N° d'ordre :………………

THESE

présentée

pour obtenir

LE TITRE DE DOCTEUR DE L'INSTITUT NATIONAL POLYTECHNIQUE DE TOULOUSE

École doctorale : Mécanique Energétique Génie civil Procédés ………..

Spécialité : Génie des Procédés et de l'Environnement …………………..……

Par : Firooz Tabkhi ……………………………………..……………

Titre de la thèse : **Optimisation de Réseaux de Transport de Gaz** ……….....

Soutenue le 13 décembre 2007 devant le jury composé de :

M.	Stephan Astier ..………………………..	Président
M.	Luc Pibouleau ………………………..	Directeur de thèse
Mme.	Catherine Azzaro-Pantel ……………..	Directeur de thèse
M.	Michel Roques …………………………	Rapporteur
M.	Arsène Isambert ………………………..	Rapporteur
M.	Jean André …………………………….	Membre
M.	Florent Bourgeois ………………………	Membre

Résumé

Le gaz naturel, considéré comme un combustible « plus propre » que le charbon et le pétrole vis-à-vis de la pollution acide et des gaz à effet de serre, est de plus en plus utilisé en tant que source énergétique. Selon la plupart des estimations, sa consommation globale doit doubler d'ici 2030. Les réseaux de gazoducs servent au transport de grandes quantités de gaz naturel pour de longues distances. Tandis que le gaz parcourt le gazoduc, sa pression et son énergie diminuent du fait de la friction entre le gaz et la surface intérieure du gazoduc et au transfert de chaleur entre le gaz et son environnement. Pour compenser ces pertes, des stations de compression sont généralement positionnées à intervalles réguliers. Elles consomment une quantité importante d'énergie qui est souvent fourni par une partie du gaz à comprimer, ce qui dans un premier temps, entraîne des coûts importants et dans un deuxième temps libère des rejets de CO_2.

Deux axes principaux sont généralement mis en évidence lorsqu'on s'intéresse aux réseaux de transmission, à savoir leur conception et leur exploitation. Ils entraînent des intérêts technico-économiques souvent basés sur la minimisation du coût d'investissement et la maximisation de la capacité de réseaux ou en gestion sur la minimisation du gaz consommé au niveau des stations de compression.

La simulation numérique basée sur les modèles de réseaux en régime permanent sert à fournir les solutions pour ces types de problèmes. Les réseaux de gazoducs ont évolué vers des systèmes compliqués de grande taille au fur à mesure que les industries de gaz ont été développées. Un réseau typique se compose de nombreuses conduites, de douzaines de stations et un bon nombre d'autres dispositifs comme vannes et régulateurs. Souvent dans les stations de compression sont réunis plusieurs et différents compresseurs qui sont installés au cours de l'extension de la capacité du système. Dans ce contexte, la simulation et l'optimisation numérique des dispositifs ci-dessus aident les concepteurs et utilisateurs tant à concevoir les réseaux de gazoducs qu'à contribuer à prédire leur comportement et à contrôler leur exploitation : de ce fait ces concepts ont mobilisé l'attention des chercheurs pendant de nombreuses années.

Des difficultés de tels problèmes d'optimisation émergent plusieurs aspects. En premier lieu, les stations de compression se composent de plusieurs unités (de compresseurs) avec leurs

propres caractéristiques fortement non linéaires et de diverses configurations. De plus, chaque compresseur peut être en (ou hors) service. Dans un deuxième temps, l'ensemble des contraintes qui définissent les conditions opératoires faisables pour les compresseurs et celles au niveau des conduites constituent un système complexe. En troisième lieu, le problème d'optimisation apparaît souvent dans une forme impliquant les variables continues et/ou entières, et il utilise une formulation de programmation non linéaire en variables mixtes.
Dans ce contexte, la recherche a pour objectif de fournir un cadre méthodologique pour modéliser et optimiser de tels réseaux et est appliqué à une série de cas d'études couvrant une gamme d'applications significatives. Elle apporte une aide pour les ingénieurs de développement des procédés et aux gestionnaires de production chargés de trouver les conditions opératoires appropriées d'un réseau de gazoduc.
Des problèmes typiques de taille industrielle pour minimiser la consommation de carburant servent d'illustration. Ensuite, une méthodologie pour des problèmes de conception de réseaux de gaz, impliquant le coût d'investissement comme critère d'optimisation est présentée. Enfin, une étude plus prospective, dédiée au transport d'un mélange GN-H2 est traitée dans une période de transition vers une économie « hydrogène ». L'idée dominante est de montrer comment l'approche présentée peut devenir intéressante pour la détermination de la quantité maximale d'hydrogène transférable par les infrastructures existantes de gazoducs.
Des conclusions et perspectives ouvrent la voie pour de futures recherches.

Mots clés:

Gaz naturel, hydrogène, réseaux, gazoduc, compresseur, optimisation, GAMS, NLP, MINLP

Preface

Natural gas (NG), viewed as a cleaner-burning alternative to coal and oil in terms of acidic and greenhouse gas pollution, is increasingly being used as an energy source and by most estimates, its global consumption will double by 2030 (Riva et al., 2006). The transport of large quantities of NG is carried out by pipeline network systems across long distances. As the gas flows through the network, pressure (and energy) is lost due to both friction between the gas and the pipe inner wall, and heat transfer between the gas and its environment. Typically, natural gas compressor stations are located at regular intervals along the pipeline to boost the pressure lost through the friction of the natural gas moving through the steel pipe. They consume part of the transported gas, thus resulting in an important fuel consumption cost on the one hand, and in a significant contribution to CO_2 emissions, on the other hand. Two main issues are generally highlighted when considering pipeline transmission networks, i.e., designing and operating a gas pipeline network. They generally involve technico-economic concerns, often based on capital cost minimization, throughput maximization, and fuel cost minimization.

Numerical simulations based on either steady state or transient models of the networks have been used to attempt to provide solutions to these problems. As the gas industry has developed, gas pipeline networks have evolved over decades into very large and complex systems. A typical network today might consist of thousands of pipes, dozens of stations, and many other devices, such as valves and regulators. Inside each station, there can be several groups of compressor units of various vintages that were installed as the capacity of the system expanded. In that context, numerical simulation and optimization of gas pipeline can be of great help to design them, to predict their behaviour and to control their operation. Over the years, many researchers have attempted this issue.

The difficulties of such optimization problems arise from several aspects. First, compressor stations are very sophisticated entities themselves. They might consist of a few dozen compressor units with different configurations and characteristics. Each unit could be turned on or off, and its behaviour is nonlinear. Second, the set of constraints that define feasible operating conditions in the compressors along with the constraints in the pipes constitute a very complex system of nonlinear constraints. Third, the considered optimization problem often involves both continuous and integer variables and uses a Mixed Integer Non Linear Programming formulation. Moreover, the treatment of industrial-size problems may render the problem highly combinatorial.

In that context, this manuscript is intended to provide a framework for modelling gas pipelines networks. It includes methods for optimizing such gas systems and illustrates their application in a series of case studies covering a range of significant problems. The work presented here attempts to provide a general methodology in a manner useful to both the scientist/engineer engaged in process development or design and the production manager who has to find the most appropriate operating conditions of a gas network.

The manuscript consists of 5 chapters.

- Chapter 1 starts with an introduction to gas pipelines, outlining their main technical features. This chapter also highlights the importance of modelling and optimization of such networks and presents the results of the literature review. The guidelines of the work are presented.
- Chapter 2 explains the modelling approach that serves as a methodology framework.
- Chapter 3 is devoted to typical problems for performance improvement of pipeline networks: fuel consumption minimization problems serve as an illustration.
- Chapter 4 considers high pressure gas pipeline design problems, involving total annualized cost as an optimization criterion.
- Chapter 5 deals with a more prospective concern, dedicated to the transport of a mixture of natural gas-hydrogen mixture in a transition period towards the so-called predicted "hydrogen economy". The underlying idea is to show how the modelling approach can be used to determine the maximal amount of hydrogen that existing pipelines infrastructure are able to transport.
- Chapter 6 gives the conclusions and recommendations for future work.

Keywords:

Natural gas, hydrogen, network, pipeline, compressor, optimization, GAMS, NLP, MINLP

تقدیم به قهرمانان حماسه‌ی تابستان ۱۳۸۵

Acknowledgments

After thanks to God, I am grateful to my family particularly to my parents who have given me the force to carry on my university studies.

This work was conducted in the department "Procédés Systèmes Industriels" of the "Laboratoire de Génie Chimique" in Toulouse. I thank this institution for its confidence and support. The Ministry of Science, Research and Technology of Iran is also gratefully acknowledged for the financial support that made this thesis possible as well as the Persian Gulf University.

I express my sincere gratitude to my supervisors Luc Pibouleau and Catherine Azzaro-Pantel, Professors at Ecole Nationale Supérieure des Ingénieurs en Arts Chimiques et Technologiques (Institut Nationale Polytechnique de Toulouse), for their great patience, help and discussions. Regardless of day or hour, they were always willing to help and answer to any question I had in such a way that I could not have asked for more. I express the same gratitude to Serge Domenech, Professor at ENSIACET. I really appreciated their competence and availability. Also I thank Mr. André Davin for all of his attention during the thesis.

I wish to show my gratitude to Michel Roques, Professor Emeritus at Université de Pau et des Pays de l'Adour and Arsène Isambert, Professor at Ecole Centrale Paris for being examiner of my thesis. I appreciate the familiarity of Professor Roques with Iran and my region of origin.

I wish to thank the referees of this thesis, especially Doctor Jean André, Direction de la Recherche (Gaz de France), also Stephan Astier, professor at ENSEEIHT (INP de Toulouse) and Florent Bourgeois, Professor at ENSIACET, for their interest in my work and for their valuable comments.

It has been very pleasant for me to work with the members of our research group who have been of friendly help to me. Particularly I am grateful to Sofiane Hocine for his encouragement to fulfil my thesis. I want also to express my appreciation to all the future doctors and actual doctors of the laboratory with whom I had fruitful discussions. I would like to thank also the laboratory staff specially Denis Plotton and Lydie Simon for their assistances.

Many friends have helped me directly or indirectly to realize this project, making it better than it otherwise would have been. Thank to my Iranian friends in Toulouse and who have supported me from a distance. Thank to the other friends in Toulouse especially to Mr. and Mrs. Charara.

Thanks to all that I have not mentioned here but they have supported me in their manners.

Contents

List of figures

List of tables

Chapter 1

Introduction and problem formulation

This chapter presents the typical features of the natural gas pipeline networks and of their main components. Then, a review of the modelling background dedicated to pipeline transmission systems is presented. The principles of the optimization techniques that can be used to tackle the problem are recalled, with a special focus on Process Systems Engineering applications. The general guidelines of this work will be then proposed and introduce the structure of the following chapter.

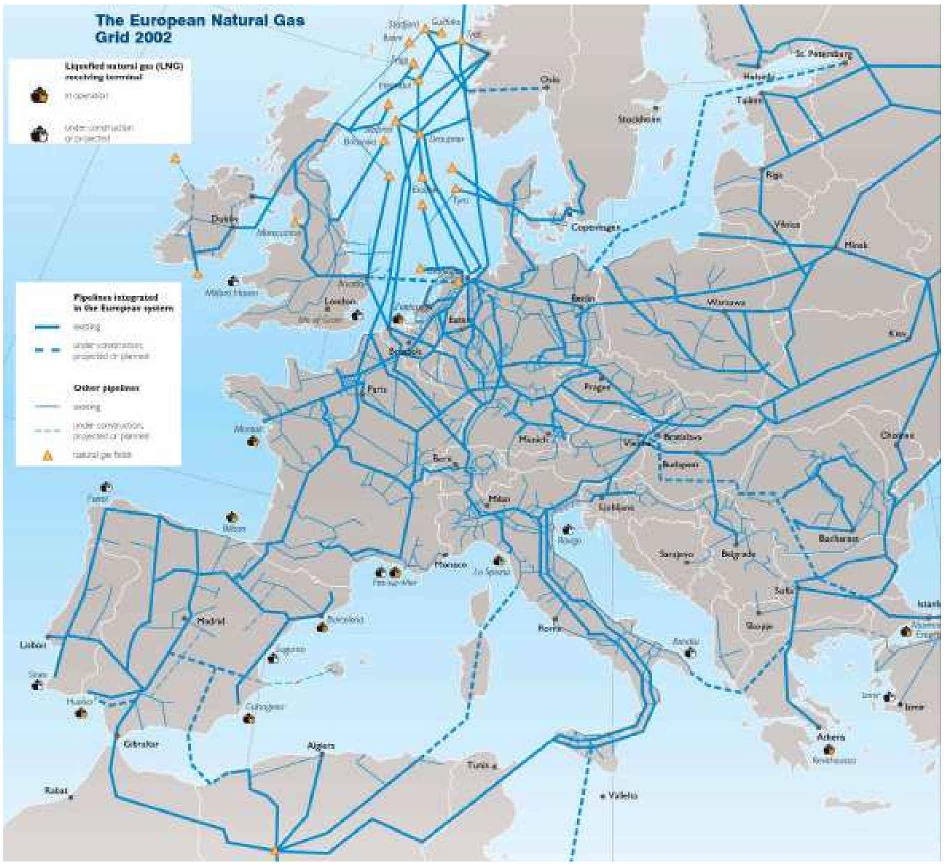

Figure (1.1) European Natural Gas Transmission System in 2002 (Eurogas, 2003 and 2004)

1.1. The natural gas system

The European natural gas system is very well developed and consists, inter alia, of 1.4 million kilometres pipelines of which 145,000 km concern high pressure transmission pipelines. In addition, 93 storage facilities with a total working volume of 60,000 million cubic meters are in operation. For the sake of illustration, Figure (1.1) presents a general overview of the European gas network.

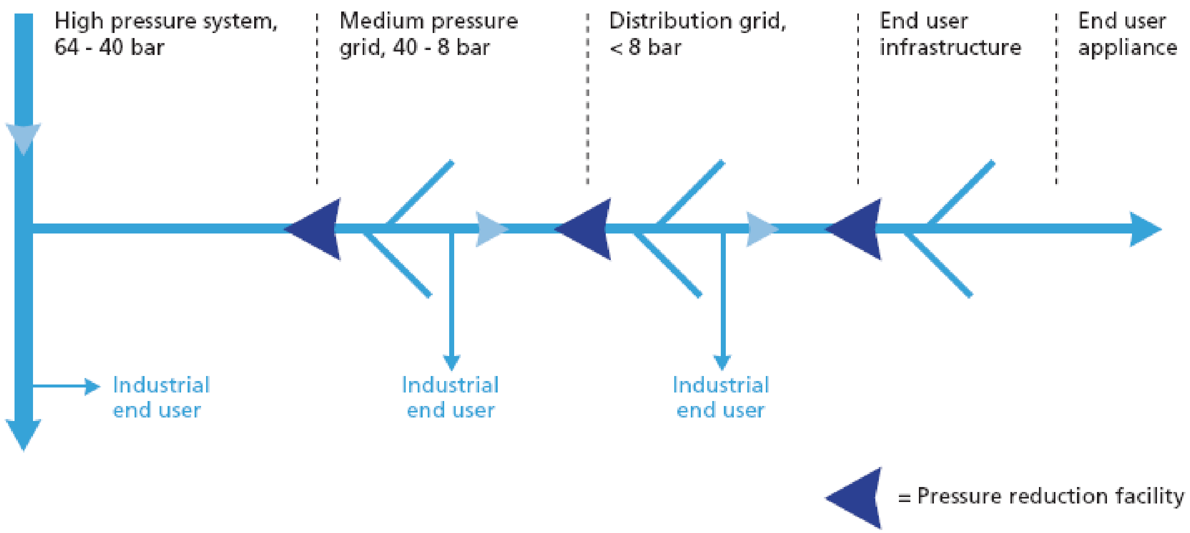

Figure (1.2) Schematic view of the different parts of a natural gas delivery system

Part of the N.G. system	Operating pressures (bar)	Active components (besides valves and pressure regulators)	Material of pipelines	Connected end users
Natural gas production and treatment facilities	> 65	Various	Steel	-
High pressure transmission system	< 70 and > 40	Compressors, LNG-facilities, underground storages, gas blending stations	Steel	Power plants, iron mills, large chemical plants using natural gas as feedstock, etc.
Medium pressure transmission system	< 40 and > 8	-	Steel	Chemical plants, ceramic industry, District heat&power plants (WKK-facilities), etc.
Distribution grid	< 8	-	Steel, cast iron, PE, PVC	Offices, domestic end users, greenhouses etc.
End user infrastructure	< 8	-	Copper, PE	-

Table (1.1) Technical features of the different parts of a natural gas delivery system

The natural gas chain is generally constituted by various components as represented in Figure (1.2) and illustrated in Table (1.1). In this table and in Figure (1.2) the pressure regimes are just indicative and may differ from country to country.
The transport lying system between the natural gas deposits and the consumers is quite complex. After the gas has been extracted, so-called trunk lines are connected with pipeline compressor stations. The natural gas is then pumped into long distance pipelines called transmission lines and sent to the take-off stations for the consumers. From there the gas is further transported to the control station of the regional distribution system. It then finally goes to industrial customers and households. A schematic view of a pipeline section is displayed in Figure (1.3) with six compression stations, delivery and supply points.

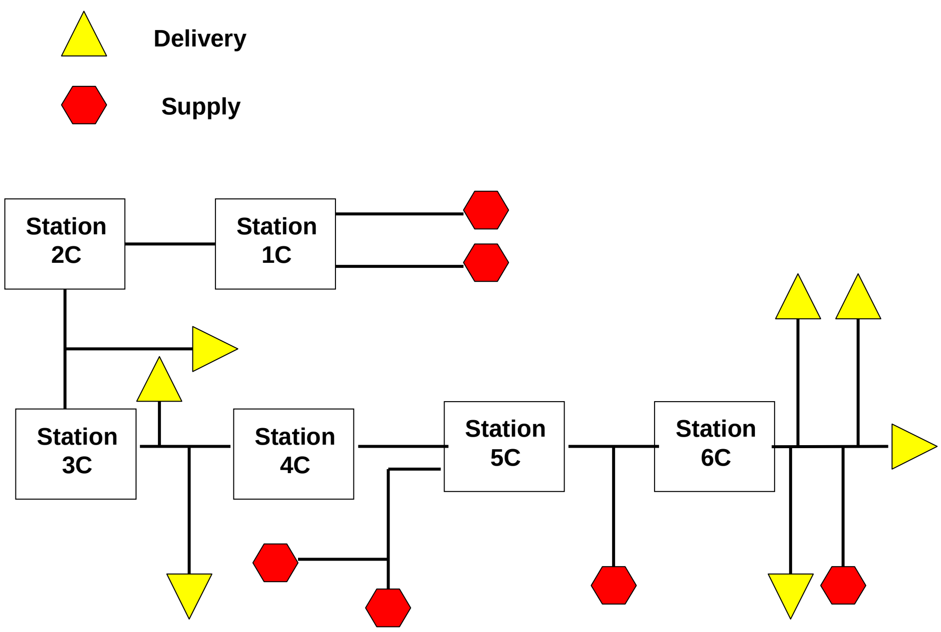

Figure (1.3) Schematic showing a selected pipeline section with six compression stations

Pipeline pressures, diameters and materials used at the different stages of transport vary considerably from country to country, with minor differences also within national systems, depending on the supplier. Table (1.2) shows some typical ranges of pressures, diameters and materials involved in the different stages of transport.

Transport stage	*Pressure, bar*	*Diameter, cm*	*Material*
To head station	70-100	40-150	LA, HSS
Long distance	60-90	50-130	LA, HSS
Local / Regional	8-40	7.5-30	LA, LCS
Customer	0.050-0.100	2.5-5	LCS, PVC, PE

Notes: LA = Low alloy, HSS = High strength steels, LCS = Low-carbon steel PVC = Poly-vinyl-chloride, PE = Polyethylene

***Table (1.2) Stages of pipeline transport (Castello et al.,* 2005)**

Several hundreds of thousands of both large-size transportation and mid/small-size distribution lines have been constructed over the last 30 years all over the world, to match with the increase in natural gas consumption (IEA, 2003).

The examples that will be treated throughout this study will mainly fall into **transmission network** domain although the approach may be extended easily to the treatment of the other distribution systems.

Compressor stations in a pipeline system can be sub-divided in two classes: the originating stations, which are positioned at the inlet to the pipeline and are usually the most complex, and the booster stations, which are located along the pipeline to compensate for the pressure decrease due to friction and elevation losses. In principle, the longer the pipeline and the elevation of the terrain crossed, the more compressor horsepower is required to achieve the required delivery pressure at destination.

However, under a fixed route and flow capacity, the number and size of booster stations can vary depending on circumstances and design. Although systems with fewer stations can be easier to operate, they have the disadvantage of introducing a need for high inlet pressures. Actual transmission systems represent a compromise between a very few powerful originating stations and a large number of small booster points.

The essential components of a compressor station are the following:

1. The gas compressors and their drivers (gas turbines, electric motors, steam turbines, internal combustion engines).
2. Measuring equipment and metering systems.
3. Inlet separators or gas scrubbers, to remove liquid and solid impurities from the gas and protect the compressors.
4. Heat exchangers and inter-stage coolers, to remove the heat of compression between subsequent compressor stages.
5. Piping manifolds, valves and controllers to direct and regulate the gas flow, valves for vent and relief.

Compressors used for gas transmission in pipelines can be divided in two categories:

- Positive displacement, or intermittent flow compressors. They can be further subdivided in reciprocating and rotary compressors. In the first type, the gas is compressed within a cylinder by a moving piston; in the other case the gas is displaced from inlet to outlet by the vanes or lobes of a turning rotor.
- Dynamic, or continuous flow compressors. They increase the pressure of the gas by increasing its velocity and converting the energy into pressure by slowing the gas flow up in a diffuser. These machines can be further sub-divided in the centrifugal and axial type, which accelerate the gas molecules respectively by subjecting them to centrifugal forces or by transferring them the energy of a spinning rotor (turbocompressors).

More detail can be found in (Gorla and Khan, 2003)

The volume of gas that a centrifugal compressor can handle depends on the size and speed of the impeller and on the discharge pressure. For a given compressor, performance curves can be drawn that define, for a given value of the impeller rotational speed, the relationship between the inlet flow and the compression work (or compressor's head), which in turn increases with the ratio between the suction and discharge pressures. The use of the so-called "performance map" will be explained in the next chapter.

1.2. Review on pipeline modelling techniques

The qualitative presentation of the pipeline network and of its components highlights that engineering design and operation studies of pipeline network are involved with the optimization of the performance of the global system. The analysis of the dedicated literature shows that there is growing interest on the subject.

The optimization of the design of a pipeline to transmit fluids involves numerous variables, which include pipe diameter, pressure, temperature, line length, space between pumping or compressor stations, required inlet and delivery pressures and delivery quantity. Each of these parameters influences the overall construction and operating cost in some degree and the selection of one or more will determine the economics of the construction and operation of the system.

This is as true for the design of a system from a clean sheet of paper (grass roots) as it is for the development and upgrading of an existing system, the only real difference between these two examples is the extent to which some of the variables are already fixed.
Because of the number of variables involved, the task of establishing the optimum can be quite difficult and in order to ensure a robust solution, many options may have to be investigated.

The two main approaches that are classically encountered in gas networks representation are numerical simulation and optimization.
The main purpose of simulation is to determine the actual behaviour of a gas network under given conditions. Simulation basically answers the question: what happens if we run our grid with given control variables and known boundary flows? Typical questions like finding a control regime which achieves several target values usually require a series of simulation runs by expert users who are familiar with the network. Two disadvantages of numerical simulation are found. First, finding an adequate regime may even take a large number of runs and second, it cannot ensure that the solution achieved is optimal.
This explains mainly why the searching process must be substituted with more sophisticated algorithms. Yet, optimization generally works with simplified models, but it yields optimum results where limits or certain target values will be achieved automatically if they are defined as optimization problem constraints. There has been a great deal of work on the optimization approach to gas pipe distribution networks. Table (1.3) presents the existing technologies that are used to model the performance and operation of the various components that collectively make up the natural gas pipeline system.

Turner and Simonson (1984,1985) Development of a computer program for compressor stations that is added to SIROGAS, which is program for solving a pipeline network for the steady-state and transient modes.
Botros et al. (1991, 1994) Use of a dynamic compressor station simulation that consisted of nonlinear partial differential equations describing the pipe flow together with nonlinear algebraic equations describing the quasi-steady flow through various valves, constrictions, and compressors.
Tian and Adewumi (1994) One-dimensional compressible fluid flow equation
Lewandowski (1994). - Application of an object-oriented methodology for modelling a natural gas transmission network using a library of C^{++} classes. - Formulation of the gas pipeline network model as a directed graph. Each arc of the directed graph represents a pipeline segment and has associated with it a partial differential equation describing the gas flow through this segment.

Osiadacz (1994) Dynamic optimization of high-pressure gas networks using hierarchical system theory.
Zhou and Adewumi (1995). One dimensional transient natural gas flow in a horizontal pipeline without neglecting any terms in the conservation of momentum equation
Surry et al. (1995) -The optimization problem is formulated with a multi-objective genetic algorithm the so-called COMOGA method (Constrained Optimization by Multi-Objective Genetic Algorithms).
Mohitpour et al.(1996) Use of dynamic simulation for the design and optimization of pipeline transmission systems. The authors explain that steady-state simulations are sufficient for optimizing a pipeline when supply/demand scenarios are relatively stable.
Glenn et al. (1996). - Use of a transient flow model based on a numerical solution of the one–dimensional; unsteady flow equations (continuity, momentum and energy), - Discretization with a highly accurate compact finite difference scheme
Osiadacz (1996) Transient pipeline models. Characterization of different transient models and existing numerical techniques to solve the transient equations.
Santos (1997) Use of transient simulation. Useful approach for staff training, on-line systems and design phase of a gas pipeline.
Boyd (1997) - Steady-state gas pipeline networks by using mathematical model over compressor station - Fuel cost minimization problem
Costa et al. (1998) - Steady state gas pipeline simulation. - One-dimensional compressible flow equation. - Flow equation and the conservation of energy equation solved in coupled fashion to investigate the differences between the isothermal, adiabatic and polytropic flow conditions. Compressors modelled by a functional relationship between the pressure increase and the mass flow rate of gas.
Sung et al. (1998) Modelling approach based on a hybrid network (HY-PIPENET) using minimum cost spanning tree. - Presentation of the results of a parametric study (source pressure, flow rate and pipeline diameter). - An optimal relationship between pipe diameter and the source pressure is established.
Doonan et al.(1998) Use of Simulink^{TM} for pipeline system simulation purpose under steady-state conditions. Investigation on the safety parameters (distance down stream from the main pressure regulating station).
Osiadacz (1998) Isothermal and non-isothermal transient models. Consideration of heat conduction effects cannot be neglected.
Carter (1998) Development of a nonsequential dynamic programming algorithm to handle loopednetworks when the mass flow rate variables are fixed. A global optimum is guaranteed to be found and that nonlinearity can be easily handled.
Sun et al.(1999) - Use of a software support system, called the Gas Pipeline Operation Advisor (GPOA). - Minimization of the overall operating costs, subject to a set of constraints such as the horsepower requirement, availability of individual compressors, types of compressor and the cycling of each compressor. The GPOA aids the dispatcher in optimizing natural gas pipeline operations in order to satisfy customer demand with minimal operating costs.
Cameron (1999) Use of an Excel-based model for steady state and transient simulation (TFlow). - Use of an interface written in Microsoft Excel's Visual Basic for Applications (VBA) and a dynamic linked library (DLL) written in C^{++}. - The robustness for general applications is not readily apparent

Rios-Mercado et al.(2001) Use of a reduction technique for natural gas transmission network optimization problems. Decisions variables: mass flow rate through each arc (pipeline segment), gas pressure level at each pipeline node.
Martinez-Romero et al. (2002). - Steady-state compressible flow through a pipeline - Use of the software package "Gas Net," - Mathematical model assuming a gas network with two elements: nodes and nodes connectors. Sensibility analysis for the most important flow equations defining the key parameters in the optimization process.
Cobos-Zaleta and Rios-Mercado (2002) Use of an MINLP model for the problem of minimizing the fuel consumption in a pipeline network. A computational experience was presented by evaluating an outer approximation with equality relaxation and augmented penalty method.
Abbaspour et al. (2005) - Nonisothermal unsteady one-dimensional compressible flow. - The compressibility factor is a function of pressure and temperature, and the friction factor as a function of Reynolds number. - Solution method : the fully implicit finite difference method - Nonlinear programming problem (NLP) formulation - Design variables: compressor speeds - Objective function to be minimized: total fuel consumption. The compressors within the compressor station are modelled using centrifugal compressor map-based polynomial equations. This modelling technique permits the designation of different models of compressors in the compressor station. The method can be easily extended to include other types of compressors.
Mora, T. and Ulieru, M. (2005) - Determination of the pipeline operation configurations requiring the minimum amount of energy (e.g. fuel, power) needed to operate the equipment at compressor stations for given transportation requirements. - Use of a genetic algorithm. The approach leads to a portfolio of feasible near-to-optimum solutions to decision makers in a timely manner. This set of solutions will represent the network operation conditions that decrease energy consumption.
C. Chauvelier-Alario et al. (2006) -Development of CARPATHE, a simulation package (GdF) for representation the behaviour of multi-pressure networks and including functionalities for both network design and network operation. - A training tool for new network operators to acquire a good knowledge of the network and its behaviour once they will be in charge.
J. André et al. (2006) -Use of optimization methods for planning reinforcement on gas transportation networks -Minimization of investment cost of an existing gas transmission network. - The size of diameter is to be taken among a discrete set of values. - Use of new heuristics to solve the large integer NLP problem

Table (1.3) Various approaches in transmission pipelines modelling

This literature analysis shows that there has been and continue to be significant effort focused on the compressible flow of natural gas through the pipeline. If the effort has been focused on steady-state flow conditions, researchers have identified the need for transient flow simulations for long. Nevertheless, it has been proven that they require a sophistication level that may be difficult to take into account as far as optimization of large systems is involved.

The objective of this work is to propose **a general framework able to embed formulations from design to operational concern**. This is why the work presented here only considers

steady-state behaviour of the gas flow. The problem is to find, for a given mathematical model of a pipeline, a numerical method that meets the criteria of accuracy and relatively small computation time, compatible with an optimization procedure.

The second significant conclusion is that very little has been done to advance the state-of-the-art of the simulation of compressor station components. For example, most references model the compressor station as a black box where the input pressure is increased by some percentage to determine the compressor station output pressure. Few approaches take into account a complete engine or compressor load map.

The third conclusion is that the formulation is based on optimization problems. Of course, the variables and objective function may differ according to the problem which is considered, either for design or operation applications.
The nature of variables may be either continuous or discrete: for instance, for a design problem: existence or not of a compression station between two nodes (discrete), set points values of compression facilities (continuous).

Typically, the aim is to minimize the total fuel used in compression or to improve the total throughput of the system as far as pipeline optimization is concerned. It must be pointed out that the goal of minimizing the energy consumption in compressor stations will have not only economic benefits but also a positive environmental impact, since pipelines emit CO_2 mainly due to energy used at compressor stations.
The design problem is generally associated with either capital cost or annualized operating cost minimization.

Fourth, it must be pointed out that the majority of the works presented are based on classical mathematical formulations. Only the problem may be highly combinatorial for industrial size problems, the literature review only mentions very few works devoted to stochastic algorithms (for instance, Simulated Annealing or Genetic Algorithms). This is probably due to the important number of constraints (inequalities and equalities) which condition the problem. The same remark is all the more valid for the treatment of multi-objective problems as they generally involve this kind of algorithm as a solution strategy : yet, the problem of pipeline operation is typically multi-objective by nature : for instance, the practitioner has to cope simultaneously with throughput maximization and fuel consumption minimization.

Finally, it must be said that the treatment of uncertainty in the demand is of major concern.

The following section proposes an analysis of the different optimization alternatives that can be examined throughout this manuscript.

1.3. Review on optimization techniques

A great variety of applications, drawn from a wide range of investigation areas, can be formulated as complex optimization problems. It covers, for instance, the famous travelling man problem studied (Padberg and Rinaldi, 1991) as well as frequencies allocation for radio-mobile networks (Hao and Dorne, 1996), process networks optimization (Lee and Grossmann, 2003), physicochemical equilibrium calculations (Teh and Rangaiah, 2003), or hydrology computing (Jain and Srinivasalu, 2005).

This large number of optimization problems arises from models that have to enable, for industrial requirements, a truly realistic representation of the system they account for. Consequently, these models tend to show an increasing sophistication degree that derives into higher complexity and, thus, solution difficulties. The complexity of the formulated models is basically due to the nature of the functions and of the variables involved in the optimization problem. The former ones may be not only non-linear, but moreover, they often prove to be non-convex, which is a strongly penalizing characteristic in the typical minimization case. Then, for constrained problem, determining the feasible space turns to be a really difficult task. With regard to variable nature, most of engineering problems consider both continuous and discrete variables, introducing discontinuities in the objective function and in the search space: those are called mixed-integer problems. Furthermore, the discrete variables induce an important combinatorial effect: this point is emphasized with NP-hard problems, for which no algorithm leading to polynomial solution times is known. Since industrial size problems have up to several thousands variables and constraints, the resulting computational times may easily become prohibitive.

In order to face these problems, a significant investigation effort has been carried out to develop efficient and robust optimization methods. At the beginning, this aim was concerned especially in the Operational Research and Artificial Intelligence areas. But the trend was subsequently followed by the Process System Engineering community, since this one provides a wide number of applications formulated as complex optimization problems. A typical reference is constituted by design problems: heat or mass exchanger networks (Zamora and Grossmann, 1998), supply chain design (Guillén et al., 2006), multiproduct (Ravemark and Rippin, 1998) or multipurpose (Dedieu et al., 2003) batch plant design or retrofitting (Montagna and Vecchietti, 2003).

