

Ordinal Optimization Technique for Three Phase Distribution Network State Estimation Including Discrete Variables

Sara Nanchian, *Member, IEEE*, Ankur Majumdar, *Member, IEEE*, Bikash C. Pal, *Fellow, IEEE*

Abstract—This paper has discussed transformer tap position estimation with continuous and discrete variables in the context of three phase distribution state estimation (SE). Ordinal optimization (OO) technique has been applied to estimate the transformer tap position for the first time in unbalanced three phase distribution network model. The results on 129 bus system model have demonstrated that OO method can generate a reliable estimate for transformer exact tap position with discrete variables in distribution system state estimation (DSSE) and also in short period of time. In this paper the node voltages and power losses are calculated for 129 bus network. It is also demonstrated that OO is much faster than other accurate methods such HPSO. The losses obtained with OO are much accurate. In view of this OO performs better than WLS as it provides higher accuracy of the loss calculation. In a distribution network where about 5-6% of electricity generated is lost, accurate estimation of this loss has significant technical and commercial value. The authors believe the technique proposed will help realize those benefits.

Index Terms— Three phase state estimation, Tap estimation, Ordinal optimization (OO).

I. INTRODUCTION

Driven by the requirement to accommodate for generation from renewables the distribution network is now required to be more active. In active distribution network the major focus is on network voltage and power flow control [1]. State estimation in power distribution is at the core of computation as all the other network control functions depend on this. The intermittency associated with these new forms of generation has made the voltage and power flow control a significant challenge [1]. Switchable or controllable shunt capacitor and reactor bank, tapped line drop compensator, on load tap changer (OLTC) in primary distribution substation are the existing options for voltage and power flow control. In the current operating situation these devices need to optimally coordinate with fast power electronics based voltage control features available with DGs. This requires the visibility of the entire network at the substation. The state estimation (SE) provides this.

This work was supported by EP/F037686/1, Power Network Research Academy by EPSRC, UK, and Scottish and Southern Energy Networks, U.K. Data supporting this publication can be obtained on request from cap-publications@imperial.ac.uk

SE is a very routinely performed in supervisory control and data acquisition (SCADA) system used for power transmission control. The transmission system is assumed balanced so the consideration of a positive sequence network data is enough for the state estimation purpose. In contrast to transmission system, the distribution segment of the , network is very unbalanced due to the presence of commonly existing single and two-phase laterals, untransposed three-phase circuits conductors and unbalanced loads. Inevitably an appropriate three-phase distribution system state estimator (DSSE) would be required to correctly obtain the states of the system [2]. The distribution part of the network has the problem of very severe voltage fluctuation owing to the variation of the outputs from the DGs. The in feed from the DG being not predictable the voltage fluctuation in the network is very significant. Naturally the control of the voltage is as important as the power flow control in the feeder. In view of such increasing power and voltage control requirement, it is important that state estimation include transformer OLTC and capacitor/reactor bank positions as discrete variables to provide effective voltage control command.

A very decent volume of literatures exist in three-phase state estimation in power distribution system. References [1], [3-5], have demonstrated the effect of various levels of unbalance in the network and topological uncertainties on the accuracy of the DSSE solution. Zero injection is modelled as nodal constraints with no measurements. The research in [6] and [7] have argued further the influence of the inaccuracy of the parameters and non-transposed lines, both on the accuracy of the estimated states and on the handling of bad data. This justified the need of synchronised phasor measurements at the distribution level.

A full three phase and linear approach has been recently reported by Houghton and Heydt [8] for DSSE. They assume that synchronised PMU based measurement and other smart measurements are available. The authors in [9] proposed graph theory for measurement placement to ascertain observability and then solve the nodal current injection equations to obtain voltage and angle. The technique is reported to produce robust

S. Nanchian, A. Majumdar and B. C. Pal are with the Department of Electrical and Electronic Engineering, Imperial College London SW72AZ, U.K. (e-mail: s.nanchian11@imperial.ac.uk; ankur.majumdar@imperial.ac.uk; b.pal@imperial.ac.uk)

estimation.

Interestingly these literature have not considered primary distribution transformer OLTC positions as variables for estimation. Such non-inclusion of transformer tap position from the formulation of state estimation motivated us to develop a three-phase state estimation algorithm including the tap as discrete variable to estimate the exact tap position which is not possible by the conventional weighted least square (WLS) technique.

