University College London

Doctoral Programme for Practising

Educational Psychologists
(DEdPsy)

Volume 1

Submitted in part fulfilment of the requirements for the Continuing Professional Development Doctorate in Educational Psychology (DEdPsy)

Paul Edward Gibbs

The influence of mathematics anxiety on pupils' choice of mental calculation strategies for two -digit addition and subtraction.

ABSTRACT

The ability to calculate mentally is a core skill in mathematics and is now a required feature of mathematics teaching. Mathematics anxiety is an established affective construct, associated with mathematical outcomes. The focus of this research is on the affective construct of mathematics anxiety and this may influence a pupil's choice of mental calculation strategy for two digit addition and subtraction. The main study (preceded by a pilot study) was divided into two parts and focused solely on Year 5 pupils; background data with permission, was obtained for each pupil. In the first part, pupils were given a mental calculation assessment, the *Myself As Learner Scale* (Burden, 1998) and the *Mathematics Anxiety Scale for Children* (Chiu & Henry, 1990; Beasley, Long & Natali, 2001). In the second part, the mental calculation strategies of pupils with either high or low mathematics anxiety was explored individually through a series of two digit addition and subtraction questions. Pupil responses were recorded, transcribed and classified.

Strategy classification particularly distinguished pupils partitioning both two digit numbers and only one of the two digit numbers. Pupils with high mathematics anxiety tend to use lower order (less effective) strategies, whereas pupils with low mathematics anxiety tend to use higher order (more effective) strategies. No gender differences were found regarding strategy use. When controlled for mathematical competence, low mathematics anxious pupils produced more accurate mental calculation, whereas high mathematics anxious pupils produced less accurate mental calculations. Implications for Educational Psychologists and teachers in schools are discussed.

TABLE OF CONTENTS

Appendices (contd)

LIST OF TABLES

ACKNOWLEDGEMENTS

I would like to thank the following people:

My wife Elizabeth, for her unwavering support and encouragement.

Dr Sean Cameron at UCL for his invaluable advice and guidance during the course of the DEdPsy programme who as a personal tutor has contributed to my professional development.

The pupils and teachers who agreed to take part in the study.

My colleagues for their genuine interest in this research.

CHAPTER 1: INTRODUCTION, BACKGROUND AND LITERATURE REVIEW CRITIQUE

1.1 Introduction

Affective issues play a central role in mathematics learning and instruction. When teachers talk about their mathematics classes, they seem just as likely to mention their students' enthusiasm or hostility towards mathematics as to report their cognitive achievements. Similarly, inquiries of students are just as likely to produce affective as cognitive responses; comments about liking (or hating) are as common as reports of instructional activities, (McLeod, 1992, p. 575).

Mathematics anxiety is a construct that has received increased attention in recent years, but for some has become a 'euphemism for debilitating test stress, low self-confidence, fear of failure and negative attitudes towards mathematical learning', (Bessant, 1995). Previous research in this area has predominantly focused on mathematics students and adults. This research focuses on mathematics anxiety in older primary school children and in particular on the influence of mathematics anxiety on children's use of mental calculation for two digit addition and subtraction. Its importance is seen in its association with negative consequences in avoidance and achievement and ultimately into longer- term consequences in adulthood.

1.1.1 Statement of the Problem

The ability to calculate 'in your head' is an important part of coping with society's demands and managing everyday events. The National Curriculum and the Framework for Teaching Mathematics make it clear that children should learn number facts by heart and be taught to develop a range of mental strategies for quickly finding from known facts a range of range of related facts they cannot recall rapidly, (QCA, 1999, p. 3).

Mental calculation is a central focus of the National Numeracy Strategy that is now in place in schools in the UK. As a consequence, school mathematics for primary children has changed quite dramatically. It establishes set time for mathematics teaching, emphasising multiplication tables knowledge and has a bias against the use of calculators. Furthermore, there is a clear emphasis on mental calculation, a structured three-part lesson, lesson objectives and on the further training of teachers. As a result, there appears to be a need to integrate affective issues into research on cognition and to question the effectiveness of teaching strategies. Furthermore,

Maths anxious individuals report disruption in their everyday activities involving number and maths, such as balancing a cheque book or figuring out a restaurant bill, as well as in school-related activities such as taking a standardised maths achievement test or in-class exams, (Ashcraft, Kirk and Hopko, 1998, p. 176).

Mathematics anxiety is now an established construct having been established in both popular and professional literature for the past thirty years. Richardson and Suinn, the original proponents of an instrument designed to measure mathematics anxiety defined this construct in the following way:

Feelings of tension and anxiety that interfere with the manipulation of numbers and the solving of mathematical problems in a variety of ordinary life and academic situations, (Richardson and Suinn, 1972, p. 551).

Mathematics anxiety is associated with serious consequences. Some researchers, for example, Martinez and Martinez (1996), suggest most mathematics anxiety is learned at a very early age often in pre-school or the early primary school age years. However, we need to question what is the effect of mathematics anxiety on a child's capacity to calculate mentally and what is its relationship to general self-perception.

1.1.2 Rationale

1.1.2.1 Changes in the Mathematics Curriculum

During the last thirty years, there have been many significant changes in the mathematics curriculum. Traditionally the content has focused heavily on arithmetic and measurement. The introduction of the Cockcroft Report (1982) and the National Curriculum consultative document (DES, 1987) heralded a call for both a revision of the mathematics taught in schools and particularly how it could be delivered to better meet the needs of modern society. The Cockcroft Report became most well known for Paragraph 243. This stated that Mathematics teaching at all levels should include opportunities for:

- Exposition by the teacher,
- Discussion between teacher and pupils and between pupils themselves,
- Appropriate practical work,
- Consolidation and practice of fundamental skills and routines,
- Problem solving, including the application of mathematics to everyday situations,
- Investigational work.

However, despite encouraging developments in mathematical investigations, in pupil methods and in handling data, there has been a genuine feeling of imposition and lack of consultation by teachers in schools. The Cockcroft report was followed by several initiatives, particularly an imposed National Curriculum, and a high level of national testing at ages 7, 11, 14, and 16. The National Numeracy Strategy now in place, provides a framework for teaching and learning, that whilst offering opportunities for some, may actually disadvantage the more vulnerable learners in schools.

1.1.2.2 Teacher Style

Teachers have the task of delivering the National Curriculum and currently receive a high level of training in mathematical techniques and practices by government enlisted consultants. Askew et al (1997) noted the lack of research on the important issues of teaching and learning mathematics. They identified three possible mathematics teaching styles (connectionist, transmission, discovery), Although the 'connectionist teacher' was seen as the most effective teacher of numeracy these styles need to be seen in context, where the social aspect of learning is also vital to the development of pupils knowledge in mathematics (Jaworski, 1999).

In order to characterise mathematics teaching in the classroom, Jaworski & Potari (1998) proposed a cyclical model in their concept of a teaching triad (management of learning/sensitivity to students/mathematical challenge). Wright, Martland & Stafford (2000, 2002) adapted the work of Cobb, Steffe and colleagues in developing a framework/model for the stages of early arithmetic learning. They identified specific techniques derived from a constructivist perspective in order to support children who may have early arithmetic difficulties.

Teachers' beliefs are very likely to influence their practice in the classroom; these beliefs may also be highly resistant to change. A range of studies has investigated teachers' subject knowledge and personal beliefs in mathematics. These include Manouchehri & Goodman (1998) who identified that teachers' pedagogical knowledge influence how they teach and learn; and Middleton & Spanias (1999) who found that mathematics teachers' personal theories are shaped by the work setting. Aubrey (1997) concluded that all primary teachers need to have a sound conceptual knowledge of mathematics and a deeper awareness of the rich, informal mathematical knowledge children bring into school. Teachers need to be aware that if primary school childrens' perceptions play an important role in their achievement in the classroom, then these perceptions are also very likely to be formative in shaping subsequent achievement over many years. It follows that teachers need to take account of these on a regular basis and to provide feedback to children in order to maximise their mathematics achievements.

It is possible to identify two broad approaches for the development of mental computational skills, (Reys, Reys, Nohda & Emori, 1995). These approaches characterise the teacher as either a transmitter of knowledge or as an intellectual coach. The former holds that proficiency with mental calculation skills is gained through direct teaching and practice, whereas the latter sees mental calculation as a process of higher order thinking in which more than the application of a mental algorithm is involved. Current research on class teaching tends to have been informed by a view of learning and knowledge development which stresses an active information processing approach which supports the active role of the learner in constructing meaning, (Aubrey, 1997).

1.1.2.3 Constructivism and Mathematics Learning

Educational practices have previously been very strongly influenced by behaviourism. Despite research reflecting a move from children being purely processors of information to acting purposefully in an evolving mathematical reality of their own making, it is apparent that a highly transmissionist (and behaviourist) model is now in place. Much of the mathematics taught in classrooms is carried out in what is called a transmission mode; the expert teacher transmits knowledge to the learner, Wright et al (2002). Their recent work in the Mathematics Recovery Programme reflects a strong constructivist theme that mathematical knowledge cannot be passed onto children. A similar theme is seen in the 'realistic' mathematics programme in the Netherlands.

A constructivist view of knowledge and knowing has been elaborated by many researchers including Cobb, Yackel & Wood (1992), Simon (1995) and Steffe (1991). The important notion that knowledge is actively constructed by the learner, not passively received from the environment, and that coming to know is a process of adaptation based on and constantly modified by a learner's experience of the world was originally encapsulated by Von Glaserfeld (1987). Skemp (1979) described knowledge as the name we give to conceptual structures built from and tested against our own experiences of reality, he made an original contribution in differentiating between instrumental learning (facts/rules) and relational learning (structures/concepts). He regarded all teaching as an intervention in the learning process; from this perspective successful co-operation between the teacher of mathematics and learner is more likely to occur if the learner chooses the long-term goals.

Skemp's view of knowledge and relational learning fits neatly with the theoretical perspective of this study, that stems in part from a constructive view of knowledge and knowing (Cobb, Yackel & Wood, 1992; Simon 1995a, Simon 1995b, Von Glaserfeld 1987) and from a Vygotskian view of teaching as creating zones of proximal development. The ways in which teachers structure classroom enquiry can greatly influence students' views of mathematics and can lead students to develop more powerful conceptual structures in the process (Cobb, Yackel & Wood, 1992). The interpretations of constructivism can also be differentiated on epistemological grounds dealing with beliefs about knowledge, ways of knowing and learning Hofer & Pintrich (1997) and Schoenfeld (1983) pointed out that general problem solving strategies are not detailed enough to account for mathematical knowledge. The

research in this study links to other research in reflecting the paradox of a cognitive problem and the search for a behavioural solution.

1.2: Background

1.2.1 The National Framework

The National Numeracy Strategy provides a clear opportunity to study the emphasis of a government/teacher imposed strategy limiting the range of mental strategies that pupils may have to call on. Clearly, the strategies that pupils do have to call upon are built on prior experience and knowledge. It has also been argued that most mathematics anxiety has its roots in the teaching and teachers of mathematics (Stodolsky, 1985). It is likely that children are not mathematics anxious before they go to school but may quickly become mathematics anxious due to the imposition of a strictly defined and target/objective driven curriculum. Their teachers and parents may also have a recurring fear of failure in this subject. Although the main focus of this research is concerned with pupils who are broadly of average ability, it is also seen against the current emphasis on encouraging the mainstreaming of children with Special Educational Needs. Social inclusion places emphasis on the social character of learning and the use of mathematics as a social tool. In general, there is a return to significant whole class teaching in classrooms, together with some withdrawal for children with particular needs. One consequence is that classroom teachers need to consider carefully when to accept a single mathematics performance as adding to their knowledge of a pupil i.e. teacher biases need to be taken into account when making judgements about pupils.

Teachers can strongly influence their pupil's performance in the classroom; achievement motivation can be effected through careful instructional design. At its best, the National Curriculum/National Numeracy Strategy provides a framework for teachers and schools in which to work; with a fuller knowledge of their pupils' mental calculation strategies, teachers should be in a better position to design effective educational experiences for their pupils. Essentially, children in classes where their teachers have high self-efficacy are more likely to demonstrate a greater use of flexible mental calculation strategies. Furthermore, it will be proposed that there is an identifiable hierarchy of mental calculation strategies for two-digit addition and subtraction. In addition, it is considered that teachers' expectations will be correlated with their pupils' ability to cope efficiently with basic mental facts.

1.2.2. Mental Calculation Strategies

However, the effect of the National Numeracy Strategy is that children do indeed need to be taught mental methods, (DfEE, 1999). Thompson's model of mental calculation strategies (Thompson, 1999) adds *attitudes* to the essential facts, skills, and understandings that are needed. It does appear to be the case however, that without the self-confidence to try, pupils will be unlikely to progress with mental calculation. The National Numeracy Strategy: Teaching Mental Calculation Strategies (QCA, 1999) provides mental calculation strategies for each year group, it also gives teaching strategies for mental calculation. From these it is possible to derive a mental calculation assessment for a year group and to deduce that there may be a hierarchy of such calculation strategies. Relevant literature sources in this area include Beishuizen (1993), Fuson et al (1997), Murray & Oliver (1989), Nicholls et al (1990) and Thompson (2000). Threlfall (2002) provides a comprehensive review of seven different mental calculation classification systems; Foxman & Beishuizen

(2002) provide a thorough re-analysis of mental calculation strategies based on a large sample of pupils in the UK. In order for children to have at their disposal a range of mental calculation strategies, they need to have a sure feel for number. An emphasis on imposed strategies is likely to limit children's responses. There is now a need to look for alternative strategies, and those based on instrumental learning and heuristic methods.

1.2.3 Self-Perception

Self-concept differs from self-esteem in that it is a context-specific assessment of competence to perform a specific task. Self-esteem is measured at a broader level, whereas self-concept has been consistently related to academic achievement. However, there is a need for a greater awareness of the possible effect of pupil self-perception in the mathematics/numeracy classroom and how teacher-pupil beliefs/values may affect learning in mathematics. This would particularly appear to be the case of gender, where further research could investigate male/female differences related to teacher influences on achievement. Few researchers have explored the relationships between self-esteem, self-concept, and academic performance; the results have been inconsistent.

Some children, who have developed mental calculation strategies, continue to experience difficulty in their execution; this may be due to either a generally low self-efficacy or a specific low self-efficacy with mental calculation. Just as there have been difficulties in capturing a teacher's self-efficacy, so there have been difficulties in developing an instrument to gain insight into children's perceptions of their own self-efficacy regarding academic performance. Understanding more about such beliefs is likely to have important implications for both curricular design and teaching strategies. Two recent inventory scales considered to be applicable for this study are Morgan-Jinks Student Efficacy Scale (Jinks & Morgan, 1999) and the Myself as A Learner Scale as constructed by Burden (1998). Both scales are appropriate for younger children, aimed at a lower age from around 8/9 years of age. Bandura (1986) observed that confidence is both a personal and a social construct. Collective systems such as classrooms/schools develop a sense of collective efficacy or groups shared beliefs in its capability to attain its goals and accomplish desired tasks. Teachers may well be best served by paying as much attention to pupils' perception of competence as to actual competence. Consequently, it should be helpful to obtain a measure of each pupil's self-efficacy in Mathematics.

1.2.4 Affect and Mathematics Anxiety

The construct of mathematics anxiety is now well established and has been shown by researchers to impact significantly on achievement and career choice. (Hembree, 1990, Ma 1997). Less attention has been given to children's mathematics anxiety and the influence it may have on their academic attainment and particularly on their mental calculation strategies. However, Ma (1999) did carry out a meta-analysis of the relationship between anxiety towards mathematics and achievement in mathematics; a negative correlation was found across a number of attributes e.g. gender, ethnicity. However, there were differences according to the instrument used. The Mathematics Anxiety Rating Scale is now an established instrument designed to capture and measure this construct. However, there has been considerable debate concerning the dimensionality of the mathematics anxiety construct and perhaps, a surprising lack of research with younger children.

It is now more widely accepted that younger children are capable of making an informed judgement regarding affective issues and their perception of themselves as learners. There is a distinction to be drawn between mathematics self-concept and mathematics anxiety. The former refers to the perceptions of personal ability to learn and perform tasks in mathematics. The latter refers to feelings of tension that interfere with the manipulation of numbers and the solving of problems in a wide variety of ordinary and academic situations. However, affective pathways for learning may be either positive or negative. It is popularly regarded that some anxiety is necessary for task completion but too much anxiety has a regressive effect. Mathematics anxiety may be either facilitative, debilitative or have no affect.

1.2.5 Important Ideas and Issues Arising

Some of the important ideas and issues that arise are:

- Research studies have shown that it is possible to classify mental calculation strategies. However, there is less evidence to say which are more influential or are of greater efficiency. Furthermore, there appears to be no research regarding the influence of mathematics anxiety on strategy choice.

- What are the implications of teaching children who are highly maths anxious? Do any common errors emerge when they are adding/subtracting mentally and do we need to examine the validity of verbal reports in children's addition and subtraction? Furthermore, do we need to reconsider the use of calculators?

- What is the relationship between working memory and children's mental addition and subtraction and how is this related to the Dutch research and The Empty Number Line?

- What is the influence of children's learning conceptions or learning styles in the final years of primary schooling? There is a need to examine research on affect, particularly pupils' beliefs about their mathematics, their emotional responses to mental calculation, and the effect of repetition. Gender effects may be important i.e. Do girls perform significantly differently from boys? Do they use different strategies? Furthermore, to what extent do the strategies used by less able pupils differ from those used by the more able? There is a need to examine possible confounding factors in this research.

- Classroom teachers frequently want immediate practical solutions to areas of concern in their classrooms. This is understandable but there is also a need to have a model for informing decisions. In this study there is a strong constructivist theme that acknowledges children as learners who actually construct and process, rather than being simply recipients of teacher (government) prescribed learning. This view is compatible with a social view of learning (Vygotsky) and is important in that children need to be encouraged mathematically to articulate their strategies.

1.3: Literature Review Critique

1.3.1 Introduction

In the research literature there now exist two bodies of research, namely mathematics anxiety and mathematical cognition; the former is regarded as an affective factor whereas the latter refers to the arithmetic and mathematical processes, for example those used in addition and subtraction. Until recently these two areas of research had been studied separately; the work of Ashcraft and colleagues, for example Ashcraft and Kirk (2001) is an example of research attempting to explore these two areas of study. This study is an attempt to explore one specific area, namely the influence of mathematics anxiety on pupils' use of mental calculation strategies for two digit addition and subtraction. The literature highlights several consequences of high mathematics anxiety and offers some alternative classifications for mental calculation. However, the research that has been carried out in integrating these areas has tended to focus on reaction times. Less attention has been paid to the range of possible strategies that could be used to carry out a calculation and none from an affective perspective involving more naturalistic research, exploring and investigating the issues involved in primary school children.

The National Numeracy Strategy, a government directed programme, is now in place in mainstream schools. This clearly raises the profile of children's mental calculation skills, with a mental/oral starter recommended for the start of each mathematics/numeracy lesson. There are also prescribed strategies in the form of 'Guidance for Teaching Mental Calculation Strategies' (QCA, 1999). This guidance firmly focuses on expectations and does not consider the affective issues that may influence strategy choice. Teachers enter classrooms with their own teaching expectations and beliefs; what they do and how they approach mental calculation strategies with their pupils will be driven by their own histories and possibly by their own self-efficacy in relation to these skills. Self-efficacy differs from self-concept in that self-efficacy is a context specific assessment of competence to perform a specific task, that is "an individual judgement of his/her capabilities to perform given actions", Schunk, (1991). Similarly, Tschannen-Moran, Woolfolk-Hoy and Hoy (1998) define self-efficacy as the self-perception of competence rather than actual competence. Although not considered individually in this research, self-efficacy is a possible factor and should be seen in the context of self-perception and mathematics anxiety.

In this country children are currently assessed using national mathematics tests more than any other country in the world. At the time of starting school children experience a Baseline Assessment, together with tests at Key Stage 1 and Key Stage 2. Local Education Authorities may in addition have their own tests of Mathematics (Numeracy) in place. In an important paper advocating formative assessment, Black & Wiliam (1998) argue that in addition to these tests, a further tier of innovations aimed as raising standards has been imposed on the educational system. They suggest that these innovations have not been successful as was anticipated because they have neglected to take into account the importance of classroom practice.

The revised Code of Practice for Children with Special Educational Needs (DfES, 2001) for schools places a great deal of emphasis on obtaining pupil views. There is now an extensive literature regarding methods of eliciting pupils' views of their classroom experiences but much less has focused on methods eliciting pupil self-perceptions, and perhaps surprisingly, academic self-concept. The National Numeracy Strategy (implemented Autumn 1999) requires all primary children to be taught a daily Mathematics lesson, where it is recommended that the whole class work together for a large proportion of the time and that oral/mental work should be significant features. Two important aspects of the National Numeracy Strategy are its 'Framework for Teaching Mathematics' and an accompanying programme of training and professional development. These and other developments have led to an emphasis on target setting; as a consequence, Black & Wiliam (1998) believe it important to move to formative, rather than summative assessment that has generally been previously used by teachers in school. Research has shown that teachers still rely on summative measures to inform them about their pupils' learning. However, they do also carry out formative assessment, although their criteria in the past whilst based on sound experience is often implicit and not clearly articulated. Consequently these judgements have been underrated, seen as subjective and accorded less status than the numerically-quantifiable results of standard written tests, although they may actually be seen as reflecting attainment with greater subtlety.

This study adopts the broad and functional definition used by the National Numeracy Strategy:

"Numeracy is a proficiency which involves confidence and competence with numbers and measures. It requires an understanding of the number system, a repertoire of computational skills and an inclination and ability to solve number problems in a variety of contexts. Numeracy also demands practical understanding of the ways in

which information is gathered by counting and measuring, and is presented in graphs, diagrams and tables.

(The National Numeracy Strategy for Teaching Mathematics from Reception to Year 6, DfEE, 1999, p. 4)

It is clear from recent surveys that both adults and pupils in this country tend to score poorly on international numeracy tests both in comparison to similar countries and in comparison to our earlier performance. Children clearly bring with them into the classroom, a variety of strengths and weaknesses. These may be more clearly evidenced through mental calculation strategies. (The term mental calculation is preferred to mental arithmetic, given the negative history that has been associated with this subject).

Theoretical models from Psychology underpin this research; broadly these are constructivism, mental calculation strategies, self-perception and mathematics anxiety. The most over-arching of these is constructivism, it is a perspective that has received substantial support from a variety of researchers in the United States but is now becoming more evident in this country's literature. Some recent models of mental calculation strategies, for example Thompson (1999) and the Dutch Realistic Mathematics Programme, for example Beishuizen (1993) can be seen from a constructivist perspective

.

1.3.2 Background Perspective: Constructivism

Curriculum models may be determined by our beliefs about how children learn; research studies that reflect a move from behaviourism, through seeing children's learning informed by developmental stage to viewing them as constructing their own learning should have significant practical implications. The literature reflects this move from seeing children purely as processors of information to acting purposefully in an evolving mathematical reality of their own making. This study stems from:

- This constructivist view of knowledge and knowing has been elaborated by Cobb, Yackel & Wood (!992), Simon (1995a), Simon (1995b) and Steffe (1991),

- The very important notion that knowledge *is actively constructed by the learner, not passively received from the environment, and that coming to know is a process of adaptation based on and constantly modified by a learner's experience of the world was* originally encapsulated by Von Glaserfeld (1987). Jaworski (1993) developed the idea of constructivist learning in mathematics and most recently, Jaworski and Potari (1998) proposed the Mathematics Teaching Triad.

Skemp (1979) described knowledge as the name we give to conceptual structures built from and tested against our own experiences of reality. Skemp made an original contribution in differentiating between instrumental learning (facts/rules) and relational learning (structures/concepts). He regarded all teaching as an intervention in the learning process, from this perspective successful co-operation between the teacher of mathematics and learner is more likely to occur if the learner chooses the long-term goals.

Skemp's view of knowledge and relational learning fits neatly with the theoretical perspective of this study, that stems in part from a constructive view of knowledge and knowing and from a Vygotskian view of teaching as creating zones of proximal development. The ways in which teachers structure classroom enquiry can greatly influence students' views of mathematics and can lead students to develop more powerful conceptual structures in the process (Cobb et al, 1991). The interpretations of constructivism can also be differentiated on epistemological grounds dealing with beliefs about knowledge, ways of knowing and learning. Hofer & Pintrich, (1997) and Schoenfeld (1983) pointed out that general problem solving strategies are not detailed enough to account for mathematical knowledge.

Steffe (1992) provided a model of young children's mathematical learning; from this perspective mathematics learning is seen as a process in which children reorganise their activity to resolve situations that they find problematic. Moos (1973) theoretical perspective concerned diverse psychosocial environments being classified along three dimensions (relationships/personal development/systems maintenance and change) leading to the consideration of characteristics of classroom environments that are likely to promote active learning and participant satisfaction.

Burden & Fraser (1983) highlighted the move from individual centred to systems oriented and consultancy based approaches. It may be inferred from the literature, that characteristics of the classroom learning environment can account for a considerable amount of variance in a number of important outcome measures, for example, achievement, attitudes, anxiety and learning style.

1.3.3 Formative Assessment

During recent years the literature suggests that there has been a shift towards formative or diagnostic assessment, reflecting a move towards greater interest in the interactions between assessment and classroom learning. Black & Wiliam (1998) produced a thorough review of the literature on formative assessment. They acknowledged the importance of expectations in social settings. Tittle (1994) offers an Educational Psychology framework for assessment of teaching and learning, along three dimensions. It is postulated that teachers' and pupils' relationships in the classroom are determined by their epistemological, psychological and pedagogical beliefs. Black and Wiliam make the important distinction between 'fit and match' in that if a student answers a teacher's question correctly, then they assume that their understanding matches the teachers that may not necessarily be the case. Treagust, Duit & Fraser (1996) leaning on Ausabel's (1968) notion that *the most important single factor influencing learning is what the learner already knows,* investigated pupil's preinstructional conceptions. They emphasise the point that these conceptions are likely to be strongly held and resistant to change. Their work was seen within the larger framework of constructivism. Torrance & Pryor (1995), Pryor and Torrance (1996) investigated teacher assessment in Key Stage 1 and Key Stage 2 classrooms. Their work arose from behaviourist and social constructivist perspectives. It was reiterated that the important aspect of teacher assessment was to find out what the child already knows and warned that well-intentioned reinforcement practices may actually decrease rather than increase pupil learning. Similarly Gipps, McCallum & Brown (1996), Tunstall & Gipps (1996) evaluated assessment practices and described four types of teacher assessment. These studies collectively call for more divergent teacher assessment, query the accuracy of teacher judgements and suggest developing more critical appraisals.

1.3.4 Mental Calculation Strategies

The National Numeracy Strategy places emphasis on mental calculation; it is debatable whether or not the strategy readily lends itself to the collection of information by teachers, in their heads or on paper, of their pupils' current mental strategies and number understanding. Children who experience significant difficulties in mathematics/numeracy may actually be further disadvantaged by this strategy, particularly where there is a heavy focus on whole class teacher directed lessons. The teachers' need for diagnostic information related to prescriptive teaching has been a long-term concern (Denvir & Brown, 1987). Approaches such as those reported by Denvir & Brown (1986) lean on Ausabel's conception starting from *what the learner already knows,* and Skemp's *relational understanding.* McIntosh, Reys & Reys (1992) proposed a framework for examining basic number sense. The National Numeracy Strategy employs the concept of an empty number line drawn from Dutch studies into arithmetic. Beishuizen (1993, 1997, and 1999) proposed the use of such a line and compared this to the 100 Number Square, commonly seen in schools. A progression can be seen in children working with physical apparatus such as blocks or counters through the 10 by 10 (100 Number Square) to the use of the empty number line. The empty number line has specific advantages as it offers opportunities both for formal and informal mental calculation strategies.

The literature on mental calculation research differentiates between simple and complex mental calculation. (Faust, Ashcraft & Fleck, 1996) Simple mental calculation is seen in working with single digit numbers and has been reported on, particularly by Ashcraft (1992, 1995). Complex mental arithmetic involves the manipulation of numbers greater than 10.

Mental Strategies for addition and subtraction have been described in the following way:

Mental Strategies are more about the application of known or quickly calculated number facts in combination with specific properties of the number system to find the solution of a calculation whose answer is not known.

(Thompson, 1999b, p. 2)

More succinctly, Fuson (1992), a formative researcher in mental calculation refers to a mental calculation strategy as a *solution procedure*. However, there now exists more theoretical work beyond simple mental facts e.g. McCloskey (1992). Not only is it important to draw a distinction between mental calculation and mental recall but also between a knowledge of facts and a knowledge of procedures (Ashcraft, Kirk & Hopko, 1998). Whereas the former refers to fact retrieval, the latter and a focus of this study involves the processes of carrying, borrowing and keeping track in a multi-step problem. Furthermore, an area for mental calculation research that has been neglected until quite recently is that of estimation. The Cockcroft Report (1982) raised the importance of this skill and differentiated between several aspects e.g. working out a rough answer or making a judgement whether or not a calculated answer is reasonable. It would appear reasonable to expect estimation and mental calculation to be related, although neuropsychological work where calculation is impaired in some patients but not estimation may be difficult to explain. Dowker (1997) studied estimation in young children aged 5 to 9 years; she found that estimation proficiency depended both on the arithmetical competence of the child and on the level of problem difficulty. However, the study pointed to a direct relationship between estimation and calculation but did contain large variances querying the trends found.

Boulton-Lewis & Tait (1994) looked at children's addition strategies and derived 13 possible strategies. They found that children preferred to use verbal/mental strategies and would only use a written algorithm if no other way were possible. Fuson et al (1997) reported on children's conceptual structures for multidigit numbers and methods of multidigit addition and subtraction. Murray & Oliver (1989) proposed a model of understanding of two-digit numeration and computation. More recently, Heid, Blume, Zbiek, & Edwards (1999) reported on factors that influence teachers' learning to do interviews to understand students' mathematical understanding. They cited that a major goal of mathematics teaching should be the development of students' understandings of deep and evolving connections between mathematical ideas, facts and procedures. However, if interviews are to be useful tools then greater understanding will be needed regarding their use. They found that teachers' beliefs and expectations affected what they were exploring. For example, concepts, the curriculum, or a particular set of skills.

Mental calculation research may conveniently be divided into *numbers less than 20* and *numbers greater than 20.* When working with numbers less than 20, these are count all, count on from first number, count on from larger number, use known number fact and derive a number fact (Denvir & Brown, 1986, Thompson, 1995). However, there is more controversy regarding a classification system for strategies involving the addition and subtraction of numbers from 20 to 100 (DfEE, 1999; QCA, 1999; Thompson, 2000a, 2000b). Research has shown that children can invent their own strategies. Carpenter et al (1997) carried out a longitudinal study about addition strategies for numbers greater than 10 and found that invented strategies led to fewer errors. The issue as to whether mental strategies should be left to spontaneous development or taught in a didactic order has been discussed, (Beishuizen & Anghileri, 1998). Their research highlights the influence of the Dutch Realistic Mathematics Education and particularly the use of the empty number line. The implication for schools in the UK is that teachers, who are likely to have a greater experience of investigative work, should consider Dutch approaches, which are likely to have a stronger influence at a metacognitive level. The realistic programme design, based on flexible solution procedures has been compared to a gradual programme design based on procedural computation, (Klein, Beishuizen & Treffers, 1998). Essentially the realistic programme design produced a higher level of flexible problem solving in children. Furthermore, there is also a strong link between theory and practice, reflecting a move from a behaviourist (traditional teaching methodology) towards a constructivist model That is, teaching is no longer seen as a treatment and learning as the effect. For the constructivist, pupils are learners who actively construct mathematics. Furthermore, mental arithmetic (calculation) plays a central role in this respect, stimulating not only conceptual understanding and procedural proficiency but also number sense and the understanding of number relations, (McIntosh, Reys & Reys, 1992)

There have been a number of attempts to produce classification schemes for two digit addition and subtraction, for example, Beishuizen (1993), Beishuizen & Foxman (2002), Fuson et al (1997) and Thompson (2000a, 2000b). Although the classification of these strategies is very similar, there are also differences. The QCA (1999) Guidance On Teaching Mental Strategies At Key Stages 1 and 2 proposes seven strategies whereas Thompson (1999) proposed four main strategies, Beishuizen (1999) proposed five and Blote et al (2000) proposed six main mental calculation strategies. Thompson (1999) suggested the following classification system: the *split* method (57+35 as 50+30=80; 7+5=12; 80+12=92) and jump method (93-35 as 93-30=63; 63-5=38). A variation of partitioning is the *mixed* method (84-46 as 80-40=40; 40+4=44; 44- 6=38), and an extension method is the *compensation* method (47+36 as 50+36=86: 86-3=83). (Thompson, 1999). It may also be possible to identify other methods, for example, in solving difference problems. Reys, Reys & Hope (1993) researched the mental computation performance of 2nd, 5th and 7th grade children. In general they found poor skills amongst these children, but saw this as a function of the time spent on task, the effect of direct instruction and a heavy focus of textbook work.

Dutch research has been particularly influential in classifying mental calculation strategies, (Beishuizen, 1997). The acronym N10, sometimes referred to as the *jump* method, is used to signify the strategy where the first number is not split up and is either added to, or subtracted from. This contrasts with the acronym 1010, sometimes referred to as the partitioning or split method, which clearly differs from the former, in that both numbers are separated and worked before finding the answer. The former can be shown to be a more efficient method but requires work to make it effective. The latter has been more commonly seen in schools, and although it appears more naturalistic to young children, quickly becomes problematic when 'borrowing' becomes involved. A proposed classification of mental calculation strategies for two-digit addition and subtraction was developed for use in this research (Appendix 2.1 & 2.2). Its purpose was to help compare possible strategies with the actual strategies reported by children. The strategies proposed draw partly on the work of leading researchers in this area, Beishuizen & Foxman (2002) and Threlfall (2002) who have published the most recent findings. However, it may also be inferred from these studies, and from the previous classification systems that have been suggested, that there is considerable difficulty in encapsulating all pupils' idiosyncratic methods.

The mental imagery that children may use when asked to perform a mental calculation strategy is a potentially fascinating element of this study. The individual work with children, seen through their verbal and written descriptions, may be a means of accessing this imagery. This is because within the National Numeracy Strategy, there are several visual aids (e.g. empty number line, 100 square models, number fans) that can used. It would be expected that these would be reflected in the pupils' own accounts of their strategies. It is also likely that the language (mathematical articulation of strategy use) used by both teacher and pupil may foster this mental imagery or representations. This in turn may effectively question what form of pictorial representation may be most appropriate when teachers are working with a class of children. 'Is it better to have a range of techniques?' or simply focus on one particular method such as the empty number line (Beishuizen, 1999) It is also interesting that children, left to their own devices, develop their own calculation strategies for number. Furthermore, the use of an individual's own idiosyncratic methods may actually be more efficient than a teacher imposed one, Thompson, (1992); Aubrey, (1993).

In this country there is a lack of tradition in teaching mental calculation, Thompson (1999). The Cockcroft Report (1982) was a very influential document of its time and effectively raised and highlighted mental calculation. Thompson (1999) offers four reasons, derived from the literature, why it is important to teach mental calculation; he states:

1. Most calculations are done in your head.
2. Mental work develops sound number sense.
3. Mental work develops problem-solving skills.
4. Mental work promotes success in later written work.

Mental arithmetic has for many people unfortunate connotations of being placed in an anxiety-inducing situation of working out a difficult problem in their head. The literature more recently suggests that there is now a need to differentiate between recall and strategic methods. It appears that, for numbers less than twenty, there is a fairly clear hierarchy of strategic methods i.e. count all, count on from the first number, count on from the larger number, use known addition fact, and use derived fact. For numbers between twenty and a hundred, there is more controversy. However, following Thompson (1999a, 2000a, 2000b), it appears possible to suggest that there is a hierarchy of levels (in increasing efficiency or effectiveness) of mental calculation strategies. This study aims to examine the strategies that pupils may use, given that an awareness of these is likely to help a teacher understand a pupil's mathematical thinking. This may be seen as an important part contributing towards formative assessment.

Thompson (1999a) offers the following model of mental calculation that links flexible mental strategies with facts, skills, understandings and attitudes. However, research is needed to test this hypothesis. Attitudes like self-confidence are an important but neglected ingredient in mental strategy use. There is a need for pupils to be self-confident and to 'take-a-risk' when they use mental strategies. This idea echoes comments found in the earlier Cockcroft Report. In order for teachers to develop confidence in their pupils, they need to develop a supportive climate in the classroom. The question that arises from this, is 'What are the factors involved in inspiring mathematical confidence in their pupils?'

If pupils are to be successful with mental calculation, they need to have a secure knowledge of number facts, have a good understanding of the number system, the ability to perform the necessary skills and have the confidence to use them, (Klein, Beishuizen & Truffers, 1998). Building pupils' confidence whilst maintaining high expectations is central to the issue of effective classroom practice. Mainstream classrooms are now more likely to include pupils with a broader range of strengths, weaknesses and Special Educational Needs than has previously been the case. Rivera (1997) provides an overview of mathematics education up to the present day. Ginsberg (1997) offers insights from Developmental Psychology and suggests involving children who experience learning disabilities in mathematics in a 'teaching experiment' (Vygotsky's zone of proximal development).

- These highlight the issue of differentiation, schools/teachers need to consider their organisation, and larger primary schools are now seen to be more likely to set or band their pupils for mathematics.

- Teachers may need to look more closely at effective questioning in the classroom, given that the expectations of teachers/pupils can be quite different and may lead to misunderstanding. Without attention to these, there may be a greater mismatch where a task is open and process oriented (constructivist approach) and less on a skills practice task (traditional approach).

The most recent revision of *Teaching Number: Advancing children's skills and strategies* (Wright et al, 2000, 2002), describes a formative assessment of children's early number skills having the advantage of generating much more detailed information than simple summative results. Assessment attempts to determine the most advanced strategy available to the child, an idea that was used in the main study of this research. The assessment scheme described offers an important advance for the teaching of early number skills, emphasises the development of children's mental strategies and provides a link between research/theory and what is taught in the classroom. However, it continues to rely heavily on a within-child deficit model and whilst noting autonomy, engagement and child reflection, does not emphasise affective issues such as self-perception or self-confidence and does not discuss the origin and emergence of mathematics anxiety.

A recent study, Ruthven (1998) examined the use of mental, written and calculator strategies of numerical computation of 56 upper primary children. The study partly examined the effect of the National Curriculum but did emphasise the 'numeracy culture' of a particular school. From this study, it can be inferred that it is important to encourage and refine the use of mental methods from an early age and the need explicitly to teach mental methods. However, the type of computational strategy was weakly related to number concept attainment. Enochs, Smith, Huinker (2000) have developed a Mathematics Teaching Efficacy Beliefs Instrument. Mathematics teaching efficacy was found to be a predictor of student achievement. In a recent study linking teachers' beliefs and practices to mathematics instruction, Stipek, Givvin, Salmon, MacGyvers (2001) found substantial coherence among teachers' beliefs and consistent associations between their beliefs and practices. Very interestingly, teachers' self-confidence as mathematics teachers was also significantly associated with their students' self-confidence as mathematics learners. Although this particular study was based on a small sample of 21 teachers, a mixture of research methods were used (survey, questionnaire, videotape); strong links were found between teacher beliefs/practices, correctness of solutions, control, teacher ability and the use of extrinsic rewards. More broadly, the study does raise the need for teachers to exercise greater reflection on their classroom experiences.

1.3.5 Mathematics Anxiety

The construct of mathematics anxiety has produced research articles and discussion from both researchers and teacher/educators over the past thirty years. The majority of this research has focused on older students, whereas little attention has been given to mathematics anxiety in primary school children, particularly in the latter years of their schooling. Key Stage 2 (at around 9 years) represents a significant change from less formal to more formal schooling. Anxieties may emerge more acutely and may also be quite resistant to change. Furthermore, as children mature, they may become increasingly aware of their own strengths/weaknesses and are likely to have a greater awareness of themselves in relation to their peers. Anxiety lies in the affective domain, is a natural human feeling and occurs when adults and children find situations threatening or difficult. Richardson and Suinn (1972) were early leaders describing mathematics anxiety. They defined mathematics anxiety as *"feelings of tension and anxiety that interfere with the manipulation of numbers and the solving of problems in a wide variety of life and academic situations"* (p. 551); Fennema and Sherman (1976) reported *"Anxiety can be debilitating and prevent one from learning. Many people report high anxiety when faced with situations involving mathematics"*(p. 34). Dew, Galassi and Galassi (1983) investigated four basic questions about mathematics anxiety. Ma (1999) made the point that mathematics anxiety can take multidimensional forms including, for example, dislike (an attitudinal element), worry (a cognitive element) and fear (an emotional element). In general, affective issues play a central role in mathematics learning and instruction, McLeod (1992, 1994). Whilst people may express a fear/dislike of mathematics based upon early school experiences, little research has been reported on development of mathematics anxiety in primary school pupils. Gierl & Bisanz (1995) found no studies prior to 1990 but did describe two distinct forms of mathematics anxiety; test anxiety and problem solving anxiety. Their work is important in the context of this report given that their youngest participants were 8 years of age. The research had the advantages of highlighting the utility of questionnaires, and suggesting focusing on constructs that influence mathematics anxiety. In contrast, two forms of mathematics are presented without much exploration and the important issue of the possible confounding effect of ability is not discussed. Although previous research has focussed on older students and adults, it would appear that the upper junior years in primary schools are crucial for the onset of mathematics anxiety. Once established, it may be resistant to change and have consequences that threaten both classroom performance and longer-term participation.

