

Comparisons of Heat Map and IFL Technique to Evaluate the Performance of Commercially Available Cloud Providers

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Abstract—Cloud service providers (CSPs) offer different Service Level Agreements (SLAs) to the cloud users. Cloud Service Brokers (CSBs) provide multiple sets of alternatives to the cloud users according to users requirements. Generally, a CSB considers the service commitments of CSPs rather than the actual quality of CSPs services. To overcome this issue, the broker should verify the service performances while recommending cloud services to the cloud users, using all available data. In this paper, we compare our two approaches to do so: a min-max-min decomposition based on Intuitionistic Fuzzy Logic (IFL) and a Performance Heat Map technique, to evaluate the performance of commercially available cloud providers. While the IFL technique provides simple, total order of the evaluated CSPs, Performance Heat Map provides transparent and explanatory, yet consistent evaluation of service performance of commercially available CSPs. The identified drawbacks of the IFL technique are: 1) It does not return the accurate performance evaluation over multiple decision alternatives due to highly influenced by critical feedback of the evaluators; 2) Overall ranking of the CSPs is not as expected according to the performance measurement. As a result, we recommend to use performance Heat Map for this problem.

Index Terms—Cloud Computing; Decision Recommendation; Cloud Brokering; Evaluation; Performance Heat Map; IFL

I. INTRODUCTION

It is a difficult task for normal cloud users to select appropriate cloud services from different Cloud Service Providers (CSPs). Cloud Service Broker (CSB) facilitates the selection of the CSPs according to users' requirements from multi-cloud environment. To ensure users that their services are reliable, CSPs define Service Level Agreement (SLA). The provisions of SLA offers may motivate cloud users during the product selection. SLA [1] is composed of different *Quality of Service* (QoS) rules, which are obligations that have to be followed by the CSPs. Generally, Cloud service users (CSUs) or CSBs consider the SLA offered by CSPs to select the cloud services [2]. In reality, CSPs may not deliver cloud services according to their SLA commitments [3]. In such situation cloud users may not receive exact services according to their expectations. Thus, it is necessary to develop a cloud brokering framework which considers both offered QoS and delivered

QoS to analyze the performance of cloud providers and recommend services to the users according to their expectations [4].

In this paper, we present the comparisons of our two approaches: Intuitionistic Fuzzy Logic (IFL), formulated in [5], initially proposed by Wang [6] for web service selection, and Heat Map Performance Table to evaluate commercially available CSPs based on service performance delivered by them [3]. For the CSP evaluation, we collect the service delivery performance of CSPs using commercially available service monitoring tools. Not all of the properties expressed in the SLA provided by CSPs are directly measurable. While evaluating performance of CSPs, we include CSUs feedback to include non-measurable parameters. IFL evaluation technique can provide the total order of CSPs as a result of the CSP evaluation system. Heat Map Table is more explanatory and less influenced by inconsistencies in users' opinions.

The paper is organized as follows: Section II gives the general overview of the related works. Framework used to measure and recommend the commercially available cloud providers with performance measurement from different CSPs under selected criteria and sub-criteria are mentioned in Section III and Section IV respectively. Section V gives the overview of IFL and Heat Map techniques to evaluate the performance of CSPs. Attribute-wise evaluation and ranking by both techniques are provided in Section VI. Result discussions with conclusions are presented in Section VII and VIII.

II. RELATED WORKS

An IFL technique for the web service selection based on consumer's vague perception which was introduced by P. Wang [6]. It was further applied for the selection of the best cloud provider according to service offer and service delivery from commercially available cloud providers [5]. S. Wang et al. [7] presented a collaborative web service selection approach to enhance the traditional trustworthy service selection approaches using two metrics: 1) reputation provided by other service's recommendation and 2) interaction frequency among its web services. It can be also implemented in cloud service selection by collaborative evaluation but this approach may not

be implemented for new cloud services or the services with no or very low reputation records.

A framework SMICloud [8] was proposed for comparing and ranking cloud services. It considers only quantifiable SLA attributes defined in CSMIC [9] and as such it does not consider the qualitative attributes. It implements an analytic hierarchy process (AHP) [10]. The main difficulty in provider ranking based on an AHP technique is assigning the hierarchy of SLA attributes. Practically, each SLA attributes are important and dependent on each user's preferences; it may be therefore difficult to assign their hierarchy.

Low-level performance metrics of Cloud services such as CPU utilization and network throughput etc. are considered in CloudCmp [11] to compare the performance of different Cloud services such as Amazon EC2, Windows Azure and Rackspace. Such low-level performance metrics could be further used to create models of high-level system properties, such as power consumption or performance [12], but it has not been included in CloudCmp. Qu et al. [13] have proposed a cloud service selection process based on consumer experience and involving the third party to avoid a biased assessment of cloud services from users, however, it does not cover the performance measurements from cloud providers. SLA offers provided by CSPs search in Service Ranking System proposed by [14] rather than service delivered by providers to rank the CSPs. Wagle et al. [3] propose the Performance Heat Map technique to evaluate the performance of commercially available CSPs.

To the best of our knowledge, in all the mentioned works, either SLA offers or low-level performance metrics by providers are considered to evaluate the performance of CSPs. Authors in [5] and [3] consider both offered SLA and delivered service performance to evaluate the CSPs based on Intuitionistic Fuzzy Logic (IFL) and Heat Map Table respectively. In this paper, we compare mentioned two evaluation techniques for the feasibility in current cloud brokering scenario.