OLTC in the primary substation is very active in effecting voltage control. So it is very important to include tap position as an estimated variable. There has not been much technical publication in tap observability detection. In [10], the authors proposed a robust algorithm based on Givens rotation of the gain matrix to estimate the state and tap position with erroneous zero injection. The method is reported to work well in Brazilian system models of varying complexity. The authors in [11] treated node voltages, angles and tap position as state variables where taps are treated as continuous variables. In reality the OLTC has discrete tap positions. Any inaccurate estimate produces less accurate computed values of other network quantities such as line flow and losses. So, the incorporation of the discrete tap variables in SE is very much required in order that SE drives efficient network operation and control.

WLS problem [12] is standard formulation in state estimation. The transformer tap positions when included in the formation, make the problem a mixed integer nonlinear one. So the solution through the normal equation framework is impossible because the objective function is not differentiable [13].

Authors in [14] and [15] have approximated continuous taps instead of discrete. The rounding technique or sensitivity approach have been taken to handle the complexity of discrete values. Nevertheless this approximation impacts the accuracy of the DSSE output at times estimating inaccurate tap positions. So, the estimation of the transformer tap positions has to rely on the solution of mixed integer non-linear optimization problem containing discrete and continuous variables. [16], [17].

The engineering application of heuristic algorithms of the like neural networks (NN), genetic algorithms (GA), honey bee mating optimization (HBMO), and particle swarm optimization (PSO) have increased off late.. The computational challenge of such complex optimization problem has been overcome through successful adoption of these tools in a wide range of optimization problems where both continuous and integer variables in the objective function are handled well in the absence of the requirement of differentiability [18]. Author in [19] used hybrid PSO (HPSO) to state estimation. But it has been applied in balanced state estimation problem. More recently published research [20] has applied the HPSO method considering an unbalanced three phase distribution network model while including discrete tap as an additional state variables [20]. However, like all other methods based on heuristic algorithm, the HPSO method takes unacceptably longer solution time (tens of minutes to hours) for a reasonably sized network model. This apparently weakens the case for

application of these heuristic methods in computation and control of smart power distribution grid.

References [16] and [21] considered ordinal optimization (OO) and Mixed-integer Quadratic Programming on a balanced three-phase network model without considering the unbalanced nature of the distribution system.

Our research proposes a full three-phase state estimator including unbalanced load network model and the discrete taps as additional estimated variables based on OO optimization technique in order to estimate the discrete value of the transformer taps and obtain more accurate value of the losses in the network.

The idea for application of OO theory in DSSE problem is to limit the search space to a good subset of search space through limited sampling [22]. This will reduce the solution time. The method helps to find a good enough solution with high probability instead of the best in solving problems with vast search space. This is the case in finding the optimal solution for DSSE problem with continuous and discrete variables combined. The performance of the proposed method has been tested on an extended version of IEEE 123-bus test system model and results are very encouraging for its potential application in real time operation.

The rest of the paper is organized as follows. Section II provides an overview of state estimation in power distribution system. A general overview of OO solution method is described in Section II. Section IV presents the OO method in the context of three -phase unbalanced state estimation for power distribution system model. Section V provides the results, performance analysis and comparison with other techniques. Also it includes detailed discussions on the value of the contribution and its usefulness for the distribution network operators. The paper is concluded in Section VI.

II. MATHEMATICAL FORMULATION

The SE with continuous and discrete variables is set up as the minimization of the following objective including the set equality and inequality constraints. The aim is to obtain the bus voltage magnitudes, angles and tap positions that seek to minimize the values of the objective function i.e. the difference between the measured quantity and their functional relationship with the estimated states. It is given as:

$$\min J(x) = \frac{1}{2} \sum_{i=1}^m w_{ii} r_i^2 \quad (1)$$

Or

$$\min J(x) = \frac{1}{2} \sum_{i=1}^m [Z_i - h_i(x_c, x_d)] R_{ii}^{-1} [Z_i - h_i(x_c, x_d)] \quad (2)$$

$$\text{Subject to: } z_i = h_i(x_c, x_d) + r_i \quad i = 1, 2 \dots m \quad (3)$$

When equation (1) is used

$$c(x_c, x_d) = 0 \quad (4)$$

$$g_{min} < g(x_c, x_d) < g_{max} \quad (5)$$

Where,

x_c : Continuous state variables voltage magnitudes and angles.

x_d : Discrete state variables (transformer taps positions).

z_i : Measured value of i^{th} measurement.

m : Number of measurements

R_{ii} : Diagonal covariance matrix of measurement error

$h_i(x_c, x_d)$: i^{th} measurement as a function of state vector variables (continuous or discrete).

w_{ii} : Weighting factor of measurement variable i ,

r_i : i^{th} measurement error.

