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ATTENUATION OF SOUND IN LINED DUCTS

by

Young-chung Cho
K. Uno Ingard

GTL Report No. 119 September 1974

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY
CAMBRIDGE, MASSACHUSETTS

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This research was carried out in the Gas Turbine Laboratory, M.I.T., supported by the Office of Noise Abatement, Department of Transportation, under Grant DOT-OS-30011.

PREFACE

This work on sound propagation in lined ducts was started under DOT Grant Agreement DOT-OS-30011 and continued under Supplement Agreement to the same grant. Some portions were reported in the first and second Interagency Symposia at Stanford University, March 1973, and at North Carolina State University, June 1974.

This Technical Report was monitored by Dr. Gordon Banerian, Office of Noise Abatement, Department of Transportation.

ABSTRACT

Extensive computations have been carried out of the attenuation characteristics of resonator and porous type duct liners in rectangular and circular ducts. First the frequency dependence of the attenuation constant and the phase velocity of the fundamental duct mode are obtained for a large number of duct and liner parameters. Then, assuming that the fundamental mode is dominant in the lined duct element, the octave band transmission losses have been computed. The effect of the shape of the input spectrum is discussed and shown explicitly for three different spectra, namely, a "flat" spectrum and spectra with slopes of + 6 dB per octave and - 6 dB per octave, respectively. Finally the effect of the length of the duct liner on the octave band transmission loss has been computed. It is found that the octave band transmission loss does not increase linearly with the length of the duct liner, particularly in regions where the attenuation varies strongly with frequency.

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1. INTRODUCTION

For the past two or three years much of the work in acoustics in the Gas Turbine Laboratory at M.I.T. under the supervision of Professor Uno Ingard has been focused on duct acoustics. A research group currently consisting of three graduate students (Vijay Singhal, William Patrick, and George Succi) and one postdoctoral fellow (Dr. Y.-C. Cho) is continuing this work, which has as the ultimate objective to provide a comprehensive account of the various practical aspects of duct acoustics that are of interest to the acoustical engineer. Numerous studies of duct acoustics have been carried out in the past, and many aspects of the field are now comparatively well understood. One of our tasks here is to use this understanding as a basis for extensive numerical computations of the acoustical duct characteristics.

There are also areas, particularly those related to the interaction of sound with duct flow, where further basic experimental and theoretical work is needed before reliable quantitative data and parametric presentation of duct performance can be provided, and our work deals also with these aspects of the problem.

In this report we present the results of one phase of the program. In this phase, which has been sponsored by

DOT under Grant Agreement DOT-OS-30011, we have carried out extensive computations of the attenuation characteristics of resonator and porous type duct liners in rectangular and circular ducts. The report is divided into four main parts. The first part deals with the frequency dependence of the attenuation constant and the phase velocity of the fundamental duct mode. In the second part we average over a finite frequency band and present values of the octave band attenuation characteristics of duct liners. Of particular interest in this connection is the effect of the shape of the input spectrum, and we demonstrate this effect by considering spectra with three different slopes, 0 dB, + 6 dB, and - 6 dB per octave. It is interesting to find that the effect of the shape of the noise spectra generally cannot be neglected even in engineering design and testing and specification of duct liner performance. In the third part we discuss a related effect, namely, the influence of the length of the duct liner on the octave band attenuation. Of considerable practical importance is the result that the octave band attenuation does not increase linearly with duct length. Actually, the attenuation increases surprisingly little with duct length in regions where the attenuation constant varies strongly with frequency (high and low frequencies), an effect that seems to have been overlooked by many practicing engineers in the field. The fourth part of the report contains the computer program.

The results presented here cover a wide range of duct parameter values that are encountered in practice. These parameters are presented in dimensionless form and are chosen as follows:

Frequency Parameter, kL

L = thickness of duct liner (depth of air cavity behind resistive sheet or thickness of uniform porous liner)

$k = \omega/c$, c = free space value of speed of sound

Duct Width Parameter, D/L

D = diameter of circular duct or separation of duct liners in a rectangular duct

Length Parameter, S/D

S = length of duct liner

Noise Spectrum Shape Slope, N

N = 0 dB, + 6 dB, and - 6 dB per octave

It should be emphasized that the results presented relate to the fundamental duct mode. The octave band attenuation of a finite duct liner is given in terms of a transmission loss, defined as the level difference between the incident sound and the transmitted sound. In calculating this quantity, a plane incident wave has been assumed, and we have accounted for the reflections from the beginning and the end of the duct liner. The "coupling" between the plane wave and the fundamental mode has been assumed frequency independent, and the possible error involved in this assumption has been estimated (see Appendix).

In view of the fact that we are calculating (octave band) attenuations for a finite duct length, the insertion loss of a duct liner rather than the transmission loss might be of interest. The insertion loss (defined as the change in the sound pressure level at a fixed location or as the change in the power level of the sound emerging from the duct), of course, depends on the characteristics of the sound source, as well as the location of the duct liner with respect to the source and the termination of the duct. Thus the number of parameters required to describe the insertion loss is quite large. For this reason we decided to carry out the parametric computations for the transmission loss rather than the insertion loss. However, we have indicated in the text how the insertion loss can be determined from the data presented in the report.

Another reason why we have chosen to compute the transmission loss is that this is the quantity that we generally measure in our experiments on duct attenuation, which will be described in a separate report.

2. ATTENUATION AND PHASE VELOCITY OF THE FUNDAMENTAL MODE IN LINED DUCTS

The attenuation and phase velocity of an acoustic mode in a duct is obtained from the real and imaginary parts of the propagation constant. Thus if the z axis is taken along the length of the duct and the spatial variation of the amplitude of a mode is given by $\exp(ik_z z) = \exp(-k_{zi} z)\exp(ik_{zr} z)$, where k_{zi} and k_{zr} are the imaginary and real parts of the propagation constant k_z, the spatial rate of decay in the duct is given by k_{zi}, which we call the attenuation constant. The corresponding attenuation in dB per unit length in the duct is then $(20 \log e)k_{zi} \cong 8.7\ k_{zi}$.

The phase velocity of the mode is given by ω/k_{zr}. The attenuation constant and the phase velocity depend on the frequency, the shape and dimensions of the duct cross section and upon the duct liner parameters.

The spatial variation $\exp(ik_z z)$ at a single frequency ω corresponds to the sound field in a (long) duct in which we need not be concerned with reflections from the end of the duct. When reflection from the end of the duct is present, the wave field contains also a reflected wave component with the spatial variation of the form $\exp(-ik_z z)$. If the fundamental mode contains a distribution of frequencies, the total pressure field will be an appropriate integral over the frequency range considered.

2.1 Rectangular Ducts

We consider first a rectangular duct with two opposite lined walls a distance 2b apart (see Figures 2.1 and 2.6). Then, if the incident wave is a plane wave, the wave function in the lined portion of the duct is symmetric with respect to the y coordinate. Thus the pressure field of the fundamental mode can be written as[1]

$$p = \int d\omega\, e^{-i\omega t} \cos(k_y y)\left[B_1(\omega)e^{ik_z z} + B_2(\omega)e^{-ik_z z}\right]. \qquad (2.1)$$

Here

$$k_y^2 + k_z^2 = k^2. \qquad (2.2)$$

($k = \omega/c$, c = free space sound speed.)

The k_y is the first root of the equation[2]

$$k_y b \tan k_y b + i\,\frac{kb}{\zeta} = 0, \qquad (2.3)$$

where ζ is the normalized boundary impedance at the lined duct wall. It follows from Eq. (2.2) that the propagation constant is

$$k_z = \sqrt{k^2 - k_y^2}. \qquad (2.4)$$

The phase velocity of the sound is $\omega/\mathrm{Re}(k_z)$, and the energy attenuation constant is $2\,\mathrm{Im}(k_z)$.

2.1.1 Resonator Liner. A "resonator" liner consisting of a resistive rigid screen with a partitioned air backing, as illustrated in Figure 2.1, has a normalized impedance ζ given by[3]

$$\zeta = \theta - i[kt' - \cot(kL)]. \tag{2.5}$$

Here θ is the dynamic acoustic resistance of the screen and kt' is the reactance accounting for the inertia of the air in the screen. For a fine-mesh rigid screen the reactance kt' generally is quite small, and we have ignored it in the computations described here. The results thus obtained for the real and imaginary parts of k_z are shown in Figures 2.2-2.5 as a function of the frequency parameter kL. It is interesting to note the variation of $Re(k_z)$ and $Im(k_z)$ in the vicinity of the resonance frequencies of the liner ($kL = \pi/2, 3\pi/2, 5\pi/2$, etc.), which is analogous to the phenomenon of anomalous dispersion of light in an optically "resonating" medium.[4] The attenuation maxima are well separated and the attenuation is zero at the "anti-resonance" frequencies ($kL = \pi, 2\pi, 3\pi$, etc.). The width of the attenuation bands increases with increasing θ. Note also that near $kL = 0$, $Re(k_z)/k$ asymptotically becomes a constant which is in general different from 1. In fact, the value is

$$\frac{k_z}{k} \underset{k \to 0}{\longrightarrow} \sqrt{1 + \frac{L}{b}}. \tag{2.6}$$

This follows because when $kL \simeq 0$, Eq. (2.3) becomes

$$(k_y b)^2 + (kL)^2 \frac{b}{L} = 0, \tag{2.7}$$

and on inserting Eq. (2.7) into Eq. (2.4), we get the result in Eq. (2.6). In other words, even in the long wavelength limit, the phase velocity of the sound in the lined duct is different from that in the rigid-walled duct. On the other hand, as the frequency becomes large, $Re(k_z)$ approaches k. The frequency dependence of the attenuation curve is much the same as for the normal absorption coefficient of the liner, at least when the wavelength is considerably larger than the duct dimension D, i.e., $kD < 2\pi$. At higher frequencies, at which the wavelength becomes smaller than D, the attenuation will decrease with frequency regardless of the behavior of the boundary.

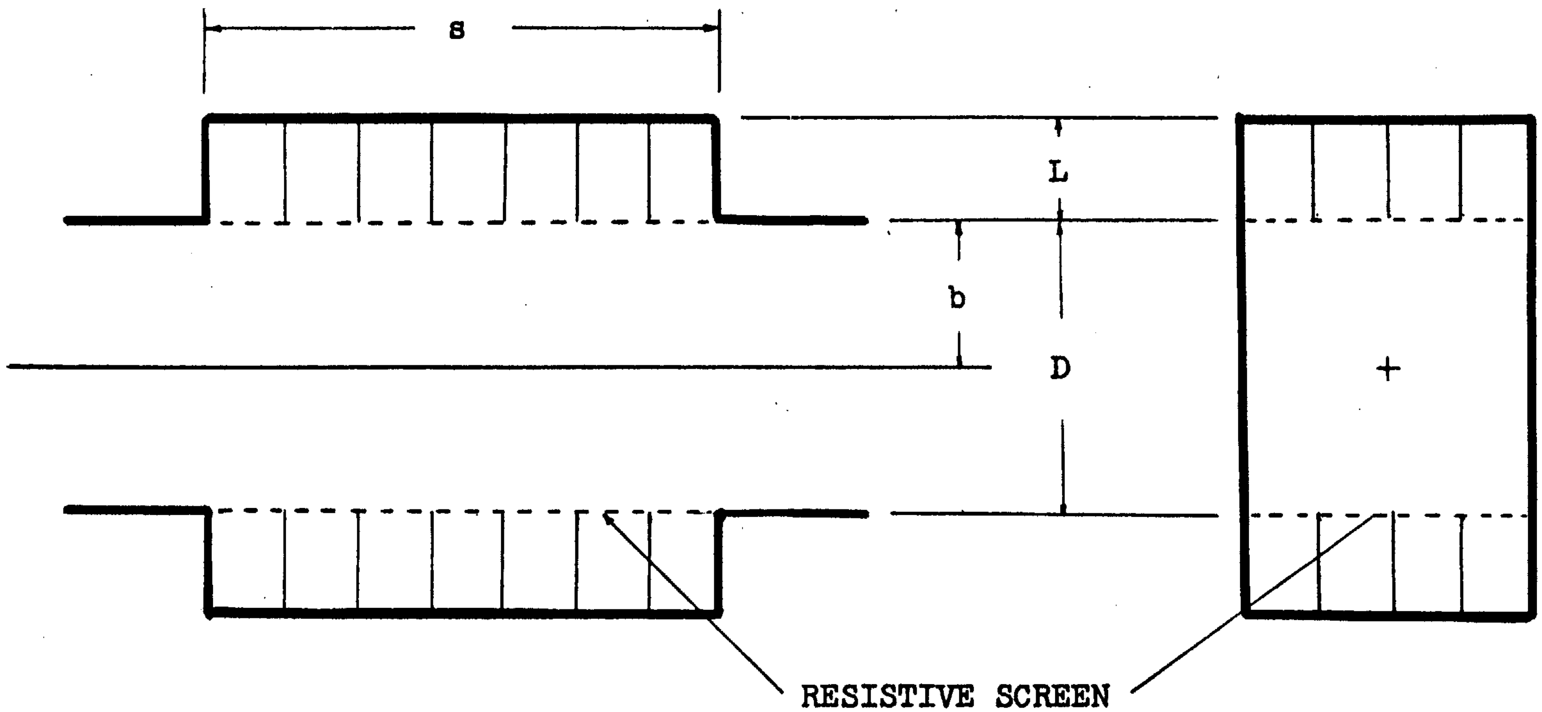


Figure 2.1. Rectangular duct lined with resonator liner and resistive screen.

Figures 2.2-2.5. Real and imaginary parts of the propagation constant k_z of the fundamental mode in a rectangular duct lined with a resistive screen type resonator lining. The real part of k_z is normalized by division by k. The imaginary part of k_z is presented in terms of $8.6859 \cdot Im(k_z) D$, which is the transmission loss in dB of a pure tone in a length D of the duct. Each figure corresponds to a different value of D/L. Each curve in a figure corresponds to a different value of the flow resistance Θ (in units of ρc) of the screen.

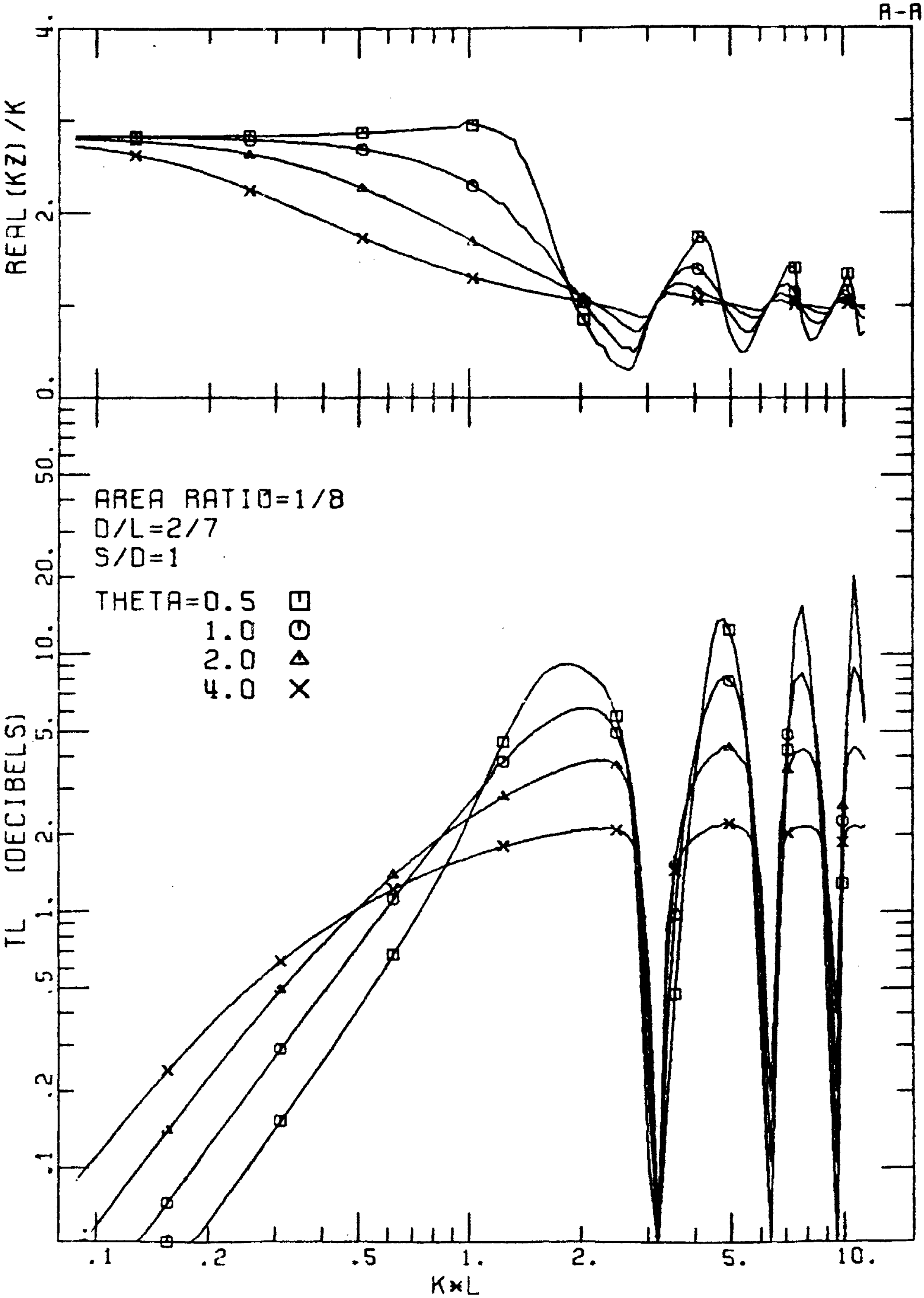

A-A
REAL (KZ) /K
4.
2.
0.
TL (DECIBELS)
50.
20.
10.
5.
2.
1.
.5
.2
.1
AREA RATIO=1/8
D/L=2/7
S/D=1
THETA=0.5
1.0
2.0
4.0
.1
.2
.5
1.
2.
5.
10.
K*L