1.3.5.1 Mathematics Anxiety Assessment

A number of instruments have been developed to measure mathematics anxiety, for example the Fennema-Sherman Mathematics Anxiety Scale (Fennema-Sherman, 1976), and the Mathematics Anxiety Rating Scale or MARS (Richardson and Suinn, 1972). The Mathematics Anxiety Rating Scale is a 98 point item inventory intended to measure mathematics anxiety and despite it being the most widely used instrument to measure mathematics anxiety questions have persisted concerning the dimensions underlying the item responses, (Alexander & Martray (1989). These researchers also produced a shortened (25 item) version of the MARS; Wigfield & Meece (1988) developed their own 22 item Mathematics Anxiety Questionnaire (for use with older primary/secondary students) and made the point that mathematics anxiety should be conceptually distinguished from perceptions of mathematics ability. This research raises the issue that test items need to avoid the possible confounding between mathematics anxiety and perceptions of ability. Suinn, Taylor and Edwards (1988) developed the Suinn Mathematics Anxiety Rating Scale for elementary school students, proposed two factors and reported anxiety scores were significantly positively correlated with achievement scores. Recent research on mathematics anxiety with its emphasis on statistical methodology and correlation analyses continues to remain subject to criticisms that have been held for many years (McLeod 1992, 1994). Significant correlations do not imply significant increases in knowledge, particularly given the difficulty in developing instruments for use in the affective domain and a seeming lack of theoretical foundation. Furthermore, the instruments developed for children are based on diluted versions of adult versions. In order to develop a suitable instrument for children, Chiu & Henry (1990) produced the Mathematics Anxiety Scale for Children. This is a 22 item, 4 point Likert scale and was reported to be an internally consistent measure of a unidimensional construct (mathematics anxiety) for school grades four to eight. More recently, Beasely, Long and Natali (2001) undertook a confirmatory factor analysis of this scale. They reported that construct validity was demonstrated i.e. correlations with state anxiety, trait anxiety and quantitative ability.

1.3.5.2 Dimensions of Mathematics Anxiety

The dimensionality of instruments such as the Mathematics Anxiety Rating Scale (MARS) is important because there is an underlying assumption particularly in younger children that mathematics anxiety is a unidimensional construct. Furthermore, mathematics anxiety has become a euphemism for debilitating test stress, low self-confidence, fear of failure and negative attitudes towards mathematics learning, (Bessant, 1995). Test anxiety appears to provide the main source of theoretical support for much of the research on mathematics anxiety, (McLeod, 1992). Evidence from a meta-analysis reveals mathematics anxiety to be distinct from attitude towards mathematics, (Ma, 1999) and distinct from either general or test anxiety. However, there has been considerable debate concerning the dimensionality of this construct. Mathematics Anxiety was regarded as a unidimensional construct by the originators of the MARS (Richardson & Suinn, 1972). This was disputed by Resnick, Viche & Segal (1982) who stated that mathematics was not unidimensional and described three forms that they labelled evaluation anxiety, arithmetic computation anxiety and social responsibility anxiety. In contrast, Gierl & Bisanz (1995) described two distinct forms, test anxiety and problem solving anxiety. In her meta-analysis, Ma (1999) reported that mathematics anxiety could take multi-dimensional forms, including for example, dislike (attitude), worry (a cognitive element) and fear (an emotional element). In their recent analysis, Beasley, Long and Natali (2001) in their confirmatory factor analysis of the Mathematics Anxiety Scale for Children, found this 22 item, shortened form of the MARS to be reliable and valid and that mathematics anxiety was a unidimensional construct.

1.3.5.2 Causes of Mathematics Anxiety

Although mathematics anxiety is unlikely to have a single cause, early mathematical experiences appear crucial for the development of mathematics anxiety. There is a consistent theme in the literature that a principal cause of the mathematics anxieties lies in the methodologies used to teach basic mathematical skills, and often originates during pupils' early educational experiences (Chui & Henry, 1990). If mathematics anxiety has its early roots in the teaching and teachers of mathematics as suggested by Stodolsky (1985) and Bush (1989) then it has followed that mathematics anxious teachers will very likely produce mathematics anxious pupils. Zatz & Chassin (1985) stressed the importance of classroom factors in test anxious children. They reported that children perform better in low threat environments, where the experience of success increases self-efficacy and in turn reduces anxiety. The confidence that pupils have in their ability to learn mathematics is affected by teacher-pupil interaction. Hart (1989) reported on gender differences and the differences experienced both by boys and by girls. The roots of mathematics anxiety are very likely to lie in the early years; its existence however, may not become apparent until the latter years with more challenging questions less based on rehearsal and memorisation of facts.

1.3.5.4 Consequences of Mathematics Anxiety

For children who are highly mathematics anxious some consequences include avoidance (Hembree, 1990), low achievement (Ma, 1999) cognitive difficulties (Ashcraft, Kirk and Hopko (1998) and restriction in conceptual thinking and memory processes, (Skemp, 1979). In a meta-analysis of 151 studies Hembree (1990) found mathematics anxiety to be related to poor performance on mathematics achievement tests, inversely related to positive attitudes towards mathematics and directly bound to subject avoidance. Pre-service arithmetic teachers were particularly prone to mathematics anxiety; females displayed higher levels than males. Hembree's findings are open to different interpretations particularly in the area of the effects of anxiety interventions. Ma (1997) carried out a meta-analysis of the relationship between anxiety toward mathematics and achievement in mathematics. This led to proposing a reciprocal relationship between attitude and achievement in mathematics and concluding that in mathematics, the feeling of enjoyment, and not the feeling of difficulty, directly affected achievement. There are clearly a range of pupil factors that influence their performance in mathematics. The research proposed will need to acknowledge/control for the other major determinants of performance, i.e. ability, gender and sociometric status. Brekelmans, Van Den Eeden, Terwel & Wubbels (1997) examined student characteristics and learning interactions in Mathematics from a resource perspective (gender, aptitude as first order resources, perceptions of their environment as second order). They suggested that if mathematics was made more interesting then gender differences would disappear.

1.3.5.5 Mathematics Anxiety and Mental Calculation Performance

It is perhaps surprising that until very recently, no studies have considered whether mathematics anxiety had any on-line effect on a pupil's mathematics performance, that is, an effect on underlying cognitive processes, as the pupil performs a mathematics task, (Ashcraft & Kirk, 2001). The notion that mental processing may differ as a feature of mathematics anxiety is a new idea and not seen in previous research. Ashcraft and Faust (1994) proposed genuine mathematics performance differences between high and low maths anxiety students. As in many studies, university undergraduates were used as subjects in reaction time based experiments. A possible confounding factor in the studies of mathematics anxiety is that of ability. In several studies, Ashcraft and colleagues ruled out maths competence as an overall confounding difference among individuals at different levels of maths anxiety. (Ashcraft and Kirk, 2001, Ashcraft, Kirk and Hopko, 1998). It may be that highly anxious children may simply present as low ability children who have learned to be test anxious because of repeated failures.

1.3.6 Working Memory

One key area of cognition is working memory. It is the term that is used to refer to the mental workplace in which information can be temporarily stored and manipulated during complex everyday activities such as understanding language and mental arithmetic, (Gathercole & Pickering, 2001). Baddeley & Hitch (1974) proposed a particular model of working memory that remains influential and is seen in many research articles. This well-known model of working memory has three sub-components (Baddeley 1986, 1992). The model comprises the central executive, the auditory rehearsal loop and the visuo-spatial sketchpad. Although these may all play a part in mathematical processes, the most important of these is the central executive; this is a highly flexible but resource-limited system responsible for a range of cognitive activities. These include the processing and storage of information, the retrieval of long-term knowledge and the scheduling of multiple concurrent cognitive activities, (Gathercole & Pickering, 2001). There is now a growing body of research attesting to the centrality of working memory to reading and reading comprehension but some although much less to mathematical cognition. However, Hitch (1978) and Adams & Hitch (1997) did explore the role of short term/working memory in mental calculation. The early study (Hitch, 1978) identified that problem solving was impaired when problems were orally presented, when the answers had to be written down and when the number of carry operations increased. The more recent study (Adams & Hitch, 1997) focused on working memory and children's mental addition, found children's mental addition spans to be higher when sums were visible throughout confirming a working memory constraint. These studies have the advantage of focusing on school aged children in the early stages of arithmetic development but raise questions about the way in which working memory deficits may explain children's arithmetic difficulties. The studies do not appear to provide direct evidence that deficits in working memory undermine children's mental calculations and that working memory deficits are specific to numeracy. Although the Baddeley and Hitch three component model of working memory has been highly influential, it has not readily captured a number of phenomena, (Baddeley, 2000). A fourth component has been proposed by Baddeley, the *episodic buffer*. This is assumed to be a limited-capacity temporary storage system that is capable of integrating information from a variety of sources. It provides a temporary interface between the slave systems (the phonological loop and the visuospatial sketchpad) and long term memory. The buffer serves as a modelling space that is separate from long-term memory but which forms an important part in long-term learning. The importance of this component is acknowledged in recent reports, for example Gathercole, Brown & Pickering (2003), Henry & MacLean (2003), Maybery & Do

(2003). However, there is little specific research regarding its functionality. Baddeley's revised model differs from the previous model particularly in focussing on the processes of integrating information, rather than on the isolation of subsystems. It appears to provide a better basis for modelling the more complex aspects of executive control in working memory.

Working memory appears to have a critical role in the proposed connection between mathematics anxiety and mathematics achievement/performance/use of mental calculation strategies. Ashcraft and Kirk (2001) demonstrated that individuals with high mathematics anxiety demonstrated smaller working memory spans, especially when assessed with a computation based span task. Overall, their results demonstrated that at an individual difference variable mathematics anxiety affects on-line performance in maths related tasks and that this effect is a transitory disruption of working memory. In particular, Ashcraft and Kirk (1994) demonstrated that high maths anxiety individuals had particular difficulty on two column addition problems due to the carry operation. Ashcraft and Kirk (2001) have found that there is not a complete confounding of mathematics anxiety and mathematics competence. For example, they reported equivalent performance across maths anxiety groups to simple one and two column addition and multiplication problems when those problems were tested in an untimed, pencil and paper format. Mental arithmetic performance relies on working memory resources. Ashcraft, Kirk and Hopko (1998) propose that this is particularly true in two circumstances; firstly when a large number of fact based problems are processed and secondly when procedural knowledge is important to the processing as in the carry operation in addition. In general, these findings are from laboratory based experiments. However, situational factors, particularly in the classroom or where others are involved are likely to be very significant factors. Furthermore, recent research studies indicate that working memory skills are closely linked with children's progress in the early years of their schooling, (Gathercole & Pickering, 2001). Consequently, it is reasonable to assert that the assessment of working memory skills should be beneficial for indicating children who may be at risk of poor progress in numeracy/mathematics.

1.3.7 Self-perception

Research on self-concept, confidence and causal attributions related to mathematics tends to focus on beliefs about the self.

McLeod (1992, p. 580)

McLeod's reconceptualization of affective issues in mathematics over the previous thirty years saw a progression from originally examining attitudes towards mathematics through beliefs about mathematics to more recent studies on the emotional and affective responses of pupils/students. Furthermore, there is now a need to complement quantitative information and data from questionnaires with potentially richer qualitative data, e.g. using the pupil's own views. Recent government legislation tends to formalise this through the emphasis on pupil participation in the Code of Practice/Special Educational Needs, (DfES, 2001). Schools are now more likely to seek pupil perceptions, although there is also likely to be considerable variation in how this is done. Recent initiatives in School Self-Evaluation and direction from OFSTED are also relevant factors. In their early years of schooling, pupils have a generally positive view of themselves and their abilities, (Hallom, Ireson & Davies, 2002). As pupils progress towards the end of primary schooling, their perceptions of themselves may be less positive. Pupils with higher attainments will very likely have a higher perception of themselves as learners than their lower attaining peers. Furthermore, in a review of the literature, Yasutake & Bryan (1995) found students with learning difficulties in mathematics to be at greater risk for negative affect. In a study using 8 to 10 year old children, Patrick, Skinner & Connell (1993) attempted to draw a distinction between children's perceived control and autonomy in the classroom. Autonomy was defined as the sense of being choiceful in one's actions and experiencing oneself as the locus of initiation of those actions. Their study used data from questionnaires but was limited in its measures of behaviour and emotion. However, it did clearly suggest that induced positive affect in the form of choice, lack of coercion and encouraging pupil views lead to more accurate outcomes and improved performances in mathematics.

Much of the fear and pain of doing mathematics has been replaced by enthusiasm and enjoyment as teachers design activities which encourage pupil participation, exploration and discovery.

Jennings & Dunne (1997, p.34)

This bold statement by two teacher educators is seen against an imposed national numeracy strategy placing a high emphasis on the efficacy of whole class teaching. It requires teachers to have detailed subject knowledge, highly refined teaching skills and particularly, an acute sensitivity to individual children. Children will bring to the classroom their own views and perceptions regarding the nature of numeracy/mathematics. Their perceptions are also likely to be greatly influenced by the culture of the mathematics classroom. This in turn is likely to be driven by a number of factors including the teachers' beliefs about numeracy/mathematics, their own numeracy/mathematics self-efficacy and their ability to connect ideas and themes that both engage and motivate the pupils in their care. It is clear that psychologists and educators place much importance on the encouragement of children's self-perceptions in the form of self-concept because of importance to the whole child and its impact on academic success.

As a result, self-perception has long been a topic of interest to psychologists. The self-construct has a long history but has also been difficult both to define and to measure. Furthermore, it does appear that the terms self-concept, self-image and self-esteem have been used interchangeably, although for Burden (1998) this was not the case. He saw self-concept most likely to be a multifaceted and possibly hierarchical construct, but also one whose research had been limited by measures and terminology. Similar ideas related to self-perception have now been held for a long period of time. For example, self-esteem may be regarded as an evaluative measure of our self-image; The classroom teacher is clearly in a powerful position to influence a child's self-esteem through a supportive learning environment and by the use of more systematic techniques. Although a lot of research has focused on this idea of effectively enhancing self-esteem, much has concerned the use of lay self-help manuals. A more productive and pertinent topic is seen in the allied topic of self-efficacy. This is an area in which there is a substantial body of thought; self-efficacy beliefs have received increasing attention in research related to education, particularly in studies of academic motivation and of self-regulation, (Pintrich & Schunk, 1995). Much of the research in this area derives from the work of the social learning theorist, Albert Bandura. Perceived self-efficacy is the sense of confidence regarding the performance of specific tasks. Bandura (1986) defines the construct as people's judgements of their capabilities to organise and execute courses of actions required to attain designated types of performances. Importantly, it is not concerned with the skills that one has but with the judgements of what one can do with whatever skills one possesses. More recently, Bandura (1993) stated that perceived self-efficacy exerted its influence through four major processes, i.e. cognitive, motivational, affective and selection. Although students' beliefs in their own perceived self-efficacy were seen to regulate their academic attainments, this work also raised the notion of collective efficacy, an important topic for schools given the likely direct effect on school achievement.

Self-concept is comprised both of descriptive and evaluative beliefs that children hold about certain characteristics, whereas self-esteem can be viewed as the global feelings and beliefs that children have about themselves as people, (Burnett, 1994). Self-concept is specific to a particular domain, for example in numeracy or mathematics, whereas self-esteem is a broader, whole child construct. Burnett & Proctor (2002) examined the relationship between learner self-concept, academic self-concept and approaches to learning. They reported that children's approaches to learning were influenced by specific personal characteristics such as their beliefs about learning, their perceived self-ability and their locus of control. It is perhaps surprising, that there have been few attempts at constructing a children's self-concept scale. A useful measure appears to be that constructed by Burden (1998). A recent alternative measure, developed in the United States, is the Children's Perceived Academic Self-Efficacy Inventory Scale, constructed by Jinks & Morgan (1999).

Burden & Proctor's study of 580 Australian pupils (mean 10.7 years) leant on earlier research in identifying three levels of approaches to learning, surface, deep and ego enhancement. Whilst the latter is not elaborated on, deep approaches to learning were associated both with low anxiety and constructivist, active learning; in contrast, surface approaches to learning were associated with high anxiety and traditional models of learning where the pupil has a passive, transmission role. It may be expected that ability is a significant factor; whilst this may be partly the case, it is not the single most important factor in determining a pupil's approach to learning. Furthermore, learner self-concept has been found to influence approaches to learning. In recent research, Burnett, Pillay & Dart (2003) reported that learner self-concept mediated between conceptions of meaning and approaches to learning. They suggest that as pupils become older then it may be necessary to have more than a set of skills and appropriate classroom environment. They need to run in parallel with beliefs about learning.

Raising self-esteem and a belief that all can succeed is a core principle of formative assessment; this is because low self-perception contributes to less effective learning in mathematics, (Ross, Hogaboan-Gray & Rolheiser, 2001). Confidence in learning mathematics/numeracy or the degree to which people feel certain of their ability to perform well, has consistently emerged as an important factor. This has been particularly so in relation to gender differences in mathematics, (Hart, 1989). Hart's study using trained observers was limited, but did indicate that boys seek more interactions than girls, with their teachers. Interestingly, high and low confidence students differed very little in their interactions with the teacher, but differences were found in interactions between different teachers. It is also clear that there are several explanations for these results; teachers may see mathematics as a 'male subject', they may also interact more with boys in order to avoid classroom disruption. It may be the case that girls are less assertive and less articulate about their mathematical reasoning than they are for example, in talking about writing. In an important study in the context of this research, Bouffard et al (2003) carried out a three-year longitudinal study regarding changes in self-perception of competence and motivation in primary school children. They report that the optimistic views of very young children decline through schooling but that as children mature and gain more schooling experience they tend to bring their self-perceptions into line with their actual performance. Some gender differences were also found where this process might happen earlier in girls than in boys.

Current classroom research in mathematics education has, at its centre, the nature of mathematics as a subject and the influence it exerts on the context in which it is taught and learned. Perceptions of mathematics as a discipline have changed among a large section of the teaching and research community, and these perceptions, in turn influence the kinds of questions being addressed in the body of research.

Nixon (2000, p. 175)

Furthermore, the roots of mathematical skill avoidance behaviours lie in mathematics classrooms, (Turner et al, 2002). For Vygotsky (1978) mediation is the main mechanism of human learning. This notion or idea represents a significant development from the Piagetian view about how pupils construct their understanding and involves taking into account social interaction in learning. Vygotsky's zone of proximal development offers a model to address these issues. This identifies the idea that someone may be able to solve a problem with help that they could not solve unaided. Vygotsky also suggested that learning may occur first through interaction with others before being internalised as a mental representation. These ideas have led to the use of the metaphor of scaffolding to capture some significant features of the teaching process. From a constructivist perspective, the teacher begins by demonstrating or modelling a new concept and than acts as a 'guide', while varying the level of support that may be needed for a particular child. It would be expected that during their interactions, pupils would construct their personal conceptual structures. Constructivism and the zone of proximal development provide models of children's learning. However, it is also clear from the literature that mathematics anxiety is an independent construct and children's attainments in mathematics are likely to be influenced by their self-perception. Although the literature supports the broad identification of strategies for two digit addition and subtraction, there is little discussion on whether or not they may be arranged hierarchically, or provide an array from which pupils may selectively choose. Furthermore, no research studies have attempted to establish the influence of self-perception or mathematics anxiety on children's use and selection of mental calculation strategies.

1.3.8 Teacher Self-Efficacy

Teachers' sense of efficacy is defined as " their belief in their ability to have a positive effect on pupil learning" (Ashton, 1985), and similarly Woolfolk, Rosoff & Hoy (1990) related this positive affect to classroom management properties. Two leading researchers this area, Woolfolk and Hoy (1990) extended the literature by examining prospective teacher's sense of efficacy. They investigated the structure of efficacy of 182 prospective teachers using the Teacher Efficacy Scale (TES; Woolfolk & Hoy, 1990). Their original TES contained 22 6-point Likert scale items, which were adapted from earlier scales developed by Gibson & Dembo (1984). In a very comprehensive review, Tschannen-Moran, Woolfolk-Hoy & Hoy (1998) examined the theoretical underpinnings of teacher efficacy; they introduced a new model and offered strategies for improving the efficacy of teachers. Their Teacher Efficacy Scale (see Appendix 6.1) was adopted for use in this study.

Brophy & Good (1970) were two of the first researchers to investigate the way in which teachers communicate different expectations to different children in their classrooms. Teacher expectations strongly influence student outcomes in high differential treatment classrooms (Jussim et al, 1998); teacher efficacy was defined as the self-perception beliefs that their efforts will bring about pupil learning. It is suggested that high efficacy teachers will differentiate instruction and will collaborate with colleagues more, they will also place more emphasis pupils' responsibility for their own learning. Ross firmly makes the point that teachers with higher expectations present an array of observable behaviours and that the most powerful source of efficacy information is the teacher's interpretation of outcomes of classroom actions. Tschannen-Moran, Hoy & Hoy (1998) provide a very comprehensive review of studies on teacher efficacy. Watson (1998) makes the point that pupils needing support may well be overlooked because their social skills or occasional successes convince the teachers that they are stronger than they really are. However, what does emerge from research/studies into teacher efficacy are two dimensions namely, *general teaching efficacy and personal teaching efficacy.*

Furthermore, teacher efficacy is higher in primary schools and classes with high ability and orderly pupils, when teacher workloads are moderate and school culture is collaborative/participatory. More recently, McKinney, Sexton & Meyerson (1999) attempted to validate empirically an Efficacy-Based Change Model. They found high efficacy was predicted by success attributions; equally Ghaith & Shabaan (1999) found that teacher efficacy was not simply about content knowledge and pedagogical skills.

The research suggests that teachers' sense of efficacy plays a powerful role in schooling. Given the importance of a strong sense of efficacy for optimal motivation in teaching, we would do well to examine how efficacy is developed, when it is most malleable, and what factors may lead to its improvement.

Tschannen-Moran, Woolfolk Hoy and Hoy, (1998, p. 234)

The literature indicates that teachers with a higher sense of efficacy exhibit greater enthusiasm for teaching have greater commitment to teaching and are more likely to stay in teaching. Clearly the study of this construct has been a fruitful area for study; the literature, however, indicates that there has been difficulty in developing a measurement tool to encapsulate it.

1.3.9 Theories emerging from the Literature Review

- Pupils actively create, experience and construct mathematical/numerical meaning which contribute to form a personal construction of the world. Pupils develop their own mental calculation strategies, which may be classified into broad groups, and these groups may be arranged in order of efficiency/effectiveness. Furthermore, where there is an identifiable hierarchy of mental calculation strategies for two digit addition and subtraction, a higher and more efficient/effective use of these strategies will be associated and linked with greater accuracy in addition or subtraction. It is apparent that there is an identifiable paradox between a cognitive problem and the search in education for a behavioural solution.

- The National Numeracy Strategy provides a clear opportunity to study the emphasis of a government/teacher imposed strategy limiting the range of mental strategies that pupils may have to call on. Clearly, the strategies that pupils do have to call upon are built on prior experience and knowledge. Current emphasis in encouraging mainstreaming of children with Special Educational Needs and social inclusion place emphasis on the social character of learning and the use of mathematics as a social tool. Teachers need to consider carefully when to accept a single mathematics performance as adding to their knowledge of a pupil i.e. teacher biases need to be taken account when making judgements about pupils.

- Mathematics Anxiety is now an established construct. However, it is apparent from a review of the literature that affective factors are both important and influential. It is hypothesised that high levels of mathematics anxiety when controlled for ability will lead to a more limited and restricted use of calculation strategies. There may be a difference between the expectations of the teacher (based on the curriculum to be delivered) and mental calculation capacity; this difference may heighten feelings of mathematics anxiety. In contrast, children with a high self-perception of themselves as learners will possess a higher use of calculation strategies leading to greater accuracy in their results. Mathematics anxiety may be the characteristic of an individual, which restricts or limits children's working memory or capacity to hold and work with skills such as two-digit addition and subtraction.

1.3.10 Under-researched Areas emerging from the Literature Review

- Self-concept differs from self-esteem in that it is context-specific assessment of competence to perform a specific task. Self-esteem is measured at a broader level, whereas self-concept has been consistently related to academic achievement. As a result, there is a need for a greater awareness of how mathematics/numeracy can affect the classroom and how teacher-pupil beliefs/values affect learning in mathematics. This would particularly appear to be so in the case of gender, where further research could investigate male/female differences related to teacher influences on achievement. Few researchers have explored the relationships between self-esteem, self-concept and academic performance; the results have been inconsistent.

- The literature review clearly raises the notion of 'collective efficacy' or 'a group's shared belief in its capabilities to realise given levels of attainment'. This appears to be a potentially useful construct, where the theoretical perspectives reviewed emphasise the social nature of the mathematical learning. The research reviewed suggests the importance of interaction is in mutually constructive situations, but more research is needed on the nature and timing of teachers' interventions. Social learning has become an area of great interest to Educational Psychologists; language for some has been seen to mediate learning. Further research could focus on whether children with different cultural backgrounds perceive the same class environment differences. Furthermore, investigating the children's mental imagery could lead to a close association with the mental calculation strategies that children may use.

- In order for children to have at their disposal a range of mental calculation strategies, they need to have a sure feel for number. An emphasis on imposed strategies is likely to limit children's responses. There is now a need to consider more closely, alternative perspectives.

Earlier research studies in mental calculation were essentially quantitative in their nature. We now have a better understanding of mental calculation; this more detailed knowledge appears to have been gained from research, which is essentially qualitative in nature. The research methodology in this study involves asking children to execute a calculation in their head and then describe how they worked it out. Ideally, interviews with children should be recorded on either video or audiotape for latter analysis.

However, an effect of the National Numeracy Strategy is that children do need to be taught mental methods, (DfEE, 1999). Thompson's model of mental calculation strategies (Thompson, 1999,) adds *attitudes* to the essential facts, skills, and understandings that are needed. It does appear to be the case however, that without the self-confidence to try, then children will be most unlikely to progress with mental calculation. The National Numeracy Strategy: Teaching Mental Calculation Strategies (QCA, 1999) provides mental calculation strategies for each year group, it also gives teaching strategies for mental calculation. From these it is possible to derive a mental calculation assessment for a year group and to deduce that there may be a hierarchy of such calculation strategies. Relevant literature sources in this area include; Beishuizen (1993), Foxman & Beishuizen (2002), Fuson et al (1997), Murray & Oliver (1989), Thompson (2000) and Threlfall (2002).

Some children who have developed mental calculation strategies, continue to experience difficulty in their execution; this may be due to either a generally low self-perception or mathematics anxiety related to mental calculation. Although mathematics anxiety has been established as a separate construct, there may be some very specific sub-divisions that are equally difficult to differentiate from each other. The literature makes it clear that there has been considerable difficulty in establishing measures for both self-perception and for mathematics anxiety. Understanding more about such beliefs may have important implications for both curriculum design and teaching strategies. Bandura (1986) observed that confidence is both a personal and a social construct. Collective systems such as classrooms/schools develop a sense of collective efficacy or a group's shared beliefs in its capability to attain their goals and accomplish desired tasks. Consequently, teachers may well be best served by paying as much attention to pupils' anxieties and self-perceptions of competence as to actual competence.

CHAPTER 2: PILOT STUDY

Title of Pilot Study

Teacher self-efficacy, pupil self-perception and their choice of mental calculation strategies.

2.1 Introduction/overview of pilot

After four years the National Numeracy Strategy had generally been well implemented and supported by schools. However, it is clear that there was a need for a review of its impact on mental calculation, particularly because of the priority given to this skill in the National Numeracy Strategy and the challenges it offers classroom teachers. Previous research, particularly that by Askew et al (1997) suggested that the most effective teachers of numeracy were connectionist and characterized by their beliefs about the nature of mathematics rather than in their content knowledge of mathematical concepts and procedures. This pilot study focused on one characteristic of effective teachers of numeracy (high self-efficacy), but particularly on pupils' self-perception of themselves as learners and their possible choice of mental calculation strategies. In addition to subject and curriculum knowledge, teachers also need pedagogic subject knowledge of pupils' understanding, attitudes and approaches towards numeracy. A positive pupil self-perception has been associated with high attainment; teachers with high self-efficacy have been associated with pupils of high self-perception. The research literature differentiates between simple (single digit) and complex (two digit) arithmetic; whilst strategies have been proposed for both, no research has sought to examine the possible influence of self-perception on strategy choice. This pilot study explores the actual mental calculation strategies that pupils say they use for two-digit addition/subtraction and that these may be arranged in increasingly flexible and efficient stages.

2.2 Purpose of Pilot Study

2.2.1 Introduction and Background

The National Numeracy Strategy provides a significant framework for pupil's mathematical learning experiences. Its main elements are a national plan and infrastructure, a substantial investment, detailed teaching programmes for children from ages 5 to 11 years, early intervention and catch-up for pupils who fall behind and support in the provision of numeracy consultants and leading mathematics teachers. However, there have been considerable concerns regarding the quality of teaching practice in primary mathematics; teacher self-efficacy is one construct that has long been considered to have significant effects on pupils' outcomes and learning experiences. Although there is substantial guidance on the teaching topics in mathematics, until recently very little attention has been given to the affective nature of pupils' learning experiences. There is now evidence that shows clear associations between self-perception and academic outcomes but less evidence on how the former influences the latter. It is known that children's mental calculation strategies may be classified; this pilot study focuses on the idea that these may be arranged in order and that high self-perception influences a more efficient ('better') choice of mental calculation strategy.

2.2.2 Teacher Self-Efficacy

This pilot study uses the teacher self-report scale developed by Tschannen-Moran & Woolfolk-Hoy (2001). Ideally, the study should control for teacher factors that may have a bearing on their efficacy; for example teaching experience, specialist qualifications or courses.

2.2.3 Mental Calculation Strategies

When pupils acquire calculation skills, they may use them well at first, but may also lose them quickly. Mental work plays a key part in keeping these skills sharp and well honed over time, (DfEE, 1998). Although quite a glib statement as a rationale for mental arithmetic, it does nevertheless reflect the importance of one aspect of a pupil's repertoire of strengths and weaknesses. Equally important was the comparison between schools in this country and elsewhere, where formal calculation methods are not introduced until a good grounding in mental skills has been achieved. However, a better starting point would be to identify mental calculation as one feature, that whilst important, could be supported by a range and variety of teaching methods. The National Curriculum Programme of Study for Year 5 makes clear demands, both on teachers and their pupils. It states that pupils should be taught the following mental methods:

d) recall all addition and subtraction facts for each number to 20;

e) work out what they need to add to any two-digit number to make 100, then add or subtract any pair of two digit whole numbers; handle particular cases of three-digit *and four digit additions and subtractions by using compensation or other methods {for example, 3000 – 1997, 4560 + 998 }*

f) recall multiplication facts to 10 x 10 and use them to derive quickly the corresponding division facts;

g) double and halve any two-digit number;

h) multiply and divide, at first in the range 1 to 100 { for example, 27 x 3, 65 divided by 5 }, then for cases of larger numbers by using factors, distribution or other methods.

Earlier research studies in mental calculation were essentially quantitative in their nature. We now have a better understanding of mental calculation, essentially based on single digit addition and subtraction. This more detailed knowledge appears to have been gained from research that is essentially qualitative in nature. The research methodology in this study involves asking children to execute a calculation in their heads and then describe how they worked it out. Interviews with individual children were recorded on audiotape for latter transcription and analysis.

Thompson's model of mental calculation strategies (Thompson, 1999) adds *attitudes* to the essential facts, skills, and understandings that are needed. It does appear to be the case however, that without the self-confidence to try, then children will be most unlikely to progress with mental calculation. The National Numeracy Strategy: Teaching Mental Calculation Strategies (QCA, 1999) provides mental calculation strategies for each year group, it also gives teaching strategies for mental calculation. From these it is possible to derive a mental calculation assessment for a year group and to deduce that there may be a hierarchy of such calculation strategies. In the literature, there is a variety of attempts at classifying children's mental calculation strategies, particularly in the work carried out in the Netherlands, over the past decade. Relevant literature sources in this area include Beishuizen (1993), Fuson et al (1997), Murray & Oliver (1989), Nicholls et al (1990), Thompson, (1999) and QCA (1999).

2.2.4 Self-perception

Some children, who appear to have developed mental calculation skills, continue to experience difficulty in their manipulation of these skills; it may be hypothesized that this may be due to either a generally low self-efficacy or a specific low self-efficacy with mental calculation. Similar to the difficulties in capturing a teachers self-efficacy, there have also been difficulties in developing an instrument to gain insight into children's perceptions of themselves as learners. Understanding more about such perceptions could have important implications for both curriculum design and teaching strategies. Teachers could be guided by acknowledging the pupils' perception of competence as well as their actual competence. Recent research suggests that gender may be an issue. As a result, it was also decided to compare boys' and girls' choice of mental calculation strategies.

2.3 Methodology

2.3.1 Research Design

Research Design refers to the type of research. The pilot study was intended to be a smaller version of the main study, to enable a trial of the various aspects; it was divided into two parts. The first part was essentially a correlation study, investigating the relationship between the variables mental calculation, attainment and pupil self-perception, as estimated by the Myself As Learner Scale, (Burden, 1998). In the second part of the main study, individual pupils were selected for work on two digit addition and subtraction. They were divided into two groups of high and low self-perception as learners, based on their scores from the Myself As Learner Scale. The pupils were then interviewed individually using an interview schedule; their responses were recorded on audiotape for later transcription.

2.3.2 Procedure/Programme

The pilot study was conducted in two parts; the first was with a whole class, the second with a sample of the pupils in each class for individual assessment of addition and subtraction questions. This pilot study was designed to explore teacher efficacy, pupils' views of themselves as learners and the types of strategy used for mental calculation by a planned sample of pupils in three schools in Suffolk. The sample was intended to be stratified by a number of different variables: type of school, gender, year group and ability group. In the first part of the study all children in each of three classes were given a mental arithmetic test and the Myself As Learner Scale.

Their teachers were given the Teacher Self-Efficacy Scale. In the second part of the pilot study a sample of 25 pupils from the three classes were classified as having, either a high or low self-perception of themselves as learners. Every pupil were asked to complete a graded series of two-digit mental calculations, designed to be commensurate with their age and ability; after each calculation each pupil was invited to describe the strategy that they had used to generate their solution. The interviews were tape-recorded for later transcription and analysis.

2.3.3 Selection of Schools

Selected schools were those that were well known and offered a variety of types i.e. rural and town. Only classes solely with Year 5 pupils and one teacher were chosen. The classes that were selected within these schools were selected with the agreement of the Headteacher. The main study will need to take into account if any setting takes place, and if the class is shared with another teacher.

2.3.4 Selection of Whole-Class Materials

In order to obtain a measure of the pupils' mental calculation attainments a mental calculation assessment of twenty questions was given to each class. The whole-class mental calculation questions were taken from those provided by the *Bristol Numeracy Project,* (see Appendix 6.3). These offer a series of tests for year groups 1 to 6. Pupils were provided with a prepared A4 sheet, space was available for workings. Instructions were made clear to all three classes at the beginning of each session; all questions were read out twice. The *Myself As Learner Scale* (Appendix 6.1) was given to each class. The questions were presented on a single sheet. Pupils were asked to read each question carefully and circle around the response letter indicating whether they definitely agreed, agreed a bit, thought the statement was true half the time, did not agree or strongly disagreed with the questions. In order to ensure understanding, the instructions and questions were read out twice to each class group. The pupils understood what was required and only a few need extra guidance in completing the forms.

2.3.5 Selection of Classes and Teachers

The Year 5 teachers in this Pilot Study were all experienced teachers with an interest in Special Educational Needs. They completed the *Teacher Self-Efficacy Questionnaire* (Woolfolk & Hoy, 1990) whilst their pupils completed the mental calculation assessment and *Myself As Learner Scale* (Burden, 1998). These were scored and reported back to teachers prior to working with individual pupils.

2.3.6 Selection of Individual Pupils for Mental Calculation Strategy Assessment

Upper and lower quartile scores from the *Myself As Learner Scale* were used to select individual pupils from the three classes. This provided a pool of 30 pupils, due to absence and timing difficulties in two cases, 25 pupils were interviewed; their replies/responses were audiotaped and transcribed during August 2001.

2.3.7 Selection of Addition and Subtraction Questions

This pilot study focuses on the addition and subtraction in the domain 20 to 100. Three questions involving the addition/subtraction of a single digit were allowed, in order to encourage pupil's on-task behaviour. All of the questions except these involved addition and subtraction of two digit numbers where the answer was less than a hundred. The questions were selected on order to provide a spectrum of difficulty, with or without carrying. (Appendix 6.5 provides the questions used). A maximum of six minutes was allowed for each pupil interview.

2.3.8 Identification of Mental Calculation Strategies

After several trials, it was decided to use an adapted form of the strategies suggested by QCA (1999). In their guidance, *Teaching Mental Calculation Strategies,* seven strategies are suggested. These are counting, reordering, partitioning using multiples of ten, partitioning through multiples of ten, partitioning with rounding and compensating, partitioning using near doubles and partitioning through numbers other than ten. For the purposes of this study, this last category was omitted given that it involves time and other quantities. An additional category was added to reflect children who work with single digit numbers but do not show an explicit understanding of their size.

Pupils' responses were recorded under the emerging six categories. These were then reduced to four and classified as four stages of mental calculation as follows:

Stage 1: Non-partitioning Strategies

The pupil simply uses counting or back strategies

e.g. 33 - 16, Pupil Y2m *"Counted on 6 then took away 1"*

Stage 2: Simple Partitioning Strategies

The pupil finds the correct answer by working with numbers but not show an awareness of the size of the tens and units

e.g. 23 + 24, Pupil Y7f, "4 add 3 is 7 and 2 add 2 is 4".

Stage 3: Partitioning using tens and units Strategies

The pupil shows an awareness of the tens and units but works with them separately in order to find the answer,

e.g. Pupil Z21, 37 + 45, *"I added 40 and 30, then I added 7 and 5*

Stage 4: Partitioning and Derived Fact Strategies

The pupil separates and works with the tens and units, and will show a subtotal prior to finding the answer, e.g. Pupil X4, 46 + 38, *"30 and 40 is 70, 8 add 6 is 14, add the answers together"*.

The pupil may also keep one number as a whole and partition the other number in order to work out an answer, e.g. Pupil Y12 46 - 28, *"It's 46 take away 20, then take away 8"*.

Please note also included in this stage were strategies involving halving or doubling.

In *Teaching Mental Calculation Strategies.* (QCA, 1999) there is included a *list of strategies for mental calculation that are appropriate for year-by-year development.* It is also stated that *the list cannot be exhaustive; the strategies chosen are widely considered to be the key strategies children need to develop their mental skills and understanding of number*. These strategies represent a progression from lower order strategies used in Year 1 through to order higher strategies used in Year 6. The four strategies that were selected for use in this pilot study were intended to represent a sequence or progression from the lower order (Stage 1) counting strategies through to higher order strategies of Stage 4. The QCA (1999) guidance (page 22) also makes the clear point that 'eventually counting-on will be replaced by more efficient method's. At Stage 2, pupils are able to work with single digits but do not show an explicit understanding of place value. This strategy stage is more efficient than counting forwards or backwards but is prone to errors particularly where subtraction is involved. Although pupils may be able to state a correct answer, this is usually in cases where no carrying is involved e.g. 23 plus 24 equals 47. At Stage 3 pupils are able to show partitioning but work with the tens and units separately. For many pupils this may be a 'common-method' but is less efficient than Stage 4 because pupils will be unable to show higher order methods and make explicit statements regarding their partitioning. At Stage 4 pupils will be able to demonstrate either clear partitioning with explicit statements and correct use of place value, or be able to partition only one number or be able to show clear examples of derived fact strategies including doubling and halving. The stages are intended to show a progression of increasingly efficient or efficacious methods or strategies. At Stage 4 pupils should be able to show a higher level of efficiency (mastery) of strategies/methods for two digit addition and subtraction mental calculation. If this is the case then their strategies should lead to greater accuracy in comparison to the other three strategies. The results section clearly demonstrates that this is the case.

2.4 Outcomes/Results

Table 2.4.1: Summary Data for Whole Class Mental Calculation (MC), Myself As Learner Scale (MALS) and Teacher Self Efficacy (TSE) scores.

