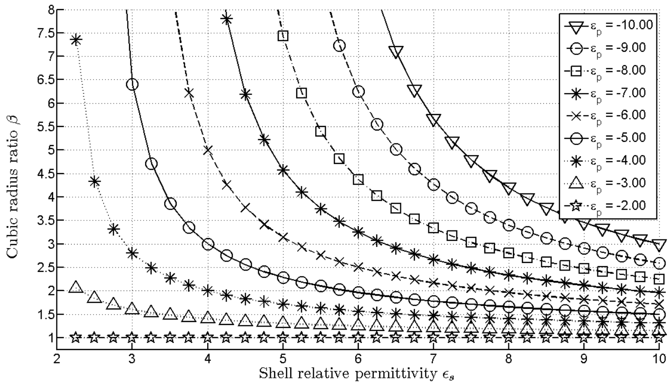

Fig. 3. Required plasma relative permittivity to enable the resonance of a plasma-shell system regarding to both shell relative permittivity ε_s and cubic radius ratio β

permittivity profile inside a sphere of radius a_p, one must find the electron density profile. Considering a sphere of plasma ignited by a radial source term $Q(r)$ proportional to the electron density n_e, as it is usually the case for weakly ionized plasma, the equation of continuity is [8]:

$$\frac{\partial n_e}{\partial t} - D\Delta n_e = Q(r) \tag{4}$$

where D is the diffusion coefficient.

In steady state, (4) becomes a Poisson-type differential equation. As a consequence of the spherical symmetry, n_e is only a function of r. Solving (4), n_e can be expressed with a Bessel function of the first kind of order $n = 0$ [8]. For small r, we approximate the Bessel function of the first kind by a polynomial function of degree 2. Assuming the density fall is about 70 % at the outer limit of the plasma, we can define a density profile for $r < a_p$ as follow:

$$n_e(r) = n_e^0 \left[1 - 0.7\left(\frac{r}{a_p}\right)^2\right] \tag{5}$$

The influence of such a density profile is studied later.

III. Influence of a Containing Layer

As shown in Figure 2, the homogeneous plasma ε_p is now surrounded by a shell made of glass (permittivity $\varepsilon_s = 5.5$) and its static behaviour is studied. One can solve the Laplace equation by considering the following boundary conditions at each interface: continuity of both scalar potential and radial component of the electric displacement field. Let a_p be the radius of the plasma sphere and e the thickness of the glass shell surrounding the plasma. The polarizability of the system is [9], [10]:

$$\alpha = 3\frac{\beta(\varepsilon_s - 1)(\varepsilon_p + 2\varepsilon_s) + (2\varepsilon_s + 1)(\varepsilon_p - \varepsilon_s)}{\beta(\varepsilon_s + 2)(\varepsilon_p + 2\varepsilon_s) + 2(\varepsilon_s - 1)(\varepsilon_p - \varepsilon_s)} \tag{6}$$

where $\beta = (\frac{a_p+e}{a_p})^3$ stands for the cubic radius ratio.

Figure 3 shows how the required plasma relative permittivity changes relatively to β and ε_s to guarantee the static resonance. For a given cubic radius ratio β, if we increase the permittivity of the shell, the system will require a higher $|\varepsilon_p|$ to resonate, which means a raise in electron density n_e. On the other hand, for a given ε_s, decreasing β will asymptotically lead to a single-layered system whose plasmonic resonance is given by $\varepsilon_p = -2$, which is consistent with (1).

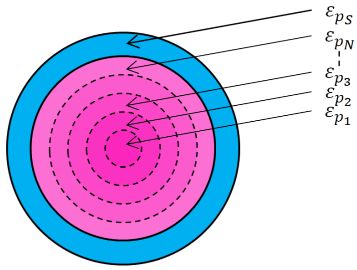

Fig. 4. Schema of the scattering problem of a N-layers plasma sphere of constant permittivity ε_{p_i}, for $i = 1$ to N surrounded by a glass shell ε_s.

Finally, the shell acts as an additional dielectric layer that offsets the resonance (regarding to the geometric and electric parameters) without cancelling it.