III. SERVICE EVALUATION FRAMEWORK

Figure 1 shows a framework for evaluating service performance delivered by CSPs including cloud users experience. For the evaluation propose, we choose three commercial cloud providers; *Amz*, *Ela* and *GMO*. Cloud auditors (*Auditor₁* and *Auditor₂*) monitor the service performance from CSP premises. *Auditor₁* and *Auditor₂* measure low-level performance metrics of CSP services of commercially available cloud providers. These values are mapped to SLA parameters so that measured performance can be compared with SLAs offered by cloud providers to validate these values against offers of CSPs. In this work, two cloud auditors for the service measurement and two cloud users for the customer feedback are chosen for the evaluation. Number of cloud providers, cloud users and auditors and criteria and sub-criteria can be modified in the proposed framework. Because of the space constraints and for the sake of simplicity, we chose only three cloud providers for comparison of the feasibility both techniques in the real scenario.

TABLE I: Criteria and sub-criteria for evaluating cloud services

Criteria	Sub-criteria	Short Name
Availability (C1)	Uptime(c11)	upT
	Downtime(c12)	dwT
	Outage Frequency(c13)	ouT
Reliability (C2)	Load Balancing(c21)	LB
	MTBF(c22)	$MTBF$
	Recoverable(c23)	Rcv
Performance (C3)	Latency(c31)	Lat
	Response time(c32)	rsT
	Throughput (c33)	tpT
Cost (C4)	Storage Cost (c41)	stC
	VM instance cost(c42)	snC
Security (C5)	Authentication(c51)	auT
	Encryption(c52)	enC
	Audit-ability(c53)	auD

To include both measurable attributes and non-measurable attributes, five main performance criteria are chosen: *Availability*, *Reliability*, *Performance* as functional criteria, *Cost* and *Security* as non-functional criteria. They measure quality of cloud computing services with respect to the important requirements of the cloud users [5]. Under each main criterion, sub-criteria are defined (See Table I) for both evaluation techniques.

IV. PERFORMANCE MEASUREMENT

Two cloud auditors (*CloudAuditor₁*, *CloudAuditor₂*) are used to evaluate the performance of the services of cloud providers. Table III and Table IV present the measurement of performance of three potential CSPs collected during seven

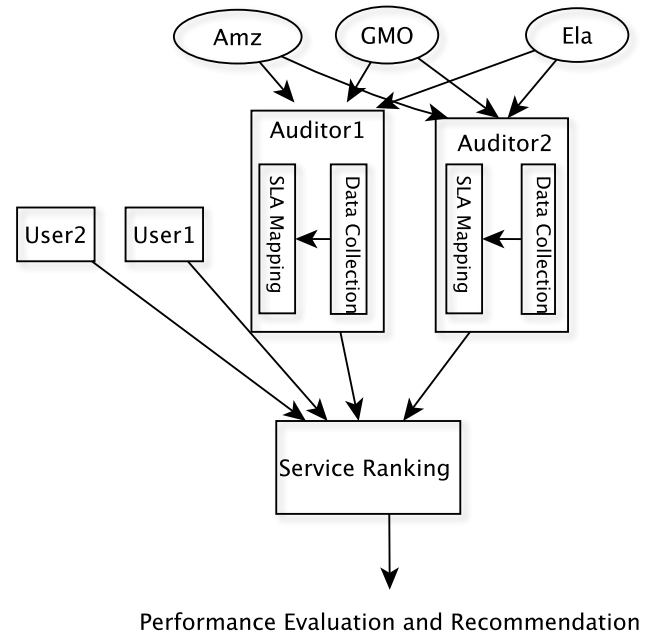


Fig. 1: CSP Evaluation Framework

TABLE II: Internet Connection between Local Test Environment and Cloud Providers

Name of CSP	Short Form	Website	Downlink (Mbps) (256 Kbps-10 Mbps)	Latency (ms)
GMOCLOUD -US	GMO	www.us.gmocloud.com	1.31	506
Amazon S3	Amz	aws.amazon.com/s3/	36.3	40.5
Elastic Host	Ela	www.elastichosts.com	12.53	34

days of observations. Presented values are subject to the specifics of each cloud auditor (which are not controlled by the proposed system), observation period length and start. These values are further used as external input data for the evaluation control as an example, but we do not guarantee its accuracy. Internet connection between local test environments to cloud providers is presented in Table II. All service users are considered based on Luxembourg. Measured values are merged performance of all the regions provided by particular cloud providers. For the user feedback, performance rating of $User_1$, $User_2$ are considered randomly. $CloudAuditor_1$ uses CloudHarmony (www.cloudharmony.com) monitoring tool and $CloudAuditor_2$ uses Monitis (www.monitis.com) monitoring tool. ‘NA’ stands for missing data. Measurements of cloud service performance of each CSP depend on cloud auditors due to differences in their methodologies and monitoring environment. In some cases there are contradictory results. Cost parameters are directly referred from CSPs websites. Service measurement of the selected CSPs covers only the computing and storage service.

TABLE III: Service Measurement by $CloudAuditor_1$

Cloud Provider	SLA Delivered													
	Auditor 1 (Measurement)													
	Availability (C1)			Reliability (C2)			Performance (C3)			Cost (C4)		Security (C5)		
	Uptime (c11) %	Downtime (c12) (min)	Outage (c13) (frequency)	Loadbalancing (c21)	MTBF (c22) (min)	Recoverable (c23)	Latency (c31) (ms)	Response Time (c32) (ms)	Throughput(c33) (Mbps)	Storage Cost (c41) (\$/month)	Snapshot cost (c42) \$	Authentication (c51)	Encryption (c52)	Audibility (c53)
GMOCLOUD -US	44.70	2.3	1	Yes	10	G	210	NA	4.24	0.17	NA	Yes	Yes	Yes
Amazon S3	100.00	1	1	Yes	H	G	47.73	NA	40.81	0.04	NA	Yes	Yes	Yes
Elastic Host	99.97	18.52	7	Yes	18hr	VP	84.05	NA	10.07	0.1	0.03	Yes	Yes	Yes

V. SERVICE QUALITY EVALUATION TECHNIQUES

A. IFL Technique

Intuitionistic Fuzzy sets (IFS) introduced by Atanassov et al. [15] and [16] incorporate the degree of hesitation called hesitation margin (π), which enables to include the confidence of opinion in decision recommendation process. In this section, we describe the CSP evaluation technique based on IFS [5], which provides the group decision based on auditors’ service measurement and consumer’s perception, which was first proposed by Wang [6] for web service selection using consumers’ vague perception.