	All Schools	School X	School Y	School Z
n	**73**	**21**	**28**	**24**
MC: Mean	**79.932**	**75.714**	**71.250**	**93.750**
: SD	**19.904**	**14.516**	**23.751**	**9.354**
MALS: Mean	**72.466**	**71.238**	**68.107**	**78.625**
: SD	**11.551**	**9.848**	**11.289**	**10.942**
rs	**0.60**	**0.07**	**0.74**	**0.66**
Significance	**$p < 0.01$**	**$p > 0.15$**	**$p < 0.01$**	**$p < 0.01$**
TSE score	**121**	**119**	**120**	**124**

<u>Key</u> n = number of pupils; MC = Mental Calculation (%); MALS = Myself As Learner Scale (Maximum, 100); rs = Spearman's Coefficient of Rank Correlation. Significance = of rs. (see Appendices 6.1, 6.2, 6.4)

The results show a generally reasonable distribution of mental calculation scores, with classes on average, accurately accessing three quarters of the test. The distribution of MALS scores is very close to the standardization results provided by Burden (1998), with mean score 71.0 and standard deviation 10.5. The literature indicates a relatively strong positive relationship between high MALS self-ratings and measured cognitive ability and basic attainments in literacy and numeracy. This appears to be reflected in the scores for Schools Y ($rs = 0.74, p < 0.0001$) and Z ($rs = 0.66, p < 0.01$) but not in the case of School X ($rs = 0.07, p > 0.15$). It is likely that this may be due School X's particularly class having a higher level of complex needs than the other two classes, which were very predominantly typical mainstream pupils. The Teacher Self-Efficacy Scale (TSE) produced scores very close to each other and as a result, did not discriminate between the three teachers.

Table 2.4.2: Strategy Stage and MALS (All Schools)

	Stage 1	**Stage 2**	**Stage 3**	**Stage 4**	**Totals**
Low	13%	4%	17%	17%	51%
High			9%	39%	48%
Total	13%	4%	26%	56%	

The table shows results pupils with either high or low learner self-perception, together with their allocated mental calculation strategy stage. (Appendix 1.5 contains the results and allocation of individual pupils to a particular mental calculation stage.)

High MALS appears to be associated with a higher stage of mental calculation strategy; low MALS appears to be associated with lower stages of mental calculation strategy. (Please see Appendix 1.4 for individual school/class data.) These results suggest that pupils with a high self-perception are likely to use (or have the potential to use) higher or more efficient mental calculation strategies or methods. Conversely low self-perception appears to be associated with lower mental calculation strategies. However, it may be the case that strategy stage is associated with, and possibly confounded by ability. This issue is addressed in the main study by limiting the sample to pupils of average ability and those without special educational needs.

Table 2.4.3: Whole Class Mental Calculation and Accuracy Scores and Strategy Stage.

	Stage 1	**Stage 2**	**Stage 3**	**Stage 4**
Mental Calculation	13%	75%	78%	86%
Accuracy	23%	65%	73%	90%

The table shows the mean mental calculation score and mean accuracy score for individual addition and subtraction questions for each stage identified. (Refer to Appendix 1.5 for the way in which these were identified.)

These scores show a close relationship, between pupils' general attainments in mental calculation and the accuracy of their answers to the two-digit addition and subtraction question. Performance in whole class and individual situations are closely linked and increase with mental calculation stage strategy. Both sets of scores show a progression where lower or more limited strategies are associated with lower mental calculation attainments and higher or more efficient strategies are associated with higher attainments. Stage 4 strategies, as predicted, appear to lead to more accurate answers or higher attainments.

Table 2.4.4: Accuracy and MALS (All Schools)

The following summary table for the three schools shows the distribution of accuracy scores for two-digit addition and subtraction questions,

School	**Low**	**High**
All Schools	59%	91%

These scores indicate that high MALS are associated with accurate addition and subtraction. They indicate that quite clearly that pupils with a low self-perception of themselves as learners have relatively low mental calculation attainments, whereas pupils with a high self-perception have high mental calculation attainments.

Table 2.4.5: Strategy Stage and Gender (All Schools)

	Stage 1	**Stage 2**	**Stage 3**	**Stage 4**
Male	9%	4%	26%	22%
Female		4%	4%	13%

The table shows the distribution of mental calculation strategy stage and gender. These results do not suggest a clear link between strategy stage and gender. However, given the concerns that have been expressed regarding female pupils' mathematical attainments, gender is a variable that should be further investigated.

2.5 Discussion

This pilot study was designed to examine the relationship between a teacher characteristic, in this case self-efficacy, pupils' views of themselves as learners, as measured by the Myself As Learner Scale and the pupils' choices of mental calculation strategies. The study was divided into two parts with whole- class and individual activities. At the outset it was hypothesized that:

1. Children in classes where their teachers have high self-efficacy will demonstrate a greater use of flexible mental calculation strategies.

The Teacher Self-Efficacy Scale used in this pilot study did not produce any significant differences between teachers. The choice of teacher was necessarily limited by focusing on a single year group (Year 5); it is potentially problematic in that teachers with high self-efficacy are likely to be much more willing to engage in a research study than teachers with low self-efficacy. Furthermore, an effective teacher of mathematics conveys information to children personally, rather than relying too much on curriculum material or textbooks, (DfEE, 1998). Although individual practice is important, leaving children to do much practice on their own gives little opportunity for interaction between the pupils and their teacher. Reluctantly, teacher self-efficacy was not directly investigated in the main study. Difficulties in obtaining a range of teacher self-efficacy scores was foreseen and furthermore, to attempt to investigate both teacher and pupil factors at the same time would appear to have too many interacting variables to yield meaningful results.

2. There is an identifiable hierarchy of mental calculation strategies for two-digit addition and subtraction.

The pilot study indicated that there is a range of mental calculation methods available to pupils; these may be hierarchical and leave scope for discussion regarding how these may be categorized. Furthermore, Threlfall (2002) suggests that strategy choice may be an inappropriate conceptualization of efficient mental calculation. Thought needs to be given to use of the word 'strategy', where 'method' or 'choice' may be alternatives. Also a pupil may have a preferred or prevailing strategy or method; care needs to given to describing subtraction strategies. The literature suggests that for single digit mental arithmetic, there are three strategies, namely bridging up and down, partitioning single digits and compensation. For two-digit addition and subtraction, QCA (1999) guidance, suggests that there are seven possible strategies, five of which may be described as 'partitioning'.

3. Teachers' expectations will be correlated with children's ability to cope efficiently with basic mental facts.

Teachers' expectations were not explored in the pilot study. This could be done by asking teachers to predict how each pupil would perform, i.e. to rank order them and compare the actual results. This remains a possibility but was omitted on this occasion to simplify the procedure.

4. Teacher enthusiasm for, and personal engagement in the processes of mental calculation will greatly enhance the learning opportunities for their pupils.

Teachers with high self-efficacy will be strongly correlated with pupils who have a high self-perception of themselves as learners (ability with flexible mental calculation strategies).

High pupil self-perception does appear to be associated both with the choice of higher mental calculation strategy and with mental calculation attainment and accuracy. A revised short-form of teacher self-efficacy remains a possibility; it may additionally/alternatively be possible to look again at the characteristics of effective teachers and to explore their belief systems, Askew et al (1997).

5. Children will be more likely to demonstrate higher level skills and techniques when their teacher has a high self-efficacy. This will be closely linked to their level of flexibility or sophistication of mental calculation strategies.

Higher skills, attainments and strategy choice do all appear to be linked, pupil ability however remains an important factor. In this pilot study found it was not possible to differentiate between different teachers' self-efficacy. However, much research does indicate that teachers make difference in the classroom. If this is so, it is reasonable to ask what are the characteristics of effective teachers of numeracy and how do these affect a pupils choice of mental calculation strategy.

2.6 Implications for the main study

It is now believed that children are born with a start-up kit for learning about mathematics as they progress through school, (Butterworth, 1999). However, children do encounter and experience significant difficulties in mathematics and numeracy. As children grow older and mature these can become highly embedded and very entrenched. There may also be several underlying reasons why children are not very good at mental calculation. There may be underlying genetic or ability factors; equally a child's attainments may reflect the fact that they have not been taught properly. Effective numeracy teaching, teacher beliefs about the nature of mathematics and their self-efficacy for mathematics/numeracy are major areas for research. Although no significant differences were found between the teachers in the pilot study, this may not be the case in a large sample. In order to investigate teacher self-efficacy, a possibly larger and differently designed research would need to be carried out; it is also likely that it would be difficult to control. This pilot study reflected the need to focus down on the affective nature of the pupils and the possible strategies that they may use.

In reviewing the literature it is apparent that much of the research that has been undertaken on difficulties in learning mathematics has focused on two dimensions of attitude, i.e. mathematics self-concept and mathematics anxiety, (Townsend et al, 1999). A distinction may be drawn between the two. Mathematics self-concept refers to perceptions of personal ability to learn and perform tasks in mathematics (Reys, 1984). Whereas mathematics anxiety may be defined as the feelings of tension that interfere with manipulation of numbers and the solving of problems in a wide variety of ordinary and academic situations, (Tobias, 1995). These are both important because of their relationship with academic achievement, (Donlan, 1998). It is also clear that much research has been carried out with older, usually university students. Less research has been carried out with younger primary school children, particularly where there has been a lack of appropriate assessment measures. Furthermore, the construct of self-perception has been frequently confused with that of self-esteem and used fairly indiscriminately in professional reports. Complementary to the shift from a behaviourist to a constructivist paradigm, has been the greater focus on pupils' self-beliefs being vital determinants of their success or failure in school. However, it is important to acknowledge that self-perception beliefs in mathematics/numeracy can be very domain-specific. There may also for very good reasons for these to be unrelated to self-perception.

There is no fixed relationship between one's beliefs about what one can or cannot do and whether one feels positively or negatively about oneself. Self-efficacy and self-perception beliefs need not be related, (Pajares, 2000). Mathematics anxiety is now an established construct; it is also unfortunately one in which may be distorted by lay or folk knowledge. Pupils may approach mathematics/numeracy mental calculation tasks without anxiety because they do not value achievements in this area; they may admit to being highly anxious when it comes to mathematics but may not feel a low self-perception simply because they do not value this activity. Pupils' decisions regarding their degree of anxiety may be a part of their self-perception of themselves as learners. Equally, the degree to which pupils can assess their mathematics anxiety and perception of themselves as learners may have different underlying reasons and influences on their learning experiences. For the main study it was therefore considered necessary to have a specific measure of mathematics anxiety and to retain a measure of general self-perception.

The idea that mental calculation strategies may be classified is seen in a few research studies. However, there is some disagreement about how these may be arranged and described. Nevertheless they provide a vehicle worthy of further research, particularly in so far as they may relate to the self-perception and mathematics anxiety of an individual child. This in turn may have implications for teaching and learning mental calculation strategies and skills and outcomes both for pupils with and without mathematics learning difficulties. Although findings regarding gender and mental calculation strategy were inconclusive, this a variable that should be further investigated because of concerns about older female pupils mathematical attainments.

CHAPTER 3:MAIN STUDY

3.1 Introduction/Overview of Main Study

The goal of the main study is to investigate the influence of children's mathematics anxiety on their choice of mental calculation strategies for two-digit addition and subtraction. The study uses two established scales, the *Myself As Learner Scale* and the *Mathematics Anxiety Scale for Children,* in order to explore any association between mathematics competence, mathematics anxiety, self-perception as a learner and strategy choice. The study focuses exclusively on Year 5 pupils of typical ability that would be found in mainstream classrooms. No pupils with special educational needs or either very high or very low ability were included in the study. It was considered highly likely that mathematics anxiety and learner self-perception would be significantly associated both with mental calculation attainment and mathematics competence. Furthermore, it was considered that the mental calculation strategies that pupils present can be arranged in order, and that pupils' selection of mental calculation strategy will be associated with their mathematics anxiety. A theme of constructivism serves as a common thread in this research. Implications are discussed at the school, teacher and pupil level. Implications are also suggested for the Educational Psychology Service.

3.2 Purpose of the Main Study

3.2.1 Hypothesis Formation

Broadly speaking there are two types of hypotheses that can be explored: causal and associative. In order to have a causal hypothesis it is necessary to think about manipulating some aspect of the system. Causal hypotheses are most easily investigated using experimental designs. Associative hypotheses describe how variables relate to each other in the absence of manipulation.

(Wright, 2003, p. 132)

In this main study it is hypothesised that:

1. Mental calculation strategies for two digit addition and subtraction may be classified in order of efficiency. It will be possible to classify these strategies as either lower order or higher order.

2. Mathematics anxiety and self-perception have been linked to academic outcomes. It is hypothesised that mathematics anxiety is inversely or negatively associated with mathematics outcomes (competence, attainment) but a pupil's self-perception of her/himself as a learner is positively associated with these same mathematical outcomes.

3. Mathematics anxiety significantly affects a pupil's choice of mental calculation strategy. Pupils with high mathematics anxiety use lower order strategies, pupils with low mathematics anxiety use higher order strategies.

4. There will be no significant differences in strategy choice for gender. Boys and girls at this stage of their education (end of Year 5) will not display significant differences.

3.2.2 Rationale

The Preliminary Report of the Numeracy Task Force (DfEE, 1998) made recommendations that it believed would improve standards and raise expectations in primary mathematics. One important desired outcome was that:

"All children have the opportunity to take part regularly in oral and mental work that develops their calculation strategies and recall skills".

(Preliminary Report of the Numeracy Task Force, DfEE, 1998, p.22)

Furthermore, the ability to calculate in your head is an important part of mathematics and of coping with society's demands and managing everyday events. (QCA, 1999). The National Curriculum and the *Framework for Teaching Mathematics* make it clear that children should learn number facts by heart and be taught to develop a range of related facts they cannot recall rapidly. It is also clear that there are several ways of calculating: using paper and pencil methods, using a calculator, working them out mentally or a combination of these. It has been seen that most children in Key Stage 2 do not use the most efficient methods. This research is concerned with finding the most efficient methods for two digit addition and subtraction and investigating if these are influenced by a child's level of mathematics anxiety.

1. It is a common misconception that Mathematics is a difficult subject to teach and to learn. It is also apparent, particularly for older children/young people that attitudes toward mathematics appear more polarised than for any other curriculum area. While many students enjoy mathematics, many others have negative attitudes (Ashcraft, Kirk, & Hopko, 1998; Fennema & Sherman, 1978; Stodolsky, 1985). The problem of negative attitudes is compounded by such attitudes being resistant to change (Tobias, 1995). Negative attitudes towards mathematics are not commonly seen in very young children, these appear to be more evident during the latter years of primary schooling. High or positive self-perception is associated with positive outcomes in mathematics; similarly low self-perception is associated with low outcomes. The pilot study confirmed this trend, although some possibly confounding evidence was found in one school. As self-perception has been firmly associated with outcomes, it was decided to carry out similar research with a larger number of schools. Would there be similar findings? Would any exceptions be found?

2. Mathematics anxiety is now an established construct, where much of the research, particularly that by Mark Ashcraft and his colleagues, has focused on reaction time experiments with older students. However, very little research has been carried out with primary school children, an exception being Gierl & Bisanz (1995). Perhaps the

most surprising finding is that these pre-service arithmetic teachers in the United States have appeared to be especially prone to mathematics anxiety, (Hembree, 1990). High mathematics anxiety appears to be associated with low outcomes. However, there does appear to be a gap in research in studies designed to assess the particular influence of mathematics anxiety on children's mental calculation methods or strategies. The main study had additional features not present in the pilot study. The main study used a reliable and valid mathematics anxiety instrument, *The Mathematics Anxiety Scale for Children.* This was used to examine the relationship between mathematics anxiety and mental calculation, mathematics competence and self-perception. The question was concerned with finding out if a relationship was evident between these variables. It was also used as the primary instrument for dividing children into high and low anxiety groups. If children can be divided into these two groups, are the mental calculation strategies that the two groups used different and if so, how are they different?

3. Informal recordings and the use of tools such as number lines and hundred number squares can be used to develop understanding of number and help to develop competence and confidence at all stages. Calculators can help to develop a better sense of number. However, this guidance makes it clear that children should not use calculators for calculations until they can at least add and subtract any pair of digit numbers in their head, (QCA, 1999). This represents a challenge for most children and a considerable challenge for children with a variety of special educational needs. In general, children in Year 5 are expected to meet this expectation and were therefore used as the focus for this research. Despite some clear demands and possibly high objectives, the research attempted to address what are the mental

calculation skills and attainments of this year group. Despite having a national strategy in place together with guidelines and expectations, the research questioned whether such a curriculum and strategy would effectively raise anxiety and influence strategy use.

3.3 Methodology

3.3.1 Research Design

Research Design refers to the type of research. The main study extended the pilot study in firstly investigating four variables namely, mathematics competence, mental calculation attainment, pupil self-perception, and mathematics. It is acknowledged that this type of investigation identifies trend, rather than cause and effect. In the second part of the main study, the pupils who were selected for two digit addition and subtraction were divided into two groups; namely those with high and those with low mathematics anxiety. In a development of the Pilot Study, pupils were chosen from the upper and lower quartiles of all of those available, rather than from the individual classes. The pupils were then interviewed individually using an interview schedule; their responses were recorded on audiotape for later transcription.

3.3.2 Measures

A number of measures were used in the main study. These were:

Mental Calculation Assessment: see Pilot Study (Chapter 2.3.2) for details of procedure.

Self-perception: In order to obtain a measure of the pupils' view of themselves as learners, the *Myself As Learner Scale* was again given to each class. (See Pilot Study for details).

Mathematics Anxiety: Chui and Henry (1990) developed the Mathematics Anxiety Scale for Children (MASC). A slightly amended version, with Mathematics/Numeracy replacing Math and pounds replacing dollars was used in this study. There were no other significant changes. (Appendix 6.3 contains the MASC version used in this study.)

NFER 8+ Scores: these scores were used as a measure or estimate of each pupil's mathematics competence. All pupils in this study were in mainstream Local Education Authority schools and all take this test; findings from these are commonly used for within county assessment. Where no NFER 8+ score was available, the pupil was excluded from the main study. In order to control for mathematics competence pupils with either very high or very low scores were also excluded from the study. Permission was obtained from the Headteacher of each school in order for these scores to be used in this research.

Individual Interview Schedule: Pupils were selected for individual interview using their Mathematics Anxiety Scale for Children score. Pupils who scored above the upper quartile on the Mathematics Anxiety Scale for Children were considered to have high mathematics anxiety. Pupils who scored below the lower quartile were considered to have low mathematics anxiety. The individual interview schedule consisted of a series of two digit addition and subtraction questions. These were alternated and balanced for the carry and no-carry operation. Each question was visually presented on a white card; the question was also read to each pupil. The pilot study had revealed that around six minutes should be allowed for each pupil. A single Sony recorder and microphone were used to record pupil responses; notes were also taken at the same time. Subsequently pupil responses were transcribed from tape for latter analysis.

3.3.3 Individual Response Analysis

Analysis of the pupils' responses gave rise to several strategies. The strategies that emerged from the pupils' own responses were also compared with other classifications, (Murray & Oliver, 1989; Beishuizen, 1993; Fuson et al, 1997, Thompson 1999, 2000b). Furthermore, the investigation of Threlfall (2002) into seven different classification systems was also considered; Foxman & Beishuizen (2002) provide the most recent comprehensive analysis of mental calculation methods.

In a study of the levels of sophistication of children's two digit mental calculation strategies in Years 4 and 5, Thompson (2000b) stated that

It is important to emphasise that the criterion for children to be allocated a specific level was that at least one of their responses should show evidence of the strategy typified by that level, whether or not all other responses were at lower levels. This fact needs to be borne in mind when interpreting the data.

Thompson (2000b, p. 8)

This study uses the same method for allocating a pupil to a specific mental calculation strategy stage. Thompson's five-stage model (Thompson, 2000b) was used as one starting point. However, the pupil responses together with a review of more recent classification schemes led to the production of an eight-stage classification.

In a recent reanalysis of data from 1987, Beishuizen & Foxman (2002) describe with examples, five strategy labels for two addition/subtraction. Threlfall (2002) describes examples taken from seven different studies. In this study, an eight-stage model is proposed:

Stage 1: Emerging Strategy

Example, 15 + 12; 15 on top of 12 and added them up.

Example, 23 – 11; 2 and the 3 and put the 1's under them.

Stage 2: Counting Strategy

Example, 15 + 12; counts up 12 using fingers.

Example, 23 – 11; counts down using fingers.

Stage 3: *Single Digits Strategy*

Example, 33 + 54; 5 + 3 = 8; 4 + 3 = 7.

Example, 58 – 19: 5 – 1; 8 - 9.

Pupil does not differentiate between tens and units. Where there is a sum greater than 10 e.g. 9 add 7 is 16, but is taken no further. This is considered as a special case of this strategy.

Stage 4: Simple Splitting Strategy

Example, 55 + 37; 50 + 30; 5 + 7; 80 + 12 = 92.

Example, 68 – 32; 60 – 30; 8 – 2; 30 + 6 = 36

Tens and units are split/partitioned and treated separately.

Stage 5: Advanced Splitting Strategy

Example, 27 + 69; 20 + 60 = 80; 80 + 7 = 87 + 9 = 96

Example, 42 –23; 40 – 20 = 20; 3 - 2 = 1; 20 – 1 = 19

The pupil finds a subtotal before stating the answer. Rounding and halving/doubling are considered to be special cases of this strategy.

Stage 6: Sequencing Strategy

Example, 57 + 14; 57 + 10 = 67; 67 + 4 = 71

Example, 42 –23; 42 – 20 = 22; 22 – 3 = 19.

The first number is retained, the pupil then splits/partitions the other number. They may or may not state a subtotal.

Stage 7: Advanced Sequencing Strategy

Example: 27 + 69; 27 + 70 –1 = 96

Example: 58 – 19; 58 – 20 – 1 = 37

The first number is retained, but more fluency is shown in dealing with the second number. (The second number is rounded-up to the nearest ten and is then either added or subtracted with a subsequent 'compensation').

Stage 8: Direct Answer Strategy

The pupil is able to go directly to a two-digit addition/subtraction answer without recourse to any of the above methods.

3.3.4 Inter-rater Reliability Agreement

In order to test the reliability and consistency of these stages a number of professionals (Educational Psychologists and Mathematics teachers) were asked to carry out an inter-rater reliability exercise. Ten examples were given. Raters were asked to identify each pupil's highest stage of mental calculation, they were particularly reminded to ignore if the answer was correct or incorrect and that only one example of each stage was needed. Raters were provided with a grid to indicate their answers and exemplars of each stage. (see Appendix 5).

3.3.5 Sample/Participants

The sample included pupils from town and country schools and schools from higher and lower socieconomic areas. Pupils were selected from mixed ability classes; all had similar experiences of the National Numeracy Strategy. All teachers were experienced, no classes who had been taught by either a newly qualified teacher or by more than one teacher were used. Pupils with significant language difficulties, serious communication difficulties, moderate learning difficulties and those with Statements of Special Educational Needs were excluded from the study. No pupils of exceptional ability or having exceptional talents in mathematics were used.

Table 3.3.5.1: Table of Selected Pupils

School	Class Size	n	NFER Excl	High Anxiety	Low Anxiety
A	26	23	3	2	10
B	26	21	5	2	7
C	21	19	2	4	0
D	24	17	7	3	2
E	25	19	6	3	2
F	22	21	1	8	1
G	29	23	6	7	6
H	23	19	4	4	3
N	196	162	34	33	31

Notes: In order for anonymity, schools were allocated a letter of the alphabet, the letter used does not necessarily indicate the initial letter of the school. Class size is the number of pupils less those with significant special educational needs or no NFER 8+ score. However, all pupils took part in the whole class sessions in their usual teaching class. N is the number of pupils taking part in the main study with NFER 8+ scores in the range 71 to 124, pupils with scores either above or below these were excluded in order to control for ability i.e. pupils with scores in either the upper or lowest 5% were excluded. NFER Excl is the number of pupils with these scores excluded from each class. High and Low Anxiety figures are the number of high and low anxiety pupils chosen from each class based on a total of 162 rather than the individual total for each class.

3.3.6 Procedure/Programme

Pupils were met in their usual teaching groups. In order to control for a number of factors, pupils were excluded from the subsequent analysis. These were as follows:

- Children with Statements of Special Educational Needs;
- Children with serious communication difficulties (Autism);
- Children identified with moderate learning difficulties
- Children with significant language difficulties
- Children of either very high or very low mathematical competence. These children were defined as those who scored in either the upper and lower 5% on their previous NFER 8+ Test..

The same general procedure that was used in the Pilot Study (Chapter 2.3.2) was used in the first part of the Main Study.

In order to determine which pupils were to be selected for the individual interview schedule, all pupils were ranked in order of size based on their MASC score. Upper and lower quartiles were calculated for the 162 pupils. An upper quartile score of more than 46 was used to define high anxiety pupils; a lower quartile score of less than 32 was used to define low anxiety pupils. A group of 63 pupils was selected for analysis. This contained 32 high anxiety pupils and 31 low anxiety pupils. All of these 63 pupils were interviewed individually. The opportunity was also taken to interview the pupils who were excluded because of their NFER 8+ scores. These were used to inform calculation strategies but were not used in the final analysis.

All questions were visually presented on white card, using a clear 20-pt Times New Roman script. All questions were read twice to each of the pupils. All pupils were asked "How did you work out your answer?". Their answers and explanations were audiotaped and subsequently transcribed. Notes were also taken when the pupil was giving his/her reply.

(Appendix 6.5 contains a list of the questions used)

3.4 Critical Evaluation of the Main Study

3.4.1 Note on Methodology

The description of the methodology allows for two important considerations. Firstly, in this study there should be sufficient information to allow a reader to replicate what has been done. Secondly, the chosen methodology should be justified. The first point is met in that there is a description of a two-part research study. In the first a whole class and secondly, an individual, one-to-one situation. The chosen methodology is justified on a number of counts.

3.4.2 The Type of Information

The first part of the study was quantitative in that the variables were measurable and a correlational approach was used in finding a relationship between a number of variables. The information or data obtained from the first part was used to explore pupils' verbal accounts of their mental calculation strategies. A qualitative approach is needed when research sets out to explore an issue in more detail; the pupils' own reported methods offer a rich source of information.

3.4.3 The Research Design

The design has been outlined and reasons given for using this particular approach. No lesser known designs or unusual approaches were used.

3.4.4 The Method of Data Collection

The method of data collection is appropriate for the type of information required. The whole class work in the first part of the study provides a method of collecting a lot of data quite quickly and ensures a 100% response rate from those involved. The self-completed questionnaires are suitable for quantitative research but specify the questions and more obviously do not allow for a free response. The questions used, particularly those in the *Mathematics Anxiety Scale for Children,* require children to make a subjective measure regarding their level of nervousness. Children may find it difficult to differentiate between being nervous, very nervous and very, very nervous. In contrast, the questions used in the *Myself As Learner Scale* are more distinctive, although pupils may still hesitate between, for example, don't agree and strongly disagree. The questions selected for the individual interviews in the second part of the study contained a equal mix of two-digit questions. Consideration was given to ensuring all pupils could at least make a start with two easier questions at the beginning. Otherwise, each pupil was allowed to make his/her own response, thus providing a rich source of data.

3.4.5 The Data Collection Tools

The data collection tools have been described, examples of which are included in the Appendices. The pupils' responses were recorded verbatim and each set of responses is included in the Appendices. An original tool in the form of a whole class mental calculation assessment was developed for use in both parts of this study. This achieved its aim of targeting quite a high (75%) mean pass rate, although the scores may not be normally distributed. The other data collection tools, NFER, MALS and MASC can all be regarded as having underlying normal distributions. The MALS and MASC have both been found to be valid and reliable instruments. The MASC is the only instrument of its type available and although the questions that it uses can be seen to readily apply to children in this country, the data were based on similar aged children in the United States. A refinement could be to standardise the questions for use in this country. This close agreement between actual and reported mean scores suggests that it was appropriate to use this scale.

In the second part of the main study children were selected based on the criteria that their mathematics anxiety scores fell outside of the upper and lower MASC quartiles. The pupils were interviewed individually using a bank of constructed two-digit addition and subtraction questions, based on data in the mental calculation guidance, (QCA, 1999). The questions were designed to elicit a variety of responses and provided appropriate replies/explanations from most pupils. It is a matter of conjecture, whether a slightly different set of questions, possibly focusing on more 'borrowing and carrying' operations should have been used. Although not explicit in the hypotheses, an assessment of working memory could have been considered at this stage. It is likely that such an assessment would need to involve using a novel technique and that this would lengthen the time with each individual pupil. The pupils used in the study were selected so that working memory difficulties could be considered to be less significant. All of the pupils had NFER scores that are typically seen in the mainstream classroom.

3.4.6 The Sample

All of the pupils used in the study were Year 5 pupils and attended typical mainstream schools. All the schools selected were large enough to have at least one Year 5 class. No mixed aged classes, commonly seen in smaller schools were used. Where more than one class was available, the class selected was the closest to average mathematical competence. Although there was a slight imbalance in boys/girls, this was not considered to be an issue given the reasonable closeness in respective pupil numbers. In order to provide a sample representative of the typical school population, a number of pupils were excluded and not used in the statistical analysis. Pupils of either very high or very low mathematical competence were excluded, i.e. those with NFER scores outside of the highest and lowest 5%. A higher percentage figure could be considered although this would reduce the pupil numbers and may restrict the spectrum of strategies available. In this case a larger number of schools would be needed. No pupils with special educational needs were used, none had moderate learning difficulties and none had been identified as experiencing specific learning difficulties. No children had language difficulties and all had English as a first language. All pupils had reading skills, as assessed by their teachers, to enable them to access the questionnaires. A further development would be to identify pupils' most recent reading scores as an additional measure. It was considered that the restrictions used provided a sample of pupils of typical ability found in mainstream classrooms. As a result, working memory, a possible mediating feature, was not considered to be significant. A consequence of the restrictions used was that there were some differences in the numbers of pupils selected for statistical analysis. This needs to be taken into account when considering differences between schools.

3.5 Ethical Considerations

- Permission was obtained from each Headteacher in both the pilot and main study. Each school was introduced to the research to ensure that staff knew the purpose of the research. All parents were informed in writing in advance of the research and were asked to indicate in writing if they did not want their children included. It was made clear to parents that some individual pupils would be asked to take part in individual mental addition and subtraction, and were asked to indicate in writing if they did not want their children to take part. Only two parents from one particular school declined the opportunity to take part in the studies.

- In general the activities were close to what could be reasonably expected in typical mainstream classrooms and schools. All schools and pupils were anonymised. The letters used to identify the schools are not necessarily their first letter; the letters/numbers used to code individual pupils do not reflect alphabetical order.

- Each class was told in advance that they would be asked to take a mental calculation assessment and complete a questionnaire in the case of the pilot study and two questionnaires in the case of the main study. In general, all of the pupils engaged appropriately at the end of the whole class session, each class was reminded that some pupils would be asked to take part in some individual work.

- There were no potential risks to the pupils in the study. The activities that were carried out were typical of mainstream schools and parental permission was obtained for the individual work. Permission was obtained from the Headteacher of each school in order for pupil NFER scores to be used.

CHAPTER 4: RESULTS

4.1 Statistical and Qualitative Methods Rationale

Whole-class data obtained from the eight schools in the first part of the study are presented in Table 4.1. Means (and standard deviations in brackets) are given for each of the eight individual schools and for all of the 160 pupils selected for this study. The selection and justification for the statistical and qualitative methods used follows from the experimental design used to test the hypotheses. Parametric tests require the data to be measured on an interval scale, be normally distributed, and have similar variability for each of the eight schools. If we cannot assume the data is normally distributed, but can assume measurement on an ordinal scale then non-parametric tests may be used. In order to assess the underlying distributions of the data, tests for normality, skewness and kurtosis were carried out on the four variables, mental competence (NFER 8+), mental calculation (MC), learner self-perception (MALS) and mathematics anxiety (MASC). This assessment indicated that it was reasonable to assume that NFER 8+ and MALS were normally distributed, but not reasonable to assume that MC and MASC scores were normally distributed. Consequently, a parametric test (ANOVA) was chosen to analyse the differences between mental competence and learner self-perception scores, whereas the non-parametric equivalent (Kruskal-Wallis) was chosen to analyse the differences between mental calculation and mathematics anxiety scores.

The NFER 8+ and MALS scores are numerical (interval scale), approximate to a normal distribution and the variability for each school is approximately the same.

This rationale allowed the use of a one-way Analysis of Variance (ANOVA) to investigate separately the differences between the eight schools' NFER and MALS scores using the "*Analyse-It*" software. This analysis formally tests for differences between independent samples. The Kruskal-Wallis One-Way Analysis of Variance is the non-parametric equivalent of the ANOVA and was used to determine whether the mental calculation and mathematics anxiety scores of the pupils in the eight schools were significantly different. The rationale for using this test is that the scores cannot be assumed to approximate to a normal distribution and are based on an ordinal scale. Because of these restrictions a non-parametric one–way analysis of variance is justified in examining the differences between scores.

It followed from this analysis, because of concerns regarding the normality of two of the variables, that it would be more appropriate to use a non-parametric measure of correlation (Spearman's Coefficient of Rank Correlation) to investigate the significance of an association between variables. The comparison of low mathematics anxiety with high mathematics anxiety is critical in linking the first and second parts of this study. This is shown in Table 4.5 where low anxiety and high anxiety scoring pupils are compared together with their mental calculation strategy, mental competence, and mental accuracy for two-digit addition and subtraction. Each pupil's strategy stage arose out of an analysis of the pupil's responses to a series of two digit addition and subtraction questions together with a review of recent work in this area. An inter-rater exercise suggested a high degree of agreement between original and inter-rater scores. Subsequent analysis allowed mathematics anxiety to be compared with strategy stage selection and by gender. In both cases, it was possible to construct two contingency tables suitable for analysis using the chi-square test.

4.2 Summary Data for Individual Schools

Table 4.2: Mean and standard deviations (in brackets) for mathematics competence (NFER 8+), mental calculation (MC), pupil self-perception (MALS) and mathematics anxiety (MASC).

School	n	NFER 8+	MC	MALS	MASC
A	23	104.70 (11.95)	14.87 (4.53)	69.70 (10.53)	34.26 (10.41)
B	21	108.48 (9.52)	15.95 (2.52)	70.19 (11.65)	36.42 (8.55)
C	19	104.11 (7.48)	13.16 (3.00)	74.11 (11.37)	41.79 (10.47)
D	17	109.35 (9.96)	15.12 (4.21)	67.71 (9.97)	41.24 (10.57)
E	19	107.11 (11.59)	15.37 (4.25)	71.00 (12.24)	39.21 (9.83)
F	19	100.84 (9.09)	13.47 (3.29)	69.42 (10.56)	45.74 (10.75)
G	23	104.78 (10.97)	17.30 (2.18)	72.17 (16.96)	39.57 (13.18)
H	19	109.05 (7.11)	15.05 (2.44)	73.47 (10.65)	41.21 (10.63)
N	160	105.98 (10.09)	15.10 (3.55)	71.00 (11.97)	39.66 (10.97)

Notes (Please refer to Appendix 4 for Main Study Summary Data and Appendices 3.1 to 3.8 for Schools A, B, C, D, E, F, G and H individual results.)

All 160 pupils were Year 5, 83 (53.9%) were male, 77 (46.1%) were female; note that n is the number of pupils in each class used in the first part of the study, but is different from the actual class size. The pupil scores used for analysis excluded either those with exceptionally high (equal to, or above the 95th percentile) or exceptionally low (equal to, or below the 5th percentile) i.e. scores in the range 70 to

125. Pupils with significant special educational needs were not included in the results. However, an examination of the scores indicated would be necessary to assess the underlying distribution of each of the variables, and as a result, tests were carried out for normality, skewness and kurtosis.

4.3 Tests for Normality, Skewness and Kurtosis

The mental competence, NFER ($k = 0.866$, $p > 0.5$) and learner self-perception, MALS ($k = 0.788$, $p > 0.05$) results indicate that the scores obtained for both variables are normally distributed. This is perhaps unsurprising since NFER and MALS are regarded as having underlying normal distributions and have been standardised on large samples. The mathematics anxiety MASC ($k = 0.957$, $p < 0.05$) and mental calculation MC ($k = 1.668$, $p < 0.01$) results do not appear to have an underlying normal distribution, although the mathematics anxiety findings may reflect more sensitivity to teacher and local effects. However, the results do indicate skewness in the distributions. In the cases of NFER, MC and MALS there is a left-tail skew with more observations in the right tail; in the case of MASC there is a right tail skew with more observations in the left-tail. No significant findings were found in any of the four variables related to kurtosis. The data suggests a tendency for pupils to be in the upper half of the mathematics competence (NFER 8+) range. MALS scores appear to be distributed normally and reasonable estimates of their intended measures. It is more difficult to draw conclusions regarding the MASC and MC scores, given the variations in findings. This analysis provided the justification for using a parametric test (ANOVA) to analyse mathematics competence and learner self-perception differences. It also provided the justification for using a non-parametric test (Kruskal-Wallis, H Statistic) to analyse mental calculation and mathematics anxiety differences.

(See Appendix 4)

4.4 Variations between Schools

4.4.1 Mathematics Competence and Learner Self-Perception

Mathematics competence and learner self-perception are both assumed to be independent samples, to be normally distributed, have similarly shaped distributions and measured on a continuous scale. The results obtained found no significant differences $F (7, 152) = 1.65$, $p > 0.05$ between mental competence, (NFER 8+) scores. The results also found no significant differences between the $F (7, 152) = 0.59$, $p > 0.05$ between the learner self-perception, (MALS) scores. This indicates that there are no significant differences in mathematical competence between the pupils in the eight schools and no significant differences between the pupils' perceptions of themselves as learners.

(See Appendices 7.1 and 7.3 respectively)

4.4.2 Mental Calculation and Mathematics Anxiety

Mental calculation and mathematics scores are assumed to be independent samples, to have similarly shaped distributions and measured on at least, an ordinal scale. Significant differences were found between the mental calculation scores ($H = 23.88$,

$p < 0.01$) and significant differences were found between the mathematics anxiety scores ($H = 17.55$, $p < 0.05$). Although, not directly comparable, the MC scores appear to show most variability between schools. There also appears to be some variability in mathematics anxiety between the eight schools. These are interesting findings in that other factors such as whole-school, teacher and national numeracy strategy/national curriculum factors may be beginning to drive pupils' mathematics anxiety and affect pupils mental calculation attainments.

(See Appendices 7.2 & 7.4)

4.5 Correlation Coefficients and their Significance

A correlation coefficient is a measure of the degree of association between the variables. If the variables are not associated then there is no linear pattern between the variables and they are independent of each other. The Spearman rank correlation coefficient (rs in the above table) was chosen to formally test for an association between the chosen variables; the significance of this correlation is given in brackets. It may be regarded as the non-parametric alternative to the parametric, Pearson correlation coefficient. The use of Spearman's coefficient is justified because of the normality concerns and skewness in the distributions; therefore, this non-parametric measure of degree of association between variables was used.

Mental competence (NFER 8+) was significantly positively correlated with mental calculation (MC) and learner self-perception (MALS), ($rs = 0.45$, $p < 0.0001$), ($rs = 0.35$, $p < 0.0001$) respectively, but significantly negatively correlated with mathematics anxiety (MASC), ($r_s = -0.32$, $p < 0.0001$). Mental calculation was significantly positively correlated with learner self-perception ($r_s = 0.57$, $p < 0.0001$) but significantly negatively correlated with mathematics anxiety ($r_s = -0.47$, $p < 0.0001$). Learner self-perception was also negatively correlated with mathematics anxiety ($r_s = -0.46$, $p < 0.0001$). Although correlation coefficients do not imply cause and effect, the scores do indicate a highly significant degree of positive association between mental calculation and mathematics competence and also a highly significant degree of positive association between pupils' self-perception of themselves as learners and mental calculation capacity and mathematical competence. In contrast, mathematics anxiety was highly significantly and negatively associated with each of the three other variables, mathematics competence, mental calculation capacity and pupil self-perception.

4.6 Distribution of Low and High Mathematics Anxiety

Table 4.6.1: Distribution of Low and High Mathematics Anxiety for NFER 8+ and Two Digit Mental Calculation Accuracy Scores.