IV. Influence of the Radial Graded Plasma

We now consider the geometry presented in Figure 4 to analyse the influence of a non homogeneous electron density. we assume n_e to be r-dependent only, according to a profile given by (5). The plasma can then be divided into N layers of constant thickness (shell number i with radius $i\frac{a_p}{N}$). For each layer, considering n_e as constant and equal to the electron density at the median radius i, we can combine (2), (3) and (5) to derive the permittivity ε_{p_i}. Therefore, we can again solve the Laplace equation to find the overall polarizability and highlight potential resonances.

The design parameters are: an outer radius a defined by $a = a_p + e = 45$ mm, a shell thickness $e = 2.14$ mm and a peak value of electron density $n_e^0 = 4.5 \times 10^9$ cm^{-3}. Figure 5 represents the resonant frequencies found for different values of N. When N increases, the observed resonant frequency approaches the converged value of 300 MHz in that case.

It is clear from Figure 5 that one might consider enough layers to get accurate results. Thereafter, we will use $N = 11$ layers to approximate the spherical resonator. It corresponds to

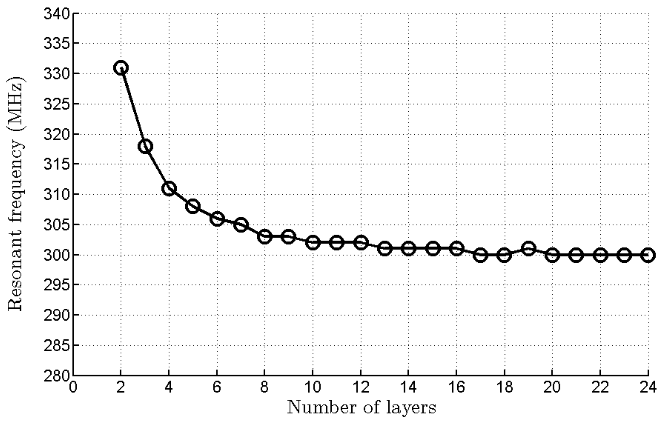

Fig. 5. Analytical resonant frequencies of a N-layered plasma-shell system for different values of N. The density profile is given by 5 with $n_e^0 = 4.5 \times 10^9$ cm^{-3} and converged value is $f_{res} = 300$ MHz.

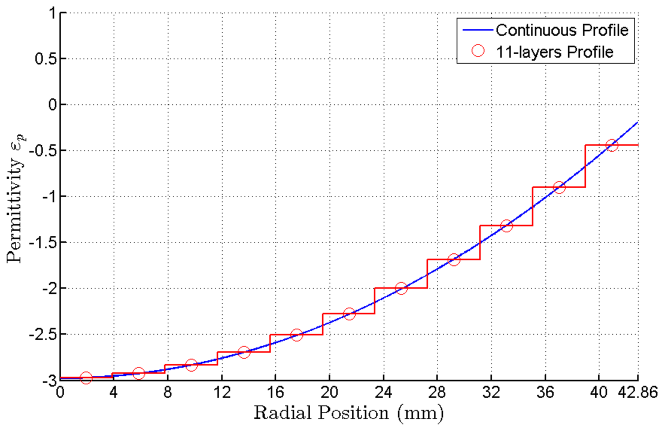

Fig. 6. Radial profile of permittivity obtained for $n_e^0 = 4.5 \times 10^9$ cm^{-3}. The blue curve represents the continuous profile and the red curve (with markers) represents the 11-layers approximated profile.

a resonant frequency $f_0 = 302$ MHz (i.e. an error of 0.6 %). Figure 6 shows the resulting permittivity profile at 302 MHz. Note that n_e^0 was obtained by parametric analysis so that the converged value is 300 MHz.

V. Impedance Matching

Now that we have settled all the geometric (a, e and N) and electric (ε_{p_i}) parameters of the spherical resonator, we can compute a full wave simulation of a plasma-based ENG antenna using commercial software Ansys HFSS. Each plasma layer is represented by a dielectric medium whose permittivity follows the Drude model from (2).

As shown in Figure 7, this antenna consists of a half sphere resonator coupled to a 50-Ω coaxial probe and placed above an infinite ground plane. The blue part represents the glass shell surrounding the 11 layers plasma sphere. The 50-Ω coaxial probe is a cylinder of radius $r = 4$ mm and length $l = 40$ mm. Note that, in each layer, we take into account plasma losses ($\nu = 40$ MHz).