Figure 2.2

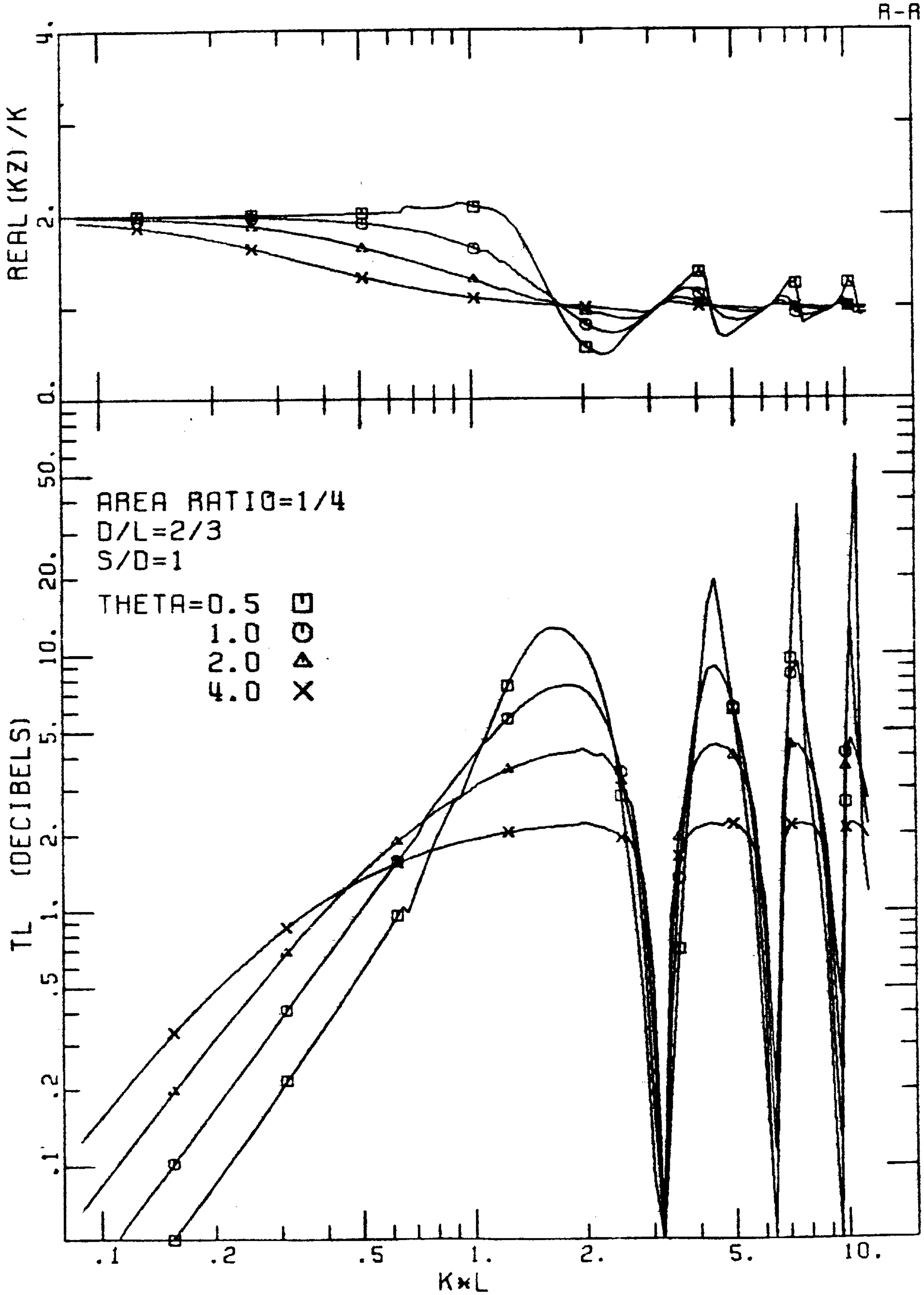

A-A
REAL (KZ) /K
TL (DECIBELS)
AREA RATIO=1/4
D/L=2/3
S/D=1
THETA=0.5
1.0
2.0
4.0
K*L

Figure 2.3

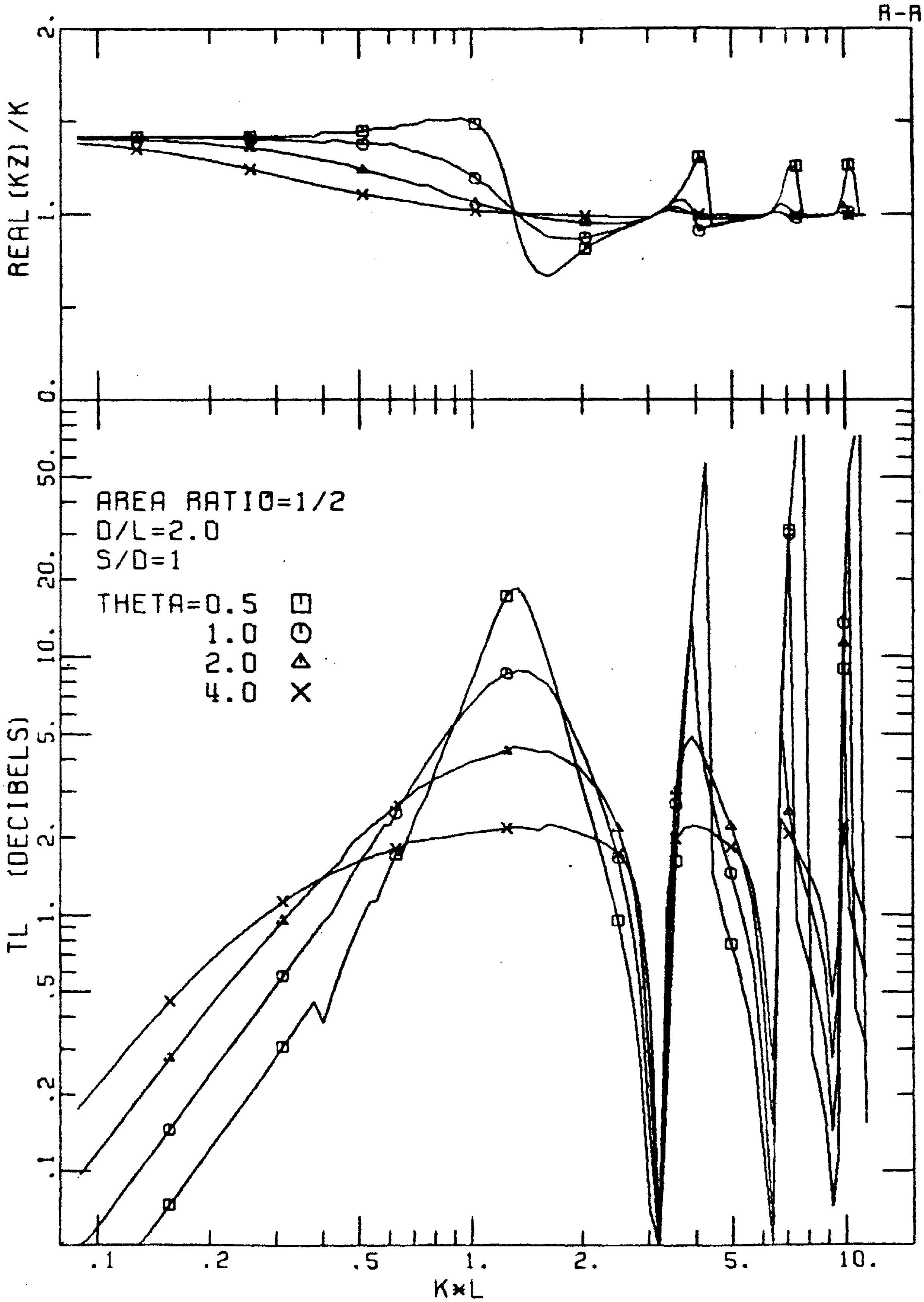

A-A
REAL (KZ) /K
2.
1.
0.
TL (DECIBELS)
50.
20.
10.
5.
2.
1.
.5
.2
.1
AREA RATIO=1/2
D/L=2.0
S/D=1
THETA=0.5
1.0
2.0
4.0
.1
.2
.5
1.
2.
5.
10.
K*L

Figure 2.4

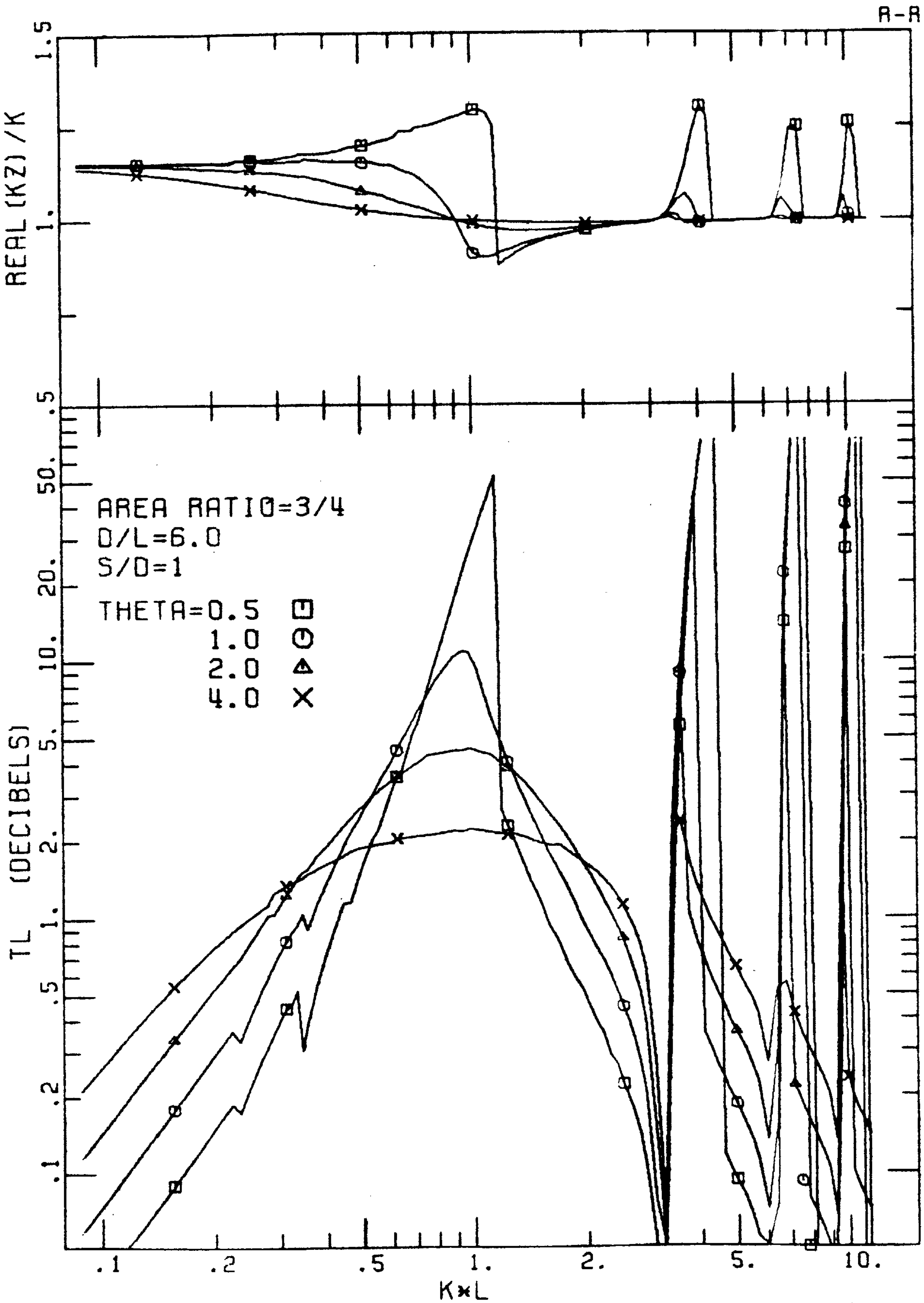

A-A
REAL (KZ) /K
1.5
1.
.5
AREA RATIO=3/4
D/L=6.0
S/D=1
THETA=0.5
1.0
2.0
4.0
TL (DECIBELS)
50.
20.
10.
5.
2.
1.
.5
.2
.1
.1
.2
.5
1.
2.
5.
10.
K*L

Figure 2.5

2.1.2 Porous Liner. On the assumption of a locally reacting liner, i.e., that the oscillatory flow in the porous boundary is normal to the porous liner, the liner impedance is[5]

$$\zeta = \frac{iq}{\Omega k} \cot(qL). \tag{2.8}$$

Here q is the wave number in the porous medium with the real and imaginary parts

$$\text{Re}(q) = k\sqrt{\frac{\gamma}{2}}\left\{\left(\sqrt{1 + \left(\frac{\theta}{kL}\right)^2} + 1\right)^{1/2}\right\} \tag{2.9}$$

$$\text{Im}(q) = k\sqrt{\frac{\gamma}{2}}\left\{\left(\sqrt{1 + \left(\frac{\theta}{kL}\right)^2} - 1\right)^{1/2}\right\}. \tag{2.10}$$

The resistance parameter is

$$\theta = \frac{\Omega \Phi L}{\gamma \rho c}, \tag{2.11}$$

where Ω is the porosity, Φ the dynamic flow resistance coefficient and γ the structure factor,[6] and in Figures 2.7-2.14 we have plotted the real and imaginary parts of k_z for a wide range of parameter values. When θ is small, the dispersion relation is similar to that obtained for a resonator liner.

If the resistance parameter is sufficiently large, say, larger than about $\theta = 1$, the attenuation increases monotonically with frequency in the range below a certain critical frequency at which the wavelength is about equal to the duct width. Above this frequency, the attenuation decreases rather sharply with frequency. This behavior at high frequencies is true for all liners, not only the porous one. However, for the resonator type liner, there is an additional limitation. For this liner the attenuation will decrease with frequency above the resonance frequency regardless of the duct width.

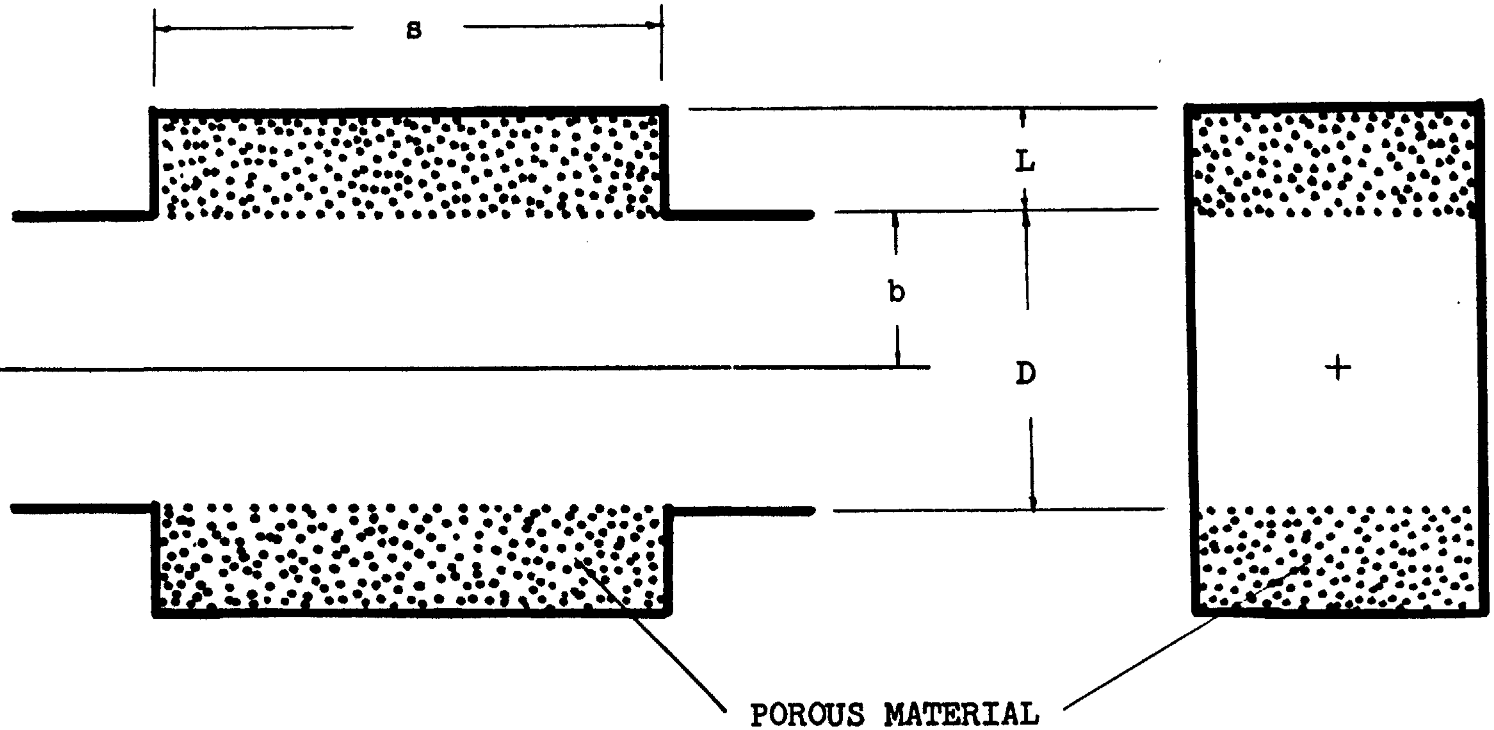


Figure 2.6. Rectangular duct lined with a porous liner.