LOW ANXIETY				HIGH ANXIETY			
Pupil	**Strategy**	**NFER**	**Acc.**	**Pupil**	**Strategy**	**NFER**	**Acc.**
A1f	S3	120	60	A17f	S6	108	100
A5f	S5	120	70	A18m	S1	81	10
A8m	S6	113	80	B4m	S4	110	70
A10m	S6	98	90	B5f	S4	94	20
A12m	S7	109	80	C8f	S4	100	40
A13m	S5	90	60	C16f	S5	103	90
A15m	S3	108	70	C18f	S5	100	80
A19f	S5	106	80	C20f	S5	98	70
A22m	S8	117	100	D8f	S5	99	50
A25f	S7	94	100	D15m	S5	120	80
B9m	S6	108	80	D20m	S4	109	40
B12m	S6	120	100	E8m	S3	97	30
B13f	S5	117	50	E22m	S6	104	60
B16f	S4	95	70	E23m	S6	94	50
B17f	S2	86	40	F4f	S2	108	40
B21m	S5	106	40	F5m	S6	111	70
B23m	S7	110	100	F7m	S5	112	40
D4f	S7	85	60	F13f	S6	89	50
D24f	S6	101	100	F17m	S4	100	80
E6f	S6	119	100	F18m	S5	97	40
E14m	S6	113	100	F19f	S5	108	80
F9f	S6	94	100	G2m	S3	89	40
G1m	S6	109	100	G3m	S6	101	70
G13m	S6	91	100	G4m	S4	102	90
G14m	S6	104	100	G5m	S5	121	80
G24f	S7	111	70	G9m	S7	100	90
G25f	S5	111	100	G10f	S3	101	40
G28m	S6	116	100	G20f	S3	88	30
H10m	S5	111	80	H1m	S6	106	90
H15m	S5	103	80	H4m	S5	102	60
H17f	S4	112	60	H5f	S5	100	70
				H11m	S6	100	100

Notes: (Appendix 8 contains transcribed pupil responses)

Table 4.5 above shows the distribution of high and low mathematics anxious pupils as determined by their scores on the Mathematics Anxiety Scale for Children. Pupils were given their own pupil code and their most recent NFER score as a measure of mathematics competence. Acc is the accuracy or percentage correct of two digit addition and subtraction questions given in individual interview. Each pupil was allocated to a strategy stage; this was based on at least one example given in individual interview of their highest (or most efficient/effective) mental calculation strategy stage. (Appendix 8 contains pupils' responses and one example of their highest or most efficient/effective strategy.)

Table 4.6.2: Testing the Difference between Mental Competence Means for High and Low Anxiety

NFER 8 +	n	Mean	SD	SE
High Anx	32	101.625	8.628	1.525
Low Anx	31	106.355	10.294	1.849

In order to investigate if there was a significant difference in mathematics competence between the two sets of NFER scores, a two-sample t-test was carried out between the means. This revealed no significant differences ($t = 1.98$, 61 df, $p > 0.05$) in competence between high and low mathematical anxious pupils. (See Appendix 7.5)

Table 4.6.3 Testing the Difference between Mental Calculation Accuracy Means for High and Low Anxiety

Accuracy	n	Rank sum	Mean rank
High Anx	32	786.5	24.58
Low Anx	31	1229.5	39.66

The accuracy scores were also compared using a Kruskal-Wallis ANOVA by ranks. This formally tests for differences between independent samples. This revealed a significant difference ($H = 10.95$, $p < 0.001$) strongly in favour of low anxiety indicating that low mathematics anxiety leads to more accurate mental calculation. (See Appendix 7.6).

4.7 Distribution of Inter-rater Scores

Table 4.7: Distribution of Inter-rater Scores

Pupil	**Original**	**Rater 1**	**Rater 2**	**Rater 3**	**Rater 4**	**Rater 5**
1.	**Stage 6**	✓	✓	✗	✓	✓
2.	**Stage 3**	✓	✓	✓	✓	✓
3.	**Stage 6**	✓	✓	✓	✓	✓
4.	**Stage 6**	✓	✓	✓	✗	✓
5.	**Stage 5**	✓	✓	✓	✗	✓
6.	**Stage 4**	✓	✓	✓	✓	✓
7.	**Stage 6**	✓	✓	✓	✓	✓
8.	**Stage 7**	✓	✓	✓	✓	✓
9.	**Stage 8**	✓	✓	✓	✓	✓
10.	**Stage 6**	✓	✓	✗	✓	✓
% Agree		**100%**	**100%**	**80%**	**80%**	**100%**

<u>Notes:</u> (Please refer to Appendix 5)

The table shows the distribution of inter-rater agreement/disagreement scores when compared with an original sample of ten questions. At least one example of each strategy stage was given. It was not considered necessary to provide examples of Stages 1 and 2. Although there are five Stage 6 strategies, this was considered very necessary given its importance amongst the eight possible strategies that could be selected. Raters 1, 2 and 3 were experienced Educational Psychologists, raters 4 and 5 were experienced teachers of mathematics.

Findings:

The findings show a high agreement of 92% between the original and the five raters' scores. It is therefore very likely that the proposed strategies offer a reasonable description of each stage.

4.8 Distribution of Mathematics Anxiety and Strategy Stage

Table 4.8: Distribution of Mathematics Anxiety and Strategy Stage

	S1	**S2**	**S3**	**S4**	**S5**	**S6**	**S7**	**S8**	**Total**
High	**1**	**1**	**4**	**6**	**11**	**8**	**1**		**32**
Low		**1**	**2**	**2**	**8**	**12**	**5**	**1**	**31**
Total	**1**	**2**	**6**	**8**	**19**	**20**	**6**	**1**	**63**

Notes:

The table shows the allocation of high and low mathematics anxiety pupils to strategy stage.

Findings:

Although there are nearly equal numbers of high and low mathematics anxious pupils, there are clear differences regarding their mental calculation strategy stage.

The strategy most commonly used by high anxiety pupils was S5 whereas the strategy most commonly used by low anxiety pupils was S6. Here is a very important distinction. S5 is still a partitioning strategy where both numbers are split whereas in S6 one number is held as a whole and the number worked on. Although the scores can be discussed, the small numbers in some cells do not readily allow for statistical analysis. The S5/S6 boundary is suitable for separating the scores because of this distinction in method and was used to produce Table 4.7.

4.9 Mathematics Anxiety and Strategy Stage

Table 4.9 Mathematics Anxiety and Strategy Stage Contingency Table

	Stages 1 to 5	**Stages 6 to 8**	**Totals**
Female	**23**	**9**	**32**
Male	**13**	**18**	**31**
Totals	**36**	**27**	**63**

The table was derived from Table 4.5. It provides a 2 x 2 contingency table suitable for analysis using the chi-square test. The table shows the distribution of high and low mathematics anxiety scores against mental calculation strategy stage. Stages 1 to 5 may be regarded as lower order (less efficient) stages, whereas Stages 6 to 8 may be regarded as higher order (more efficient) stages.

Findings:

The chi-square analysis revealed a significant (χ^2 = 4.61, 1df, $p < 0.05$). This suggests that lower order mental calculation strategies are associated with high mathematics anxiety, whereas higher order mental calculation strategies are associated with low anxiety.

4.10 Gender and Strategy Stage

Table 4.10: Gender and Strategy Stage Contingency Table

	Stages 1 to 5	**Stages 6 to 8**	**Totals**
Female	**20**	**6**	**26**
Male	**20**	**17**	**37**
Totals	**40**	**23**	**63**

Notes: (Please refer to Appendix 7.2, Chi-Square Analysis (2)

The table was derived from Table 4.5. It provides a 2 x 2 contingency table suitable for analysis using the chi-square test. (This is a special where there is one degree of freedom.)

Findings:

The chi-square analysis did not reveal a significant difference ($\chi^2 = 2.53$, 1 df, $p > 0.05$). This suggests that boys and girls do not differ significantly in their choice of mental calculation strategies.

Note: All findings in Tables 4.1, 4.2, 4.3, 4.4, 4.5, 4.8 and 4.9 were performed using *Analyse-it* (2003) software, General Statistics, version 1.69, downloaded from www.analyse-it.com.

CHAPTER 5: DISCUSSION

5.1 A New National Framework?

This study focused on two very important aspects of children's mathematical learning experiences i.e. their mental calculation strategies for two-digit addition and subtraction and mathematics anxiety. The National Curriculum and the National Numeracy Strategy established in 1999 provide a framework of what teachers should teach and what pupils should learn. However, they are also highly directive and prescriptive programmes of study together with an extensive range of tests that some regard as being very limiting to enriching pupils' mathematical learning experiences. The National Numeracy Strategy places much emphasis on mental calculation; indeed the Medium Term Plans within the NNS emphasise that every day, pupils should practise and develop oral and mental skills (e.g. counting, mental strategies and rapid recall of addition, subtraction, multiplication and division facts). Mental calculation strategies are clearly a primary feature of the National Numeracy Strategy and very important for its success. Teaching pupils to use mental calculation strategies, based on guidance provided by QCA (1999) is an established feature of the NNS seen in schools. Whilst the National Numeracy Strategy provides a clear opportunity to study the emphasis of a government/teacher imposed strategy, it may also limit or restrict the range of mental strategies that pupils have to call on. The strategies that pupils do possess are built on prior experience and knowledge; but the current emphasis on encouraging mainstreaming of all children, together with the raised profile of social inclusion, places emphasis on the social character of learning and the use of mathematics as a social tool. In the classroom, teachers need to consider carefully when to accept a single mathematics performance as adding to their knowledge of pupils' learning. This means that teacher biases need to be taken into account when making judgements about pupils. Central to this study is the notion that behaviourism no longer provides a suitable model for teaching and learning because learning is no longer seen as the simple effect of teaching. A study of constructivism shows us that pupils are learners who actively construct mathematics; their teachers may be seen as guides who facilitate their learners' experiences. However, there is also a tension in schools, where teachers are employed to deliver a prescribed curriculum and pupils may be seen as enduring a heavy regime of standardised tests, often accompanied by an associated stress from both teachers and parents to achieve somebody else's perhaps arbitrary levels of attainment. It is clear that pupils in the upper years of our primary schools are now required to experience a higher level of mathematics than was the case prior to the inception of the National Numeracy Strategy in 1999. It is also clear, particularly in the larger primary schools that broad ability groupings are now in place. However, informal discussion with teachers reveals a lack of confidence on their part and particularly, a very likely weakness in their subject content and pedagogical knowledge in mathematics.

In order for the NNS to succeed there is a need to increase teacher capacity, for them to be more highly skilled and knowledgeable about mathematics than is currently the case. Although the NNS has engendered some successes, many teachers have not yet had the sustained learning experiences necessary to develop a thorough understanding of the strategy or of the best ways to teach mathematics to their pupils, (Earl et al, 2003). If teaching is to be effective, this should be reflected not only in the pupils' competence and attainments in this study, but as importantly, in their self-perception, mathematics anxiety and, crucially, the way in which pupils can articulate their mathematical experiences. Mental arithmetic or mental calculation plays a central role in the way in which pupils construct their mathematics, stimulating not only conceptual understanding and procedural proficiency but also number sense and the understanding of number relations, (Klein, Beishuizen & Truffers, 1998). The importance of developing number sense as an essential element of mathematics education has been recognised in the literature. Mental computation may be seen as a subset of number sense. People who are good at mental computation use self-developed strategies based on conceptual knowledge (Reys, Reys, Nohda & Emori, 1995) and the ability to compute mentally is an indicator of the possession of number sense (McIntosh, 1992). Despite a raft of various initiatives in education, there continues to be considerable concern regarding very low level procedural competency in numeracy and mathematics, also reflected in informal, everyday teacher and parental views. This was one starting point for the study. Although the underlying nature of the mathematics that is taught in schools has not changed, there is now a very firm emphasis on numeracy and the manipulations of the four rules of number. This represented another starting point. However, it is also clear that there are now higher expectations together with access to, as pupils become older, various forms of information technology. Consequently, conceptual and procedural understanding is now increasingly important, as is the capacity of pupils to articulate their mathematical experiences.

5.2 The Classification of Mental Calculation Strategies

A central hypothesis of this study was that:

Mental calculation strategies for two digit addition and subtraction may be classified in order of efficiency. These will range from lower order to higher order strategies.

It has been argued that previous research has shown that mental calculation (computation) is a valid method, which contributes to mathematical thinking as a whole. It is also a process for which young children have exhibited a variety of proficient spontaneous strategies contrary to instruction, (Heirdsfield & Cooper, 1997). Although pupils may create their own individual or idiosyncratic strategies for two digit addition and subtraction, this study illustrated the notion that the strategies demonstrated by pupils may be classified in stages and that these stages may be arranged in a way that increasingly facilitates calculation. This study followed previous research that had yielded much information on the two digit addition and subtraction mental calculation strategies used by pupils; in particular the work of Threlfall (2002) and Foxman & Beishuizen (2002) has been significant. Earlier research evidence had been important in that it provided a knowledge base to inform teachers' practice, (Thompson, 1999). Furthermore, an awareness of mental calculation strategies will help them better understand children's explanations and provide appropriate support to develop, where appropriate, more efficient strategies, (Thompson, 1999).

This research had the advantage in using the pupils' articulation of their own strategies where importantly, all had had access to the National Numeracy Strategy. In broad terms, all pupils in each of the eight schools had a very similar mathematics curriculum; all had followed a similar programme of work. Furthermore, all pupils were in the Year 5 age range, no pupils from mixed age classes took part. However, three points arise. Competence factors were not significant in this study, but clearly schools do differ in their attainments in mathematics. Following the exclusion criteria of not using children with significant special educational needs nor those pupils with either very high or very low NFER scores no significant differences in competence were found between schools. Furthermore, no significant differences in competence were found in the second part of the study, between high and low mathematics anxious pupils. However, the results/findings do need to be treated cautiously, given the closeness of competence findings to significance. Secondly, ability grouping (broad banding seen in School G, where an "average competence" class was selected) of itself has no clear impact on attainment. Some evidence supports ability grouping, whereas some does not; the effects on high/middle/low attaining pupils have been inconsistent, (Hallom, Ireson & Davies, 2002). Thirdly, any effects on attainment would appear to be mediated by other factors, such as the nature of the curriculum and highlighted in the quality of the teaching. In this study it was important to have pupils all of a similar age (Year 5) and similar mathematical competence (mean NFER score, 106). An extension in a larger study would be to consider dividing the year group to account for the extremes of the age range and/or to divide pupils into competence groups. For example, high or low, high, middle or low or if possible to use a measure of ability in the form a cognitive score. In this study all pupils had NFER 8+ scores; an alternative arrangement would be to consider using a more restricted range of competence, for example consider using pupils having standard scores between 119 (90th percentile) and 81 (10th percentile), although this would clearly reduce the number of pupils in this study and would require a larger number of schools in a further study.

It was important to develop a classification of mental strategies for two digit addition and subtraction that reflected the theoretical (proposed) possibilities, but also one that used the strategies presented by the pupils. There have also been several attempts in the literature to classify mental calculation strategies for two digit addition and subtraction. Threlfall (2002) examined seven strategy classifications and made the point that each of the systems seemed reasonable in themselves as an approach to classification, *'yet it was noticeable that none of them is adequate to capture the diversity found in the calculations of a small sample of ordinary primary children',* (Threlfall, 2002, p.35). Foxman & Beishuizen (2002) made the point that Beishuizen et al (1997) published similar schemes based on research respectively in Holland and the USA. The following table was used as a framework by Foxman and Beishuizen (2002). They also added Thompson's (2000a) strategy labels; Threlfall's (2002) description of each strategy is presented additionally for clarification.

Table 5.2.1: Mental Calculation Strategy Classifications

Beishuizen	Thompson	Threlfall
N10	**Sequencing**	*The tens of the second number are added to the first number followed by the ones*
N10C	**Compensation**	*The second number is rounded up to ten, which is then added to the first number, with a subsequent adjustment or compensation.*
A10	**Complementary Addition**	*The second number is split to provide a quantity that takes the first number to the next ten, and then the remainder of the second number is added.*
10S	**Mixed Method**	*The two tens are dealt with, then the ones are added sequentially*
1010	**Partitioning**	*Tens and units are dealt with separately*

All of these strategies and the others reviewed by Threlfall (2002) focus on either partitioning/splitting or sequencing strategies. However, this quite narrow view does not fully reflect the range of strategies presented by the pupils in this study. Thompson's (2000b) five stage mental calculation strategy model identified the use of low level strategies for addition and subtraction (counting in ones and/or tens, and manipulating digits), but also emphasised the partitioning and sequencing strategies seen in other research. Drawing on Wright at al (2000), Thompson's (2000b) five stage model was used to develop the eight stages in this study by proposing levels of mental calculation strategies where 'sequencing strategies' could be regarded as the most efficient. This idea of a progression from lower to higher level strategies was important in developing the eight stage model. Although there are differences in the terms used, by researchers in describing mental calculation strategies, Foxman &

Beishuizen (2002, p. 5) state that *'The terms used may differ but they have a recognisable comparable meaning'.* The National Numeracy Strategy has led to greater attention being given to mental calculation skills. The eight-stage model is justified by drawing on a body of research where there is reasonable agreement in the mental calculation strategies that may be available across several classifications. The model has a foundation of previous research, is in agreement with previous classification findings, and takes the opportunity to present these more broadly. Research studies suggest agreement in the types of strategies but also raise questions regarding strategy choice and the underlying affective factors that may inform that choice. To extend the work in this area, it was important to propose a system that (a) reflected pupil methods, (b) encapsulated a broader classification system and (c) reflected a progression from lower to higher order methods. The following eight-stage model was proposed:

Table 5.2.2: Eight Stage Model of Mental Calculation Strategies

Strategy Stage	Name	Description
Stage 1	**Emerging Strategy**	*Words such as 'top' and 'under' may be used to suggest an emerging strategy.*
Stage 2	**Counting Strategy**	*Counting is used, up or down from the first or second number*
Stage 3	**Single Digits Strategy**	*Tens and units are not differentiated, but are treated in a similar way.*
Stage 4	**Simple Splitting Strategy**	*Tens and units are split or partitioned separately*
Stage 5	**Advanced Splitting Strategy**	*A subtotal is found before stating the answer. Rounding, halving, doubling are considered to be special cases.*
Stage 6	**Sequencing Strategy**	*The first number is retained as a whole, the other number is split or partitioned.*
Stage 7	**Advanced Sequencing Strategy**	*The first number is retained but fluency is shown in dealing with the second. (Derived fact implicit in this strategy, no subtotal needed.*
Stage 8	**Direct Answer**	*The answer is given without recourse to any of the above.*

The eight stages proposed are methods for classifying increasingly effective mental calculation strategies. Note that in this model Thompson's (2000b) counting and manipulating digits levels are equivalent to Stages 2 and 3 respectively. Partitioning (1010) is equivalent to Stage 4. Mixed Method (10S) and Complementary Addition (A10) are included in Stage 5. Sequencing (N10) is included in Stage 6 and Compensation (N10C) in Stage 7. Stages 6 and 7 are similar strategies but differ in the mental calculation fluency of the pupil. Although Stages 1 and 8 may not be considered to be typical of the other strategies where one or both of the numbers in a two-digit addition/subtraction calculation are manipulated, they do appear to provide opposing ends of a calculation continuum or spectrum. Stages 1 to 5 are considered to be lower order strategies (including partitioning strategies) whereas Stages 6 to 8 are considered to be higher order strategies (including sequencing strategies). Sequencing (jump) methods are more fluent and successful than partitioning (split) methods which are more time consuming and vulnerable to errors. The Partitioning (split) methods comprise more steps than the sequencing (jump) method as some intermediate answers have to be stored in short term (working) memory, (Foxman & Beishuizen, 2002). Using this formulation, higher order strategies are more efficient than lower order strategies in that they require fewer steps in finding an answer. The inter-rater reliability exercise carried out indicated that the strategies and their descriptions are reasonably robust in that a high agreement between the original and the raters was obtained. The strategies above were obtained through an analysis of typical two digit addition and subtraction questions. An extension or alternative procedure as suggested by Beishuizen, Van Putten and Van Mulken (1997) could also be used. In their research, they investigated mental arithmetic and strategy use with indirect number problems up to one hundred. Missing addend problems such as $27 + x = 65$ were used and children were asked to find the value of x. The children involved were Dutch Third Grade (mean age 9y 3m); interestingly most (78%) attempted to carry this out by indirect addition, rather than the minority (22%) who attempted to transform this in to a subtraction problem i.e. $65 - 27 = x$. It is a straightforward matter to assert that addition is easier than subtraction, particularly where there is a no-carry operation involved. Individual addition and subtraction problems in the main study, in common with those found in much of the research literature, were presented in horizontal rather than vertical format. The way in which a problem is presented visually may have an influence on the mental calculation that is carried out.

Although a number of classification schemes for two digit addition and subtraction are possible, evidence from the main study points to the use of two main types that pupils are likely to use. For the 63 high and low anxiety pupils reported in this study 37% used a sequencing strategy whereas 29% used an Advanced Splitting Strategy. These strategies differ in one important respect. In the sequencing strategy the first number of the two digit is held in place and the second number separated and worked on. The Advanced Splitting Strategy is a partitioning strategy in which both two digit numbers are split and worked on. The main study also found a division between lower order and higher order strategies; these represented a division or boundary between advanced splitting (partitioning both numbers) and sequencing (partitioning only one number) strategies, where 55% of the pupils used lower order strategies and 45% used higher order strategies. It would therefore seem reasonable to assert that based on this research, that pupils have a variety of strategies from which to choose in order to solve calculation problems. If this is the case (controlling for factors such as ability, competence, reading and working memory), higher order strategies are likely to be more efficient and produce a higher degree of mental calculation accuracy.

In this study higher order mental calculation strategies such as the sequencing strategy were better facilitators of mental calculation because they produced more accurate results. In contrast, lower order strategies were poorer facilitators because they produced less accurate results. Furthermore, the stages as described appear not only to represent a broad successive array on which a pupils 'best case' mental calculation strategy may be placed, but also reflect a move from declarative to procedural knowledge. The *empty number line* seen in much of the Dutch research provides one method of how these discussions may be realised in practice. It has the advantage of easily presenting and enhancing mental strategies and can accept pupils' informal or idiosyncratic strategies and may be seen as a more natural method. However, there continue to be concerns. Whilst teachers have had over the last four years a much higher level of support regarding mathematics/numeracy teaching than was previously the case, *Have their teaching practice and more importantly their underlying beliefs regarding the nature of mathematics changed?* The recent report *Watching and Learning: Final Report of England's National Literacy and Numeracy Strategies* (Earl et al, 2003) offers both successes and challenges for the NNS. Although there is a general acceptance of policies and practices of the NNS, there continues to be a major weakness in actually changing teachers' practices from a tranmissionist to connectionist perspective. In this study pupils were able to articulate their views and were able, in some cases, to offer some concise explanations of their methods. However, the evidence provided does not support new or alternative methods being embedded. For example, only one pupil mentioned the word 'partitioning', no child mentioned, or made reference to the use of the empty number line. It is therefore evident that based on this study and the wider evaluation of the NNS, that more work is needed in changing the way mathematics is taught in the classroom. One method could be through teachers reflecting on their mathematical attitudes and beliefs (philosophies). For teachers to be effective they need to be able to work with the children's knowledge that they bring with them to the classroom but to be able to readily connect the various ideas and concepts. There are many teacher factors that were not considered in this study, for example, teacher style, attitude/beliefs, and knowledge/qualifications and as was suggested in the pilot study, teacher efficacy. Reform of both instructional/teaching practice and curriculum modification would be hoped to have a positive effect on mental calculation skills. A practical step may be less emphasis on traditional paper/pen algorithms and more emphasis on developing and working with children's spontaneous strategies. It is likely that whatever curriculum systems and reforms that teachers have to manage and deliver, these will best be engendered through enhancing teachers' perceptions of their capacity to work with the various aspects of mathematics and numeracy.

5.3 Mathematics Anxiety and Self-Perception

In this study, it was hypothesised that:

Mathematics anxiety and self-perception have been linked to academic outcomes. It is hypothesised that mathematics outcomes are inversely or negatively associated with mathematics outcomes (competence, attainment) but a pupil's self-perception of her/himself as a learner is positively associated with these same mathematical outcomes.

This study was conceived out of a concern that when pupils begin school and in their early years and through Key Stage 1, they display no apparent anxiety regarding mathematics and have generally postive views about school and schooling.

However, as pupils grow older and mature there are more concerns, expressed by teachers and parents regarding their children's self-esteem, self-confidence and self-image. Unfortunately, these 'constructs' tend to be used quite indiscriminately. There is a danger of attempting to apply a simple label for these constructs that may have more complex underlying origins. Sometimes they are used as an 'excuse' for poor academic attainment or performance; sometimes a child may be described as having a low self-esteem and would therefore be expected to have low attainments in, for example literacy or numeracy. The Local Education Authority (for the schools in this study) has concerns regarding aspects of pupil's mathematics at Key Stage 2. These concerns focus on raising numeracy attainment, but there is also a general acceptance that mathematics/numeracy can be difficult subjects both to teach and to learn. This view appears to be derived from poor parental perspectives that are based on their own school experiences. Mathematics anxiety is now an established construct, mainly due to the work of Ashcraft and his colleagues. Although this work did focus mainly on reaction time experiments with older students, it has been closely linked with mathematical outcomes and achievements. There is now a substantial body of research on mathematics anxiety. The literature suggests that the origin of this anxiety lies in the attitudes and beliefs of teachers (Martinez and Martinez, 1996). There is a recurrent theme in the literature which suggests that whatever the nature of the learning material, the way in which it is delivered is likely to be as important as the content.

This theme has been reflected in the work of Albert Bandura (Bandura, 1993), referred to in the Pilot study, who believed that self-beliefs of efficacy played a key role in the self-regulation of motivation; most human motivation, he said, was cognitively generated. Beliefs are critical to actions. Constructs such as self-efficacy, self-perception and anxiety are closely linked either positively or negatively to actions. Furthermore, self-concept is most likely to be multi-faceted and possibly hierarchical in nature, (Burden, 1998). However, there is also a need to draw a distinction between the terms that are used. Mathematics anxiety and self-perception have both been associated with academic outcomes. In this study, mathematics anxiety was significantly negatively associated with competence, mental attainment and self-perception. In contrast, competence, mental attainment and self-perception were positively associated with each other. These findings appear highly consistent with other studies. For example, Burden (1998) reports a correlation of 0.34 (significant at the $p < 0.001$ level) between the MALS and numerical ability.

In a meta-analysis Hembree (1990) reported a negative correlation of –0.31 between mathematics anxiety and mathematics competence and suggested a uniform reduction in competence as a function of mathematics anxiety. Ashcraft, Kirk and Hopko (1998) attempted to argue that this was not the case. They discussed the effects of anxiety interventions, content-specific competence and on-line analysis of simple arithmetic. They found that competence and anxiety effects for simple, whole number arithmetic did not confound mathematics anxiety. However, they did find marked anxiety effects when the carry operation was involved. Historically, research on mathematics anxiety has tended to focus more on global measures such as standardised attainment scores. This research, whilst supporting the previous broad correlations that have been found, indicates a need to be more specific and investigate the cognitive processes involved.

Pupils differ in the degree to which they assume control over their cognitive processes. Highly mathematics anxious pupils and those with a low self-perception of themselves as learners are more likely to attribute their reasons for their learning externally by relying more on the teacher. In contrast, low anxious mathematics pupils and those with a high self-perception are more likely to attribute their reasons for learning internally by relying less on the teacher. Young pupils such as those in the study are able to make judgements about their learning that relate to internal regulation, for example in the pleasure and satisfaction of successfully completing a task independently, (Klatter et al, 2001). Pupils also believe that shared control of learning is helpful when the learning content is difficult or when new learning is introduced. It is interesting that none of the 20 items on Burden's *Myself As Learner Scale* has a social question that asks how well they interact with others or do other pupils influence their own learning. In contrast, 4 of the 22 questions on the *Mathematics Anxiety Scale for Children* contain references to interactions with others in the classroom. In the context of this study, these questions are relevant since younger pupils who experience their learning within generally whole-class prescribed and directed situations are likely to be heavily reliant on their teacher. It may well be the case that for pupils of this age, mathematics anxiety and learner self-perception are opposite ends of a broad affective spectrum, (a highly significant negative correlation was found between MALS and MASC scores). Whilst it is enticing to suggest this with a high self-perception/low mathematics anxiety at one end and low self-perception/high mathematics anxiety at the other, ultimately mathematics anxiety and self-perception are most likely to be distinct constructs. However, it may account for the skewness seen in the respective distributions and be a function of the pupils' ages. Clearly, an extension to the study would be to investigate different age groups using both of the scales.

The National Numeracy Strategy has had the advantage in its implementation of a variety of supporting activities. This aspect is important since the way mathematics is introduced is a key factor to changing perceptions and anxiety about mathematics, particularly with a view to improving pupil achievement. Schools are also now more likely to actively encourage parents to become involved in their pupils' mathematics supporting the necessary links between home and school. Cognitive factors are likely to have a direct effect on learning, whereas social factors may be more indirect; a pupil's beliefs about mathematics learning, expressed as either anxiety or self-perception are highly likely to influential factors contributing to learning outcomes. In investigating the relationship between academic self-concept, learner self-concept and approaches to learning in elementary students, Burnett & Proctor (2002) found deep approaches to learning were highly correlated with learning self-concept. They reported that a deep approach (e.g. find that most schoolwork is interesting once begun) to learning was characterised by an intention to seek meaning from the material being studied and relating to it in ways that elaborate and transform the material. In contrast, a surface approach (e.g. only doing the homework that is set and none extra) to learning is one in which the intention is to reproduce the material being studied through routine procedures. Burnett and Proctor (2002) developed the Approaches to Learning Inventory, a 20 item, and 5 Point Likert-type scale. This scale, similar in some ways to Burden's MALS was reported to contain both surface strategy/motivation and deep strategy/motivation items; deep approaches to learning were positively correlated with school self-concept and with learning self-concept. In the context of this study, it is likely that higher order mental calculation strategies are associated with deep approaches to learning, whereas lower order mental calculation strategies may be associated with surface approaches to learning. Furthermore, Burnett, Pillay & Dart (2003) demonstrated that learner self-concept mediated approaches to learning.

It is a relatively recent conception that mathematics anxiety has specific cognitive consequences on mental processing and representation, (Ashcraft, Kirk and Hopko, 1998). Previous research, as reflected in some parts of this study, focused on the negative association between mathematics competence and attainment. What is different in this study is the notion that mental processes for addition and subtraction calculations may vary as a function of a pupil's level of mathematics anxiety. Ashcraft's and colleagues' research touches on these themes. However, this study would appear to be an original piece of research investigating the most (or least efficient) mental calculation strategy for two digit addition and subtraction with a pupil's level of mathematics anxiety.

5.4 The Role of Mathematics Anxiety in Pupils' Mental Calculation Strategies

It was hypothesised that

Mathematics anxiety significantly affects a pupil's choice of mental calculation strategy. Pupils with high mathematics anxiety will tend to use lower order strategies, whereas pupils with low mathematics anxiety use higher order strategies.

Findings from this study showed quite a clear and important separation between high anxiety and low anxiety pupils. The primary finding was that low anxiety pupils tend to use higher order strategies such as sequencing whereas high anxiety pupils tend to use lower order strategies involving partitioning both of the numbers. When the results were presented in a 2 x 2 contingency table, a significant association was obtained. From this we may conclude that the results indicate a significant relationship between pupils' mathematics anxiety and their mental calculation strategies.

Studies of affective issues have always been central to the goals of mathematics education. From attitudes towards mathematics through beliefs to emotional and affective responses of pupils and students. Although not a primary focus of this study, it has also been found that pupils with learning disabilities in mathematics are at greater risk for negative affect, (Yasutake & Bryan, 1995). It is likely that the association found in this study between lower order mental calculation strategies and low self-perception and high mathematics anxiety is also seen in children who experience learning difficulties or disabilities (dyscalculia). McCloskey (1992) suggested that an analysis of acquired dyscalculia (mathematics learning difficulties) contributes to an understanding of normal processing. An understanding of mathematics anxiety has been shown to be a separate construct associated with lower outcomes in number. A better understanding of this construct should contribute to a better understanding and the raising of pupils attainments/achievements in mathematics. Furthermore, the influence of behaviourism in educational psychology in this century has been an important factor in the neglect of the affective domain, (McLeod, 1992). Pupils' beliefs about themselves are closely related to ideas of metacognition, self-regulation and self-awareness. However, we have seen that concepts like self-efficacy, self-perception etc overlap and that pupils are likely to see computational situations from a variety of perspectives. However, for pupils to become efficient at mental calculation, they appear to need the flexibility that comes from constructing their own procedures for computation. Consequently, this should be significant in reducing their mathematics anxiety. Furthermore, it appears that there has been an emphasis on measurement issues rather than on building concepts. Mathematics anxiety is separate from test anxiety, although the significant correlations that have been found do not necessarily indicate cause and effect.

The dimensionality of the *Mathematics Anxiety Rating Scale (MARS)* and the *Mathematics Anxiety Scale for Children (MASC)* are important because there is an underlying assumption that mathematics anxiety is a unidimensional construct. Earlier studies had suggested that this may not be the case e.g. Resnick, Viche & Segal (1982) differentiated three factors, evaluation anxiety, computation anxiety and social responsibility anxiety. However, Beasley et al (2002) demonstrated that for the age range of pupils in this study, mathematics anxiety is a unidimensional construct and is not necessarily related to intelligence (Hadfield, Merton & Wooden, 1992) and not completely confounded by ability, (Ashcraft, Kirk & Hopko, 2002). However, children do show preferences for particular strategies. This study focused on at least one example of their 'best case' strategy; responses from the children did suggest that they will use the strategy that for them is easiest to use and as a result is most likely associated with lower mathematics anxiety. Blöte, Klein and Beishuizen (2000) found that students' preference for certain mental calculation strategies or procedures depended on the number characteristics of the problem. In this study, it is clear that a relatively high number of pupils are able to confidently use a higher order strategy (e.g. sequencing) and that highly mathematically anxious pupils are more likely to resort to lower order strategies (e.g. partitioning both numbers) such as simple splitting and advanced splitting. It may also be the case that lower order strategies engender relatively more anxiety than the higher order strategies because of the greater number of steps involved. Less mathematics anxious pupils are more likely to find a correct answer because of the generally fewer steps in lower order strategies leaving less room for errors.

Finally, the mental calculation strategies that pupils may use may be framed in terms of their knowledge of facts or procedures. Pupils displaying less efficient, lower order strategies with more steps may nevertheless be demonstrating procedural knowledge. This is assumed to consist of the processes such as carrying and borrowing, as well as the more loosely defined functions such as keeping track in a multistep problem, rule application etc, (Ashcraft, Kirk & Hopko, 2002). Pupils displaying more efficient, higher order strategies with fewer steps (or possibly no steps) offer declarative knowledge or knowledge of facts. The ability to go directly to answer with no step or only one step, that may involve the use of an embedded, derived fact knowledge is very likely to be accompanied by less anxiety simply because the pupil can go directly to answer without the need for unnecessary steps. Clearly, as the complexity (three digit addition/subtraction and above) of problems increases, then there is likely to be a need for greater procedural knowledge. Although fact knowledge and procedural knowledge describe these strategies, a more adequate model is needed. In order to solve problems such as mental calculations, we need two kinds of mental skills, one for routine or lower order questions, and one requiring problem-solving skills for new, novel or more difficult questions. Problem solving skills may also become routine, so we need also to be able to learn new methods and then routinize them without detracting from the possibility of future adaptation, (Skemp, 1993). Skemp's articulation of mathematical understanding has the potential for making a significant impact on the strategies that primary teachers adopt in constructing effective learning environments for pupils. Skemp (1979) made a strong statement in differentiating between two forms of mathematical understanding; instrumental understanding (for routine problems) involves the ability to carry out operations, whereas relational understanding (for non-routine/problem solving situations) involves understanding structures and connections between concepts. In Skemp's model, knowledge is organised through a schema; a schema is an organised structure of knowledge, which deals with knowledge and experiences; a pupil may have many schemas, which may interrelate with each other. Mathematics anxiety, it may be argued, is a facilitator or inhibitor for finding the appropriate schema/knowledge structure to carry out a problem such as a mental calculation. In a study of adult numerate practices, Evans (2000) addressed the inseparability of thinking and emotion, and the consequent ways in which mathematics is emotional and not cognitive. This study addressed similar issues in highlighting the important influence of mathematics anxiety on pupils' mental calculation strategies.

5.5 Working memory

Mental arithmetic (calculation) is a common everyday skill, (Adams & Hitch, 1997). The ability to work with numbers and mentally calculate is a skill that is given much priority in the National Numeracy; as has been noted, pupils are expected to have daily practice in this area. Routine mental calculation problems such as typical single digit addition and subtraction may be retrieved on demand, from a long-term storage. However, two digit mental addition and subtraction calculations are not usually retrieved form a long-term storage. It is usual that these are obtained by using a particular algorithm or mental calculation strategy that are a focus of this study. Working memory is the theoretical flexible operating space in which ongoing processes are combined with the handling of partial results, (Adams & Hitch, 1998). Routine, single digit questions may generate very little mathematics anxiety, whereas two digit questions involving either 'carrying' or 'borrowing' may generate more anxiety. It is likely that two-digit involving addition and subtraction with 'carrying' and 'borrowing' places most effort on working memory resources and this effort has relatively more mathematics anxiety. The problem size effect is also significant in this area of research. Although working memory was not a primary focus of this study, other research indicates clear links with calculation performance. Several of these studies focused on adults/students/older pupils and consequently the results are not directly relevant to the pupils in this study. Working memory is typically closely associated with reading. No children used in the main study had significant reading difficulties, all had standardised mathematics competence scores well within two standard deviations of the mean, and none were described as having specific learning difficulties. It was therefore assumed that working memory was not a significant factor in their acquisition of basic academic skills. The individual addition and subtraction questions used, were presented both visually and orally, and all the pupils were assumed to have adequate working memory resources.

Working memory studies do show higher addition spans for visual rather than oral presentation; the main finding from research studies is that working memory is a major factor limiting children's mental calculation. Complex working memory skills are closely linked with children's academic progress within the early years of school, (Gathercole & Pickering, 2000). The assessment of working memory skills has been considered as a possible method of screening young children likely to be at risk of poor academic progress. An additional feature of the main study could have been to assess the individual pupil's working memory using a *working memory battery* assessment as suggested by Gathercole and Pickering. This may provide some useful data but would clearly lengthen the time with each pupil and may have an effect on each pupil's willingness/capacity to articulate their strategies, the core of the individual work. Furthermore, the assessment of working memory would need to be carried out using novel or new items, rather than the forward or backward digit recall that is typically used to assess short-term (working memory) skills.

In 1974, a three component model of working memory was proposed by Baddeley and Hitch. This has become a widely known and studied model, with a substantial body of background research. However, it became apparent to Baddeley that there were phenomena that were not captured by this model. He consequently proposed a new component, called the *episodic buffer*. Its importance has been seen in recent research on memory and education, for example Gathercole, Brown & Pickering (2003), Henry & MacLean (2003) and Maybery & Do (2003). Baddeley's working memory model, particularly in the light of affective research has considerable weaknesses. If the central executive does play a role in planning and decision making, 'How does such a mechanistic model explain these?' The model cannot explain the more complex phenomena and is limited to explaining prose recall in amnesia, chunking and meaningfulness and indeed the relationship between working memory and long term memory. The episodic buffer provides a link between working and long term memory and is thought to be a limited capacity temporary storage system that is capable of integrating information from a variety of sources. The working memory model together with the episodic buffer remains open to unresolved questions from behavioural research. An earlier but limited study (Darke, 1988) was important in that it highlighted the conclusion that high levels of anxiety reduces both the storage and processing capacity of working memory. If Baddeley's revised model better describes working memory with the addition of the episodic buffer, then it may be reasonable to assert that high anxiety limits the functioning of this mechanism.

5.6 Some Other Issues

5.6.1 Gender

In many studies and in educational research in general, it is typical to look for gender differences:

The pilot study indicated that there are likely to be no differences in gender between pupils' choice of mental calculation strategy; the main study also has the hypothesis that for Year 5 pupils of average competence, there will be no significant differences in their choice of strategies.

In the main study, it was considered that there would be no significant differences between boys and girls. This was partly found to be the case. After the pupils had been divided into high and low anxiety groups, it was found that these two groups contained very nearly equal numbers of boys and girls. In the whole study there were slightly more boys than girls i.e. 52% compared with 48%. There were no significant anxiety differences between boys and girls. Furthermore, it may have been anticipated that there would be a difference in the strategies used by boys and girls. However, the findings revealed no significant differences between mental calculation strategies that boys and girls may use. It would be interesting to investigate if any gender differences did become apparent as the pupils became older or when they transferred to high school.

5.6.2 Teachers

All kinds of teaching have this in common, that they are an intervention in the learning process. This learning process is inaccessible to direct observation by an outside person, such as a teacher, in the same sort of way as one's digestive processes are inaccessible to direct observation by a medical practitioner. In both cases, a person who intervenes without any clear picture of what is going on inside is as likely to do harm as good.