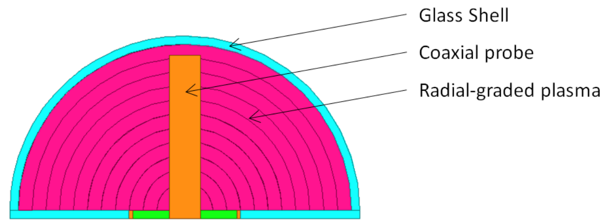

Fig. 7. Cut-view of a N-layered plasma surrounded by a glass shell and coupled to a 50-Ω coaxial probe. Only one quarter was used as xz and yz symmetry planes were used to reduce the number of tetrahedra (close to 90000)

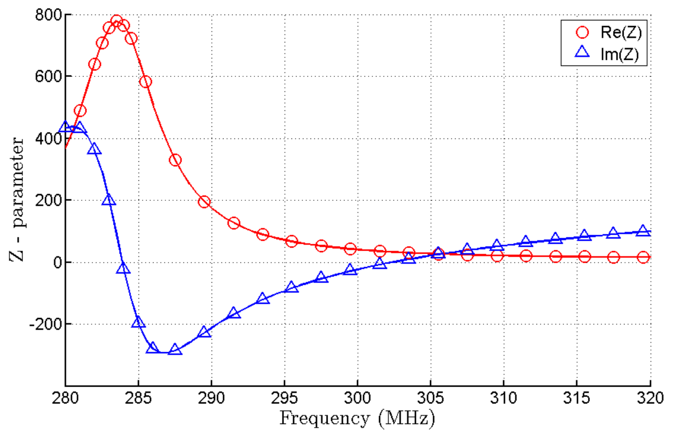

Fig. 8. Simulated input impedance of the coaxially fed ENG resonator around its resonant frequency.

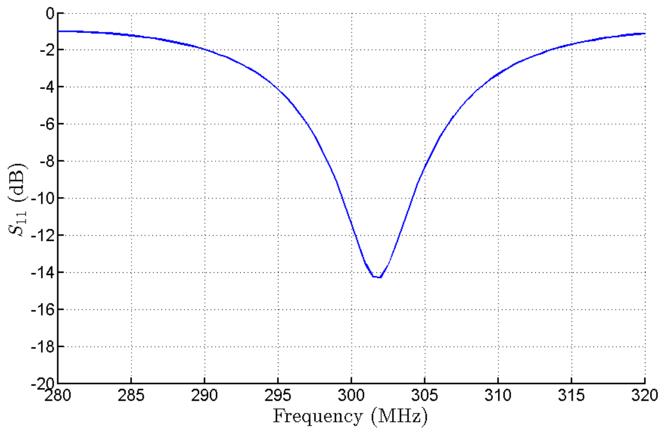

Fig. 9. Simulated S-Parameter of the coaxially fed ENG resonator around its resonant frequency.

The corresponding input impedance and S-parameter are given in Figure 8 and Figure 9, respectively. As predicted by the analytical model, a resonance occurs at $f = 302$ MHz. This resonance is well-adapted to a 50-Ω transmission line. We also observe in Figure 8 a strong antiresonance at $f = 284$ MHz.

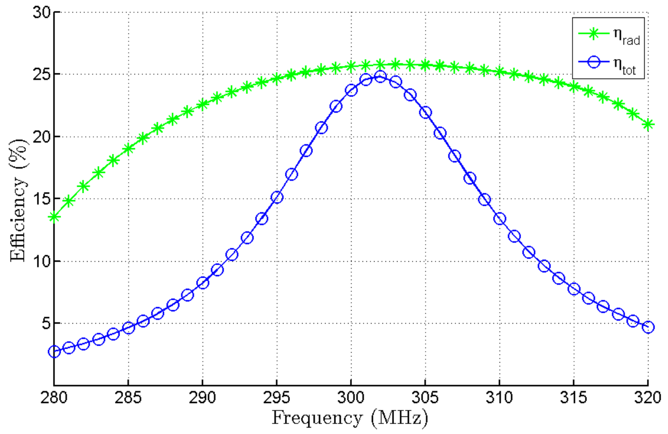

Fig. 10. Simulated radiation efficiency (green curve with stars) and total efficiency (blue curve with circles) of the coaxially fed ENG resonator around its resonant frequency.