As a consequence, a great diversity of optimization methods was implemented to meet the industrial stakes and provide competitive results. But if they prove to be well fitted to the particular case they purchase, these techniques performance cannot be constant whatever the treated problem is. Actually, method efficiency for a particular example is hardly predictable, and the only certainty we have is expressed by the *No Free Lunch* theory (Wolpert and Macready, 1997): there is no method that outdoes all the other ones for any considered problem. This feature generates a common lack of explanation concerning the use of a method for the solution of a particular example, and usually, no relevant justification for its choice is given *a priori*.

Among the diversity of optimization techniques, two important classes have to be distinguished: deterministic methods and stochastic ones. Complete reviews are proposed in literature for the two classes (Grossmann, 2002; Biegler and Grossmann, 2004; Hao et al., 1999).

The deterministic methods involve the verification of mathematical properties of the objective function and constraints, such as continuity or derivability. This working mode enables them to ensure to get an optimum, which is a great advantage. Among the deterministic class, the following ones stand out: the Branch & Bound methods (Gupta and Ravindran, 1985; Ryoo and Sahinidis, 1995; Smith and Pantelides, 1999); the Generalized Benders Decomposition (Geoffrion, 1972) and the Outer Approximation (Duran and Grossmann, 1986) algorithms; the Extended Cutting Plane method (Westerlünd and Petterson, 1995); Disjunctive Programming methods (Raman and Grossmann, 1994). Even though most of the above-mentioned methods keep being at academic level, some (commercial or free) computational codes are available: the *SBB*, *BARON*, *DICOPT++* and *LOGMIP* solvers within the *GAMS* modelling environment (Brooke et al. 1998), *MINLP_BB* (Leyffer, 19999) and *αECP* (Westerlünd and Lundqvist, 2003).

The second class, namely metaheuristics or stochastic methods, is based on the evaluation of the objective function at different points of the search space. These points are chosen through the use of a set of heuristics, combined with generations of random numbers. Thus, metaheuristics cannot guarantee to obtain an optimum. They are divided into neighbourhood techniques such as Simulated Annealing (Kirkpatrick et al., 1982), Tabu Search (Teh and Rangaiah, 2003) and evolutionary algorithms comprising genetic algorithms (Holland, 1975), evolutionary strategies (Beyer and Schwefel, 2002) and evolutionary programming (Yang et al., 2006).

A thorough analysis of both classes was previously studied by (Ponsich, 2005) with the support of batch plant design problems.
Since the number of constraints associated with the formulation of the problem may be important, **the deterministic way is adopted in this study**. Nevertheless, it must be kept in mind that the other approach may present some advantages related to the treatment of the underlying combinatorial aspect for industrial-size problems and to its easy extension to the consideration of multi-objective problems. It will be examined as potential perspectives of this work (Baez Senties, 2007).
To represent the deterministic class, solvers of the *GAMS* environment were chosen, since this optimisation tool is widely used, and even stands as a reference for the solution of problems drawn from Process Engineering. The principle of the module used will be presented in chapter 2 of this manuscript.

1.4. Conclusions and general outline of the manuscript

The objective of this chapter was to review the typical problems of natural gas transmission pipelines problems and to present the general methodology of this work. The idea is to propose an **optimization-oriented framework for gas transmission pipeline modelling**.

The formulation must be general enough to embed various problems that are of tremendous importance, **pipeline design and operation**. For this purpose, **steady state behaviour** of the gas is considered and will be assumed in the momentum and mass balances that will be presented in detail in the subsequent chapter. Although various optimization techniques can be used, the choice of a **deterministic** one is selected, since it is generally recognized that this kind of methods is particularly well-fitted to take into account the important number of constraints that are likely to be involved in the problem formulation. **Adequate solvers within *GAMS* environment** will be used since this optimisation tool is often considered as a standard for the solution of Process Systems Engineering problems and experience was previously acquired in our research group for batch plant design problems. This chapter is now logically followed by the **formal presentation of the pipeline model** that will serve as a basis throughout this work.

Chapter 2

Model formulation for gas pipeline networks

A gas pipeline network consists of connected pipelines where gas is derived from sources to exploitation areas by using compressors regrouped in compressor stations. The purpose of this chapter is to provide a gas transportation model taking into account the elements of the network under steady-state conditions. It will serve as a basis for the remaining chapters of this manuscript, either for operation condition optimization or design targets. The idea is to develop a model general enough to take into account various gases. Yet, the physical properties that are given here are relative to natural gas for illustration purpose. The extension to other gas mixtures such as H_2-CH_4 will be presented in a dedicated chapter. This one is organized as follows.

- In Section 1, the gas pipeline hydraulics is recalled with conservation of mass and momentum balance.
- The compressor characteristics are provided in Section 2.
- Then in Section 3, the principle of incidence matrices that model the links between the elementary sections of a network is defined to facilitate the transmission network design.
- A classification of the related optimization problems is proposed in Section 4 and the outline of the thesis objective is presented in Section 5.
- The optimization techniques that are used in this work conclude this chapter.

The GAMS environment is used as programming software. The SBB and the CONOPT solvers are used to solve encountered MINLP and NLP problems respectively.

2.1. Gas pipeline hydraulics

Due to operating problems, a gas transmission line is not normally designed to handle two-phase flow. Exceptions lie for example in oil/gas wells, gathering systems and separation units. The formulation presented here is only valid for single phase gas flow.

The pressure drop in a gas pipeline, i.e., the essential parameter to determine the required compression power for the transmission is derived from the differential momentum balance. Friction between fluid boundary layer and interior surface of the tube induces energy losses and, consequently, reduces the gas pressure.

The material balance and the equations of momentum conservation on the basic elements of the network as well as the other governing equations constitute the modelling core. The necessary equations and inequalities in the system of the gas transmission network in order to determine the dynamic conditions such as pressure and flow rate are developed. First of all, the momentum balance for a single pipeline is given.

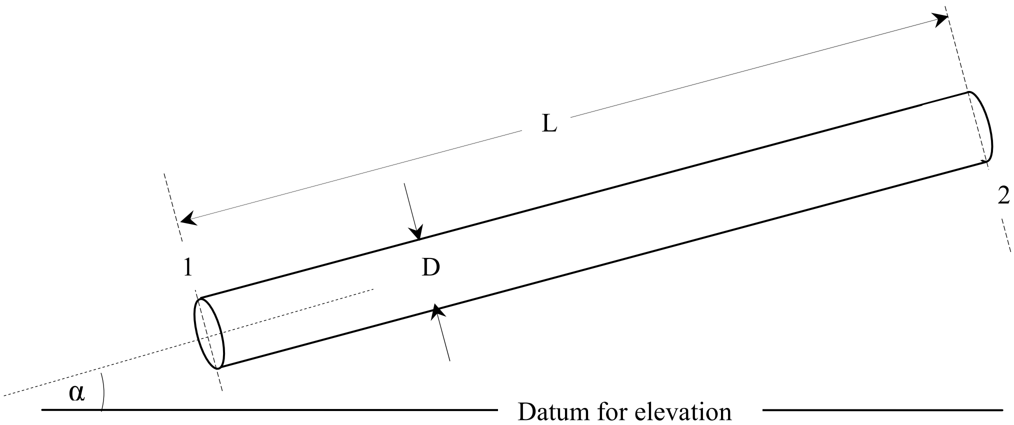

Figure (2.1) Schematic representation of an inclined pipe segment

2.1.1. One dimensional compressible gas flow

The application of one-dimensional flow model to gas pipeline pressure drop calculation, in which the fluid conditions vary only along the pipe, is a good approximation which is usually adopted in the dedicated literature (Osiadacz, 1987). A reason for this application is that the cross section area is constant and the curvature of the pipe centre-line is very large compared with the cross-sectional dimensions. Figure (2.1) shows a general schema of a pipe segment. In general, basic equations describing the flow of gas in pipes are derived from a momentum balance that is named also equation of motion, equation of continuity, energy balance and equation of state.

In general, basic equations describing the flow of gas in pipes are derived from a momentum balance that is named also equation of motion, equation of continuity, energy balance and equation of state. In practice, the form of the mathematical models varies with the assumptions made corresponding to the conditions of the operation. Simplified models are based on neglecting some terms in the basic model.

2.1.2. Conservation of mass: continuity equation

Generally, the one-dimensional conservation of mass is expressed in the form of following equation where ρ is the gas density in (kg/m^3), v is the average gas velocity in (m/s) in x direction and t is the time in (second):

$$\frac{\partial(\rho v)}{\partial x} + \frac{\partial \rho}{\partial \mathrm{t}} = 0 \tag{2.1}$$

The relation between mass flow rate, m in (kg/s), also called pipe throughput, the density and the velocity of gas is expressed in Equation (2.2).

Unlike a liquid pipeline, due to compressibility, the gas velocity depends upon the pressure and, hence, will vary along the pipeline even if the pipe diameter is constant.

$$m = \frac{\pi}{4} D^2 \rho v \tag{2.2}$$

In this expression, A, the cross section area of the pipe in (m^2), remains constant over its entire length. D is the pipe internal diameter in m.

Gas density and pressure are related as represented in the form of the following equation by introducing the compressibility factor, Z in the model.

$$\rho = \frac{pM}{ZRT} \tag{2.3}$$

R is universal gas constant, equal to 8314 J/kmol-K and M is the average molecular mass of the gas and depends on its composition. Molecular mass of the gas is calculated using a simple mixing rule expressed in the form of the following equation in which y_i's and M_i's are respectively the mole fractions and the molecular masses of species.

$$M = \sum M_i y_i \tag{2.4}$$

The compressibility factor, Z, is used to alter the ideal gas equation to account for the real gas behaviour. Traditionally, the compressibility factor is calculated using an equation of state. Yet, for natural gas, it may be estimated from the empirical relationship proposed for simulation goals in the literature (Mohring et al., 2004). For example, this factor can be expressed as a function of the critical properties of the gas mixture, average pressure of the pipe segment and the temperature that have been considered as constant:

$$Z = 1 + (0.257 - 0.533\frac{T_c}{T})\frac{p_{ij}}{p_c} \tag{2.5}$$

$$T_c = \sum T_{ci} y_i \tag{2.6}$$

$$p_c = \sum p_{ci} y_i \tag{2.7}$$

The pseudo-critical temperature of natural gas, T_c, and its pseudo critical pressure, p_c, can be calculated using an adequate mixing rule starting from the critical properties of the natural gas components. The critical point of a material is the point where the distinction between the liquid and vapour phases disappears. In this work, average pseudo-critical properties of the

gas are determined from the given mole fractions of its components by Kay's rule which is a simple linear mixing rule shown in Equations (2.6) and (2.7).
Average pressure, p_{ij}, can be calculated from two end pressures (Mohring et al., 2004):

$$p_{ij} = \frac{2}{3}(p_i + p_j - \frac{p_i p_j}{p_i + p_j}) \tag{2.8}$$

Using Equation (2.3), the continuity equation can be rearranged in the basis of mass flow rate and pressure expressed as the following equation:

$$\frac{1}{A}\frac{\partial m}{\partial x} + \frac{M}{R}\frac{\partial}{\partial t}(\frac{p}{ZT}) = 0 \tag{2.9}$$

2.1.3. Equation of motion: momentum balance

The conservation law of momentum is employed to a cylindrical control volume in steady state to derive the pattern of the pressure changes along a pipe and with time. So the governing equation to calculate the pressure at each point of a pipe can be derived as follows:

$$\frac{\partial p}{\partial x} + \frac{f}{2D}\rho v^2 \pm g\rho \sin\alpha + \frac{\partial(\rho v^2)}{\partial x} + \frac{\partial(\rho v)}{\partial t} = 0 \tag{2.10}$$

In this equation, p is pressure in Pa, g is the acceleration of gravity in m/s^2 and α is the acute angle between the horizon and the pipe centreline direction x. The sign of gravity term in the equation above is positive if the gas flows upward and is negative when the gas flows downward.
The Darcy friction factor, f, is a dimensionless value that is a function of the Reynolds number, Re, and relative roughness of the pipeline, ε/D. Darcy friction factor is numerically equal to four times of the Fanning friction factor that is preferred by some engineers. The

Reynolds number quantifies the ratio of inertial forces to viscous forces for given flow conditions and helps to identify different flow regimes, such as laminar or turbulent flow:

$$\mathrm{Re} = \frac{\rho v D}{\mu} \tag{2.11}$$

Traditionally, to characterize roughness of pipelines the equivalent sand-grain roughness has been used. The sand-grain roughness refers to the rough pipe experiments of Nikuradse and as common practice the hydraulic properties of a pipeline are compared to Nikuradse's work to arrive at an equivalent roughness (Sletfjerding and Gudmundsson, 2003).

In turbulent flow, the wall roughness is often a limiting factor as compared with the Reynolds number *Re* to find out the value of the friction factor. In offshore gas pipelines, for example, where *Re* has an order of magnitude of 13000, the wall roughness will strongly influence the pipeline pressure drop. In such pipelines, it is a common practice to apply coating on pipe walls to reduce wall roughness and pressure drop (Sletfjerding and Gudmundsson, 2003). Another example concerns flow around merchant ships where the viscous drag dominates the resistance, and the wall roughness has a significant influence on drag (Grigson, 1992).

Since the flow is considered fully developed here, which is the case concerning in gas pipelines, the friction factor is estimated through the equation deduced by Prandtl-von Karman (Romeo et al., 2002) in which the friction factor depends only on the relative roughness:

$$f = (-2\log\frac{\varepsilon / D}{3.71})^{-2} \tag{2.12}$$

The momentum balance in terms of pressure and throughput can be written with the following equation:

$$\frac{\partial p}{\partial x} + \frac{f}{2D}\frac{ZRT}{pMA^2}m^2 \pm g\frac{pM}{ZRT}\sin\alpha + \frac{2mR}{A^2M}\frac{ZT}{p}\frac{\partial m}{\partial x} + \frac{m^2R}{A^2M}\frac{\partial}{\partial x}(\frac{ZT}{p}) + \frac{1}{A}\frac{\partial m}{\partial t} = 0 \tag{2.13}$$

The derivation of this equation is presented in the appendix. In the case of the steady state, the flow properties do not change with time at each point of the pipe. This clause can be presented mathematically as Equations (2.14) and (2.15). Therefore, according Equation (2.9), the mass flow rate through the pipe remains constant across it:

$$\frac{\partial m}{\partial t} = 0 \tag{2.14}$$

$$\frac{\partial}{\partial t}\left(\frac{p}{ZT}\right) = 0 \tag{2.15}$$

$$\frac{1}{\mathrm{A}}\frac{\mathrm{d}m}{\mathrm{d}x} = 0 \Rightarrow m = cte \tag{2.16}$$

Consequently, Equation (2.13) which is a general equation can be written in steady state as follows:

$$\frac{\mathrm{d}p}{\mathrm{d}x} + \frac{f}{2D}\frac{ZRT}{pMA^2}m^2 \pm g\frac{pM}{ZRT}\sin\alpha + \frac{m^2R}{A^2M}\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{ZT}{p}\right) = 0 \tag{2.17}$$

In gas transmission lines, changes in elevation may seem to have a negligible contribution to the overall pressure drop, but it turns out that, particularly in high pressure lines this contribution could be appreciable. The associated equation for the pressure drop calculation in a pipe segment with the change in elevation is shown in the appendix.

For a horizontal pipe, by assuming that the temperature and compressibility factor stay constant between the points 1 and 2 of the pipe, the steady-state pressure drop can be calculated using the following expression.

$$({p_2}^2 - {p_1}^2) - \frac{32m^2ZRT}{\pi^2D^4M}\ln\left(\frac{p_2}{p_1}\right) + \frac{16f}{\pi^2D^5}\frac{ZRT}{M}m^2L = 0 \tag{2.18}$$

In general, when considering compressible flow, as pressure changes along the line so does the density. A variation in density implies a variation in the Reynolds number on which the friction factor is dependent. A rigorous calculation of pressure loss for long pipelines involves dividing it into segments, performing the calculation for each segment (considering variable parameters) and integrating over the entire length. The relationship between pressure and flow exhibits a high degree of nonlinearity. Equation (2.14) evaluates the pressure drop corresponding to a given flow magnitude and direction. This equation is used to estimate the pipeline's pressure profile and can incorporate the pressure head that occurs due to the location of the pipeline via the elevation changes as presented in appendix as well as for the other cases. Introducing flow direction, pressure loss equation yields to the form below:

$$(p_1^{\,2} - p_2^{\,2}) - \frac{32m^2 ZRT}{\pi^2 D^4 M}\ln(\frac{p_1}{p_2}) = \frac{16f}{\pi^2 D^5}\frac{ZRTL}{M}m^2 sign(m) \tag{2.19}$$

2.1.4. Maximum allowable operational pressure

The internal pressure in a pipe causes the pipe wall to be stressed, and if allowed to reach the yield strength of the pipe material, it could cause permanent deformation of the pipe and ultimate failure. In addition to the internal pressure due to gas flowing through the pipe, the pipe might also be subjected to external pressure which can result from the weight of the soil above the pipe in a buried pipeline and also by the probable loads transmitted from vehicular traffic. The pressure transmitted to the pipe due to vehicles above ground will diminish with the depth of the pipe below the ground surface. In most cases involving buried pipelines the effect of the internal pressure is more than that of external loads. Therefore, the necessary minimum wall thickness will be dictated by the internal pressure in a gas pipeline.

The pressure at all points of the pipeline should be less than the maximum allowable operating pressure (*MAOP*) which is a design parameter in the pipeline engineering. This upper limit is calculated using Equation (2.21):

$$p < MAOP \tag{2.20}$$

$$MAOP = SMYS\frac{2t}{D - t}f_F f_E f_T \tag{2.21}$$

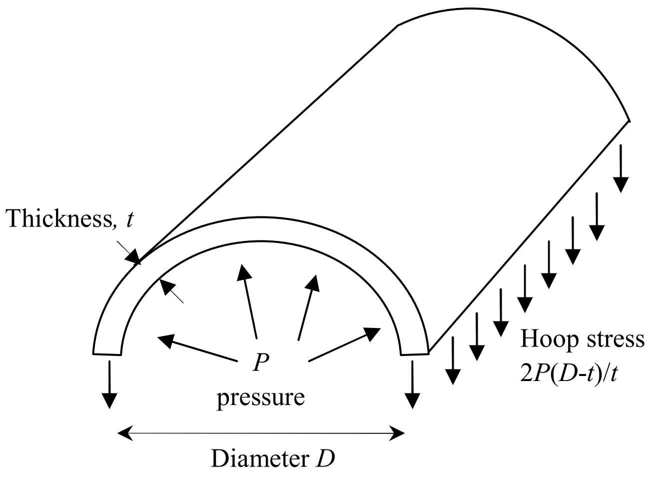

Figure (2.2) Stress in pipeline subjected to internal pressure due to gas flow

The derivation of this equation is shown in the appendix. According to this equation, to withstand the internal pressure in a gas pipeline, the required minimum wall thickness depends upon the pipe diameter and pipe material. In addition other factors such as population density of the region wherein the pipeline goes through are introduced (Menon, 2005).

The yield stress used in Equation (2.21), is called the specified minimum yield strength (SMYS) of pipe material. *SMYS* is a mechanical property of the construction material of the gas pipeline. The factor f_F has been named the design factor. This factor is usually 0.72 for cross-country or offshore gas pipelines, but can be as low as 0.4, depending on class location and type of construction. The class locations, in turn, depend on the population density in the vicinity of the pipeline. The seam joint factor, f_E, varies with the type of pipe material and joint type. Seam joint factors are between 1 and 0.6 for the most commonly used material types. The temperature deration factor, f_T, is equal to 1 for the gas temperature below 120ºC but it arrives to 0.867 at 230ºC. These three factors are explained more in appendix.

2.1.5. Critical velocity

The gas velocity is directly related to the flow rate. As flow rate increases due to the augmentation in pressure drop, so does the gas velocity. An important factor in the treatment of compressible fluid flow is the so-called critical flow. For a compressible flow, the increase in flow owing to the pressure drop increase is limited, to the velocity of sound in the fluid, i.e., the critical velocity. Sonic or critical velocity is the maximum velocity which a compressible fluid can reach in a pipe. For trouble-free operation, velocities maintain under a half of sonic velocity. Sonic velocity in a gas, c is calculated with a satisfactory approximation using Equation (2.23). Here κ is the average isentropic exponent of the gas. C_p is the heat capacity at constant pressure in J/(kmol.K).

$$v < c/2 \tag{2.22}$$

$$c = \sqrt{\frac{\kappa ZRT}{M}} \tag{2.23}$$

$$\kappa = \frac{\sum (C_{p_i} y_i)}{M \sum (C_{p_i} y_i) - R} \tag{2.24}$$

Increasing gas velocity in a pipeline can have a particular effect on the level of vibration and increase the noises too. Moreover, higher velocities in the course of a long period of time will cause the erosion of the inside surface of the tubes, elbows and other joints. The upper limit of the velocity range should be such that erosion-corrosion cavitations or impingement attack will be minimal. The upper limit of the gas velocity for the design purposes is usually computed empirically with the following equation (Menon, 2005). In pipeline design domain, the erosional velocity, v_e, falls always underneath the speed of sound in the gas.

$$v < v_e \tag{2.25}$$

$$v_e = 122\sqrt{\frac{ZRT}{PM}} \tag{2.26}$$

Consideration should be given such that the flow velocity remains within a range where corrosion is minimized. The lower limit of the flow velocity range should be so that the impurities keep suspended in the pipeline, thereby minimizing accumulation of corrosion matter within the pipeline.

2.2. Compressor characteristics

As shown in Figure (2.3), a centrifugal gas compressor is characterized by means of its delivered flow rate and its pressure ratio, the ratio between suction side pressure of the compressor and its discharge pressure.
The compression process in a centrifugal compressor can be well formulated using isentropic process aiming for calculating horsepower for a compressor station. The pressure ratio of a centrifugal compressor is usually linked with a specific term named "head" carried over from pump design nomenclature and expressed in m even for compressors. The "head" developed by the compressor is defined as the amount of energy supplied to the gas per unit mass of gas.

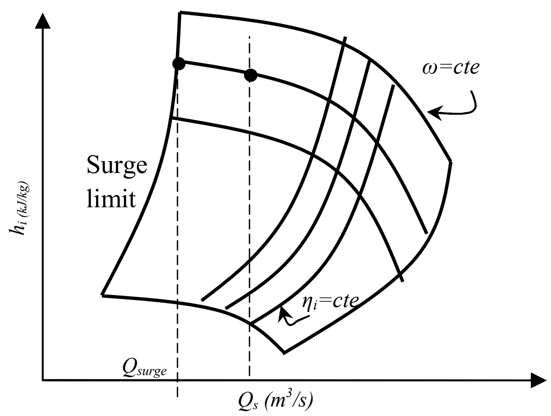

Figure (2.3) A typical centrifugal compressor map

Therefore, by multiplying the mass flow rate of compressed gas, m_c in kg/s, by the compressor isentropic head, h_i in meter, the total energy supplied to the gas is calculated in watts. Dividing this by compressor isentropic efficiency, η_i, the required power to compress the gas is obtained. Thus, the equation for power calculation can be expressed as follows:

$$P = \frac{m_c h_i}{\eta_i} \tag{2.27}$$

This equation is obtained by considering compression adiabatic process that is a reasonable assumption because the heat transfer between gas and the outside is very low. For adiabatic compressor firstly the adiabatic efficiency is defined:

$$\eta_i = \frac{P_{ideal}}{P} \tag{2.28}$$

As shown in the following equation, considering adiabatic compression, head is an index of the pressure ratio across the compressor. In this equation, p_d is the discharge pressure of the compressor and p_s is the suction pressure and κ is isentropic exponent and will be calculated using equation (2.24). The compressibility factor and the temperature here are considered at suction side of the compressor (Smith and Van Ness, 1988).

$$h_i = \frac{Z_s R T_s}{M} \frac{\kappa}{\kappa - 1} \left[(\frac{p_d}{p_s})^{\frac{\kappa - 1}{\kappa}} - 1 \right] \tag{2.29}$$

Centrifugal compressors drives are commonly electric motors, steam turbine or internal combustion engines. Combustion turbines can also supply the required energy for compression process. Turbine compressors gain their energy by using up a small proportion of the natural gas that they compress. The turbine itself serves to operate a centrifugal compressor, which contains a type of fan that compresses and pumps the natural gas through the pipeline. Some compressor stations are operated by using an electric motor to turn the same type of centrifugal compressor. This type of compression does not require the use of any of the natural gas from the pipe; however it does require a reliable source of electricity nearby. Reciprocating natural gas engines are also used to power some compressor stations. These engines are similar to a very large automobile engine, and are powered by natural gas provided from the pipeline. The combustion of the gas powers pistons on the outside of the engine, which serves to compress the natural gas.

In this work, centrifugal compressors in the station are assumed to be driven by turbines whose supply energy is provided from a line of the gas derived from the pipeline passed through the station in order to be compressed as shown in Figure (2.4).

The flow rate of the consumed gas as fuel for the compression process in each compressor is obtained by dividing required power for compression, *P*, by the mechanical efficiency, η_m, driver efficiency, η_d and *LHV* (low heating value):

$$m_f = \frac{10^6 m_c h_i}{\eta_i \eta_m \eta_d LHV} \tag{2.30}$$

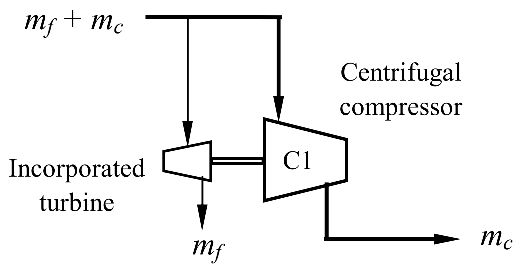

Figure (2.4) Representation of the centrifugal compressor and its incorporated turbine

Here *LHV* represents the quantity of energy released by mass unity of the gas during complete combustion. It is considered at 25ºC and 1 bar in (kJ/kg) and is calculated from the mass lower heating values, LHV_i of the molecules composing the gas:

$$LHV = \frac{\sum y_i M_i LHV_i}{\sum y_i M_i} \tag{2.31}$$

HHV (high heating value) is not introduced here as the released water after the combustion reactions is assumed to be in gaseous state. Feasible operating domain of a single compressor is constituted using the inequalities. The compressors within the compressor station are modelled using compressor map-based polynomial equations. Normalized head, $h_i/\omega^2$and normalized flow rate, Q_a/ω, are used to describe the performance map of the compressors (Odem, 1990). Q_a is volumetric flow rate expressed in m^3/s at suction side and ω is rotational speed in rpm. The set of polynomial equations identify using constant coefficients. If the compressor's driver allows, the compressor speed can be varied to control the pressure ratio. Applying standard polynomial curve-fit procedures for each compressor, the normalized head can thus be obtained under the form of the following equation (Abbaspour et al., 2005).

$$\frac{h_i}{\omega^2} = b_1 + b_2 \frac{Q_s}{\omega} + b_3 (\frac{Q_s}{\omega})^2 \tag{2.32}$$

As well, contours of constant isentropic efficiency could be fitted in the polynomial form of second degree shown in Equation (2.33):

$$\eta_i = b_4 + b_5 \frac{Q_s}{\omega} + b_6 (\frac{Q_s}{\omega})^2 \tag{2.33}$$

The rotation speed of all compressors is comprised between lower and upper boundaries as represented below. The lower limit on flow is marked by surge or pumping phenomenon that is an unsteady flow condition marked by increased noise and flow reversal through the machine. To prevent from surge phenomenon, by considering surge margin, λ_{surge}, the following constraint is introduced (Odom, 1990).

$$\omega_l \leq \omega \leq \omega_u \tag{2.34}$$

$$\lambda_{surge} \leq \frac{Q_s \quad Q_{surge}}{Q_s} \tag{2.35}$$

There is a surge flow rate, Q_{surge}, corresponding to each compressor rotational speed (see Figure (2.3). The line joining the surge points at different speeds gives the surge line. The surge line will be sketched using the following equation (Pugnet, 1999):

$$Q_{surge} = b_7 \left(\left(\frac{Z_s R T_s}{M {p_s}^2} \frac{\kappa - 1}{\kappa} h_{surge} + \left(\frac{Z_s R T_s}{p_s M} \right)^2 \right)^{\frac{\kappa}{\kappa-1}} - \left(\frac{Z_s R T_s}{p_s M} \right)^2 \right)^{1/2} \tag{2.36}$$

In this equation, h_{surge} is the surge head at specified compressor speed and can be calculated using following equation:

$$\frac{h_{surge}}{\omega^2} = b_1 + b_2 \frac{Q_{surge}}{\omega} + b_3 \left(\frac{Q_{surge}}{\omega} \right)^2 \tag{2.37}$$

Considering a fixed value for the surge pseudo efficiency, it will be introduced as a parameter during optimization procedure. Previous equation represents a nonlinear correlation between surge flow rate and rotational speed of the compressor.

The right portions of the head-flow characteristics curves drop because of choking. Choking phenomenon which occurs at high flow rates also limits the compressor's operating range. At a given speed, the upper limit on flow is set by stall in the inlet, diffuser or impeller passages. To avoid chocking occurrence at inlet, the following inequality should be considered.

$$Q_s \le A_s c_s \left[\frac{2}{\kappa + 1} \right]^{\frac{\kappa+1}{2(\kappa-1)}} \tag{2.38}$$

In this inequality, A_s is cross sectional area and c_s is gas sonic velocity at the compressor inlet. Another inequality is introduced corresponding to the protection of a compressor against chocking phenomenon in impeller passages as shown in Inequality (2.39). In this expression, impeller radius, r in m and A, flow rate area in m^2 are considered at the section of rotating passages as well as Q, Z, T, p. The index 1 indicates impeller inlet state.

$$Q \le \frac{ZRT}{pM} \rho_1 c_1 A \left[\frac{2 + (\kappa - 1)(r\omega)^2 / {c_{01}}^2}{\kappa + 1} \right]^{\frac{\kappa+1}{2(\kappa-1)}} \tag{2.39}$$

To stay away from diffuser choking, another inequality similar to that of the compressor inlet is considered, but as shown below, in this relation the gas properties are in the conditions of the diffuser and index 2 is used for diffuser inlet. The derivations of these three latter inequalities are presented in literature (Gorla and Khan, 2003). It is clear that the conditions at diffuser inlet are dependent on the impeller process. Thus:

$$Q_f \leq \frac{Z_f R T_f}{p_f M} \rho_{02} c_{02} A_f \left[\frac{2}{\kappa + 1} \right]^{\frac{\kappa+1}{2(\kappa-1)}} \quad (2.40)$$

2.3. Incidence matrices

The different links between the elementary sections of a network can be defined using incidence matrices. So, all the relation between the variables of the system such as the material balances at steady state around the nodes of a pipeline network can be expressed under the very concise forms by using different types of incidence matrices such as the arc-node matrix (Suming et al., 2000).

In the model, each pipe, each compressor and each fuel stream are represented by an arc. Consider a network with *n* nodes, *l* pipe arcs and *m* compressor arcs. Therefore, there will be *m* fuel streams since for each compressor unit there is a stream that carries fuel to it. Because in a compressor, compression process is carried out, a compressor unit can be named an active arc. In this way, a pipe segment, in where the pressure decreases, may be called a passive arc. Let us note that the fuel streams have been considered as inert arcs regarding pressure change through them. A flow direction is assigned preliminarily to each pipe that can be or not coincide with the real flow direction of the gas that running through the arc.

Let A be a matrix with the dimension of $n \times (l + m)$, to each of its elements, a_{ij}, is given the following attribution:

$$a_{ij} = \begin{cases} 1 & \text{if arc j comes out from node i} \\ -1 & \text{if arc j goes into node i} \\ 0 & \text{otherwise} \end{cases}$$

A is called the node-arc incidence matrix and make easier to describe the material balance around the nodes. Similarly, let B be another matrix with the dimension of $l \times m$ whose elements, b_{ij}, are defined below and is named the pipe-compressor incidence matrix:

$$b_{ij} = \begin{cases} 1 & \text{if pipe i is connected to discharge node of compressor j} \\ -1 & \text{if pipe i is connected to suction node of compressor j} \\ 0 & \text{otherwise} \end{cases}$$

The third defined matrix is the node-fuel incidence matrix which describes the existing fuel stream derivations from a node. The dimension of this matrix is $n \times m$ and its elements are defined like below:

$$c_{ij} = \begin{cases} 1 & \text{if fuel stream i be derived from node j} \\ 0 & \text{otherwise} \end{cases}$$

The last considered incidence matrix is the compressor-fuel matrix. This matrix is also defined as previously, indicating which fuel stream belongs to which compressor.