(Skemp, 1979, p. 251)

A key factor in developing pupils' mental calculation strategies is having a thorough understanding of what they know already. This tenet has been seen previously, particularly in formative assessment. Askew et al (1997, 1998) established that connectionist teachers or those who were able to clearly link the various parts of the subject were the most effective teachers of numeracy. However, the findings from this study indicate that mathematics anxiety is a significant mediating factor, although for this age group of children, learner self-perception is also likely to be a mediating factor. Variations between teachers were not investigated in the main study as no differences between them were found in the Pilot Study. However, it is clear that teachers can strongly influence pupils' performance in the classroom. Mathematics anxiety has the potential to be affected through careful instructional design. The National Numeracy Strategy provides the framework for teachers and schools in which to work. With a fuller knowledge of their pupils' mental calculation strategies teachers will be in a better position to design effective educational experiences for their pupils. Consequently, teachers need to encourage pupils to articulate their strategies. Furthermore, teachers need to have a thorough working knowledge of the Dutch research and more specifically how to encourage children to consistently use sequencing rather than partitioning strategies. They also need to be aware of their own self-efficacy beliefs, particularly that these may have a strong predictive and mediational role.

The ways in which teachers construct meaning in the classroom is influenced by their own self-efficacy, their expectations, their goals, and their knowledge of pedagogy and knowledge of their pupils. In contrast, teachers do not work within independent environments and many teachers possess a highly dependent orientation towards their own teaching and their pupils' learning. Teachers' beliefs, expectations and particularly their efficacy, strongly influence how they teach. There appears to be a clear link between orientation and practice and these may result in different learner outcomes. Attitudes in the classroom are very important. Teacher enthusiasm for personal engagement in the processes of mental calculation will greatly enhance the learning opportunities for their pupils. Teachers with high self-efficacy are likely to be strongly associated with pupils who have a high self-perception of themselves as learners. Pupils will be more likely to demonstrate higher level skills and techniques when their teacher has a high self-efficacy and they have a sound self-perception of themselves as learners.

Mental calculation is a specific area of the school curriculum in which teachers need to seek to understand their pupils' current strategies (and best case strategy) for use in subsequent lessons. Teachers need to value their pupils' questions and support them by allowing their own possibly idiosyncratic strategies and encourage the use of oral articulation through focused group work. This research provides direct support for emphasising the use of the '*Empty Number Line*' in primary classrooms together with a move away from the use of the '*The Hundred Square*'. There is a need for teachers to appropriately challenge both the less able and the less confident pupils who may struggle to acquire mental calculation skills. These pupils may need quite a lot of support in making the shift from a physical to mental representation of the calculation. There is clearly a range of strategies but also a need to focus on a limited range of methods that can be shared by the majority of a class. Although techniques are important, this research indicates that equal importance needs to be given to affective factors in the classroom that may equally affect performance and attainment.

5.6.3 Affect

Self-concept differs from self-esteem in that it is context-specific assessment of competence to perform a specific task. Self-esteem is measured at a broader level, whereas self-concept has been consistently related to academic achievement. As a result, there is a need for a greater awareness of how mathematics/numeracy can affect the classroom and how teacher-pupil beliefs/values affect learning in mathematics. This may appear to be a case of gender in older pupils where further research could investigate male/female differences related to teacher influences on strategy choice. Research has generated some consistent findings in single and two digit mental calculation. However, there remains a lack of research in multiplication and division, and in those areas that involve more novel problem solving.

The literature review clearly raises the notion of 'collective efficacy' or 'a group's shared belief in its capabilities to realise given levels of attainment'. This appears to be a potentially useful construct, where the theoretical perspectives reviewed emphasise the social nature of the mathematical learning. The research reviewed suggests the importance of interaction in mutually constructive situations; more research is needed on the nature and timing of teachers' interventions. Further research in these areas could focus on whether children with different cultural backgrounds perceive the same class environment differences. Investigating the mental imagery that pupils may use could show an association with their level of mental calculation strategies. In order for children to be adaptable in their use of mental calculation strategies, they need to have a sure feel for number. It is suggested that teachers will benefit both from a reworking and update on the work of Richard

Skemp and being introduced to constructivism, as the model of learning that drives their practice in the classroom.

It follows from the above that an important element in the broader teaching of mathematics and a more specific area of mental calculation strategies is the encouragement of pupils' mathematical articulation. Although 44% of pupils used higher order strategies, whereas 56% used lower order strategies, the implication is that in order to move pupils on from using a partitioning, advanced splitting strategy to a sequencing strategy is to work to reduce any feelings of anxiety regarding mathematics. Although this is likely to be a global feeling, it may nevertheless, differentiate between pupils' willingness to express themselves socially or individually and whether their anxieties are concerned with numbers on their own or in context. In contrast, there is little direct evidence of the impact of the NNS on this group of pupils. Only one pupil mentioned the word 'partitioning' if pupils were conversant with the strategies, for example as suggested by QCA (1999), then these would be more in evidence in the pupils' own articulations or descriptions of the strategies that they actually used.

5.7 A Model of Mathematics Anxiety and Mental Calculation Strategies

Affective factors play an important role in the development of children's mathematics skills, but continue to be undervalued by not being evidenced in research studies. Mathematics anxiety is an affective factor that has become established as a stable construct but also one where research studies have tended to focus on older students and adults. Despite an association in the research between mathematical anxiety and mathematical outcomes, there appears to be a significant gap in showing how mathematics anxiety influences pupils' mental calculation strategies. Although a number of researchers (and the QCA in the UK) have produced mental calculation systems, less attention has been paid to how these are influenced by other factors, particularly gender, working memory and learner self-perception.

Pupils need to have an adequate learner self-perception in order to gain a range of skills; this construct appears to be negatively associated with mathematics anxiety but may not simply be part of an 'affective spectrum'. Although young children do not generally express mathematics anxiety, this does become more evident and quite strongly present in older students and adults. Crucially, the influence of a heavily prescribed curriculum and banding/setting arrangements, now in place in primary schools, may actually heighten a child's potential for being mathematically anxious. An effect of this may be seen in the further marginalisation of pupils susceptible to learning difficulties (dyscalculia) in mathematics. It is also likely that pupils may habitually resort to a particular strategy in which they feel successful. However, this may also have the effect of pupils being either reluctant or unwilling to try alternative strategies.

Earlier studies have shown the role of working memory in mental calculation; it is reasonable to suggest that it also plays a part in the use of mental calculation strategies. An established model of working memory (Baddeley, 1986, 1992) has gained acceptance amongst researchers. Using this model, there is now an opportunity to suggest that mathematics anxiety limits a pupil's working memory by restricting the ability of the episodic buffer (Baddeley, 2000) to act as a transitory place to carry out (two digit) mental calculation. As a result, pupils with low mathematics anxiety should have more working memory capacity for higher order mental calculation whereas pupils with high mathematics anxiety will have reduced working memory capacity and be limited to more lower order strategies.

Although gender may not appear to be a significant factor, it should not be discounted given boys and girls may vary in age in their awareness of themselves as learners and their willingness to acknowledge their level of mathematics ability. Simply focusing on techniques and skills without giving due consideration to affective factors may be insufficient to develop confident pupils. Furthermore, there is also a need to include within the model the ability to account for the routine, and contrast with having a strategy or strategies to cope with new or novel calculations. Teachers need to be aware that mathematics anxiety is a stable construct and that it has a particular influence on pupils' mental calculation strategies and outcomes. Working to reduce mathematics anxiety (and enhancing learner self-perception) should enable a greater enjoyment of mathematics and lead to improved outcomes

5.8 The Role of the Educational Psychologist

Educational Psychologists can have a pivotal role to play in meeting the needs of all children in schools. Their role was explored in the wide-ranging reports, *Educational Psychology Services: Current Role, Good Practice and Future Directions* (DfEE, 2000a, 2000b). Core functions and best practices were identified which were valued by stakeholders, (Lown et al, 2001). However, the report was very general in its findings, partly because of variations and levels of staffing between services and partly because the work of the Educational Psychologist has historically been driven by requests for statutory assessment. Indeed, the culture of many schools and Local Education Authorities focuses the role of the Educational Psychologist using a very narrow definition of special needs that ultimately lead to a formal assessment. For some children and their families this is very important but may disadvantage the Educational Psychologist for a broader preventative role. Educational Psychologists' assumptions regarding children's learning (Hardman & Worthington, 2000), pursuit of quality assurance/role of service level agreements, (Bartram & Wolfendale, 1999) and the future direction of the profession of Educational Psychology (MacKay, 2002) rest on the more obvious factors such as time, but also on the opportunity to develop creativity within a research practitioner model, that acknowledges the needs and wishes of different clients. Teachers and parents, and possibly schools and Local Education Authorities value the work of well-informed Educational Psychologists who are able to contribute to meeting the needs of children in school.

Behaviourism has long held a strong influence over educational practise. In contrast, the constructivist theory of learning and view of knowledge that underpins this research emphasises the belief that pupils create their own knowledge. Educational Psychologists taking this perspective would see an active process, adapted and modified by experience. Mathematics in general and mental calculation in particular have been traditionally neglected in the work of the Educational Psychologist. They do have available a few standardised assessments of written numeracy but are limited diagnostically, particularly for the less able or those with significant difficulties in numeracy. The proposed eight stage model offers Educational Psychologists a framework to assess mental calculation strategies and a method for fine tuning individual assessments. It was noticeable that the teachers and Educational Psychologists used in the inter-rater exercise commented on the potential usefulness of the model. Skemp (1979) suggested that all teaching was an intervention in the learning process. The proposed model provides guidance for specific teaching strategies; For example, introduction of halving/doubling, the empty number line with or without manipulatives. It also highlights the importance of affective issues such as mathematics anxiety in the development of mental calculation strategies.

The introduction of the National Numeracy Strategy raised the profile of numeracy and mathematics. This led to an increased number of referrals locally, to Educational Psychologists for assessment/advice regarding mathematics/numeracy skills but particularly dyscalculia (or specific difficulties in numeracy). This research has been highly influential in supporting teachers in three specific ways. It has contributed substantially to local inservice training in dyscalculia, in emphasising the importance of mathematics anxiety and learner self-perception (how they can be assessed and where necessary, supported) and contributing towards the completion of an early years number assessment instrument. The research offers Educational Psychologists a framework or model with direct implications for practice. It raises the importance of affective issues with specific implications for practice in the use of mathematics anxiety and learner self-perception scales. In general, it has the potential to contribute significantly in a historically neglected area.

5.9 Originality and Distinctive Contribution of this Study to the Knowledge Base of Educational Psychology

Mathematics is a core subject of the National Curriculum, but presents heightened feelings of dislike and anxiety amongst a large number of people. Although young primary school pupils do not typically present such strong feelings; they do appear to become evident towards the end of primary schooling and much more evident during the secondary years, when much disaffection with the subject can be presented. Although mathematics anxiety is an established construct, there appears to be a significant gap in the research base in addressing this issue in the primary phrase. Mental calculation can provoke various feelings of anxiety in many people; mental calculation strategies are now required to be taught in schools. Several researchers have suggested different mental classification systems and some have attempted to address the issue of flexible mental calculation strategies. However, there has been a significant gap in the research exploring the possible association between mathematics anxiety in primary school children and the mental calculation strategies that pupils may use.

In this study, the ability to communicate mathematically is underpinned by learner self-perception and driven particularly by anxiety in relation to calculation skills. The research addresses why children may use a particular strategy and offers support for an alternative in order to improve mathematical outcomes. There appears to be a strong theme in the research literature that whilst we can describe the mechanics of calculation, this is not enough, particularly given the variety of idiosyncratic mental calculation methods that pupils can present. Children need to be empowered to think mathematically, both for an enjoyment of the broader subject and for the specific task of mental calculation. Taking a constructivist approach towards a cognitive difficulty, this study offers a significant contribution to the research base in demonstrating how mathematics anxiety influences a pupil's choice of one type of mental calculation strategy in preference to another.

CHAPTER 6: EVALUATION AND IMPLICATIONS

Educational Psychology has traditionally been intimately involved in the assessment and development of literacy skills in children, particularly because of the need for these skills across all curriculum areas and ultimately in the lives of pupils outside of school. Since the implementation of the National Numeracy Strategy (NNS) in 1999, Mathematics/Numeracy has gained a similar standing. All children are now required to have a daily Mathematics/Numeracy lesson. In the context of this study, all pupils are required to have daily practice in order to develop oral and mental skills, for example, counting, mental strategies, rapid recall of addition, subtraction, multiplication and division facts. Furthermore, the government is committed to raising standards and has required that 75% of Year 6 children were required to obtain a level 4 in their mathematics SAT's by 2002. However, it would not seem desirable to focus just on one indicator of success. In Mathematics, 73% of children reached the expected level, short of the target of 75%, but a considerable increase from the 61% of 1997, (Earl et al, 2003). There is a perception in schools that NNS has improved pupil learning. However, there are also concerns regarding what constitutes raising standards and that schools will simply focus on priming their pupils for the standard assessment tasks. Educational Psychologists are well placed in their regular contact with schools to support teachers and pupils in the development of mathematics and numeracy skills. The NNS has seen a better range and balance of mathematical elements, an increase in the amount of whole class teaching, more attention given to the pace of lessons and planning based on objectives rather than activities. However, there remains considerable concern regarding variations in teacher's subject knowledge, weaknesses in pedagogy in mathematics/numeracy and weaknesses in the delivery of the strategy itself.

The main study provided a good opportunity to investigate the possible influence of mathematics anxiety on pupils' mental calculation strategies for two-digit addition and subtraction. Previous research had largely focused on the classification of these strategies, with much evidence and associated findings coming from Dutch research in the 1990's. In the main study, pupils were drawn from typical mainstream classes and were controlled for competence, language, reading/working memory, ability/special needs and age. They were approximately equal numbers of boys and girls with slightly more boys. No significant differences were found between learner self-perception scores, but some differences were found between schools mathematics anxiety scores suggesting there may be some unusual variations in mathematics anxiety between individual schools. Although there were differences in the numbers of pupils from each of the eight schools contributing to the second part of the study, all pupils had had a similar curriculum. The differences found may well be due to broader school or more likely, teacher effects. Findings from the main study indicate that it is possible to classify mental strategies for two digit addition and subtraction in a way that increasingly facilitates calculation. A clear distinction was found between the partitioning of both numbers (lower order) and the partitioning of only one of the numbers (higher order). Furthermore, low mathematics anxiety was related to the higher order mental calculation strategies and more accurate results, whereas high mathematics anxiety was related to lower order strategies and less accurate results. No significant gender differences were found.

Mathematics anxiety is now an established construct, together with a substantial body of research. The roots and origins of mathematics anxiety have been discussed. This study indicates that a particular consequence of mathematics anxiety is the limiting effect that it may have on pupils' choice of mental calculation strategy and reduced accuracy in mental calculation. An equally interesting approach would be to select pupils based on either high or low self-perception. Given the strong inverse association found between self-perception and mathematics anxiety, it may well be the case that self-perception produces similar (but opposite) results and could be the focus for a further study. Understanding more about affective issues should have important implications for both curricula design and teaching strategies. The Mathematics Anxiety Scale for Children could be used to identify children at risk for underachievement/learning disabilities in mathematics. It could also be used support high schools in making decisions about setting arrangements rather than relying on a one off test or teacher opinion. Although the MASC can be regarded having an underlying normal distribution and does provide a reliable and valid instrument, it does not necessarily indicate cause. Pupils' mathematics anxiety may well be driven by home, culture and social factors, and not simply by school or teacher factors. Bandura (1986) observed that confidence is both a personal and a social construct, collective systems such as classrooms/schools develop a sense of collective efficacy or a groups shared beliefs in its capability to attain their goals and accomplish desired tasks. Similar arguments are likely to apply to mathematics anxiety, self-perception and other affective constructs. Teachers may well be best served by paying as much attention to these affective issues that may influence competence as to actual competence itself.

The NNS has raised the profile of mathematics and has clearly significantly effected classroom practice and pupils learning experiences. There is value in having a common framework between schools but there are concerns that it may lessen opportunities for teachers making professional judgements about the learning experiences of their pupils. Furthermore, a closely prescribed curriculum together with a very high level of national testing may have the unfortunate consequence of raising pupil mathematics anxiety and thereby reducing their opportunity for achievement, because of the limitations placed on their calculation strategies. Mathematics anxiety may be an outcome of educational experience but may also be a factor determining future learning. The use of a high level of testing in this country may result in a narrowing of the curriculum that further increases a pupil's mathematics anxiety. It is likely that only those pupils with very low mathematics anxiety and who feel very confident enjoy taking SAT's (Standard Assessment Tasks). The repeated practice of these seen in some schools may further disadvantage children susceptible to high mathematics anxiety.

In mainstream primary schools, Educational Psychologists have historically focused much of their efforts on two areas; the development of children's literacy skills and the management of behaviour. Mathematics and numeracy have been largely ignored, partly because of a lack of insightful tools, few referrals from schools and an anxiety for this curriculum area reflected by some teachers, parents and pupils. The NNS provides an opportunity for Educational Psychologists to develop their skills in this area both for pragmatic reasons and for broader reasons such as investigating thinking skills. Mental calculation is one specific area, Educational Psychologists need to be aware of the strategies that may be used and of the way in which mathematics anxiety may limit and restrict pupils to using lower order strategies. A number of questions and issues arise:

- How to address the legitimate use of pupils' personal mental calculation strategies and given a prescribed curriculum, how can schools be supported in the management of government and QCA strategies?

- How to raise the awareness of mental calculation strategies in the classroom? When to introduce specific techniques such as the empty number line and raise both whole class teaching methods and appropriate individual support for pupils, who find this work particularly demanding,

- What are the implications of pupils' errors in mental calculation, what are the implications for instruction, individual, small-group and whole-class work?

- What are the implications for children who have low self-perception and high mathematics anxiety and vice versa? What support arrangements are needed?

- How does this study relate to the individual, for example specific learning difficulties in the form of dyscalculia?

The broader role of the Educational Psychologist was explored in the discussion section of this study. More specifically Educational Psychologists should have at the very least, a working knowledge of the above, particularly that children in school now spend a significant part of their day on mathematics and numeracy. The theme of this study underpinning many of the features, has been constructivism. In a surprisingly strong statement, (Hardman & Worthington, 2000) declared that their research indicated that the majority of Educational Psychologists are supportive of inclusion and integration, and adopt a social constructivist philosophy towards the nature and process of children's learning. Pupils do actively process their learning, rather than having it given to them, teachers can be facilitators for that learning, rather than just transmitters of a fixed body of knowledge. However, the currently very prescribed curriculum, focusing on the superficial benefits of whole class teaching, together with a high level of associated testing is likely to increase mathematics anxiety in both teachers and their pupils, and potentially limit the opportunities for flexible use of mental calculation strategies.

Although mental calculation is important, it needs to be reinforced that it is one aspect of mathematics and numeracy. It has the unfortunate quality that many people both children and parents feel extremely threatened, vulnerable and often defensive when asked to take part in this activity. It is also clear that teachers may have too often have relied on very similar techniques and methods across the variety of school mathematics. Children benefit enormously from consistency in approach but also need both explicit and implicit stimulation. The range of methods that classteachers can and do use has much potential. However, it would be expected that these would be more reflected in the pupils' articulation of their strategies, than was the case in this study. For young pupils at Key Stage 1, their general perception of themselves as learners is as likely to be as critical as any other affective factor. As they become older, pupils become increasingly aware of their strengths and weaknesses, by Key Stage 2 they are able to make judgements about their mathematics anxiety. The early identification of mathematics anxious pupils, possibly using a modified form of the

Mathematics Anxiety Scale for Children should prove to be a better way of addressing individual difficulties and may also be extremely useful in supporting pupils as they progress through school. Educational Psychologists need to continually and firmly reinforce the beauty of mathematics in its many forms and that the enjoyment that comes from being involved will drive the progress of all.

REFERENCES

ADAMS, J.W. & HITCH G.H. (1997) Working Memory and Children's Mental Addition *Psychology Journal of Experimental, 67,* 21-38

ALEXANDER, L. & MARTRAY, C. (1989) The development of an abbreviated version of the Mathematics Anxiety Rating Scale *Measurement and Evaluation in Counselling and Development, 22,* 143-150.

ANALYSE-IT (2003) General Statistics Module Version 1.69. (downloaded from < www.analyse-it.com).

ASHCRAFT M.H. (1992) Cognitive Arithmetic : A Review of data and theory. *Cognition, 44*, 75-106.

ASHCRAFT M.H. (1995) Cognitive psychology and simple arithmetic : a review and summary of new directions. *Mathematical Cognition, 1*, 3-34

ASHCRAFT M.H. & FAUST M.W. (1994) Mathematics Anxiety and mental Arithmetic performance : an exploratory investigation. *Cognition and Emotion, 8,* 97-125

ASHCRAFT, M. H. & KIRK, E. P. (2001) The Relationship Among Working Memory, Math Anxiety and Performance. *Journal of Experimental Psychology, 130(2),* 224-237.

ASHCRAFT, M. H., KIRK, E. P. & HOPKO, D. (1999) On the cognitive consequences of mathematics anxiety. In Donlan, C. (Ed) *The Development of Mathematical Skills.* Hove, East Sussex, UK: Psychology Press.

ASHTON, P. T. (1985) Motivation and teachers' sense of efficacy. In C. Ames & R. E. Ames (Eds) Research on motivation in education: Vol. 2: the classroom milieu (141-148). Orlando, USA: Academic Press.

ASKEW, M., BROWN, M., RHODES, V., JOHNSON, D. & WILIAM, D. (1997) *Effective Teachers of Numeracy: Final Report (February 1997)_Report of a study carried out for Teacher Training Agency 1995-96 by the School of Education, King's College London.* London: Kings College.

ASKEW, M., BIBBY T. &BROWN M. (1998) The Co-Construction of Mathematical Knowledge: The Effect of an Intervention Programme on Primary Pupils' Attainment *Proceedings of the 22nd Conference for the International Group for the Psychology of Mathematics Education 2,* 40-47

AUBREY, C. (1997) *Mathematics Teaching in the Early Years: An Investigation of Teachers Subject Knowledge* London & Washington: Falmer Press

AUSABEL, D. P. (1968) *Educational Psychology: a cognitive view,* New York: Holt, Rinehart and Wilson.

BADDELEY, A. & HITCH G.J.(1974) Working Memory. In G.H. Bower (Ed) *The psychology of learning and motivation* (Vol. 8, pp. 47-89) New York: Academic Press

BADDELEY, A. (1986) *Working memory* Oxford: Oxford University Press.

BADDELEY, A. (1992) Is Working Memory Working? - The Fifteenth Bartlett Lecture *The Quarterly Journal of Experimental Psychology, 44A (1),* 1-31.

BADDELEY, A. (2000) The episodic buffer: a new component of working memory? *Trends in Cognitive Sciences* 4(11) (November 2000), 417-423.

BANDURA, A. (1986) *Social Foundation of Thought and Action: A Social Cognitive Theory* Eaglewood Cliffs, New Jersey, USA: Prentice-Hall

BANDURA, A.(1993) Perceived Self-Efficacy in Cognitive Development and Functioning. *Educational Psychology, 28(2),* 117-148.

BARTRAM, P. & WOLFENDALE, S. (1999) Educational Psychology Services *Educational Psychology in Practice, 15(1),* 51-56.

BEASLEY, T. M., LONG, J. D. & NATALI, M. (2001) A Confirmatory Factor Analysis of the Mathematics Anxiety Scale for Children. *Measurement and Evaluation in Counselling and Development* 34 (April), 15-26.

BEISHUIZEN, M. (1993) Mental strategies and materials or models for addition and subtraction upto 100 in Dutch second graders. *Journal for Research in Mathematics Education, 24 (4),* 294-323.

BEISHUIZEN, M. (1997) *Two types of mental arithmetic and the Empty Number Line.* Paper presented at the British Society for Research into Learning Mathematics conference, 7th June 1997. University of Oxford, Department of Educational Studies.

BEISHUIZEN, M. (1999) The empty number line as a new model. In Thompson, I. *Issues in Teaching Numeracy in Primary Schools.* Buckingham: Open University Press.

BEISHUIZEN, M. & ANGHILERI, J. (1998) Which mental calculation strategies in the early number curriculum? A comparison of British ideas and Dutch views, British Educational Research Journal, *24,* 519-538.

BEISHUIZEN, M., VAN PUTTEN, C.M. & VAN MULKEN, F. (1997) Mental Arithmetic and Strategy Use with Indirect Number Problems up to One Hundred *Learning and Instruction, 7(1),* 87-196.

BESSANT K. (1995) Factors associated with types of mathematics anxiety in college students. *Journal for Research in Mathematics Education, 20(4),* 327-345.

BLACK, P. & WILIAM, D. (1998) Assessment and Classroom Learning *Assessment in Education, 5(1),* 7-73.

BLÖTE, A.W., KLEIN, A.S., BEISHUIZEN, M. (2000) Mental Computation and Conceptual Understanding, *Learning and Instruction, 7(1),* 87-106.

BOUFFARD, T., MARCOUX, M-F., VEZEAU, C. & BORDELEAU, L. (2003) Changes in Self-Perceptions of Competence and Intrinsic Motivation among Elementary Schoolchildren. *British Journal of Educational Psychology, 73(2),* 171-186.

BOULTON-LEWIS, G. M. & TAIT, K. (1994) Young children's representations and strategies for addition *British Journal of Educational Psychology, 64,* 231-241.

BRAINERD, C. J. (1983) Young children's mental arithmetic errors: a working memory analysis *Child Development, 54,* and 814-830.

BRAINERD, C. J. (1987) sources of working memory error in children's mental arithmetic. In G.Deloche & X. Serena (eds), *Mathematical Disabilities : a cognitive neuropsychological perspective.* Hillsdale NJ : Erlbaum.

BREKELMANS, M., VAN DAN EEDEN, P., TERWEL, J. & WUBBELS, T. (1997) Student characteristics of environment interactions in Mathematics and Physics Education : A Resource Perspective. *International Journal of Educational Research, 27(4),* 283-292.

BROPHY, J. E. & GOOD, T. L. (1970) Teachers' Communications of Differential Expectations for Children's Classroom Performance : Some behavioural data. *Journal of Educational Psychology,* 61(5), 366-378.

BURDEN, R. (1998) Assessing Children's Perceptions of Themselves as Learners and Problem Solvers: The construction of the Myself-As-Learner Scale (MALS) *School Psychology International, 19(4),* 291-305.

BURDEN, R.L. & FRASER, B.J. (1993) Use of Classroom Environment Assessments in School Psychology : A British Perspective *Psychology in the Schools, 30 (July),* 232-240.

BURNETT, P.C. (1994) Self-concept and self-esteem in elementary school children. *Psychology in the Schools, 31 (April),*164-171.

BURNETT, P. C. & PROCTOR, R. M. (2002) Elementary School Student's Learner Self-Concepts and Approaches to Learning. *Educational Psychology in Practice, 18(4),* 325-334.

BURNETT, P. C., PILLAY, H., DART, B. C. (2003) The Influence of Conceptions of Learning and Learner Self-Concept on High School Students' Approaches to Learning. *School Psychology International, 24(1*), 54-66.

BUSH, W.S. (1989) Mathematics Anxiety in Upper Elementary School Teachers, *School Science and Mathematics, 89(6), (October),* 499-509.

BUTTERWORTH B. (1999) *The Mathematical Brain* London: Macmillan.

CARPENTER, T. P., FRANKE, M. L., JACOBS, V. R., FENNEMA, E. and EMPSON, S. B. (1998) 'A longitudinal study of intervention and understanding children's addition and subtraction.' *Journal for Research in Mathematics Education, 29, (1),* 3-20.

CHIU, L. & HENRY L.H. (1990) Development and Validation of the Mathematics Anxiety Scale for Children *Measurement and Evaluation in Counselling and Development, 23,* 121-127.

CLARKE D. M, (1996) The Changing Role of the Mathematics Teacher *Journal for Research in Mathematics Education 27(1),* 23-32.

COBB, P. (1988) The tension between theories of learning and instruction in mathematics education *Educational Psychologist, 23(2),* 87-103

COBB, P., YACKEL, E., & WOOD, T. (1992) A Constructivist Alternative to the Representational View of Mind in Mathematics Education *Journal for Research in Mathematics Education* *23(1),* 2-33

COBB, P., STEPHAN, M., McCLAIN, K. & GRAVEMEIJER, K. (2001) Participating in Classroom Mathematical Practices. *The Journal of Learning Sciences, 10,* 113-163.

COCKCROFT, W.H. (1982) *Mathematics Counts : Report of Inquiry into the Teaching of Mathematics in Schools under the Chairmanship of Dr. W .H. Cockcroft* London : H.M.S.O.

DARKE, S. (1988) Anxiety and Working Memory Capacity *Cognition and Emotion, 2(2),* 145-154.

DENVIR, B. & BROWN, M. (1986) Understanding of number concepts in low attaining 7-9 yr. olds: Part 2. The teaching studies *Educational Studies in Mathematics, 17*, 143-164.

DENVIR, B. & BROWN, M. (1987) The feasibility of class administered diagnostic assessment in mathematics *Educational Research, 29(2),* 95-107.

DES (1987) *The National Curriculum 5 – 16: a Consultation Document* London: HMSO.

DEW K, GALLASSI J. P. & GALASSI M.D. (1983) Mathematics Anxiety : Some Basic Issues *Journal of Counselling Psychology, 30(3),* 443-446

DfEE (1998) *Numeracy Matters: The Preliminary Report of the Numeracy Task Force.* Reynolds, D. (Chair) London: DfEE.

DfEE (1999) *The National Numeracy Strategy Framework for Teaching Mathematics from Reception to Year 6.* Sudbury: DfEE Publications.

DfEE (2000a) Educational *Psychology Services (England): Current Role, Good Practice and Future Directions.* Report of the Working Group. Nottingham DfEE publications.

DfEE (2000b) Educational *Psychology Services (England): Current Role, Good Practice and Future Directions.* The Research Report. Nottingham DfEE publications.

DfES (2001) *Special Educational Needs Code of Practice* Nottingham: DfES.

DONLAN C. (ED) (1998) *The Development of Mathematical Skills* London Psychology Press

DOWKER, A. D. (1997) Young children's addition estimates *Mathematical Cognition, 3,* 141-180

EARL, L, WATSON, N., LEVIN, B., LEITHWOOD, K., FULLAN, M. and TORRANCE, N. with JANTSI, D., MASCALL, B., VOLANTE, L. (2003) *Watching and Learning 3. Final Report of the External Evaluation of England's National Literacy and Numeracy Strategies.* Ontario Institute for Studies in Education, University of Toronto. Nottingham: DfES Publications.

ENOCHS, L.G., SMITH, P. L. & HUINKER, D. (2000) Establishing Factorial Validity of the Mathematics Teaching Efficacy Beliefs Instrument *School Science and Mathematics, 100(4) (April 2000),* 194-202.

EVANS, J. (2000) *Adults' Thinking and Emotions: A Study of Numerate Practices London*: Routledge Falmer.

FAUST M.W., ASHCRAFT M.H. & FLECK D.E. (1996) Mathematics Anxiety Effects in Simple and Complex Addition, *Mathematical Cognition, 2(1),* 25-62.

FENNEMA, E. & SHERMAM, J. (1976) Fennema-Sherman Mathematics Attitudes Scales: Instruments Designed to Measure Attitudes Toward the Learning of Mathematics by Females and Males *Abstracted in the JSAS Catalog of Selected Documents in Psychology, 6(1),* 31-71.

FOXMAN, D. & BEISHUIZEN, M. (2002) Mental Calculation Methods Used by 11-Year Olds in Different Attainment Bands: a reanalysis of data from the 1987 APU survey in the UK. *Educational Studies in Mathematics, 51(1/2),* 41-69.

FUSON, K. (1992) Research on whole number addition and subtraction. In D. A. Grouws (Ed) *Handbook of Research on Mathematics Teaching and Learning, 243-275, New York: Macmillan.*

FUSON, K., WEARNE, D. HIEBERT, J.C., MURRAY, H.G., HUMAN, P.G., OLIVIER, A.L., CARPENTER, T.P., & FENNEMA, E. (1997) Children's conceptual structures for multidigit numbers and methods of addition and subtraction *Journal for Research in Mathematics Education, 28(2),*130-162

GATHERCOLE, S.E. & PICKERING, S.J. (2000) Working memory deficits in children with low achievements in the national curriculum at 7 years of age *British Journal of Educational Psychology, 70*,177-194.

GATHERCOLE, S. E., BROWN, L. E. & PICKERING, S. J. (2003) Working Memory Assessments at school entry as longitudinal predictors of National Curriculum attainment levels. *Educational and Child Psychology, 20*(3), 109-122.

GHAITH, G. & SHAABAN, K. (1999) The relationship between perceptions of teaching concerns, teacher efficacy, and selected teacher characteristics. *Teacher and Teacher Education, 15,* 487-496.

GIBSON, S. & DEMBO, M. (1984) Teacher Efficacy: A construct validation. *Journal of Educational Psychology, 76(4),* 569-582.

GIERL, M.J. & BISANZ, J. (1995) Anxieties and attitudes related to mathematics in grades 3 and 6. *Journal of Experimental Psychology, 63,* 139-158.

GINSBERG, H.P. (1997) Mathematics Learning Disabilities: A View from Developmental Psychology *Journal of Learning Disabilities, 30(1),* 20-33.

GIPPS, C. & MCCALLUM, B & BROWN, M. (1996) Models of teacher assessment among primary school teacher in England *The Curriculum Journal*, 7 (2),167-183.

HADFIELD O. D., MARTIN J. V. & WOODEN S. (1992) Mathematics Anxiety and Learning Style of Navajo Middle School Students *School Science and Mathematics, 92 (4),* 171-176.

HALLOM, S., IRESON, J. & DAVIES, J. (2002) *Effective Pupil Grouping in the Primary School: A Practical Guide.* London: David Fulton Publishers.

HARDMAN, M. & WORTHINGTON, J. (2000) Educational Psychologists' Orientation to Inclusion and Assumptions about Children's Learning. *Educational Psychology in Practice, 16(2),* 349-360.

HART, L.E. (1989) Classroom processes, sex of student, and confidence in learning mathematics *Journal for Research in Mathematics Education, 20(3),* 242-260.

HEID, M.K., BLUME, G.W., ZBIEK, M.K. EDWARDS, B. S. (1999) Factors that influence teachers learning to do interviews to understand students' mathematical understandings *Educational Studies in Mathematics, 37,* 223-249

HEIRDSFIELD, A. & COOPER, T. J. (1997) The architecture of mental addition and subtraction. *Paper presented at the Annual Conference of the Australian Association for Educational Research, Brisbane, Australia.* (downloaded from < http://.aare.edu.au/97pap/heira395.htm)

HEMBREE, R. (1990) The nature effects and relief of mathematics anxiety. *Journal for Research in Mathematics Education, 21*, 33-46

HENRY, L. A. & MacLEAN, M. (2003) Relationships between working memory, expressive vocabulary and arithmetical reasoning in children with and without intellectual disabilities. *Educational and Child Psychology, 20(3),* 51-64.

HITCH, G. J. (1978) The role of short-term working memory in mental arithmetic. *Cognitive Psychology, 10,* 302-323.

HITCH G.J. & TOWSE J.N. (1995) Working Memory : What Develops? in F.E.Weinhart & W. Schneider (Eds) *Memory, performance and competencies : Issues in Growth and Development,1-22,* Hove,UK:Erlbaum.

HOFER, B.K. & PINTRICH P.R. (1997) The Development of Epistemological Theories : Beliefs about Knowledge and Knowing and their Relation to Learning *Review of Educational Research, 67(1)*, 88-140.

JAWORSKI, B. (1999) The Plurality of Knowledge Growth in Mathematics Education, in Jaworski, B., Wood, T. & Dawson S. (eds) *Mathematics Teacher Education : Critical International Perspectives* (pp180-209) London : Falmer Press.

JAWORSKI, B. & POTARI, D. (1998) Characterising Mathematics Teaching Using the Teaching Triad *Proceedings of the 22nd Conference of the International Group for the Psychology of Mathematics Education, 3,*17-24.

JENNINGS, S. & DUNNE, R. (1997) Improving the quality of teaching and learning Mathematics *Mathematics Teaching,158,* 34-37.

JINKS, J. & MORGAN, V. (1999) Children's Perceived Academic Self-Efficacy: An Inventory Scale. *http://www.coe.ilstu.edu/portfolios/students/elmanwa/MORGAN.doc*

JUSSIM, L., SMITH, A., MADON, S. & PALUMBO, P. (1998) Teacher Expectations in Advances in Research on Teaching, *Volume (7),* 1-48.

KLATTER, E.B., LODEWIJKS, H.G.L.C. & AARNOUTSE, C.A.J. (2001) Learning Conceptions of Young Students in the Final Year of Primary Education *Learning and Instruction, 11,* 485-516.

KLEIN, A.S., BEISHUIZEN, M. & TREFFERS, A. (1998) The Empty Number Line in Dutch Second Grades: Realistic versus Gradual Program Design *Journal for Research in Mathematics Education, 29(4),* 443-464.

LOWN, J., FOX, G., GERSCH, I., MORRIS, S. & STOKER, R. (2001) Reconstructing Educational Psychology Practice Support *for Learning, 16(3),* 135-139.

MA, X. (1997) Reciprocal Relationships Between Attitude Toward Mathematics and Achievement in Mathematics *The Journal of Educational Research, 90(4),* 221-229.

MA X. (1999) A Meta-Analysis of the Relationship between Anxiety Towards Mathematics and Achievement in Mathematics *Journal for Research in Mathematics Education, 30(5),* 520-540

MACKAY, T. (2002) Discussion Paper – The Future of Educational Psychology *Educational Psychology in Practice, 18(3),* 245-254.

MANOUCHEHRI, A. & GOODMAN, T. (1998) Mathematics Curriculum Reform and Teachers: Understanding the Connections *The Journal of Educational Research, 92(1),* 27-41.

MARTINEZ, G. R., & MARTINEZ, N. C. (1996). *Math without fear: A guide for preventing math anxiety in children.* Needham Heights, USA: Allyn and Bacon.

MAYBERY, M. T. & DO, N. (2003) Relationships between facets of working memory and performance on a curriculum-based mathematics test in children. *Educational and Child Psychology, 20(3),* 77-92.

McLEOD, D. B. (1992) Research on Affect in Mathematics Education: A Reconceptualisation. In D. A. Grouws (Ed) *Handbook of Research on Mathematics Teaching and Learning.* New York: Macmillan.

McLEOD, D.B. (1994) Research on Affect and Mathematics Learning in the JRME: 1970 To The Present *Journal for Research in Mathematics Education, 25(6)*, 637-647.

McCLOSKEY, M. (1992) Cognitive mechanisms in numerical processing Evidence from acquired dyscalculia. *Cognition, 44,*107-157.

McKINNEY, M., SEXTON, T. & MEYERSON, M. J. (1999) Validating the Efficacy-Based Change Model *Teacher and Teacher Education, 15*,471-485.

McINTOSH, A., REYS, B. & REYS, R. (1992) A propose framework for examining number sense. *For the Learning of Mathematics, 12*, 2-8.

MIDDLETON, J.A. & SPANIAS, P.A. (1999) Motivation for Achievement in Mathematics : Findings, Generalisations, and Criticisms of the Research *Journal for Research in Mathematics Education, 30(1),* 66-88

MOOS, R. H. (1973) Conceptualisations of Human Environments *American Psychologist (August) 28*, 652-665.

MURRAY, H. & OLIVIER, A. (1989) A model of understanding two-digit numeration and computation. In G. Vergnaud (Ed), *Proceedings of the Thirteenth International Conference for the Psychology of Mathematics Education, Volume 3*, 3-10, G. R. Didactique, Paris.

NICHOLLS, J. G., COBB, P., WOOD, T. YACKEL, E. & PATASHNICK, M. (1990) Assessing Students' Theories of Success in Mathematics: Individual and Classroom Differences. *Journal for Research in Mathematics Education, 21(2),* 109-122.

NIXON, M. (2000) *Teaching and Learning Mathematics* London & New York : Cassell Education

PAJARES F. (2000) *Current Directions in Self-efficacy Research* (downloaded from < www.emory.edu/EDUCATION/mtp/ettchapter.html >*)*

PATRICK, B. C., SKINNER, E.A. & CONNELL, J.P. (1993) What Motivates Children's Behaviour and Emotion? Joint Effects of Perceived Control and Autonomy in the Academic Domain. *Journal of Personality and Social Psychology, 65(4),* 781-791.

PINTRICH, P. & SCHUNK, D. (1996) *Motivation in Education: Theory, Research & Applications,* Englewood Cliffs, NJ, USA: Prentice-Hall.

PRYOR, J. & TORRANCE, H. (1996) Teacher-pupil interaction in formative assessment: assessing the work or protecting the child? *The Curriculum Journal, 7 (2),* 205-226

QCA (1999) *The National Numeracy Strategy: Teaching mental calculation, guidance for teachers at key stages 1 and 2.* Sudbury: DfEE Publications.