Where, in the three-phase unbalanced distribution system,

$$x_c = [\delta_2^{abc} \dots \delta_i^{abc} \dots V_1^{abc} \dots V_i^{abc}]^T$$

is the concatenation of three-phase voltage angles and magnitudes at all buses, where $\delta_i^{abc} = [\delta_i^a \delta_i^b \delta_i^c]^T$ represents the concatenation of vector of phase angles of all buses except the reference bus and $V_i^{abc} = [V_i^a V_i^b V_i^c]^T$ expresses the vector of voltage magnitudes and $x_d = [\dots t_i^{abc} \dots]^T$

Where $t_i^{abc} = [t_i^a t_i^b t_i^c]^T$ is the vector of all taps.

The equality constraints $c(x_c, x_d)=0$ are the set of equations modelling the virtual measurements. The details can be found in [20] and [23].

The inequality constraints ($g_{min} < g(x_c, x_d) < g_{max}$) are the set of continuous and discrete constraints representing the system operational and security limits. It captures the upper and lower limits for decision/control variables such as voltage, angle and tap positions.

Real measurements $h_i(x_c, x_d)$: are powers, bus voltages, and branch currents. The expressions in three-phase format are detailed in [20].

III. ORDINAL OPTIMIZATION (OO)

Ordinal optimization is a method to obtain an approximate solution to the problem of minimising or maximising an objective function over a large set of designs. The main idea behind OO is to reduce the search space to the good enough subset by limited sampling. Ordinal optimisation is based on two concepts stating that the optimisation of complex problems can be made much easier by order comparison and goal softening [22], [24].

1. Order comparison: It is easier to find order than value. For instance finding whether A is bigger than B is much easier than finding the exact value of A and B.

2. Goal softening: For many practical problems, it makes the problem easier and it is sufficient to find good enough solution with high probability instead of getting the best.

A. Application of OO to complex optimisation problems

The procedure for the practical application of OO to complex optimization problems is as follows [25]:

- Step 1. Uniformly sample N designs from θ to form θ^N .
- Step 2. Estimate the performance of the designs in θ^N using a crude and computationally fast model.
- Step 3. Reducing the size of search space to good enough subset, G , by sampling a small number of alternatives (top- g_d). In such a way that the reduced search space contain at least one solution inside the top $n\%$ with the probability level of P .
- Step 4. Evaluation of the objective function value for each sampled alternatives using an approximate but computationally efficient model.
- Step 5. The estimated top-s designs from an approximate model are compared via an exact model to form the selected subset, S .
- Step 6. OO theory ensures that S contains at least k truly good enough designs from among the true top- g_d samples.

Where

θ : search space of optimisation variables

θ^N : set of N chosen designs

N : number of designs uniformly chosen from θ

G : good enough subset, usually the true top- g_d designs in θ^N , which is a subset of the search space in which the members fulfil some of the design standards that are set out by the designer

S : selected subset, usually the estimated top-s designs in θ^N , which is a subset of search space in which the members are selected by using certain evaluation methods

$G \cap S$: set of truly good enough designs in S

k : alignment level which is the degree of matching between G and S

P : alignment probability = $Pr[|G \cap S| \geq k]$, the probability that there are actually k truly good enough designs in S .

IV. DISTRIBUTION SYSTEM STATE ESTIMATION BY ORDINAL OPTIMIZATION (OO)

The OO has been applied for the placement of meters in power distribution planning [26]. To our understanding it has not been tried to estimate states in unbalanced power networks with discrete variables such as OLTC and switched capacitor. This section describes the various steps we have devised to adopt the application of OO method in three-phase state estimation for distribution system in order to obtain the accurate estimate of transformer tap positions. The steps are as follows:

1. Network configuration & Network data

The first step is modelling of various network components of distribution system such as lines, transformers, switches and

load for DSSE. This is due to the presence of unsymmetrical-network components and unbalanced-load in distribution systems. Therefore, it is required to consider the exact model of the system components (three-phase model). It includes the information about the line resistance, reactance, tap setting connectivity information etc. The comprehensive three phase model of the various components of the network such as the distribution line, transformers, switches and load model can be found in references [27] and [28].