Figures 2.7-2.14. Real and imaginary parts of k_z for rectangular ducts lined with a porous liner. The format is the same as in Figures 2.3-2.6. The resistance parameter θ is the total flow resistance of the porous layer. (See Eq. 2.11 for definition of θ.)

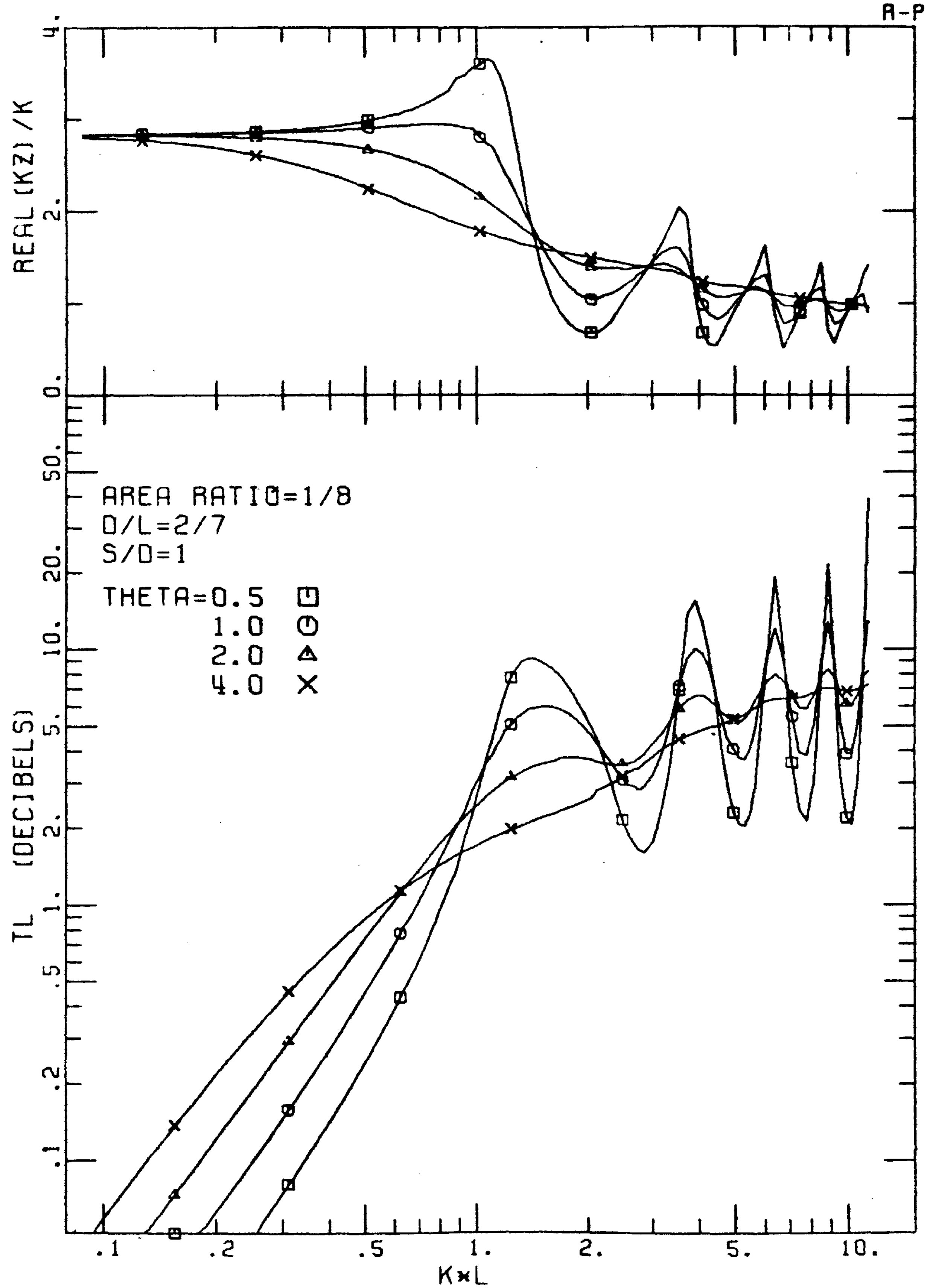
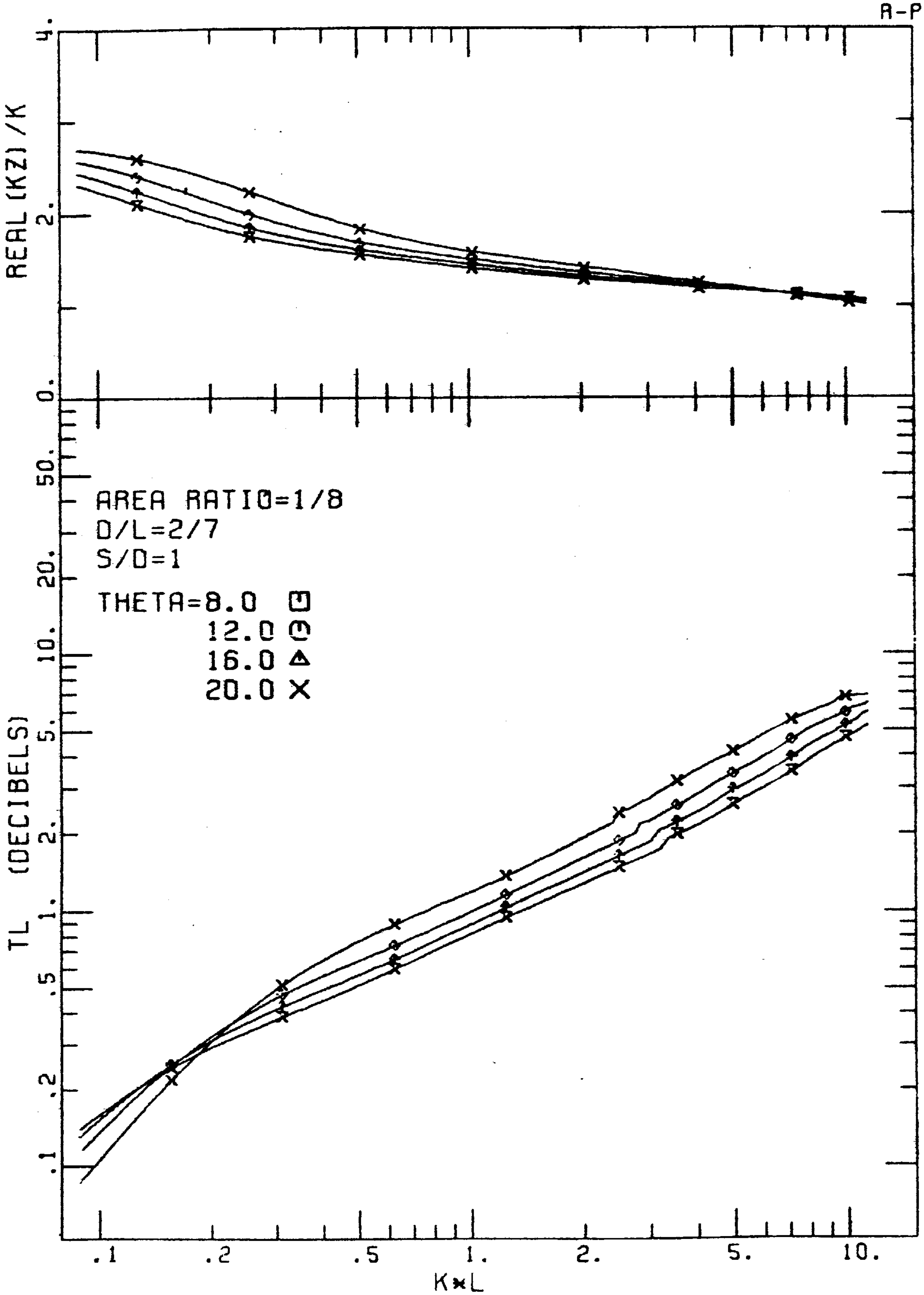

A-P
REAL(KZ)/K
4.
2.
0.
50.
20.
10.
5.
2.
1.
.5
.2
.1
TL (DECIBELS)
AREA RATIO=1/8
D/L=2/7
S/D=1
THETA=0.5
1.0
2.0
4.0
.1
.2
.5
1.
2.
5.
10.
K*L

Figure 2.7



A-P
REAL (KZ) /K
4.
2.
0.
TL (DECIBELS)
50.
20.
10.
5.
2.
1.
.5
.2
.1
AREA RATIO=1/8
D/L=2/7
S/D=1
THETA=8.0
12.0
16.0
20.0
.1
.2
.5
1.
2.
5.
10.
K*L

Figure 2.8

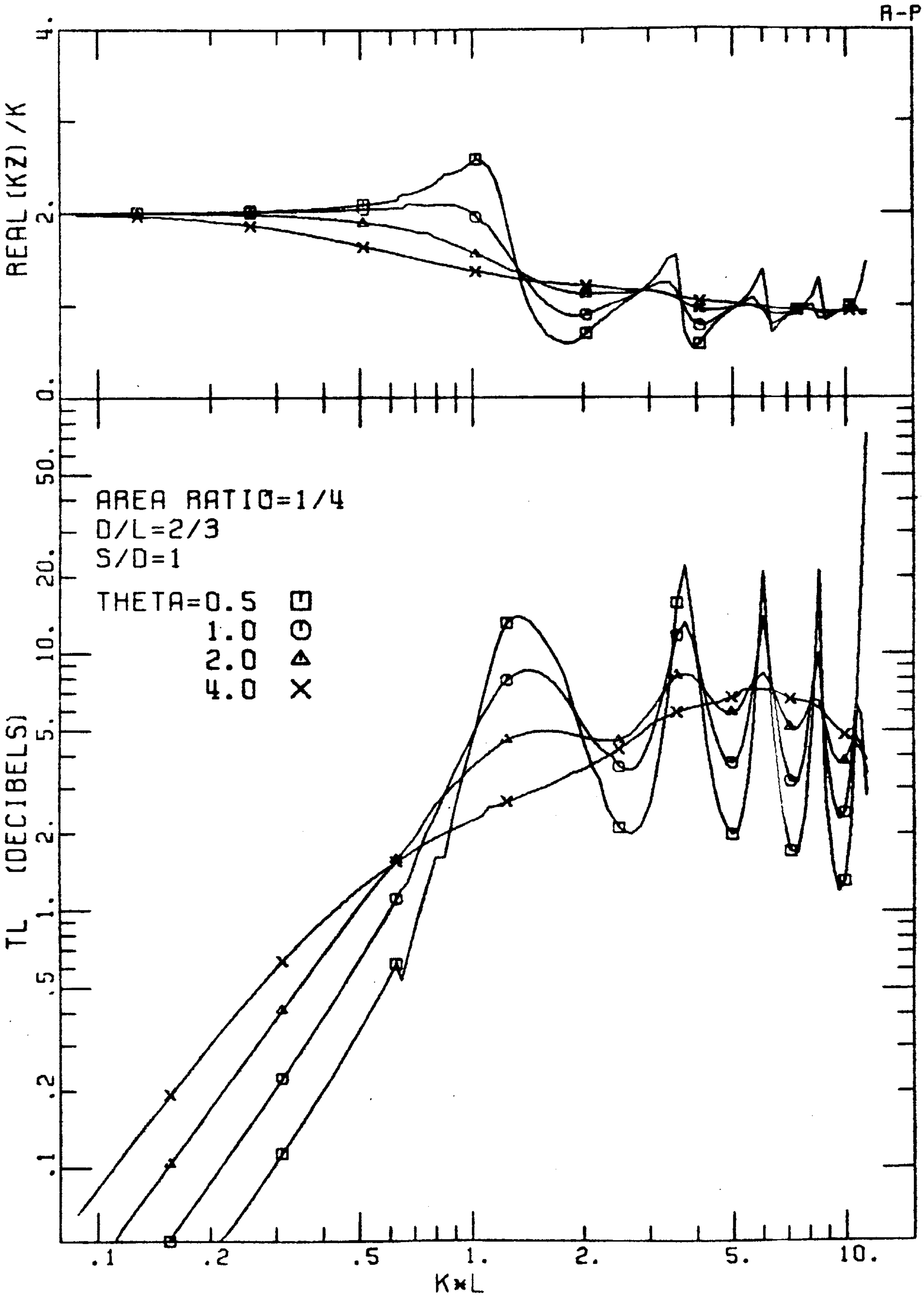
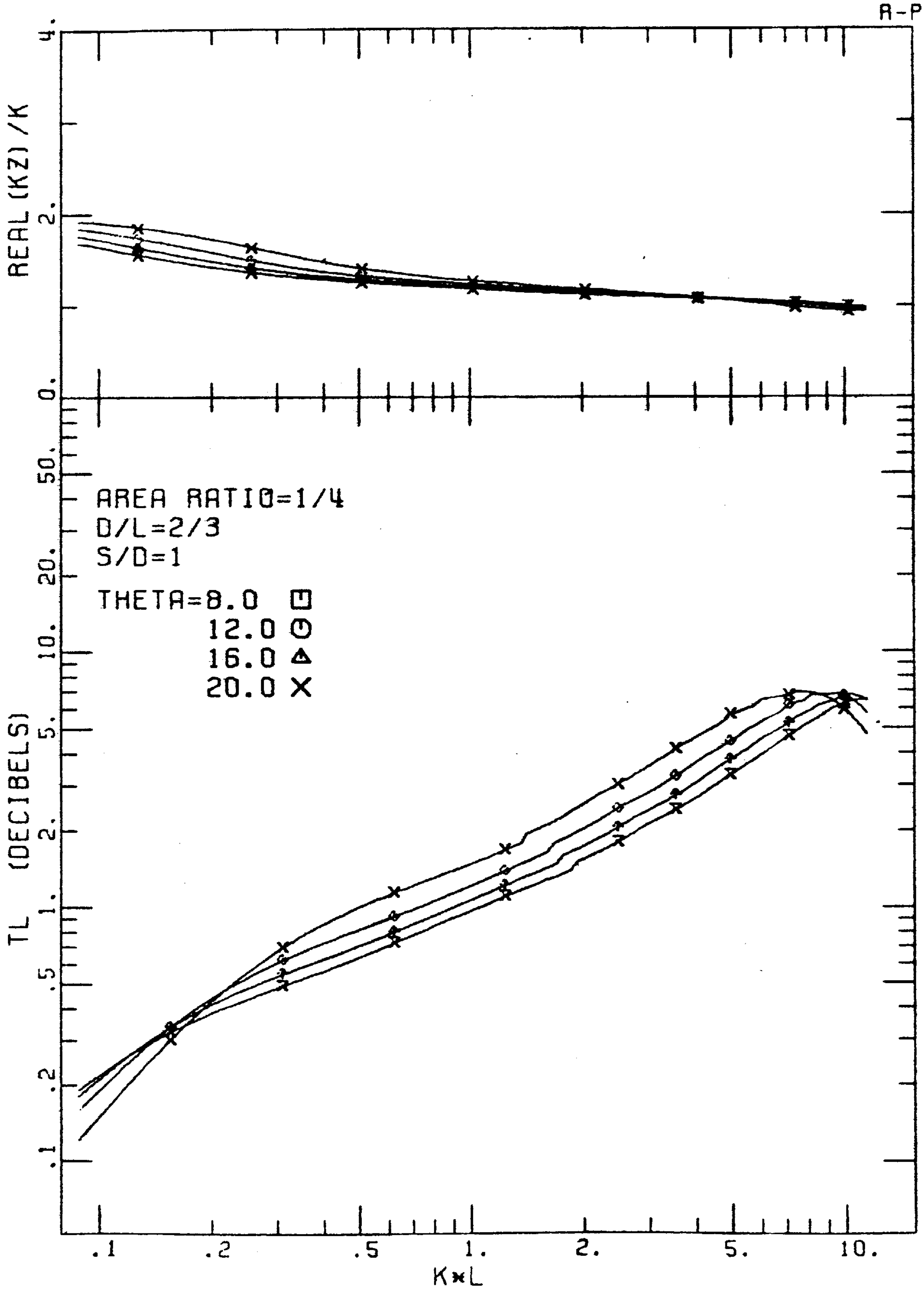