RESNICK, H., VIEHE, J. & SEGAL, S. (1982) Is Math Anxiety a Local Phenomenon? A Study of Prevalence and Dimensionality *Journal of Counselling Psychology, 29(1),* 39-47.

REYS, L. H. (1984) Affective variables and mathematics education. *Elementary School Journal, 84,* 558-581.

REYS, B.J., REYS, R.E. & HOPE, J.A. (1993) Mental Computation: A Snapshot of Second, Fifth and Seventh Grade Student Performance. *School Science and Mathematics, 93 (6),*306-315.

REYS, R.E., REYS, B. J., NOHDA, N. & EMORI, H. (1995) Mental computation performance and strategy use of Japanese students in Grades 2, 4, 6 and 8. *Journal of Research in Mathematics Education, 22(1),* 304-326.

RECH J., HARTZELL J. & STEPHENS L. (1993) Comparisons of Mathematical Competencies and Attitudes of Elementary Majors with Established Norms of a General College Population *School Science and Mathematics, 93(3),* 141-144.

RESNICK H. VIEHE J. & SEGAL S. (1982) Is Math Anxiety a Local Phenomenon? A Study of Prevalence and Dimensionality *Journal of Counselling Psychology,29(1),* 39-47.

RICHARDSON F. & SUINN R. (1972) The Mathematics Anxiety Rating Scale : Psychometric Data *Journal of Counselling Psychology, 19(6),* 551-554.

RIVERA, D. P. (1997) Mathematics Education and Students with Learning Disabilities: Introduction to the Special Series *Journal of Learning Disabilities, 30(1),* 2-19.

ROSS, J. (1998) Teacher Expectations, in Brophy, J & Good, T.L. (Eds) *Advances in Research on Teaching, Vol. 7,* New York: JAI Press Inc

ROSS, J.A., HOGABOAM-GRAY, A. & ROLHEISER, C. (2001) Student self-evaluation in Grades 5-6 Mathematics Effects on Problem Solving Achievement *Paper presented at the annual meeting of the American Educational Research Association* Seattle, U.S.A., April 2001.

RUTHVEN, K. (1998) The Use of Mental, Written and Calculator Strategies of Numerical Computation by Upper Primary Pupils within a 'Calculator-aware' Number Curriculum *British Educational Research Journal, 24(1),* 21-42.

SCHOENFELD, A. H. (1983) Beyond the Purely Cognitive: Belief Systems, Social Cognitions, and Metacognitions As Driving Forces in Intellectual Performance *Cognitive Science, 7,* 329-363.

SCHUNK, D. H. (1991) Self-efficacy and academic motivation. *Educational Psychologist, 26,* 207-231.

SIMON, M. A. (1995a) Reconstructing Mathematics Pedagogy from a Constructivist Perspective *Journal for Research in Mathematics Education, 26(2),* 145-166.

SIMON, M. A. (1995b) Elaborating Models of Mathematics Teaching : A Response to Steffe & D'Ambosio *Journal for Research in Mathematics Education 26(2),* 160-162.

SKEMP, R. R. (1979) *Intelligence, Learning and Action* Chichester, UK: John Wiley & Sons.

SKEMP, R. R. (1993) Theoretical Foundations of Problem Solving: A Position Paper. University of Warwick (1993.) Downloaded from (< http: //www.skemp.org.uk >)

STEFFE, L. P. (1991) The Constructivist Teaching Experiment : Illustrations and Implications. In von Glaserfeld, E. (ed) *Radical Constructivism in Mathematics Education* Dordrecht, The Netherlands : Kluwer Academic

STIPEK, D. J., GIVVIN, K. B., SALMON, J. M. & MacGYVERS V. L. (2001) Teachers' beliefs and practices related to mathematics instruction. *Teaching and Teacher Education, 17,* 213-226.

STODOLSKY, S. S. (1985) Telling Math: Origins of Math Aversion and Anxiety *Educational Psychologist* 20(3), 125-133.

SUINN, R. M., TAYLOR, S. & EDWARDS, R. W. (1988) Suinn Mathematics Anxiety Rating Scale for Elementary School Students (MARS-E): Psychometric and Normative Data, *48(4),* 979-986.

SWANSON, H.L. (1993) Working memory in learning disability subgroups. *Journal of Experimental Child Psychology, 56*, 87-98.

THOMPSON, I. (1995) The role of counting in the idiosyncratic mental calculation algorithms of young children. *European Early Childhood Education Research Journal, 3(1),* 5-16.

THOMPSON, I. (1999a) Getting your head around mental calculation, In Thompson, I., (Ed) *Issues in teaching numeracy in primary schools* Buckingham, UK: Open University Press.

THOMPSON, I (1999b) Mental Calculation Strategies for Addition and Subtraction: Part 1 *Mathematics in School, 28(5), 2-4.*

THOMPSON, I. (2000a) Mental Calculation Strategies for Addition and Subtraction: Part 2 *Mathematics in School, 29(1),* 24-26.

THOMPSON, I. (2000b)*Understanding and Assessing Children's Thinking through Mental Calculation.* Downloaded from (< www.nku.edu/~sheffield/ithompson.html >)

THRELFALL, J. (2002) Flexible Mental Calculation. *Educational Studies in Mathematics, 50(1),* 29-47.

TITTLE, C.K. (1994) Toward an Educational Psychology of Assessment for Teaching and Learning: Theories, Contexts and Validation Arguments *Educational Psychologist, 29(3),* 149-162.

TOBIAS, S. (1995) *Overcoming Math Anxiety* New York, W.W. Norton.

TORRANCE, H. & PRYOR, J. (1995) Investigating Teacher Assessment in Infant Classrooms: methodological problems and emerging issues *Assessment in Education, 2(3),* 305-320.

TOWNSEND, M., LAI, M. K., LAVERY, L., SUTHERLAND, C. & WILTON, K. (1999) Mathematics Anxiety and Self-Concept: Evaluating Change Using The "Then-Now" Procedure. *Paper Presented at the Joint Conference of the Australian Association for Research in Education the New Zealand Association for Research in Education, Melbourne, Australia, December 1999.*

TREAGUST, D. F., DUIT, R. & FRASER, B. (1998) *Improving Teaching and Learning in Science and Mathematics* Columbia University, USA: Teachers College.

TSCHANNEN-MORAN, M. & WOOLFOLK-HOY, A. (2001) Teacher Efficacy: capturing an elusive construct. *Teaching and Teacher Education, 17(7),* 783-808.

TSCHANNEN-MORAN, M. & WOOLFOLK-HOY, A. & HOY, W. (1998) Teacher Efficacy: Its Meaning and Measure *Review of Educational Research, 68(2),* 202-248.

TUNSTALL, P. & GIPPS, C. (1996) 'How does your teacher help you to make your work better?' *The Curriculum Journal, 7(2)*,185-203.

TURNER, J.C., MIDGLEY, C., MEYER, D.K., ANDERMAN, E.M., KANG, Y. & PATRICK, H. (2002) The Classroom Environment and Students' Reports of Avoidance Strategies in Mathematics: A Multimethod Study *Journal of Educational Psychology, 94(1),* 88-106.

VON GLASERFIELD, E. (1987) 'Constructivism', In Husen, T. & Postlewaite, N. (Eds) *International Encyclopaedia of Education, Vol 1,* Oxford; Pergamon

VYGOTSKY, L. S. (1978) *Mind in Society* Cambridge: Harvard University.

WATSON, A. (1998) What makes a mathematical performance noteworthy in informal teacher assessment *Proceedings of the 22nd Conference for the Psychology of Mathematics Education, 4,*169-175.

WIGFIELD, A. & MEECE, J.L. (1988) Math anxiety in elementary and secondary school students. *Journal of Educational Psychology, 80,* 210-216.

WILIAM, D. (1999) Formative assessment in Mathematics: Part 1: Rich Questioning, *Equals* 5(2).

WOOD, D. (1988) *How Children Think and Learn: The Social Contexts of Cognitive Development* Oxford: Blackwell Publishers.

WOOLFOLK, A. E. & HOY, W. K. (1990) Prospective teachers' sense of efficacy and beliefs about control . *Journal of Educational Psychology, 82,* 81-91.

WOOLFOLK, A. E., ROSOFF, B. & HOY, W.K. (1990) Teachers' Sense of Efficacy and their beliefs about managing students. *Teaching and Teacher Education, 6(2),* 137-148.

WRIGHT, D. B. (2003) Making friends with your data: Improving how statistics are conducted and reported. *British Journal of Educational Psychology, 73,* 123-136.

WRIGHT, R.W., MARTLAND, J. & STAFFORD, A.K. (2000) *Early Numeracy : Assessment for Teaching and Intervention* London : Paul Chapman Publishing.

WRIGHT, R.J., MARTLAND, J., STAFFORD, A.K. & STANGER, G. (2002) *Teaching Number: Advancing children's skills and strategies.* London: Paul Chapman Publishing.

YASUTAKE, D. & BRYAN, T. (1998) The Influence of Affect on the Achievement and Behaviour of Students with Learning Disabilities, *Journal of Learning Disabilities,* 28(6), 329-334.

YATES, S.M. (1998) Teacher Perceptions, Learned Helplessness and Mathematics Achievement *Proceedings of the 22nd Conference of the International Group for the Psychology of Mathematics Education, 4,* 216-223

ZATZ, S. & CHASSIN, L. (1985) Cognitions of Test Anxious Children Under Naturalistic Test-Taking Conditions, *Journal of Counselling and Clinical Psychology 53(3),* 393-401.

APPENDICES

Appendix 1.1: School X - Mental Calculation (MC) and Myself As Learner Scale (MALS) Scores

School X	MC Score/%	MALS Score/100
X1m	60	66
X2m	85	72
X3f	85	64
X4f	85	74
X5f	75	85
X6m	80	80
X7m	90	66
X8m	95	83
X9m	65	88
X10m	70	81
X11m	50	65
X12m	65	86
X13f	90	64
X14m	95	58
X15m	60	59
X16m	45	54
X17f	70	61
X18f	80	72
X19f	75	72
X20f	75	73
X21m	95	73

Appendix 1.2: School Y - Mental Calculation (MC) and Myself As Learner Scale (MALS) Scores

School Y	MC Score/%	MALS Score/100
Y1m.	30	41
Y2m.	5	58
Y3f.	65	65
Y4m.	65	62
Y5m.	80	82
Y6m.	95	81
Y7f.	55	62
Y8m.	65	62
Y9f	100	72
Y10m.	95	69
Y11m.	90	82
Y12f.	90	61
Y13.f.	25	49
Y14f.	85	81
Y15f.	90	74
Y16m.	70	62
Y17f.	90	69
Y18f.	55	60
Y19f	85	81
Y20f.	80	63
Y21m.	50	61
Y22f.	70	73
Y23f.	80	89
Y24f	45	59
Y25f.	65	67
Y26f.	85	65
Y27m.	100	88
Y28m.	85	69

Appendix 1.3: School Z - Mental Calculation (MC) and Myself As Learner (MALS) Scores

School Z	MC Score/%	MALS Score/100
Z1f.	95	85
Z2f.	85	71
Z3m.	100	79
Z4m.	95	95
Z5m.	85	70
Z6f.	90	71
Z7m	100	91
Z8f.	100	79
Z9m.	100	85
Z10m.	100	91
Z11m.	85	67
Z12m.	100	89
Z13m.	100	94
Z14m.	100	88
Z15m.	100	94
Z16m.	95	79
Z17f.	60	58
Z18m.	100	80
Z19m.	100	72
Z20f.	85	76
Z21m.	85	57
Z22m.	95	71
Z23m.	100	69
Z24f.	95	76

Appendix 1.4: Pilot Study Summary Data

Comparative Descriptives

School	n	Mean	SD
X - MC	21	75.714	14.5160
MALS	21	71.238	9.8484
Y – MC	28	71.250	23.7512
MALS	28	68.107	11.2886
Z - MC	24	93.750	9.3541
MALS	24	78.625	10.9418
All Schools			
MC	73	79.932	19.9042
MALS	73	72.466	11.5516

Spearman's Rank Correlations

School	n	rs	p
X	21	0.07	p > 0.15
Y	28	0.74	p < 0.0001
Z	24	0.66	p < 0.01
All Schools	73	0.60	p < 0.0001

Appendix 1.5: Individual Pupils (Schools X, Y, Z)

School X

Pupil	Gender	MC Score	MALS	Stage	Accuracy
X8m	m	95%	High	4	93%
X9m	m	65%	High	4	77%
X12m	m	75%	High	3	77%
X15m	m	60%	Low	3	54%
X17f	F	70%	Low	4	93%

School Y

Pupil	Gender	MC Score	MALS	Stage	Accuracy
Y1	m	30%	Low	1	25%
Y2	m	5%	Low	1	20%
Y6	m	95%	High	3	100%
Y7	f	55%	Low	4	80%
Y11	m	90%	High	5	100%
Y12	f	90%	Low	5	93%
Y14	f	85%	High	4	100%
Y18	f	55%	Low	2	69%
Y19	f	85%	High	5	100%
Y21	m	50%	Low	3	40%

School Z

Pupil	Gender	MC Score	MALS	Stage	Accuracy
Z4	m	95%	Low	1	60%
Z7	m	100%	High	4	100%
Z10	m	100%	High	4	80%
Z13	m	100%	High	5	80%
Z15	m	100%	High	4	100%
Z17	f	60%	Low	3	70%
Z21	m	85%	Low	3	60%
Z23	f	100%	Low	4	93%

Appendix 2.1: A Proposed Classification of Two Digit Addition Strategies for Mental Calculation

Example Question 55 + 37

	Proposed Strategy Stage		
Emerging	**Child attempts to show an emerging strategy**		
Counting	**Child attempts to count or add on 37 from 55**		
Single Digits	**5 + 3 = 8;**	**5 + 7 = 12;**	**8 + 12 = 92**
Simple Splitting	**50 + 30 = 80;**	**5 + 7 = 12;**	**80 + 12 = 92**
Advanced Splitting	**50 + 30 = 80;**	**80 + 5 = 85;**	**85 + 7 = 92**
Sequencing	**55 + 30 = 85;**	**85 + 7 = 92;**	
or	**55 + 7 = 62;**	**62 + 30 = 92;**	
Advanced Sequencing	**55 + 40 – 3 = 92;**		
Direct Answer	**Child goes directly to answer without explicit strategy**		

There are clearly several variations or possibilities within each of the stages, the above is intended show that in general, the higher order of strategy, the fewer number of steps are involved. Please see Beishuizen & Foxman (2002) for a recent analysis; Threlfall (2002) in his analysis of seven different classification systems states that:

Each of these systems seems in itself reasonable as an approach to classification, yet it is noticeable that none of them is adequate to capture the diversity found in the calculations of a small sample of ordinary primary children.

The classification above drew on the long history of mental calculation in the Netherlands, on the work of Beishuizen & Foxman (2002) and on the work of Threlfall (2002).

Appendix 2.2: A Proposed Classification of Two Digit Subtraction Strategies for Mental Calculation

Example Question 42 - 23

	Proposed Strategy Stage		
Emerging	Child attempts to show an emerging strategy		
Counting	Child attempts to count back or subtract 23 from 42		
Single Digits	4 – 2 = 2;	2 – 3 = N/A (1);	2 (0) – (1) = 19
Simple Splitting	40 – 20 = 20;	3 - 2 = 1 (or 2 – 3);	20 – 1 = 19
Advanced Splitting	40 – 20 = 20;	20 + 2 = 22;	22 – 3 = 19
Sequencing	42 – 20 = 22;	22 – 3 = 19;	
or	42 – 3 = 39;	39 – 20 = 19	
Advanced Sequencing	42 – 22 - 1 = 19		
Direct Answer	Child goes directly to answer without explicit strategy		

There are clearly several variations or possibilities within each of the stages, the above is intended show that in general, the higher order of strategy, the fewer number of steps are involved. However, subtraction strategies also need to account for (2 –3) etc. Please see Beishuizen & Foxman (2002) for a recent analysis; Threlfall (2002) in his recent analysis of seven different classification systems states that :

It seems that general ways of making sense of mental calculation struggle to map onto the variations found in calculating particular problems. This begins to suggest that more may be involved in flexible mental calculation than the choice of an identifiable general strategy.

The classification above drew on the long history of mental calculation in the Netherlands, on the work of Beishuizen & Foxman (2002) and on the work of Threlfall (2002). Classification schemes for subtraction need to account for the possibility of taking away a larger number from a smaller number.

Appendix 3.1: School A - Mental Competence (NFER 8+), Mental Calculation (MC), Myself As Learner Scale (MALS) and Mathematics Anxiety Scale for Children (MASC) Scores

School A	NFER 8+	MC Score/20	MALS Score/100	MASC Score/88
A1f.	120	18	73	26
A2f.	102	15	68	34
A3f.	106	15	77	37
A4f.	82	9	54	43
A5f.	120	20	81	26
A6m.	102	12	62	40
A7f.	111	7	59	32
A8m.	113	17	81	26
A9m.	112	19	72	43
A10m.	98	18	69	28
A11f.	81	11	50	33
A12m.	112	19	72	22
A13m.	90	16	73	30
A14f.	106	16	59	32
A15m.	108	13	69	29
A16f.	110	18	76	33
A17f.	108	15	73	63
A18m	81	2	62	58
A19f.	106	18	88	24
A21m.	117	18	77	42
A22m.	117	19	63	22
A24f.	115	17	90	36
A25f.	94	10	55	29

M = 10
F = 13
T = 23

Original Total = 26
Analysis Total = 23, 3 pupils not included in analysis

A20f: NFER 8+ no score
A23f: NFER 8+ 130
A26f: NFER 8+ 130

Appendix 3.2: School B – Mental Competence (NFER 8+), Mental Calculation (MC), Myself As Learner Scale (MALS) and Mathematics Anxiety Scale for Children (MASC) Scores

School B	NFER 8+	MC Score/20	MALS Score/100	MASC Score/88
B1m.	107	18	63	37
B2f.	105	12	47	43
B3m.	112	15	63	46
B4m.	110	15	57	57
B5f.	94	11	65	47
B6f.	120	19	79	43
B7m.	120	18	69	32
B8f.	108	15	66	46
B9m.	108	15	73	25
B11m.	113	17	79	38
B12m.	120	19	92	22
B13f.	117	11	57	31
B15f.	104	19	71	39
B16f.	95	17	75	27
B17f.	86	13	55	30
B19m.	123	18	90	34
B20m.	115	16	62	39
B21m.	106	18	82	28
B22f.	98	15	70	37
B23m.	110	17	77	30
B25f.	107	17	82	34

M = 11
F = 10
T = 21

Original Total = 26

Analysis Total = 21, 5 pupils not included

B10m: NFER 8+ score 128
B14m: NFER 8+ score 126
B18f: NFER 8+ score 128
B24m NFER 8+ score 131
B26f NFER 8+ score 130

Appendix 3.3: School C - Mental Competence (NFER 8+), Mental Calculation (MC), Myself As Learner Scale (MALS) and Mathematics Anxiety Scale for Children (MASC) Scores

School C	NFER 8+	MC Score/20	MALS Score/100	MASC Score/88
C1m.	104	16	88	36
C2m.	102	7	59	43
C4f.	95	10	55	43
C5f.	99	9	70	41
C6m.	96	12	83	35
C7m.	106	16	73	44
C8f.	100	11	72	62
C9f.	123	18	79	32
C10m.	106	12	86	34
C11f.	98	11	71	39
C13f.	105	16	83	42
C14m.	117	16	84	32
C15m.	117	16	83	32
C16f.	103	11	49	66
C17f	100	12	63	38
C18f.	100	13	67	59
C19m.	106	13	76	32
C20f.	98	14	80	50
C21m.	103	17	87	34

M = 10
F = 9
T = 19

Original Total = 21 No pupils with Statements included

Analysis Total = 19, 2 pupils not included

C3m: NFER 8+ No score
C12f: NFER 8+ No score

Appendix 3.4: School D - Mental Competence (NFER 8+), Mental Calculation (MC), Myself As Scale Learner (MALS) and Mathematics Anxiety Scale for Children (MASC) Scores

School D	NFER 8+	MC Score/20	MALS Score/100	MASC Score/88
D1f.	106	11	62	34
D2m.	113	14	61	34
D3f.	122	9	59	40
D4f.	85	20	80	31
D6f.	120	14	65	40
D8f.	99	14	56	69
D10m.	104	8	67	44
D11m.	104	19	76	37
D12m.	119	20	77	32
D14f.	118	16	60	39
D15m.	120	10	47	59
D17m.	120	19	74	38
D20m.	109	18	77	53
D21m.	106	18	81	42
D22f.	102	17	68	34
D23m.	111	10	61	46
D24f.	101	20	80	29

M = 8
F = 9
T = 17

Original Total = 24

Analysis Total = 17, 7 pupils not included

D5m: NFER 8+ score 128
D7m: NFER 8+ score 130
D9m: NFER 8+ score 127
D13f: NFER 8+ score 128
D16f: NFER 8+ score 126
D18m NFER 8+ score 130
D19f: NFER 8+ score 130

Appendix 3.5: School E - Mental Competence (NFER 8+), Mental Calculation (MC), Myself As Learner Scale (MALS) and Mathematics Anxiety Scale for Children (MASC) Scores

School E	NFER 8+	MC Score/20	MALS Score/100	MASC Score/88
E1f.	99	12	57	43
E3f.	118	18	86	42
E4f.	111	15	77	32
E5f.	112	20	72	32
E6f.	119	20	77	22
E8m.	97	13	66	49
E10m.	108	18	81	37
E11f.	125	19	70	26
E12f.	114	18	85	43
E13f.	80	7	53	36
E14m.	113	20	91	24
E15f.	117	13	58	46
E18f.	109	18	78	34
E19f.	110	15	63	46
E20m.	113	13	86	37
E21m.	107	18	63	40
E22m.	104	18	76	54
E23m.	94	11	60	60
E24f	85	6	50	42

M = 7
F = 12
T = 19

Original Total = 25

Analysis Total = 19, 6 pupils not included

E2f: NFER 8+ No score
E7f: NFER 8+ score 130
E9m: NFER 8+ score 126
E11f: NFER 8+ score 125
E16m: NFER 8+ score 127
E17f: NFER 8+ score No score

Appendix 3.6: School F – Mental Competence (NFER 8+), Mental Calculation (MC), Myself As Learner Scale (MALS) and Mathematics Anxiety Scale for Children (MASC) Scores

School F	NFER 8+	MC Score/20	MALS Score/100	MASC Score/88
F1m.	108	14	62	35
F2f.	93	14	60	46
F3f.	84	8	67	46
F4f.	108	10	69	52
F5m	111	9	62	61
F6f	109	17	65	33
F7m.	112	17	68	34
F8m.	100	17	90	27
F9f.	94	13	77	49
F10m.	113	19	85	42
F12f.	92	13	73	48
F13f.	89	13	68	46
F14m.	112	14	73	35
F15m.	90	14	64	53
F16m.	100	15	73	45
F17m.	100	11	69	61
F18m.	97	14	40	70
F19f.	108	7	79	41
F20m.	96	17	75	45

M = 11
F = 8
T = 19

Original Total = 20

Analysis Total = 19, 1 pupil not included

F11f: NFER 8+ score 70-

Appendix 3.7: School G – Mental Competence (NFER 8+), Mental Calculation (MC), Myself As Learner Scale (MALS) and Mathematics Anxiety Scale for Children (MASC) Scores.

School G	NFER 8+	MC Score/20	MALS Score/100	MASC Score/88
G1m.	109	17	85	25
G2m.	89	14	42	60
G3m.	101	17	79	56
G4m.	102	16	59	49
G5m.	121	19	69	48
G6m.	82	13	58	38
G7f.	100	14	66	39
G8f.	117	17	92	35
G9m.	100	15	68	48
G10f.	101	15	63	52
G13m.	91	19	86	23
G14m.	104	19	83	31
G16f.	112	19	73	41
G17f.	119	19	80	33
G19f.	100	17	53	44
G20f.	88	14	25	73
G23m.	100	18	80	33
G24f.	111	19	95	22
G25f.	111	19	88	22
G26f.	115	19	76	36
G27f.	121	20	66	40
G28m.	116	19	88	22
G29f.	121	20	86	40

M = 11
F = 12
T = 23

Original Total = 29

Analysis Total = 23, 6 pupils not included

G11f: NFER 8+ score 131
G12m NFER 8+ score 130
G15f: NFER 8+ score 131
G18m: NFER 8+ score 126
G21m: NFER 8+ score 128
G22m: NFER 8+ score 126

Appendix 3.8: School G – Mental Competence (NFER 8+), Mental Calculation (MC), Myself As Learner Scale (MALS) and Mathematics Anxiety Scale for Children (MASC) Scores.

School H	NFER 8+	MC Score/20	MALS Score/100	MASC Score/88
H1m.	106	16	45	71
H2m.	120	17	77	35
H4m.	102	11	63	48
H5f.	100	10	68	51
H7m.	108	18	76	33
H9m.	113	16	74	36
H10m.	111	16	89	24
H11m.	100	15	74	51
H12f.	118	15	61	46
H13f.	110	16	83	36
H14f.	99	10	68	44
H15m.	103	17	82	31
H16m.	120	16	70	46
H17f.	112	16	77	28
H18m.	107	17	86	46
H20m.	110	17	69	39
H21m.	101	15	66	44
H22m.	120	16	85	32
H23m.	112	12	83	42

M = 14
F= 5
T = 19

Original Total = 23

Analysis Total = 19, 4 pupils not included

NFER 8+ score 131
NFER 8+ score 131
NFER 8+ score 127
NFER 8+ score 126

Appendix 4.2: Main Study Summary Data – Associations

Spearman Rank Correlation's and Significance

School		MC	MALS	MASC
A -	NFER	0.68 (p = 0.0004)	0.59 (p = 0.0033)	- 0.32 (p = 0.1381)
	MC	-	0.61 (p = 0.0021)	- 0.49 (p = 0.0167)
	MALS	-	-	- 0.18 (p = 0.4166)
B -	NFER	0.39 (p = 0.0827)	0.30 (p = 0.1793)	- 0.04 (p = 0.8796)
	MC	-	0.71 (p = 0.0003)	- 0.28 (p = 0.2201)
	MALS	-	-	- 0.45 (p = 0.0422)
C -	NFER	0.66 (p = 0.0023)	0.45 (p = 0.0520)	- 0.55 (p = 0.0157)
	MC	-	0.70 (p = 0.0008)	- 0.49 (p = 0.0336)
	MALS	-	-	- 0.61 (p = 0.0052)
D -	NFER	- 0.32 (p = 0.2075)	0.78 (p = 0.0003)	0.23 (p = 0.3782)
	MC	-	0.78 (p = 0.0003)	- 0.61 (p = 0.0097)
	MALS	-	-	- 0.50 (p = 0.0404)
E -	NFER	0.67 (p = 0.0017)	0.59 (p = 0.0077)	- 0.45 (p = 0.0561)
	MC	-	0.65 (p = 0.0027)	- 0.58 (p = 0.0097)
	MALS	-	-	- 0.39 (p = 0.0978)
F -	NFER	0.27 (p = 0.2681)	0.28 (p = 0.2376)	- 0.55 (p = 0.0140)
	MC	-	0.16 (p = 0.5001)	- 0.55 (p = 0.0140)
	MALS	-	-	- 0.38 (p = 0.1068)
G -	NFER	0.63 (p = 0.0012)	0.62 (p = 0.0016)	- 0.33 (p = 0.1247)
	MC	-	0.62 (p = 0.0016)	- 0.50 (p = 0.0154)
	MALS	-	-	- 0.80 (p < 0.0001)
H -	NFER	0.39 (p = 0.0973)	0.35 (p = 0.1393)	- 0.44 (p = 0.0572)
	MC	-	0.45 (p = 0.0545)	- 0.51 (p = 0.0246)
	MALS	-	-	- 0.67 (p = 0.0017)
All Schools				
	NFER	0.45 (p < 0.0001)	0.35 (p < 0.0001)	- 0.32 (p < 0.0001)
	MC	-	0.57 (p < 0.0001)	- 0.47 (p < 0.0001)
	MALS	-	-	- 0.46 (p < 0.0001)

Appendix 4.3: Continuous Summary Descriptives (MASC)

n	160
Mean	39.731
95% CI	38.021 to 41.441
Variance	119.9713
SD	10.9531
SE	0.8659
CV	28%
Median	38.500
95.2% CI	36.000 to 41.000
Range	51
IQR	14

Percentile	
25th	32.000
50th	38.500
75th	46.000

Note:

Low Mathematics Anxiety pupils were those scoring below the lower quartile (25^{th} percentile) i.e. those scoring 31 or less on the Mathematics Anxiety Scale for Children.

High Mathematics Anxiety pupils were those scoring above the upper quartile (75^{th} percentile) i.e. those scoring 47 or more on the Mathematics Anxiety Scale for children.

This is equivalent to arranging the 160 Mathematics Anxiety scores in order from lowest to highest and using the formula, $\frac{1}{4}(n + 1)^{th}$ and $\frac{3}{4}(n + 1)^{th}$ items for lower and quartiles respectively, where n is the total number of scores.

Appendix 4.4: Tests for Normality, Skewness and Kurtosis

All Schools	NFER 8+	MC	MALS	MASC
K-S Probability	k = 0.886 p > 0.05	k = 1.668 p < 0.01	k = 0.788 p > 0.05	k = 0.957 p < 0.05
Skewness Probability	Sk = - 0.476 p < 0.05	Sk = - 0.834 p < 0.01	Sk = - 0.538 p < 0.01	Sk = 0.743 p < 0.01
Kurtosis Probability	Kg = -0.159 p > 0.15	Kg = 0.382 p > 0.15	Kg = 0.565 p > 0.15	Kg = 0.474 p > 0.15

In order to test the variables for normality, skewness and kurtosis, analysis was carried out using the Analyse-It computer software. There are a number of methods that may be used to determine whether observations are normally distributed. The Kolmorogov-Smirnov test (k statistic) tries to determine if two data sets differ significantly. It has the advantage that it makes no assumption about the distribution of the data; k statistic values close to 1 indicate that the data are normally distributed. The null hypothesis that the data values are a random sample is rejected if the probability is less than the chosen significance level, in this case, $p < 0.05$. The data were examined for skewness and kurtosis, or measures of the form of a distribution. Skewness (test statistic, Sk) is a measure of the symmetry of a distribution around its mean; kurtosis (test statistic, Kg is a measure of the peakiness of a distribution or the way in which items are clustered around the mean or distributed in the tails.

Appendix 5: UCL Doctoral Programme: Inter-rater Reliability

Dear Colleague,

I would be very grateful if you could complete the following table by identifying with a tick (✓) the highest 'stage of mental calculation' according to the following criteria that each pupil has attained.

Please note: Ignore whether the answer is correct or incorrect.
Only one example of the highest stage is needed.
Please tick only one box for each pupil.

Strategy Stage	**Stage 1; Emerging**	**Stage 2: Counting**	**Stage 3: Single Digits**	**Stage 4: Simple Splitting**	**Stage 5: Advanced Splitting**	**Stage 6: Sequencing**	**Stage 7: Advanced Sequencing**	**Stage 8: Direct Answer**
Example						✓		
Pupil 1								
Pupil 2								
Pupil 3								
Pupil 4								
Pupil 5								
Pupil 6								
Pupil 7								
Pupil 8								
Pupil 9								
Pupil 10								

Example: The pupil shows evidence of a sequencing strategy.

For example: 57 + 14: I added the 4 to make 61 and then I added the 10 to make 71.

Notes: Mental Calculation Strategy Stages for the Addition and Subtraction of Two Digit Numbers

Stage 1: Emerging Strategy

The pupil shows evidence that he/she is attempting to show an emerging strategy, possibly by 'picturing' the calculation

Example, 15 + 12; *I just put 15 on top of 12 and added them up and down.*
Example, 23 – 11; *I looked at the 2 and the 3 and put the 1's under them. I took them away*

Stage 2: Counting Strategy

The pupil simply counts up or down in tens or units. The pupil may or may not say that they are using their fingers.

Example, 15 + 12; *I got the 15 and counted on another 12 on my fingers.*
Example, 23 – 11; *I just went down 11.*

Stage 3: Single Digits Strategy

The pupil adds/subtracts single digits. They do not differentiate between tens/units, there is no notion of relative size.

Example, 33 + 54; *5 plus 3 equals 8 and 4 plus 3 equals 7.*
Example, 58 – 19: *I took away 1 from 5 and 8 from 9.*

NB Where pupils calculate a sum greater than 10, e.g. 9 + 7 is 16, but is taken no further, is considered as a special case of this strategy.

Stage 4: Simple Splitting Strategy

The pupil recognises and works with multiples of 10. The tens and units are split and added/subtracted separately.

Example, 55 + 37; *I added the 50 and 30 and then added the 5 and 7, and then added the answers together.*
Example, 68 – 32; *Take away 30 from 60 and then take away 2 from 8.*

Stage 5: Advanced Splitting Strategy

The pupil splits both pairs of numbers. There is an intermediate number calculated in working out the answer.

Example, 27 + 69; *I added the 20 and the 60 which made 80 and then added the 7 which made 87 and the 9 which made 96.*
Example, 42 –23; *Take away 20 from 42 to give 22 and then take away 3 to give 19.*

NB Pupils showing evidence of either rounding up or down, or halving or doubling are considered as a special cases of this strategy.

Stage 6: Sequencing Strategy

The pupil keeps the first number as a whole and than splits or partitions the second number. They may or may not state a subtotal.

Example, 57 + 14; *I added 10 to the 57 and then I added 4*
Example, 52 –23; *Take away 20 from 52 to give 32 and take away 3 to give 29*

Stage 7: Advanced Sequencing Strategy

The pupil keeps the first number as a whole but is able to show more fluency in dealing with the second number, other than a simple partitioning of the second number. For example, the pupil may use a fact such 70 – 1 instead of 69 to work out an answer without the need for a subtotal

Example: 27 + 69; 27 add 70 take 1
Example: 58 – 19; 58 take 20 take 1

Stage 8: Direct Answer Strategy

The pupil is able to go directly to a two-digit addition/subtraction answer without recourse to one of the other seven strategies.

The following pupils were selected. Their verbatim accounts were presented on 10 separate sheets and coded as follows.

Pupil 1: Pupil B12m;
Pupil 2: Pupil G2m;
Pupil 3: Pupil D24f;
Pupil 4: Pupil H1m;
Pupil 5: Pupil B13f;
Pupil 6: Pupil G4m;
Pupil 7: Pupil E6f;
Pupil 8: Pupil B23m;
Pupil 9: Pupil A22m;
Pupil 10: Pupil H11m.

Appendix 6.1: Teacher Efficacy Scale

On the following sheets please attached a number of statements about organisations, people and teaching. The purpose is to gather information regarding the actual attitudes of teachers concerning these statements. There are no correct or incorrect answers. We are interested only in your frank opinions. Your responses will remain confidential.

INSTRUCTIONS: Please indicate your personal opinion about each statement by circling the appropriate response at the right of each statement.

KEY:

1 = Strongly Agree

2 = Moderately Agree

3 = Agree slightly more than agree

4 = Disagree slightly more than agree

5 = Strongly Disagree

1. When a student does better than usually, many times it is because I exert a little extra effort.	1	2	3	4	5	6
2. The hours in my class have little influence on students compared to the influence of their home environment.	1	2	3	4	5	6
3. The amount a student can learn is primarily related to their family background.	1	2	3	4	5	6
4. If students aren't disciplined at home, they aren't likely to accept any discipline.	1	2	3	4	5	6
5. I have enough training to deal with almost any learning problem.	1	2	3	4	5	6
6. When a student is having difficulty with an assignment, I am usually able to adjust his/her level.	1	2	3	4	5	6
7. When s student gets a better grade than he/she usually gets, it is usually because I found better ways of teaching that student	1	2	3	4	5	6
8. When I really try, I I can through to most difficult students.	1	2	3	4	5	6
9. A teacher is very limited in what he/she can achieve because a students home environment has a large influence on his/her achievement	1	2	3	4	5	6
10. Teachers are not a very powerful influence on student achievement when all factors are considered.	1	2	3	4	5	6
11. When the grades of my students improve, it is usually because I found more effective approaches.	1	2	3	4	5	6
12. Is a student masters a new concept quickly, this might be because I knew the necessary steps in teaching.	1	2	3	4	5	6
13. If parents would do more for their children, I could do more.	1	2	3	4	5	6
14. If a student did not remember information I gave in a previous lesson , I would know how to increase his/her retention in the next lesson.	1	2	3	4	5	6
15. The influences of a student's home experiences can be overcome by good teaching.	1	2	3	4	5	6
16. If a student in my class becomes disruptive and noisy, I feel assured that I know some techniques to redirect him/her quickly.	1	2	3	4	5	6
17. Even a teacher with good teaching abilities may not reach many students.	1	2	3	4	5	6
18. If one of my students couldn't do a class assignment, I would be able to accurately assess whether the assignment was at the correct level of difficulty.	1	2	3	4	5	6
19. If I really try hard, I can get through to even the most difficult or unmotivated students.	1	2	3	4	5	6

20. When it comes right down to it, a teacher really can't do much because most of a student's motivation and performance depends on his or home environment.	1	2	3	4	5	6
21. Some students need to be placed in slower groups so they are not subjected to unrealistic expectations.	1	2	3	4	5	6
22. My teacher training programme and/or experience has given me the necessary skills to be an effective teacher.	1	2	3	4	5	6

From Woolfolk, A. E. & Hoy, W. K. (1990) Prospective teachers' sense of efficacy and beliefs about control *Journal of Educational Psychology,* 82, 81-91. Originally based on the Teacher Efficacy Scale, developed by S. Gibson & M. Dembo (1984). Teacher Efficacy: a construct validation. *Journal of Educational Psychology,* 76, 569-582.

Appendix 6.2: Myself As Learner Scale (Burden, 1998)	Definitely Agree	Agree a bit	True half the time	Don't agree	Strongly Disagree
1. I'm good at doing tests.	a	b	c	d	e
2. I like having problems to solve.	a	b	c	d	e
3. When I'm given new work to do, I usually feel confident I can do it.	a	b	c	d	e
4. Thinking carefully about your work helps you to do it better.	a	b	c	d	e
5. I'm good at discussing things.	a	b	c	d	e
6. I need lots of help with my work.	a	b	c	d	e
7. I like having difficult work to do.	a	b	c	d	e
8. I get anxious when I have to do new work.	a	b	c	d	e
9. I think that problem solving is fun.	a	b	c	d	e
10. When I get stuck with my work I can usually work out what to do next.	a	b	c	d	e
11. Learning is easy.	a	b	c	d	e
12. I'm not very good at solving problems.	a	b	c	d	e
13. I know the meaning of lots of words.	a	b	c	d	e
14. I usually think carefully about what I have got to do.	a	b	c	d	e
15. I know how to solve the problems that I meet.	a	b	c	d	e
16. I find a lot of schoolwork difficult.	a	b	c	d	e
17. I'm clever	a	b	c	d	e
18. I know how to be a good learner.	a	b	c	d	e
19. I like using my brain.	a	b	c	d	e
20. Learning is difficult	a	b	c	d	e

Appendix 6.3: Mathematics Anxiety Scale for Children (Chiu & Henry, 1990; Beasley, Long & Natali, 2001)	Very, very nervous	Very nervous	A little bit nervous	Not nervous
1. Being given a new Maths book	4	3	2	1
2. Reading and interpreting graphs or charts	4	3	2	1
3. Listening to another pupil explain a Numeracy problem	4	3	2	1
4. Watching a teacher work on a Numeracy problem on the white board	4	3	2	1
5. Walking into a Numeracy lesson	4	3	2	1
6. Looking through the pages in a book about Maths or Numeracy	4	3	2	1
7. Starting a new chapter in a Maths book	4	3	2	1
8. Thinking about Numeracy outside of the classroom	4	3	2	1
9. Working on Mathematics or Numeracy homework	4	3	2	1
10. Working on a Maths problem, such as "If I spend £3.87 at the shop, how much change will I get from a £5 note?".	4	3	2	1
11. Reading a formula in science	4	3	2	1
12. Listening to the teacher in a Numeracy Lesson	4	3	2	1
13. Using tables that have a lot of numbers	4	3	2	1
14. Being told how to interpret Mathematical statements	4	3	2	1
15. Being given homework with many difficult Maths problems	4	3	2	1
16. Thinking about a Maths test one day before the test	4	3	2	1
17. Doing a long division problem such as 3476 ÷ 35	4	3	2	1
18. Taking part in a Maths test	4	3	2	1
19. Getting ready to study for a Maths test	4	3	2	1
20. Being given a Maths test that you were not told about	4	3	2	1
21. Waiting to get the result of a Maths test in which you expect to do well	4	3	2	1
22. Taking an important test in a Numeracy lesson.	4	3	2	1

Appendix 6.4: Whole Class Mental Calculation Questions

1. What is half of sixteen?
2. What is six multiplied by three?
3. Write the number three hundred and seven in figures.
4. What is three hundred subtract one hundred and fifty.
5. What is the remainder when eighteen is divided by four?
6. In a class there are twenty eight children, sixteen are boys. How many are girls?
7. Write down any odd number between twenty eight and forty three.
8. What is fifty percent of £6.
9. What is 19 fewer than 43.
10. Write 1.6m in centimetres.
11. How many seconds are there in a minute?
12. What is 100 more than 622?
13. Write down two numbers which have a difference of 13.
14. Look at the board. Choose the one to use to work out how many cans of coke there are in three packs with ten cans in each pack (*Write on board 3 + 10, 10 – 3, 3 x 10, 10 ÷ 3).*
15. One ice cream costs 20p. How much would four cost?
16. How long is the perimeter of a 5 cm by 5cm square?
17. A box holds 60 biscuits. How many are left if you eat 18?
18. There are one hundred and thirty nine books on a shelf. If I remove 56, How many will be left? (*Write 139*).
19. I think of a number, add 2, then multiply by 3, the answer is 15. What is the number?
20. You start to read a book on Tuesday. On Wednesday you reach 10 more pages than on Tuesday. You read page 60. How man pages did you read on Tuesday?

