

- Title:** Some innovative types of fuzzy bi-ideals in ordered semigroups
- Author/Authors:** Faiz Muhammad Khan, Nor Haniza Sarmin, Asghar Khan, Hidayat Ullah Khan
- Abstract:** An ordered semigroup (algebraic structure) is a semigroup together with a partial order that is compatible with the semigroup operation. In many applied disciplines like computer science, coding theory, sequential machines and formal languages, the use of fuzzified algebraic structures especially ordered semigroups play a remarkable role. A theory of fuzzy sets in terms of fuzzy points on ordered semigroups can be developed. In this paper, we introduce the concepts of  $(\alpha, \beta)$ -fuzzy bi-ideals and  $(\overline{\beta}, \overline{\alpha})$ -fuzzy bi-ideals of ordered semigroups, where  $\alpha, \beta$  is an element of  $\{ \text{is an element of } (\gamma), q(\delta), \text{ is an element of } (\gamma) \text{ boolean AND } q(\delta), \text{ is an element of } (\gamma) \text{ boolean OR } q(\delta) \}$ ,  $(\overline{\beta}, \overline{\alpha})$  is an element of  $\{ (\text{is an element of } (\gamma)) \text{ over bar}, (q(\delta)) \text{ over bar}, (\text{is an element of } (\gamma)) \text{ over bar boolean AND } (q(\delta)) \text{ over bar}, (\text{is an element of } (\gamma)) \text{ over bar boolean OR } (q(\delta)) \text{ over bar} \}$ ,  $\alpha \neq$  is an element of  $(\gamma)$  boolean AND  $q(\delta)$  and  $\beta \neq$  is an element of  $(\gamma)$  over bar  $(q(\delta))$  over bar and some related properties are investigated. The important milestone of this paper is, to link ordinary bi-ideals and fuzzy bi-ideals of types  $(\text{is an element of } (\gamma), \text{ is an element of } (\gamma) \text{ boolean OR } q(\delta))$  and  $((\text{is an element of } (\gamma)) \text{ over bar}, (\text{is an element of } (\gamma)) \text{ over bar boolean OR } (q(\delta)) \text{ over bar})$  using level subset  $U(\mu; r)$ . Special attention is paid to  $(\text{is an element of } (\gamma), \text{ is an element of } (\gamma) \text{ boolean OR } q(\delta))$ -fuzzy bi-ideals and  $((\text{is an element of } (\gamma)) \text{ over bar}, (\text{is an element of } (\gamma)) \text{ over bar boolean OR } (q(\delta)) \text{ over bar})$ -fuzzy bi-ideals.