2. Select N designs in search space

N designs in the search space θ are chosen as follows. For instance the size of search space with m discrete numbers of transformer tap positions which can change between the steps of -16 and 16 would be 32^m . This is a large search space. In order to reduce the size of search space the conventional method of WLS state estimation should be applied while setting all the discrete values of the network as continuous variables [16]. Since all the variables are continuous the problem alters to the convex problem and the standard solution for WLS estimator can obtain the optimal solution for each state variables. The mathematical formulation of state estimation with only continuous values can be written as follows.

$$\min J(x) = \frac{1}{2} \sum_{i=1}^m [Z_i - h_i(x)] R_{ii}^{-1} [(Z_i - h_i(x))] \quad (6)$$

$$c(x_c, x_{cd}) = 0 \quad (7)$$

$$g_{\min} < g(x) < g_{\max} \quad (8)$$

Where x is concatenation of x_c and x_{cd} .

$$x = [x_c, x_{cd}]^T \quad (9)$$

and x_{cd} denotes the continuous tap ratios.

This problem is solved with the previous method of non-linear programming algorithm such as interior point method to obtain the estimated values of states variables. The corresponding continuous values of the discrete variables now will be set to either the nearest lower bound integer or upper bound integer value of that component. After estimating the continuous values of transformer tap positions and setting them to the lower bound or upper bound integer values, the size of N design will be obtained $N = 2^m$.

$$\bar{x}_{cdj} = (\bar{x}_{cj} \text{ or } \underline{x}_{cj}) \quad (10)$$

Where

\bar{x}_{cdj} : is the j^{th} discrete value of variable x_{cd} .

However, the number of these 2^m samples is still larger than the representative set N , which is $\cong 1000$ found by uniform selection. The method of minimum norm deviation is employed to further reduce the size of the representative set to around

1000. The weighted norm deviation given by $\|\hat{x}_{cdj} - \bar{x}_{cdj}/\hat{x}_{cdj}\|$ of each tap is calculated, where \hat{x}_{cd} is the solution of (5)-(7). The deviation $(\hat{x}_{cdj} - \bar{x}_{cdj}) = \hat{x}_{cdj} - \underline{x}_{cj}$ or $\bar{x}_{cj} - \hat{x}_{cdj}$ corresponding to the lower bound or upper bound. If this weighted norm deviation corresponding to any one of the bounds is less than the pre-assigned value val , then the transformer tap position is fixed to the corresponding bound. By this way, some K components are fixed such that $2^{m-K} = N$; $m - K = 10$.

3. Evaluate and order N designs

In order to evaluate N samples of designs an approximate but computationally efficient model is applied. At this step the sensitivity analysis of the discrete variables with respect to the objective function is applied [16], [29]. The first derivative of the objective function with respect to discrete variables for transformer taps has been calculated. Therefore, the value of the sensitivity function for different combinations of transformer tap positions in the system will be obtained. The following shows the equations relating to obtaining the sensitivity values for transformer tap positions.

$$J(x_c, x_d) = \frac{1}{2} \sum_{i=1}^m [(Z_i - h_i(x_c, x_d))^T] R_{ii}^{-1} [(Z_i - h_i(x_c, x_d))] \quad (11)$$

$$\frac{\partial J}{\partial T_j} = \sum_{i=1}^m [(Z_i - h_i(x_c, x_d)) \cdot R_{ii}^{-1} \cdot \frac{\partial h_i(x_c, x_d)}{\partial T_j}] \quad (12)$$

Where

$J(x_c, x_d)$: is the objective function

$\frac{\partial J}{\partial T_j}$: is the first derivative of objective function with respect to transformer taps

Consequently, the sensitivity function is specified by

$$\sum_{j=1}^{N_{\text{trans}}-k} T_j \cdot \frac{\partial J}{\partial T_j} = - \sum_{j=1}^{N_{\text{trans}}-k} \left[T_j \sum_{i=1}^m [(Z_i - h_i(x_c, x_d)) \cdot R_{ii}^{-1} \cdot \frac{\partial h_i(x_c, x_d)}{\partial T_j}] \right] \quad (13)$$

T_j : j^{th} component of the discrete value of transformer tap position.

4. Select the best subset

The absolute value of the sensitivity function is evaluated for the 2^{10} tap combinations and is organised in ascending order to identify the top- s samples set in the $N \cong 1000$ designs. The size of the top- s sample set, called the selected subset, depends on the number of alignments k ; the size of the good enough subset,

g_d ; the noise characteristic of modelling error; the shape of the ordered performance curve (OPC); and the size of the representative set and alignment probability. The top- s samples with smallest absolute sensitivity function values will form the selected subset [29].

$$s = e^{\alpha_0} k^\gamma g_d^{\beta_0} + \vartheta \quad (14)$$

Where, k is the alignment level, g_d is the number of good enough designs, ϑ is the noise factor and $\alpha_0, \beta_0, \gamma$, are coefficients whose values depend on the OPC curve and noise characteristics. The values for this case are given in the next section. The top- s samples with smallest absolute sensitivity function values will form the selected subset S [29].