(< Downloaded from http://harpo.bristol-lea.org.uk/teaching/primary/maths/mentaltests.html >)

Appendix 6.5: Two Digit Addition and Subtraction Questions

(a) List of Two Digit Addition and Subtraction Questions used in the Pilot Study

1. 23 + 9;
2. 25 – 6;
3. 18 + 5;
4. 33 – 16;
5. 23 + 24;
6. 37 – 18;
7. 28 + 29;
8. 46 - 28;
9. 39 + 25;
10. 38 – 24;
11. 46 + 23;
12. 32 – 21;
13. 37 + 45;
14. 68 – 32;
15. 46 + 37.

(b) List of Two Digit Addition and Subtraction Questions used in the Main Study

1. 15 + 12;
2. 23 – 11;
3. 33 + 54;
4. 42 – 23;
5. 55 + 37;
6. 68 – 32;
7. 57 + 14;
8. 58 – 19;
9. 27 + 69;
10. 64 – 27;
11. 38 + 35;
12. 71 – 22;
13. 45 + 37;
14. 65 – 49;
15. 38 + 59;
16. 97 – 58.

Appendix 7.1: One-Way between subjects ANOVA

Comparison All Schools MALS: Schools, A, B, C, D, E, F, G, H.

n	160

All Schools Myself As Learner Scale	n	Mean	SD	SE
School A	23	69.696	10.529	2.1954
School B	21	70.190	11.647	2.5417
School C	19	74.105	11.372	2.6089
School D	17	68.118	9.867	2.3931
School E	19	71.000	12.243	2.8087
School F	19	69.421	10.564	2.4235
School G	23	72.609	16.708	3.4839
School H	19	73.474	10.648	2.4428

Source of variation	SSq	DF	MSq	F	P
All Schools Myself As Learner Scale	598.685	7	85.526	0.59	0.7610
Within cells	21926.509	152	144.253		
Total	22525.194	159			

Appendix 7.2 One-Way Between Subjects ANOVA

Comparison: Mathematics Anxiety – All Schools A, B, C, D, E, F, G, H.

n	160

Mathematics Anxiety	n	Rank sum	Mean rank
School A	23	1228.5	53.41
School B	21	1414.0	67.33
School C	19	1667.0	87.74
School D	17	1453.5	85.50
School E	19	1550.5	81.61
School F	19	2051.0	107.95
School G	23	1827.5	79.46
School H	19	1688.0	88.84

Kruskal-Wallis statistic	17.55	
p	0.0142	(chisqr approximation, corrected for ties)

Appendix 7.3 One-Way Between Subjects ANOVA

Comparison: NFER 8+ – All Schools A, B, C, D, E, F, G, H.

n	160

NFER 8+	n	Mean	SD	SE
School A	23	104.696	11.949	2.4914
School B	21	108.476	9.516	2.0766
School C	19	104.105	7.483	1.7166
School D	17	109.353	9.962	2.4162
School E	19	107.105	11.595	2.6600
School F	19	100.842	9.0937	2.0862
School G	23	104.783	10.967	2.2867
School H	19	109.053	7.106	1.6303

Source of variation	SSq	DF	MSq	F	P
NFER 8+	1166.944	7	166.706	1.69	0.1158
Within cells	15010.956	152	98.756		
Total	16177.900	159			

Appendix 7.4 Kruskal-Wallis ANOVA

Comparison: Mental Calculation Scores – All Schools A, B, C, D, E, F, G, H.

n	162

Mental Calculation Scores	**n**	**Rank sum**	**Mean rank**
School A	23	1901.5	82.67
School B	21	1871.5	89.12
School C	19	986.0	51.89
School D	17	1418.5	83.44
School E	19	1670.0	87.89
School F	21	1076.0	56.63
School G	23	2544.0	110.61
School H	19	1412.5	74.34

Kruskal-Wallis statistic	23.88	
p	0.0012	(chisqr approximation, corrected for ties)

Appendix 7.5: Testing the Difference between NFER Means for High and Low Mathematics Anxiety

In order to test the difference between the differences between two unpaired samples, we first make the assumption that that we have two samples chosen randomly from two normal distributions. There is also an important second assumption in the knowledge that the scores come from a population with known mean and variance, where the two samples in this case were not taken from the whole population i.e. there were some exclusions. The variances of the two distributions are assumed equal to σ^2 and that this variance has to be estimated from the samples.

Using a t-test and pooled estimate of variance with 61 degrees of freedom, using the Analyse-It (2003) General Statistics Module Version 1.69, we obtain the following table:

n	63

NFER 8+	n	Mean	SD	SE
High Anx	32	101.625	8.628	1.5252
Low Anx	31	106.355	10.294	1.8489

Difference between means	-4.730
95% CI	-9.509 to 0.049

t statistic	-1.98
2-tailed p	0.0523

As p the probability of the t statistic is greater than 0.05, we accept the Null Hypothesis that there is no difference between the means ($p > 0.05$) and conclude that there is no difference at this level of significance in the mathematical competence between the two groups of pupils.

Appendix 7.6: Kruskal-Wallis ANOVA

Comparison: Accuracy – High Anxiety, Low Anxiety

n	63

Accuracy	**n**	**Rank sum**	**Mean rank**
High Anx	32	786.5	24.58
Low Anx	31	1229.5	39.66

Kruskal-Wallis statistic	10.95	
p	0.0009	(chisqr approximation, corrected for ties)

Appendix 7.7: Chi-Square Analysis (1)

	Stages 1 to 5	Stages 6 to 8	Totals
High Anxiety	**23** (18.3)	**9** (13.7)	**32**
Low Anxiety	**13** (17.7)	**18** (13.3)	**31**
Totals	**36**	**27**	**63**

$$\chi^2 = \sum \frac{(|O - E| - 0.5)^2}{E}$$

Using Yate's Correction for Continuity

O is the observed result; E is the expected result.

χ^2 = 4.61 (1 df), result is significant at the 5% ($p < 0.05$) level.

Appendix 7.8: Chi-Square Analysis (2)

	Stages 1 to 5	Stages 6 to 8	Totals
Female	**20** **(16.5)**	**6** **(11.111)**	**26**
Male	**20** **(23.5)**	**17** **(13.5)**	**37**
Totals	**40**	**23**	**63**

$$\chi^2 = \sum \frac{(|O - E| - 0.5)^2}{E}$$

Using Yate's Correction for Continuity

O is the observed result; E is the expected result.

$\chi^2 = 2.53$, (1 df) result is not significant at the 5% ($p > 0.05$) level.

Results obtained using *Analyse-It* software.

Appendix 8: Main Study Two Digit Addition and Subtraction Interviews

School/Pupil: A1f **LA** **Strategy Stage = S3; see Q5.**

No	Qu	Ans	✓✗	Explanation
1.	15 + 12	26	✗	I added the 5 and the 2, and the two tens
2.	23 – 11	12	✓	I took the 1 away from the 3, and the 1 away from the 2
3.	33 + 54	87	✓	Same again
4.	42 – 23	79	✗	I had to carry the 4, one of the tens over to the units, cos there wasn't enough to take away 3 and 2, the 3 away from the 12
5.	55 + 37	92	✓	I had to add the 7 to the 5 which is 12, so I had to carry 10 over to whatever the 3 and the 5 came too
6.	68 – 32	36	✓	I made, I did 2 take away from 8, and I knew that half of 6 is 3,
7.	57 + 14	61	✗	I did, Well I know that 3 and 7 make 10, so 7 and 4 is 11, so 5 and 6 is 60
8.	58 – 19	39	✓	You can't subtract 9 from 8, so I had to take/carry 10 off the 5 and put it onto the units
9.	27 + 69	96	✓	Well you can't subtract, well 9 plus 7 is over 10, so you have to put that over to the tens
10.	64 – 27	36	✗	You can't subtract 7 from 4, so you have to take one of the tens from the 6 and put it on the units, and then I subtracted it all

% Correct = 60%

School/Pupil: A5f LA Strategy Stage = S5 see Q4

No	Qu	Ans	✓✗	Explanation
1.	15 + 12	27	✓	I added the 2 and the 5 and the two tens
2.	23 – 11	12	✓	I did 3 subtract 1 which is 2, and 20 subtract 10 which is 10
3.	33 + 54	87	✓	Same as the other two
4.	42 – 23	19	✓	43 subtract 23 which is 20 subtract 1
5.	55 + 37	92	✓	I did 5 plus 7 and 50 plus 30
6.	68 – 32	36	✓	60 subtract 30 and 8 subtract 2
7.	57 + 14	71	✓	7 plus 4 and 50 plus 10 and added them together
8.	58 – 19	59	✗	59 subtract 19 subtract 1
9.	27 + 69	100	✗	9 and the 7 and the 2 and the 6
10.	64 – 27	43	✗	I did 67 subtract 27 subtract 3

% Correct = 70%

School/Pupil: A8m LA Strategy Stage = S6, see Q8

No	Qu	Ans	✓✗	Explanation
1.	15 + 12	27	✓	10 add 10 is 20 and 5 add 2 is 7 and then you add the 20 and 7 that's 27
2.	23 – 11	12	✓	Take 10 from 23 it makes 13 and if you take 1 from 13 it makes 12
3.	33 + 54	88	✗	50 add 30 is 80 and 3 add 4 is 8 and you add the 80 and 8 to get 88
4.	42 – 23	19	✓	3 from 42 its 39, if you take 20 from 39 (its 19)
5.	55 + 37	92	✓	50 add 30 is 80, and 5 add 7 is 12, and if you add 12 onto the 80 its 92
6.	68 – 32	36	✓	2 take 8 is 6 and 60 take 30 is 30
7.	57 + 14	71	✓	50 add 10 is 60, and then you've got 7 add 4 is 11, which you add onto the 60, that's 71
8.	58 – 19	39	✓	58 take 10 is 48 take 9 is 39
9.	27 + 69	96	✓	7 add 9 is 16, and 20 add 60 is 80, and you add the 16 onto the 80 that's 96
10.	64 – 27	33	✗	60 take 20 is 40; 64 take 20 is 64 and take away the 7 would leave you with the answer

% Correct = 80%

School/Pupil: A10m LA **Strategy Stage = S6; see Q5**

No	Qu	Ans	✓✗	Explanation
1.	15 + 12	27	✓	I added 10 to 15 and then I added 2
2.	23 – 11	12	✓	I got 23 and then minused 10 and then minused another 1
3.	33 + 54	87	✓	I got 54 and I added 30 and then I added another 3
4.	42 – 23	19	✓	I got 42 minused 20 and then minused another 3
5.	55 + 37	92	✓	55 plus 30 and then plus another 7
6.	68 – 32	36	✓	68 minus 30 and minus another 2
7.	57 + 14	70	✗	57 plus 10 then plus another 4
8.	58 – 19	39	✓	I got 58 minus 20 and then I added another 1
9.	27 + 69	96	✓	69 plus 20 plus 7
10.	64 – 27	37	✓	64 minus 20 and then plus 7

% Correct = 90%

School/Pupil: A12m LA Strategy Stage = S7; see Q8

No	Qu	Ans	✓✗	Explanation
1.	15 + 12	27	✓	10 plus 10 is 20 and 5 plus 2 is 7, so I put them together to make 27
2.	23 – 11	14	✓	23 take 10 is 13 and then take 1 is 13 no 11, no 12
3.	33 + 54	87	✗	I did 54 plus 39 plus 3
4.	42 – 23	19	✓	42 take 10 take 10 take 3
5.	55 + 37	91	✓	55 plus 10 plus 10 plus 7
6.	68 – 32	36	✓	68 take 10 take 20 take 2
7.	57 + 14	71	✓	57 plus 10 plus 4
8.	58 – 19	39	✓	58 take 20 plus 1
9.	27 + 69	96	✓	69 plus 20 plus 7
10.	64 – 27	37	✗	64 take 20, 64 take 10 is 54 take 10 is 44 take 7 is 37
				% Correct = 80%

School/Pupil: A13m LA **Strategy Stage = S5, see Q1.**

No	Qu	Ans	✓✗	Explanation
1.	15 + 12	27	✓	10 plus 10 equals 20 and 5 plus 2 equals 7
2.	23 – 11	12	✓	1 minus 2 equals 1, 3 minus 1 equals 2
3.	33 + 54	87	✓	5 plus 3 equals 8 and 3 plus 4 equals 7
4.	42 – 23	19	✓	20 from 40 equals 20 but you can't do 3 from 2, yeah you can, but it will equal 19
5.	55 + 37	92	✓	5 plus 3 equals 80 and 7 plus 5 equals 11, so add the 10 on to the 80 makes 90 and add the 2 makes 92
6.	68 – 32	36	✓	3 from 6 equals 3 and 2 from 8 equals 6
7.	57 + 14	72	✗	1 plus 5 equals 60, and 7 plus 4 equals 11and you add that to the 80
8.	58 – 19	48	✗	10 from 50 equals 40 and 9 from 8 equals, you can't do it, so you would have to go down a number, then it would come up as 48
9.	27 + 69	92	✗	20 plus 60 equals 80 and 7 plus 9 equals 16, so you add the 10 to the 80 to get 90 and add the 2 to get 92
10.	64 – 27	32	✗	6 minus 2 equals 4, 4 minus 7 you can't do , so you would have to go down, then it would equal …
				% Correct = 60%

School/Pupil: A15m LA **Strategy Stage = S3, see Q3.**

No	Qu	Ans	✓✗	Explanation
1.	15 + 12	27	✓	I added the 2 and the 5 together that made 7 and both of the tens that made 20.
2.	23 – 11	12	✓	I took the 1 off the 2 and 1 off the 3
3.	33 + 54	87	✓	dk I did like the first time, with the 3 and the 5 and the 3 and the 4
4.	42 – 23	19	✓	I took the 3 off the 2, said that was like a 10, so that was like....
5.	55 + 37	92	✓	I added the 7 and the 5 that made 12, and I put a 10 on the 3 that made 4 and I put the 4 and the 5 together that made 90 and the last digit 92
6.	68 – 32	110	✗	I added the 2 and the 8 that made 10, and the 3 and the 6, that's not right its 100, I added the 3 and the 6, no sorry the 2 and the 8 that made 40, and the 3 and the 6
7.	57 + 14	71	✓	I added the 1 on the 5 and the 4 on the 7
8.	58 – 19	39	✓	I took the 9 off the 8 that made it minus 1, so I knew I add to take it ten away, that's 5
9.	27 + 69	-	✗	Unable to respond
10.	64 – 27	-	✗	Unable to respond

% Correct = 70%

School/Pupil: A17f HA **Strategy Stage = S6, see Q5.**

No	Qu	Ans	✓✗	Explanation
1.	15 + 12	27	✓	10 plus 10 which is 20 and I then added the others, which is 5 add 2 which is 7
2.	23 – 11	12	✓	I took 10 away from 23 which is 13, and I took 1 away which 12
3.	33 + 54	87	✓	I worked out 50 plus 30 which is 80, and 4 add 3 which is 7
4.	42 – 23	19	✓	40 take away 20 which is 20, and then put the 2 on again which is 22 and take away 3
5.	55 + 37	92	✓	55 add 30 equals 85 and plus 7 which is 92
6.	68 – 32	36	✓	60 take away 30 which is 30 and 8 take away 2 which is 6
7.	57 + 14	71	✓	50 add 10 is 60, add 7 is 67, and add 4 is 71
8.	58 – 19	39	✓	50 take away 10 which is 40, and then added the 8 on which is 48 and take away 9
9.	27 + 69	96	✓	60 add 20 is 80, and 9 add 7 is 16
10.	64 – 27	37	✓	60 take away 20 is 40 add 4 is 44, and take away 7

% Correct = 100%

School/Pupil: A18m HA **Strategy Stage = S1, see Q2.**

No	**Qu**	**Ans**	**✓✗**	**Explanation**
1.	15 + 12	9	✓	I just went down 12
2.	23 – 11	12	✓	I just went down 11
3.	33 + 54	15	✗	Just added it
4.	42 – 23	-	✗	Unable to respond
5.	55 + 37	-	✗	Unable to respond
6.	68 – 32	-	✗	Unable to respond
7.	57 + 14	-	✗	Unable to respond
8.	58 – 19	-	✗	Unable to respond
9.	27 + 69	-	✗	Unable to respond
10.	64 – 27	-	✗	Unable to respond
				% Correct = 10%

School/Pupil: A19f **Strategy Stage = S5, see Q1.**

No	Qu	Ans	✓✗	Explanation
1.	15 + 12	27	✓	I added 10 plus 10 equals 20, 5 plus 2 is 7, then added them together
2.	23 – 11	12	✓	10 take away 20 is 10, 1 take away 3 is 2
3.	33 + 54	87	✓	50 plus 30 is 80, 4 plus 3 is 7
4.	42 – 23	19	✓	3 take away 2 is zero, and you have to take away 1 from 40 which is 39, 39 take away 20 is 19
5.	55 + 37	93	✗	30 add 50 is 80, 7 add 5 is 12
6.	68 – 32	100	✗	No it is'nt, 36, 30 take away 60 is 30, 2 take away 8 is 6
7.	57 + 14	71	✓	10 plus 50 is 60, 4 plus 7 is 11
8.	58 – 19	39	✓	10 take away 50 is 40, 9 take away 8 you can't do, so you take 1 from 40 which you call 39
9.	27 + 69	96	✓	6 plus 20 is 80, 9 plus 7 is 16
10.	64 – 27	37	✓	60 take away 20 is 40, 7 take away 4 you can't do, so you take 3 away from 40, which is 37
				% Correct = 80%

School/Pupil: A22m LA **Strategy Stage = S8, see Q6.**

No	Qu	Ans	✓✗	Explanation
1.	15 + 12	27	✓	I added the 10 and 10 together and then the 5 and the 2, add together
2.	23 – 11	12	✓	Taking the 1 away first and then the 10
3.	33 + 54	87	✓	3 plus 4 and then 30 plus 50 and add them together
4.	42 – 23	19	✓	Take away the 20 and then take away the 3
5.	55 + 37	92	✓	30 plus 50, and then the 5 plus 7 and adding them together
6.	68 – 32	36	✓	I just knew it
7.	57 + 14	71	✓	57 just plus 14
8.	58 – 19	39	✓	Take away 20 and add 1
9.	27 + 69	96	✓	70 plus 27, then I took away 1
10.	64 – 27	37	✓	Took away 20 then took away 7

% Correct = 100%

School/Pupil: A25f LA Strategy Stage = S6, see Q8.

No	Qu	Ans	✓✗	Explanation
1.	15 + 12	27	✓	I added the 2 and the 5, and the 10 and the 10
2.	23 – 11	12	✓	I took 1 away, and 10 away
3.	33 + 54	87	✓	4 plus 3, and 50 plus 30
4.	42 – 23	19	✓	I took the 3 away, and then took the 9 away
5.	55 + 37	92	✓	7 plus 5 and 50 plus 30
6.	68 – 32	36	✓	8 take 2, and 60 take 30
7.	57 + 14	71	✓	7 plus 4, and 50 plus 10
8.	58 – 19	39	✓	58 take 9, then take 10
9.	27 + 69	96	✓	7 plus 9 and 60 plus 20
10.	64 – 27	37	✓	64 take 7, then take 20
				% Correct = 100%

School/Pupil: B4m HA Strategy Stage = S4, see Q5.

No	Qu	Ans	✓✗	Explanation
1.	15 + 12	27	✓	I added the 5 and 2 which is 20, and the 10 and 10 which is 20
2.	23 – 11	12	✓	I took 1 off 3 which is 2, and 10 off 20 which is 10
3.	33 + 54	87	✓	I added 4 and 3 which is 7, and then I added 5 and 3 which is...and then I added 50 and 3 which is 8
4.	42 – 23	21	✗	I took 2 off 3 which is 1, and took 2 off 4 which is 2
5.	55 + 37	92	✓	I added 7 and 5 which is 12, and then added the 50 and 30 which was 80, and then I remembered it was 12 and added 10 on the 12 to the 80
6.	68 – 32	36	✓	I took 2 off 8 which is 6, and took 3 off 6 which is 3
7.	57 + 14	71	✓	4 add 7 which is 11, and then 50 add 10 which is 60 and then I put them together
8.	58 – 19	41	✗	I took 9 off 8 which 1 and 1 off 5 which is 4
9.	27 + 69	96	✓	I done 9 add 7 which is 11, and then I done 6 add 2 which is 8,and then I remembered it was 11, so 6 add 2 is 7 is 8, 80, and then I had to add on 10
10.	64 – 27	43	✗	I took 7 away from 4 which is 3 and than I took 2 away from 6 which is 4, 43

% Correct = 70%

School/Pupil: B5f HA Strategy Stage = S4, see Q5

No	Qu	Ans	✓✗	Explanation
1.	15 + 12	27	✓	1 plus 1 is 2 and 5 plus 2 is 7
2.	23 – 11	12	✓	I did 3 take away 1, and 2 take away 1
3.	33 + 54	78	✗	I did 4 add 3 and 5 add 3
4.	42 – 23	10	✗	I did 2 take away 2 is nothing and 4 take away 3 is 1
5.	55 + 37	87	✗	5 add 3 is 8, 50 add 30 is 70, 5 add 7 which makes 12
6.	68 – 32	45	✗	6 take away 2 is 4, 8 take away 3 is 5
7.	57 + 14	98	✗	5 add 4 which is 9, and 7 add 1 which equals 8
8.	58 – 19	47	✗	I did 9 take away 5 which is 5, and then I did 8 take away 1 which is 7
9.	27 + 69	-	✗	Unable to respond
10.	64 – 27	-	✗	Unable to respond
				% Correct = 20%

School/Pupil: B9m LA Strategy Stage = S6, see Q4.

No	**Qu**	**Ans**	**✓✗**	**Explanation**
1.	15 + 12	27	✓	I added the tens, then the 5 and the 7,
2.	23 – 11	12	✓	First I took the 10 away, then I took the 1 away
3.	33 + 54	87	✓	I added the tens, then I added the units
4.	42 – 23	19	✓	I took the 3 away from the 42 that equals 39, then I took the 20 away, leaves 19
5.	55 + 37	92	✓	I rounded the 37 up to 40, added the 50, took the 3 away and added 5
6.	68 – 32	36	✓	I took 2 from 8 and that equalled 6, and I took 30 from 60 and that equalled 36
7.	57 + 14	61	✗	I rounded 57 up to 60, then added 14 and took away 3
8.	58 – 19	39	✓	First I rounded 58 up to 60, then I took 10 away and that equalled 50, and then I took another 10 away, added 1 and then I took the 2 away and that equalled 39
9.	27 + 69	104	✗	First I added 2 and 6 and that equalled 80, then I added the 9 and 7, that's how I worked it out
10.	64 – 27	37	✓	I took 7 and 4 away, no I took 4 away from 7, then I took 20 away from 60

% Correct = 80%

School/Pupil: B12m LA Strategy Stage = S6, see Q7.

No	**Qu**	**Ans**	**✓✗**	**Explanation**
1.	15 + 12	27	✓	5 add 2, and 10 add 10 and add them together
2.	23 – 11	12	✓	23 take away 10 is 13, then take away the extra 1 is 12
3.	33 + 54	87	✓	4 plus the 3 is 7, 30 plus 50 is 80 and 80 plus 7 is 87
4.	42 – 23	19	✓	42 take away 3 is 39,then 39 take away the 20
5.	55 + 37	92	✓	7 plus the 5 is 12, 50 plus 30 is 80, then 80 plus the 12 is 92
6.	68 – 32	36	✓	8 take away 2 is 6, 60 take away 30 is 30, then 30 and 6 is 36
7.	57 + 14	71	✓	57 plus the 4 is 61, and add the 10 is 71
8.	58 – 19	39	✓	58 take away the 9 equals 49 and take away the extra 10 is 39
9.	27 + 69	96	✓	69 plus the 7 is 76, plus the extra 20 is 96
10.	64 – 27	37	✓	64 take the 7 is 57, and take away the extra 20 is 37

% Correct = 100%

School/Pupil: B13f LA Strategy Stage = S5, see Q3.

No	Qu	Ans	✓✗	Explanation
1.	15 + 12	27	✓	You get 15, then you add 6 on, then you add 6 on
2.	23 – 11	13	✗	Well, you make it 21 which is double eleven, and then you take 11 away which would make it 11, then add on 2
3.	33 + 54	87	✓	Add 50 and 30 which is 80,and 4 and 3 which is 7
4.	42 – 23	21	✗	You take away 20 from 40, and take away 2 from 3
5.	55 + 37	91	✗	7 add 5 is 11, and 50 and 30 is 80 and you add them together its 81
6.	68 – 32	36	✓	2 take away from 8 is 6 and 3 take away 60 is 30
7.	57 + 14	71	✓	7 add 4 is 11, 50 add 1 is 60 and add them together its 71
8.	58 – 19	41	✗	10 take away 50 is 40 and 9 take away 8 is 1
9.	27 + 69	96	✓	60 add 20 is 80, and 9 add 7 is 16 and add them together
10.	64 – 27	39	✗	60 take away 20 is 40 and 4 take away 7 is ….no 3
				% Correct = 50%

School/Pupil: B16f LA Strategy Stage = S4, see Q1.

No	Qu	Ans	✓✗	Explanation
1.	15 + 12	27	✓	I added the 5 and 2 which made 7, and the ten and the ten which made 20, made 27
2.	23 – 11	12	✓	I took 1 away from 3 and the 1 away from the 2
3.	33 + 54	87	✓	I added the 3 and the 4 together, and then I added the 3 and the 5 together
4.	42 – 23	21	✗	I took the 2 away from the 3, then the 2 away from the 4
5.	55 + 37	92	✓	I added the 5 and the 7 together which made 12, and then I put the 1 on to the 3 which made 4 and then I added the 5 and the 4 together
6.	68 – 32	36	✓	I took 2 away from the 8 and then I took 3 away from the 6
7.	57 + 14	71	✓	I did the 4 and the 7 together which made 11, then I put the 1 under the 1 which made 2, than I put the 5 and the 2 together
8.	58 – 19	41	✗	I took 8 from 9 which made 1, then I took the 1 away from the 5
9.	27 + 69	96	✓	I added 7 and 9 together which made 16, then I put the 1 onto the 6 which made 7, then I put the 2 on which made 9
10.	64 – 27	34	✗	I took 4 from 7 which made 3, and took 2 from 6 which made 4

% Correct = 70%

School/Pupil: B17f LA Strategy Stage = S2, see Q2.

No	Qu	Ans	✓✗	Explanation
1.	15 + 12	27	✓	I used my fingers
2.	23 – 11	12	✓	I had 23 in my head and took away 11 by counting backwards
3.	33 + 54	87	✓	I lost count
4.	42 – 23	20	✗	I done the same as the other one, take away (used my fingers)
5.	55 + 37	92	✓	I done it by my fingers
6.	68 – 32	-	✗	Unable to respond
7.	57 + 14	-	✗	Unable to respond
8.	58 – 19	-	✗	Unable to respond
9.	27 + 69	-	✗	Unable to respond
10.	64 – 27	-	✗	Unable to respond

% Correct = 40%

School/Pupil: B21m LA Strategy Stage = S5, see Q5.

No	Qu	Ans	✓✗	Explanation
1.	15 + 12	27	✓	15 add 15 is 40, take off 3 (27)
2.	23 – 11	34	✗	20 add 10 then add 4
3.	33 + 54	87	✓	30 add 50, then 4 add 3
4.	42 – 23	15	✗	40 take away 20 is 20 then take away 5
5.	55 + 37	92	✓	50 add 30 is 80, then 7 add 5 is 12
6.	68 – 32	20	✗	60 take away 30 is 30, then take away 10
7.	57 + 14	71	✓	50 add 10, then 7 add 4
8.	58 – 19	37	✗	I rounded 58 to 60, and 19 to 20 and I took them off, and put the 3 back on
9.	27 + 69	98	✗	60, 90, 70 then add 7 and take away 1
10.	64 – 27	31	✗	60 take away 20 and 7 take 4

% Correct = 40%

School/Pupil: B23m LA Strategy Stage = S6, see Q9.

No	Qu	Ans	✓✗	Explanation
1.	15 + 12	27	✓	I added 10 from the 12 onto the 15, and then I added the 2
2.	23 – 11	12	✓	I took off the 10 and then I took off the 1
3.	33 + 54	87	✓	I added the 50 and the 30 which made 80 and the 3 and the 4 which made 7
4.	42 – 23	19	✓	I took off the 20 from 42 which is 22 and took off the 3 which is 19
5.	55 + 37	92	✓	I added the 30 and the 50 which made 80 and the 7 and the 5 which made 12, and added them together
6.	68 – 32	36	✓	I took off 30 from 60 which made 30, and 2 off 8 which made 6
7.	57 + 14	71	✓	I added the 10 to the 57 which made 67 and added 4 which was 71
8.	58 – 19	39	✓	I took off the 10 from 58, and then the 9
9.	27 + 69	96	✓	I did 20 plus 69 which was 69 plus the 7 which was 96
10.	64 – 27	37	✓	I too off the 20 from 64 then the 7

% Correct = 100%

School/Pupil: C8f HA Strategy Stage = S4, see Q3

No	Qu	Ans	✓✗	Explanation
1.	15 + 12	27	✓	I added the two tens and then I added the 5 and the 2
2.	23 – 11	7	✗	I took the 10 away from the 20 and then I took 3 and then took 1
3.	33 + 54	87	✓	I added the two tens and then added the two units
4.	42 – 23	17	✗	Took the two units away, and then the two tens
5.	55 + 37	92	✓	Added the two tens and added the two units
6.	68 – 32	31	✗	Took the two tens away from each other, and then the 2 away from the 8
7.	57 + 14	80	✗	Added the two tens and added the two units
8.	58 – 19	35	✗	Took the 1 from the 5 and then took away the 8 from the 9
9.	27 + 69	96	✓	Added the two tens, and the two units together
10.	64 – 27	29	✗	Took the two tens away and then took the two things (units) away
				% Correct = 40%

School/Pupil: C16f HA Strategy Stage = S5, see Q6.

No	Qu	Ans	y/n	Explanation
1.	15 + 12	27	✓	Just added the two tens and the five and the 2
2.	23 – 11	12	✓	I took a 10 away and a 1 from 23
3.	33 + 54	87	✓	I just added the 30 and the 50 which is 80 and the 3 and the 4 which is 7
4.	42 – 23	19	✓	I don't know really, just guessed it
5.	55 + 37	92	✓	I just added the tens and the units
6.	68 – 32	36	✓	I took the 30 from the 60 which is 30 and took the 2, took away the units
7.	57 + 14	71	✓	Just added the tens and the units again
8.	58 – 19	39	✓	I just took the units from the tens, as usual what I do
9.	27 + 69	96	✓	I just added the 20 and the 60 which is 80 and then the 7 and then the 9
10.	64 – 27	43	×	I took them two (4 & 7) which is 3, I took them two (60 & 20) which is 40, and took 2 away from 40 which is 3

% Correct = 90%

School/Pupil: C18f HA Strategy Stage = S5, see Q5.

No	Qu	Ans	✓✗	Explanation
1.	15 + 12	27	✓	I added the 10's and then I added the units
2.	23 – 11	12	✓	I took 1 away from the 20 and 1 away from the 3
3.	33 + 54	87	✓	I added the tens, and then the units
4.	42 – 23	21	✗	Took 1 away from the 3 and 2 away from the 4
5.	55 + 37	92	✓	The 5 and the 7 equals 12 and the 3 and the 5 equals 8, so I added 10 to the 80 made 90 and 2 made 92
6.	68 – 32	36	✓	Take 3 away from the 6 and 2 away from the 8
7.	57 + 14	71	✓	I added the 4 and 7 that equals 11, then I added 1 onto the 5 that equals 6, then I added 1 to the 60 that made 70 and 1 to the 70 that made 71
8.	58 – 19	41	✓	The 8 is less than the 9 and if you take 8 away from 9 that would leave you with 1, and 5 and 1, and 5 take away 1 leaves 4
9.	27 + 69	96	✓	The 9 and the 7 equals 16 and the 6 and the 2 equals 8, and the 6 and the 2 equals 80, and than I added I added the 10 to the 80 that equals 90 and just added the 7 makes 96
10.	64 – 27	43	✗	Took 4 away from the 7 and took 2 away from the 6

% Correct = 80%

School/Pupil: C20f HA Strategy Stage = S5, see Q5.

No	Qu	Ans	✓✗	Explanation
1.	15 + 12	27	✓	I added 10 and 10 which is 20 and 5 and 2 which is 7
2.	23 – 11	12	✓	Just went 20 take away 10 is 10 and 3 take away 1 is 2
3.	33 + 54	87	✓	4 add 3 and then 5 add 3
4.	42 – 23	21	✗	dk
5.	55 + 37	92	✓	I took 3 from the 55 and then I put the 50 and the 30, and then I put 10 on, and then I put an extra 2 on.
6.	68 – 32	36	✓	6 take away 3 is 3 and 8 take away 2 is 6
7.	57 + 14	71	✓	5 and 1 is 6 and too make it 7 there is 1 left over
8.	58 – 19	41	✗	dk
9.	27 + 69	96	✓	6 and 2 is 8, 7 and 9 is 16
10.	64 – 27	43	✓	60 take away 2 is 40, and 4 take away 7 is 3

% Correct = 70%

School/Pupil: D4f LA Strategy Stage = S7, see Q8.

No	Qu	Ans	✓✗	Explanation
1.	15 + 12	23	✗	I added the 5 and the 2 together to make7, and I then added the two tens to make 20 and added them together
2.	23 – 11	12	✓	I took away 1 to make 22 and I took away 10
3.	33 + 54	88	✗	I did 5 and the 3 tens to make 80 and the 5 and the 4
4.	42 – 23	29	✗	I took away the 3 to make 49 and I took away the 20
5.	55 + 37	91	✓	I did the 7 and the 5 together, and I did the 3 and the 5
6.	68 – 32	36	✓	I took away the 30 from the 60 and the 2 from the 8
7.	57 + 14	71	✓	I knew that the 3 would make the 10, so I did that to make 60 from the 50 and I did the 1 and added the 10
8.	58 – 19	39	✓	Take away 20 and add 1
9.	27 + 69	96	✓	I added the 9 and 7 together and the 6 and the 2
10.	64 – 27	38	✗	I took away the 4 to make 60 cos its 7, and then I took away another 3 to make 58 and I took away 20
				% Correct = 60%

School/Pupil: D8f HA Strategy Stage = S5, see Q5.

No	**Qu**	**Ans**	**✓✗**	**Explanation**
1.	15 + 12	27	✓	10 add 10 is 20 plus 5 plus 2
2.	23 – 11	12	✓	20 away 10 is 10, 1 take away 3 is 2
3.	33 + 54	88	✗	30 plus 50 is 80 and 3 plus 4
4.	42 – 23	21	✗	40 take away 20 is 20 and 3 take away 2
5.	55 + 37	92	✓	50 plus 30 and then 5 plus 7
6.	68 – 32	36	✓	60 take away 30 is 30 and then 8 take away 2
7.	57 + 14	77	✗	50 plus 14, 4, I mean 10, and then 7 plus 4
8.	58 – 19	41	✗	50 take away 10 and then 8 take away 9
9.	27 + 69	96	✓	20 plus 60 and 9 plus 7
10.	64 – 27	43	✗	60 take away 20 and 4 take away 7
				% Correct = 50%

School/Pupil: D15m HA Strategy Stage = S5, see Q3.

No	Qu	Ans	✓✗	Explanation
1.	15 + 12	27	✓	I got the two tens and put them together and I knew that 5 add 2 equals 7 so it equals 27
2.	23 – 11	12	✓	I take it as 1 take away 2 equals 1 and 1 take away 3 equals 2
3.	33 + 54	87	✓	I got 5 and added it onto 3 equals 80, and 4 add onto 3 equals 7, cos 1 add 4 equals 5 and 2 add 5 equals 7
4.	42 – 23	19	✓	Taking away 2 from 4 equals 2, and 3 is higher than 2 so it must be 1 and one higher than 2 must make it 19
5.	55 + 37	82	✗	Got 5 add 3 equals 8, no its 93, cos 5 add 3 equals 8 and 5 add 7 equals 12, so that 80 to 12 was equals 92
6.	68 – 32	36	✓	2 take away 8 equals 6 cos 2,4,6,8 and three times table equals 3,6, so you take away half of it equals 3
7.	57 + 14	71	✓	5 add 1 equals 60, so you …get the other one, ten equals 67 add 4 the 10 and the 4 equals 50, so equals 51
8.	58 – 19	39	✓	5 take away 1 equals 4, and 8 take away 9 cos 9 is bigger than 8, its one higher that makes it 41
9.	27 + 69	96	✓	2 add 6 equals 8 and 7 add 9 equals 16 so that's 16 add 80 equals 91
10.	64 – 27	43	✗	I knew that 1 take away 6 equals 5, so 2 take away 6 must equal 4 and 4 take away 7 must be 3, so it must be 43

% Correct = 80%

School/Pupil: D20m **HA** **Strategy Stage = S4, see Q7.**

No	Qu	Ans	✓✗	Explanation
1.	15 + 12	27	✓	5 plus 2 and 10 plus 10
2.	23 – 11	12	✓	Cos I know, I just halved it, half 2, cos it was 10 and I just took 1 off the 3
3.	33 + 54	87	✓	4 plus 3 and 50 plus 30
4.	42 – 23	21	✗	Cos I know 3 take away 2 is 1 and 40 take away 2 is 20
5.	55 + 37	97	✗	7 add 5, 3 add 5
6.	68 – 32	100	✗	I know that 8 plus 2 is 10 and 6 plus 3 is 9
7.	57 + 14	71	✓	4 plus 7 and 50 plus 10
8.	58 – 19	41	✗	9 take away 8 and 10 plus 15
9.	27 + 69	92	✗	9 plus 7, and 6 plus 2
10.	64 – 27	43	✗	7 take away 4, and 2 take away 6

% Correct = 40%

School/Pupil: D24f LA Strategy Stage = S6, see Q8.

No	Qu	Ans	✓✗	Explanation
1.	15 + 12	27	✓	I added the two tens and added the 5 and the 2 and added them together
2.	23 – 11	12	✓	I took away the 1 to make 22 and took away the 10 to make 12
3.	33 + 54	87	✓	I just did the 4 and the 3 to make 7 and the 50 and the 30 to make 80 and added them together
4.	42 – 23	19	✓	Took 2 away from 42 to make 40 and took another one away to make 39 and took 20 away to make 19
5.	55 + 37	92	✓	Added the 5 and 7 to make 12 and added the 50 and 30 to make 80 and added them together
6.	68 – 32	36	✓	Took away 2 from 8 to make 6, took 30 away from 66 to make 36
7.	57 + 14	71	✓	Added 10 to 57 to make 67 and 3 to make 70 and 1 to make 71
8.	58 – 19	39	✓	Took 8 away from 58 to make 50 and 1 away to make 48 and 10 away to make 39
9.	27 + 69	96	✓	I did 9 and 7 to make 16 and 16 and 80 and added them together
10.	64 – 27	37	✓	Took 3 away from 64 to make 60, then took the other 3 away to make 57, then took the other 20 away to make 37

% Correct = 100%

School/Pupil: E6f **LA** **Strategy Stage = S6, see Q9.**

No	Qu	Ans	✓✗	Explanation
1.	15 + 12	27	✓	I did the 10 plus the 10 and the 5 plus the 2 and added those two answers together
2.	23 – 11	12	✓	I did the 1 from 3 leaves 2, I did the 10 from , the two tens leaves one 10
3.	33 + 54	87	✓	I did 30 add 50 and 3 add 4 and added those two answers together
4.	42 – 23	19	✓	I took 20 away from 42 and that 22 and I took 3 away from 22 and that left 19
5.	55 + 37	92	✓	I did 50 plus 30 is 80 and 5 plus 7 is 12 and added those two answers together
6.	68 – 32	36	✓	I took 2 away from the 8 and that left 66, and I took 30 away from that and that left 36
7.	57 + 14	71	✓	I did 50 add 10 and 7 add 4 and added them together
8.	58 – 19	39	✓	I did 9 from 8 and that's em.. No, I did 9 from 58 and that's 49 and took away the 10 is 39
9.	27 + 69	96	✓	I did 69 add 7 equals 76 and add the 20
10.	64 – 27	37	✓	I took the 7 away from 64 equals 57 and took away the 20

% Correct = 100%

School/Pupil: E8m HA Strategy Stage = S3, see Q3.