A network is shown in Figure (2.5) where gas flow direction is considered from node N1 to node N4 thereby the arcs direction will be obtained. They coincide with real directions in this example if there is no gas sink or source in the nodes excepting node N1 and N4 which is the case in the treatment of the networks in the following chapters. This network has $n = 4$ nodes, $l = 2$ pipe arcs and $m = 1$ compressor arc as well as $m = 1$ stream carrying fuel to incorporated turbine of the compressor. All the nodes, pipes and compressor units have been labelled in Figure (2.5). The corresponding matrices to this example are shown:

$$A = \begin{array}{c} \begin{matrix} G1 & G2 & C1 \end{matrix} \\ \begin{bmatrix} +1 & 0 & 0 \\ -1 & 0 & +1 \\ 0 & +1 & -1 \\ 0 & -1 & 0 \end{bmatrix} \end{array} \begin{matrix} N1 \\ N2 \\ N3 \\ N4 \end{matrix} \qquad B = \begin{array}{c} C1 \\ \begin{bmatrix} -1 \\ +1 \end{bmatrix} \end{array} \begin{matrix} G1 \\ G2 \end{matrix} \qquad C = \begin{array}{c} F1 \\ \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \end{array} \begin{matrix} N1 \\ N2 \\ N3 \\ N4 \end{matrix}$$

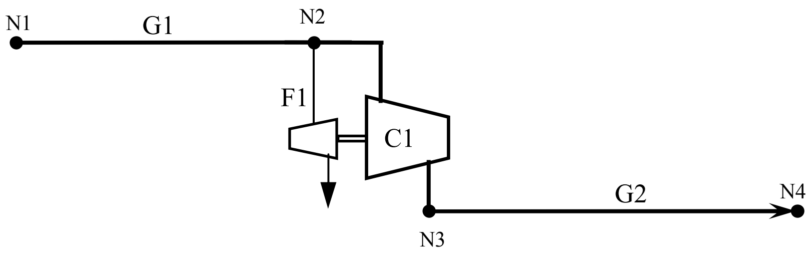

Figure (2.5) Demonstration of a simple network comprising a compressor unit situated between two pipelines and its corresponding incidence matrices

For the various optimization problems treated in the thesis and presented below, the applied equations can be summarized as follows:

- Estimation of the required properties of gas such as LHV as a function of its composition and its components.
- Material balances around all the nodes.
- For each pipe segment:
    - Equation of movement.
    - Calculation of MAOP if it is not given in advance.
    - Velocity estimation and its upper bound, that is erosional velocity.
    - Finding the average gas density as a function of the average pressure.
    - Average compressibility factor estimation.
- For each compressor unit:
    - Isentropic head and isentropic efficiency calculations.
    - Power consumption estimation and consequently fuel consumption rate.
    - Surge and pumping limits considerations via appropriate inequalities.
    - Calculation of the conditions of the gas such as density in suction side.

Because the majority of the variables are in vector form, they are linked together during computer programming using the incidence matrices. For example the material balance arround the node i is expressed as Equation (2.41). In this equation S_i represents the gas delivery or supply relative to this node. It is negative if the node is delivery node and positive for a supply node where the gas is injected to the node.

$$\sum_{j \in arcs} a_{i,j} m_j + \sum_{j \in compressors} b_{i,j} m_{f\,j} = S_i \tag{2.41}$$

Let us note at that level that the number of variables and constraints depends on the number of nodes and arcs which vary according to the size of the network. This aspect will be explained in detail in the applications that will be tackled in the following chapters.

2.4. Optimization problems

As previously presented, a gas transmission network is usually made up of conduits and compressor stations. Whereas the conduits cause the pressure drop in flowing gas, a compressor station allows its pressure increasing to provide the sufficient delivery pressures. The hydraulics of the pipes, the constraints related to compressors and the fuel gas consumption of the compressors are described by nonlinear functions. Besides, a lot of constraints are imposed in order to guarantee satisfying operating conditions (MAOP, critical gas velocity). Please note at that level that additional constraints may be taken into account for special mixtures, for instance hydrogen-natural gas.

In general, two types of optimization problems are of interest in pipeline networks: design optimization and operation optimization. In design optimization, the goal is to optimize the pipeline design by selecting appropriate layouts, equipment, etc. In operation optimization, the network and station configuration is given and the goal is to operate the pipeline network in an optimal manner.

2.4.1. Optimization problem classification

The following classification of optimization problems is based on the process engineering point of view. The search for one or several solution(s) to these problems requires different techniques which range from standard methods for linear programming to nonlinear optimization and even mixed-integer nonlinear programming techniques.

2.4.1.1. Optimal configuration of the gas transmission networks

Most of the time, the objective is the optimal design for the pipeline projects by selecting the appropriate devices and equipment units according to a technico-economic criterion.
More precisely, gas pipeline design involves: determination of the optimal number of compressor stations and their locations and design; selection of the optimal pipe diameter and maximum allowable operating pressure. The following design variables need to be determined: number of compressor stations, compressor station locations, lengths of pipeline segments between compressor stations, diameters of pipeline segments, and suction/discharge pressures at each compressor station.
The objective generally taken into account may be minimization of investment cost of compressors and pipes or maximization of the net present value of the studied project.

2.4.1.2. Finding the optimal operating conditions of gas networks

The management of gas transmission networks involves a lot of decisions which may be difficult to take, particularly because of the great number of action variables. These problems arise when the operating conditions and the parameters related to the exploitation of the gas pipeline infrastructures must be set. In this problem, different technical or economic criteria, which may be difficult to formulate must be taken into account such as dynamical feasibility, fuel consumption minimization on the network, maximization of flow rates or pressures in particular points such as delivery terminals, minimization of output gas temperature of compressors or carbon dioxide emission minimization. They can be considered either as the constraints or the objective function of the optimization process. The choice of an objective function as well as its evaluation has a very important influence on the algorithm process and on the quality of the final solution.

One main concern of this work is to find the optimal operating conditions of gas networks with an imposed gas structure. The operating conditions that can be found are mentioned as the optimization variables.

In what follows, we present some optimization problems which are particularly interesting for gas transmission pipelines and which may be relevant to both classes of optimization problems.

2.4.2. Fuel consumption minimization

Literature analysis reveals that in a gas transmission network, the overall operating cost of the system is highly dependent upon the operating cost of the compressor stations in a network. A compressor station's operating cost, however, is generally measured by the fuel consumed at the compressor station. The operating cost of running the compressor stations represents between 25% and 50% of the total company's operating budget (Luongo et al., 1989). Hence, the objective for a transmission network is to minimize the total fuel consumption of the compressor stations while satisfying specified delivery flow rates and minimum pressure requirements at the delivery terminals.

Reduction of the energy used in pipeline operations will not only have a tremendous economical impact, but nevertheless an environmental one: the more efficient, the use of compressor stations is, the less greenhouse emissions are dissipated in the atmosphere.

The objective function of the problem is the summation of the fuel consumption mass flow rates over all the drivers of the compressor units existing in the stations of the network. Total fuel consumption, m_{ft} is determined by the following equation:

$$m_{f_t} = \sum m_{f_i} \qquad i \in E_c \tag{2.42}$$

Here, the optimization is performed for steady-state. The following inequality or equality constraints dictate this problem (optimization constraints):

(1) mass balance around all nodes

(2) gas flow equations through each pipe section

(3) equipment pressure limit consideration

(4) gas velocity limits in conduits

(5) operational constraints on compressor units such as surge and chocking phenomena

(6) the rate of demand and supply of gas at end points and their pressures.

While the mass balance equations are linear, the majority of other equations and inequalities, as it was shown in the previous sections, are nonlinear. In defining the action variables, it is important to choose only those variables whose values can be directly controlled while operating the actual network; otherwise, a solution impossible to implement may be found. At the same time, the set of variables must be large enough to provide a reasonably extensive research space so that there is more scope for optimization. With these considerations in mind, we have selected the compressor speeds to be the action variable in this formulation. It will be referred as the compressor speed adjustment problem for an existing network.

For each pipe segment, a variable describing the gas flow rate is introduced as well as for each compressor unit. Pressure variables describing the pressure of the gas in each node, are considered.

From flow rates and pressure variables, dependent variables can be computed: compressors fuel gas consumption of compressors, whose summation is to be minimized, average gas velocity, gas density, gas compressibility factor, sonic velocity and erosional velocity at each pipe segment, average pressure across each of them and isentropic head, isentropic efficiency, rotational speed of the compressor units and volumetric flow rate at their suction sides.

The friction factor for pipe segments, as shown in Equation (2.12), is not computed from the Reynolds number, but is considered fixed since the diameter of pipes as well as their roughness are fixed parameters of the problem.

Gas composition, temperature, geometrical dimensions of the network such as pipe diameters and compressor surge margin are other parameters. In addition, there are the transport and thermodynamic properties of gas that will be calculated in the model formulation. In Table (2.1) these parameters are reported generally accompanied with their order of magnitude.

Parameter	*Order of magnitude*	*unit*
Gas molecular weight	18 – 25	g/mol
Gas heating value at standard conditions	35 – 50	MJ/m^3
Gas critical pressure	45 – 50	bar
Critical temperature	200 – 250	K
Gas heat capacity at constant pressure	35 – 45	J/(mol.K)
Gas isentropic exponent	1.2 – 1.4	-
Specified Minimum Yield Strength	2000 – 5000	bar
Design factor	0.4 – 0.7	-
Seam joint factor	0.6 – 1.0	-
Temperature deration factor	0.85 – 1.0	-
Pipeline internal roughness	50 – 100	µm
Network temperature	260 – 315	K
Compressor mechanical efficiency	80 – 98	%
Compressor driver efficiency	25 – 45	%

Table (2.1) Parameters and their order of magnitude

Generally, NLP-type problems are involved when all the variables are continuous but in some cases, the application of an MINLP formulation may be more interesting.

Typically, the situation may become more complex if the flow directions can not easily be deduced from the network structure, for instance when some loops are involved. In that case, the problem can be formulated with binary variables to interpret flow directions, thus leading to an MINLP (Mixed Integer Nonlinear Programming) problem.

A parallel problem appears for design purpose: in other words, the location of the necessary compressor station and their size as well as the number of compressor units in each station can be determined in order to minimize the amount of fuel used in their drivers.

Here the challenge is situated at the investment phase of a gas pipeline project. At that stage, the involved variables involve the diameter and length of the pipes and may be considered as discrete variables. In addition, the variables defining the compressor size can also be considered, such as compressor isentropic heads and compressor capacities.

In what follows, the problems involve the same variables but differ from the objective function considered.

2.4.3. Fuel cost minimization

A related problem to fuel consumption minimization is minimization of consumed fuel cost. It should be mentioned that the variable operating costs of a pipeline network are dominated by the cost of the gas transport that is the fuel consumption of compressors. The objective function of the problem is the sum of the fuel costs over all the compressor stations in the network. The formulation of the fuel cost objective function is more complex than that of fuel consumption because the cost of fuel usage may not change linearly related to the amount of fuel. Furthermore, there may be start-up costs which have to be taken into account in this case.

2.4.4. Throughput Maximization

The maximum amount of gas that can be transported by the network is determined while satisfying supply, delivery and transport obligations and other physical constraints. From an energetic viewpoint, the power that is transmitted by the pipeline, P_p (in Watt) can be considered as an objective function to be maximized: the transmitted power of a pipeline is a linear function of its throughput, m_p in (kg/s), via Equation (2.43) when the composition of the gas is constant during the optimization procedure:

$$P_p = m_p LHV \tag{2.43}$$

This type of problem is met usually in the domain of intrastate or interstate pipeline transmission projects to determine maximum capacity of the concerned pipeline.

2.4.5. Delivery gas maximization

The maximum amount of gas that can be transported to the clients is determined while satisfying supply, delivery and transport obligations and other physical constraints. The flow rate of delivery gas at one or (several) considered node(s) can be taken as an objective function. This can also be expressed in the form of constraints in the optimization procedure.

2.4.6. Profit maximization

Another related problem is the determination of the optimal quantities of supply gases and deliveries in order to maximize profit while satisfying physical constraints.

2.5. Outline of the thesis objectives

Finally, it must be pointed out that the problems presented in the classification are strongly interconnected:

- For instance, the design problem also involves the determination of the operating conditions of the main components of the networks.
- The determination of the operating conditions of an existing pipeline infrastructure may also involve the determination of the compressors which are actually in operation for a given gas transportation problem.

In this work, one problem belonging to each class will be tackled:

- The former is related to the improvement of an existing pipeline network with fuel cost minimization problem as an objective function : it will be the core of chapter 3 of this manuscript.

- The latter concerns an investment cost problem and is devoted to the determination of a pipeline infrastructure (number and location of compressor stations, pipeline diameter, flow rate in pipelines and pressures at nodes): in chapter 4 will present the main results and analysis of this investigation.

A more prospective study devoted to the case of hydrogen addition to existing natural gas infrastructure will be presented in chapter 5. More precisely, the conditions under which hydrogen can be added to natural gas with acceptable performance of end-user appliances will be studied.

In this kind of problem, the fraction of molecular hydrogen in gas mixture is introduced as an additional variable. It must be noted that the addition of hydrogen to the natural gas strongly modifies its transport and thermodynamic properties.

The previous presentation has shown that all the previous problems are formulated as optimization problems. This approach is quite common in the chemical engineering community, namely within the scope of Process Systems Engineering. The problems may be relevant to Nonlinear Programming (NLP) or Mixed Integer Nonlinear Programming (MINLP). A deterministic approach was selected to treat the previous problems using the GAMS environment which is identified as a standard in the chemical engineering discipline. The modules which will be used in this work are briefly described in what follows.

2.6. GAMS as a programming environment for optimization purpose

The optimization procedures implemented in this work use the classical modules such as CONOPT and SBB within the General Algebraic Modelling System (GAMS) environment. GAMS has been designed upon the principles drawn from relational database theory and mathematical programming and has combined these ideas to satisfy the needs of the modellers (Brooke et al., 2004). Relational database theory provides a structured framework for developing general data organization and transformation capabilities. Mathematical programming provides a way of describing a problem and the methods to solve it. GAMS creates models that will be solved with the optimization modules which should be an LP, NLP or MINLP … etc. algorithm. To handle MINLP (Mixed Integer Nonlinear Programming) problems, GAMS environment proposes modules such as SBB (Standard Branch and Bound) and DICOPT (Discrete and Continuous Optimizer).

2.6.1. Nonlinear programming and CONOPT algorithm

Several equality and inequality expressions presented in the previous sections establish that the constraints which govern the model are nonlinear. Thus, the associated optimization problem is solved by nonlinear programming (NLP) methods. Therefore, the problem takes the standard form presented below:

Determine optimization variable $\mathbf{x}^T = [\, x_1 , x_2 , \ldots , x_r \,]^T$
In order to minimize the objective function $f(\mathbf{x})$
Subject to the constraints $g_n(\mathbf{x}) = 0 \quad , n = 1 , 2 , \ldots$
$h_m(\mathbf{x}) \leq 0 \quad , m = 1 , 2 , \ldots$

CONOPT is based on GRG (Generalized Reduced Gradients) algorithm first suggested by Abadie and Carpentier (1969). The GRG algorithm used by CONOPT is a feasible path algorithm. CONOPT is specifically designed for large nonlinear programming problems. Details on the algorithm can be found in literature (Drud, 1985 and 1992). To improve the rate of convergence, CONOPT tries to estimate second order information in the form of an estimated reduced Hessian using the BFGS formula. When the model continues to appear linear CONOPT uses Sequential Linear Programming (SLP) technique to find a search direction. The SLP procedure is only used to generate good directions; the usual feasibility preserving steps in CONOPT are maintained, so CONOPT is still a feasible path method. The second order information is used in a Sequential Quadratic Programming (SQP) procedure that much like the SLP procedure finds a good search direction when progress is determined by nonlinearities. CONOPT allows using a Steepest Edge Algorithm (SEA) as an alternative for certain models during linear mode iterations. The initial experience has been shown that the SEA will give less number of iterations for most nonlinear models (Drud, 2004).

CONOPT is compatible with models with very nonlinear constraints, which is the case in this thesis.

2.6.2. The Branch and Bound Algorithm

SBB is a GAMS solver based on a combination of the standard Branch and Bound method known from Mixed Integer Linear Programming (MILP) and some of the standard NLP solvers already supported by GAMS such as CONOPT and SNOPT (Sparse Nonlinear Optimizer). SBB supports all the types of discrete variables including binary, integer and some other specific ones (GAMS, 2004).

The Relaxed Mixed Integer Nonlinear Programming (RMINLP) model is initially solved using the starting point. A RMINLP model can contain both discrete variables and general nonlinear terms but the discrete requirements are relaxed. In other words, the integer and binary variables can assume any values between their bounds thus an RMINLP model will be just as a NLP problem. If all discrete variables in the RMINLP model are integer, SBB will return this solution as the optimal integer solution. Otherwise, the current solution is stored and the Branch and Bound procedure will start.

During the Branch and Bound process, the feasible region for the discrete variables is subdivided, and bounds on discrete variables are tightened to new integer values to cut off the current non-integer solutions. Each time a bound is tightened, a new tighter NLP sub-model is solved starting from the optimal solution to the previous looser sub-model. The objective function values from the NLP sub-model is assumed to be lower bounds on the objective in the restricted feasible space (assuming minimization), even though the local optimum found by the NLP solver may not be a global optimum.

2.6.3. Overview of DICOPT

DICOPT solver is based on the extensions of the Outer Approximation algorithm for the equality relaxation strategy (Grossmann et al., 2004). The algorithm in DICOPT is based on another key idea that is Augmented Penalty (AP). It is a program for the MINLP problems that involve linear binary or integer variables and linear and nonlinear continuous variables. The MINLP algorithm inside DICOPT solves a series of NLP and MIP sub-problems which can be solved using any NLP or MIP (Mixed Integer Programming) solver that runs under GAMS.

An equality constraint can be relaxed to be an inequality constraint. This property is used in the MIP master problem to accumulate linear approximations. Augmented Penalty refers to the introduction of non-negative slack variables on the right hand sides of the just described inequality constraints and the modification of the objective function when assumptions concerning convexity do not hold (Grossmann et al., 2004).

2.7. Conclusions

This chapter was devoted to the presentation of the problem formulation involved in gas pipeline modelling, that typically takes into account the use of mathematical optimization methods:

- **Modeling equations and problem constraints**
  - ✓ Mass and transport equations ;
  - ✓ Problem constraints such as the Maximum admissible operational pressure to satisfy;
  - ✓ Compressor modelling through characteristic curves;
  - ✓ Incidence matrices to model the interconnection between the network components (arcs-nodes).

- **Objective functions**

Various objectives can be considered according to the problem class, either improvement of operating conditions of a gas network or network design.

In the following chapters, two types of objective functions will be selected for illustration purpose:

- ✓ fuel consumption minimization;
- ✓ total annualized cost minimization

- **Optimization methods and tools**

Since the problems involved are formulated as either nonlinear or mixed integer nonlinear optimization problems, GAMS environment which is well-known as a high-level modelling system for mathematical programming and optimization has been selected in the following chapters to describe the pipeline models.

Chapters 3 and 4 are based on this framework to treat the various problems that the pipelines managers or designers have to cope with.

Using the same methodology, a prospective study is carried out in chapter 5, to determine a feasibility study on the impact of hydrogen addition in existing natural gas infrastructures.

Chapter 3

Improving the performance of pipeline networks: Fuel consumption minimization problems

As previously shown in Chapter 2, one of the most important elements of natural gas pipeline stations is the compressor station. Thus, efficient operation of compressor stations is of major importance for enhancing the performance of the pipeline network. This chapter is devoted to the presentation of a systematic approach for optimizing the performance of compressor stations using a mathematical programming approach.

Of course, any possible objective functions can be used to define our sense of optimality of the network operation, including minimization of fuel consumption, minimization of emissions, minimization of maximum pressure, for instance, etc. Any of these can be employed in the method described in this paper, but for the examples treated in this chapter, the total fuel consumption was selected as the objective function.

Indeed, as the gas industry has developed, gas pipeline networks have evolved over decades into very large and complex systems. A typical network today might consist of thousands of pipes, dozens of stations, and many other devices, such as valves and regulators. Inside each

station, there can be several groups of compressor units of various vintages that were installed as the capacity of the system expanded. The compressor stations typically consume about 3 to 5% of the transported gas (Suming et al. 2000). It is estimated that the global optimization of operations can save considerably the fuel consumed by the stations. Hence, the problem of minimizing fuel cost is of tremendous importance.
Besides, reduction of the energy used in pipeline operations will not only have a beneficial economical impact but also an environmental one: the more efficient the use of compressors stations is the less greenhouse emissions are dissipated in the atmosphere. It is estimated that about 70% of greenhouse gas emissions from natural gas occur when natural gas is burned to produce heat or energy. Pipelines emit CO_2 mainly due to energy used at compressor stations. Combustion of natural gas generates mostly carbon dioxide (CO_2) and water vapour. For each megawatt (MW) of energy produced (35% thermal efficiency) 5 tonnes of CO_2 are generated per year (Mora and Ulieru, 2005).
Consequently, the objective is to operate a given compressor station or a set of compressor stations so that the total fuel consumption is reduced while maintaining the desired throughput in the line. This problem is identified in the following part as compressor steady-state adjustment problem. Let us note, at this level, that several requirements are imposed to the network, for instance, pressures at the endpoints of the network. The constraints imposing that the compressors always work inside their feasible region were previously described in the second section of Chapter 2.

Two case studies illustrate the methodology and are presented in the following sections.

- Case study 1 was chosen for its simple and small-size design, developed for the sake of illustration. The implementation of the methodology is thoroughly presented and typical results are analyzed.
- Case study 2 was provided by Gaz de France. It is a more complex network containing several loops, supply nodes and delivery points, referred as a multi-supply multi-delivery transmission network. The key points of implementation of an optimization framework are presented.

The treatment of both case studies provides some guidelines for optimization of the operating performances of pipeline networks, according to the complexity of the involved problem.

3.1. Compressor adjustment problem

The first example is a didactic one inspired from a work within literature (Abbasspour et al., 2005) (see Figure 3.1), but is enough representative of the elements that may take place in natural gas transport. The network consisted of three long pipelines of 100 kilometers. There are two compressor stations that operate to compensate for pressure drop in the pipelines. Each compressor station includes three parallel centrifugal compressors. In each station, there are six short pipe segments of about a hundred meters linked to the entrances and outlets of the compressors. Although the length and the diameter of these pipes is lower than those of the three major pipelines, their role in the pressure change through the network may not be negligible and may even sometimes become bottleneck of the system. Therefore, these pipelines are also considered in the model. The technical features of the pipeline system corresponding to Figure (3.1), considered as fixed parameters for the optimization problem, are proposed in Table (3.1).

Pipeline tag	G1	G2	G3	G4	G5
Diameter (m)	0.787	0.889	0.330	0.381	0.330
Length (m)	1E+5	1E+5	200	300	100
Pipeline tag	G6	G7	G8	G9	G10
Diameter (m)	0.330	0.330	0.330	0.381	0.330
Length (m)	200	100	200	100	100
Pipeline tag	G11	G12	G13	G14	G15
Diameter (m)	0.432	0.330	0.330	0.330	0.838
Length (m)	100	100	400	100	1E+5

Table (3.1) Technical features of the pipelines of the system shown in Figure (3.1)

The network includes18 nodes, 15 pipes arcs and 6 compressor arcs. As for each compressor unit, there is a stream that carries fuel to it; there are 6 fuel streams which have not been shown in Figure (3.1) to avoid complexity. For each compressor, this stream originates from suction node. Node 0 is a supply node and gas flows from this node towards node 17. There is no input or output in the other nodes. A flow direction is assigned to each pipe so the gas flows from 0 to 17. Yet, this preliminary direction corresponds evidently with the real flow direction of the gas running through the arc.

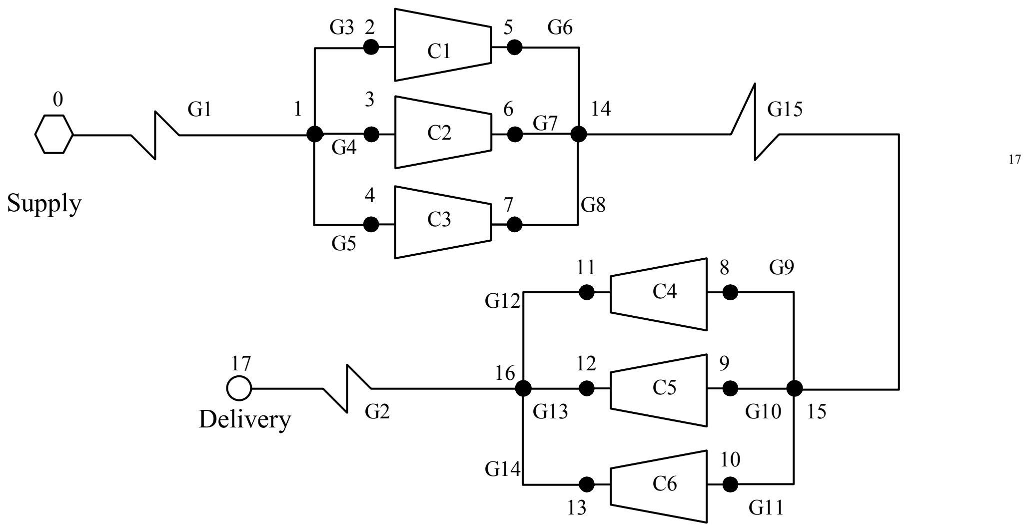

Figure (3.1) Schema of the considered pipeline network in case study 1

The node-arc incidence matrix, named *A* (see Figure 3.2) is a matrix with a dimension of 18 × (15+6). The material balance around the nodes can be stated in a very concise way by using this kind of matrices.

Pipe-compressor incidence matrix, *B*, has a dimension of 15 × 6 and is shown in Figure 3.3. It is important to note that this matrix is not independent of matrix *A* but is introduced directly on this example formulation for simplicity reasons.

$A =$

Arc:	G1	G2	G3	G4	G5	G6	G7	G8	G9	G10	G11	G12	G13	G14	G15	C1	C2	C3	C4	C5	C6	*Node*:
	+1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	−1	0	+1	+1	+1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
	0	0	−1	0	0	0	0	0	0	0	0	0	0	0	0	+1	0	0	0	0	0	2
	0	0	0	−1	0	0	0	0	0	0	0	0	0	0	0	0	+1	0	0	0	0	3
	0	0	0	0	−1	0	0	0	0	0	0	0	0	0	0	0	0	+1	0	0	0	4
	0	0	0	0	0	+1	0	0	0	0	0	0	0	0	0	−1	0	0	0	0	0	5
	0	0	0	0	0	0	+1	0	0	0	0	0	0	0	0	0	−1	0	0	0	0	6
	0	0	0	0	0	0	0	+1	0	0	0	0	0	0	0	0	0	−1	0	0	0	7
	0	0	0	0	0	0	0	0	−1	0	0	0	0	0	0	0	0	0	+1	0	0	8
	0	0	0	0	0	0	0	0	0	−1	0	0	0	0	0	0	0	0	0	+1	0	9
	0	0	0	0	0	0	0	0	0	0	−1	0	0	0	0	0	0	0	0	0	+1	10
	0	0	0	0	0	0	0	0	0	0	0	+1	0	0	0	0	0	0	−1	0	0	11
	0	0	0	0	0	0	0	0	0	0	0	0	+1	0	0	0	0	0	0	−1	0	12
	0	0	0	0	0	0	0	0	0	0	0	0	0	+1	0	0	0	0	0	0	−1	13
	0	0	0	0	0	−1	−1	−1	0	0	0	0	0	0	+1	0	0	0	0	0	0	14
	0	0	0	0	0	0	0	0	+1	+1	+1	0	0	0	−1	0	0	0	0	0	0	15
	0	+1	0	0	0	0	0	0	0	0	0	−1	−1	−1	0	0	0	0	0	0	0	16
	0	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17

Figure (3.2) Node-arc incidence matrix A

A compressor-fuel matrix, C is also considered (see Figure 3.4) in order to identify which fuel stream belongs to a compressor. It is worthwhile mentioning that the eliminated lines corresponding to nodes, from which no fuel stream exists, are not represented since all their components are equal to zero.

As already mentioned, we consider here operation optimization only. The optimal operation of a network involves meeting all performance requirements while minimizing an objective function that is fuel consumption.

The formulation of the optimization problem is carried out in four steps. First, we define the operational variables whose values are to be determined by the optimization process. Next, we specify the objective function whose value is to be minimized. Then, we formulate the necessary constraints to ensure that the optimal solution found is meaningful. Finally, the parameters of the problem are presented. Each of these steps is discussed in brief below.

$$
\begin{array}{c}
 \\ \\ \\ \\ \\ \\ \\ B = \\ \\ \\ \\ \\ \\ \\ \\
\end{array}
\begin{array}{c}
\begin{array}{cccccc} C1 & C2 & C3 & C4 & C5 & C6 \end{array} \\
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 \\
+1 & 0 & 0 & 0 & 0 & 0 \\
0 & +1 & 0 & 0 & 0 & 0 \\
0 & 0 & +1 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & +1 & 0 & 0 \\
0 & 0 & 0 & 0 & +1 & 0 \\
0 & 0 & 0 & 0 & 0 & +1 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\end{array}
\quad
\begin{array}{c}
\textit{Node:} \\ G1 \\ G2 \\ G3 \\ G4 \\ G5 \\ G6 \\ G7 \\ G8 \\ G9 \\ G10 \\ G11 \\ G12 \\ G13 \\ G14 \\ G15
\end{array}
$$

Figure (3.3) Pipe compressor incidence matrix B

Fuel stream corresponding to the compressor: *Node*:

$$
C = \begin{array}{c}
\begin{array}{cccccc} C1 & C2 & C3 & C4 & C5 & C6 \end{array} \\
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\end{array}
\quad
\begin{array}{c}
2 \\ 3 \\ 4 \\ 8 \\ 9 \\ 10
\end{array}
$$

Figure (3.4) Compressor-fuel incidence matrix C

<u>*Optimization variables*</u>: the basic continuous variables of this problem are: 18 pressure variables governing the nodes and 21 flow rate variables (including fuel streams) corresponding to pipes and compressors. The pressures at nodes 0 and 17 are not prefixed but they have a thin domain of variation. So practically, there are 16 pressure variables.

<u>*Objective function*</u>: Total sum of the fuel consumption in compressors is the objective function as expressed in Equation (3.1) analytically. For each compressor, fuel consumption flow rate, m_{fi}, is obtained using Equation (2.30).

$$f_{obj} = \sum_{i \in compressors} m_{f\,i} \tag{3.1}$$

<u>*Constraints*</u>: obviously there are two kinds of constraints consisting of:

- *Equality constraints*: 18 mass balances around nodes, 15 equations of motion for the pipe arcs, 6 isentropic head equations for compressors as shown in Equation (2.29), 6 relationships between rotational speed, suction volumetric flow rate and head of each compressor as Equation (2.32), 6 equations to calculate isentropic efficiency according to Equation (2.33), 6 equations to determine fuel consumption at each compressor unit.

- *Inequality constraints*: a lower bound for delivery flow rate (flow rate in arc G2) equal to 150 kg/s, an upper bound as well as a lower bound for the pressures of the nodes: MAOP as an upper bound and atmosphere pressure as a lower bound, sonic velocity and erosional velocity in the role of upper bounds of the velocities through pipes, lower and upper boundaries on the rotation speed of all compressors (166.7 and 250 rpm respectively), a lower bound on compressor throughput taken in account to avoid pumping phenomenon, an upper bound on compressor throughput to prevent from chocking phenomenon.

Parameters: The pressure is considered to be equal to 60 bars with a margin of ±2% at the entrance point of the network, node 0, as well as the delivery pressure, at node 17 (in other words the lower bound is 58.8 bar and the upper one is 61.2 bar). The typical composition of natural gas considered in the numerical runs is presented in Table (3.2) along with thermodynamic properties of gas components. Let us recall that while natural gas is mainly methane, it can also include ethane, propane, butane and pentane. It is generally accepted that the composition of natural gas can vary widely from lean (70% methane) to rich gas (approximately 90% methane).

Component	*Methane*	*Ethane*	*Propane*
Mole percent	70	25	5
Molecular weight	16.04	30.07	44.1
Critical temperature, K	190.6	305.4	369.8
Critical pressure, bar	46	48.8	42.5
Lower Heating Value, kJ/kg	50009	47794	46357
Heat capacity at constant pressure, kJ/(kmol.K)	35.663	52.848	74.916

Table (3.2) Parameters of the optimization problem related to the components of gas flowing in the pipeline of case study 1

These values are introduced in the formulation as additional parameters of the problem. Dimensions of the network pipelines presented in Table (3.1) also constitute the parameters of

the optimization problem. The incidence matrixes can be assimilated to the structure parameter of the network. Roughness of inner surface of the pipes is considered to be equal to 46×10^{-6} (traditional value reported for stainless steel). The temperature is assumed to be isothermal and equal to 330 K all over the system. Mechanical efficiency and driver efficiency for the compressors are assumed roughly to be 0.90 and 0.35 respectively according to values proposed in the dedicated literature (Menon, 2005).
The coefficients of the isentropic head equation in addition to the coefficients of the isentropic efficiency equation of the compressors are shown in Table (3.3).

Coefficient:	b_1	b_2	b_3	b_4	b_5	b_6
Value:	3.8113×10^{-4}	3.849×10^{-6}	-6.3985×10^{-9}	17.269	0.3237	-4.1789×10^{-6}
Unit:	m^2	m^{-1}	m^{-4}	-	m^{-3}	m^{-6}

Table (3.3) Coefficients of the isentropic head equation and coefficients of the isentropic efficiency equation of the compressors

The total number of variables in this optimization problem is 39. The 6 rotational speeds of the compressors have not been explicitly considered as variables, since the flow rates of the fuel streams have been already considered as variables and for each compressor this latter is directly dependent on its rotational speed as shown in Equation (2.30). In total, there are 57 equality constraints and 76 inequality constraints.

Initialization point: The initialization of the variables is performed directly through the software (GAMS/CONOPT) under the condition that the problem is well-scaled and that bounds are assigned adequately. Yet, in this problem, the search for an initial point is a little difficult, because the variables are linked together implicitly within the strongly nonlinear constraints.

In order to show the interest of the optimization process, another preliminary optimization problem is solved via GAMS/CONOPT to produce an initial point for the main problem. In this problem, the isentropic efficiency of the compressor C6 is maximized under the same conditions explained above. The whole results for this problem are not presented but some significant ones are presented below in Table (3.4).

Total fuel consumption rate	0.863 kg/sec
Isentropic efficiency at the compressor C6	79.954
Rotational speed of the compressor C6	166.7 round/sec
Pressure subjected to the node 0	60.988 bar
Pressure subjected to the node 17	61.200 bar

Table (3.4) Initial value corresponding to some variables obtained using an auxiliary optimization problem

Results: Once the network operation problem has been formulated as an optimization problem as outlined above, it was solved using GAMS environment. Since the problem is nonlinear, the CONOPT solver has been chosen. The resolution takes less than 1s CPU on a PC which is quite acceptable.