5. Good enough solution

Each sample of selected subset S will be evaluated using the standard WLS objective function. Once the value of the objective function for each sample is obtained, the good enough solution will be the one (i.e. alignment level $k=1$) that produces the minimum value of the objective function [16].

V. RESULTS AND DISCUSSIONS

A. Case study

The proposed OO approach was programmed in MATLAB for DSSE. It has been tested on 129 bus distribution network model (extended model of IEEE-123 bus distribution network) and section of Scottish and Southern Energy Network (SSE) model to validate the proposed algorithms. It is required to have at least 10 tap changers for OO to be effective. So, for 129 bus test system, two additional transformers have been installed between nodes 26, 125 with 3 additional phases and one between 62, 119 with 2 additional phases. The system now consists of 129 buses and 123 lines and 11 transformer tap changers. The SSE network consists of 26 bus and 27 lines and 11 transformer tap changers.

The network parameters including load data for 129 bus system are obtained from [30] and [31]. The configuration of these model systems are displayed in Fig. 1 and Fig. 2. The 129 bus model system is configured and modelled as an unbalanced three phase system. There are both three-phase and single-phase loads. Three phase transformers are modelled as three individual single-phase units. The 26 bus SSE network is a balanced system as it does not have single phase or two phase laterals.

The power flow programme was used to generate the measurements. Normally distributed noise components were added represent measurement with noise. The error for real measurement was assumed 3% while the error for pseudo measurement (load data) was taken to be 20%.

A representative set of $N=1000$ designs samples was assumed, with alignment probability of $P=0.95$. It is assumed that the good enough subset consists of the top 5% of the representative

set and the number of alignments to be 1 as mentioned in Section IV. Taking into account the noise of modelling error to be very large and the worst OPC which has a lot of bad designs, according to equation (14), the coefficients are given as $\alpha_0 = 8.2995$, $g_d = 50$ (5% of 1000), $k=1$, $\gamma=1.3777$, $\beta_0 = -1.4986$, $\vartheta=8$. The values of the coefficients are explained by Lau and Ho in [25]. The value of s is chosen to be equal to 20.

The lower and upper limit of control variables correspond to the coding on the OO is set in such a way that the inequality constraints of the control variables are satisfied.

- $0.95 pu < V_i^a, V_i^b, V_i^c < 1.05 pu$
- $-30^\circ < \delta_i^a, \delta_i^b, \delta_i^c < +30^\circ$
- $0.9 < t_i^a, t_i^b, t_i^c < 1.1$

The transformer tap ratios are specified in the range of 0.9 to 1.1.

B. Results and comparison

Fig. 3 and Fig. 4 displays the true and estimated voltages for the 129 bus model network and the 26 bus SSE distribution network respectively. The value of voltages in Fig. 3 has been taken as 1.0 where there is no phase available for the given bus on the system diagram. Within the tolerable error range the results are found satisfactory. The transformer tap positions obtained through OO are shown in Table I and Table II for 129 buses and 26 bus system respectively. They are also compared with the results obtained through WLS rounding technique. A close look at the results reveals that continuous variables assumption in WLS based estimation result in an inaccurate tap positions while OO produces the exact position of transformer taps through discrete optimization. The voltage and angle estimate from OO are more accurate. Obviously they help to obtain more accurate calculated value of line and transformer loading and total losses. This is very useful in setting the price for efficient electricity market operation. It also helps for optimal voltage and var control scheduling.

The power losses in the lines for 129 node and 26 bus system models have been computed and the results are shown in Fig. 5 and 6. The real power losses for model 129 bus system obtained by WLS are 73.83, 17.16, 36.83 kW for three phases and the total is 127.82 kW. The respective values obtained by OO method are 67.17, 13.16, 31.2 kW and the total is 111.53 kW. The difference in total computed losses by the two methods are 16.3 kW. The corresponding value for the total active power losses obtained by WLS for 26 bus system is 1.255 MW and by OO method is 1.238 MW. The difference in total computed losses by the two methods are 17.4 kW. Such differences in losses by both methods are significant for the systems whose total capacity is 4879 kVA and 42.7 MVA.

It is clearly seen that OO based technique provides more accurate loss information. Since the customers are charged for the cost of the losses, the OO approach to estimate voltage and angles and tap positions when used to obtain the operational

losses for costing, will result in fairer electricity price. In the UK, 6% of the total power generated is lost in the system. A significant part of the losses is in distribution networks and power distribution transformers at different voltage levels. The annual cost of the network losses is about £1b pounds. The accurate estimation of losses clearly helps to develop tools that can be used to minimize the losses. . Southern Electric Power Distribution (SEPD), one distribution network operator (DNO) in the UK has already commissioned a work to adopt this method to accurately estimate the losses in their model network. The 26 node system results is an initial outcome of that activity. These results, discussions and insight justifies the novelty, value and benefit of this research contribution.