VI. Radiation Parameters

Using Ansys HFSS, we compute the radiation efficiency at $f = 302$ MHz and compare it to the analytical value that can be found in [11]:

$$\eta_{rad} = \frac{1}{1 + \frac{3\sqrt{3}\nu}{2(ka)^3\omega_p}} \tag{7}$$

where $ka = 0.28$ and $\nu = 40$ MHz in our case.

Note that the plasma angular frequency ω_p is not constant in the sphere. E-field and H-field plots on Ansys HFSS confirm however that the resonance is located at the outer plasma layer. Thus, ω_p is taken at $r = a_p$, that is to say a value of 2.2×10^9 rad.s^{-1}. The resulting analytical radiation efficiency is $\eta_{rad} = 33$ %, which is close to numerical results plotted in Figure 10.

Finally, the quality factor Q of the simulated antenna is derived and compared with the well-known fundamental limit [12]:

$$Q_{min} = \eta_{rad}\left(\frac{1}{(ka)^3} + \frac{1}{ka}\right) = 12.1 \tag{8}$$

The simulated Q-factor is calculated using Yaghijan and Best approach [13]. Using the simulated antenna input impedance $Z_{in}(\omega) = R_A(\omega) + jX_A(\omega)$ (see Figure 8), we can determine the following Q-factor at resonance:

$$Q_{Yag}(\omega_0) = \frac{\omega_0}{2R_A}\sqrt{(R'_A)^2 + \left(X'_A + \frac{|X_A|}{\omega_0}\right)^2} \tag{9}$$

At the resonant frequency, we find $Q_{Yag} = 43.1$ which represents about 3.57 times the fundamental limit. This value is not optimum and suggests that the geometry can be improved to enhance the bandwidth and the radiation efficiency.

Figure 11 represents the gain obtained at resonance frequency for both ϕ and θ planes. It exhibits the classical radiation patterns of a small electric dipole.

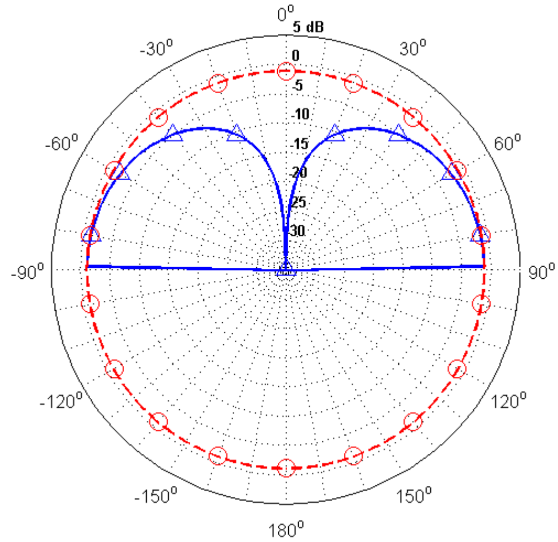

Fig. 11. Simulated radiation pattern of the coaxially fed ENG resonator at resonance frequency $f = 302$ MHz. Blue curve (triangles) represents the realized gain at $\phi = 0°$ and red curve (circle) represents the realized gain at $\theta = 0°$

VII. Conclusion

A plasma-based spherical ENG resonator coupled to a coaxial probe was studied here. We have considered a lossy dispersive plasma whose permittivity is not homogeneous and that is surrounded by a dielectric shell. Analytical approach and numerical simulations confirmed that the static resonance observed in small homogeneous sphere still occurs but the frequency response is shifted regarding to the plasma gradient and the shell properties.

Numerical results are in good agreement with analytical presizing of such a resonator. The antenna radiation efficiency was found to be close to analytical limits and a good matching was reached.

Future work will focus on optimizing the current design to improve Q-factor and enhance impedance matching which remains a non trivial challenge for this kind of antennas. Furthermore, a prototype is currently in development to confirm the feasibility of a plasma-based electrically small antenna.

Acknowledgment

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