A-P
REAL (KZ) /K
TL (DECIBELS)
AREA RATIO=1/4
D/L=2/3
S/D=1
THETA=0.5
1.0
2.0
4.0
K*L
.1
.2
.5
1.
2.
5.
10.
4.
0.
50.
20.
10.
.1

Figure 2.9



A-P
REAL (KZ) /K
0.
2.
4.
AREA RATIO=1/4
D/L=2/3
S/D=1
THETA=8.0
12.0
16.0
20.0
TL (DECIBELS)
.1
.2
.5
1.
2.
5.
10.
20.
50.
K*L

Figure 2.10

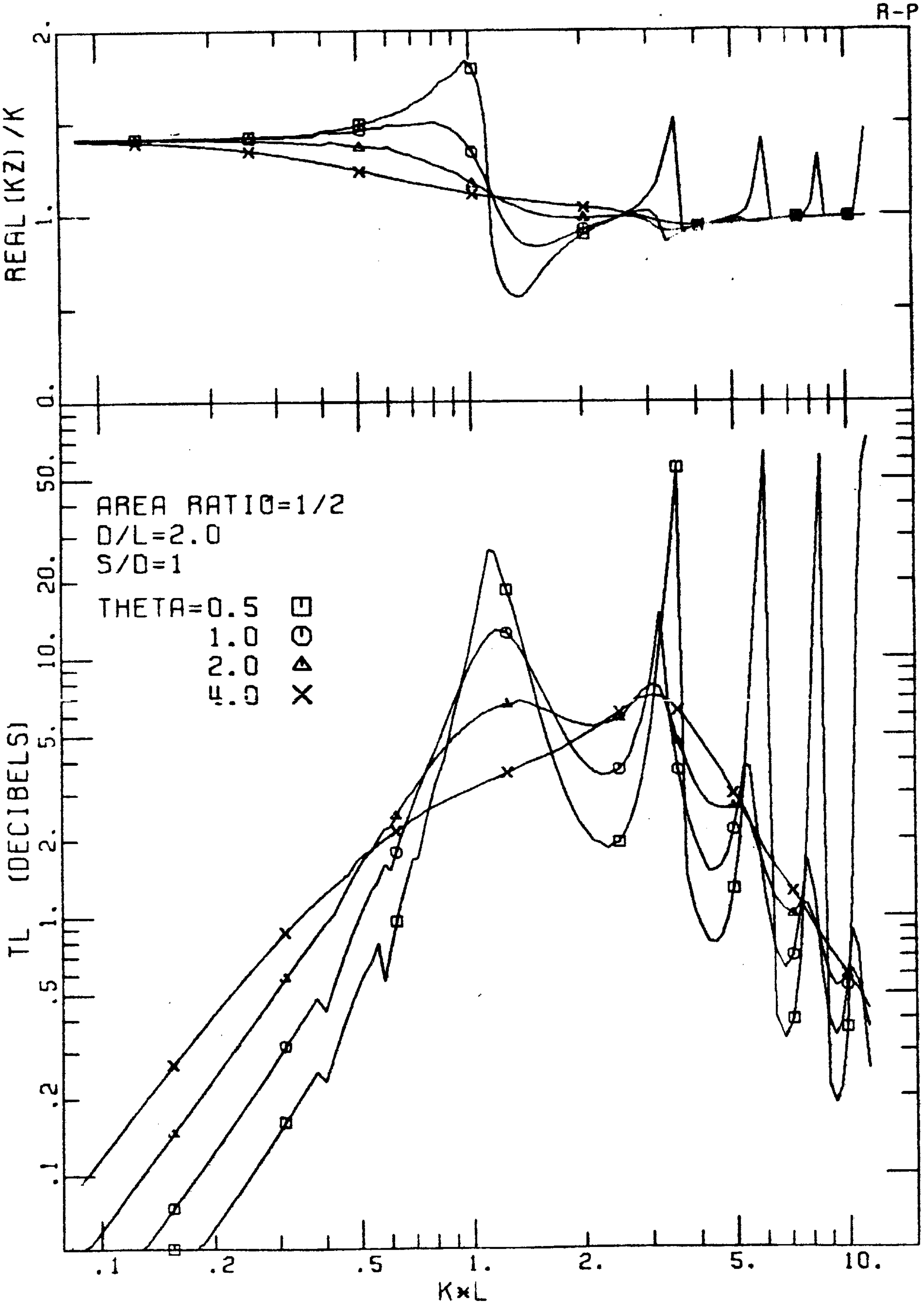
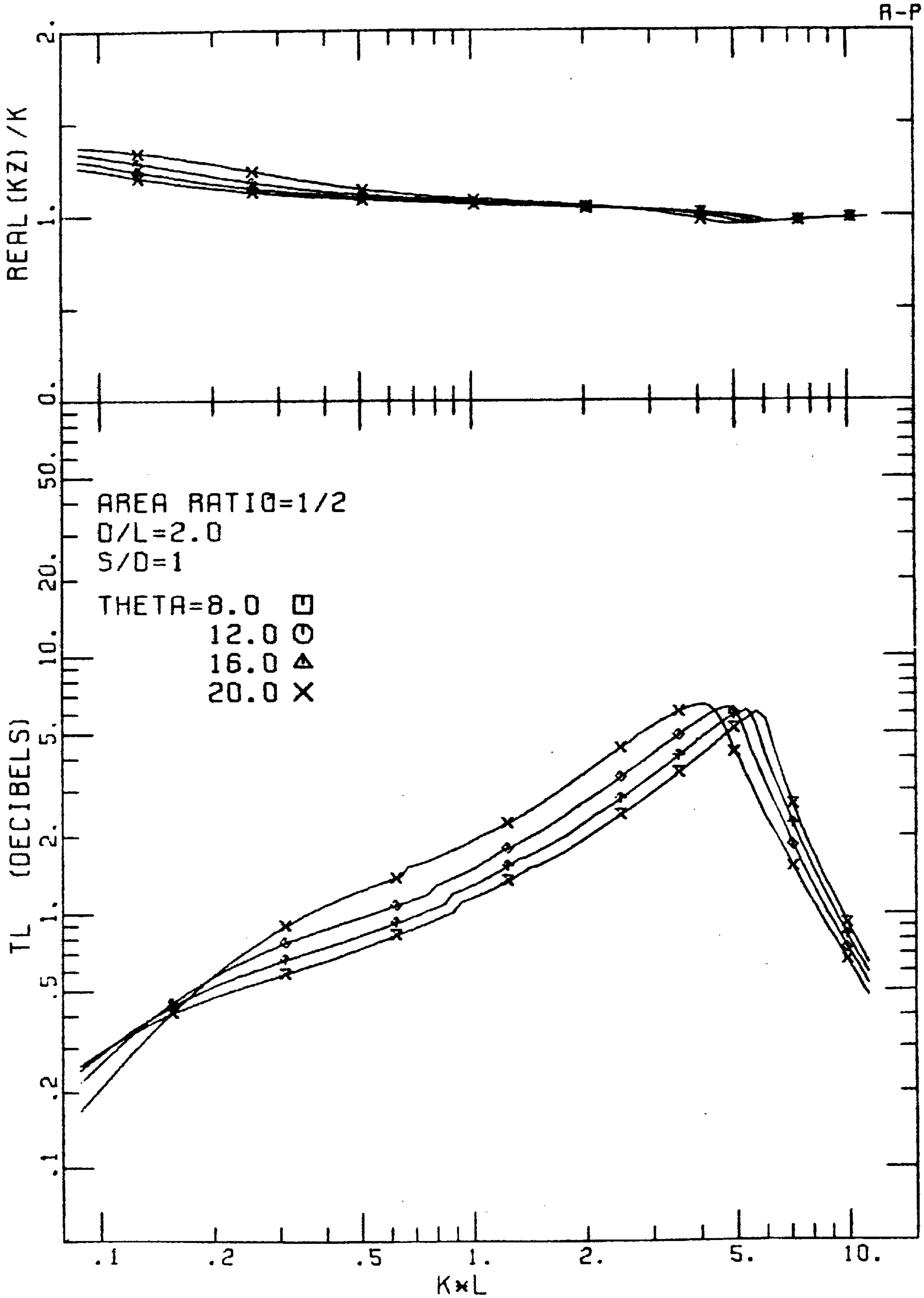

A-P
REAL (KZ) /K
2.
1.
0.
AREA RATIO=1/2
D/L=2.0
S/D=1
THETA=0.5
1.0
2.0
4.0
TL (DECIBELS)
50.
20.
10.
5.
2.
1.
.5
.2
.1
.1
.2
.5
1.
2.
5.
10.
K*L

Figure 2.11



A-P
REAL (KZ) /K
2.
1.
0.
AREA RATIO=1/2
D/L=2.0
S/D=1
THETA=8.0
12.0
16.0
20.0
TL (DECIBELS)
50.
20.
10.
5.
2.
1.
.5
.2
.1
.1
.2
.5
1.
2.
5.
10.
K*L

Figure 2.12

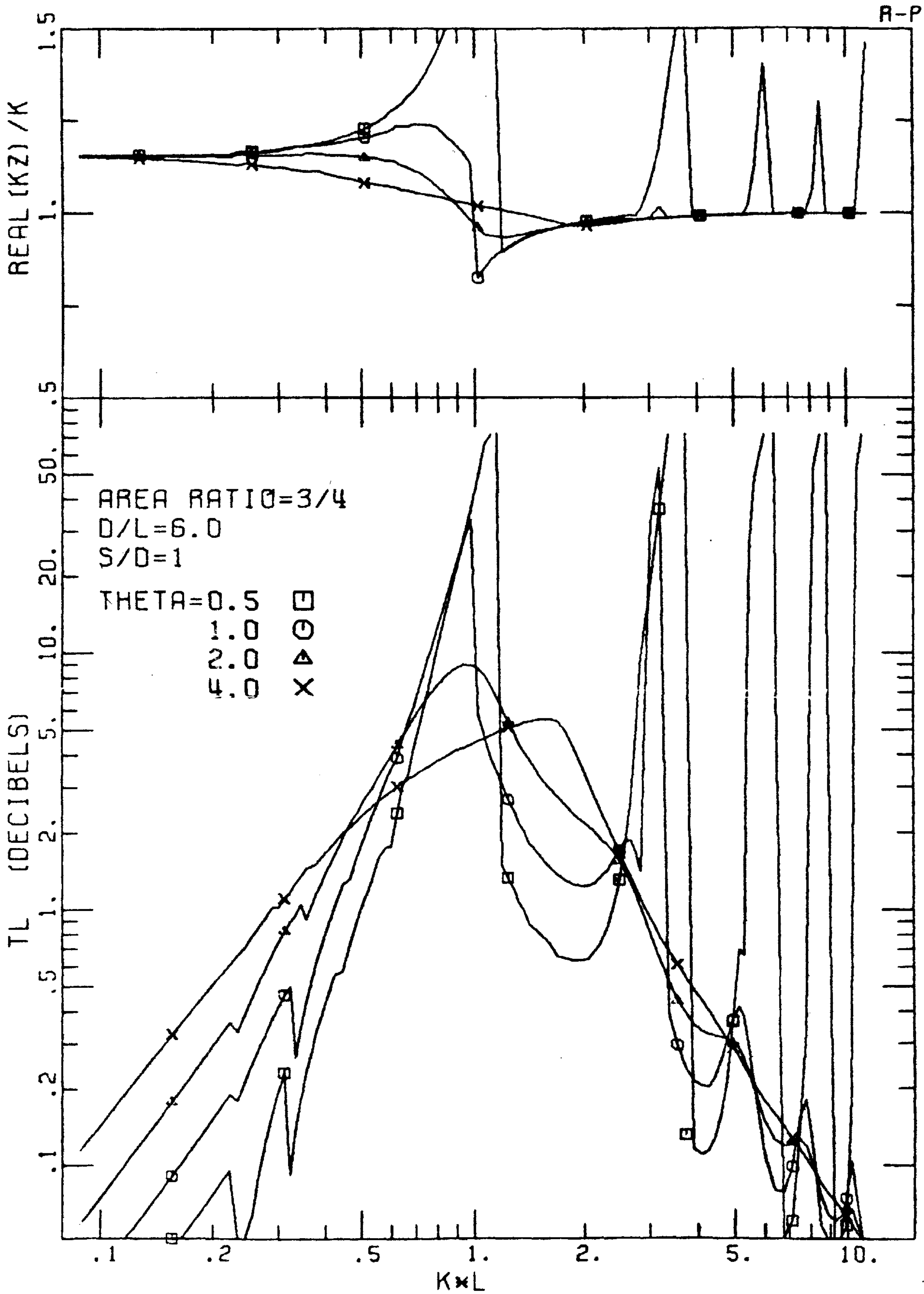
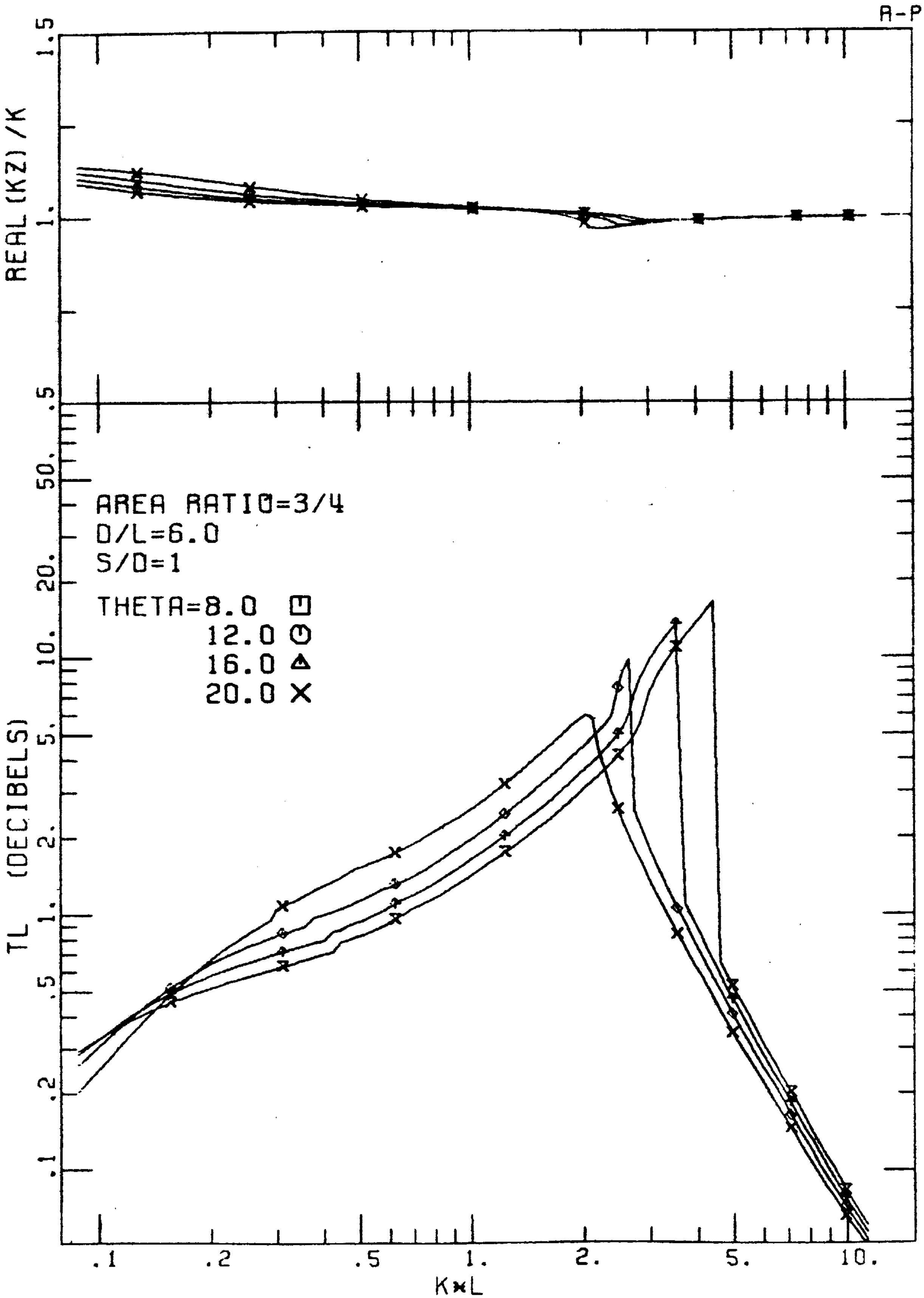

A-P
REAL (KZ) /K
1.5
1.
.5
TL (DECIBELS)
50.
20.
10.
5.
2.
1.
.5
.2
.1
AREA RATIO=3/4
D/L=6.0
S/D=1
THETA=0.5
1.0
2.0
4.0
.1
.2
.5
1.
2.
5.
10.
K*L

Figure 2.13



A-P
REAL (KZ) /K
1.5
1.
.5
TL (DECIBELS)
50.
20.
10.
5.
2.
1.
.5
.2
.1
AREA RATIO=3/4
D/L=6.0
S/D=1
THETA=8.0
12.0
16.0
20.0
.1
.2
.5
1.
2.
5.
10.
K*L

Figure 2.14

2.2 Circular Ducts

With reference to Figures 2.15 and 2.20 illustrating lined circular ducts, the pressure field of the fundamental mode is written as[1]

$$p = \int d\omega\, e^{-i\omega t}\, J_o(k_r r) \left\{ B_1(\omega) e^{ik_z z} + B_2(\omega) e^{-ik_z z} \right\} , \qquad (2.12)$$

where

$$k_z = \sqrt{k^2 - k_r^2} \qquad (2.13)$$

and k_r is the first root of the equation

$$\frac{k_r b J_1(k_r b)}{J_o(k_r b)} + \frac{ikb}{\zeta} = 0. \qquad (2.14)$$

2.2.1 Resonator Liner. As for the rectangular duct, we shall consider first a resonator liner with an impedance given by Eq. (2.5). This impedance applies, at least approximately, to the liner shown in Figure 2.15. The approximation relates to the fact that this expression for the impedance does not account for the cylindrical spreading of the wave in the annular region between the resistive sheet and the outer rigid wall. However, under most conditions, the impedance in Eq. (2.5) is an adequate approximation. The modification to Eq. (2.5) becomes important only for large values of L/D. This question will be discussed in an addendum to the present report.