TABLE IV: Service Measurement by $CloudAuditor_2$

Cloud Provider	SLA Delivered													
	Auditor 2(Measurement)													
	Availability (C1)			Reliability (C2)			Performance (C3)			Cost (C4)		Security (C5)		
	Uptime (c11) %	Downtime (c12) (min)	Outage (frequency)(c13)	Loadbalancing (c21)	MTBF (c22) (min)	Recoverable (c23)	Latency (c31) (ms)	Response Time (c32)(ms)	Throughput (C33)(Mbps)	Storage Cost (c41) (\$/month) \$	Snapshot cost (c42) \$	Authentication (c51)	Encryption (c52)	Audibility (c53)
GMOCLOUD -US	98.9	5	4	Yes	H	F	NA	282.1	NA	0.17	NA	Yes	Yes	Yes
Amazon S3	98.82	8	6	Yes	H	G	NA	505.71	NA	0.041	NA	Yes	Yes	Yes
Elastic Host	100	0	0	Yes	VH	VG	NA	59.81	NA	0.1	0.025	Yes	Yes	Yes

The general steps followed in an IFL technique are: 1) Evaluators first select the evaluation criteria, 2) evaluators involved in evaluation provide the importance of each criteria in the form of linguistic terms (in IFN value), 3) Similarly, evaluators also give the performance evaluation in the form of linguistic terms. These two parameters (IFN values) are further manipulated according to criteria mentioned in this section to reach into the final performance evaluation and ranking.

1) *Ranking Algorithm:* Decision makers involved in service evaluation determine the importance of each criteria according to the linguistic terms defined in Table V and calculate aggregated importance of the criterion [6], $w(c_j)$, where $w(c_j)$ is calculated using intersection operator of Intuitionistic Fuzzy Number (IFN) number defined by Atanassov [16] with all the IFN values provided by decision makers. To provide the aggregate weight of the sub-criteria under the same criteria [5], aggregate weights of the subcriteria is calculated using $W(c_j) = c_{i1} \cap c_{i2} \cap \dots \cap c_{in}$. Performance Matrix (X)(See Table XI) is created to evaluate each of the alternatives and gives the corresponding score for each alternative.

Finally, Max-Min-Max composition (T) defined by Biswas [17] and De et. al [18], $Z^k(CSP_i)$ is calculated from Performance Matrix (X) and $w(c_j)$ using $Z^k(CSP_i) = T(x_{ij}^k, w(c_j))$, where x_{ij} are the elements of Performance Matrix (X) and k is the number of evaluators. Average of $Z^k(CSP_i)$ is denoted as $Z(CSP_i)$, to aggregate the score of the decision makers/evaluators. To evaluate the performance and rank order of alternatives, score functions $S_w(C_j)$ and $S_{CSP}(CSP_i)$ are calculated using $S_w(C_j) = \mu_w(C_j) - \pi_w(C_j) \cdot \nu_w(C_j)$ and $S_{CSP}(CSP_i) = \mu_z(CSP_i) - \pi_z(CSP_i) \cdot \nu_z(CSP_i)$. The values of $S_{CSP}(CSP_i)$ can be interpreted as a total order, or ranking, of CSPs, e.g. the highest value maps to the highest rank and lowest value maps to the lowest rank

TABLE V: Linguistic terms for the Importance of a Criteria and Performance Rating

Importance of Criteria	Performance rating	IFNs	Measured Value
Very unimportant (VU)	Very Poor (VP)	$[0.1 \ 0.9 - \pi]$	$<min_{value}$
Unimportant (U)	Poor (P)	$[0.3 \ 0.7 - \pi]$	min_{value}
Medium (M)	Fair (F)	$[0.5 \ 0.5 - \pi]$	th_{value}
Important (I)	Good (G)	$[0.7 \ 0.3 - \pi]$	max_{value}
Very Important (VI)	Very Good (VG)	$[0.9 \ 0.1 - \pi]$	$>max_{value}$
I do not know (N)	I do not know (N)	$[0.0 \ 0.0]$	Data not received

for that particular SLA parameter. μ and ν represent a degree of membership and degree of non-membership to the element [15] respectively.

2) *Auditors' Measurement and Cloud User's Feedback Mapping in IFN*: CSPs define typically multiple breaking points, which define the level of quality of delivered service. Value of service credit, i.e. a discount, is offered by a CSP for each of the levels. The minimum value (min_{value}), the maximum value (max_{value}) and the threshold value (th_{value}) are set in order to compare SLA offer of CSPs. These values are used to determine the linguistic term for the measured value (See Table V). For example, if a measured value is greater than max_{value} , it is interpreted as Very Good (VG). If measured value is less than min_{value} it is interpreted as very poor (VP). If cloud provider does not allow collecting the service status or some parameters are missing from cloud provider, it is interpreted as I do not know (N).

B. HeatMap Technique

Heat Map table evaluation approach sorts the CSPs into marginal performance quantile classes to rank the providers with multiple performance criteria in an increasing or decreasing order [19], [20], [3]. The general steps of the Heat Map Technique followed in this evaluation are: 1) sort the potential cloud providers into marginal quantile classes, 2) cloud providers are ranked, according to the defined multiple ordinal performance criteria. The performance criteria are sorted in decreasing order of correlation with the ranked position, 3) the order of providers is visualized in the form of performance heat map.