Table (3.5) presents the results relative to pressure computation at each node. Observe that at node 0 (i.e., supply node), the algorithm has taken the maximum possible pressure (61.2 bar) whereas it has taken the minimum possible value (58.8 bar) at node 17 (i.e., delivery node).

Node	*Pressure (bar)*	*Node*	*Pressure (bar)*	*Node*	*Pressure (bar)*
0	**61.200**	6	66.919	12	65.510
1	47.359	7	67.030	13	65.186
2	47.042	8	58.324	14	66.809
3	47.122	9	58.260	15	58.386
4	47.192	10	58.354	16	65.072
5	67.018	11	65.185	17	**58.800**

Table (3.5) Pressure of natural gas at all of the nodes of the pipeline network

The value of objective function that is the total fuel consumption in the compressor stations is equal to 0.750 kg/s (sum of individual compressor consumptions, see Table 3.7) which represents a significant reduction of 15% from the initial solution (0.863 kg/s) which may represent a viable solution for the practitioner. The values of the optimal flow rates through pipelines are presented in Table (3.6).

Arc	*Flow rate (kg/s)*	*Arc*	*Flow rate (kg/s)*	*Arc*	*Flow rate (kg/s)*
G1	150.750	G6	49.186	G11	50.343
G2	150	G7	50.450	G12	50.200
G3	49.367	G8	50.559	G13	49.521
G4	50.637	G9	50.264	G14	50.279
G5	50.746	G10	49.587	G15	150.195

Table (3.6) Optimal values of the flow rate for each pipeline

The optimum percentage of the input gas that is consumed in the stations can thus be calculated and is found equal to 0.497 %.

Additional information concerning compressor operating conditions can be deduced from pressure and flow rate optimal values (see Table 3.7): discharge flow rate, rotational speed, consumption ratio, isentropic head, isentropic efficiency and individual fuel consumption of course. For each compressor, consumption ratio is defined as the fuel consumption divided by the input mass flow rate.

Compressor	C1	C2	C3	C4	C5	C6
Discharge flow rate (kg/s)	49.186	50.450	50.559	50.200	49.521	50.279
Rotational speed (rpm)	244.348	246.482	246.558	166.7	166.7	166.7
Fuel consumption (kg/s)	**0.182**	**0.186**	**0.187**	**0.064**	**0.066**	**0.064**
Consumption ratio (%)	0.369	0.367	0.369	0.127	0.133	0.127
Isentropic head (kJ/kg)	42.592	42.188	42.201	12.664	13.367	12.607
Isentropic efficiency (%)	74.917	74.215	74.207	64.195	65.331	64.101

Table (3.7) Optimal values of discharge flow rate, rotational speed, fuel consumption, isentropic head and isentropic efficiency for the compressor units of the network

Let us mention in this example that compressors involved in the second station work at their minimum rotational speeds whereas the compressors of the first station work close to their maximum speeds.

Finally, the transmitted power of the pipeline, that is the product of the pipeline delivery throughput (150 kg/sec) and the lower heating value (LHV) of the natural gas (48830 kJ/kg) is calculated equal to 7324 MW at this optimum point.

We may, therefore, conclude that in this example, optimization is a viable alternative to find adequate operating conditions for pipeline network and this low-size example provides very encouraging results. The methodology is now applied to larger networks, to illustrate the capability of the approach to treat real problems involving a great number of variables.

3.2. Multi-supply multi-delivery transmission network

Case study 2 was provided by Gaz de France and is inspired from real data. The multi-supply multi-delivery transmission network is presented schematically in Figure (3.5). The combinatorial aspect of this example is more complex than case study 1, namely because of the existence of three loops and seven compressor stations.

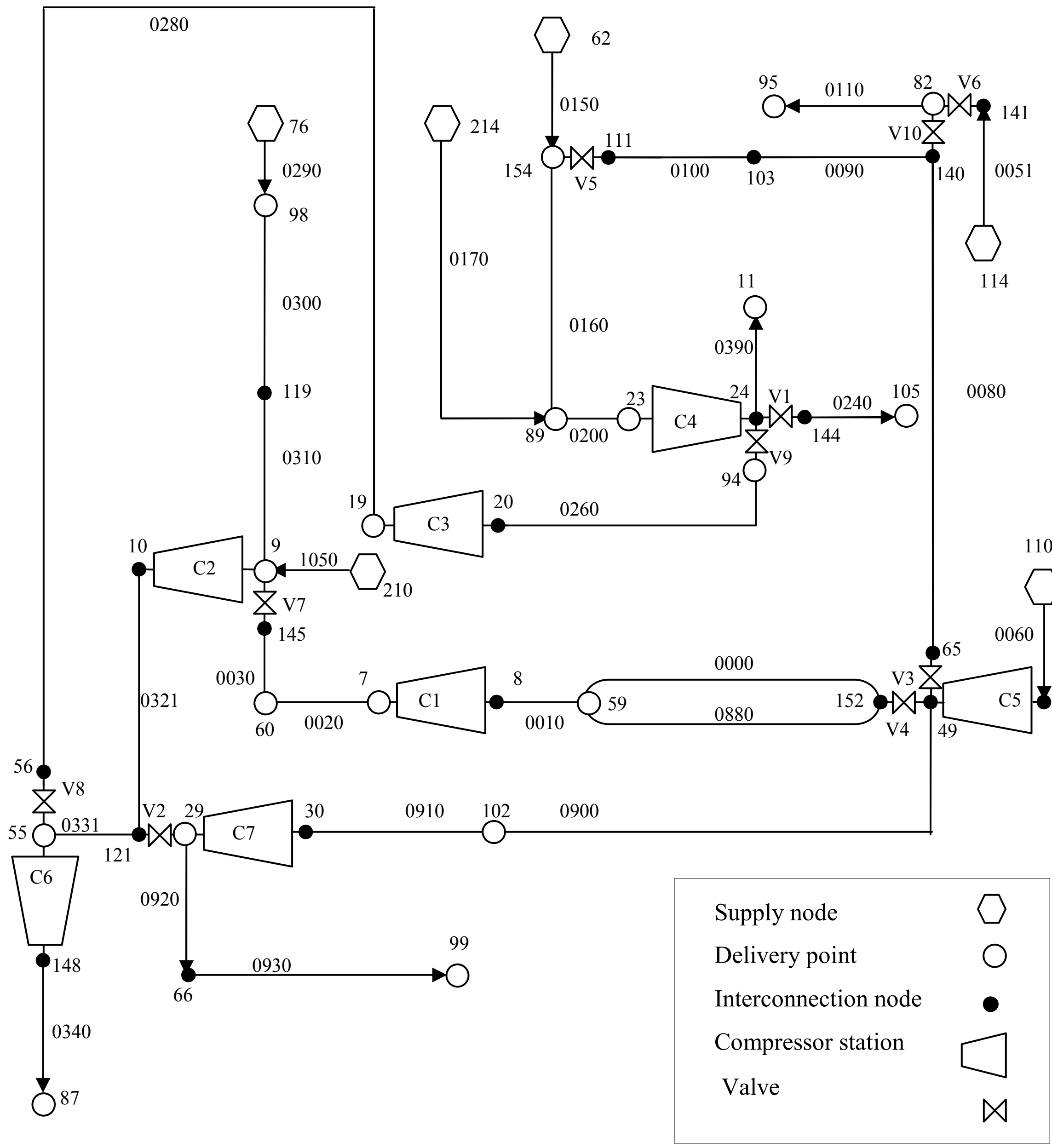

Figure (3.5) Representation of the multi-supply multi-delivery pipeline network (by courtesy of Gaz de France)

3.2.1. Network characteristics

Natural gas flowing through the network is assumed to contain 91 percent methane and 9 percent ethane. Its properties are summarized in Table (3.8). Once more, isothermal conditions are considered in the optimization framework.

Higher heating value (HLV) in kJ/m^3	4.18×10^4
Specific gravity in relation to air	0,6
Gas temperature in K	278.15
Heat capacity ratio	1.309
Assumed composition	*methane* : 91% , *ethane* : 9%

Table (3.8) Characteristics of natural gas in case study 2

The system is composed of 19 delivery points at which gas comes out from the network. These delivery points are symbolized by small empty circles as shown in the figure legend. Gas can be supplied from six points symbolized by hexagons. Moreover, 20 intermediate nodes are considered to provide necessary interconnections or in some cases to specify explicitly some changes in design parameters: for example, node 103 is considered to introduce the effect of diameter change between arcs 0100 and 0090 in the model. For the same purpose, node 66 is introduced because the pressure limits are different at the endpoints of its neighboring arcs. Globally, the network consists of 45 nodes and 30 pipe arcs. In addition, seven compressors are located to compensate for pressure losses through the network. Ten valves permit to break the pressure between some pairs of points in order to balance the network. In some cases, a valve or more can be positioned after a compressor to regulate the output pressure of two or more streams that originate from discharge side of the compressor.

The node-arc incidence matrix corresponding to this network is written similarly to the previous example. This matrix has a dimension of 45 × (30+7+10) and is used in the problem formulation step. Since it involves too many terms and follows the same principles as already presented, it will not be presented exhaustively. Table (3.9) that maps well the network structure is used for illustration purposes. Let us precise that term $a_{i,j}$ is equal to (+1) if downstream node of arc *j* is node *i* and is equal (-1) if its upstream node is node *i*. Upstream and downstream assignments have been proposed by implementation of a rough free-hand material balance calculation around the nodes with flow rate assumptions in the level of a few arcs.

Arc	*Downstream node*	*Upstream node*	*Arc*	*Downstream node*	*Upstream node*	*Arc*	*Downstream node*	*Upstream node*
0000	152	59	0280	19	56	C3	20	19
0010	59	8	0290	76	98	C4	23	24
0020	7	60	0300	98	119	C5	50	49
0030	60	145	0310	119	9	C6	55	148
0051	114	141	0321	10	121	C7	30	29
0060	110	50	0331	121	55	V1	24	144
0080	65	140	0340	148	87	V2	29	121
0110	82	95	0390	24	11	V3	49	65
0090	140	103	0880	152	59	V4	49	152
0100	103	111	0900	49	102	V5	111	154
0150	62	154	0910	102	30	V6	141	82
0160	154	89	0920	29	66	V7	145	9
0200	89	23	0930	66	99	V8	56	55
0170	214	89	1050	210	9	V9	24	948
0240	144	105	C1	8	7	V10	82	140
0260	94	20	C2	9	10			

Table (3.9) Downstream and upstream node of arcs from node-arc incidence matrix

Pressure bounds that have to be respected for each node are presented in Table (3.10). As shown by a preliminary sensitivity analysis, these data have an important effect on the optimization process and on its final result. The pressure of the supply node 114 should be considered invariable at 85 bars prefixed already by gas supplier. Therefore its lower bound like its upper bound is fixed at this pressure.

Node	*Pressure lower bound, bar*	*Pressure upper bound, bar*	*Node*	*Pressure lower bound, bar*	*Pressure upper bound, bar*
29	40	86	105	40	56,8
30	40	86	110	40	67
62	40	49	114	85	85
66	40	86	119	40	81
76	40	81	141	60	86
98	40	81	144	40	56,8
99	61	86	*Others*	40	68.7

Table (3.10) Bounds on node pressures

The principal characteristics of pipe arcs including their length, internal diameter and inner surface roughness as well as maximum admissible operating pressure for each of them are displayed in Table (3.11). As already mentioned in chapter 2, MAOP is a function of diameter and wall thickness of the pipes as well as of material construction type. Pipe segments are normally designed using different type of steels depending on the treated fluid, availability, economic limitations, etc.

Pipe arc	*Length* (*km*)	*Diameter* (*mm*)	*MAOP* (*bar*)	*Roughness* (μm)
0000	64.1	754	68	20
0010	101.6	688	68	20
0020	80.4	681	68	10
0030	27.1	617	68	10
0051	172.699	1090	85	10
0060	4.9	1167	68	10
0080	122.2	1069	68	10
0090	41.6	1069	68	10
0100	28.4	1054	68	10
0110	81.3	895	68	10
0150	21.6	874	68	10
0160	14.2	954	68	10
0170	46.8	595	68	10
0200	43.3	948	68	10
0240	27.9	588	56.8	10
0260	95.701	744	68	10
0280	119.715	744	68	10
0290	4.9	892	80	10
0300	30.9	1167	80	10
0310	53.4	892	80	10
0321	54.5	892	68	10
0331	77	892	68	10
0340	89.0	794	68	10
0390	63.9	493	68	20
0880	64.1	994	68	10
0900	204.5	994	68	10
0910	36.2	994	68	10
0920	125.8	891	85	10
0930	67.7	891	85	10
1050	0.001	1000	68.7	10

Table (3.11) Principal characteristics of pipe arcs

Gas consumption flow rates at delivery nodes are set at the values proposed in Table (3.12). These amounts should be guaranteed for the different customers. From the point of view of flow rates, the most important delivery nodes are 99, 89, 95 and 9 respectively. Besides, from pressure standpoint, a more elevated pressure value must be respected at node 99 according to its pressure lower bound that has been given in Table (3.10).

Node	*Delivery gas, kg/s*	*Node*	*Delivery gas, kg/s*
7	16.15	87	42.064
9	119.988	89	146.964
11	42.596	94	19.987
19	20.436	95	126.622
23	36.824	98	73.574
29	6.866	99	172.76
55	41.693	102	0.393
59	63.628	105	23.011
60	59.507	154	75.678
82	62.274	*Total*	1 151.014

Table (3.12) Amount of gas consumptions delivered by the pipeline network

Delivery gases are provided from six supply locations situated on the network. The maximum gas flow rate demands from each supply node are presented in Table (3.13). During the optimization process, these amounts serve as an upper limit of the supply flow rate at the corresponding nodes. As previously seen, the principal supply nodes are nodes 110 and 114. If design or operational constraints allow it, these two nodes are able to satisfy 75 percent of gas demands.

Node	*Maximum rate, kg/s*	*Node*	*Maximum rate, kg/s*
62	78.406	114	400.564
76	190.786	210	53.377
110	474.331	214	68.652

Table (3.13) Maximum rate of the gas provided from the supply nodes

The valves permit to break the pressure between two points of the network in order to inter a gas to a point whose pressure is below than gas pressure or to prevent the gas flowing in an undesirable direction. These pieces of equipment are sometimes oriented after a compressor station. The valve characteristics are given by the maximum flow rate that can go through them. This capacity must be respected for each valve in the model. Here, the maximum volumetric flow rate is 10 million cubic meter per hour for all of them.

Considering compressors, some constraints exist related to their maximum capacity, maximum admissible operating pressure, maximum pressure ratio and maximum power consumption (see Table 3.14). The adiabatic efficiency and the product of mechanical and driver efficiency of the stations in percentage are assumed to be equal to 75 and 35 respectively.

Compressor	*MAOP, bar*	*Capacity, standard m^3/hr*	*Maximum pressure ratio*	*Maximum power, MW*
1	80	5.60×10^5	1.35	3.9
2	80	1.75×10^6	1.50	17.6
3	80	7.50×10^5	1.34	5.3
4	80	9.60×10^5	1.39	9.4
5	80	2.40×10^6	1.46	34
6	80	1.36×10^6	1.56	14
7	86	9.50×10^5	1.90	22

Table (3.14) Principal characteristics of the compressors

3.2.2. Problem formulation with mixed integer nonlinear programming

First, to tackle the problem, flow directions for each pipe arc as well as for each compressor and valve arc are not imposed in advance. Let us note that some arc directions are considered to be imposed without ambiguity during the optimization process. They consist of the following elements: pipes 0051, 0060, 0110, 0150, 0170, 0240, 0290, 0340, 0390, 0920, 0930, 1050, compressors C5 and C6, valves V1 and V6 as shown by the arrows in Figure (3.5). In order to solve the problem via an MINLP model, for each arc, a binary variable, *d*, is assigned to identify its flow direction. This binary can be 0 or 1: when it is equal to 0, the gas flows in the arc opposite to the preliminary direction which is the direction conform to the corresponding node-arc incidence matrix.

The mixed integer nonlinear problem is formulated as below using the relationships presented in chapter two:

Determine optimization variables $\mathbf{x}^T = [\, x_1 , x_2 , \ldots , x_r \,]^T$ *and* $\mathbf{y}^T = [\, y_1 , y_2 , \ldots , y_r \,]^T$

In order to minimize the objective function $f(\mathbf{x} , \mathbf{y})$

Subject to the constraints:

$g_n(\mathbf{x}) = 0 \qquad n = 1 , 2 , \ldots , 60$

$h_m(\mathbf{x}) \leq 0 \qquad m = 1 , 2 , \ldots , 236$

$p_k(\mathbf{y}) = 0 \qquad k = 1 , 2 , \ldots , 12$

$Q_l(\mathbf{x} , \mathbf{y}) = 0 \qquad l = 1 , 2 , \ldots , 89$

$S_t(\mathbf{x} , \mathbf{y}) \leq 0 \qquad t = 1 , 2 , \ldots , 10$

$\mathbf{x}^T$ is the set of continuous variables existing in the model and $\mathbf{y}^T$ is the set of binary variables. The average pressure at each pipeline is obtained using an expression in the form of $g_n(\mathbf{x}) = 0$ as the average gas velocities. The inequalities that introduce the erosional velocities for each pipe, are in the form of $h_m(\mathbf{x}) \leq 0$ as well as the lower and upper bounds on different variables without considering that all the variables except for the supply/delivery flow rate at nodes, are already inferiorly bounded at zero by assuming them as positive. Note that some arc directions are necessarily similar, for instance arcs 0090 and 0100, so these related constraints are introduced in the formulations via simple 12 linear expressions as $p_k(y) = 0$. There are 45 material balances around nodes, 30 equations of motion for pipes, 7 equations to express compressor pressure ratios and 7 other equations to calculate fuel rate consumption in them in the form of $Q(\mathbf{x} , \mathbf{y}) = 0$ (altogether 99). For each valve, there is a relationship in the form of $S_t(\mathbf{x}, \mathbf{y}) \leq 0$.

The constraint related to a valve is considered as a linear inequality satisfying just that the downstream pressure is lower than or equal to the upstream pressure:

$$\sum_i p_i (2d_j - 1) a_{ij} \geq 0 \quad j \in Valves \quad i \in Nodes \qquad (3.1)$$

In this equation p_i refers to pressure, d_j is a binary to define flow direction through valve and a_{ij}'s are the components of the node-arc incidence matrix A corresponding to the associated valve. This equation is only used for MINLP formulation. For NLP case, flow directions are imposed in advance and the coefficients ($2d_j$-1) are always equal to one.

Using SBB solver within GAMS modelling system, a program module for the optimization problem has been developed. The optimum results are dependent on the initial assumed values of flow directions in arcs, probably due to the presence of local optima.

Optimization variables: the main continuous variables are pressures at nodes and flow rates corresponding to pipes, valves and compressors in addition to the gas injection flow rates at supply nodes. They all take positive values. There are 44 pressures at nodes without taking into account the pressure subjected to the node 114 that is fixed at 85 bar. The total number of flow rates corresponding to arcs and the total number of the supply flow rate variables are 47 and 6 respectively. As aforementioned, a total number of 47 binary variables are introduced for flow direction of each arc. Due to the problem definition, fixed flow directions are assumed for some arcs, (for instance arc 0051 where the gas flows always from the supply node 114 towards the node 141), so that the number of *influencing* binary variables finally reduces to 31.

Constraints: there are equality constraints that are:

- mass balance around nodes
- equation of motion for each pipe arc
- fuel consumption calculation at each compressor.

Moreover, there are inequality constraints that are:

- upper limits for supply gases
- limits on the gas pressure subjected to different nodes and arcs
- erosional velocity as an upper limit for the average gas velocity through pipes
- maximum value for pressure ratio as well as power consumption of each compressor
- maximum throughput for valves.

Altogether, there are 31 binary variables and 97 continuous variables. The number of equality constraints as well as inequalities has been explained above.

Objective function: the minimization of the totality of gas consumption in compressor stations is considered as the optimization criterion. As for the previous example, each compressor station uses a low amount of the gas that passes through it. So the optimization process is performed to minimize these consumptions.

The objective function follows the same formulation as previously mentioned for the small-size example. It must be pointed out that no penalty term related to the complexity of the network is introduced.

Initialization procedure:

Mainly due to the complexity (for instance, nonlinearities) involved in the MINLP models, good initial values and bounds are essential in order to achieve convergence.

The selected initialization scheme lies on the generation of a set of binary variables provided by the user whereas the continuous variables are automatically initialized by the solver because they are well - bounded and scaled.

Five initializations configurations of binary variables the algorithm successfully referred as case 1 to 5 were selected:

1. In case 1, all binary variables are initially chosen equal to one, except that of pipeline 0030. Consequently, the binary variable corresponding to valve V7 are also equal to zero.
2. In case 2, the independent binary with a zero value is only that of pipeline 0010 and its associated compressor C1.
3. In case 3, a zero value is attributed to the binary value corresponding to pipeline 0280 and to the related valve V8.
4. In case 4, a zero value is assigned to the binary variables related to pipelines 0010, 0030, 0260 and 0321. Consequently, in this case, the binaries related to compressors C1, C2, C3 and the valve V7 are also equal to zero.
5. Finally, in the most dispersed situation that is case 5, the binaries corresponding to pipelines 0010, 0030, 0160, 0260, 0321, 0910 and that of compressor C4 are set initially at zero, which also leads to the same value for dependent binaries related to compressors C1, C2, C3 and C7 as well as to valve V7.

Table (3.15) presents a summary of the different initialization cases.

Case 1	0030	V7			
Case 2	0010	C1			
Case 3	0280	V8			
Case 4	0010 C2	0030 C3	0260 V7	0321	C1
Case 5	0010 0910 C7	0030 C4 V7	0160 C1	0260 C2	0321 C3

Table (3.15) The arcs whose binary variables are given zero in the initial solution

<u>*Results*</u>: The MINLP algorithm was run with the different initialization strategies using the SBB solver within GAMS.

It was observed that, with these initial guess schemes, a solution is also obtained. The same structure is always obtained (Compressors C4 and C5) but two sets of operating conditions were found.

In order to guarantee gas flow in the network without violation of any constraint, the results show that compressors C4 and C7 work with different power consumptions. The values of the different variables related to these two stations are shown in Table (3.16) for all optimal points. It has been verified that none of the values presented in this table hits one of its bound. Note that the other compressors are bypassed.

Case	*Compressor*	*Pressure ratio*	*Input pressure Bar*	*Throughput Kg/s*	*Power consumption kW*	*Fuel consumption kg/s*
	C4	1.102	44.180	139.186	6090	0.111
1, 4	C7	1.262	57.155	179.946	17587	0.320
					Total (objective function):	0.431
	C4	1.065	45.680	106.03	3013	0.055
2	C7	1.262	57.170	179.945	17567	0.320
					Total (objective function):	0.375
	C4	1.062	45.822	102.239	2761	0.050
3	C7	1.262	57.177	179.945	17557	0.320
					Total (objective function):	0.370
	C4	1.102	44.179	139.295	6010	0.109
5	C7	1.263	57.156	179.946	17586	0.320
					Total (objective function):	0.430

Table (3.16) 0perating conditions of the compressors in work obtained by MINLP using different initialization configuration in the level of binary variables corresponding to flow direction in arcs

More precisely, for cases 1, 4 or 5, the optimum structure of the studied gas network exhibits a lot of similarities with the assumed flow directions considered in Table (3.9) except that of pipeline 0030 and valve V7.

Besides, in these study cases, no gas flows through pipeline 0030, valves V2 and V7 whereas valve V5 is totally open. Let us note that only for initialization cases 1 and 4, valve V8 is also totally open. This result is obtained by comparing the end-point pressures of each valve.

Using case 2 as an initialization scheme, it was found that all binary variables are equal to unity which means that all flow directions are the same as those are given in Table (3.9). Yet in the optimal structure, there is no gas flow in pipeline 00280 as well as through valves V2 and V8. In addition, valves V3, V9 and V10 are totally open.

In case 3, flow directions in the optimal solution are in agreement with those of Table (3.9) except for pipeline 0280 and valve V8. Here, the calculation gives a zero value for gas flow rate through valve V2 at optimal state and valves V4, V5, V7, V8 and V10 are totally open.

The best solution for the problem was obtained using case 3 as an initialization guess: the value of objective function is 0.370 kg/sec, which is very near from the value obtained for case 2.

Node	*Pressure (bar)*	*Node*	*Pressure (bar)*	*Node*	*Pressure (bar)*
7	53.364	59	65.562	105	40
8	53.364	60	45.041	110	67
9	44.275	62	49	111	48.961
10	44.275	65	54.71	114	85
11	40	66	65.178	119	46.501
19	40.976	76	47.332	121	42.949
20	40.976	82	54.324	140	54.324
23	45.822	87	40	141	69.17
24	48.671	89	47.619	144	40.441
29	72.175	94	41.206	145	44.275
30	57.177	95	51.053	148	40.991
49	66.596	98	46.813	152	66.596
50	66.596	99	61	154	48.961
55	40.991	102	58.705	210	44.275
56	40.991	103	51.311	214	52.452

Table (3.17) Pressure at diverse nodes of the transmission network according to the best optimum solution obtained by MINLP method (Case 3)

The computed pressures at all nodes for the best solution are presented in Table (3.17). Having a look at Table (3.10), it is observed that the pressures at delivery nodes 11, 99 and 105 are obtained at their lower bounds. In the same time, the pressures of supply nodes 62 and 110 are calculated to be at their upper limits: with this operating mode, the number of compressors is minimized and the compressors that are in operation consume smaller amounts of energy. Consequently, the value of objective function is reduced.

Table (3.18) presents the flow rates of each arc for the best optimum solution (case 3) where valve 2 is closed.

Arc	*Flow rate (kg/s)*	*Arc*	*Flow rate (kg/s)*	*Arc*	*Flow rate (kg/s)*
0000	55.758	0280	3.791	C3	16.645
0010	112.604	0290	190.786	C4	102.239
0020	96.454	0300	117.212	C5	413.802
0030	36.947	0310	117.212	C6	42.064
0051	400.564	0321	87.548	C7	179.945
0060	413.802	0331	87.548	V1	23.011
0080	57.233	0340	42.064	V2	0
0090	268.902	0390	42.596	V3	57.233
0100	268.902	0880	120.473	V4	176.231
0110	126.622	0900	180.338	V5	268.902
0150	24.202	0910	179.945	V6	400.564
0160	217.426	0920	172.76	V7	36.947
0170	68.652	0930	172.76	V8	3.791
0200	139.113	1050	53.377	V9	36.632
0240	23.011	C1	112.604	V10	211.669
0260	16.645	C2	87.548		

Table (3.18) Flow rate through pipes, compressors and valves obtained by MINLP (Case 3)

Flow directions in arcs for the best optimum point are outlined in Figure (3.6a) with the associated mass balances on the different components of the network. In addition, pressures subjected to nodes for this point are reported in Figure (3.6b). The simplified sketch of this optimum solution also is outlined in Figure (3.6c). Comparing end-point pressures of each valve, it can be deduced that valves 1, 3, 6 and 9 are only partially open. Only these valves are displayed in Figure (3.6c).

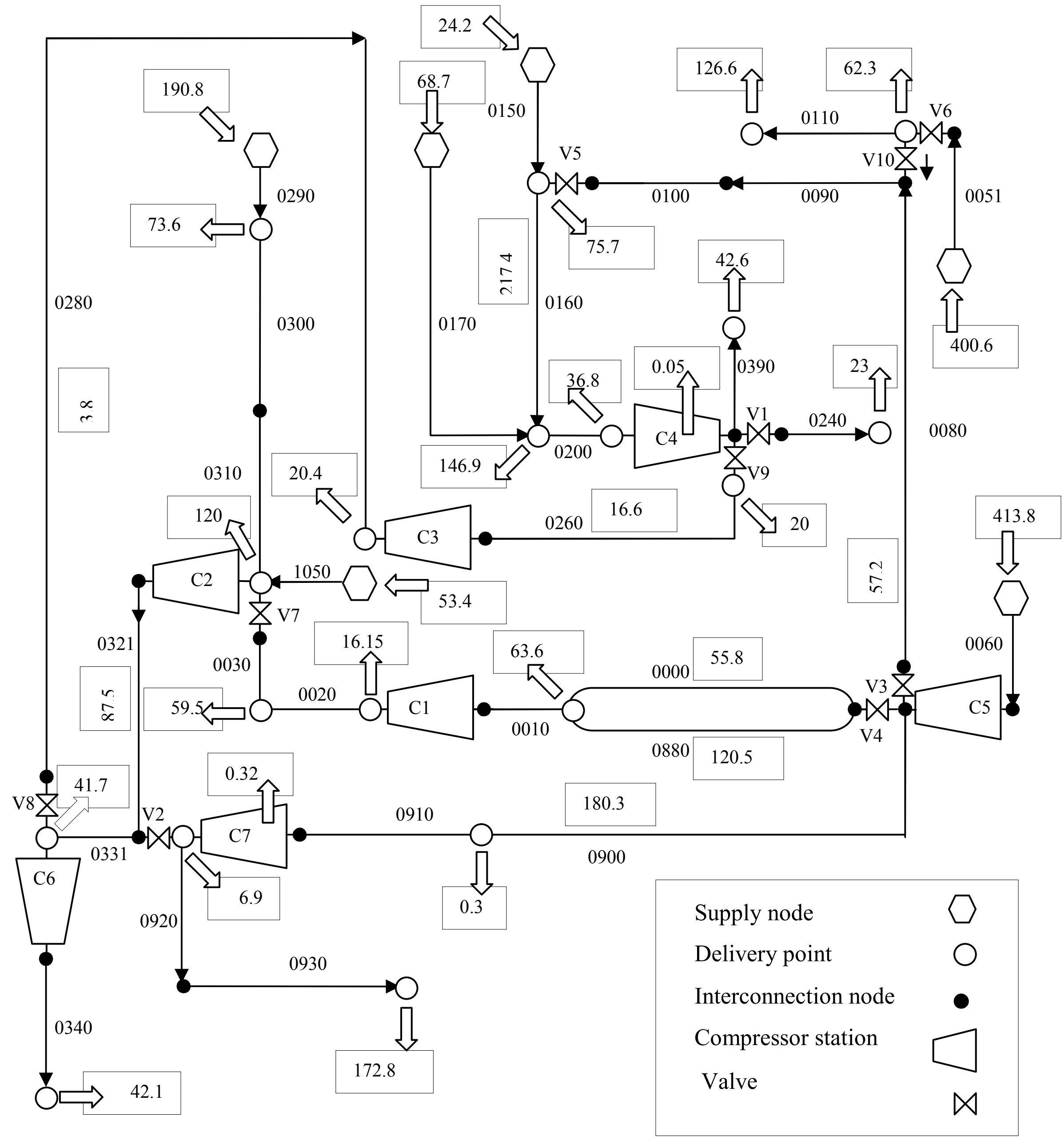

Figure (3.6a) Flow sheet of the best solution obtained by MINLP with material balance (Case 3)

- Note: flow rates are reported in the cubes

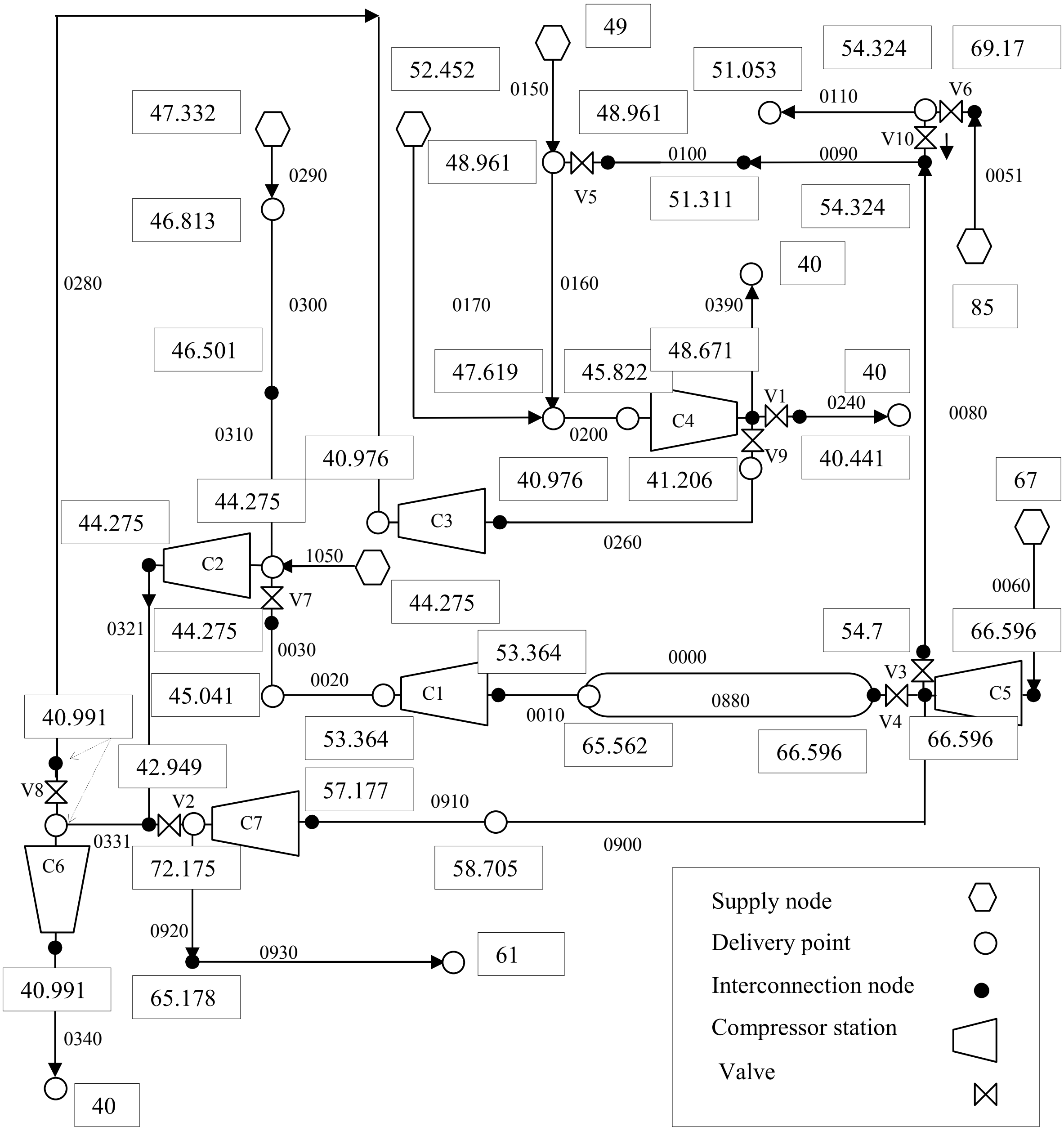

Figure (3.6b) Flow sheet of the best solution obtained by MINLP with pressures (Case 3)