The real and imaginary parts of the propagation constant k_z in Eq. (2.14) are shown in Figures 2.16-2.19.

The overall features of the dispersion relation are similar to those of the rectangular duct with a resonator liner.

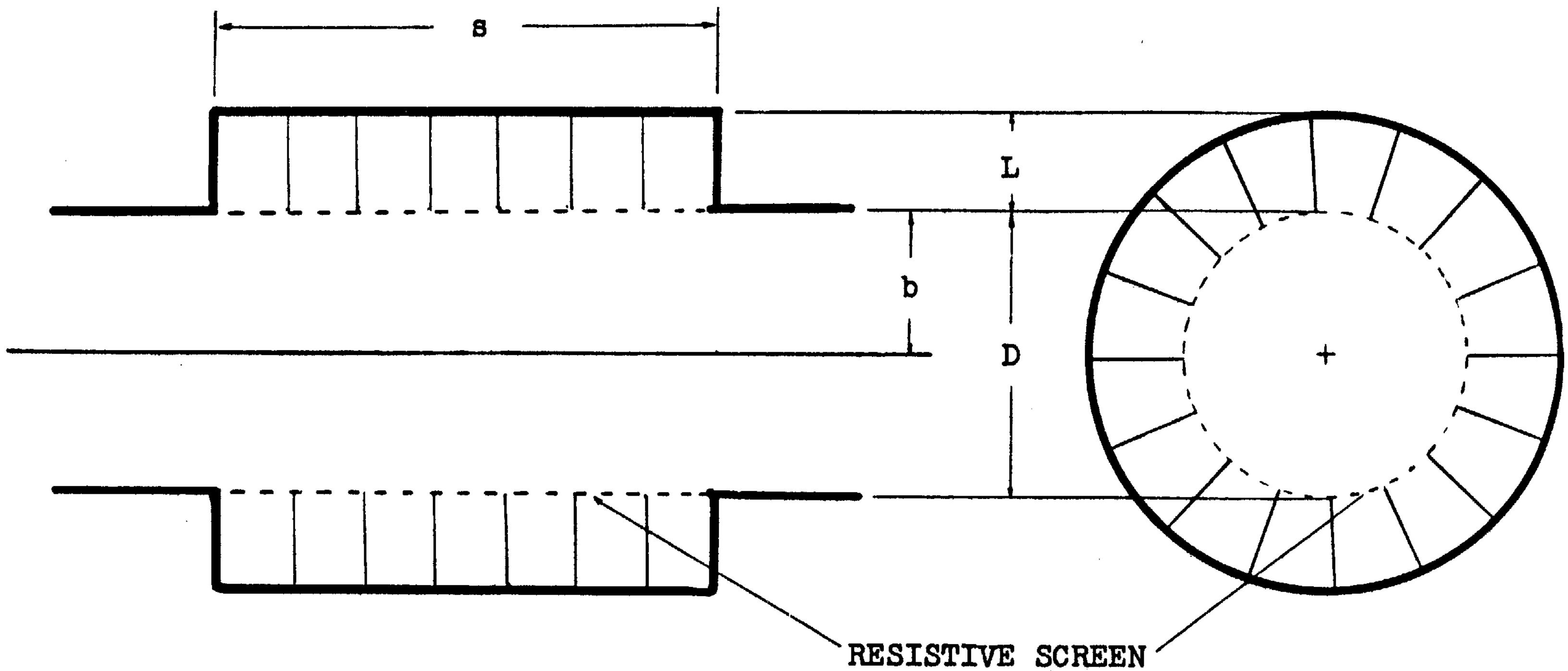


Figure 2.15. Circular duct lined with a resistive screen type resonator liner.

Figures 2.16-2.19. Real and imaginary parts of k_z for circular ducts lined with a resistive screen type resonator liner. The format is the same as in Figures 2.3-2.6.

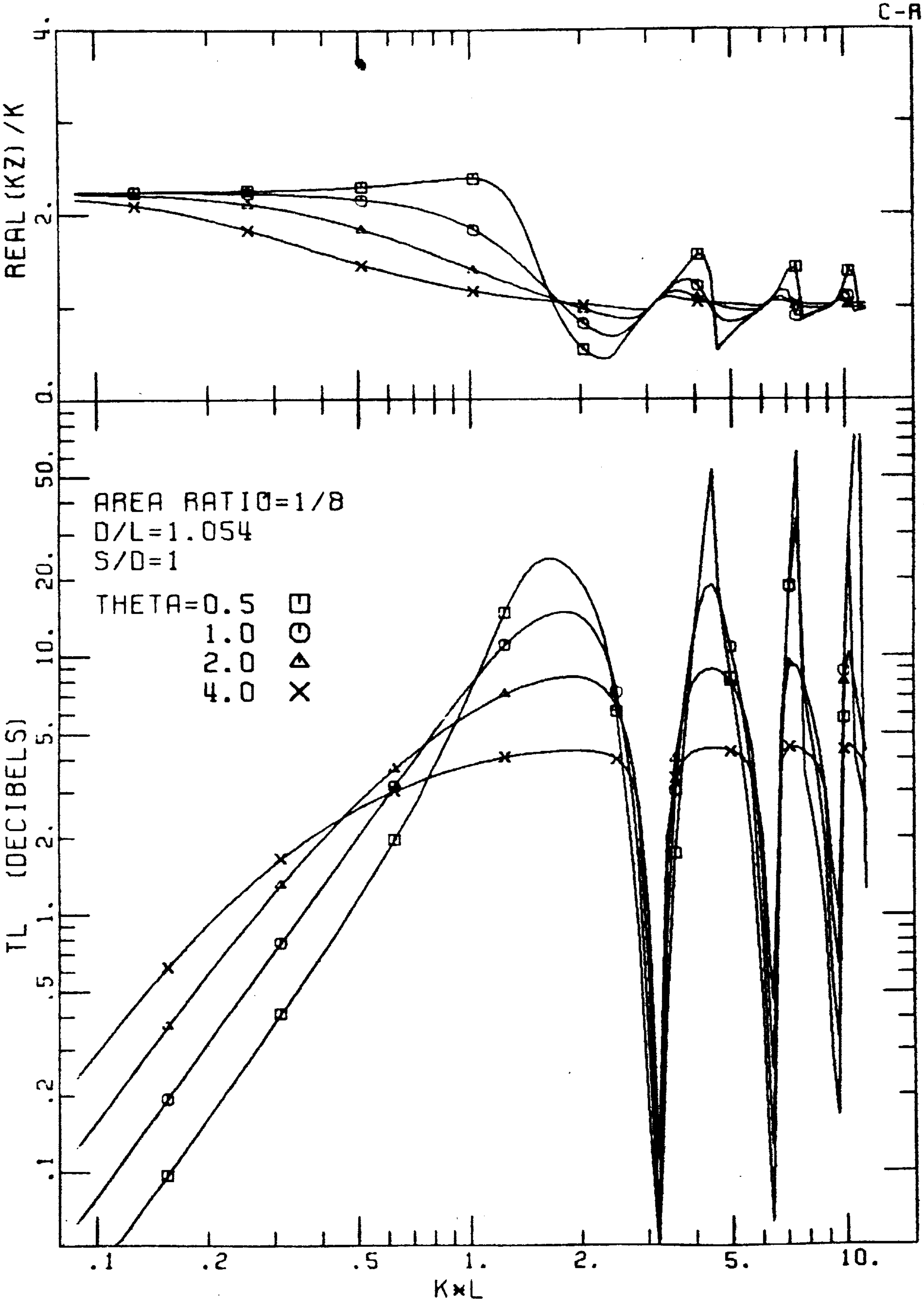
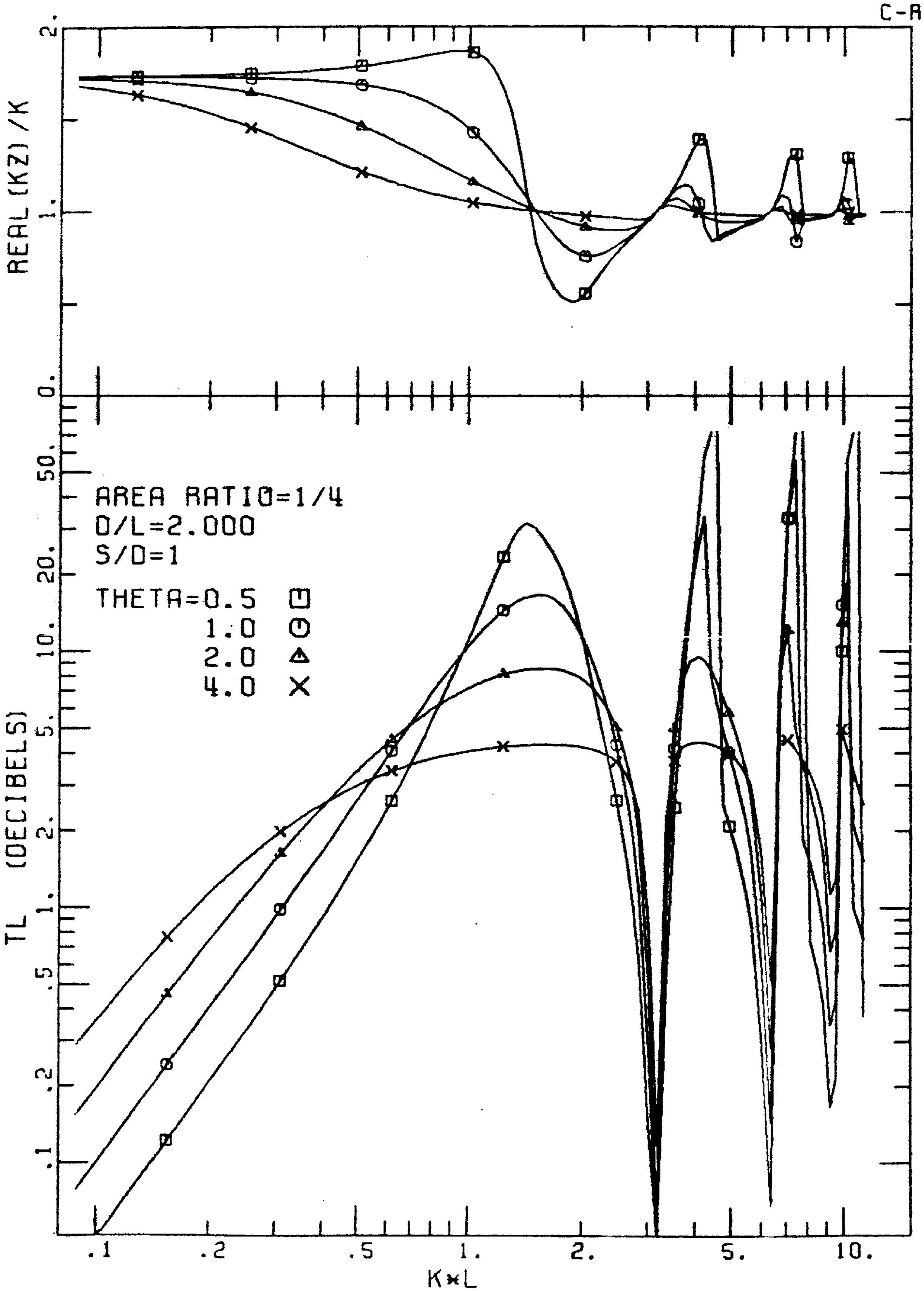

C-A
REAL (KZ)/K
4.
2.
0.
TL (DECIBELS)
50.
20.
10.
5.
2.
1.
.5
.2
.1
AREA RATIO=1/8
D/L=1.054
S/D=1
THETA=0.5
1.0
2.0
4.0
.1
.2
.5
1.
2.
5.
10.
K*L

Figure 2.16



C-A
REAL (KZ) /K
2.
1.
0.
AREA RATIO=1/4
D/L=2.000
S/D=1
THETA=0.5
1.0
2.0
4.0
TL (DECIBELS)
50.
20.
10.
5.
2.
1.
.5
.2
.1
.1
.2
.5
1.
2.
5.
10.
K*L