Performance quantile classes are mapped to colors on a scale from *dark red* (worst) to *dark green* (best) in the performance Heat Map visualization (See the color legend for 7-tiles in Table XVII). Let x, y, \dots are the observed performances of CSPs in X where X is the set of CSPs evaluated on a single real performance criteria. The performances such that $p\%$ of the observed n in X are less or equal to $q(p)$ where $q(p)$ is estimated by linear interpolation from the cumulative distribution of the performances in X .

Obviously, it is a multiple criteria ranking. In this technique, CSPs are ranked in decreasing order from the overall best performing to the overall worst performing. It is also possible to rank providers in an increasing order from the

TABLE VI: Ordinal Level and Interpretation of auditor measurement in Ordinal Value

Linguistic terms	Ordinal Value	Measured Value
Very poor(VP)	0	$<min_{value}$
Poor (P)	1	min_{value}
Fair (F)	2	th_{value}
Good(G)	3	max_{value}
Very Good(VG)	4	$>max_{value}$
No Value	NA	Data not received

worst to the best performing. Overall ranking is computed using bipolar outranking situations Bisdorff [21], which define that alternative x outranks an alternative y when there is a *significant majority of criteria* that warrant an '*better than*' relation between them and there is no *considerable counter-performance* observed between when considering x in place of y [3].

1) *Auditors' Measurement and Cloud User's Feedback Mapping in Heat Map Table*: As shown in table VI collections of measured values are mapped into an ordinal performance scale consisting of five levels: from *Very Poor* (0) to *Very Good* (4). In this method, decision makers can provide the importance of the criteria using different values of weights. In our evaluation, weights are assigned to each sub-criteria to make equally significant with weight assigned in IFL evaluation (See Table X) to make comparable with both techniques. Average weights are assigned for combined evaluations of all cloud auditors and users. We obtain the following sets of significance weights of the criteria and sub-criteria for cloud auditors and users:

1) For cloud auditors

$$w_A = w_{c11} + w_{c12} + w_{c13} = 6.0; w_R = w_{c21} + w_{c22} + w_{c23} = 6.0; w_P = w_{c31} + w_{c32} + w_{c33} = 6.0; w_C = w_{c41} + w_{c42} = 4.0; w_S = w_{c51} + w_{c52} + w_{c53} = 3.0$$

2) For $user_1$

$$w_A = w_{c11} + w_{c12} + w_{c13} = 3.0; w_R = w_{c21} + w_{c22} + w_{c23} = 3.0; w_P = w_{c31} + w_{c32} + w_{c33} = 3.0; w_C = w_{c41} + w_{c42} = 4.0; w_S = w_{c51} + w_{c52} + w_{c53} = 3.0$$

3) For $user_2$

$$w_A = w_{c11} + w_{c12} + w_{c13} = 6.0; w_R = w_{c21} + w_{c22} + w_{c23} = 3.0; w_P = w_{c31} + w_{c32} + w_{c33} = 3.0; w_C = w_{c41} + w_{c42} = 4.0; w_S = w_{c51} + w_{c52} + w_{c53} = 3.0$$

4) Average of cloud auditors and users for combined evaluations

$$w_A = w_{c11} + w_{c12} + w_{c13} = 5.25; w_R = w_{c21} + w_{c22} + w_{c23} = 4.5; w_P = w_{c31} + w_{c32} + w_{c33} = 4.5; w_C = w_{c41} + w_{c42} = 4.0; w_S = w_{c51} + w_{c52} + w_{c53} = 3.0$$

where $w_{c_{ij}}$ represents the significance weight assigned to sub-criterion j under criterion i . As monitoring environment of $CloudAuditor_1$ and $CloudAuditor_2$ are different, their mapping reference tables are independent. Values of Max_{value} , min_{value} and th_{value} for each sub-criteria are defined for each cloud auditor basing on their SLAs. Ordinal values obtained by the mapping values from Tables III and IV are presented in Tables VII and VIII, respectively.

TABLE VII: Service Mapping to ordinal value measurement by *CloudAuditor*₁

Cloud Provider	SLA Delivered														
	Auditor 1 (Mapped)														
	Availability (C1)			Reliability (C2)			Performance (C3)			Cost (C4)			Security (C5)		
	Uptime (c11) %	Downtime (c12) (min)	Outage (c13) (frequency)	Loadbalancing (c21)	MTBF (c22) (min)	Recoverable (c23)	Latency (c31) (ms)	Response Time (c32)(ms)	Throughput (c33) (Mbps)	Storage Cost (c41) (GB/month) \$	Snapshot cost (c42) \$	Authentication (c51)	Encryption (c52)	Auditability (c53)	
GMOCLOUD -US	0.00	3	3	4	1	3	2	NA	1	3	NA	4	4	4	
Amazon S3	4.00	3	3	4	3	3	4	NA	4	4	NA	4	4	4	
Elastic Host	2.00	2	1	4	3	1	3	NA	2	3	4	4	4	4	

TABLE VIII: Service Mapping to ordinal value measurement by *CloudAuditor*₂

Cloud Provider	SLA Delivered														
	Auditor 2(Mapped)														
	Availability (C1)			Reliability (C2)			Performance (C3)			Cost (C4)			Security (C5)		
	Uptime (c11) %	Downtime (c12) (min)	Outage (c13) (frequency)	Loadbalancing (c21)	MTBF (c22) (min)	Recoverable (c23)	Latency (c31) (ms)	Response Time (c32)(ms)	Throughput (c33) (Mbps)	Storage Cost (c41) (GB/month) \$	Snapshot cost (c42) \$	Authentication (c51)	Encryption (c52)	Auditability (c53)	
GMOCLOUD -US	1	3	3	4	3	2	NA	4	NA	3	NA	4	4	4	
Amazon S3	2	2	2	4	3	3	NA	3	NA	4	NA	4	4	4	
Elastic Host	4	4	4	4	4	4	NA	4	NA	3	4	4	4	4	