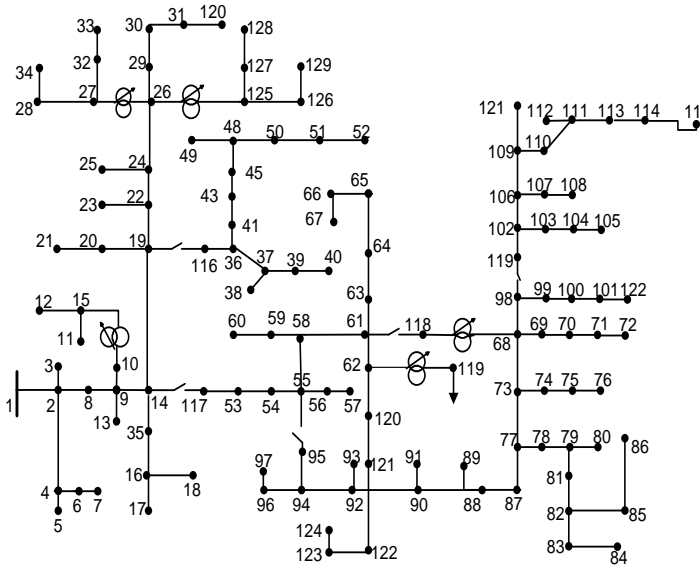


Fig 1. 129 bus distribution system

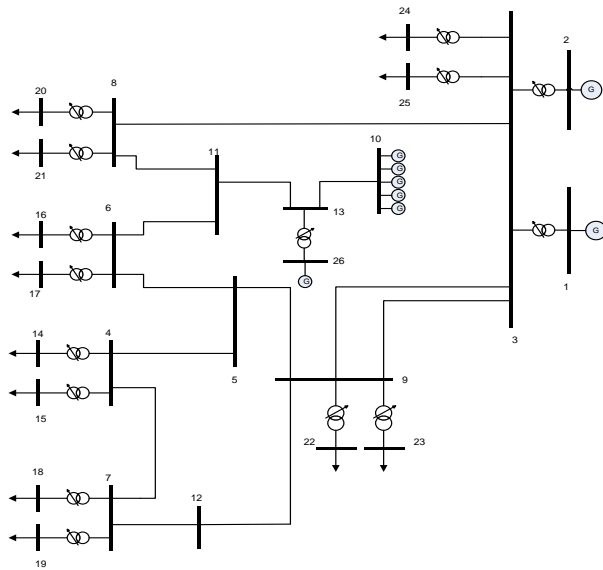


Fig 2. 26 bus distribution system

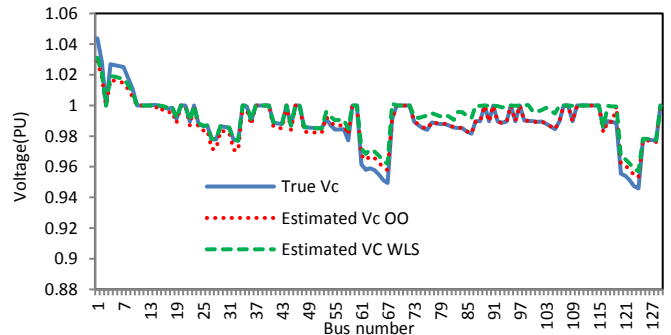
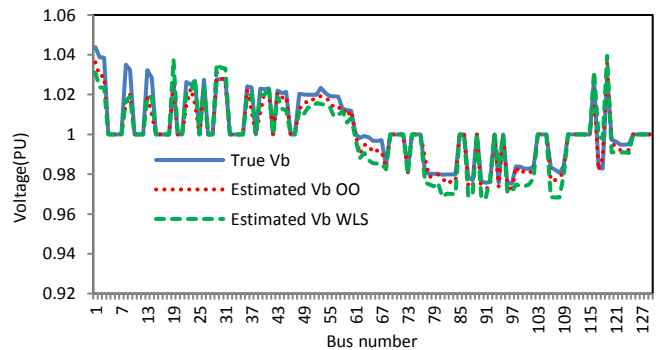
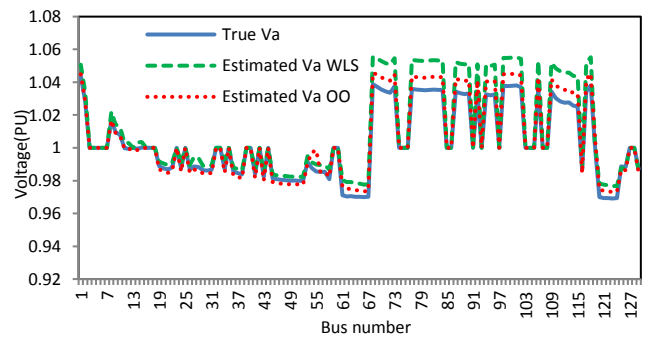