Figure 2.17

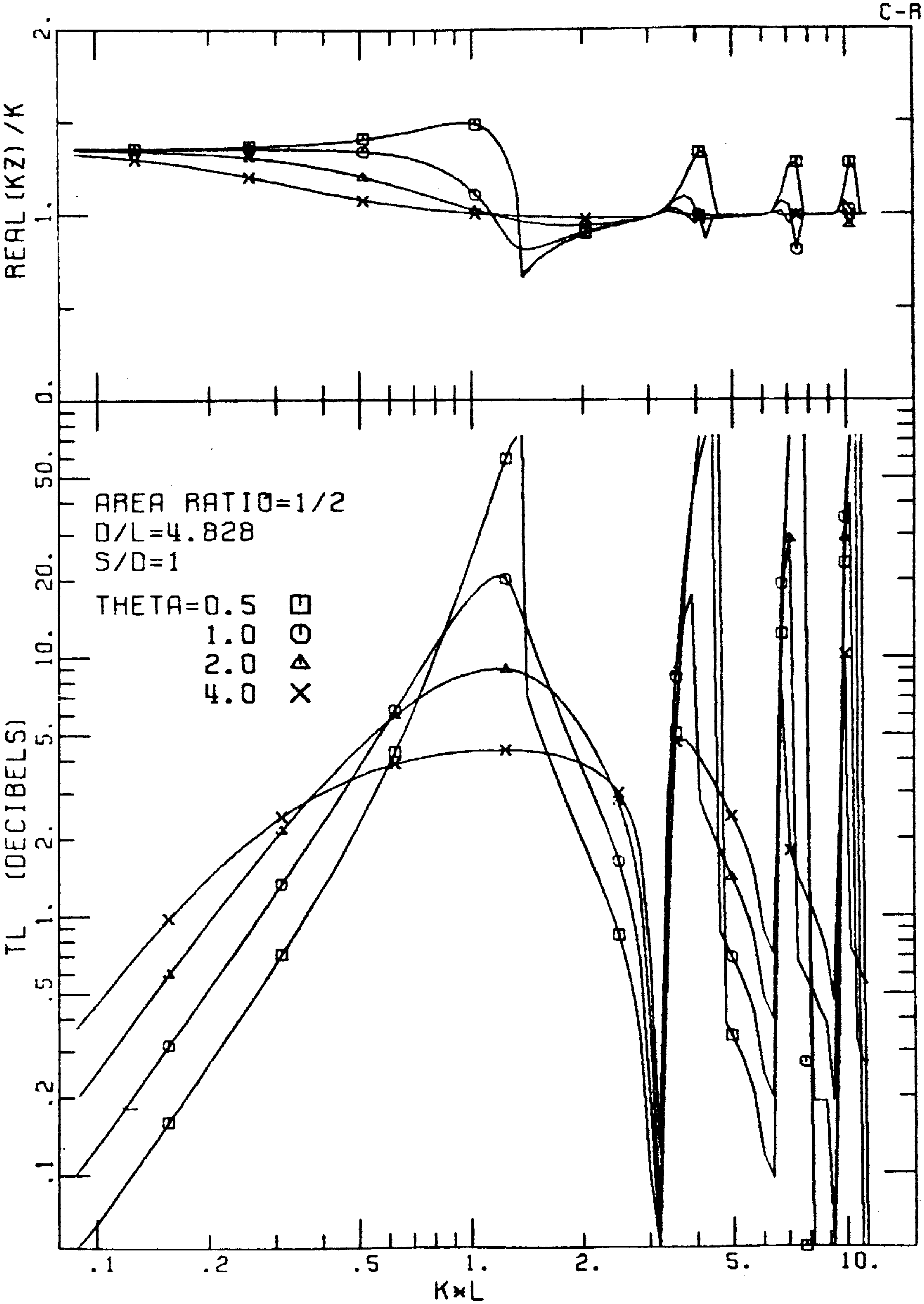

C-A
REAL (KZ) /K
TL (DECIBELS)
AREA RATIO=1/2
D/L=4.828
S/D=1
THETA=0.5
1.0
2.0
4.0
K*L

Figure 2.18

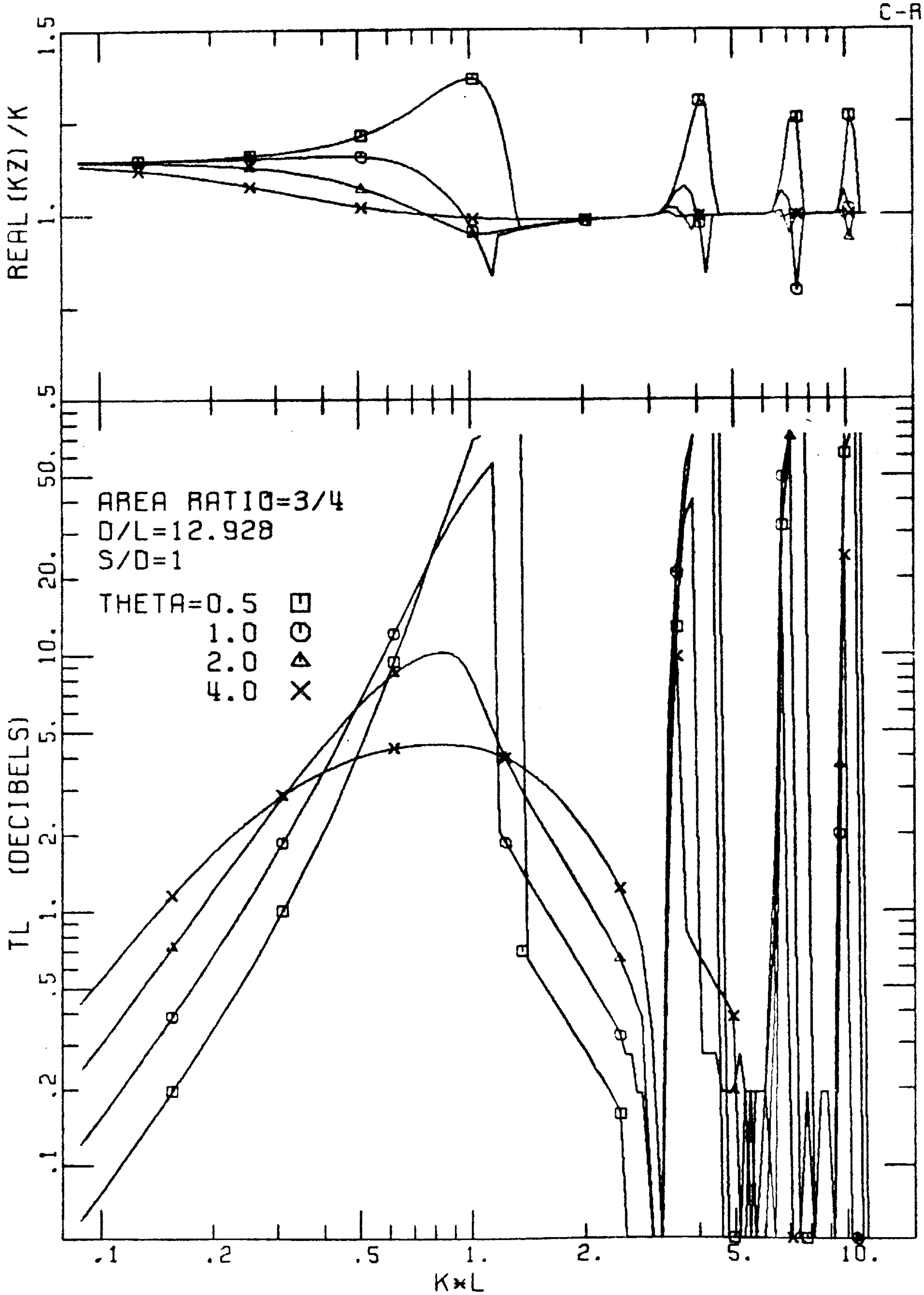

C-A
REAL(KZ)/K
TL (DECIBELS)
AREA RATIO=3/4
D/L=12.928
S/D=1
THETA=0.5
1.0
2.0
4.0
K*L
.1
.2
.5
1.
2.
5.
10.
1.5
1.
.5
50.
20.
10.
5.
2.
1.
.5
.2
.1

Figure 2.19

2.2.2 Porous Liner. A circular duct with a porous liner is illustrated in Figure 2.20.

For the boundary impedance we shall use the expression given in Eq. (2.8). Although this expression is a good approximation for the impedance of the duct liner in Figure 2.12, it does not account for the cylindrical divergence of the wave field within the porous material. The modification required to account for this spreading is important only for large values of L/D, as will be discussed in an addendum to this report.

The results of the numerical computations of the real and imaginary parts of the propagation constant are shown in Figures 2.21-2.28.

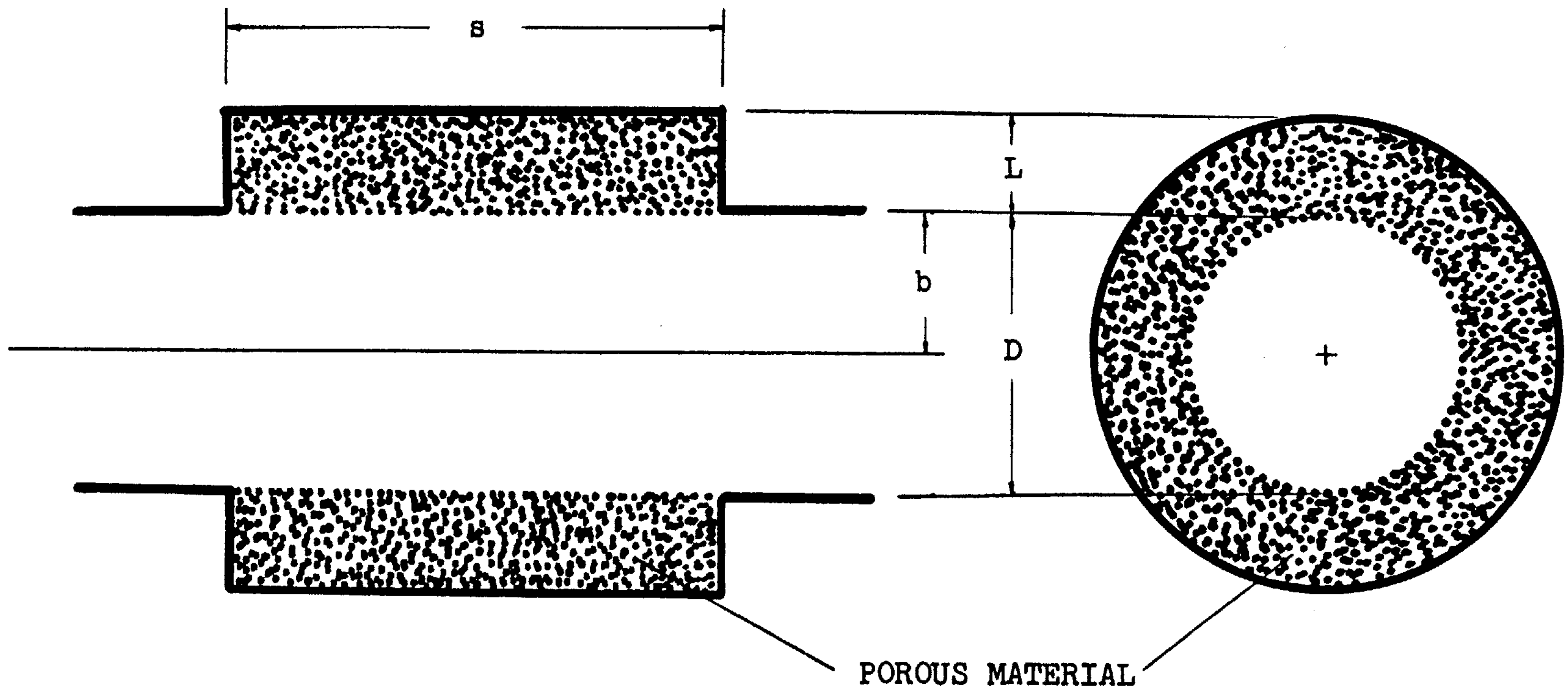


Figure 2.20. Circular duct lined with a porous liner.

Figures 2.21-2.28. Real and imaginary parts of k_z for circular ducts lined with a porous liner. The format is the same as in Figures 2.3-2.6.

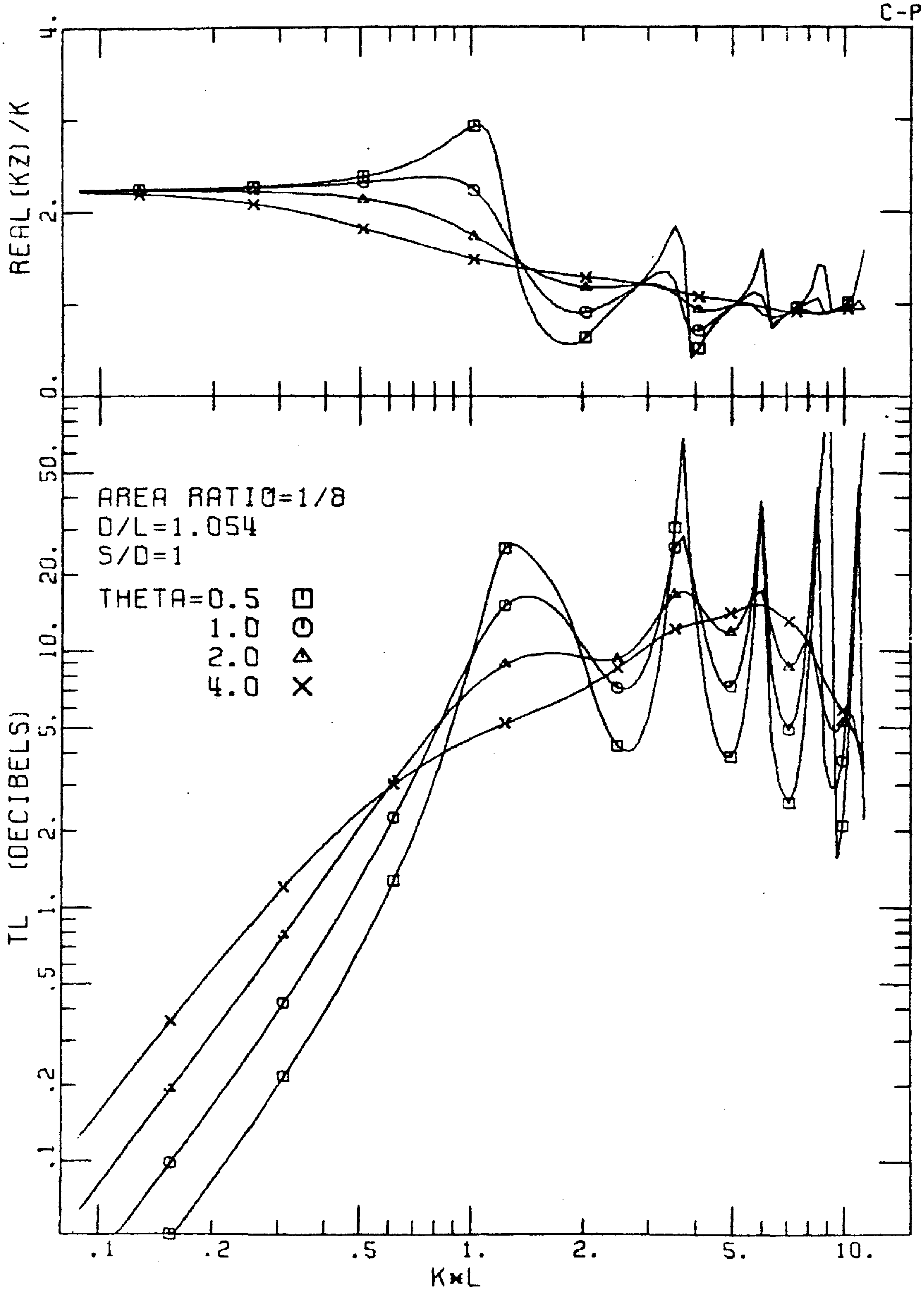
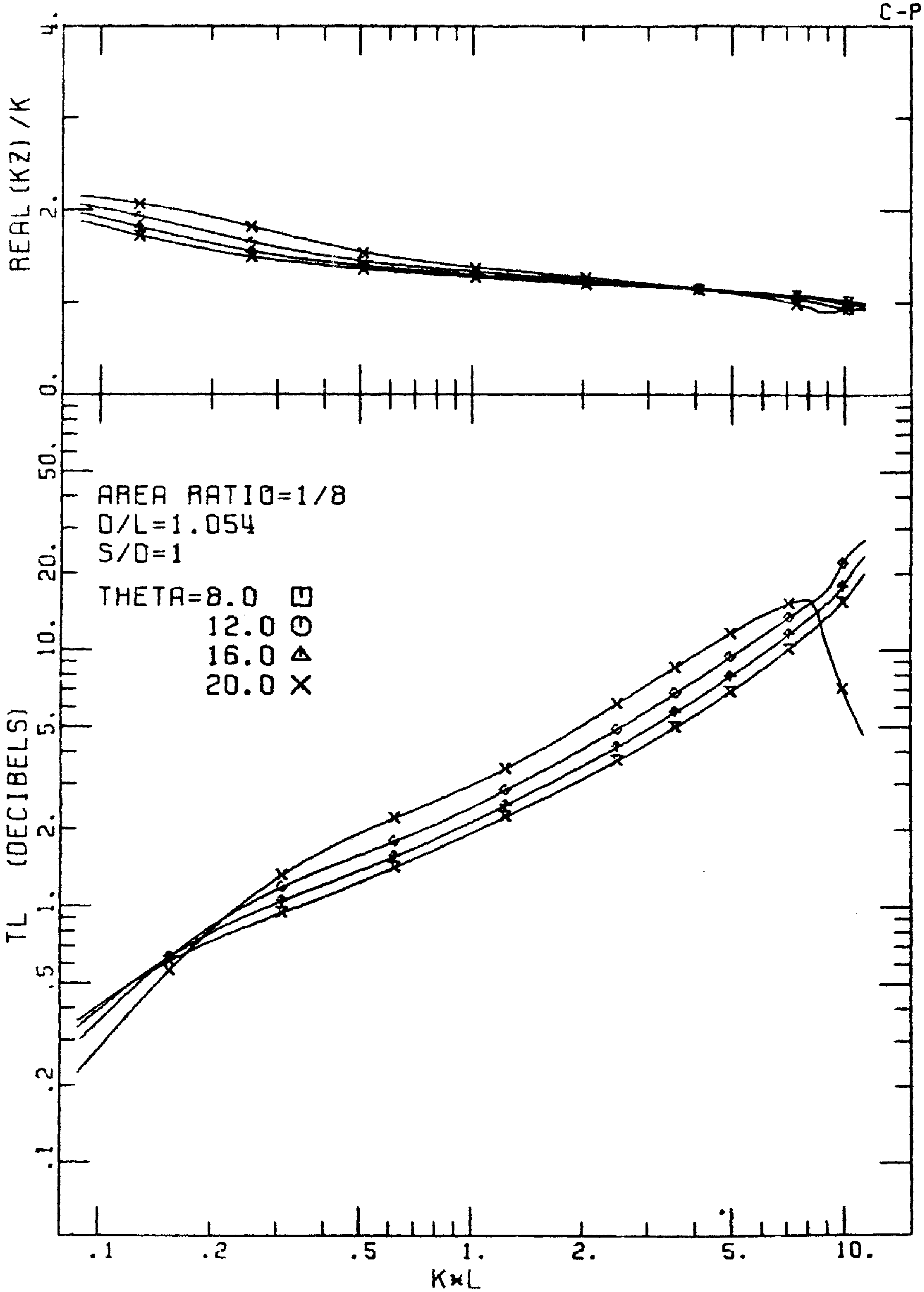

C-P
REAL (KZ)/K
0.
2.
4.
TL (DECIBELS)
.1
.2
.5
1.
2.
5.
10.
20.
50.
AREA RATIO=1/8
D/L=1.054
S/D=1
THETA=0.5
1.0
2.0
4.0
K*L
.1
.2
.5
1.
2.
5.
10.