- Note: pressures are reported in the cubes

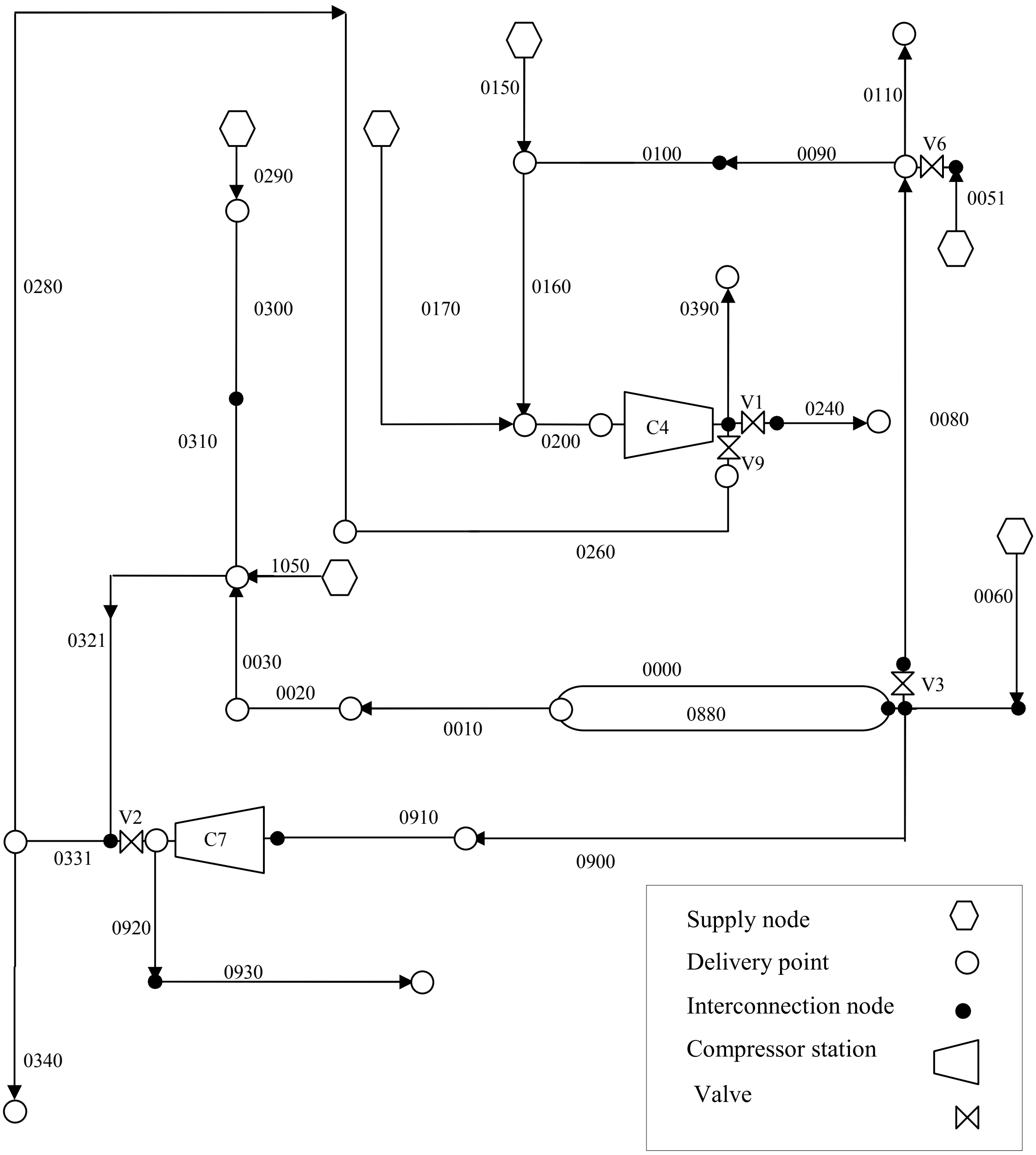

Figure (3.6c) Simplified sketch of the best solution by MINLP (Case 3)

Note that all valves are presented in Figures (3.6a) and (3.6b), however they must be completely open or close. In addition, the bypassed compressors for which pressure ratios are equal to one are shown in these latter figure as well; but there is no gas consumption in them. Comparing the division of end-point pressures corresponding to each compressor and their pressure ratios shown in Table (3.16), it is observed that these two values are in agreement for all compressors.

Finally, Table (3.19) presents the flow rates corresponding to each supply node. Comparing these operational supply rates with Table (3.13), notice that all nodes are used with their maximum supply rates except for nodes 62 and 110. Besides, having a look at Table (3.12), it is observed that the totality of the supply gases is equal to the summation of the delivery gases and gas consumption in compressor stations.

Node	*Flow rate, kg/s*	*Node*	*Flow rate, kg/s*
62	24.202	114	400.564
76	190.786	210	53.377
110	413.802	214	68.652
		Total :	1151.383

Table (3.19) Mass flow rate of the gas provided from the supply nodes obtained by MINLP

Another interesting result concerns the average gas velocity that is obtained by performing related calculations for the different pipelines. They range between 1 to 8 m/sec, which means that it is far enough from the upper limit given by erosional velocity. The latter also are obtained during the optimization process because they are dependent on gas pressure according Equation (2.26). Their values at optimum point are located between 14 and 21 m/sec. For check purpose, the Reynolds numbers were also calculated roughly and exhibit an order of magnitude of 10^{+7} which confirms the regime of turbulence for gas flow in pipelines.

3.2.3. Nonlinear programming with imposed flow directions

In this section, with the previous MINLP results in mind, flow directions for each pipe arc as well as for each compressor and valve arc are imposed in advance. The flow directions in these arcs are shown using arrow symbols in Figure (3.6). Other directions are assumed as presented in Table (3.9).

The nonlinear problem involving only continuous variables (NLP) is formulated as below using the relationships presented in Chapter 2. The flow directions through arcs are predetermined.

Determine optimization variables $\mathbf{x}^T = [\, x_1 , x_2 , \ldots , x_r \,]^T$
In order to minimize the objective function $f(\mathbf{x})$
Subject to the constraints

$g_n(\mathbf{x}) = 0 \qquad n = 1 , 2 , \ldots , 149$

$h_m(\mathbf{x}) \leq 0 \qquad m = 1 , 2 , \ldots , 246$

$\mathbf{x}^T$ is the set of continuous variables existing in the model. Average pressure and average gas velocities calculations at pipes, material balances around nodes, equations of motion for pipes, equations to express compressor pressure ratios and equations to calculate their fuel rate consumption are obtained using an expression in the general form of $g_n(\mathbf{x}) = 0$. The inequalities that introduce the erosional velocities for each pipe and the inequality related to each valve are in the general form of $h_m(\mathbf{x}) \leq 0$ as well as the lower and upper bounds.

Initialization: in this case, the continuous variables are automatically initialized between their bounds by the solver because they are well bounded and scaled.

Results: The results obtained with the NLP code are presented in Table (3.20). They also lead to an optimum solution in which only the compressors C4 and C7 work. Not surprisingly, it is found that the other compressors are bypassed because their pressure ratio is equal to unity. The values of their corresponding variables are also shown in Table (3.20) and none of them is at its boundary.

Compressor	*Pressure ratio*	*Input pressure Bar*	*Throughput kg/s*	*Isentropic power consumption, kW*	*Fuel consumption kg/s*
C4	1.066	45.674	106.03	3019	0.055
C7	1.262	57.172	179.625	18442	0.336
				Total (objective function):	**0.391**

Table (3.20) Operating conditions of the compressors in work obtained by NLP

As for the MINLP strategy, the results concerning pressure and flow rates networks are presented in detail. The computed pressures at all nodes are presented in Table (3.21). Having a look at Table (3.9), it is observed that the pressures at delivery nodes 11, 99 and 105 are obtained at their lower bounds. In the same time, the pressures of supply nodes 62 and 110 are calculated to be at their upper limits.

Node	*Pressure (bar)*	*Node*	*Pressure (bar)*	*Node*	*Pressure (bar)*
7	52.828	59	64.415	105	40
8	52.828	60	45.104	110	67
9	44.489	62	49	111	48.965
10	44.489	65	54.972	114	85
11	40	66	65.178	119	46.704
19	41.513	76	47.531	121	43.284
20	41.513	82	54.518	140	54.518
23	45.674	87	40.536	141	69.17
24	48.671	89	47.575	144	40.441
29	72.175	94	41.853	145	44.489
30	57.172	95	51.261	148	41.513
49	66.594	98	47.014	152	65.427
50	66.594	99	61	154	48.965
55	41.513	102	58.701	210	44.489
56	41.513	103	51.402	214	52.413

Table (3.21) Pressure at diverse nodes of the transmission network obtained by NLP

In Table (3.22), the flow rates corresponding to each pipeline, compressor and valve are shown. The valves 2 and 8 are closed since their throughputs are equal to zero.
According to the values given in Table (3.21), the valves 1, 3, 4, 6 and 9 are partially open and the valves 5, 7 and 10 are completely open.

Finally, the flow rates corresponding to each supply node are presented in Table (3.23). Comparing these operational supply rates with Table (3.13), it is seen that all nodes are used with their maximum supply rates except for nodes 62 and 110.
Once more, the same trends as previously observed are highlighted with only low differences on the absolute values of flow rates and pressures.

Arc	*Flow rate (kg/s)*	*Arc*	*Flow rate (kg/s)*	*Arc*	*Flow rate (kg/s)*
0000	54.559	0280	0	C3	20.436
0010	108.813	0290	190.786	C4	106.03
0020	92.663	0300	117.212	C5	415.061
0030	33.156	0310	117.212	C6	42.064
0051	400.564	0321	83.757	C7	179.625
0060	415.061	0331	83.757	V1	23.011
0080	64.266	0340	42.064	V2	0
0090	275.935	0390	42.596	V3	62.266
0100	275.935	0880	117.882	V4	172.441
0110	126.622	0900	180.354	V5	273.935
0150	22.965	0910	179.961	V6	400.564
0160	221.221	0920	172.76	V7	33.156
0170	68.652	0930	172.76	V8	0
0200	142.909	1050	53.377	V9	40.423
0240	23.011	C1	108.813	V10	211.669
0260	20.436	C2	83.757		

Table (3.22) Flow rate through pipes, compressors and valves obtained by NLP

Node	*Flow rate, kg/s*	*Node*	*Maximum rate, kg/s*
62	22.965	114	400.564
76	190.786	210	53.377
110	415.061	214	68.652
		Total :	1151.405

Table (3.23) Mass flow rate of the gas provided from the supply nodes obtained by NLP

3.2.4. Comparison between MINLP and NLP algorithm

All these results are in agreement with the previous observations of the MINLP strategy, which validates the problem formulation. It must be observed that the value of the objective function is slightly higher in the NLP case than in the MINLP formulation, probably due to the infeasible pathways followed by both procedures which may be different.

All the numerical tests performed were not reported here. It must be clearly said that the NLP algorithm does not always lead to an optimal solution for complex networks. For instance, if the incidence matrix is not well defined, an optimization failure is observed systematically. This can be all the more observed as the network is complex and exhibits several loops.

For instance, if the incidence matrix is changed such that the upstream and downstream nodes of pipeline 0020 take now the opposite directions of those given in Table (3.9), the algorithm can not succeed in reaching an optimal solution.
This reinforces the interest of implementing an MINLP framework, with binary variables assigned to flow directions for tackling complex structure networks.

3.3. Conclusions

The optimization examples presented in this chapter have been carefully selected to illustrate specific points for the optimization of operating conditions of gas pipeline networks for fuel consumption minimization problems.
First, a didactical example was presented to show in detail the associated formulation. The different steps of the methodology are introduced: implementation of the so-called node-arc, compressor-fuel, pipe-compressor incidence matrices, definition of the optimization variables, parameters and constraints of the problems and formulation of the objective function. Since the flow directions can be easily predicted, an NLP procedure was implemented taking into account only continuous variables. Typical results are analyzed and the characteristic values for compressor stations of some key parameters that may be useful for the practitioner (isentropic head, isentropic efficiency…) are computed. The numerical results obtained show that numerical optimization is an effective tool for optimizing compressor speeds, and can yield significant reductions in fuel consumption.
Second, a multi-supply multi-delivery transmission network was treated to represent the typical features of real distribution networks which are highly meshed and made of large numbers of equipment, which renders their behaviour difficult to anticipate intuitively. For this purpose, an MINLP procedure is introduced with binary variables representing flow directions. The results obtained show that this kind of procedure is robust as compared to an NLP one which often leads to failure cases.
Finally, the global framework can help decision making for optimizing the operating conditions of gas networks and anticipating the changes that may occur i.e. gas quality, variation in supply sources availability and consequences in maintenance.

Chapter 4

High pressure gas pipeline optimization

Total cost minimization using an MINLP formulation

4.1. Introduction

This chapter deals with a high pressure gas pipeline optimization, where the problem is to find the design properties of the pipelines and necessary compressor stations to satisfy customer requirements, using available supply gas and storage capacities. The considered objective function is the total annualized cost, including the investment and operating costs. The binary variables used to represent the flow direction of pipelines lead to an MINLP problem, solved by using the GAMS package. Two different optimization problems are solved. In the former case, the pipeline diameters are considered as continuous variables all along the problem solution strategy, and corrected after the optimization procedure by rounding them up to the closest commercial size used in practice. In the latter case, which appears to be more realistic, logic constraints are included into the constraint set of the MINLP problem, to force the pipeline diameters to their commercial sizes during the optimization phase.

4.2. Network structure

The schematic structure of the studied network, taken from the literature (De wolf, 2000) is shown in Figure (4.1). This network, inspired from a real problem comprises of 20 nodes linked together with 20 arcs. The position of nodes in this figure is selected such that they are consistent with their geographical location in the real network.

The properties of flowing gas through the network are given in Table (4.1). Other necessary properties, such as gas density at standard condition or its lower heating value can be calculated as explained before in chapter 2 of the manuscript.

Gas component	CH_4	C_2H_6	C_3H_8
Mole percent in gas	70	25	05
Molecular mass (*g/mol*)	16.04	30.07	44.10
Lower heating value at 15 °C and 1 bar (*MJ/m^3*)	37.706	66.067	93.936
Critical pressure (*bar*)	46.0	48.8	42.5
Critical temperature (*K*)	190.6	305.4	369.8
Heat capacity at constant pressure (*J/mol-K*)	35.6635	52.848	74.916

Table (4.1) Properties of the flowing gas through the network

There are two supply points 1 and 8 to satisfy the delivery requirements. Gas contracts contain flexibility clauses stating that supply rates may have certain fluctuations in the neighbourhood of the daily average quantities. The daily gas inputs at supply nodes 1 and 8 should not exceed their lower and upper bounds as shown in Table (4.2). Pressure of the gas supplied from node 1 is not limited inferiorly.

Supply node	*Minimum value* MMSCMD (kg/s)	*Maximum value* MMSCMD (kg/s)	*Relative minimum pressure* (bar)	*Relative maximum pressure* (bar)
1	08.870 (087.918)	11.594 (114.917)	00.0	77.0
8	20.344 (201.646)	22.012 (218.178)	50.0	66.2
Total	29.214 (289.564)	33.606 (333.095)		

Table (4.2) Supply nodes characteristics

- Note 1. Relative pressure means absolute pressure added to atmospheric pressure.
- Note 2. MMSCMD is the abbreviation of million standard cubic meters per day.
- Note 3. Standard condition is taken at 25 °C and 101325 Pa

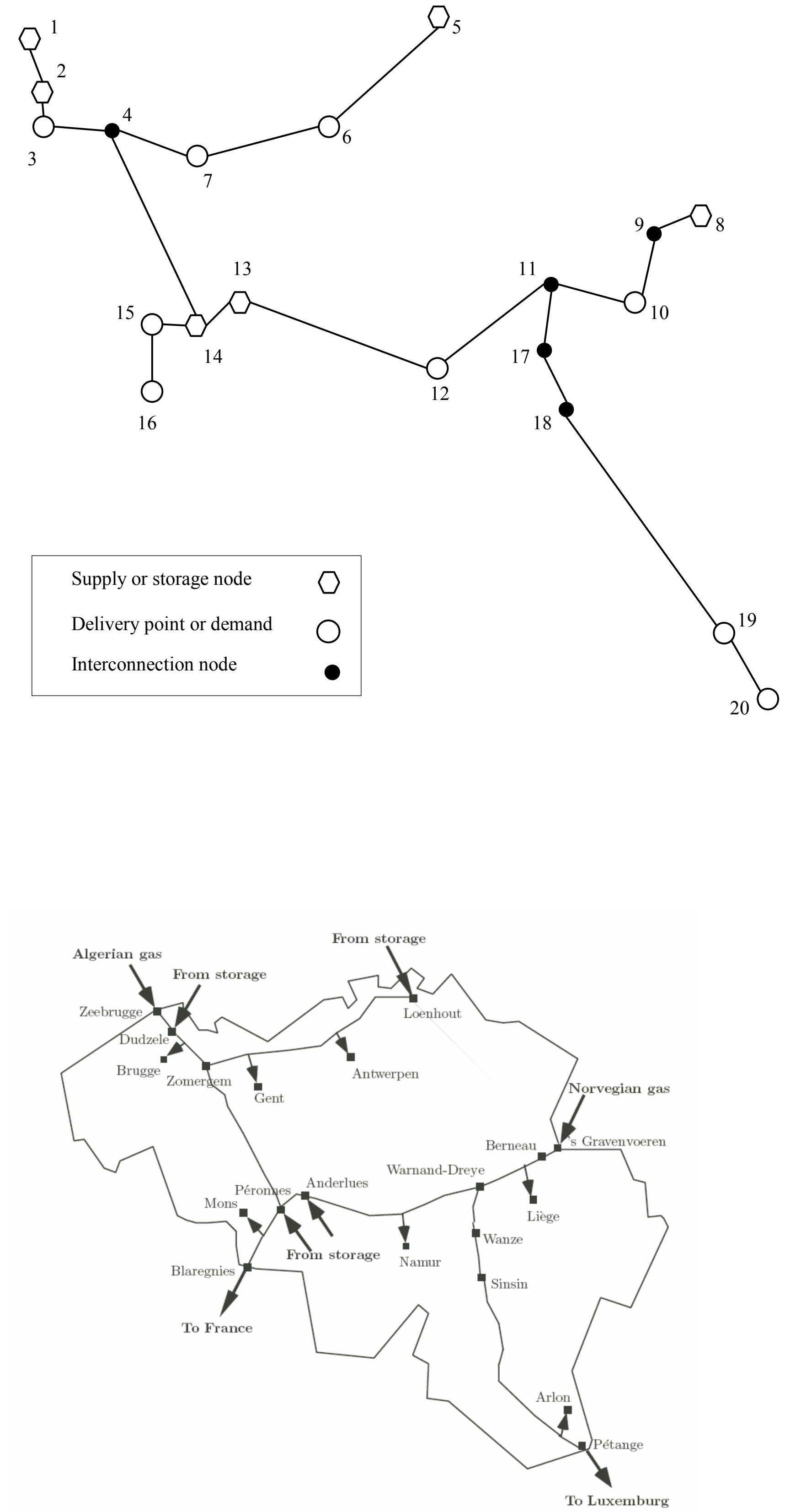

Figure (4.1) Natural gas transmission network inspired from (de Wolf, 2000)

Available gas flows from the nodes where gas can be injected from storage tanks in Table (4.3). Storage tanks are located in nodes 2, 5, 13 and 14. Each storage facility has a maximum daily delivery capacity to gas network reported in Table (4.3).

Storage node	*Maximum outflow* MMSCMD (kg/s)	*Relative minimum pressure* (bar)	*Relative maximum pressure* (bar)
2	08.40 (083.259)	0.0	77.0
5	04.80 (047.577)	0.0	77.0
13	01.20 (011.894)	0.0	66.2
14	00.96 (009.515)	0.0	66.2
Total	15.36 (152.245)		

Table (4.3) Characteristics related to gas storage nodes

Daily gas demand at the nine delivery nodes are given in Table (4.4). It is assumed that the mass flow rates of gas demands are fixed. Pressure boundaries imposed to deliver gases should be respected. Node 16 has the most important delivery gas flow rate. The range of the required pressure at this node is also the more restricted compared with the others.

Delivery node	*Gas demand*		*Relative minimum*	*Relative maximum*
	MMSCMD	(kg/sec)	*pressure* (bar)	*pressure* (bar)
3	03.918	(038.834)	30.0	80.0
6	04.034	(039.984)	30.0	80.0
7	05.256	(052.096)	30.0	80.0
10	06.365	(063.089)	30.0	66.2
12	02.120	(021.013)	00.0	66.2
15	06.848	(067.876)	00.0	66.2
16	15.616	(154.783)	50.0	66.2
19	00.222	(002.200)	00.0	66.2
20	01.919	(019.021)	25.0	66.2
Total	46.298	(458.896)		

Table (4.4) Daily gas demands and their pressure restrictions at delivery nodes

There is no lower limit on the pressure related to interconnection nodes. The upper limits on these pressures are given in Table (4.5).

Interconnection node	*Relative maximum pressure* (bar)	*Interconnection node*	*Relative maximum pressure* (bar)
4	80.0	17	66.2
9	66.2	18	63.0
11	66.2		

Table (4.5) Upper limits on the pressures of interconnection nodes

In addition, the pressure at all nodes including interconnection nodes is limited superiorly with the maximum admissible operating pressure (MAOP) of the arcs which link them together. The maximum admissible operating pressures for pipelines are not given in advance as problem data, but they are computed during the optimization phase by implementing the procedure given in chapter two.

In Table (4.6) upstream and downstream nodes for all arcs of the network are defined. For each arc, at the beginning of the optimization procedure, it is assumed that gas flows from upstream node to downstream node. Then the direction of the flow may be changed by the optimizer.

Arc (*upstream to downstream*)	*Length* (km)	*Arc* (*upstream to downstream*)	*Length* (km)	*Arc* (*upstream to downstream*)	*Length* (km)
1-2	04.0	8-9	05.0	15-16	25.0
2-3	06.0	9-10	20.0	11-17	10.5
3-4	26.0	10-11	25.0	17-18	26.0
5-6	43.0	11-12	42.0	18-19	98.0
6-7	29.0	12-13	40.0	19-20	06.0
7-4	19.0	13-14	05.0		
4-14	55.0	14-15	10.0		

Table (4.6) Arc description including their end-points as well as their length

The corresponding node-arc incidence matrix, A of this case study can be simply written using the data related to end-point nodes presented in odd columns of Table (4.6). For example $a_{ij} \in A$ is equal to +1 when i is node 1 and j is arc 1-2, equal to -1 when i is node 2 and otherwise it is equal to zero.

4.3. MINLP Optimization

4.3.1. General formulation

The optimization problems treated in this chapter aims at searching for the design properties of the pipelines and necessary compressor stations of the network presented in Figure (4.1) to satisfy customer requirements using available supply gas and storage capacities.

Yet, the topology of the transmission network is set, i.e., the links between two nodes and the distance of the corresponding arc. As previously for the optimization problems related to the determination of the operating conditions with a given network, a degree of freedom is introduced in the procedure, which is no gas flow direction is assigned a priori.

The pipe diameters as well as the power of required compressor stations are decided via an MINLP algorithm within GAMS modeling environment. In addition, locations of necessary stations are also determined by the optimizer. To sum up, the continuous optimization variables are the pipeline diameters, the location of the compressor stations as well as their horsepower, pressures and gas flow rates at different nodes, pressure ratios and gas consumption at compressor stations. In addition, a set of binary variables is used to define the flow directions in the pipelines. The objective function to be minimized is the total cost of the network, consisting of investment and operating costs.

4.3.2. Basic design conditions

Design temperature is assumed equal to 281 K. All the other required properties of the components of natural gas are given in Table (4.1).

For the used steel, SMYS (specified minimum yield strength) which is needed to determine pipeline MAOP is supposed to be equal to 2000 bar that is within conventional typical values. Design factor, another parameter to calculate MAOP via Equation (2.21) is considered equal to 0.4 is a low enough and, consequently, safe value. Note that this factor depends on the class location of the pipeline which depends also on the population density around it. Seam joint factor which represents weld efficiency, is assumed to be equal to one. The last factor present

in Equation (2.21), temperature derating factor, is considered equal to one too since the temperature is lower than 120 °C.
The distances between nodes as given before in Table (4.6) are predetermined. Absolute roughness of the interior surface of pipes is about 50 μm for steel pipes.
In order to compute fuel consumption, total efficiency of compressor stations is considered equal to 25 % for all stations. As explained in chapter two, total efficiency is the product of three values comprising isentropic, mechanical and driver efficiencies.
We assume that a compressor station is at least one km far from the neighbouring nodes. This assumption is realistic to introduce the role of the area occupied by the station in addition to the required installations to link the pipelines at a node.

4.3.3. Optimization variables

Two types of variable including continuous and binary variables are introduced in the optimization problem formulation. All of the continuous variables are positive except the flow rates of gas feeding or leaving the nodes of the network. This operating variable is negative for demand nodes, positive for supply or storage nodes and zero for interconnection nodes.
There are basic design variables that are related to the equipment size including pipe diameters, the location of the compressor stations as well as their horsepower. Note that for each pipeline, if a compressor station must be considered on a line, the diameter remains unchanged after the compressor station.
The other kind of continuous variables consists of operating variables such as pressures at different nodes, in addition to the gas flow rates feeding or leaving the nodes. None of the pressures subjected to nodes is fixed and all of them are bounded inferiorly and superiorly, so they must be decided during the optimization process.
Like as the mass flow rates through arcs, there are other operating variables such as pressure ratio at compressor stations which are dependent variables which are computed when the previously nominated basic variables are fixed.
Gas consumption at each station is another dependent variable. Because its amount is low enough compared to compressor throughput, the pipeline diameter will not be changed after a compressor station.

Average gas velocity through a pipe as well as its average compressibility factor is also dependent on other variables such as the pressures at its end-point nodes.
Finally, a set of binary variable is defined to assign flow direction of pipelines. So a MINLP optimization problem will be encountered. If a binary variable would be equal to unity, flow direction will coincide with the one reported in Table (4.6) and if it would be equal to zero, gas flows in the direction opposite to that the one given in this Table.

4.3.4. Objective function

The goal in this optimization problem is to minimize the total annual cost. The objective function, total annualized cost as given in Equation (4.1) after (Edgar and Himmelblau, 2001), includes the sum of the investment cost discounted over 10 years and the operating annual cost for all arcs:

$$ATC = \sum_{j \in arcs} (ICP_j + ICS_j + OCS_j) \tag{4.1}$$

In this equation *ATC* is the annualized total cost expressed in euros/year. The investment cost consists of that of the pipelines named *ICP* and that of the required compressor stations named *ICS*. The investment cost corresponding to the pipelines depends obviously on their lengths and diameters as shown in Equation (4.2) where C_s, d and L are yearly unit pipe capital cost, pipe diameter and pipe length respectively. C_s is assumed 15778 €/(km-m-year). (We have considered an exchange rate of 1 € = 1.35 $ that is the market rate of August 2007):

$$ICP_j = (C_s \; d \; L)_j \tag{4.2}$$

The annualized investment cost of the compressor stations are divided into fixed and variable costs as shown in Equation (4.3). The former term of the right-hand side is independent of station horsepower, *P*, and takes into account installation costs, civil engineering, etc. The latter term increases linearly by the horsepower:

$$ICS_j = (C_f \, sign(P) + C_b P)_j \tag{4.3}$$

The terms C_f and C_b in Equation (4.3) are fixed and variable unit capital costs respectively and are supposed equal to 7410 €/year and 70 €/kW in this order (Edgar and Himmelblau, 2001). If no compressor exists on an arc, its corresponding horsepower and so the term ICS_j is equal to zero. The sign function is introduced in the formulation to take in count that for the arc *j*, if the compressor station horsepower *P* is equal to zero then *ICS* is also equal to zero.
The only operating and maintenance cost that comes into account is due to the compressor stations. It is supposed that pipelines have no operating costs compared with compressor stations (Edgar and Himmelblau, 2001). The yearly operating cost of a compressor station is also a linear function of its horsepower, so it can be calculated according Equation (4.4) where unit operating cost, C_o, is assumed equal to 8.2 €/(kW-year) (Edgar and Himmelblau, 2001):

$$OCS_j = (C_o P)_j \tag{4.4}$$

4.3.5. Constraint definition

Calculation of MAOP using Equation (2.21) for each pipeline requires its wall thickness that is obtained from the following equation in meter. This equation is obtained using the scheduled dimensions provided by ASME B36.19M standard that concerns stainless steel pipes.

$$t = 52\times 10^{-3} d + 989\times 10^{-7} \tag{4.5}$$

For each pipeline, using Equation (2.18), continuity equation corresponding to the segment before compressor is written in the form of Equation (4.6) by introducing the binary variable b that indicates the flow direction. Let us note that the introduction of these binary variables leads to an MINLP problem (although the objective function only involves continuous variables).

$$\frac{\pi^2 MD^5}{16ZRTfL_c}(2b-1)(P_i^2 - P_1^2) = [b\dot{m}^2 + (1-b)(\dot{m}-\dot{m}_c)^2][1+\frac{2D}{fL_c}\ln(\frac{p_i}{p_1})] \tag{4.6}$$

Friction factor is calculated using Equation (2.12). Similarly, continuity equation for the section of pipeline after compressor station is written in the form of Equation (4.7):

$$\frac{\pi^2 MD^5}{16ZRTf(L-L_c)}(2b-1)(P_2^2-P_j^2)=[(1-\mathrm{b})\dot{m}^2+b(\dot{m}-\dot{m}_c)^2][1+\frac{2D}{f(L-L_c)}\ln(\frac{p_2}{p_j})] \qquad (4.7)$$

Using Equation (2.41), the material balance around the node *i* can be written as Equation (4.8). In this equation S_i represents the gas delivery or supply to this node. It is negative if the node is a delivery node, positive for a supply or storage node where the gas is injected to the gas network, and zero otherwise.

$$\sum_{k\in arcs}\lfloor a_{i,j}m_j(2b_{\mathrm{j}}-1)-m_{cj}(a_{i,j}(2b_j-1)-1)/2\rfloor=S_i \qquad (4.8)$$

The existence of compressor station on an arc is decided when pressure ratio is more than one, its lower bound. Pressure ratio is obtained using following equation where p_1 and p_2 are compressor station end-point pressures. Note that if b = 1 then p_1 and p_2 are suction and discharge pressures respectively and if b = 0 vice versa.

$$ratio=b(\frac{p_2}{p_1})+(1-b)(\frac{p_1}{p_2}) \qquad 1\le ratio\le 2 \qquad (4.9)$$

As mentioned, the pressure ratio is comprised between a lower bound that is obviously equal to one and its upper bound considered equal to two. This value is a classical one adopted in the dedicated literature.

The horsepower of a compressor station can be calculated starting from its pressure ratio and its throughput. Gas consumption rate in a station is obtained using Equation (2.30). The gas consumption at each station depends linearly on its power consumption from Equations (2.27) and (2.30).

Note that the optimization problem in this chapter is fall into preliminary design phase in which the rotational speed of the required compressors is not decided. Therefore the characteristic curves are not taken in consideration like the first part of chapter 3 where the problem concerns operating conditions optimization.

It has been imposed that if a compressor station must be considered on a line, it must work with a power greater than a lower value assumed here equal to P_-=1000 kW. This constraint is taken in account by declaring a special variable type called *semi-continuous* in the GAMS input file. Semi-continuous variables are those, whose values if non-zero, must be above a given minimum level. This can be expressed algebraically as the logic relation (4.10) (Brooke et al., 2004):

$$P = 0 \quad \vee \qquad P_{-} \leq P \leq P^{+} \quad P_{-} \neq 0 \tag{4.10}$$

Finally, the average gas velocity through pipelines is restricted by erosional velocity obtained following Equation (2.26).

4.3.6. Optimization process

4.3.6.1. Case 1

In this first case, pipeline diameters are considered as continuous variables. However, in practice, these diameters must take values imposed by the market, where the pipelines have standardized diameters. So the diameters found by the optimizer will be corrected during a post-optimization procedure by rounding them up to the closest commercial size following logic relations presented in Equation (4.11). In this relationship, *d* represents the pipeline internal diameter expressed in meter.