Fig 3. True & estimated V with WLS & OO for phase a,b,c for 129 bus system

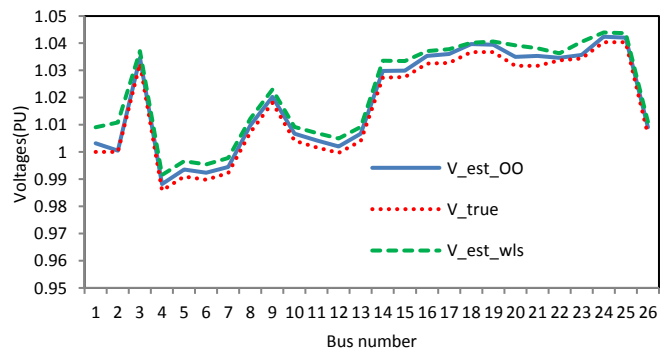


Fig 4. True & estimated V with WLS &OO for 26 bus system

TABLE I
TRUE & ESTIMATED TAP POSITION FOR 129 BUS SYSTEM BY OO & WLS

Trans	Phase	True	OO	WLS
T10-15	Phase a	1	1	1
T26-27	Phase a	0	0	0
T26-27	Phase c	1	1	1
T61-68	Phase a	-8	-8	-8
T61-68	Phase b	-1	-1	-1
T61-68	Phase c	-5	-5	-5
T62-119	Phase a	-5	-5	-4
T62-119	Phase b	-5	-5	-5
T62-119	Phase c	-5	-5	-4
T26-125	Phase a	0	0	0
T26-125	Phase c	1	1	1

TABLE II
TRUE & ESTIMATED TAP POSITION FOR 26 BUS SYSTEM BY OO & WLS

Trans	True	OO	WLS
T1-3	-3	-3	-3
T2-3	-3	-3	-3
T4-14	-4	-4	-4
T4-15	-4	-4	-4
T6-16	-5	-5	-5
T6-17	-5	-5	-5
T7-18	-4	-4	-4
T7-19	-4	-4	-4
T8-20	-3	-3	-3
T8-21	-3	-3	-3
T9-22	-3	-3	-4
T9-23	-3	-3	-4
T3-24	-2	-2	-2
T3-25	-2	-2	-2

The accurate solution of OLTC positions has been attempted in our earlier research [20] for 129 bus network model by a custom designed HPSO. The same method is applied to 129 bus unbalanced and 26 bus balanced distribution network models. The results obtained from HPSO in respect of tap position matches with that obtained by OO. The solution times are displayed in Table III. It is clearly seen that the OO is faster. Therefore, the ordinal optimization method has been compared with WLS method. Since OO method provides significant practical engineering advantages such as accurate estimates of transformer tap positions and losses when compared to WLS solution (with continuous taps) which will result in fairer value of electricity for customers. In case of large unbalanced network, it is important to have state estimation solved in seconds not in minutes. That way WLS is the best but at the cost of accuracy of the solution. However, after massive deployment

of smart meters in the system – the data transmission to the control centre is very slow- maximum at the rate of every few hours. The reference [32] supports this understanding. In this context OO method can be used for operational planning such as contingency analysis, loss estimation for price calculation rather than merely requiring it to drive real time network control.

Moreover, changing the tap position every few seconds will reduce the transformer operating life since the transformers tap changing mechanism requires to be changed after 30000 operations. One way to deal with this is to run WLS as main algorithm and in every few hours OO can run to obtain the correct tap positions.