Figure 2.21



C-P
REAL (KZ) /K
4.
2.
0.
TL (DECIBELS)
50.
20.
10.
5.
2.
1.
.5
.2
.1
AREA RATIO=1/8
D/L=1.054
S/D=1
THETA=8.0
12.0
16.0
20.0
.1
.2
.5
1.
2.
5.
10.
K*L

Figure 2.22

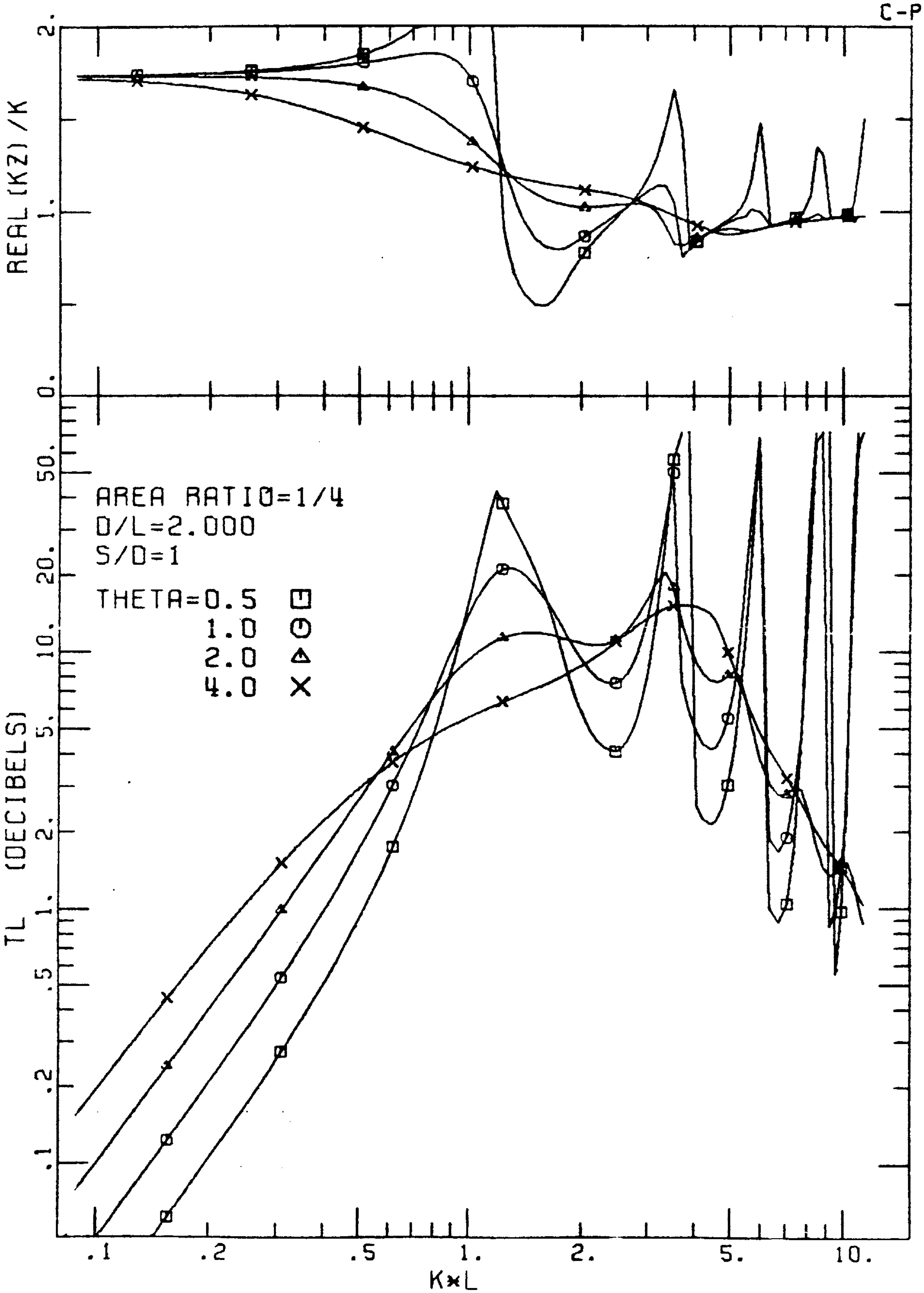

C-P
REAL (KZ) /K
2.
1.
0.
TL (DECIBELS)
50.
20.
10.
5.
2.
1.
.5
.2
.1
AREA RATIO=1/4
D/L=2.000
S/D=1
THETA=0.5
1.0
2.0
4.0
.1
.2
.5
1.
2.
5.
10.
K*L

Figure 2.23

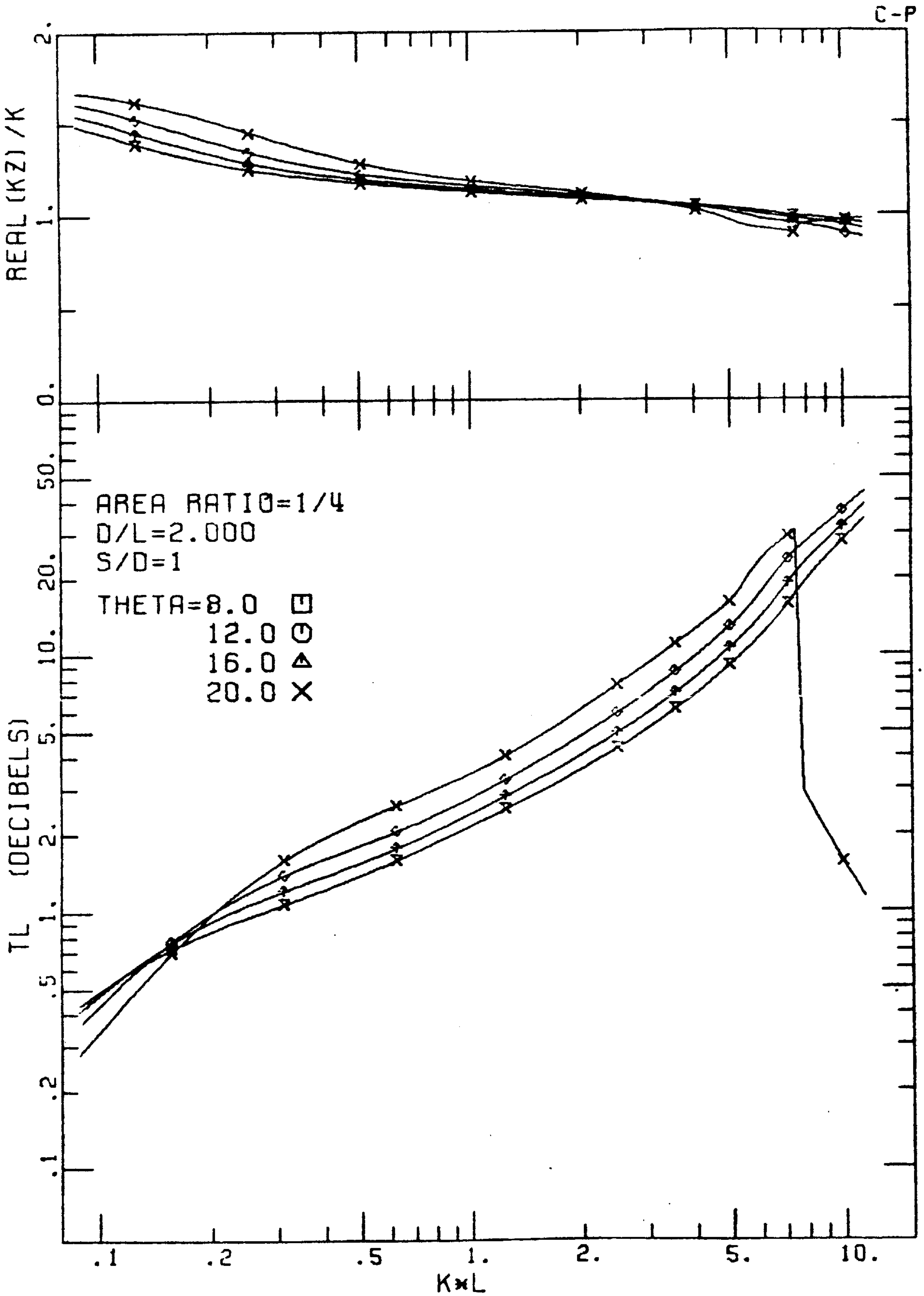

C-P
REAL (KZ) /K
2.
1.
0.
50.
20.
10.
5.
2.
1.
.5
.2
.1
TL (DECIBELS)
AREA RATIO=1/4
D/L=2.000
S/D=1
THETA=8.0
12.0
16.0
20.0
.1
.2
.5
1.
2.
5.
10.
K*L