TABLE IX: Performance Evaluation by *CloudAuditor*₁

criteria	tpT	Lat	upT	stC	MTBF	auD	enC	auT	snC	rsT	Rcv	LB	ouT	dwT
weights	2.00	2.00	2.00	1.00	2.00	1.00	1.00	1.00	2.00	2.00	2.00	2.00	2.00	2.00
tau(*)	1.00	1.00	1.00	0.67	0.67	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Amz	4	4	4	4	3	4	4	4	NA	NA	3	4	3	3
Ela	2	3	2	3	3	4	4	4	4	NA	1	4	1	2
GMO	1	2	0	3	1	4	4	4	NA	NA	3	4	3	3

Color legend:

quantile	0.14%	0.29%	0.43%	0.57%	0.71%	0.86%	1.00%
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(*) tau: Ordinal (Kendall) correlation between marginal criterion and global ranking relation.

TABLE X: The ratings of importance weight of each criteria by decision makers

Criteria	Sub-criteria	Auditor1	Auditor2	User1	User2	w(cj)	W(cj)
Availability(C1)	Uptime(c11)	VI(0.0)	VI(0.0)	I(0.1)	M(0.1)	[0.5,0.4]	[0.5,0.5]
	Downtime(c11)	VI(0.0)	VI(0.0)	I(0.1)	M(0.1)	[0.5,0.4]	
	Outage(c13)	VI(0.0)	VI(0.0)	I(0.1)	M(0.0)	[0.5,0.5]	
Reliability(C2)	Loadbalancing(c21)	VI(0.0)	VI(0.0)	I(0.0)	I(0.2)	[0.7,0.3]	[0.5,0.4]
	MTTB(c22)	VI(0.0)	VI(0.0)	I(0.0)	I(0.0)	[0.7,0.3]	
	Recoverable(c23)	VI(0.0)	VI(0.0)	I(0.0)	M(0.1)	[0.5,0.4]	
Performance(C3)	Latency(c31)	VI(0.0)	VI(0.0)	I(0.0)	M(0.0)	[0.5,0.5]	[0.5,0.5]
	Responsetime(c32)	VI(0.0)	VI(0.0)	I(0.1)	I(0.0)	[0.7, 0.3]	
	Throughput(c33)	VI(0.0)	VI(0.0)	VI(0.0)	VI(0.0)	[0.9,0.1]	
Cost(C4)	Installation cost(c41)	U(0.1)	U(0.1)	VI(0.0)	VI(0.0)	[0.3,0.6]	[0.3,0.6]
	Running cost(c42)	U(0.1)	U(0.1)	VI(0.0)	VI(0.0)	[0.3,0.6]	
Security(C5)	Authentication(c51)	VI(0.0)	VI(0.0)	M(0.1)	M(0.1)	[0.5,0.4]	[0.3,0.7]
	Encryption(c52)	I(0.0)	U(0.1)	U(0.1)	U(0.0)	[0.3,0.7]	
	Auditability(c53)	VI(0.0)	VI(0.0)	M(0.2)	U(0.1)	[0.3,0.6]	

TABLE XII: Decision Matrix $Z(CSP_i)$

	AMZ	GMO	Ela
Auditor1	[0.9, 0.1]	[0.7, 0.3]	[0.7, 0.3]
Auditor2	[0.7, 0.1]	[0.7, 0.1]	[0.7, 0.1]
User1	[0.9, 0.1]	[0.9, 0.1]	[0.9, 0.1]
User2	[0.9, 0.1]	[0.9, 0.1]	[0.9, 0.1]
Avg	[0.24992, $3e^{-10}$]	[0.24977, $7.5e^{-5}$]	[0.24977, $7.5e^{-5}$]

VI. EVALUATION

A. Evaluation by IFL technique

Using performance matrix assigned from decision makers from Table XI and weight ratings of each criteria $W(c_j)$ from Table X, $Z(CSP_i)$ is calculated to assign the scoring value for each criteria (See Table XII). The corresponding values of $Z(CSP_i)$ with each SLA criteria C_j signifies the group decision provided by 4 decision makers for 3 alternative CSPs. To evaluate the performance of the CSPs, aggregated value of all the criteria is calculated. Scoring values of each SLA criteria are calculated in Table XIII which provide the scoring for each SLA parameter under different CSPs. On the basis of scoring value, cloud user selects the appropriate cloud provider according to his/her requirement in each SLA parameter. Average value of $Z(CSP)$ is high for the CSP *Amz* and low for the CSP *Ela* and *GMO* and (See Table XII), however, difference is very low. So, the overall ranking is of the selected CSPs is: $Amz > Ela = GMO$. Performance Criteria *Cost*, *Security* and *Availability* of *Amz* show better performance than for *GMO*. *Ela* shows better performance in *Availability*, *Reliability* and *Performance* than others. However, the performance of *GMO* is very low in comparisons with *Ela* and *Amz* (See Table XIII) in most of the performances and does not seem equal with provider *Ela* in overall performance.