No	Qu	Ans	✓✗	Explanation
1.	15 + 12	27	✓	I put the 15 at the top and the 12 at the bottom and added them up
2.	23 – 11	13	✗	I put the 23 at the top and the 11 at the bottom and took it away
3.	33 + 54	87	✓	I put the 33 at the top and the 54 at the bottom and added them together
4.	42 – 23	64	✗	I put the 42 at the top and the 23 at the bottom
5.	55 + 37	81	✗	I put the 55 at the top and the 37 at the bottom and added them together
6.	68 – 32	36	✓	68 at the top, 32 at the bottom and took it away
7.	57 + 14	61	✗	Got the 57 at the top and the 14 at the bottom and added them together
8.	58 – 19	41	✗	Take the 9 from the 8 and the 1 from the 5
9.	27 + 69	-	✗	Unable to respond
10.	64 – 27	-	✗	Unable to respond
				% Correct = 30%

School/Pupil: E14m LA Strategy Stage = S6, see Q6.

No	**Qu**	**Ans**	**✓✗**	**Explanation**
1.	15 + 12	27	✓	5 and 2 which equals 7, and 10 and 10 which equals 20, then add them together
2.	23 – 11	12	✓	Two eleven's are 22, so take away the one left over
3.	33 + 54	87	✓	50 add 30 is 80, and 4 add 3 is 7, then add them together
4.	42 – 23	19	✓	40 take 20 is 20, add the 2 take away 3 makes 19
5.	55 + 37	92	✓	5 add 7 is 12 and 50 add 30 is 80
6.	68 – 32	36	✓	68 take away 30 is 38, take away 2 is 36
7.	57 + 14	71	✓	57 add 4 is 61, add 10 is 71
8.	58 – 19	39	✓	Because if the 58 take away 9 is 49 and take away 10
9.	27 + 69	96	✓	Because 7 add 9 is 16, and 20 add 60
10.	64 – 27	37	✓	64 take away 20 is 44 and take away 7is 37

% Correct = 100%

School/Pupil: E22m HA Strategy Stage = S6, see Q8.

No	Qu	Ans	✓✗	Explanation
1.	15 + 12	17	✗	I done 10 plus 10 and 5 plus 2 and added them together
2.	23 – 11	12	✓	I took away the 10 and took away the 1
3.	33 + 54	87	✓	50 plus 30 and I done 3 plus 4
4.	42 – 23	19	✓	I took away 20 from 40 which was 20, and I took away 3 from 22
5.	55 + 37	92	✓	50 plus 30 and 5 plus 7 and added it together
6.	68 – 32	36	✓	I took away 30 from 60 which was 30, I took 30 away from 60 which was 38 and I took away 2
7.	57 + 14	51	✗	50 plus 10 and 7 plus 4 and I added them together
8.	58 – 19	39	✓	I took away the 10 from 58 and I took away the 9
9.	27 + 69	-	✗	Unable to respond
10.	64 – 27	-	✗	Unable to respond
				% Correct = 60%

School/Pupil: E23m HA Strategy Stage = S6, see Q5.

No	Qu	Ans	✓✗	Explanation
1.	15 + 12	27	✓	Well I took the 2 and 5 away and put zero there, so I added 10 and 10 together equals 20, then I added the 5 and 2 together equals 7 and put them together
2.	23 – 11	7	✗	I got 20 and took away 11 gave me 9, and I took away the 3 from 9 gave me 6
3.	33 + 54	87	✓	I done 30 and 50 gave me 80, and then I took 1 away from the 3 and put it to the 4 what gave me 5 and then I added 2 what gave me 7, I mean 87
4.	42 – 23	14	✗	I took away 20 from 42, that gave me 18 and then I took away 3 and that gave me 15
5.	55 + 37	92	✓	Well I done 55 add 30 is 85 and I added 7 and that gave me the answer
6.	68 – 32	22	✗	Well I took away 30 from 60 that gave me 30, and I took away 8 and that gave me 22, and then I took away 2 and that gave me 19
7.	57 + 14	71	✓	Well I done 50 add 14 and that gave me 64 and I added 14 and that gave me 71
8.	58 – 19	39	✓	Well I took away 10 from 50 and that gave me 40, and then I took away 8 and 9
9.	27 + 69	-	✗	Unable to respond
10.	64 – 27	-	✗	Unable to respond

% Correct = 50%

School/Pupil: F4f HA **Strategy Stage = S2, see Q7.**

No	Qu	Ans	✓✗	Explanation
1.	15 + 12	27	✓	Got 15 just added on 12 (counted)
2.	23 – 11	12	✓	Got 23 then take away 11 (counted)
3.	33 + 54	-	✗	Don't know
4.	42 – 23	19	✓	Count up to 42 from 23
5.	55 + 37	18	✗	55 plus 37 (counted)
6.	68 – 32	-	✗	Don't know
7.	57 + 14	71	✓	Just 57 add on 14 (counted)
8.	58 – 19	42	✗	Take 58 take away 19 (counted)
9.	27 + 69	92	✗	I thought of 69 if that was on top of like the 27 then add the 9 and the 7 and carry 1 and add the 6 and the 2
10.	64 – 27	-	✗	I don't know
				% Correct = 40%

School/Pupil: F5m HA Strategy Stage = S6, see Q7.

No	Qu	Ans	✓✗	Explanation
1.	15 + 12	27	✓	I first add the tens together, then I knew I had 5, and then I add 2 on the 5
2.	23 – 11	12	✓	I had 23, then I take away 10, then I had 13 and take away 1 and I had 12
3.	33 + 54	87	✓	I started off with the 50 and I add 30 that gave me 80, then I went to the 4 and add 3 on that gave me 7
4.	42 – 23	17	✗	I had 40 and took away 20 what gave me 20, and then I add 3 and took away 2
5.	55 + 37	92	✓	I had 50 and add the 30 what gave me 80, then I looked at the 7 and looked at the 5 and I add 7 onto the 5
6.	68 – 32	36	✓	I had 60 and took away 30 what gave me 30 and then I looked at the 8 and took away 2 from the 8
7.	57 + 14	71	✓	I had 57 and then I add 10 onto the 57 what gave me 67, and then I add 4
8.	58 – 19	49	✗	I had 50 then I took away 10 what gave me 40, so then I had 9 so I took away 8
9.	27 + 69	96	✓	I had 60 and I add 20 on what gave me 80 and then I looked at the 9 and I add 7 on what gave me the answer
10.	64 – 27	43	✗	I looked at the 60 and took away 20 what gave me 40 then I had the 7 and took away 3 and I took away 1 what have me the answer

% Correct = 70%

School/Pupil: F7m HA Strategy Stage = S5, see Q7.

No	Qu	Ans	✓✗	Explanation
1.	15 + 12	27	✓	I just added the two tens and the 5 and the 2 which made 7 and put them together
2.	23 – 11	12	✓	I took away 10 from 2 and I took 1 away from 3 and put them together
3.	33 + 54	87	✗	I added the 50 and the 30 which made 80 and I added the 4 and 3 which made 7 and put them together
4.	42 – 23	21	✗	I took, I halved the 40 which makes 20 and then I took then I put the 3 on the end and took 2 away makes the answer
5.	55 + 37	93	✗	I added the 50 and 30 which makes 80, and then I added 7 to the 80 and then I added 5 to the 7 which makes I know the answer
6.	68 – 32	36	✓	I halved the 6 which makes 30, then I took 2 away from the 8 then put it together
7.	57 + 14	71	✓	I added the 50 and 10 which makes 60 and then I added the 7 and 4 together and put them together and they make, keep on forgetting the answers
8.	58 – 19	41	✗	I took the 10 from the 50 which makes 40 and then I put the 9 on the end and took 8 away
9.	27 + 69	86	✗	I added the 60 and 20 which makes 70 and then I added 7 to the 70 which makes 77 and then I added the 9
10.	64 – 27	43	✗	I took the 20 away from the 60 which makes 40 and then I put the 7 on the end of the 40 which so that makes 47 and I took away 4

% Correct = 40%

School/Pupil: F9f LA Strategy Stage = S6, see Q2.

No	Qu	Ans	✓✗	Explanation
1.	15 + 12	27	✓	I added 10 add 10 and 5 add 2 and I than added 20 add 7 that's 27
2.	23 – 11	12	✓	I took away 10 from 23 what left me with 13, then I took 1 what left me 12
3.	33 + 54	87	✓	I added 50 plus 30 what was 80, and I added 4 add 3 what was 7
4.	42 – 23	19	✓	I took 20 from 40 what made 20 and I took 3 from…and I knew what was 22 take away 3 that made 19
5.	55 + 37	92	✓	I added 50 add 30 was 70 and 5 add 7 and added them together
6.	68 – 32	36	✓	I took 30 from 60 what was 38 then I took 2 from the 8
7.	57 + 14	71	✓	I added 50 and 10 what made 60 and I added 4 and 7, and then I added them together
8.	58 – 19	39	✓	I took 10 from 50 what gave me 48 and then I took 9 from 48
9.	27 + 69	96	✓	I added 60 and 20 what was 80 and I added 7 and 9 and added them together
10.	64 – 27	37	✓	I took 20 from 60 what was 40 and I took 7 from 44

% Correct = 100%

School/Pupil: F13f HA Strategy Stage = S6, see Q7.

No	Qu	Ans	✓✗	Explanation
1.	15 + 12	27	✓	I just added the two tens which made 20 then I added 5 and the 2 which made 7
2.	23 – 11	12	✓	Well I took away 10 from 23 which made 13 and I just took away 1 which was 12
3.	33 + 54	92	✗	I just added 30 onto 54 and added the 4 and the 3
4.	42 – 23	21	✗	I took away 20 from 42 then took away 1
5.	55 + 37	92	✓	Just added 30 onto 50 and then added the 5 and 7
6.	68 – 32	30	✗	I took away 30 then took away 2 from 8
7.	57 + 14	71	✓	I added 10 onto 57 which was 67 then added 4
8.	58 – 19	39	✓	I took 10 from 58 which was 48 then took away 9
9.	27 + 69	98	✗	I added 20 onto 69 then added 9 to 7
10.	64 – 27	37	✗	I took 20 from 64 then added 7 onto 4

% Correct = 50%

School/Pupil: F17m HA Strategy Stage = S4, see Q3.

No	Qu	Ans	✓✗	Explanation
1.	15 + 12	27	✓	I added the 10 and the 10 that equals 20, then I added the 5 and the 2 that equals 7 and I added 20 from 7, 27
2.	23 – 11	12	✓	I took away from the 2 and that leaves me with 1, 10, then I took 1 away from the 3, 2, so that leaves me with 12
3.	33 + 54	87	✓	I add the 50 and the 3c equals 80, then I added the 4 and the 3 equals 7 and then I added the 8 and the 7 together, 87
4.	42 – 23	19	✓	I took 2 away from the 4 that leaves me with 2, 3 from the 2 I could'nt do so I had to take 1 from the 27, so leaves 19
5.	55 + 37	91	✗	I added the 5 and the 3 that comes to 80, and then I added the 7 and the 5 that comes to 12, so that should be 92
6.	68 – 32	36	✓	I took the 3 away from the 6 that leaves me with 3, and the 2 away from the 8 that leaves me with 6
7.	57 + 14	71	✓	I added the 5 and 1 that leaves me with 6, and I add the 7 and the 4 that equals 11
8.	58 – 19	39	✓	I took 1 away from the 5 that leaves me 4, took 10 from the 50 that leaves me 40, took 9 from the 8 could'nt do, so I took 9 away from the 40 that leaves me 39
9.	27 + 69	96	✓	I added the 6 and the 2 equals 80, then I added the 9 and the 7 equals 16, then I added them together
10.	64 – 27	41	✗	I took 2 away from the 6 that leaves me 40 and I, so I could'nt take 7 away from 4, so I take 7 away from 40 that leaves me 33

% Correct = 80%

School/Pupil: F18m HA Strategy Stage = S5, see Q1

No	Qu	Ans	✓✗	Explanation
1.	15 + 12	27	✓	I added 15 and 15 which made 30 and took away 3 which gave me 27
2.	23 – 11	12	✓	I got 23 and took away, and took away something, can't remember now, how I done it, got a short a very short memory, I can't remember how I took it away
3.	33 + 54	87	✓	I added the 5 and the 3 which made 8, and the 4 and the 3 which made 7, so it was 87
4.	42 – 23	15	✗	I done like the same as the last one but took it away. So I like, took the 4 and the 2 away and then I knew if I took that (3) away, that would be 17, then I took the 2 which made it 15
5.	55 + 37	92	✓	I got the 5, added 3, this time added 7 cos it was the highest number, which the highest is easiest and then I added 5, which made me did, so I take, so like if it was in a mental test, and I would have to do it really quickly, I would get the answer quicker
6.	68 – 32	31	✗	The 8 and the 2 made, 8 take away 2 is 6, and then I took that and 3 away, which that made 3 and then, I took, I added them two on again and I took away the 8 and took away the 2
7.	57 + 14	81	✗	I added 10 to the 5 which made 67 and then in my head I counted 14 which made it the answer 81
8.	58 – 19	33	✗	I took away 1 from that, 1 from 5 and then I added up the 8, did 20 to 19, so I gave me an extra 1, so I took an extra 1 away which gave me the answer
9.	27 + 69	95	✗	I added the 2 to the 6, added 7 which gave me 87 and then I added up to 9 which gave me the answer
10.	64 – 27	29	✗	I done the 6 and 2 to make 4, took away the 7 and done the 4

% Correct = 40%

School/Pupil: F19f HA **Strategy Stage = S5, see Q3.**

No	Qu	Ans	✓✗	Explanation
1.	15 + 12	27	✓	I added the two tens together which made 20 and then added the 2 and the 5 which made 7 and then added 20 and 7 which made 27
2.	23 – 11	12	✓	I took 10 away from 20 which leaves me with 10 and then I took 1 away from 3 which leaves me 2 and added 10 and 2 together to make 12
3.	33 + 54	87	✓	I added 30 and 50 together to make 80 and added 4 and 3 together and added the two together
4.	42 – 23	21	✗	I took away 2 from 4, I took 20 away from 40 which made 20, and I took 3 away from 2, I mean 2 away from 3 which makes 1
5.	55 + 37	92	✓	I added 50 and 30 which equals 80 and 5 and 7 which equals 11, 12, and added them together
6.	68 – 32	36	✓	I took 30 away from 60 which leaves 30 and then subtracted 8 away from 2 I mean 2 away from 8
7.	57 + 14	71	✓	I added 7 and 4 which makes 11, and then 50 add 10 which equals 60 and added the two together
8.	58 – 19	39	✓	I done 50 take away 1 which leaves me 40, then 9 take away 8 which you can't do, so I done 58 take away 9 and then I took away 10 and that's my answer
9.	27 + 69	96	✓	I done 20 and 60 which equals 80 and 9 and 7 and added the two answers together
10.	64 – 27	43	✗	I done 60 take away 20 which left me with 40, and 7 take away 4, and that's 3, and added them together

% Correct = 80%

School/Pupil: G1m LA Strategy Stage = S6, see Q2.

No	Qu	Ans	✓✗	Explanation
1.	15 + 12	27	✓	Added the two tens and added the 5 and 2
2.	23 – 11	12	✓	Took away the 10 from the 23 which equals 13 then took away the 1
3.	33 + 54	87	✓	Added the 30 and the 50 and added the 3 and the 4
4.	42 – 23	19	✓	I took away the 20 from the 40 and I took away the 3 from the 22
5.	55 + 37	92	✓	Added the 30 to the 50 and added the two five's together
6.	68 – 32	36	✓	I took away the 30 from the 60 and the 2 from the 8
7.	57 + 14	71	✓	I added 10 to the 50 and 4 to the 7
8.	58 – 19	39	✓	I took away the 10 from the 50 and the 9 from the 8
9.	27 + 69	96	✓	I added the 20 to the 60 and the 7 to the 9
10.	64 – 27	37	✓	I took away the 20 from the 60 and 7 from 4

% Correct = 100%

School/Pupil: G2m HA Strategy Stage = S3, see Q6.

No	**Qu**	**Ans**	**✓✗**	**Explanation**
1.	15 + 12	27	✓	I added 1 and 1 is 2 and 5 and 2 is 7
2.	23 – 11	34	✗	Added 2 and 1 and 3 and 1
3.	33 + 54	77	✗	5 add 3 and 3 add 4
4.	42 – 23	21	✗	4 and 2, and 2 and 3
5.	55 + 37	80	✗	7 and 5 and 5 and 3
6.	68 – 32	36	✓	6 take away 3 and 8 take away 2
7.	57 + 14	71	✓	Added 7 and 4 and 5 and 1
8.	58 – 19	39	✓	5 take away 9 and 8 take away 1
9.	27 + 69	110	✗	2 and 6, and 7 and 9
10.	64 – 27	33	✗	6 and 7 and 4 and 2

% Correct = 40%

School/Pupil: G3m HA Strategy Stage = S6, see Q4.

No	Qu	Ans	✓✗	Explanation
1.	15 + 12	27	✓	I just added 10 and 10 which I knew was 20 and 5 and 2 which I knew is 7
2.	23 – 11	12	✓	I minused 10 and then I minused 1
3.	33 + 54	87	✓	5 add 3, add a zero so that was 80, and add the 4 and the 3 which was 7
4.	42 – 23	19	✓	Well I minused 20 from 42 which was 22, and take away 3
5.	55 + 37	92	✓	5 add 3 is 9, so I added a zero so that was 90, and added 5 add 7
6.	68 – 32	36	✓	Take 2 from 68, then take 3 from 6 and add an 0
7.	57 + 14	71	✓	7 and 4 is 11 and add 5 which equals 71
8.	58 – 19	29	✗	Minus 10 from 58, then minus 9
9.	27 + 69	86	✗	6 and 2 equals 80 then add 7 and 9
10.	64 – 27	43	✗	Take 2 from 6 which is 4, then add an 0 which is 40, then add a 7 which is 47 and add a 4

% Correct = 70%

School/Pupil: G4m HA Strategy Stage = S4, see Q6.

No	**Qu**	**Ans**	**✓✗**	**Explanation**
1.	15 + 12	27	✓	Added the two tens together and added the 5 and the 2
2.	23 – 11	12	✓	Took away the 10 from 20 and 1 from 3
3.	33 + 54	87	✓	30 add 50 is 80 and 3 add 4
4.	42 – 23	19	✓	Took away 20 from 40 and 3 from 2
5.	55 + 37	92	✓	Add 30 to 50 and 7 to 5
6.	68 – 32	36	✓	Take away 30 from 60 and 2 from 8
7.	57 + 14	71	✓	10 add 5 and 4 add 7
8.	58 – 19	39	✓	Take away 10 from 50 and 8 from 9
9.	27 + 69	96	✓	20 add 60 and 7 add 9
10.	64 – 27	39	✗	20 take away 60 and 7 take away 4
				% Correct = 90%

School/Pupil: G5m HA Strategy Stage = S5, see Q3.

No	**Qu**	**Ans**	**✓✗**	**Explanation**
1.	15 + 12	27	✓	I did 10 add 10 which is 20 and 5 add 2 which is 7, put them together 27
2.	23 – 11	12	✓	Take away 10 is 13, and take away 1 is 12
3.	33 + 54	87	✓	30 add 50 is 80 and 4 add 3 is 7
4.	42 – 23	19	✓	40 take 20 is 20 and 3 take away 2 is 1
5.	55 + 37	92	✓	50 add 30 is 80 and 5 add 7 is 12
6.	68 – 32	36	✓	60 take away 30 is 30 and 8 take away 2 is 6
7.	57 + 14	71	✓	50 add 10 is 60 and 7 add 4 is 11
8.	58 – 19	31	✗	50 take away 10 is 40 and 9 take away 8 is 1
9.	27 + 69	96	✓	60 add 20 is 80 and 7 add 9 is 16
10.	64 – 27	33	✗	60 take away 20 is 40, and 7 minus 3
				% Correct = 80%

School/Pupil: G9m HA Strategy Stage = S7, see Q8.

No	**Qu**	**Ans**	**✓✗**	**Explanation**
1.	15 + 12	27	✓	I added 10 to the 15 and I added 2
2.	23 – 11	12	✓	I took away 10 which would leave 13 and 1, 12
3.	33 + 54	87	✓	I added 33 to 54, and just kept on (adding tens) and added 3
4.	42 – 23	19	✓	I took away the 20 and took away the 3
5.	55 + 37	87	✓	I added the 3 tens, to make 85 and I added 3
6.	68 – 32	36	✓	I took away 30 from 68 and I took away 2
7.	57 + 14	71	✓	I added 15 and took away 1
8.	58 – 19	47	✗	I took away the 20 and took the 1
9.	27 + 69	96	✓	I added the 27 to the 69, I added 20 and I added 7
10.	64 – 27	37	✓	I took away the 20 and I took away the 7
				% Correct = 90%

School/Pupil: G10f HA Strategy Stage = S3, see Q5.

No	Qu	Ans	✓✗	Explanation
1.	15 + 12	21	✗	I done 15 add 6 and add another 6
2.	23 – 11	12	✓	I took the 1 from the 3 which gave me 2 and the 1 from the 2 which gave me 1
3.	33 + 54	88	✗	I done 4 add 3 and 5 add 3
4.	42 – 23	21	✓	I took the 2 from the 3 which gave me 1 and the 2 from the 4 which gave me 2
5.	55 + 37	92	✓	5 add 3 is 8, 5 add 7 is 12 and add the two together 8
6.	68 – 32	36	✓	I took the 2 away from the 8 which gave me 6 and the 3 away from the 6 which gave me 3
7.	57 + 14	81	✗	7 add 4 which is 11, and the 5 add 1 which is 6
8.	58 – 19	41	✗	9 take away 8 which gave me 1 and 5 take away 1 which gave me 4
9.	27 + 69	78	✗	9 add 7 is 16, and then add 2 which is 8 and add 2
10.	64 – 27	43	✗	7 away 4 and 6 take away 2
				% Correct = 40%

School/Pupil: G13m **LA** **Strategy Stage = S6, see Q7.**

No	Qu	Ans	✓✗	Explanation
1.	15 + 12	27	✓	15 add 10 is 25 add 2 is 27
2.	23 – 11	12	✓	23 take away 10 equals 13 take away 1 equals 12
3.	33 + 54	87	✓	3 add 4 equals 7, 30 add 50 equals 80
4.	42 – 23	19	✓	20 take away 42 equals 22, take away 3 equals 19
5.	55 + 37	92	✓	50 add 30 equals 80, 5 add 7 equals 12
6.	68 – 32	36	✓	Half of 60 is 30, so half 60 is basically 60 take away 30,take away 2 from 8 is 6
7.	57 + 14	71	✓	57 add 4 equals 61 add 10
8.	58 – 19	39	✓	Half of 19 is 8 which is 9 take away 58 which would equal 49 take away 10
9.	27 + 69	96	✓	7 add 69 equals 76 add 20
10.	64 – 27	37	✓	20 take away 60 is 40, 7 take away 4 is minus 3, so take away 3 from another 10

% Correct = 100%

School/Pupil: G14m **LA** **Strategy Stage = S6, see Q7.**

No	Qu	Ans	✓✗	Explanation
1.	15 + 12	27	✓	I added 10 and 10 which is 20 and 5 and 2 which is 7
2.	23 – 11	12	✓	I took away 10 which is 13 and I took away 1 which is 12
3.	33 + 54	87	✓	I added 50 and 30 which is 80 and 4 and 3 which is 7
4.	42 – 23	19	✓	I took 20 away from 42 which is 22 and 3 away from 22 which is 19
5.	55 + 37	92	✓	I added 30 to 55 which 85 and I added 7 to 85 which is 92
6.	68 – 32	36	✓	I took 30 away from 68 which is 38 and 2 away from 38 which is 36
7.	57 + 14	71	✓	I added 10 to 57 which is 67 and 4 to 67 which is 71
8.	58 – 19	76	✓	I took away 20 from 58 which is 38 and added 1
9.	27 + 69	96	✓	I added 20 to 69 which is 89 and 7 to 89 which is 96
10.	64 – 27	37	✓	I took 20 away from 64 which is 44 and 7 away from 44 which is 37

% Correct = 100%

School/Pupil: G20f HA Strategy Stage = S3, see Q7.

No	Qu	Ans	✓✗	Explanation
1.	15 + 12	27	✓	I added 2 add 5 which is 7 and 1 add 1 which is 2
2.	23 – 11	2	✗	Well I took away 1 from 2 which is 2 and 2 from 1 which is nothing
3.	33 + 54	77	✗	Well 4 add 3 is 7 and 5 add 3 is 7
4.	42 – 23	21	✗	Well I done 2 take away 3 which is 1 and 2 take away 2 which is 2
5.	55 + 37	73	✗	Well I done 7 add 5 is 13, and 3 add 5 is 7
6.	68 – 32	36	✓	Well 2 take away 8 is 6 and then 3 take away 6 is 3
7.	57 + 14	71	✓	Well I done 4 add 7 which is 11 and 5 and 1 which is 6
8.	58 – 19	21	✗	I took 8 from 9 which is 1, and then I from 5 which is 4
9.	27 + 69	91	✗	I added 9 and 7 which is 16 and 6 and 2 which 8
10.	64 – 27	43	✗	Well I took 7 from 4 which is 3, and 2 from 6 which is 4
				% Correct = 30%

School/Pupil: G24f **LA** **Strategy Stage = S7, see Q8.**

No	Qu	Ans	✓✗	Explanation
1.	15 + 12	27	✓	I just did 15 plus 12 cos its quite a simple one
2.	23 – 11	12	✓	I took 1 from 3 and 1 from 2
3.	33 + 54	87	✓	I did 30 add 50 and 3 add 4
4.	42 – 23	19	✓	I took away 3 from 42 and 20 away from 39
5.	55 + 37	92	✓	I did 7 add 5 and that equal 12 and added that onto 20
6.	68 – 32	36	✓	30 take away 60 and 8 take away 2
7.	57 + 14	61	✗	I did 7 add 4 equal 11, and I added; I did 57 add 10 equals 67 and then I added 4
8.	58 – 19	37	✗	I did 58 take away 20 and take away 1
9.	27 + 69	96	✓	I did 27 add 70 and took away 1
10.	64 – 27	31	✗	I took away 30 from 64 and added 3
				% Correct = 70%

School/Pupil: G25f LA Strategy Stage = S5, see Q7.

No	Qu	Ans	✓✗	Explanation
1.	15 + 12	27	✓	I added 2 the 5 which is 7 and the 10 to the other 10 which is 20 and added them together
2.	23 – 11	12	✓	I took 1 away from 3 and 10 away from 20 and put the two numbers together
3.	33 + 54	87	✓	I partitioned the numbers again, I added 4 to the 3 and 50 to the 30 and put them together
4.	42 – 23	19	✓	I took away 3 from 2 which but I put took away another 10 from the 40 to put it onto the 2 so I could take away 3 from 2 so its 12 take away 3, and then I took away 20 away from 30 which was 10 and then I added the 10 and 9 together
5.	55 + 37	92	✓	Well add 7 to the 5 which was 12, then I added 3, 30 to the 50 which was 80 and then I added both numbers together
6.	68 – 32	36	✓	Took away 2 from 8 and 30 from 60 and put those two numbers together
7.	57 + 14	71	✓	I added 4 to the 7 which was 11 and then I added 10 to the 50 which was 60 and I put those two numbers together
8.	58 – 19	39	✓	I took away, I put from the 50, I took 10 from the 50 and put it onto the 8, and I then took away 9 from the 18 which I made, and then I took 10 away from the 40 and I put those two numbers together
9.	27 + 69	96	✓	I added 1 to the 69 and I and then I added 27 to the 70 and took away 1 and put those two numbers together
10.	64 – 27	37	✓	I took away 7, I added, I took away a 10 from the 60, and I put it onto the 4 which made it 14, and I took away the 7 from the 14 which made it 7, and I took away 2 from 5 which was 30 and then I put those two numbers together

% Correct = 100%

School/Pupil: G28m LA Strategy Stage = S6, see Q4.

No	**Qu**	**Ans**	**✓×**	**Explanation**
1.	15 + 12	27	✓	I knew what 10 add 10 is, and then 2 add 5 and added them together
2.	23 – 11	12	✓	I took 1 from the 3 and 10 from the 20
3.	33 + 54	87	✓	I added 30 to the 50 and 3 to the 4
4.	42 – 23	19	✓	I done 20 take 42 and I which is 20 and took another 1
5.	55 + 37	92	✓	I done 30 add which is 80 and than 5 add 7 which is 12 and added them together
6.	68 – 32	36	✓	I took 30 from the 60 and 2 from the 8
7.	57 + 14	71	✓	I done 4 add 7 which is 11 added it to the 50 add 10 which is 60
8.	58 – 19	39	✓	I done 20 take 58 and added 1
9.	27 + 69	96	✓	I done 70 add 27 and took 1
10.	64 – 27	37	✓	I done 27 add 3 equals 30 and then I took 30 from 64 and added 3
				% Correct = 100%

School/Pupil: H1m HA Strategy Stage = S6, see Q8.

No	Qu	Ans	✓✗	Explanation
1.	15 + 12	27	✓	Added the two tens and added the 5 and 2
2.	23 – 11	9	✓	Take 10 away from 20, then take 1 away from 13 leaves 12
3.	33 + 54	87	✓	Add the 50 and 30 together, then add 4 and 3 together then add them together
4.	42 – 23	19	✓	Take the 20 away from the 40, take 32 away from the 2 which you cannot do, so you take it away from 22 which leaves 19
5.	55 + 37	92	✓	50 add 30 is 80, 5 add 7 is 12, then add them together
6.	68 – 32	36	✓	Take 30 away from 60 then take 2 away from 8
7.	57 + 14	71	✓	Add 10 to 50 is 60, then take 2 away from 8
8.	58 – 19	39	✓	Take 10 away from 58, then take 9 away from 48
9.	27 + 69	96	✓	Add 20 and 6(0) equals 80 add 9 and 7 is 16
10.	64 – 27	33	✗	Take 20 from 40, take 7 from 44

% Correct = 90%

School/Pupil: H4m HA Strategy Stage = S5, see Q3.

No	Qu	Ans	✓✗	Explanation
1.	15 + 12	27	✓	I had 15 added on 10 to make 25 then I added on 2
2.	23 – 11	12	✓	I had 32 take away 11, I took away 10 which made 13, and then 1 which made 12
3.	33 + 54	87	✓	I added 30 to 50 which made 80 and I then added 3 to 4 which made 7, added them together
4.	42 – 23	-	✗	Unable to respond
5.	55 + 37	92	✓	I got 3 and added 5, put 3 on which made 80 and I then added 3 to 4 which made 7, added them together
6.	68 – 32	16	✗	Put 3 on to 6, look at 3, take off which made, got 2, took 2 off which made 6
7.	57 + 14	71	✓	Put 10 on to 50 which made 60, put 4 on to 7 which made 11, then put 10 on to 60 which made 70, then put 1 onto units which made 61
8.	58 – 19	-	✗	Unable to respond
9.	27 + 69	96	✓	Put 60 on to 20 which made 80, put on 7 which made 26, put 10 on 80 which made 90, put 6 on units column which made 96
10.	64 – 27	-	✓	Unable to respond
				% Correct = 60%

School/Pupil: H5f HA Strategy Stage = S5, see Q7.

No	Qu	Ans	✓✗	Explanation
1.	15 + 12	27	✓	I added 5 and 2, and I then added the tens, than added them together
2.	23 – 11	13	✗	I took away 11 from 23 (shows counting)
3.	33 + 54	87	✓	I added 30 and 50 together, I then added the 3 and 4 together, then added them together
4.	42 – 23	19	✓	I looked at 20 and 40, took 20 from 40, added them together, took 30 from 20 and added together
5.	55 + 37	92	✓	I added the 50 and 30 which made 80, I added the 5 and the 7 which made 12, I then added 10 to the 80
6.	68 – 32	36	✓	Took 30 away from 60 which gave me 30 and took 2 away from 8 which made 6
7.	57 + 14	71	✓	I added the 50 and the 10 which gave me 60, I added the 7 and the 4 which gave me 11, and added them together
8.	58 – 19	49	✗	Took 19 away from 59
9.	27 + 69	96	✓	I added the 20 and the 60, I then added the 7 and the 9
10.	64 – 27	31	✗	Took 20 away from 60, I then took 4 from 7
				% Correct = 70%

School/Pupil: H17f LA Strategy Stage = S4, see Q2.

No	Qu	Ans	✓✗	Explanation
1.	15 + 12	27	✓	Add the tens up that makes 20, add the 5 and the 2 to make 7, add them together
2.	23 – 11	12	✓	Take away the 20 and the 10, take away the 3 and the 1, then add
3.	33 + 54	87	✓	Add the 3 and the 4 which make 7, then add the 5 and the 3 which make 8, then add them together
4.	42 – 23	21	✓	Take the 2 from the 4 which make 2 and take the 2 from the 3 which makes 1
5.	55 + 37	92	✓	Add the 5 and the 7 which makes 12, then carry the 1 on to the 5 which makes 6 then add the 3 which makes 9, then add them together
6.	68 – 32	36	✓	Take 3 from 6 which makes 3, 2 from 8 which makes 6
7.	57 + 14	70	✗	4 and 7 make 10, so add that on to 5 which makes 6 and add the 1 which makes 70
8.	58 – 19	41	✗	Take 1 from 5 which makes 4 and 8 from 9 which makes 1, gives 41
9.	27 + 69	63	✗	9 and 7 make 16, put that on to the 9 which makes 1, gives 41
10.	64 – 27	43	✗	4 from 7 makes 3, 2 from 6 which makes 4
				% Correct = 60%

School/Pupil: H10m LA **Strategy Stage = S5, see Q7.**

No	Qu	Ans	✓✗	Explanation
1.	15 + 12	27	✓	Add the 2 on to the 15 to make 17, add the 10 on to the 17 to make 27
2.	23 – 11	12	✓	Take 10 away from 20 that's 10, took 1 away from 3 that's 12
3.	33 + 54	87	✓	50 add 30 is 80, 4 add 3 is 7
4.	42 – 23	21	✗	40 take away 20 that's 20, 3 take away 2 that's 1
5.	55 + 37	92	✓	50 add 30 that's 80, 5 add 7 that's 12
6.	68 – 32	36	✓	60 take 30 that's 30 and 8 take 2 that's 6
7.	57 + 14	71	✓	50 add 10 is 60, 4 add 7 is 11
8.	58 – 19	39	✗	50 take 10 is 40, 9 take 8 is 1
9.	27 + 69	96	✓	20 add 60 that's 80, add 1 on to 9 that makes 90, add the 6 that's 96
10.	64 – 27	43	✓	60 take 20 is 40 and 7 take 4 is 3

% Correct = 80%

School/Pupil: H11m HA Strategy Stage = S6, see Q8.

No	Qu	Ans	✓✗	Explanation
1.	15 + 12	27	✓	I added both of the tens then both of the units
2.	23 – 11	12	✓	I took 1 from the 2 and the 1 from the 3
3.	33 + 54	87	✓	I added the units which is 7 and the tens which is 8
4.	42 – 23	19	✓	I took away 20 from 42 to give 22, then took away the 3 to give 19
5.	55 + 37	92	✓	I added the 50 to the 30 which is 80 and I added the 5 and the 7 which is 12 and added them together which is 92
6.	68 – 32	36	✓	I took 30 away from 60 and the 2 away from the 8
7.	57 + 14	71	✓	I added the 50 and the 10 and then the 7 and the 4, and added them together
8.	58 – 19	39	✓	I took away the 10 from the 58 which is 48 and took 9 away from 48 which is 39
9.	27 + 69	96	✓	I added 27 to the 60 which is 87 and added 9 to 87 which is 96
10.	64 – 27	37	✓	I took away 20 from the 64 which is 44 and took away 7 from 44 which is 37

% Correct = 100%

School/Pupil: H15m LA Strategy Stage = S5, see Q9.

No	Qu	Ans	✓✗	Explanation
1.	15 + 12	27	✓	10 add 10 is 20, 5 add 2 is 7, 20 add 7 is 27
2.	23 – 11	12	✓	I got 23, took away 10 that's 13 and I've got 1 left, so I took 1 away, that's 12
3.	33 + 54	87	✓	I done the 30 and the 50 is 80, 3 add 4 is 7, 80 add 7 is 87
4.	42 – 23	15	✗	I had the 40 and the 20, so I took that away is 20, then I did the 3 and 2 is 5, so I took it away from 20, that's 15
5.	55 + 37	92	✓	I done the 50 and the 30 that's 80, then I done the 5 and the 7 that's 12, and 80 and 12 is 92
6.	68 – 32	36	✓	60 and 30 is 30, 30 from 60 is 30, 2 from 8 is 6 that gives 34
7.	57 + 14	71	✓	50 and 10 is 60, 7 and 4 is 11, then I add 60 and 11 is 71
8.	58 – 19	33	✗	I done the 5 and take away 1 is 50, take away 10 is 40, then the 8 and the 7 added together was 17, then I done took away was 33
9.	27 + 69	96	✓	I done the 20 and the 60 is 80, then I done the 7 and 9 is 16, and 80 and 16 is 96
10.	64 – 27	37	✓	20 take away 60 is 40, 4 add 7 is 11, then I took that away is 37

% Correct = 80%

School/Pupil: H17f LA Strategy Stage = S4, see Q2.

No	Qu	Ans	✓✗	Explanation
1.	15 + 12	27	✓	Add the tens up that makes 20, add the 5 and the 2 to make 7, add them together
2.	23 – 11	12	✓	Take away the 20 and the 10, take away the 3 and the 1, then add
3.	33 + 54	87	✓	Add the 3 and the 4 which make 7, then add the 5 and the 3 which make 8, then add them together
4.	42 – 23	21	✓	Take the 2 from the 4 which make 2 and take the 2 from the 3 which makes 1
5.	55 + 37	92	✓	Add the 5 and the 7 which makes 12, then carry the 1 on to the 5 which makes 6 then add the 3 which makes 9, then add them together
6.	68 – 32	36	✓	Take 3 from 6 which makes 3, 2 from 8 which makes 6
7.	57 + 14	70	✗	4 and 7 make 10, so add that on to 5 which makes 6 and add the 1 which makes 70
8.	58 – 19	41	✗	Take 1 from 5 which makes 4 and 8 from 9 which makes 1, gives 41
9.	27 + 69	63	✗	9 and 7 make 16, put that on to the 9 which makes 1, gives 41
10.	64 – 27	43	✗	4 from 7 makes 3, 2 from 6 which makes 4

% Correct = 60%

Appendix 8: Index of Pupil Interviews

Pupil	Strategy	Page	Pupil	Strategy	Page
A1f	S3	227	A17f	S6	234
A5f	S5	228	A18m	S1	235
A8m	S6	229	B4m	S4	239
A10m	S6	230	B5f	S4	240
A12m	S7	231	C8f	S4	248
A13m	S5	232	C16f	S5	249
A15m	S3	233	C18f	S5	250
A19f	S5	236	C20f	S5	251
A22m	S8	237	D8f	S5	253
A25f	S7	238	D15m	S5	254
B9m	S6	241	D20m	S4	255
B12m	S6	242	E8m	S3	258
B13f	S5	243	E22m	S6	260
B16f	S4	244	E23m	S6	261
B17f	S2	245	F4f	S2	262
B21m	S5	246	F5m	S6	263
B23m	S7	247	F7m	S5	264
D4f	S7	252	F13f	S6	266
D24f	S6	256	F17m	S4	267
E6f	S6	257	F18m	S5	268
E14m	S6	259	F19f	S5	269
F9f	S6	265	G2m	S3	271
G1m	S6	270	G3m	S6	272
G13m	S6	277	G4m	S4	273
G14m	S6	278	G5m	S5	274
G24f	S7	280	G9m	S7	275
G25f	S5	281	G10f	S3	276
G28m	S6	282	G20f	S3	279
H10m	S5	286	H1m	S6	283
H15m	S5	288	H4m	S5	284
H17f	S4	289	H5f	S5	285
			H11m	S6	287