TABLE III
SPEED OF SOLUTION BY HPSO AND OO

Method	129 bus unbalanced system	26 bus balanced system
HPSO	260 mins	20 mins
OO	10 mins	68 seconds

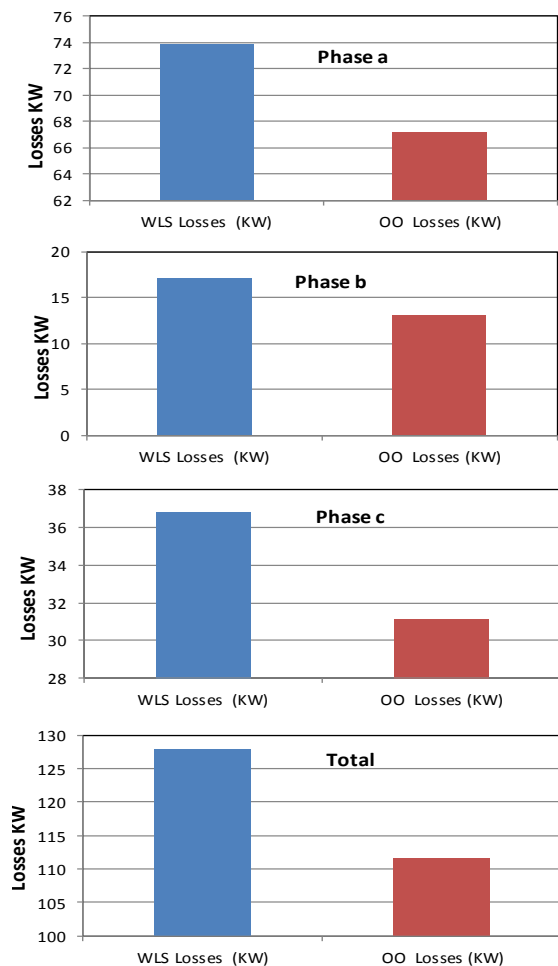


Fig 5. The estimated value of losses by OO and WLS for 129 bus system

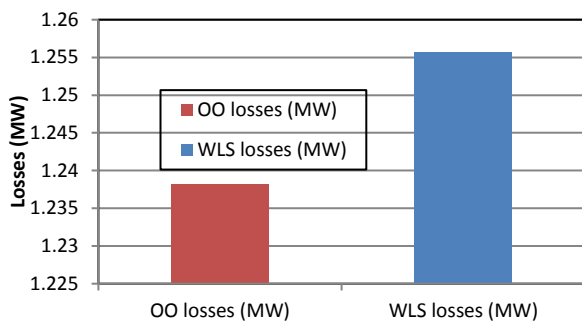


Fig 6. The estimated value of losses by OO and WLS for 26 bus system

VI. CONCLUSIONS

This paper discussed transformer taps estimation as discrete variables in the context of three phase distribution state estimation (SE). The OO technique has been applied to estimate the transformer tap position for the first time in unbalanced three phase distribution network. The simulation results on 129 bus and 26 bus distribution systems showed that the OO method can generate a reliable estimate for transformer taps with discrete variables in distribution network in short period of time. In this paper the node voltages and power losses are calculated for 129 bus and 26 bus model networks. It is also demonstrated that OO is much faster than other accurate methods such HPSO.

The calculated losses from OO are more reflective of the real situation than that obtained by WLS method. In this sense OO performs better over WLS. Accurate and lower losses bring the overall cost of electricity down. Even in a well-managed distribution network about 5% of generated power is lost. So the correct estimation of this loss has significant technical and commercial benefits. The technique proposed in this paper potentially help realize those benefits.

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Sara Nanchian (S'12-M'16) received her PhD in Electrical Power Engineering in March 2016 from Imperial College London and her M.Sc. degree in Electrical Power Engineering from University of Manchester, in 2008. Currently, she is working as a Research Associate at control and Power Research Group at Imperial College London. She is research active in smart distribution system planning, design, operation and control.

Ankur Majumdar (S'12-M'16) received his PhD in Electrical Power Engineering in June 2016 from Imperial College London and his B.E.E.(with honours) degree from Jadavpur University, Calcutta, India, in Electrical Engineering in 2009 and the M.E. degree from the Indian Institute of Technology, Delhi, India in Electric Power Systems in 2011. Currently, he is a Research Associate in the Department of Electrical and Electronic Engineering, Imperial College London. His research interests include state estimation, smart grid security and power system analysis.

Bikash C. Pal (M'00-SM'02-F'13) received the B.E.E. (with honours) degree from Jadavpur University, Calcutta, India, the M.E. degree from the Indian Institute of Science, Bangalore, India, and the Ph.D. degree from the Imperial College London, London, U.K, in 1990, 1992, and 1999, respectively, all in electrical engineering. Currently, he is a Professor in the Department of Electrical and Electronic Engineering, Imperial College London, London, U.K. His current research interests include state estimation, power system dynamics, and Flexible ac transmission system controllers. He is the Editor-in-Chief of IEEE Transactions on Sustainable Energy and a Fellow of the IEEE for his contribution to power system stability and control.