B. Evaluation by Heat Map Table

In Table IX it can be observed that, for *CloudAuditor*₁, providers *Amz* shows on sub-criteria *tpT*: *Performance*, *upT*: *Availability*, *MTBF*: *Reliability* the best performance(4), whereas cloud provider *GMO* shows the worst performance (0) on sub-criteria *upT*: *Availability* and bad performance (1) on sub-criteria *tpT*:*Performance*, *MTBF*: *Reliability* but cloud provider *GMO* shows bad performance only on

TABLE XI: Performance Matrix

Performance Evaluator	Cloud Provider	Performance Measurement													
		Availability (C1)			Reliability (C2)			Performance (C3)			Cost (C4)		Security (C5)		
		Uptime (c11) %	Downtime (c12) (min)	Outage (c13) (frequency)	Loadbalancing (c21)	MTBF (c22) (min)	Recoverable (c23)	Latency (c31) (ms)	Response Time (c32)(ms)	Throughput (c33) (Mbps)	Storage Cost (c41) (GB/month) \$	Snapshot cost (c42) \$	Authentication (c51)	Encryption (c52)	Audibility (c53)
Auditor1	Amazon S3	VG(0,0)	P(0,0)	G(0,0)	VG(0,0)	G(0,0)	G(0,0)	VG(0,0)	N(0,0)	VG(0,0)	VG(0,0)	N(0,0)	VG(0,0)	VG(0,0)	VG(0,0)
	GMOCLOUD -US	VP(0,0)	G(0,0)	G(0,0)	VG(0,0)	P(0,0)	G(0,0)	F(0,0)	N(0,0)	P(0,0)	G(0,0)	N(0,0)	VG(0,0)	VG(0,0)	VG(0,0)
	Elastic Host	VG(0,0)	F(0,0)	G(0,0)	VG(0,0)	F(0,0)	P(0,0)	G(0,0)	N(0,0)	F(0,0)	G(0,0)	VG(0,0)	VG(0,0)	VG(0,0)	VG(0,0)
Auditor2	Amazon S3	F(0,0)	F(0,0)	P(0,0)	VG(0,0)	G(0,0)	G(0,0)	N(0,0)	G(0,0)	N(0,0)	VG(0,0)	N(0,0)	VG(0,0)	VG(0,0)	VG(0,0)
	GMOCLOUD -US	P(0,0)	G(0,0)	G(0,0)	VG(0,0)	G(0,0)	F(0,0)	N(0,0)	VG(0,0)	N(0,0)	G(0,0)	N(0,0)	VG(0,0)	VG(0,0)	VG(0,0)
	Elastic Host	VG(0,0)	VG(0,0)	VG(0,0)	VG(0,0)	VG(0,0)	VG(0,0)	N(0,0)	VG(0,0)	N(0,0)	VG(0,0)	G(0,0)	VG(0,0)	VG(0,0)	VG(0,0)
User1	Amazon S3	P(0,1)	F(0,2)	G(0,0)	G(0,0)	VP(0,1)	VG(0,0)	G(0,1)	G(0,0)	VG(0,0)	G(0,0)	G(0,1)	F(0,0)	G(0,0)	G(0,0)
	GMOCLOUD -US	G(0,0)	VG(0,0)	G(0,0)	P(0,0)	P(0,1)	G(0,1)	F(0,1)	G(0,0)	VG(0,0)	F(0,0)	F(0,2)	G(0,1)	VG(0,0)	F(0,0)
	Elastic Host	F(0,0)	G(0,0)	G(0,0)	F(0,0)	VG(0,1)	G(0,1)	G(0,0)	VG(0,0)	VG(0,0)	G(0,0)	G(0,0)	F(0,1)	VP(0,1)	VP(0,1)
User2	Amazon S3	VG(0,0)	VG(0,0)	P(0,0)	G(0,0)	P(0,1)	G(0,1)	G(0,0)	VG(0,0)	VG(0,0)	VG(0,0)	G(0,0)	G(0,1)	F(0,1)	F(0,0)
	GMOCLOUD -US	G(0,1)	F(0,0)	G(0,0)	G(0,0)	VP(0,1)	F(0,0)	VG(0,0)	P(0,1)	VG(0,0)	P(0,0)	F(0,0)	VG(0,0)	G(0,0)	G(0,1)
	Elastic Host	G(0,1)	G(0,0)	VG(0,0)	VG(0,0)	F(0,1)	G(0,0)	VG(0,0)	F(0,2)	VG(0,0)	G(0,0)	P(0,0)	VG(0,0)	VG(0,0)	G(0,0)

TABLE XIII: Scoring Value for CSP based on SLA Parameters $S_W(C_j)$

Amz					GMO					Ela				
C1	C2	C3	C4	C5	C1	C2	C3	C4	C5	C1	C2	C3	C4	C5
0.1964	0.1796	0.1255	0.2376	0.20498	0.10985	0.1383	0.142	0.044	0.145	0.219	0.215	0.1750	0.161	0.12

TABLE XIV: Performance Evaluation by *CloudAuditor2*

criteria	ouT	dwT	rsT	MTBF	Rev	upT	auD	enC	auT	snC	tpT	Lat	LB	stC
weights	2.00	2.00	2.00	2.00	2.00	2.00	1.00	1.00	1.00	2.00	2.00	2.00	2.00	2.00
tau(*)	1.00	1.00	0.67	0.67	0.33	0.33	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.67
Amz	4	4	4	4	4	4	4	4	4	4	NA	NA	4	3
GMO	3	3	4	3	2	1	4	4	4	NA	NA	NA	4	3
Ela	2	2	3	3	3	2	4	4	4	NA	NA	NA	4	4

Color legend:

quantile	0.14%	0.29%	0.43%	0.57%	0.71%	0.86%	1.00%
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(*) tau: Ordinal (Kendall) correlation between marginal criterion and global ranking relation.