Pipeline internal diameters in meter according to standard schedule number 40				
0.154	0.178	0.203	0.227	0.254
0.279	0.305	0.336	0.387	0.438
0.489	0.540	0.590	0.641	0.692
0.743	0.794	0.844	0.895	

Table (4.7) Commercial pipe sizes employed in the model

Commercial pipe sizes for schedule number 40 based on the standard ASME B36.19M Stainless Steel Pipe as shown in Table (4.7), are employed to compute the pipe diameters. Regardless of schedule number, pipes of a particular nominal size all have the same outside diameter. As the schedule number increases, the wall thickness increases, and the actual internal diameter is reduced.

$$d = \begin{cases} 0.154 & d \le 0.154 \\ 0.178 & 0.154 < d \le 0.178 \\ 0.203 & 0.178 < d \le 0.203 \\ : & : \\ 0.844 & 0.794 < d \le 0.844 \\ 0.895 & 0.844 < d \end{cases} \tag{4.11}$$

Optimization process for this case is performed as follows. First, the pipeline diameters are optimized as continuous variables; next, the resulted diameters are rounded up towards commercial sizes presented in Table (4.7) following the logic relationship above; finally, the pressure variables as well as the flow rates and all other variables are computed again performing another optimization process in GAMS where all the diameters are fixed.

4.3.6.2. Case 2

In this second case, another optimization problem is solved in which pipeline diameters must only take values falling in the range of the commercial sizes. In this approach, the commercial sizes of pipes are used during the optimization phase. In fact, in the problem formulation, the diameters are defined as continuous variables that can be between 0.154 and 0.895 meter. By utilizing logic relationships given by Equation (4.11) as a logic constraint added to the others in the GAMS input file, pipeline diameters will be forced to receive only the discrete values presented above.

Another way to proceed for considering only commercial sizes for pipe diameters, would be to represent each diameter by an integer variable taking the values 1 to 19, each integer value being associated with a commercial size (for example, the value 2 would represent the size 0.178 meter. Yet, in this case, the problem combinatorial aspect explodes because the problem would involve 19 additional integer variables, each one taking 19 values, that is to say 19^{19} (in order of magnitude of 2×10^{24}) additional possibilities in the problem search space.

4.3.7. Some guidelines on initialization and scaling

A very important and recurrent concern in nonlinear programming that the "modeler" has to cope with includes to help the solver by specifying a narrow range of possible values for each continuous variable by giving adequate lower and upper bounds. It is also very helpful to specify a "good" initial solution from which the solver can start searching for the optimum.

The assignment procedure of initial values to variables is carried out as follows:

Firstly, initial supply gas flow rates for concerned nodes are assigned within their bounds according to Tables (4.2) and (4.3). Using these values within simple material balances around all nodes, flow rates through arcs and delivery gas flow from node 20 are calculated one after the other. In the next step, adequate values are assigned to binary variables depending on the sign of the calculated gas flow rates through arcs. Initial values for pressures at different nodes are given arbitrarily according to the pressure bounds shown in Tables (4.3) to (4.5) and to the directions through pipelines which link the nodes. Compressor station suction pressures are also assigned like that. Pressure ratios are initialized arbitrarily between their bounds. After that, pipeline diameters are obtained; giving an arbitrary gas velocity to each pipe between 5 to10 m/sec. Note that, gas velocity can not exceed the erosional velocity. For the problem considered here, gas velocities can not go above 13 m/sec more or less. Finally, station locations are considered equal to their lower limit initially.

One of the best ways to improve the efficiency of the solver is that the developed model would be a well-scaled model, that is to say a model where all the variables have similar orders of magnitude and similar ranges of variation. For this purpose gas consumption flow rate is introduced in the model in *g/s* while gas flow rates through arcs are in kg/sec: for instance, both variables are present in mass balance equations for example. Internal diameters and pipe lengths are brought in relationships expressed in m and km respectively. Node pressure is expressed in bar and power consumption in kW.

4.4. Results

The results of the two optimization cases are presented in this section. Their performance is compared from the objective function viewpoint. Case 1, taking into account the two-stage strategy (i.e., pipeline diameters are optimized as continuous variables then rounded up towards commercial sizes as presented in Table (4.7)) gives a total cost, discounted over 10 years, of 4.024 million euros. After the post-treatment of the diameters, the pressure variables as well as the flow rates and all the other variables are computed again by solving another optimization problem in which the diameters are fixed. In case 2 where the specified logic relationship is respected during optimization as a constraint, the value of objective function is 29.23 % higher that is 5.202 million euros. The computed diameters are shown in Table (4.8) for both cases.

Arc	*Diameter* (mm)		*Arc*	*Diameter* (mm)		*Arc*	*Diameter* (mm)	
	Case 1	*Case 2*		*Case 1*	*Case 2*		*Case 1*	*Case 2*
1-2	489	844	8-9	844	895	15-16	743	743
2-3	743	590	9-10	844	743	11-17	305	794
3-4	692	590	10-11	692	743	17-18	279	743
5-6	305	438	11-12	641	489	18-19	336	743
6-7	154	178	12-13	590	590	19-20	305	844
7-4	305	489	13-14	590	489			
4-14	590	641	14-15	844	743			

Table (4.8) Optimum pipeline diameters

Note. The wall thickness of the pipelines can be calculated following Equation (4.4)

In both cases, the resulting binary variables are equal to unity except for arc 7-4 where the gas flows in opposite direction to that supposed in Table (4.6) Therefore, the flow direction in arcs obtained using optimization process is such as their upstream and downstream are the same as that has been presented in this table except for the arc connecting node 7 and node 4. According to the given results, gas should flow from node 4 to node 7 to assure the mass balance in the network.

The optimum pressures subjected to the nodes are shown for both cases in Table (4.9). Average compressibility factor in each pipe can be obtained for the optimum solutions by applying the results presented in this Table, using Equations (2.5) and (2.8).

Node	*Relative Pressure* (bar)		*Node*	*Relative Pressure* (bar)		*Node*	*Relative Pressure* (bar)	
	Case 1	*Case 2*		*Case 1*	*Case 2*		*Case 1*	*Case 2*
1	74.54	75.00	8	65.60	66.20	15	54.03	54.03
2	72.56	75.01	9	65.07	65.83	16	50.00	50.00
3	71.52	73.97	10	62.91	61.62	17	66.21	58.95
4	67.11	64.57	11	60.58	58.97	18	55.61	58.88
5	77.00	77.00	12	64.52	64.16	19	33.52	58.62
6	31.33	71.00	13	56.76	59.97	20	30.92	58.61
7	30.51	62.71	14	55.64	57.10			

Table (4.9) Pressure subjected to the nodes for the two cases

The mass flow rate of delivery gas for the demand nodes are fixed as a constraint shown in Table (4.4) but the optimum flow rate of injected gas to the network at the supply nodes or the storages are calculated by the algorithm. They are shown in Table (4.10) for each case study.

Node	*Relative Pressure* (bar)		*Node*	*Relative Pressure* (bar)	
	Case 1	*Case 2*		*Case 1*	*Case 2*
1	114.92	114.92	8	201.65	201.65
2	83.26	77.99	13	10.02	11.78
5	40.93	46.22	14	9.52	9.52

Table (4.10) Mass flow rate of delivery gas or supply gas at the concerned nodes

The optimum mass flow rates across the different arcs are reported in Table (4.11). Using the information given in Tables (4.8), (4.9) and (4.11), optimum average gas velocity and erosional velocity for each arc can be calculated simply, following Equations (2.2) and (2.26) respectively.

Arc	*Flow rate* (kg/s)		*Arc*	*Flow rate* (kg/s)		*Arc*	*Flow rate* (kg/s)	
	Case 1	*Case 2*		*Case 1*	*Case 2*		*Case 1*	*Case 2*
1-2	114.85	114.77	8-9	201.58	201.49	15-16	154.85	154.94
2-3	198.04	192.59	9-10	201.51	201.33	11-17	21.51	21.86
3-4	159.13	153.60	10-11	138.36	138.08	17-18	21.43	21.70
5-6	40.86	46.06	11-12	116.83	116.21	18-19	21.36	21.54
6-7	0.81	5.92	12-13	95.70	94.91	19-20	19.09	19.18
7-4	51.36	46.34	13-14	105.65	106.52			
4-14	107.70	107.10	14-15	222.80	222.98			

Table (4.11) Gas mass flow rate through pipelines

For the pipelines on which a compressor station exists, the value presented in Table (4.11) expresses the gas flow rate before the station. In case 1, there are compressor stations on the pipelines 10-11, 11-12 and 11-17. Their operating conditions are shown in Table (4.12).
In case 2, the pipelines 1-2, 11-12 and 12-13 require a compressor station to assure the gas flowing through the network to satisfy all the constraints. The optimum values for their main design and operating conditions are also outlined in Table (4.12).

Case 1			
Pipeline that requires a compressor station	**10-11**	**11-12**	**11-17**
Suction pressure (bar)	63.39	61.42	59.04
Pressure ratio	1.022	1.147	1.142
Station throughput (kg/s)	138.34	116.73	21.49
Fuel consumption (g/s)	18.5	101.8	18.5
Station power consumption (kW)	1000	5520	1000
Position from upstream node (km)	3.56	1.00	9.50
Case 2			
Pipeline that requires a compressor station	**1-2**	**11-12**	**12-13**
Suction pressure (bar)	75.98	59.29	65.00
Pressure ratio	1.028	1.442	1.032
Station throughput (kg/s)	114.75	115.93	94.89
Fuel consumption (kg/s)	18.5	279.8	18.5
Station power (kW)	1000	15160	1000
Position from upstream node (km)	1.00	1.00	1.00

Table (4.12) Optimum conditions related to necessary compressor stations

The compressor stations with low pressure ratio can not be eliminated because their throughput is high enough to increase the station power consumption significantly.
The discharge pressure for each station can be easily obtained using equation (4.3). For each compressor station, the sum of the throughput and the fuel consumption is equal to its gas flow rate for each pipeline presented in Table (4.11). Obviously the mass flow rate of the gas through a pipeline after its compressor station is equal to the station throughput.

4.5. Conclusions

In this chapter, we have developed a strategy to solve an important engineering problem: the optimal design of gas transmission network when the network topology is known.
The solution of the MINLP problem resulting from the minimization of the total annualized cost, including the investment and operating costs, is obtained by using the GAMS package. The optimization strategy provides the main design parameters of the pipelines (diameters, pressures, flow rates) and the characteristics of compressor stations (location, suction pressure, pressure ratio, station throughput, fuel consumption, station power consumption) to satisfy customer requirements. Concerning the commercial sizes for pipeline diameters, two types of problems have been solved. In the former, the diameters have been considered as continuous variables during the optimization process. Then, a post-optimization procedure is implemented to round up these diameters towards the nearest commercial sizes. The corresponding objective function value is 4.024 millions euros. In the latter problem, diameters are forced to take commercial size values by introducing an additional constraint (see Equation 4.11) at each iteration of the optimization algorithm in the GAMS input file. The new value of the optimal objective function increases up to 5.202 million euros. From a strictly mathematical point of view, this increase of the objective function value is a logical consequence of the search space reduction due to the additional constraint concerning the commercial sizes for diameters. Yet, this significant increase reaches up 30%, mainly due to an over-estimation of the pipeline diameters in the second case. Indeed, compared with the first case, the diameters are over-estimated on 13 arcs among the 19 arcs of the network, corresponding to 385.5 km from the total length 494.5 km.
It can be recommended for the practitioner to follow these two strategies at design stage of gas transmission networks.
Case 1 can be viewed as a rough estimate of the total cost at preliminary design and follows the classical order of magnitude traditionally adopted at that stage in project development, that is 30%. It can be viewed as a potential estimation of the gain that can be performed on the system.
Case 2 is a more realistic strategy following the standard pipes used in gas transport and reduces the search space.

Chapter 5

Impact of hydrogen injection in natural gas infrastructures

5.1. Introduction

Confronting more and more today's urgent environmental challenges, such as the control of the release of the gases with greenhouse effect, and facing with the ever-increasing shortage in the fossil resources, the radical changes in energy policies seem now inevitable. Among the various domains, hydrogen is one of the energy alternatives full of promise. Preliminary studies have shown that the transport of a mixture of natural gas and hydrogen is possible through the existing natural gas networks without pipeline modification as long as the mass fraction of hydrogen remains sufficiently low. Although hydrogen substitution perturbs little the fluid mechanics constraints of the system, the limitations fall into the tolerance of the construction materials of the pipes, compressors and other elements of the natural gas infrastructures.

This work is exclusively interested in the transmission pipeline of hydrogen gas and more exactly in the adaptation of the current transmission networks of the natural gas to hydrogen transport.

The transition towards the situation in which hydrogen becomes an important energy carrier, needs decades but worldwide great efforts are made in the field of hydrogen production, delivery, storage and utilization. In this view, an analysis of the potential of using the actual natural gas pipeline systems for the delivery of hydrogen is a valid argument. Defining the conditions under which hydrogen can be added to natural gas constitutes a first step of this investigation.

The chemical and physical properties of hydrogen and natural gas differ significantly, which have an effect on safety related to gas transport and its utilization as well as on the integrity of the network.

The study is aimed to develop a mathematical model for such transmission networks. The quantitative estimation and analysis of the maximal hydrogen contents that are acceptable in the transported gas passing through the existing infrastructures of the natural gas transmission and distribution are proposed in this chapter. As previously, the optimization procedure has been implemented by means of a nonlinear programming method involving the CONOPT resolution module within the GAMS environment. Some typical results are then presented and discussed.

5.2. General context

5.2.1. Towards a hydrogen economy

In a world where energy demand is growing at unprecedented rates, pipelines will continue to play an important role in safely and efficiently transporting oil and gas from often remote areas to their markets. Hydrogen is foreseen as an important and reliable energy carrier in the future sustainable energy society. This energy vector, which can be produced from different primary sources among which the renewable energies, is exploitable in different stationary or portable applications.

Hydrogen deployment scenarios can be based on one of two different fundamental assumptions concerning the level of decentralization in production. Regardless of the primary energy sources and technologies used, hydrogen can be produced at large scale facilities and then distributed to individual customers over a range of few tens to some hundreds kilometers (centralized production), or it can be produced in the immediate proximity of dispensing facilities or end-use appliances (on-site generation). Consequently, this yields principally to two separate families of production and distribution pathways made of neighbouring stages allowing the adoption of different technologies.

Gaseous hydrogen can be transported using several modes like pipeline, railroad, tanker truck, and tanker ship. The chosen method depends on the distance of transportation, the production method, the use, etc. Regarding transportation of hydrogen, along with conventional means, transportation via pipelines has been employed to make hydrogen available to a specific range of mass consuming users.

The current aggregated length of pipelines for hydrogen transport that are known to be either in service, under construction, or under planning reaches almost 2500 km comprising a total of some 1500 km in Europe as a whole and at least 700 km in North America. The oldest hydrogen pipeline is a 220 km started in 1938 in the German Ruhr Valley (Whaley, 2001). The longest hydrogen pipeline in Europe runs more than 400 km between France and Belgium (Kruse et al., 2002). The most extensive hydrogen pipeline network in the U.S.A. is about 720 km long and runs almost continuously along the Gulf Coast from Corpus Christi, Texas to New Orleans, Louisiana (Mintz et al., 2002). Other shorter hydrogen pipelines include an 80 km pipeline in South Africa and two short pipelines in Texas that supply hydrogen to industrial users. NASA has piped hydrogen through short pipelines at their space centres for several years (Whaley, 2001).

Of course, the idea of adding hydrogen to gas via pipelines to satisfy the increased demand for energy will require changes in the natural gas pipeline infrastructure to enhance the reliability of the existing systems.

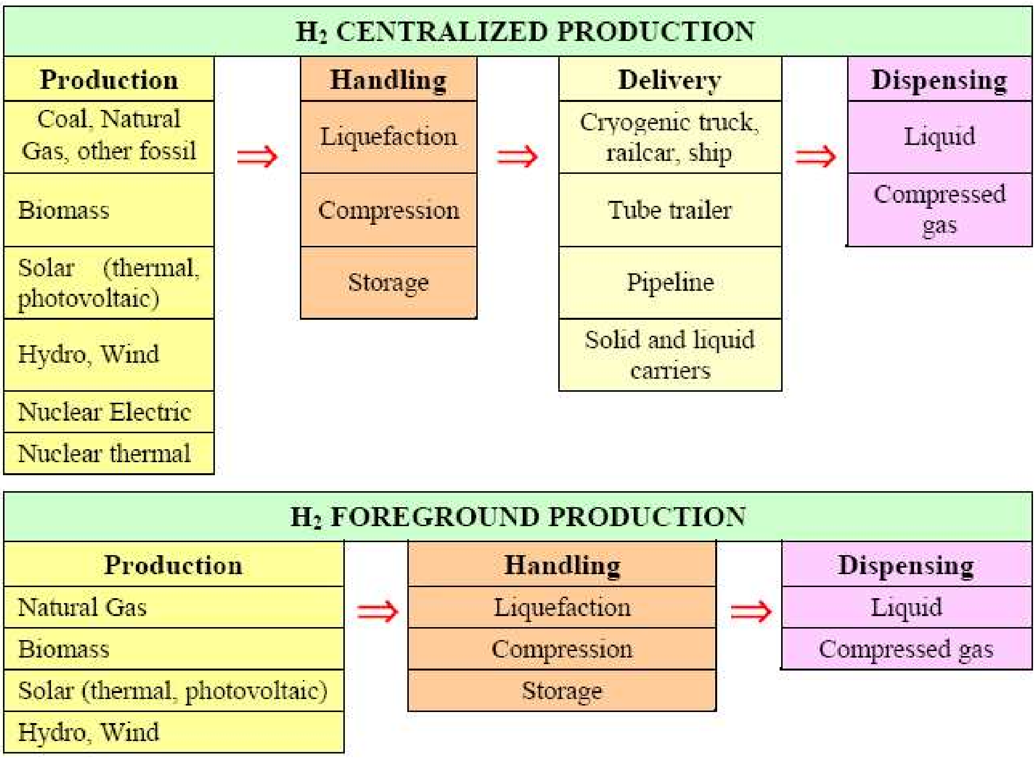

Figure (5.1) Overview of technologies for the deployment of hydrogen production and distribution, for centralized and foreground production (Castello et al., 2005)

According to the analysis of the dedicated literature concerning hydrogen, it is foreseeable that the hydrogen economy will have to rely on a combination of different delivery options and the share of application of each option will change and evolve with time. This study only considers hydrogen-natural gas mixture transmission via pipeline networks.

Thorough technical and economic studies on the whole energy chain including production, storage, transport, distribution and utilization are the first steps to provide new industrial perspectives.

	Hydrogen, H_2	*Methane, CH_4*	*Unit*
Molecular weight	2.02	16.04	g/mol
Critical temperature	33.2	190.65	K
Critical pressure	13.15	45.4	bar
Acentric factor	-0.215	0.008	-
Liquid density at normal boiling point	70.8	422.6	kg/m^3
Vapour density at normal boiling point	1.34	1.82	kg/m^3
Vapour density at 293 K and 1 bar	0.0838	0.651	kg/m^3
139 bar	10.58	111.2	kg/m^3
250 bar	17.81	189.0	kg/m^3
340 bar	23.43	230.0	kg/m^3
Boiling point	20.4	111	K
Heat capacity at constant pressure at 25°C	28.8	35.5	J/mol-K
Specific heat ratio (C_p/C_v)	1.4	1.31	
Lower heating value, weight basis	120	48	MJ/kg
Higher heating value, weight basis	142	53	MJ/kg
Lower heating value, volume basis at 1 atm	11	35	MJ/m^3
Higher heating value, volume basis at 1 atm	13	39	MJ/m^3
Maximum flame temperature	1800	1495	K
Explosive (detonability) limits	18.2-58.9	5.7-14	vol% in air
Flammability limits	4.1-74	5.3-15	vol% in air
Autoignition temperature in air	844	813	K
Dilute gas viscosity at 299 K	9×10^{-6}	11×10^{-6}	Pa.sec
Molecular diffusivity in air	6.1×10^{-5}	1.6×10^{-5}	m^2/sec

Table (5.1) Comparison between physical properties of hydrogen and methane as the principal constituent of natural gas (Baade et al., 2001; Smith and Van Ness, 1988; Padró and Keller, 2001; Rivkin, 2006; Zéberg-Mikkelsen, 2001; Randelman and Wenzel, 1988)

5.2.2. Differences between the properties of hydrogen and natural gas

The physical and chemical properties of hydrogen differ significantly from those of natural gas. Table (5.1) shows some indicative values of relevant properties for the gas chain from source to end user.

As a result of these contrasting properties, a system designed for natural gas cannot be used without appropriate modifications for pure hydrogen, and vice versa. Even the addition of a certain percentage of hydrogen to natural gas will have a direct impact on the combustion properties, diffusion into materials and the behaviour of the gas mixture in air. These aspects are considered further below.

The addition of hydrogen to the natural gas modifies its transport and calorific properties (Schouten et al., 2004). Besides, a gas with higher hydrogen content can have an impact on the safety of the transmission-distribution-utilization chain, the durability and the reliability of the gas pipeline and the utilization performances for the end user.

5.2.3. The impact of hydrogen on the natural gas system

In principle, hydrogen can be added to natural gas in the high-pressure grid, in the medium pressure grid, or in the low pressure distribution grid, but it must be remembered that the existing system was designed and constructed specifically for natural gas and, as explained above, the physical and chemical properties of hydrogen differ significantly from those of natural gas. In particular, the addition of hydrogen to natural gas may have an impact on the following aspects:

- **Safety related to the transmission, distribution and use of gas**

  Aspects of pipeline systems, such as location, materials, wall thickness, safety devices, etc., are designed on the basis of risk assessments. For instance, the design criteria for a pipeline in a populated area differ from the criteria for a pipeline in the countryside. As hydrogen is added, it will change the gas properties and, as a consequence, the related risks will change. An additional safety risk of using a natural gas system for hydrogen may arise from the fact that the potential leakage rate of hydrogen is much larger than that of natural gas through the same sized leak.

- **Integrity of pipelines**

Hydrogen may diffuse into materials and change their mechanical properties. For example, hydrogen embrittlement of steel, leading to an accelerated growth of micro cracks, is a well recognised phenomenon. Hydrogen may also diffuse through polymers and thus result in a significant loss of hydrogen. This may affect the integrity of the system and could also have an impact on safety. A related issue concerns condition monitoring and repair techniques of the delivery system.

- **Gas quality management**

It should be ensured that end users will remain supplied with gas that meets the contractual specific cautions in order to guarantee their safety, performance of end user appliances, and billing accuracy. Moreover, this is an issue if hydrogen is extracted from the mixture, and the remaining gas is supplied to end users further downstream.

- **The performance of end user appliances**

As the combustion properties change when hydrogen is added to natural gas, this may also affect the performance of end user appliances.

- **The energy capacity of the delivery system**

The natural gas system is designed for the maximum capacity that may be required. As energy demand shows a pattern over the day, over the seasons and over the year, dynamic simulations are routinely used to optimize the layout and the dimensions of the systems. The delivery system not only moves gas from production to end user, but it also adapts to the different patterns of supply and demand and it must be capable of coping with fluctuations in gas composition of the gases entering the system. “Capacity” is the key issue of a natural gas system to ensure a sufficiently high level of security of supply, both volume and gas quality.

If an existing pipeline system could be switched from natural gas to hydrogen and still be operated at the same maximum pressure, its maximum capacity (measured in energy terms) would be approximately one third less with hydrogen than with natural gas (the calorific value of hydrogen in volume basis is about 1/3 of the value for natural gas, but hydrogen can be transported with lesser friction resistance than natural gas). For the same reason, it is anticipated that the addition of hydrogen to natural gas will reduce the capacity of a pipeline.

Pipelines are usually not continuously loaded up to their full capacity and so, for most of the time, there will be, in principle, room for the addition of hydrogen, without limiting the energy transmission and distribution capacity of the delivery system.

- **Gas and energy losses**

During the transmission, storage and distribution, the permeability of the walls of underground storages and of pipeline materials, etc. is higher for hydrogen than for natural gas. In addition, leakage from small leaks will be increased. Next to feasible safety aspects, these losses also have economic and environmental aspects.

Some authors have examined hydrogen transport by pipeline and a few reports discuss the use of existing natural gas pipelines to transport hydrogen or hydrogen-natural gas blends.
These are also the main objectives of the NATURALHY-project (supported by the European Commission within a Thematic Priority on Sustainable Energy Systems of the Sixth Framework Programme) which investigates the conditions under which hydrogen can be added to natural gas with acceptable consequences for safety, life cycle and socioeconomic aspects, durability of the system, gas quality management and performance of end-user appliances (Florisson et al., 2006).
Among the recent works, the influence of hydrogen on the pressure drop in the pipelines has been calculated by Schouten et al. (2004). In Parker (2004), the construction costs of natural gas transmission pipelines have been analyzed and the impact of hydrogen in the global cost has been studied. From an economic viewpoint, the cost of natural gas pipelines is a function of pipe diameter and the cost of a hydrogen pipeline can be 50%-80% higher than that of a natural gas pipeline of the same size (Veziroglu and Barbir, 1998). Regional transportation costs could be as much as five times higher than natural gas, primarily because of the lower volumetric energy density of hydrogen (Whaley, 2001). Besides, hydrogen embrittlement of the steel under the high pressures environment of hydrogen constitutes a major concern: consequently, the transportation of a hydrogen-rich gas requires a great attention since hydrogen embrittlement is characterized by a loss of ductility of a steel (Sherif et al., 2005).
The remaining sections are devoted to the modelling of gas pipeline networks and to the influence of hydrogen injection in natural gas infrastructures. This chapter has not the ambition to give an answer to all questions that may arise, but may help to approach the potential challenges of the exploitation of hydrogen as an energy carrier using current pipeline systems. The possibility of low amounts of hydrogen injection into natural gas pipelines will be analysed from a process engineering viewpoint.

5.3. Model extension to hydrogen-natural gas mixtures

A mathematical modelling of the gas transportation problem in networks was presented in chapter two. The model is generalized enough to take into account various gases. As abovementioned, the case of mixtures of natural gas and hydrogen, is particularly examined in this chapter. The pressure drop in a gas pipeline, i.e., the essential parameter to determine the required compression power for the transmission, has been derived from the differential momentum balance. Friction between fluid boundary layer and interior surface of the tube induces energy losses and, consequently, reduces the gas pressure.
The material balance and the equations of momentum conservation on the basic elements of the network as well as the other governing equations constitute the modelling core. The necessary equations in the system of the gas transmission network in order to determine the conditions such as pressure and flow rate are developed. The momentum balance for a single pipeline, the formulation related to a compressor and the associated incidence matrix deduced to facilitate the transmission network design are still applied to optimize the operating conditions.
The different links between the elementary sections of the network are defined with the use of incidence matrices as explained in section 3 of chapter 2. So, the material balances around the nodes of a pipeline network can be expressed under a very concise form by using different types of incidence matrices such as the arc-node matrix (see Equation (2.41) and Figure (2.5)).

5.3.1. Gas pipeline hydraulics

The governing equation to calculate the pressure at each point of a straight pipe can be derived as follows:

$$\frac{dP}{dx} + \frac{f}{2D}\rho\bar{v}^2 + \frac{d}{dx}(\rho\bar{v}^2) = 0 \tag{5.1}$$

This relationship is obtained from the one dimensional momentum balance around a horizontal cylindrical control volume in steady state. The Darcy friction factor, *f*, is a dimensionless value that is a function of the Reynolds number, *Re*, and relative roughness of the pipeline, *ε/D*. As mentioned before, the Darcy friction factor is numerically equal to four times of the Fanning friction factor that is preferred by some engineers.

Since the regime of the gas passing through pipelines lies in turbulent ranges when the flow is considered fully developed, here again it is assumed that the wall roughness is the limiting factor compared with the Reynolds number to find out the value of the friction factor. Therefore the friction factor is estimated through Equation (2.12) deduced by von Karman (Romeo et al., 2002).

The relation between gas density and pressure is represented by Equation (2.3) and mass flow rate (pipe throughput) can be expressed as a function of average gas velocity using Equation (2.2). Considering these last equations, the momentum balance in terms of pressure and throughput can be written in the form of the following equation:

$$\frac{dP}{dx}+\frac{8fZRTm^2}{\pi^2 MD^5 P}+\frac{16Rm^2}{\pi^2 MD^4}\frac{d}{dx}\left(\frac{ZT}{P}\right)=0 \qquad (5.2)$$

This equation was presented before in chapter 2 for a further generalized case as Equation (2.17). By integrating the differential Equation (5.2) between the points *i* and *j*, the following equation is obtained and will be used in the numerical formulations. By assuming constant temperature and pseudo-constant compressibility factor between the points *i* and *j*, the following expression can be deduced:

$$P_i^2 - P_j^2 - \frac{32ZRTm^2}{\pi^2 MD^4}\ln\frac{P_i}{P_j}+\frac{16fZRTm^2 L}{\pi^2 D^5 M}=0 \qquad (5.3)$$

This relationship between pressure and flow rate exhibits a high degree of nonlinearity. It evaluates the pressure drop corresponding to a given flow magnitude and direction. This equation is used to estimate the pipeline's pressure profile and can incorporate the pressure head that occurs due to the location of the pipeline via the elevation changes as shown in chapter two.

The compressibility factor can be evaluated using thermodynamics experimental data or calculated from appropriate equations of state such as presented in Figure (5.2) for pure hydrogen (Zhou and Zhou, 2001). The isotherms for the compressibility factor are shown as a function of pressure expressed in psi (14.706 psi = 1 bar).

In our study, it is estimated from an empirical equation proposed for simulation goals in the literature (Mohring et al., 2004):

$$Z = 1 + (0.257 - 0.533 \frac{T_c}{T}) \frac{p_{ij}}{p_c} \tag{5.4}$$

During the optimization procedure, the compressibility factor is considered as a function of the critical properties of the gas mixture, average pressure of the pipe segment and the temperature. The temperature has been considered as constant. Compared with the data presented in Figure (5.2) corresponding to the temperatures confronted for hydrogen pipelines, the trend of this equation is in agreement with the values obtained from state equation. Average pressure is calculated from two end pressures (see Equation (2.8)).

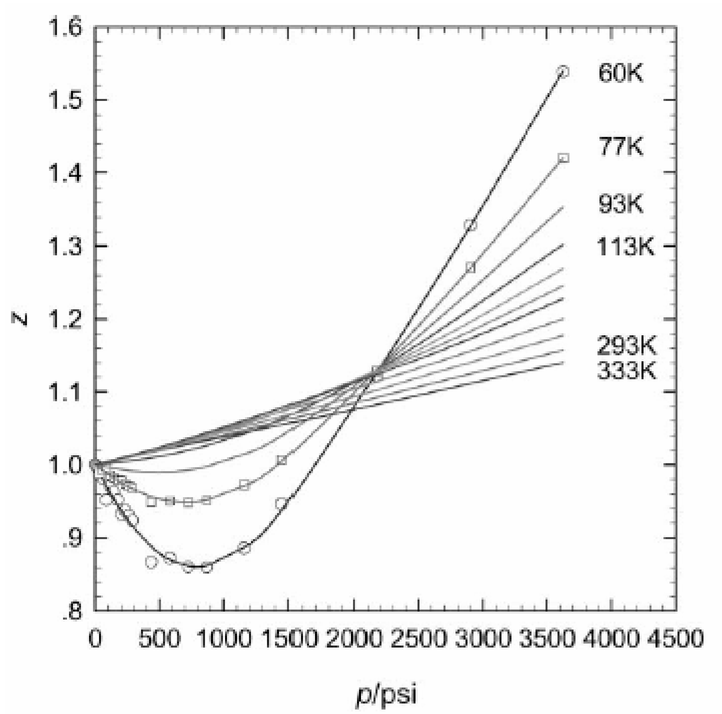

Figure (5.2) Compressibility factor of pure hydrogen evaluated from the experimental p-V-T data and calculated from the Soave-Redlich-Kwong and Bennedict-Webb-Rubin equations of state (Zhou and Zhou, 2001)

The influence of the presence of hydrogen on the pipeline hydraulic is reflected in molecular weight and compressibility factor in Equation (5.3). Note that the effect of the former is more significant than the latter. Since the presence of hydrogen reduces the molecular weight of the gas mixture, according to Equation (5.3), gas transportation by a fixed mass flow rate demands a higher pressure difference. For this reason, the pipelines transporting hydrogen require higher pressures.

The pressure at all points of the equipments should be less than maximum admissible operational pressure (MAOP) which is a design parameter in the pipeline engineering. As shown in Figure (2.2), this upper limit is calculated using Equation (2.21) which linked it with the so-called specified minimum yield strength (SMYS). *SMYS* is a mechanical property of equipment construction material.

To calculate MAOP, the wall thickness of the pipelines is considered dependent on their internal diameter according to hypothetical Equation (5.5). This equation gives the thickness, *t*, in cm for different diameters also in cm. This equation is obtained using the scheduled dimensions provided by ASME B36.19M standard that concerns stainless steel pipes.

$$t = 52\times10^{-3}d + 989\times10^{-5} \tag{5.5}$$

Recall that the MAOP depends also on the population density in the vicinity of the pipeline, the type of pipe material and employed welding and the temperature deration factor (Menon, 2005). Since the operating pipeline pressure is higher in the case of hydrogen, its transport requires thicker pipelines compared with natural gas.

An increase in flow rate due to an increase in pressure drop raises the gas velocity. An important factor in the treatment of compressible fluid flow through pipelines is the erosional velocity. This velocity is sufficiently lower than sonic or critical velocity that is the maximum velocity which a compressible fluid can reach in a pipe. In a pipeline, higher velocities in the course of a long period of time will cause the erosion of the inside surface of the tubes, elbows and other joints. Moreover, increasing gas velocity can have a particular effect on the level of vibration and increase the noises. Once more, the upper limit of the gas velocity is computed empirically with Equation (2.26).