TABLE XV: Performance Evaluation by *User1*

criteria	dwT	upT	auT	enC	rsT	MTBF	tpT	ouT	auD	snC	stC	Lat	Rev	LB
weights	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	2.00	2.00	1.00	1.00	1.00
tau(*)	1.00	1.00	0.67	0.33	0.33	0.33	0.00	0.00	-0.20	-0.67	-0.67	-0.67	-0.67	-1.00
Amz	4	3	3	4	3	1	4	3	2	2	2	2	3	1
Ela	3	2	2	0	4	4	4	3	0	3	3	3	3	2
GMO	2	0	2	3	0	0	4	3	3	3	3	3	4	3

Color legend:

quantile	0.14%	0.29%	0.43%	0.57%	0.71%	0.86%	1.00%
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(*) tau: Ordinal (Kendall) correlation between marginal criterion and global ranking relation.

sub-criteria *upT:Availability* observed by *Auditor2* (see Table XIV). Cloud provider *Ela* shows the worst performance (0) on sub-criteria *enC:Security* whereas *AMZ* shows the best

performance (4) on the same sub-criteria by *User1*. Cloud provider *GMO* shows the worst performance (0) only on sub-criteria *MTBF:Reliability* by *User2* (see Table XV) and Table XVI). In the same way, we observed that in Table XVII, for combining result of both auditors, provider *AMZ* shows the best performance (4) on sub-criteria *tpT:Performance*, *MTBF:Reliability*, *upT:Availability* as by *CloudAuditor1* whereas provider *GMO* shows the worst performance (0) the sub-criteria *upT:Availability* and bad performance (1) on sub-criteria *tpT:Performance*, *MTBF:Reliability*. Ranking position by *CloudAuditor1* and *User1* among selected CSPs is: *Amz>Ela>GMO* whereas the ranking is *Ela>Amz>GMO* by *CloudAuditor2* and *User2*.

The overall ranking by both cloud auditors is: *Amz>Ela>GMO* and it is the same when considering opinions of all auditors and users. It is important to mention that the quantile sorting result has not to be considered as a kind of service rating. When observing in the Heat Map table that a CSP is evaluated best on a criteria, this only means that its performance is to be considered best relatively to the actually given set of potentially available CSPs [3]. That is why, in case of a same evaluation for all potential CSPs, the performance will be sorted into the best quantile for all of them (see for instance in last the column in Table IX). Dark

TABLE XVI: Performance Evaluation by $User_2$

criteria	MTBF	Rcv	LB	enC	stC	rsT	ouT	dwT	auD	auT	tpT	Lat	upT	snC
weights	1.00	1.00	1.00	1.00	2.00	1.00	2.00	2.00	1.00	1.00	1.00	1.00	2.00	2.00
tau ^(*)	1.00	0.67	0.67	0.33	0.33	0.33	0.33	0.33	0.00	0.00	0.00	0.00	0.00	-0.33
Ela	2	3	4	4	3	2	4	3	3	4	4	4	3	1
GMO	1	3	3	2	4	4	1	4	2	3	4	3	4	3
Amz	0	2	3	3	1	1	3	2	3	4	4	4	3	2

Color legend:

quantile	0.14%	0.29%	0.43%	0.57%	0.71%	0.86%	1.00%
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(*) tau: Ordinal (Kendall) correlation between marginal criterion and global ranking relation.

green, respectively dark red marked performances indicate best ones; respectively worst ones, on each sub-criterion.

Ordinal correlation τ provides the dominancy role in sorting order, i.e. higher the τ value, higher the dominancy role (for instance the third row of Heat Map table in Table IX). *Performance* sub-criteria *Throughput* (tpT) and *Latency* (Lat), *Availability* sub-criteria *Uptime* (upT) evaluated by *CloudAuditor*₁, *Reliability* sub-criteria *Recoverable* (Rcv), *Availability* sub-criteria *Uptime* (upT) evaluated by *CloudAuditor*₂ and *Performance* sub-criteria *Throughput* (tpT) and *Latency* (Lat), *Reliability* sub-criteria *MTBF*, *Availability* sub-criteria *Uptime* (upT) evaluated by both auditors do influence most of the ranking recommendation. Similarly, *Availability* subscription *Uptime* (upT) and *Downtime* (dwT) evaluated by $User_1$ and *Reliability* subscription *MTBF* evaluated by $User_2$ dominated the ranking decision respectively. Table XVIII returns the complete evaluation by all evaluators where sub-criteria with $\tau=1.00$ do influence the ranking order.

VII. RESULT ANALYSIS

Both Heat Map and IFL techniques provide evaluations of three potential CSPs using the same input data. In the overall evaluation, IFL techniques gives the ranking order of CSPs: $Amz > Ela = GMO$ whereas Heat Map technique gives the overall ranking order: $Amz > Ela > GMO$ by both auditors. Similar performance evaluation is assigned in Heat Map individually for $User_1$ and $User_2$ as in IFL technique. In general, the overall position of CSPs remains the same. CSPs *Ela* and *GMO* have the same performance ranking in the overall IFL evaluation (See Table XII) but performance evaluation in individual criteria is clearly different for overall evaluation by IFL technique for *Ela* and *GMO*. Individual evaluation of *GMO* is comparatively lower than the overall evaluation of provider *Ela*.

Performance evaluation by Heat Map does not only provide the performance ranking of the CSPs but it also provides a transparent visualization of individual criteria, which separates evaluations of individual decision makers, but it still combines result of all the decision makers. In overall performance ranking (see for instance in Table XVIII), it gives convincing results according to the performance measured by cloud auditors. For instance, cloud providers *Amz* and *Ela* look comparatively close in ranking position of performance measurement. *Availability* factor sub-criteria *downtime* (dwT) and

Uptime (upT), *Performance* factor sub-criteria *Throughput* (tpT) and *Latency* (Lat), *Reliability* factor sub-criteria *MTBF* (MTBF) played dominating role for the position determination as represented by highest τ value (See Table XVIII). In case of provider *GMO*, it is clear that it has lower performance than the other cloud providers.