5.3.2. Compressor characteristics

The compressor stations compensate for the pressure drops due to friction in the pipelines, valves and other joints, as well as those due to elevation changes. In pipeline networks, compressor stations consume a small fraction of transported gas. The relation between suction and discharge pressures of a centrifugal compressor and the power transported to gas is represented using definition of the isentropic height of the compressed fluid (see Equations (2.28) and (2.29)). As shown in chapter 2, the flow rate of the consumed fuel in the turbine of each compressor is obtained using Equation (2.30).

In this chapter, it is assumed that the compressors with the characteristic curves presented in chapter two, are compatible with the case of NG-H_2. So the normalized parameters h_i/ω^2, Q_a/ω, and η_i are used again to describe the characteristic curves of the compressors obtained under the form of Equations (2.32) and (2.33). The rotation speed of all compressors is comprised between two bounds.

The constraint that explains the surge limit of the compressors, represented by Equations (2.35), (2.36) and (2.37), are taken in account once again.

5.4. Case study

The addition of hydrogen is examined in this section for the pipeline network studied in section 3 of chapter 3, that is recalled in Figure 5.3. Hydrogen is added to natural gas. The composition of the natural gas is considered constant during the optimization procedure.

This didactic network consists of three long pipelines of 100 kilometers. There are two compressor stations between these pipelines that operate to compensate for pressure drop in the transportation system. Each compressor station includes three parallel centrifugal compressors. In each station, there are six pipelines of about one hundred meters that link the compressors together in parallel. As for each compressor unit, there is a stream that carries fuel to it, there will be 6 fuel streams which have not been shown in Figure (5.3) to avoid complexity. For each compressor, this stream originates from suction node. The set of the specifications of the pipelines of this system which have been introduced here as the parameters of the optimization problem; is proposed in Table (5.2). In addition, the wall thickness of each pipeline is calculated according Equation (5.5).

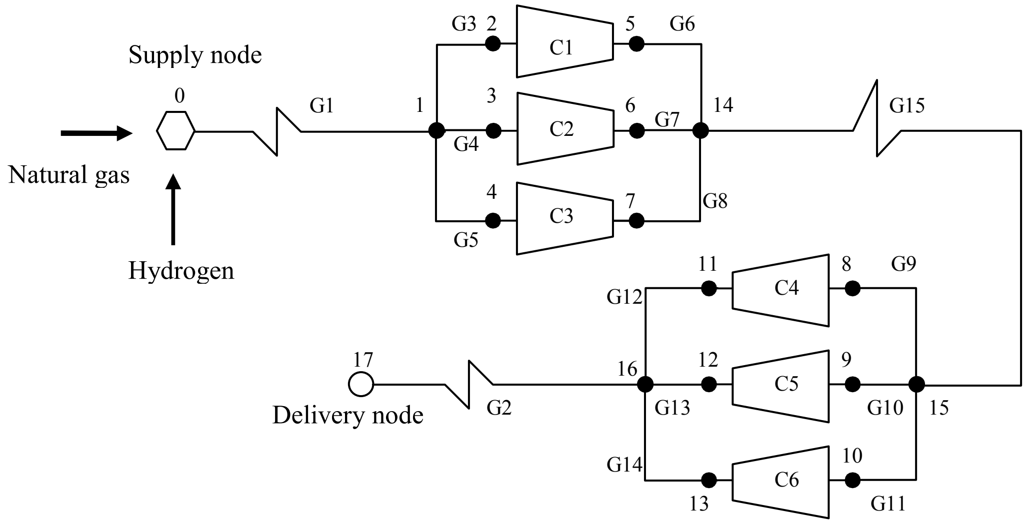

Figure (5.3) Schema of the considered pipeline network to study the influence of hydrogen injection

Pipeline tag	G1	G2	G3	G4	G5
Diameter (m)	0.787	0.889	0.330	0.381	0.330
Length (m)	1E+5	1E+5	200	300	100
Pipeline tag	G6	G7	G8	G9	G10
Diameter (m)	0.330	0.330	0.330	0.381	0.330
Length (m)	200	100	200	100	100
Pipeline tag	G11	G12	G13	G14	G15
Diameter (m)	0.432	0.330	0.330	0.330	0.838
Length (m)	100	100	400	100	1E+5

Table (5.2) Technical features of the pipelines of the system shown in Figure (5.3)

Node 0 is the supply node and gas flows from this node towards node 17. There is no input or output in rest of the nodes.

The composition of natural gas is considered as shown in Table (5.3) where the thermodynamic properties of the components of gas are also presented.

Component	*Methane*	*Ethane*	*Propane*
Mole percent	70	25	5
Molecular weight	16.04	30.07	44.1
Critical temperature, K	190.6	305.4	369.8
Critical pressure, bar	46	48.8	42.5
Lower Heating Value, kJ/kg	50009	47794	46357
Heat capacity at constant pressure, kJ/(kmol.K)	35.663	52.848	74.916

Table (5.3) Composition of the natural gas and the thermodynamic properties of its components

The required thermodynamic properties of hydrogen are given in Table (5.1) as well. They are introduced in the formulation as additional parameters of the problem. Dimensions of the pipelines of the network constitute other structural parameters of the optimization problem. The incidence matrixes are fixed parameters of the problem to define the configuration of the network. Roughness of inner surface of the pipes is considered to be equal to 46×10^{-6}, which is a reported value for stainless steel. The temperature is 330 K all over the system. Mechanical efficiency and driver efficiency for the compressors are assumed roughly 0.90 and 0.35 respectively according to literature. The coefficients of the isentropic head equation in addition to the coefficients of the isentropic efficiency equation for the compressors as shown in Table (3.3) are also the data which determine the working domain of the compressors.

5.5. Results and discussion

In this section, the results of the optimization problem considering the variables of the network under different operating conditions are proposed. The maximal amount of hydrogen in the natural gas is calculated for different energetic capacities of the pipeline. One of the considered optimization criterion is based on the minimization of fuel consumption in the compressor stations. Another standpoint is the estimation of the maximal capacity of the pipeline in various cases.

As in the previous chapters, the procedure of optimization is implemented by means of a nonlinear programming method by using the module of resolution CONOPT within the environment GAMS. Let us recall that GAMS was designed upon the principles of relational database theory and mathematical programming methods. Relational database theory provides a structured framework for developing general data organization and transformation capabilities (Brooke et al., 2004). GAMS creates nonlinear models that will be solved with the CONOPT module, which is a NLP algorithm. GAMS/CONOPT is well-suited for models with very nonlinear constraints, which is the case in our work. CONOPT uses Sequential Linear Programming (SLP) and Sequential Quadratic Programming (SQP) algorithms (Drud, 2004). The approach is illustrated here by typical results obtained on several examples of applications.

5.5.1. Fuel consumption-pipeline capacity optimization (without hydrogen) as a reference

The first preliminary optimization problem is the minimization of the total rate of fuel consumption in two compressor stations simultaneously (objective function) at a constant pipeline throughput. The pressure is considered to be equal to 60 bars with a margin of ±2% at the entrance point of the network, node 0, as well as the delivery point, node 17. So the lower bound of their pressure is 58.8 bar and the upper bound is 61.2 bar at these nodes.

Variables and constraints: In these two problems, the basic optimization variables are the pressures governing at the nodes and the flow rates corresponding to pipes and compressors. There are two kinds of constraints consisting of equalities and inequalities. Equality constraints are the mass balances around nodes, the equation of motion for the pipe arcs, the isentropic head equations for compressors, the relationships between rotational speed, suction volumetric flow rate and head of each compressor as Equation (2.32), the equations to calculate isentropic efficiency according to Equation (2.33) and the equations to determine fuel consumption at each compressor unit. Inequality constraints are the upper bound as well as the lower bound on the pressures of the nodes: MAOP as an upper bound and atmosphere pressure as a lower bound, the superior bounds of the velocities through pipes, the lower and the upper boundaries on the rotation speed of all compressors (166.7 and 250 rpm respectively), lower limit on compressor throughput taken in consideration to avoid pumping phenomenon, upper bound on compressor throughput to stay in shelter from chocking phenomenon.

Evidently to minimize the gas consumption, it is enough to there is no gas to transport, but firstly the raison of the pipeline existence is the gas flow and secondly the model must fulfill the criteria to have the indicated pressures at the endpoints of the network. There are also constraints specifying that compressors work always inside their feasible region.

The results related to this optimization problem were previously presented in section 1 of chapter 3 when the pipeline throughput is constant at 150 kg/sec. Note that the network throughput is equal to the gas mass flow rate of the arc G2. At this case the computed mass percentage of the input gas that is consumed in the stations is equal to 0.50. The transmitted power of the pipeline is equal to 7324 MW at this optimum point. This quantity is the product of the pipeline delivery mass flow rate and the weight basis lower heating value of the gas (48830 kJ/kg):

$$P_n = LHV.m_{G2} \tag{5.6}$$

Neglecting the constraint that states the pipeline throughput is constant at 150 kg/sec, the computed minimum mass percentage of the input gas that is consumed in the stations is equal to 0.33. The transmitted power of the pipeline is equal to 5630 MW at this optimum point. The result of this problem corresponds to the left tip point of the curve in Figure (5.4).
If the pressure is considered equal to 50 bar at the entrance point of the network as well as at the delivery point, the network is not able to transport 150 kg/sec. Performing another optimization process with the transmitted power of the network as an objective function, it is observed that the maximum pipeline capacity is only 130.81 kg/sec at end-point pressures of 50 bar. It corresponds to a transmitted power of 6387 kW. For this case, the mass percentage of the input gas consumed in the stations is equal to 0.683.

Pressure, bar	50	60
Throughput, kg/sec	130.81	150
Transmitted power, kW	6387	7324
Fuel consumption, % of the input gas	0.683	0.50

Table (5.4) Maximum capacity of the pipeline at two different end-point pressures

.

Another problem is treated to maximize the transmitted power at the same end-point pressure conditions, 60 bar, like for the fuel consumption minimization problem above. Here, the objective function considered is the mass flow rate of the pipeline G2. The result of the latter problem named also pipeline capacity maximization is equal to 7778 MW at these end point pressures. It corresponds to a network throughput of 159.3 kg/sec. At this maximum pipeline throughput, 0.677 percents of the supply gas are burnt in the turbines of the pressure stations. The result of this problem corresponds to the right tip point of the curve in Figure (5.4).
Verifying the constraints at this optimum point, it is observed that the reason why the pipeline capacity can not increase more; is that all compressors are working around their maximum revolution speed of 250 rotation/sec. Removing this limitation by changing this upper value to 350, the pipeline capacity maximization results in a value of 169.75 kg/sec. At this optimum

point, the compressors C1 to C5 are working at rotational speeds of 294.9, 343.8, 293.6, 187.9 and 171.0 rotation/sec respectively whereas the compressor C6 is working at its minimum possible speed that is 166.7 rotation/sec. At this optimum point, the gas velocities in the pipelines G3, G4 and G5 are around their maximum possible values (erosional velocity). These examinations confirm that these short pipelines are the bottleneck of the system due to their relatively small diameters as well as the compressors due to their characteristic working domains.

The curve in Figure (5.4) represents the results of the fuel consumption minimization problem at several fixed transmitted power of the pipeline system along with the results of the maximization problem of the transmitted power through the pipeline system.

In the latter problem, the fuel consumption has been kept constant in the form of a constraint. The curve in Figure (5.4) expresses the optimal values of the consumed fraction of delivery gas as a function of the transmitted power at network end-points. According to this graph, increasing the pipeline transmitted energy increases the fraction of transported gas that is consumed in the compressor stations. It is necessary to say that beyond the tip point of the curve, the optimization procedure falls into an infeasible solution.

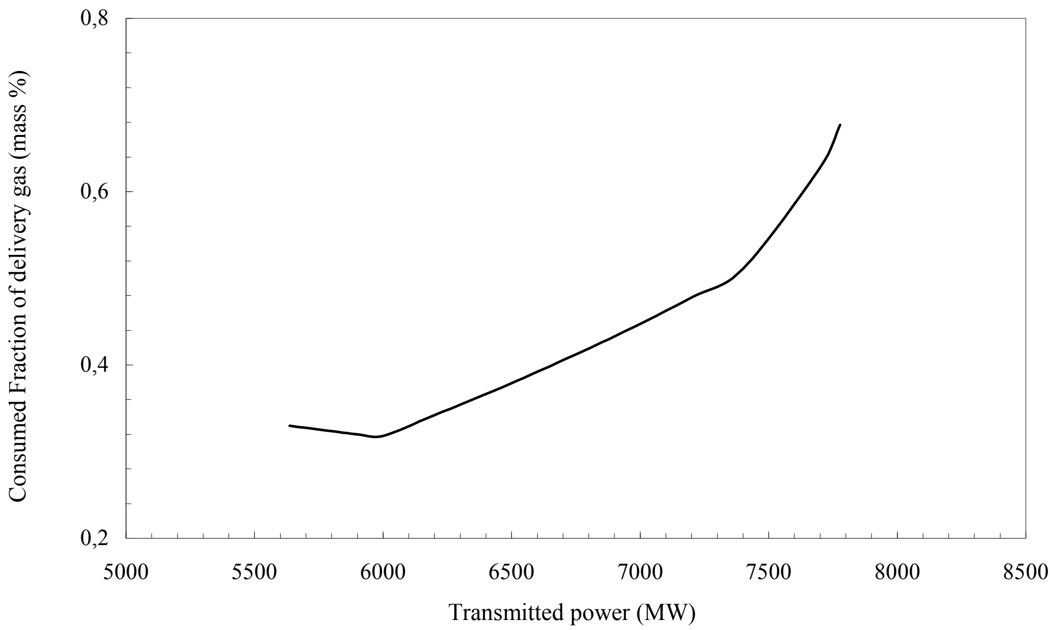

Figure (5.4) Optimal values of the consumed fraction of delivery gas as a function of the transmitted power at network end-points

5.5.2. Optimization problems in the presence of hydrogen

In this section, the gas running through the pipeline network is assumed to be a mixture of hydrogen and natural gas. The composition of the natural gas is the same as the reference problem presented above. Different operational conditions such as delivery pressure or hydrogen fraction in NG-H_2 mixtures can be considered as the objective function. The same structure of the network and specifications as those mentioned in Figure (5.3) and Table (5.2) are used.

Initially, the gas passing through pipeline is considered pure hydrogen without natural gas. Here, the pressure is considered to be equal to 60 bars with a margin of ±2% at the entrance point of the network, node 0, as well as the delivery point, node 17. After performing an optimization process whose objective function is the network transmitted power, the maximum achievable pipeline transmittable power is obtained 1272 MW that is equal to 16% of its value in the case of pure natural gas (7778 MW). In this case, the hydrogen consumption as station fuel is 0.176% of the input hydrogen (10.534 kg/sec).

The observed reduction in the transmitted energy by the pipeline can be mainly attributed to the low molecular weight of hydrogen, i.e., about 10% of the value of natural gas (see Table (5.1) and the role of molecular weight in the equation of motion). Because the mass basis *LHV* of hydrogen is about 2.5 times of the corresponding value for natural gas, it reduces the impact of the low molecular weight of hydrogen on the reduction of the transmitted energy by the pipeline. Other parameters such as compressibility factor play a relatively minor role. Yet, another factor can be here highlighted: the diameters of the pipelines existing in the compressor stations are so small that it is observed that the gas average velocity tends to its upper limits (erosional velocity) in the case of pure hydrogen transport. Consequently, the mass flow rate can not increase any more.

The amount of hydrogen that can be added to natural gas is maximized at these end-pressure conditions. The results of this optimization problem are presented in Figure (5.5). The maximum amount of added hydrogen is presented in mass and volume fraction versus transmitted power. Each computation point is obtained by considering the power transmitted by the pipeline as a constant via a constraint introduced in the optimization procedure. The objective function is the mole fraction of hydrogen in NG-H_2 mixtures. For example if the transmitted power is fixed at 5000 MW i.e. 65% of the maximum pipeline capacity (7778 MW), the maximum fraction of hydrogen in the transported gas is 6.6% in mass basis.

According to Figure (5.5), the maximum transmitted power decreases linearly as hydrogen mole fraction (volume fraction) in the gas increases.

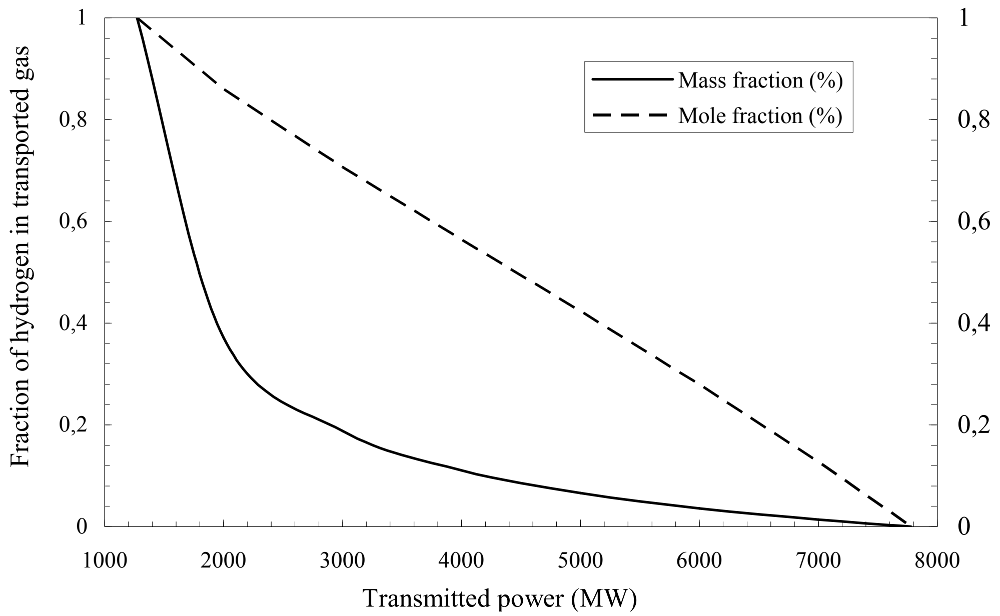

Figure (5.5) Maximum hydrogen fraction in the mixture of the natural gas and hydrogen at different pipeline transmitted power for end-point pressures of 60 bar

Additional problems related to the optimization of the operating conditions can be treated with the same formulation by only changing the objective function. For instance, delivery pressure optimization for different hydrogen fractions in NG-H_2 mixtures is another interesting problem. Only some quantitative results are shown in Figure (5.6) which corresponds to a supply pressure of 60 bar. In Figure (5.6), each point corresponds to a hydrogen mass fraction considered as constant taken into account via a constraint and the optimization procedure has been performed by taking the supply pressure as the objective function. The transmitted power corresponding to each delivery pressure of the pipeline network is shown on the right side axis in Figure (5.6) as well.

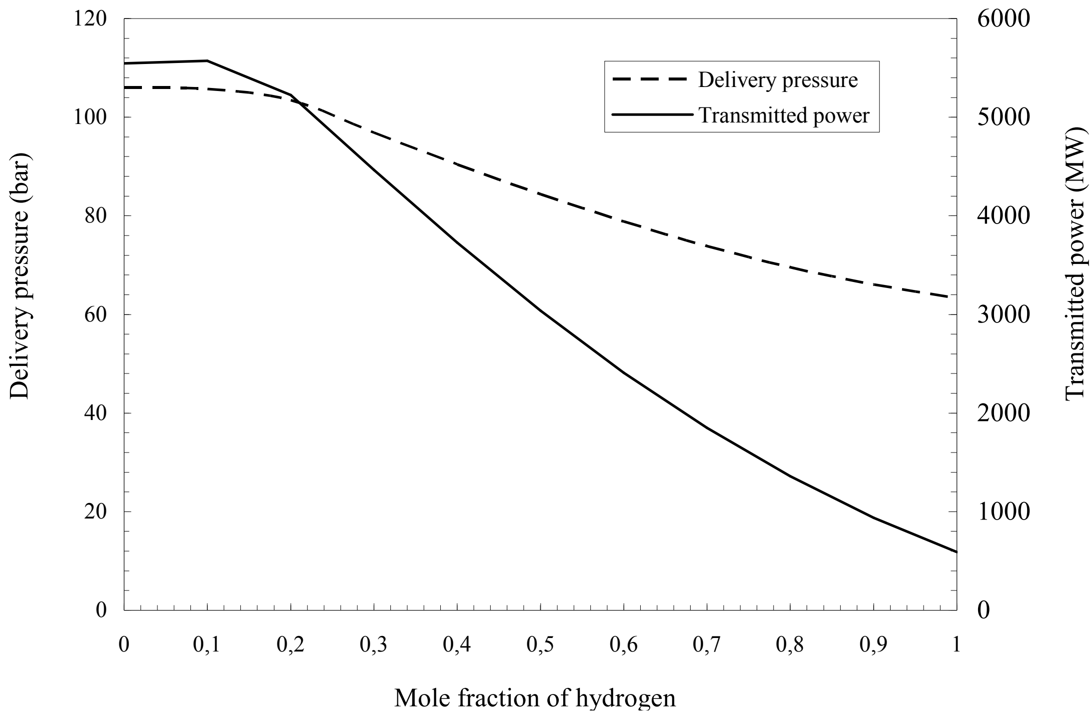

Figure (5.6) Optimized delivery pressures versus added hydrogen to natural gas and corresponding transmitted powers for constant supply pressure of 60 bar

According to Figure (5.6), for pure hydrogen transport, optimum delivery pressure will be 105.98 bar and the transmitted power at this pressure is only 590.5 MW. This amount is extremely lower than the transmitted power of the pipeline for natural gas without hydrogen which is 5547 MW at this supply pressure.

5.6. Conclusions

A mathematical modelling framework for gas pipeline networks was proposed in this study. A major interest of this work is to take into account the amount of hydrogen that can be added to the pipeline network traditionally devoted to the transportation of natural gas without any modification in the system. Defining the conditions under which hydrogen can be added to natural gas constitutes a key point of this investigation.

Several operational variables were selected as decision variables of gas pipeline optimization problem. Optimization procedures including fuel consumption minimization, amount of added hydrogen, transmitted power and delivery pressure maximization were performed for different gas mixtures of natural gas and hydrogen using CONOPT/GAMS. It has been shown that the maximum achievable fraction of hydrogen that can be added to natural gas is around 6 mass percent. Addition of hydrogen to natural gas decreases the transmitted power significantly. According to this study, an adaptation of the current networks of transmission of natural gas to the transport of hydrogen seems to be possible until this limit.

Chapter 6

Conclusion and perspectives

The objective of this PhD work was to develop a general methodology for **gas transmission pipeline modelling** based on an **optimization-oriented framework** in order to embed various problems that are of tremendous importance, i.e., **pipeline design and operation**. For this purpose, **steady state behaviour** of the gas is considered and assumed in the momentum and mass balances.

Although various optimization techniques can be used, the choice of a **deterministic** one has guided the solution strategy, since it is generally recognized that this kind of methods is particularly well-fitted to take into account the important number of constraints that are likely to be involved in the problem formulation. Adequate **solvers** within **GAMS environment** are selected since this optimisation tool is often considered as a standard for the solution of **Process Systems Engineering** problems. Besides, experience was previously acquired in our research group for batch plant design that presents some similarities with the problems that were addressed here. The **conclusions of this work** imply both **theoretical** and **practical aspects**.

6.1. Theoretical viewpoint

The **formal presentation of the pipeline model,** that serves as a basis throughout this work takes typically into account.

- *Modeling equations and problem constraints*
    - ✓ Mass and transport equations;
    - ✓ Problem constraints such as the Maximum Admissible Operational Pressure *to satisfy*:
    - ✓ Compressor modelling through characteristic curves;
    - ✓ Incidence matrices to model the interconnection between the network components (arcs-nodes).

In this work, compressor stations which consist of several identical centrifugal compressor units in parallel are considered, since this type of station is very common in today's gas industry, and having an understanding of this type of station is fundamental for modelling more complex station configurations.
Note that the model can take into account various compositions of gas mixtures.

- *Objective functions*

Various objectives can be considered according to the problem class, either **improvement of operating conditions of a gas network** or **network design**. Two types of objective functions are chosen for illustration purposes, **fuel consumption minimization** and **total annualized cost minimization,** respectively.

From the theoretical viewpoint, the following items must be highlighted:

6.1.1. Some guidelines for improvement of operating conditions of a gas network under fuel consumption minimization

Typically, this kind of problems involves **continuous** variables, i.e., pressures at nodes and flow rates through pipes. When the flow directions can be easily predicted, the formulation is based on a **non linear-programming** procedure (NLP with CONOPT procedure).

In this case, compressor modelling occurs through the **use of characteristic curves** as previously mentioned and the search for their optimal operating conditions is carried out in the feasible operating domain for the unit.

For most complex networks (for instance multi-supply multi-delivery transmission grids) which may be highly meshed, exhibit some loops and made of large numbers of equipment, the flow direction may be difficult to anticipate intuitively. For this purpose, a **mixed integer non linear programming approach** is recommended (MINLP with SBB solver) with binary variables representing flow directions. The results obtained on the treatment of a real-case problem show that this kind of procedure is much robust than the NLP one which often leads to failure cases.

6.1.2. Some guidelines for gas network design under total annualized cost minimization

In that case, the optimization strategy provides the main design parameters of the pipelines (diameters, pressures, flow rates) and the characteristics of compressor stations (location, suction pressure, pressure ratio, station throughput, fuel consumption, station power consumption) to satisfy customer requirements. Due to the complexity of the problem, the use of a compressor map is not recommended at this stage.

The solution of the resulting **MINLP problem** under the minimization of the total annualized cost, including the investment and operating costs, is obtained by using the GAMS package (SBB solver).

A rigorous and realistic way to tackle the problem is to consider the pipeline diameters as **discrete variables** since their choice is generally decided from a range of available market sizes.

6.1.3. Some guidelines for model extension to the case of hydrogen-natural gas mixtures

The previous approach applied to natural gas was easily extended to the case of hydrogen-natural gas mixture. The hydrogen properties are taken into account in the model: for instance, the compressibility factor is calculated from appropriate equations of state.

Several operational variables were selected as decision variables of gas pipeline optimization problem. Optimization procedures including fuel consumption minimization, amount of added hydrogen, transmitted power and delivery pressure maximization were performed for different gas mixtures of natural gas and hydrogen using CONOPT/GAMS.

6.2. Practical viewpoint

From the practical viewpoint, the following key points have been shown:

6.2.1. Strategies for improvement of operating conditions of a gas network under fuel consumption minimization

The use of this objective function is particularly interesting since reduction of the energy used in pipeline operations will not only have a beneficial economical impact but also an environmental one: the more efficient the use of compressors stations is the less greenhouse emissions are dissipated in the atmosphere.

The use of the proposed strategy can help the gas network manager to answer these recurrent questions:

Knowing that I need to deliver a certain volume of gas at certain key points, how do I utilize the compressors at my disposal most efficiently to reduce fuel gas consumption?

How do I set the consequent pressures and flow rates?

Let us mention that characteristic values for compressor stations of some key parameters that may be useful for the practitioner (isentropic head, isentropic efficiency…) are systematically computed. The numerical results obtained show that numerical optimization is an effective tool for optimizing compressor speeds, and can yield significant reductions in fuel consumption.

Finally, the global framework can help decision making for optimizing the operating conditions of gas networks, anticipating the changes that may occur (i.e. gas quality, variation in supply sources availability and consequences in maintenance) and quantifying CO_2 emissions.

6.2.2. Gas network design under total annualized cost minimization

For this purpose, the methodology offers once more a decision-making tool to the designer, since the problem exhibits a highly combinatorial aspect mainly due to the diameters of the pipelines that can take discrete values.
Obviously, the two strategies can be combined, gas network design followed by an optimization of the operating conditions to have a complete description of the optimal gas system, since the two aspects are interconnected. This two-stage methodology will have the advantage to confirm parameter tuning obtained at the previous stage.

It can be also mentioned that different examples of various complexity serve as a test bench from them the practitioner can find some similarities with its own problem, in order to follow the most adequate solution strategy.

6.2.3. Hydrogen-natural gas mixtures

The maximum fraction of hydrogen that can be added to natural gas is calculated to be about 6 mass percent. Addition of hydrogen to natural gas decreases the transmitted power significantly. According to this study, an adaptation of the current networks of transmission of natural gas to the transport of hydrogen seems to be possible until this limit.

This work also suggests perspectives that concern both **theoretical and practical** implications:

6.3. Optimization strategy: towards the use of stochastic procedures

6.3.1. Monobjective methodology

A natural extension to this methodology is to explore stochastic algorithms which have proven their ability to treat high combinatorial problems for long. For this purpose, the choice of Genetic Algorithms acts as a good candidate. Indeed, among the diversity of optimization methods, the choice of the relevant technique for the treatment of a given problem keeps being a thorny issue.

Gas network design and operation problems provide a good framework to test the performances of various optimisation methods : on the one hand, Mathematical Programming techniques – such as those studied here, implemented in the GAMS environment – and, on the other hand, one stochastic method, i.e. a genetic algorithm. Several examples, showing an increasing complexity, can be designed and solved with these techniques. The result comparison will enable to evaluate their efficiency in order to highlight the most appropriate method for a given problem instance.

A particular attention has to be focused on the treatment of constraints.

6.3.2. Multiobjective methodology

The use of stochastic algorithms such as Genetic Algorithms is all the more interesting as it is well recognized that they can be easily adapted to multiobjective problems.

Optimization of pipeline operations may require optimization of more than one objective of conflicting nature: for instance, one objective can be the minimization of energy used to operate the pipeline system and at the same time a second objective might be maximization of the throughput. Optimal solutions to one objective may contradict optimal solutions of the other objective; therefore, a solution to the problem will entail mutual sacrifice (trade-off) of objectives. Pareto dominance was studied and tested with common test functions available in optimization literature and with Process Systems Engineering problems in previous studies (Baez Senties, 2007).

6.3.3. Uncertainty modelling

Another extension that could increase the realism of the model is to consider uncertainty in the demand.

The most common approaches treated in the dedicated literature represent the demand uncertainty with a probabilistic frame by means of Gaussian distributions. Yet, this assumption does not seem to be a reliable representation of the reality, since in practice the parameters are interdependent, leading to very hard computations of conditional probabilities, and do not follow symmetric distribution rules.

Fuzzy concepts and arithmetic constitute an alternative to describe the imprecise nature on product demands. This reinforces the interest of using Genetic Algorithms, since similar problems were treated previously by extension of a multiobjective genetic algorithm for Multiproduct Batch Plant Design (Aguilar Lasserre, 2006)

6.4. Retrofit strategy

A direct extension to the gas pipeline design strategy could be to find the necessary adaptations to an existing network (the "so-called" retrofit problem) in order to face different demand conditions.

6.5. Hydrogen pipelines

The suggested perspectives are particularly relevant to the case of hydrogen. A major concern is to take into account safety constraints or criteria in the design and operation phase.

We hope that this contribution has provided an efficient methodology basis to explore these investigation areas.

References and citations

Abadie J. and J. Carpentier, 1969, Generalization of the Wolfe reduced gradient method to the case of nonlinear constraints, Optimization, R. Fletcher edition, Academic Press, New York, USA, pp. 37-47

Abbaspour M., K.S. Chapman and P. Krishnaswami, 2005, Nonisothermal compressor station optimization, Journal of energy resources technology, vol. 127, no. 2, pp. 131-141

Aguilar Lasserre A.A., 2006, Approche multicritère pour la conception d'ateliers discontinus dans un environnement incertain, PhD thesis, Institut National Polytechnique de Toulouse, France

André J., F. Bonnans, L. Cornibert, 2006, Planning reinforcement on gas transportation networks with optimization methods, Process Operation Research Models and Methods in the Energy Sector Conference, ORMMES, Coimbra, Portugal

API Specification 5L, 2000, American Petroleum Institute, Washington D.C, USA.

ASME B36.19M, 2004, Stainless Steel Pipe, The American Society of Mechanical Engineers, New York, USA

Baade W.F., U.N. Parekh and V.S. Raman, 2001, Hydrogen, Kirk-Othmer encyclopaedia of chemical technology, vol. 13, pp. 759-808, John Wiley & Sons Inc., New York, USA

Baez Senties O., 2007, Méthodologie d'aide à la décision multicritère pour l'ordonnancement d'ateliers discontinus, Ph.D. thesis, Institut National Polytechnique de Toulouse, France

Beyer H.G., H.P. Schwefel, 2002, Evolution strategies, a comprehensive introduction, Natural Computing, vol. 1, no. 1, pp. 3-52

Biegler L.T. and I.E. Grossmann, 2004, Retrospective on optimization, Computers and Chemical Engineering, 28, 1169

Botros, K.K., 1994, Transient phenomena in compressor stations surge, Journal of Engineering for Gas Turbines and Power, vol. 116, no. 1, pp. 133–142

Botros K.K., P.J. Campbell and D.B. Mah, 1991, Dynamic simulation of compressor station operation including centrifugal compressor and gas turbine, Journal of Engineering for Gas Turbines and Power, vol. 113, no. 2, pp. 300–311

Boyd E. A., L.R. Scott and Wu S.S., 1997, Evaluating the quality of pipeline optimization algorithms, 29th annual meeting of Pipeline Simulation Interest Group, 15-17 October, Tucson, USA

Brooke A., D. Kendrick, A. Meeraus and R. Raman, 2004, GAMS: a user's guide, GAMS Development Corporation, Washington, USA

Cameron, I., 1999, Using an excel-based model for steady state and transient simulation, 31st annual meeting of Pipeline Simulation Interest Group, 20-22 October, Missouri

Carter, R.G., 1996, Compressor station optimization: computational accuracy and speed, 28th annual meeting of Pipeline Simulation Interest Group, 23-25 October, San Francisco

Castello P., E. Tzimas, P. Moretto and S.D. Peteves, 2005, Techno-economic assessment of hydrogen transmission & distribution systems in Europe in the medium and long term, The Institute for Energy, Petten, Netherlands

Chauvelier-Alario C., B. Mathieu and C. Toussaint, 2006, Decision making software for Gaz de France distribution network operators: Carpathe, 23rd World Gas Conference, Amsterdam, Netherlands

Claudel B., 1996, Propriétés thermodynamiques des fluides, Techniques de l'ingénieur, Génie énergétique, vol. 1, no. B8020

Cobos-Zaleta D. and R.Z. Rios-Mercado, 2002, A MINLP model for a minimizing fuel consumption on natural gas pipeline networks, XI Latin-Ibero-American conference on operations research, 27-31 October, Concepión, Chile

Costa A.L.H., J.L. de Medeiros and F.L.P. Pessoa, Steady-state modelling and simulation of pipeline networks for compressible fluids. Brazilian Journal of Chemical Engineering, 1998, vol. 15, no. 4, pp. 344-357

De Wolf D., Y. Smeers, 2000, The gas transmission problem solved by an extension of the simplex algorithm, Management Science, vol. 46, no. 11, pp. 1454-1465.

Dedieu S., L. Pibouleau, C. Azzaro-Pantel and S. Domenech, 2003, Design and retrofit of multiobjective batch plants via a multicriteria genetic algorithm, Computers and chemical engineering, vol. 27, no. 12, pp. 1723-1740

Doonan A.F., 1998, Evaluation of an AGI control strategy using SIMULINKTM, International Conference on Simulation, 2 October, 457, pp. 305-312

Drud A., 1985, A GRG code for large sparse dynamic nonlinear optimization problems, Mathematical programming, vol. 31, pp. 153-191

Drud A.S., 1992, CONOPT - A large-scale GRG code, ORSA Journal on computing, vol. 6, pp. 207-216

Drud A., 2004, GAMS: the solver manuals, CONPT, GAMS Development Corporation, Washington

Duran M.A. and I .E. Grossmann, 1986, An outer-approximation algorithm for a class of mixed-integer nonlinear programs, Mathematical Programming: series A and B, vol. 36, no. 3, pp. 307-339

Edgar, T.F., Himmelblau D.M., 2001, Optimization of chemical processes, McGraw Hill, Singapore

Eurogas Annual report 2003-2004, Eurogas, Bruxelles, Belgium, http://www.eurogas.org

Florisson O., I. Alliat, B. Lowesmith, G. Hankinson, 2006, The value of the existing natural gas system for hydrogen, the sustainable future energy carrier (progress obtained in the NATURALHY-project), 23rd World gas conference, Amsterdam, Netherlands

Geoffrion A.M., 1972, Generalized benders decomposition, Journal of Optimization Theory and Applications, vol. 10, no. 4, pp. 237-260

Glenn R.P. et al., 1996, Evaluating the effective friction factor and overall heat transfer coefficient during unsteady pipeline operation, International Pipeline Conference, ASME, volume 2, pp. 1175-1182

Gorla R.S.R. and A.A. Khan, 2003, Turbomachinery design and theory, Marcel Dekker Inc., New York, USA

Grigson, C., 1992, Drag losses of new ships caused by hull finish, Journal of ship research, vol. 36, no. 2, pp. 182-196

Grossmann I.E., 2002, Review of nonlinear mixed-integer and disjunctive programming techniques, Optimization and Engineering, vol. 3, 227-252

Grossmann I.E., J. Viswanathan, A. Vecchietti, R. Raman and E. Kalvelagen, 2004, DICOPT: A discrete continuous optimization package, Engineering Research Design Center of Carnegie Mellon University, Pennsylvania, and GAMS Development Corporation, Washington DC, USA

Guillén G., M. Badell, A. Espuña and L. Puigjaner, 2006, Simultaneous optimization of process operations and financial decisions to enhance the integrated planning/scheduling of chemical supply chains, Computers and Chemical Engineering, vol. 30, no. 3, pp. 421-436

Gupta O.K., V. Ravindran, 1985, Branch and bound experiments in convex nonlinear integer programming, Management Science, vol. 31, no. 12, pp.1533-1546

Hao J.K. and R. Dorne, 1996, Study of genetic search for the frequency assignment problem, Lecture Notes in Computer Science, Springer Verlag, 1063, 333

Hao J.K., P. Galinier and M. Habib, 1999, Métaheuristiques pour l'optimization combinatoire et l'affectation sous contrainte, Revue d'intelligence Artificielle, vol. 13, no. 2, pp. 283-324

Holland J.H. Adaptation in natural and artificial systems, University of Michigan Press, Ann Arbor: MI, 1975

IEA Statistics, 2003, Natural gas information, International Energy Agency, Paris, France

Jain A., S. Srinivasalu and R.K. Bhattacharjya, 2005, Determination of an optimal unit pulse response function using real-coded genetic algorithms, Journal of Hydrology, vol. 303, no. 1-4, pp. 199-214

Ke S.L. and H.C. Ti, 2000, Transient analysis of isothermal gas flow in pipeline network, Chemical Engineering Journal 76, pp. 169–177

Kirkpatrick S., J.C.D. Gelatt, M.P. Vecchi, 1982, Optimization by simulated annealing, IBM Research Report, RC9355

Kruse B., S. Grinna and C. Buch, 2002, Hydrogen statu sog muligneter, rapport no. 6, Bellona Foundation, Oslo, Norway

Lee S., I.E. Grossmann, 2003, Global optimization of nonlinear generalized disjunctive programming with bilinear equality constraints: applications to process networks, Computers and Chemical Engineering, vol. 27, no. 11, pp. 1557-1575

Lewandowski, A., 1994, Object-oriented modelling of the natural gas pipeline network, 26th annual meeting of Pipeline Simulation Interest Group, 13-14 October, Sandiego, USA

Leyffer S., 1999, User manual for MINLP_BB, University of Dundee, Numerical Analysis Report NA/XXX, Dundee, Scotland

Luongo C.A., B.J. Gilmour, D. W. Schroeder, 1989, Optimization in natural gas transmission networks: a tool to improve operational efficiency, 3rd SIAM conference on optimization, Boston

Martinez-Romero N., O. Osorio-Peralta and I. Santan-Vite, 2002, Natural gas network optimization and sensibility analysis, Proceedings of the SPE International Petroleum Conference and Exhibition of Mexico, pp. 357-370

Menon, E.S., 2005, Gas pipeline hydraulics, CRC Press, Taylor & Francis Group, Boca Raton, Florida

Mintz M., S. Folga, J. Gillette and J. Molburg, 2002, Hydrogen: On the Horizon or Just a Mirage, paper no. 02FCC-155, Society of Automotive Engineers Inc., USA

Mohitpour M, W. Thompson and B. Asante, 1996, The importance of dynamic simulation on the design and optimization of pipeline transmission systems, Proceedings of ASME International Pipeline Conference, vol. 2, pp. 1183-1188, Calgary, Canada

Mohring J., J. Hoffmann, T. Halfmann, A. Zemitis, G. Basso, P. Lagoni, 2004, Automated model reduction of complex gas pipeline networks, 36th annual meeting of pipeline simulation interest group, Palm Springs, California, USA

Montagna J.M. and A.R. Vecchietti, 2003, Retrofit of multiproduct batch plants through generalized disjunctive programming, Mathematical and computer modelling, vol. 38, no. 5-6, pp. 465-479

Mora T. and M. Ulieru, 2005, Minimization of energy use in pipeline operations - an application to natural gas transmission systems, Industrial Electronics Society, IECON, 31st annual conference of IEEE, ISBN: 0-7803-9252-3

Odom F.M., 1990, Tutorials on modelling of gas turbine driven centrifugal compressors, 22nd annual meeting of pipeline simulation interest group, Baltimore, Maryland, USA

Osiadacz A.J., 1987, Simulation and analysis of gas networks, E. & F.N. Spon, London

Osiadacz A. J, 1994, Dynamic optimization of high Pressure gas Networks using hierarchical systems theory, 26th annual meeting of Pipeline Simulation Interest Group, 13-14 October, Sandiego, USA

Osiadacz A.J., 1996, Deferent transient models - limitations, advantages and disadvantages, 28th annual meeting of Pipeline Simulation Interest Group, 23-25 October, San Francisco, USA

Osiadacz A.J. and M. Chaczykowski, 1998, Comparison of Isothermal and Non-Isothermal Transient Models, 30th annual meeting of Pipeline Simulation Interest Group, 28-30 October, Colorado, USA

Osiadacz A.J. and M. Chaczykowski, 2001, Comparison of isothermal and non-isothermal pipeline gas flow models, Chemical engineering journal, vol. 81, no. 1-3, pp. 41-51

Padberg M. and G. Rinaldi, 1991, A branch-and-cut algorithm for the resolution of large scale symmetric traveling salesman problems, SIAM Review, vol. 33, no. 1, pp. 60-100

Padro C.E.G. and J.O. Keller, 2001, Hydrogen energy, Kirk-Othmer encyclopaedia of chemical technology, vol. 13, pp. 837-866, John Wiley & Sons Inc., New York, USA

Parker N., 2004, Using natural gas transmission pipeline cost to estimate hydrogen pipeline cost, Masters thesis, Institute of transportation studies, University of California, USA

Perry R.H. and C.H. Chilton, 1973, Chemical engineers' handbook, 5th ed., McGraw-Hill Book Co., New York

Ponsich A., 2005, Stratégies d'optimisation mixte en Génie des Procédés - Application à la conception d'ateliers discontinus, PhD thesis, Institut National Polytechnique de Toulouse, France

Pugnet J.M., 1999, Pompage des compresseurs, Techniques de l'ingénieur, Génie mécanique, vol. BL2, no. BM 4 182, pp. BM4182.1-BM4182.18

Raman R., I.E. Grossmann, 1994, Modeling and computational techniques for logic based integer programming, Computers and Chemical Engineering, vol. 18, no. 7, pp. 563-578

Randelman R.E. and L.A. Wenzel, 1988, Joule-Thomson coefficients of hydrogen and methane mixtures, Journal of chemical engineering data, vol. 33, pp. 293-299

Rangwala A.S., Reciprocating Machinery Dynamics: Design and Analysis, 2001, Marcel Dekker, New York, USA

Ravemark D.E. and D.W.T. Rippin, 1998, Optimal design of a multi-product batch plant, Computers and Chemical Engineering, vol. 22, no. 1-2, pp.177-183

Rivkin C., 2006, Hazards and hazard mitigation techniques for natural gas and hydrogen fueling operations, Process safety progress, vol. 26, No.1, pp. 27-34

Rios-Mercado R.Z., S. Wu, L.R. Scott and E.A. Boyd, 2002, A Reduction technique for natural gas transmission network optimization problems, Annals of Operations Research, vol. 117, pp. 217-234

Riva A., S. D'Angelosante and C. Trebeschi, 2006, Natural gas and the environmental results of lifecycle assessment, Energy, vol. 31, no. 1, pp. 138-148

Romeo E., C. Royo and A. Monzon, 2002, Improved explicit equations for estimation of the friction factor in rough and smooth pipes, Chemical engineering journal, vol. 86, no. 3, pp. 369-374

Ryoo H.S. and N.V. Sahinidis, 1995, Global optimization of nonconvex NLPs and MINLPs with applications in process design, Computers and Chemical Engineering, vol. 19, no. 5, pp. 551-566

Santos S.P., 1997, Transient analysis, a must in gas pipeline design, 29th annual meeting of Pipeline Simulation Interest Group, Arizona, USA

Schouten J.A., J. P.J. Michels and R.J. Rosmalen, 2004, Effect of H_2-injection on the thermodynamic and transportation properties of natural gas, International journal of hydrogen energy, vol. 29, no. 11, pp. 1173-1180

Schroeder D.W., 1996, Hydraulic analysis in the natural gas industry, Advances in industrial engineering applications and practice I, edited by Chen J. J. W. and A. Mital, pp. 960-965, International journal of industrial engineering, Cincinnati, USA

Sherif S.A., F. Barbir and T.N. Veziroglu, 2005, Towards a hydrogen economy, The electricity journal, vol. 18, no. 6, pp. 62-76

Sletfjerding E. and J.S. Gudmundsson, 2003, Friction factor directly from roughness measurements, Transactions of the ASME, vol. 125, pp. 126-130

Smith E.M.B., C.C. Pantelides, 1999, A symbolic reformulation/spatial branch-and-bound algorithm for the global optimization of nonconvex MINLPs, Computers and Chemical Engineering, vol. 23, no. 4-5, pp. 457-478

Smith J.M. and H.C. Van Ness, 1988, Introduction to chemical engineering thermodynamics, 4th ed., McGraw-Hill Book Company, Singapore

Suming W., R.Z. Rios-Mercado, E. A. Boyd and L. R. Scott, 2000, Model relaxations for the fuel cost minimization of steady-state gas pipeline networks, Mathematical and computer modelling, vol. 31, no. 2-3, pp. 197-220

Sun C.K., U. Varanon, Christine W. Chan and P. Tontiwachwuthikul, 1999, An integrated expert system/operations research approach for optimization of natural gas pipeline operations, Engineering Applications of Artificial Intelligence, vol. 13, no. 4, pp. 465-475

Sung W. et al, 1998, Optimization of pipeline networks with A hybrid MCST-CD networking model, SPE Production and Facilities, August, volume 13, no. 3, pp. 213-219

Surry P.D., N.J. Radcliffe and I.D. Boyd, 1995, A multi-objective approach to constrained optimization of gas supply networks: The COMOGA method, Lecture Notes in Computer Science, Evolutionary Computing, Springer Berlin/Heidelberg, volume 993, pp. 166-180

Teh Y.S. and G.P. Rangaiah, 2003, Tabu search for global optimization of continuous functions with application to phase equilibrium calculations, Computers and Chemical Engineering, vol. 27, no. 11, pp. 1665-1679

Tian S. and M.A. Adewumi, 1994, Development of analytical design equation for gas pipelines, SPE Production and Facilities, vol. 9, no. 2, pp. 100-106

Turner W.J. and M.J. Simonson, 1984, A compressor station model for transient gas pipeline simulation, 16th annual meeting of Pipeline Simulation Interest Group, Tennessee, USA

Turner W.J. and M.J. Simonson, 1985, Compressor station transient flow modelled, Oil and Gas Journal, 20 May, pp. 79–83

Veziroglu T., and F. Barbir, 1998, Emerging technology series, Hydrogen energy technologies. United Nations Industrial Development Organization, Vienna

Westerlünd T. and K. Lundqvist, Alpha-ECP, version 5.04, An interactive MINLP-solver based on the extended cutting plane method, Report 01-178-A, Process Design Laboratory, Abo Akademi University, Abo, Finlande, 2003

Westerlünd T., F. Petterson, 1995, An extended cutting plane method for solving convex MINLP problems, Computers and Chemical Engineering, vol.19, no. sup.1, pp. S131-S136

Whaley T.P., 2001, Pipelines, Kirk-Othmer Encyclopaedia of Chemical Technology, John Wiley & Sons Inc., New York, USA

Wolpert D.H. and W. G. Macready, 1997, No free lunch theorems for optimization, IEEE Transactions on Evolutionary Computation, vol. 1, no. 1, pp. 67-82

Yang Y.W., J.F. Xu, C.K. Soh, 2006, An evolutionary programming algorithm for continuous global optimization, European Journal of Operational Research, vol. 168, no. 2, 354-369

Zamora M.Z. and I.E. Grossmann, 1998, A global MINLP optimization algorithm for the synthesis of heat exchanger networks with no stream splits, Computers and Chemical Engineering, vol. 22, no. 3, pp. 367-384

Zéberg-Mikkelsen C.K., 2001, Viscosity study of hydrocarbon fluids at reservoir conditions, modeling and measurements, Ph.D. thesis, Department of chemical engineering, Technical University of Denmark

Zhou J., et al., 1995, Simulation of transient flow in natural gas pipelines, 27th annual meeting of Pipeline Simulation Interest Group, 18-20 Oct, Albuquerque, USA

Zhou L. and Y. Zhou, 2001, Determination of compressibility factor and fugacity coefficient of hydrogen in studies of adsorptive storage, International journal of hydrogen energy, vol. 26, pp. 597–601

Appendices

Appendix 1

Derivation of the general form of the equation of movement

Writing momentum balance, the equation of movement is obtained as Equation (A.1) (Osiadacz, 1987):

$$\frac{\partial p}{\partial x} + \frac{f}{2D}\rho v^2 \pm g\rho \sin\alpha + \frac{\partial(\rho v^2)}{\partial x} + \frac{\partial(\rho v)}{\partial t} = 0 \qquad \text{(A.1)}$$

Having Equation (A.2) which declares the relationship between gas pressure and gas density, the latter is eliminated following Equations (A.3) and (A.4). Note that the cross section area of the pipe, *A*, is constant as well as gas molecular weight, *M* and universal gas constant, *R*.

$$\rho = \frac{pM}{ZRT} \qquad \text{(A.2)}$$

$$\frac{\partial p}{\partial x} + \frac{f}{2D}\frac{pM}{ZRT}\left(\frac{m}{\frac{pM}{ZRT}A}\right)^2 \pm g\frac{pM}{ZRT}\sin\alpha + \frac{\partial}{\partial x}\left(\frac{pM}{ZRT}\left(\frac{m}{\frac{pM}{ZRT}A}\right)^2\right) + \frac{\partial}{\partial t}\left(\frac{pM}{ZRT}\frac{m}{\frac{pM}{ZRT}A}\right) = 0 \qquad \text{(A.3)}$$

$$\frac{\partial p}{\partial x} + \frac{f}{2D}\frac{ZRT}{pMA^2}m^2 \pm g\frac{pM}{ZRT}\sin\alpha + \frac{R}{A^2M}\frac{\partial}{\partial x}\left(\frac{ZT}{p}m^2\right) + \frac{1}{A}\frac{\partial m}{\partial t} = 0 \qquad \text{(A.4)}$$

Since mathematically we have:

$$\frac{\partial}{\partial x}(\frac{ZT}{p}m^2) = 2m\frac{ZT}{p}\frac{\partial m}{\partial x} + m^2\frac{\partial}{\partial x}(\frac{ZT}{p}) \qquad (A.5)$$

Thus Equation (A.4) is developed as Equation (A.6) and then Equation (A.7):

$$\frac{\partial p}{\partial x} + \frac{f}{2D}\frac{ZRT}{pMA^2}m^2 \pm g\frac{pM}{ZRT}\sin\alpha + \frac{R}{A^2M}[2m\frac{ZT}{p}\frac{\partial m}{\partial x} + m^2\frac{\partial}{\partial x}(\frac{ZT}{p})] + \frac{1}{A}\frac{\partial m}{\partial t} = 0 \qquad (A.6)$$

$$\frac{\partial p}{\partial x} + \frac{f}{2D}\frac{ZRT}{pMA^2}m^2 \pm g\frac{pM}{ZRT}\sin\alpha + \frac{2mR}{A^2M}\frac{ZT}{p}\frac{\partial m}{\partial x} + \frac{m^2R}{A^2M}\frac{\partial}{\partial x}(\frac{ZT}{p}) + \frac{1}{A}\frac{\partial m}{\partial t} = 0 \qquad (A.7)$$

The relationship above is still too general for the isotherm gas flow. However the isotherm assumption is usually used to simulate a gas pipeline but sometimes in gas transmission lines the gas temperature might decrease even below ambient temperature when excessive pressure drop is encountered. This phenomenon called "Joule-Thomson effect in pipelines" should be watched particularly when the gas contains water vapor and hydrate formation is suspected.

Appendix 2

Continuity equation simplification

Material balance is generally express differentially as Equation (A.8) for the gas flow through a pipeline with constant hydraulic cross section area:

$$\frac{1}{\mathrm{A}}\frac{\partial m}{\partial x} + \frac{\partial \rho}{\partial t} = 0 \qquad (A.8)$$

Replacing gas density using Equation (A.2) as shown in Equation (A.9), continuity equation then is easily obtained as Equation (A.10):

$$\frac{1}{A}\frac{\partial m}{\partial x}+\frac{\partial}{\partial t}(\frac{pM}{ZRT})=0 \tag{A.9}$$

$$\frac{1}{A}\frac{\partial m}{\partial x}+\frac{M}{R}\frac{\partial}{\partial t}(\frac{p}{ZT})=0 \tag{A.10}$$

Equation (A.10) accompanied with Equation (A.7) can be used to simulate gas pipeline transients.

Appendix 3

Continuity equation and equation of movement in steady state

In steady state at each section of the pipe, the mass flow rate is constant as well as the gas density:

$$\frac{\partial m}{\partial t}=0 \tag{A.11}$$

$$\frac{\partial \rho}{\partial t}=0 \tag{A.12}$$

Thus it is resulted from Equation (A.8) that the gas mass flow rate remains constant across the pipeline length:

$$\frac{1}{A}\frac{\mathrm{d}m}{\mathrm{d}x}=0 \Rightarrow m=cte \tag{A.13}$$

Equation (A.7) is expressed as below in steady state:

$$\frac{\mathrm{d}p}{\mathrm{d}x}+\frac{f}{2D}\frac{ZRT}{pMA^2}m^2 \pm g\frac{pM}{ZRT}\sin\alpha+\frac{m^2R}{A^2M}\frac{\mathrm{d}}{\mathrm{d}x}(\frac{ZT}{p})=0 \tag{A.14}$$

Appendix 4

Resolution of the equation of movement for isothermal condition

Taking two suppositions we will try to simplify and then solve the Equation (A.14) algebraically:

- Isothermal gas flow through pipe segment.
- Constant compressibility factor.

The first assumption is applicable in a grand category of industrial cases and the second assumption is realistic for relatively short pipelines for instance.

Based on these suppositions, Equation (A.14) will be simplified as shown in Equations (A.15), (A.16) and (A.17).

$$\frac{\mathrm{d}p}{\mathrm{d}x}+\frac{f}{2D}\frac{ZRT}{pMA^2}m^2 \pm g\frac{pM}{ZRT}\sin\alpha+\frac{m^2ZRT}{A^2M}\frac{\mathrm{d}}{\mathrm{d}x}(\frac{1}{p})=0 \tag{A.15}$$

$$\frac{\mathrm{d}p}{\mathrm{d}x}+\frac{f}{2D}\frac{ZRT}{pMA^2}m^2 \pm g\frac{pM}{ZRT}\sin\alpha-\frac{m^2ZRT}{A^2Mp^2}\frac{\mathrm{d}p}{\mathrm{d}x}=0 \tag{A.16}$$

$$(1-\frac{m^2ZRT}{A^2Mp^2})\frac{\mathrm{d}p}{\mathrm{d}x}+\frac{f}{2D}\frac{ZRT}{pMA^2}m^2 \pm g\frac{pM}{ZRT}\sin\alpha=0 \tag{A.17}$$

Ordinary differential equation (A.17) will be solved for three cases comprising inclined, vertical and horizontal pipeline. Considering that $m = \rho v A$ and $A = \pi D^2 / 4$ Equation (A.17) can be rearranged as Equation (A.18) and then in the form of Equation (A.19):

$$\frac{\mathrm{d}p}{\mathrm{d}x} = \frac{\frac{f}{2D}\frac{ZRT}{pMA^2}m^2 \pm g\frac{pM}{ZRT}\sin\alpha}{\frac{m^2 ZRT}{A^2 M p^2} - 1} \tag{A.18}$$

$$\frac{\mathrm{d}p}{\mathrm{d}x} = \frac{ap \pm bp^3}{c - p^2} \quad a = \frac{f}{2D}\frac{ZRT}{MA^2}m^2 \quad b = \frac{gM\sin\alpha}{ZRT} \quad c = \frac{m^2 ZRT}{A^2 M} \tag{A.19}$$

The mathematical procedure to solve this latter parameterized equation is included a polynomial development and an integration operation between the pipe end-points 1 and 2 as following:

$$\mathrm{d}x = \frac{c - p^2}{p(a \pm bp^2)}\mathrm{d}p = (\frac{c/a}{p} - \frac{(1 \pm cb/a)p}{a \pm bp^2})\mathrm{d}p = (\frac{c}{ap} - \frac{1 \pm cb/a}{\pm 2b}\frac{\pm 2bp}{a \pm bp^2})\mathrm{d}p \tag{A.20}$$

$$\int_1^2 \mathrm{d}x = \int_1^2 (\frac{c}{ap} - \frac{1 \pm cb/a}{\pm 2b}\frac{\pm 2bp}{a \pm bp^2})\mathrm{d}p = \frac{c}{a}\int_1^2 \frac{\mathrm{d}p}{p} - \frac{1 \pm cb/a}{\pm 2b}\int_1^2 (\frac{\pm 2bp}{a \pm bp^2})\mathrm{d}p \tag{A.21}$$

As shown below, the result of this integration operation gives a pseudo-explicit equation to compute the pressure drop:

$$L = \frac{c}{a}\ln(\frac{p_2}{p_1}) - \frac{1 \pm cb/a}{\pm 2b}\ln(\frac{a \pm bp_2^{\,2}}{a \pm bp_1^{\,2}}) \tag{A.22}$$

Replacing the parameters a, b and c; this equation shows the role of each parameter on the pressure drop through the pipe segment limited between points 1 and 2:

$$L = \frac{2D}{f}\ln(\frac{p_2}{p_1}) - \frac{fZRT \pm 2DgM\sin\alpha}{\pm 2fgM\sin\alpha}\ln(\frac{f(ZRT)^2 m^2 \pm 2DgM^2\sin\alpha A^2 p_2^{\,2}}{f(ZRT)^2 m^2 \pm 2DgM^2\sin\alpha A^2 p_1^{\,2}}) \tag{A.23}$$

However this equation is not valuable for horizontal pipes where sin $\alpha = 0$ but it can be more simplified for the following cases where sin $\alpha = 1$:

- Vertical upward gas flow:

$$L = \frac{2D}{f}\ln(\frac{p_2}{p_1}) - (\frac{ZRT}{2gM} + \frac{D}{f})\ln(\frac{f(ZRT)^2 m^2 + 2DgM^2A^2{p_2}^2}{f(ZRT)^2 m^2 + 2DgM^2A^2{p_1}^2}) \tag{A.24}$$

- Vertical downward gas flow:

$$L = \frac{2D}{f}\ln(\frac{p_2}{p_1}) + (\frac{ZRT}{2gM} - \frac{D}{f})\ln(\frac{f(ZRT)^2 m^2 - 2DgM^2A^2{p_2}^2}{f(ZRT)^2 m^2 - 2DgM^2A^2{p_1}^2}) \tag{A.25}$$

For a horizontal pipe, it should start from Equation (4.17) as follows through Equations (A.26) to (A.30):

$$(1 - \frac{m^2 ZRT}{A^2 M p^2})\frac{\mathrm{d}p}{\mathrm{d}x} + \frac{f}{2D}\frac{ZRT}{pMA^2}m^2 = 0 \tag{A.26}$$

$$(p - \frac{m^2 ZRT}{A^2 M p})\frac{\mathrm{d}p}{\mathrm{d}x} + \frac{f}{2D}\frac{ZRT}{MA^2}m^2 = 0 \tag{A.27}$$

$$\int_1^2 (\frac{m^2 ZRT}{A^2 M p} - p)\mathrm{d}p = \int_1^2 \frac{f}{2D}\frac{ZRT}{MA^2}m^2 \mathrm{d}x \tag{A.28}$$

$$\frac{m^2 ZRT}{A^2 M}\ln(\frac{p_2}{p_1}) - \frac{1}{2}({p_2}^2 - {p_1}^2) = \frac{f}{2D}\frac{ZRT}{MA^2}m^2 L \tag{A.29}$$

$$({p_2}^2 - {p_1}^2) - \frac{32 m^2 ZRT}{\pi^2 D^4 M}\ln(\frac{p_2}{p_1}) + \frac{16f}{\pi^2 D^5}\frac{ZRT}{M}m^2 L = 0 \tag{A.30}$$

Equation (A.30) is the principal equation in this thesis to compute pressure drop for a pipe segment limited between points 1 and 2.

Appendix 5

Maximum allowable operating pressure (MAOP)

Consider a circular pipe of length L, and internal diameter d as shown below in Figure (A.1). The cross section of one-half portion of this pipe is considered. The pipe is subjected to an internal pressure of p. Within the pipe material, the hoop stress σ_h and the axial stress σ_a act at right angles to each other as shown. For balancing the forces in the circumferential direction, hoop stress acting on the two areas $L\times t$, balances the internal pressure on the projected area $D\times L$. Therefore:

$$p\times D\times L=\sigma_h\times L\times t\times 2 \quad\Rightarrow\quad p=\sigma_h\frac{2t}{D} \tag{A.31}$$

Looking at the balancing of longitudinal forces, the internal pressure acting on the cross section area $\pi D^2/4$, produces bursting force that is balanced with by the resisting force of the axial stress acting on the area $\pi D\times t$. therefore:

$$p\times\pi D^2/4=\sigma_a\times\pi D\times t \quad\Rightarrow\quad p=\sigma_a\frac{4t}{D} \tag{A.32}$$

Because the stresses within the pipe body should not exceed *SMYS* (specified minimum yield stress) of its material, comparing these two equations above and by introducing design factor, seam joint factor and temperature derating factor Equation (2.21) is concluded that state that the internal pressure should not go above a certain value i.e. *MAOP*:

$$p<MAOP=SMYS\frac{2t}{D}f_F f_E f_T \tag{A.33}$$

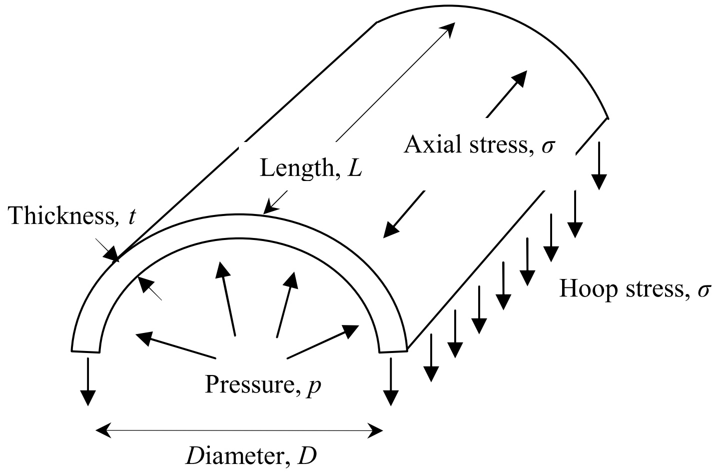

Figure (A.1) Stress in pipeline subjected to internal pressure due to gas flow

In table (A.1) the design factor is reported for various class locations:

Class location	*Design factor, f_F*
1	0.72
2	0.60
3	0.50
4	0.40

Table (A.1) Design factors for steel pipes, f_F

Temperature, K	*Derating factor*
121 or less	1.000
149	0.967
177	0.933
204	0.900
232	0.867

Table (A.2) Temperature derating factor, f_T

In Table (A.3) some examples of class 1 through class 4 are given (Menon, 2005). (The class location unit (CLU) is defined as an area that extends 220 yards on either side of a 1 mile pipe):

Class	Examples
1	Offshore gas pipelines, onshore pipelines with CLUs having 10 or fewer buildings intended for human occupancy
2	CLUs having more than 10 but fewer than 46 buildings
3	CLUs having 46 or more
4	CLUs where buildings with four or more stories above ground exist

Table (A.3) Examples of pipeline class locations

The temperature derating factor f_T is equal to 1.00 to gas temperature 121°C as indicated in Table (A.2). In Table (A.4) the seam joint factor, f_E, for some cases are given after ASTM and API specifications:

Specification	*Pipe type*	*Seam joint factor*
ASTM A53	Seamless	1
ASTM A691	Electric fusion welded	1
ASTM A53	Furnace lap welded	0.8
ASTM A134	Electric fusion arc welded	0.8
API 5L	Furnace lap welded	0.8
ASTM A53	Furnace butt welded	0.6
API 5L	Furnace butt welded	0.6

Table (A.4) Pipe seam joint factors, f_E

Appendix 6

Surge volumetric flow rate

The surge line of a centrifugal compressor has a parabolic function in the form of Equation (A.34) (Pugnet, 2006):

$$\frac{p_d}{p_s} = 1 + \lambda m_{surge}^2 \qquad \text{(A.34)}$$

λ is a constant coefficient that is a characteristic of the centrifugal compressor. Pressure ratio between discharge and suction sides at surge point, p_d/p_s can be obtained from Equation (A.36) as a function of surge isentropic head, h_{surge}:

$$h_{surge} = \frac{Z_s R T_s}{M} \frac{\kappa}{\kappa - 1} \left[\left(\frac{p_d}{p_s}\right)^{\frac{\kappa-1}{\kappa}} - 1 \right] \qquad \text{(A.35)}$$

$$\frac{p_d}{p_s} = \left[\frac{M}{Z_s R T_s} \frac{\kappa - 1}{\kappa} h_{surge} + 1 \right]^{\frac{\kappa}{\kappa-1}} \qquad \text{(A.36)}$$

By replacing this ratio in Equation (A.34) as shown in Equation (A.37), surge mass flow rate can be calculated as a function of the surge isentropic head. By substituting volumetric flow in the place of mass flow rate using Equation (A.38), it is obtained as presented in Equation (A.39).

$$1+\lambda m_{surge}{}^{2}=\left[\frac{M}{Z_s RT_s}\frac{\kappa-1}{\kappa}h_{surge}+1\right]^{\frac{\kappa}{\kappa-1}} \tag{A.37}$$

$$m_{surge}=\frac{p_s M}{Z_s RT_s}Q_{surge} \tag{A.38}$$

$$Q_{surge}=b_7\left(\left(\frac{Z_s RT_s}{Mp_s{}^{2}}\frac{\kappa-1}{\kappa}h_{surge}+\left(\frac{Z_s RT_s}{p_s M}\right)^{2}\right)^{\frac{\kappa}{\kappa-1}}-\left(\frac{Z_s RT_s}{p_s M}\right)^{2}\right)^{1/2} \qquad b_7=\frac{1}{\lambda^{1/2}} \tag{A.39}$$