Positions of cloud provider *Ela* and *GMO* are the same by IFL and 2nd and 3rd in Performance Heat Map Table. IFL technique is less convincing, because individual performance is in fact very low in provider *GMO* (See Table XIII). In our observation, it is because the IFL technique is heavily guided by the opinion of the most critical performance evaluators. If any of the users provides very bad feedback of a CSP, it has strongly negative impact on the final position of that CSP. Even if other decision makers judge this CSP as the one with the best performance, the position of that CSP may be lower due to single inconsistent performance evaluator. Beside the fact that IFL can include hesitation degree, it does not increase the precision of the ranking. In spite of high multiple decision alternatives, a new HPC linear ranking algorithm for very big performance tables (up to several thousand of decision alternatives) gives the results less than 3 seconds [22] in Heat Map Technique. So, performance Heat Map is highly computationally scalable for the multiple decision alternatives. In both performance evaluation techniques (see Table X and Section V-B1) decision makers can provide the importance of their requirements in specific selection criteria.

VIII. CONCLUSIONS AND FUTURE WORK

In this paper, we analyzed the performance of three commercial providers using data from measurements and opinions on cloud providers, using IFL and Heat Map techniques. These two techniques return different overall performance rankings. Performance Heat Map provides a convincing, consensually ranked and transparent multiple criteria performance ranking of commercially available providers, contrary to the IFL technique. In our observation, the performance Heat Map is a tool that is more expressive and precise than the IFL technique in case of recommendation of cloud services to the cloud users according to their specific requirements by cloud service brokers. The performance Heat Map technique provides more convincing results when numbers of alternatives increases to more than few alternatives. Our future work includes an evaluation of big numbers of cloud providers with multiple decision criteria and sub-criteria of commercially available cloud providers using Heat Map technique.

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TABLE XVII: Performance Evaluation by all Auditors

criteria	tpT	Lat	MTBF	upT	stC	stC	Rcv	upT	auD	enC	auT	snC	tpT	Lat	MTBF	LB	auD	enC	auT	snC	rsT	Rcv	LB	ouT	dwT	ouT	dwT	rsT
weights	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	1.00	1.00	1.00	2.00	2.00	2.00	2.00	2.00	2.00	1.00	1.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
tau ^(*)	1.00	1.00	1.00	1.00	0.67	0.67	0.33	0.33	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.33	-0.33	-0.67
Amz	4	4	3	4	4	4	3	2	4	4	4	NA	NA	NA	3	4	4	4	4	NA	NA	3	4	3	3	2	2	3
Ela	2	3	2	2	3	3	4	4	4	4	4	4	NA	NA	4	4	4	4	4	4	NA	1	4	1	2	4	4	4
GMO	1	2	1	0	3	3	2	1	4	4	4	NA	NA	NA	3	4	4	4	4	NA	NA	3	4	3	3	3	3	4

Color legend:

quantile	0.14%	0.29%	0.43%	0.57%	0.71%	0.86%	1.00%
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(*) tau: Ordinal (Kendall) correlation between marginal criterion and global ranking relation.

TABLE XVIII: Performance Evaluation of All Auditors and Users

criteria	dwT	upT	tpT	Lat	MTBF	upT	auD	enC	auT	Lat	upT	auT	stC	stC	ouT	enC	rsT	MTBF	Rcv	upT	tpT	LB	tpT	ouT	auD	enC	auT	snC
weights	1.75	1.75	1.50	1.50	1.50	1.75	1.00	1.00	1.00	1.50	1.75	1.00	2.00	2.00	1.75	1.00	1.50	1.50	1.50	1.75	1.50	1.50	1.75	1.00	1.00	2.00	2.00	
tau ^(*)	1.00	1.00	1.00	1.00	1.00	1.00	0.67	0.67	0.67	0.67	0.67	0.67	0.67	0.67	0.33	0.33	0.33	0.33	0.33	0.33	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Amz	4	3	4	4	3	4	3	4	4	4	3	3	4	4	3	4	3	1	3	2	4	3	4	3	4	4	4	NA
Ela	3	2	2	3	2	2	3	4	4	4	3	2	3	3	4	0	4	4	4	4	4	4	4	3	4	4	4	4
GMO	2	0	1	2	1	0	2	2	3	3	1	2	3	3	1	3	0	0	2	1	4	3	4	3	4	4	4	NA

tpT	Lat	MTBF	LB	auD	enC	auT	snC	rsT	Rcv	LB	ouT	dwT	snC	MTBF	auD	ouT	dwT	Rcv	dwT	stC	Lat	Rcv	rsT	stC	rsT	snC	LB
1.50	2.00	1.50	1.50	1.00	1.00	1.00	2.00	1.50	1.50	1.50	1.75	1.75	2.00	1.50	1.00	1.75	1.75	1.50	1.75	2.00	1.50	1.50	1.50	2.00	1.50	2.00	1.50
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.33	-0.33	-0.33	-0.33	-0.33	-0.67	-0.67	-0.67	-0.67	-0.67	-0.67	-1.00	-1.00	-1.00	-1.00
NA	NA	3	4	4	4	4	NA	NA	3	4	3	3	2	0	2	2	2	2	2	2	2	3	3	1	1	2	1
NA	NA	4	4	4	4	4	4	NA	1	4	1	2	1	2	0	4	4	3	3	3	3	3	4	3	2	3	2
NA	NA	3	4	4	4	4	NA	NA	3	4	3	3	3	1	3	3	3	3	3	3	3	4	4	4	4	4	3

Color legend:

quantile	0.14%	0.29%	0.43%	0.57%	0.71%	0.86%	1.00%
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(*) tau: Ordinal (Kendall) correlation between marginal criterion and global ranking relation.

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