Figure 2.24

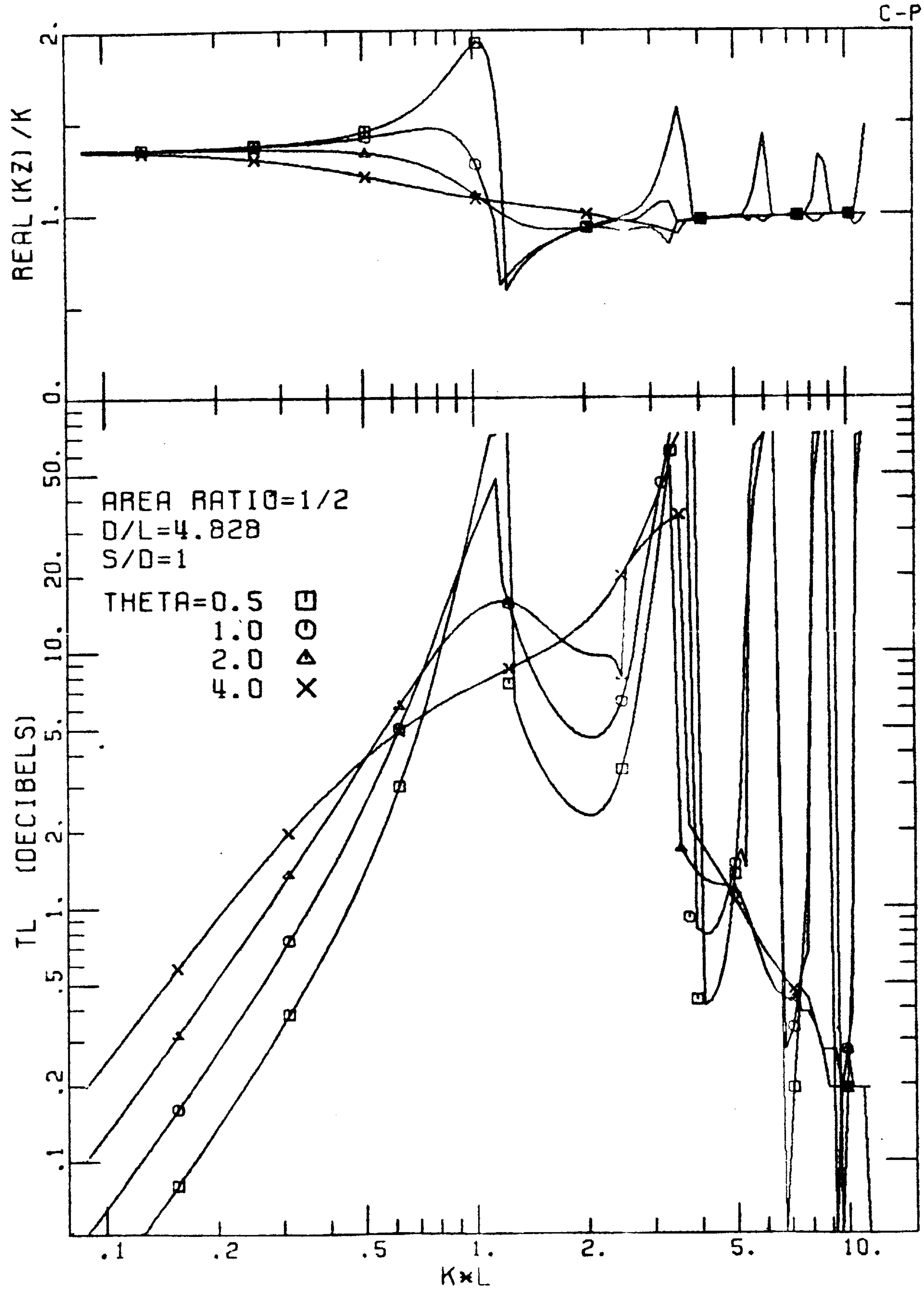

C-P
REAL(KZ)/K
TL (DECIBELS)
AREA RATIO=1/2
D/L=4.828
S/D=1
THETA=0.5
1.0
2.0
4.0
K*L

Figure 2.25

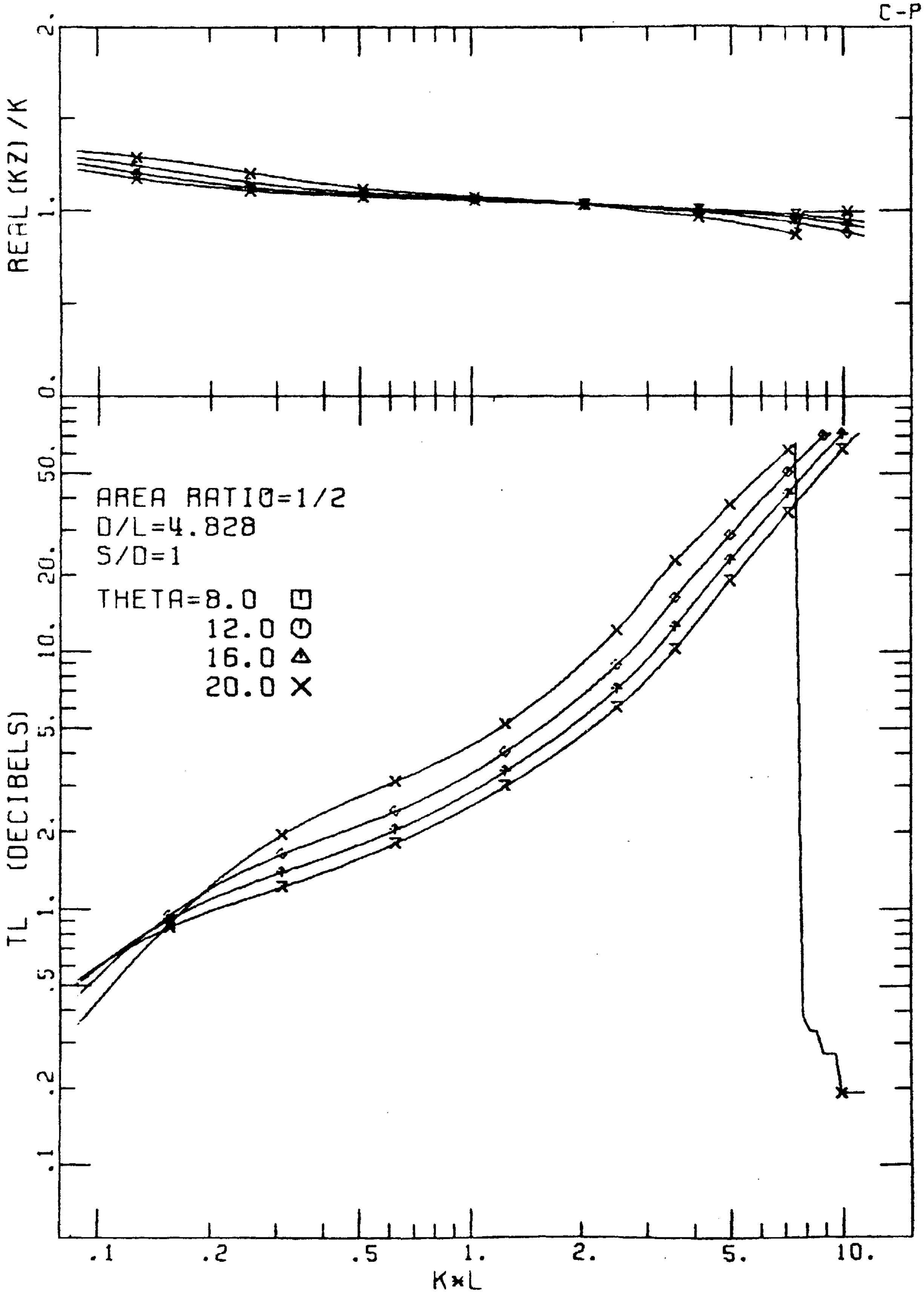
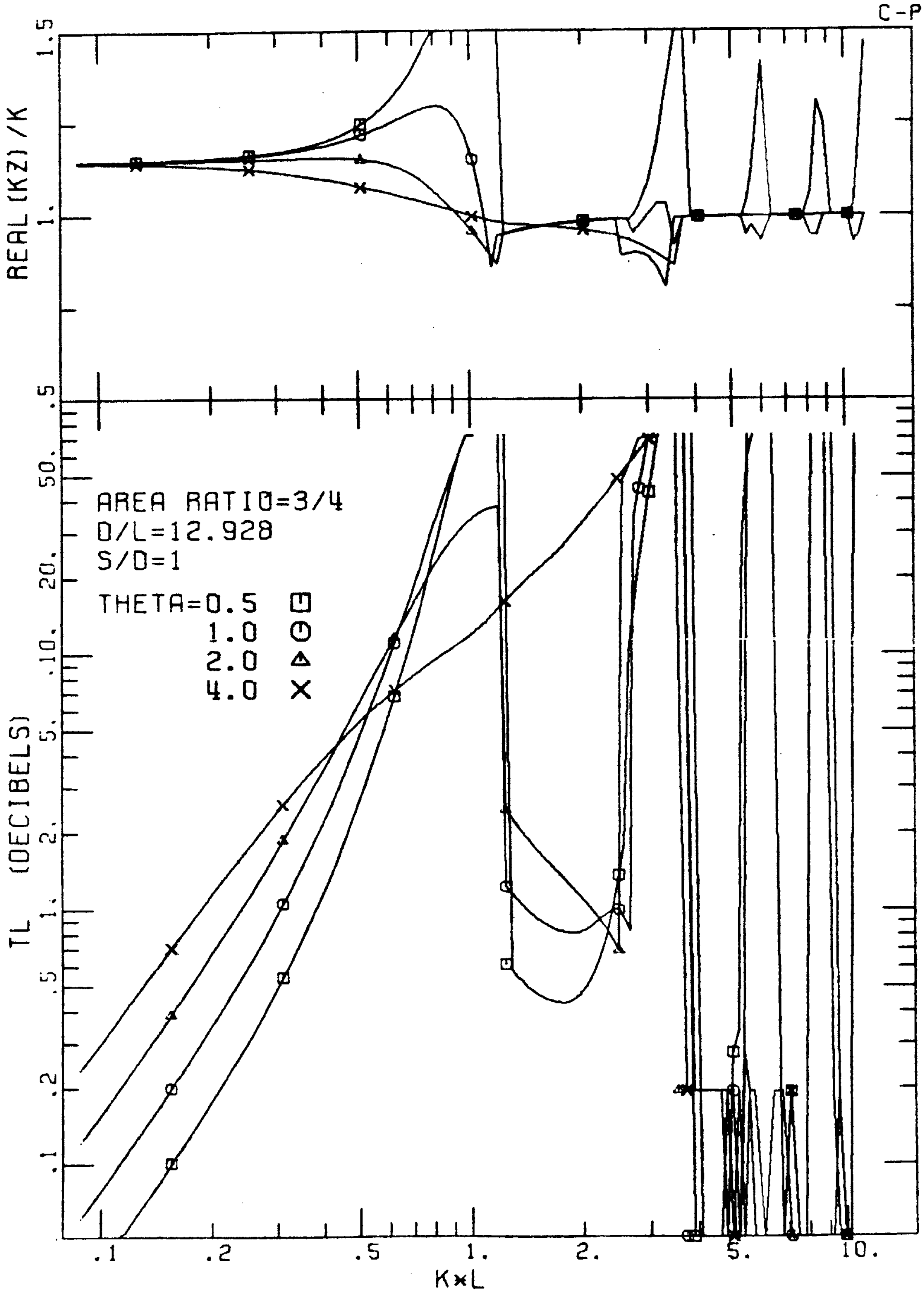
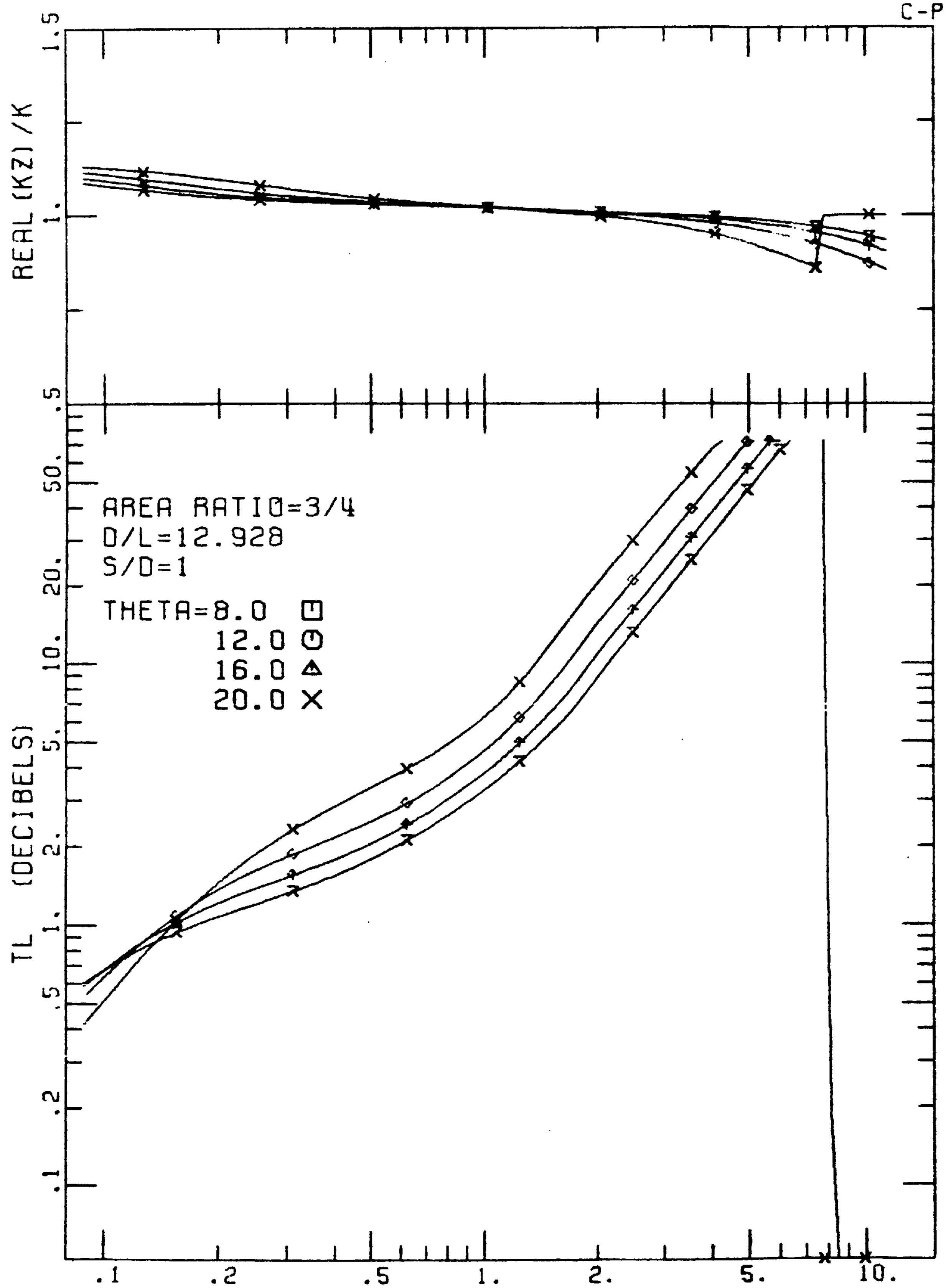

C-P
REAL (KZ)/K
2.
1.
0.
TL (DECIBELS)
50.
20.
10.
5.
2.
1.
.5
.2
.1
AREA RATIO=1/2
D/L=4.828
S/D=1
THETA=8.0
12.0
16.0
20.0
.1
.2
.5
1.
2.
5.
10.
K*L

Figure 2.26



C-P
REAL (KZ)/K
1.5
1.
.5
AREA RATIO=3/4
D/L=12.928
S/D=1
THETA=0.5
1.0
2.0
4.0
TL (DECIBELS)
50.
20.
10.
5.
2.
1.
.5
.2
.1
.1
.2
.5
1.
2.
5.
10.
K*L

Figure 2.27



C-P
REAL (KZ) /K
1.5
1.
.5
TL (DECIBELS)
50.
20.
10.
5.
2.
1.
.5
.2
.1
AREA RATIO=3/4
D/L=12.928
S/D=1
THETA=8.0
12.0
16.0
20.0
.1
.2
.5
1.
2.
5.
10.
K*L

Figure 2.28

3. OCTAVE BAND TRANSMISSION LOSS OF A DUCT ELEMENT

If the lined duct element is infinitely long, so that we can neglect the reflections from discontinuities in the wall impedance of the duct, the pressure field within the duct is of the form $\exp(ik_z z - i\omega t)$, and the pressure attenuation per unit length of the lined duct is

$$\begin{array}{c}\text{Pressure}\\ \text{Attenuation}\\ \text{per unit length}\end{array} = (20 \log_{10} e)\text{Im}(k_z)D \cong 8.6859 \text{ Im}(k_z).$$

This quantity, together with $\text{Re}(k_z)$, was presented as a function of the frequency parameter kL for rectangular and circular ducts for a wide range of duct parameters in the previous section. Although these quantities are of fundamental interest, expressing a basic characteristic of the fundamental acoustic mode in the lined duct, the quantity of more immediate practical importance is the performance of a finite duct element inserted in a hard-walled duct. Furthermore, in most engineering applications one is interested in the attenuation characteristics averaged over an octave band. This section will be devoted to this particular aspect of the problem, and we shall start by first deriving the expression for the transmission loss of a finite duct element in a hard-walled duct.

3.1 Calculation of Reflected and Transmitted Waves

With reference to Figure 3.1, the duct element under consideration is located between $z = 0$ and $z = S$. A plane wave is incident from the left. As it encounters the lined section there will be a distortion of the wave front, as, in principle, an infinite number of duct modes are excited. The amplitude of the fundamental mode in the duct will be dominant, however, and it is a good approximation to use as a boundary condition at the entrance and the exit of the duct, continuity of the average pressure amplitude and average velocity amplitude (average over the cross section of the duct). We shall denote these average amplitudes at the entrance and exit of the duct by (p_1, u_1) and (p_2, u_2) respectively.

The propagation constant for the fundamental wave in the lined duct is k_z, and in the hard-walled portion of the duct it is $k = \omega/c$. The relationship between the acoustic amplitudes at the entrance and the exit of the duct element is related by the equation

$$\begin{pmatrix} p_1 \\ \rho c u_1 \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \begin{pmatrix} p_2 \\ \rho c u_2 \end{pmatrix} \tag{3.1}$$

where

$$\begin{aligned} T_{11} &= \cos k_z s \\ T_{12} &= - \frac{ik}{k_z} \sin k_z s \\ T_{21} &= - \frac{ik_z}{k} \sin k_z s \\ T_{22} &= \cos k_z s. \end{aligned} \tag{3.2}$$

In Eq. (3.1) p_2 is the same as the transmitted pressure amplitude and we have

$$p_2 = \rho c u_2. \qquad (3.3)$$

The quantity p_1 is the sum of the incident pressure amplitude p_i and the reflected amplitude

$$p_r = R p_i, \qquad (3.4)$$

or

$$p_1 = (1 + R) p_i, \qquad (3.5)$$

where R is the reflection coefficient. This reflection coefficient can be expressed as

$$R = \frac{\zeta_1 - 1}{\zeta_1 + 1}, \qquad (3.6)$$

where ζ_1 is the input impedance of the duct element at $z = 0$.

It follows from Eq. (3.5) that

$$p_1 = \frac{2\zeta_1}{1 + \zeta_1} p_i. \qquad (3.7)$$

The impedance ζ_1 is obtained from Eq. (3.1)

$$\zeta_1 = \frac{p_1}{\rho c u_o} = \frac{T_{11} + T_{12}}{T_{21} + T_{22}}, \qquad (3.8)$$

and it follows from Eqs. (3.5) and (3.7) that

$$R = \frac{T_{12} - T_{21}}{T_{21} + T_{22} + T_{11} + T_{12}}. \qquad (3.9)$$

The relation between p_1 and p_2, as obtained from Eq. (3.1), is simply

$$\frac{p_1}{p_2} = T_{11} + T_{12}. \tag{3.10}$$

Together with Eqs. (3.7) and (3.9) this leads to the following expressions for the pressure transmission coefficient and the transmission loss

$$\tau = \frac{p_2}{p_1} = \frac{2}{(T_{21} + T_{22} + T_{11} + T_{12})} \tag{3.11}$$

$$TL = 10 \log \left| \frac{p_1}{p_2} \right|^2 = 10 \log \frac{1}{|\tau|^2} . \tag{3.12}$$

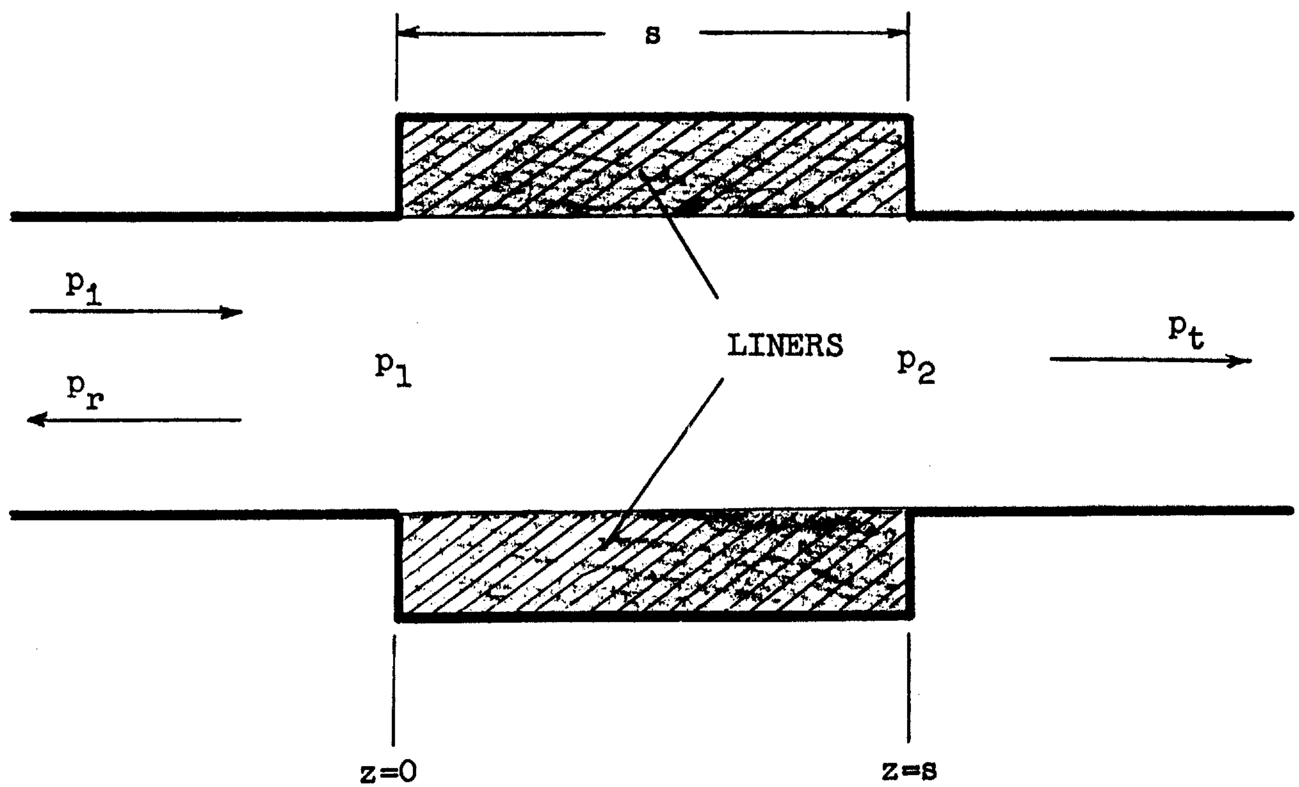


Figure 3.1. Illustration of quantities used in calculation of transmission loss.

3.2 Octave Band Transmission Loss. Effect of Input Spectrum

We now turn to the calculation of the average transmission loss over an octave band. The incident plane wave is now assumed to have a random time dependence with a power density spectrum $f(\omega)$. The total incident power in an octave band with a center frequency ω_o is then

$$W(0) = \int_{\omega_o/\sqrt{2}}^{\sqrt{2}\omega_o} f(\omega) d\omega. \tag{3.13}$$

With the assumption that the coupling to the fundamental mode component is independent of frequency in the octave band under consideration, the transmitted power can be written approximately as

$$W(S) = \int_{\omega_o/\sqrt{2}}^{\sqrt{2}\omega_o} f(\omega)|\tau|^2 d\omega. \tag{3.14}$$

The transmission coefficient τ, given by Eqs. (3.11) and (3.2), is a function of frequency and of the duct length S, and it follows then that the octave band transmission loss

$$TL = 10 \log \frac{W(0)}{W(S)} \tag{3.15}$$

will be a function of the length S of the lined duct element. It is also clear that the octave band transmission loss depends on the shape of the input spectrum.

In the numerical computations of the octave band transmission loss, we have considered five different duct lengths

(S/D = 1, 2, 4, 8, and 16) and a spectrum of the form $f(\omega) = \text{const}\ \omega^N$, with three different values of N, N = 0, 2, and -2, corresponding to a spectrum slope of 0 dB, 6 dB, and -6 dB per octave. The results are shown in Figures 3.2-3.97. The transmission loss is determined mainly by the sound absorption in the duct liner, and the reflections at the ends of the duct play a relatively minor role.

For a resonator liner, the octave band transmission loss is maximum in the octave band with the center frequency corresponding to kL = 1, which contains the first resonance of the liner ($kL = \pi/2$). For small values of D/L, the maximum is shifted toward kL = 2, an effect which is particularly pronounced for short duct liners.

For the porous duct liner with the resistance parameter θ greater than unity, the attenuation increases monotonically up to a frequency at which the wavelength is approximately equal to the diameter D.

It is interesting to note the influence of the shape of the input spectrum. The transmission loss is greatest for the input spectrum that has the same slope as the frequency dependence of the attenuation constant. Consequently, at low frequencies, at which the attenuation increases with frequency, the spectrum with N = 2 gives the highest octave band TL, and the same applies to N = -2 at high frequencies.

Figures 3.2-3.17. Octave band TL vs kL for a rectangular duct lined with a resistive screen type resonator liner. Each figure corresponds to a different combination of values of the screen resistance Θ and D/L. Each figure contains three frames corresponding to different spectra of the incident wave as indicated by N (see Eq. 3.32). In each frame five curves are given corresponding to five values of the duct length parameter S/D, which are given at the corner of each figure.

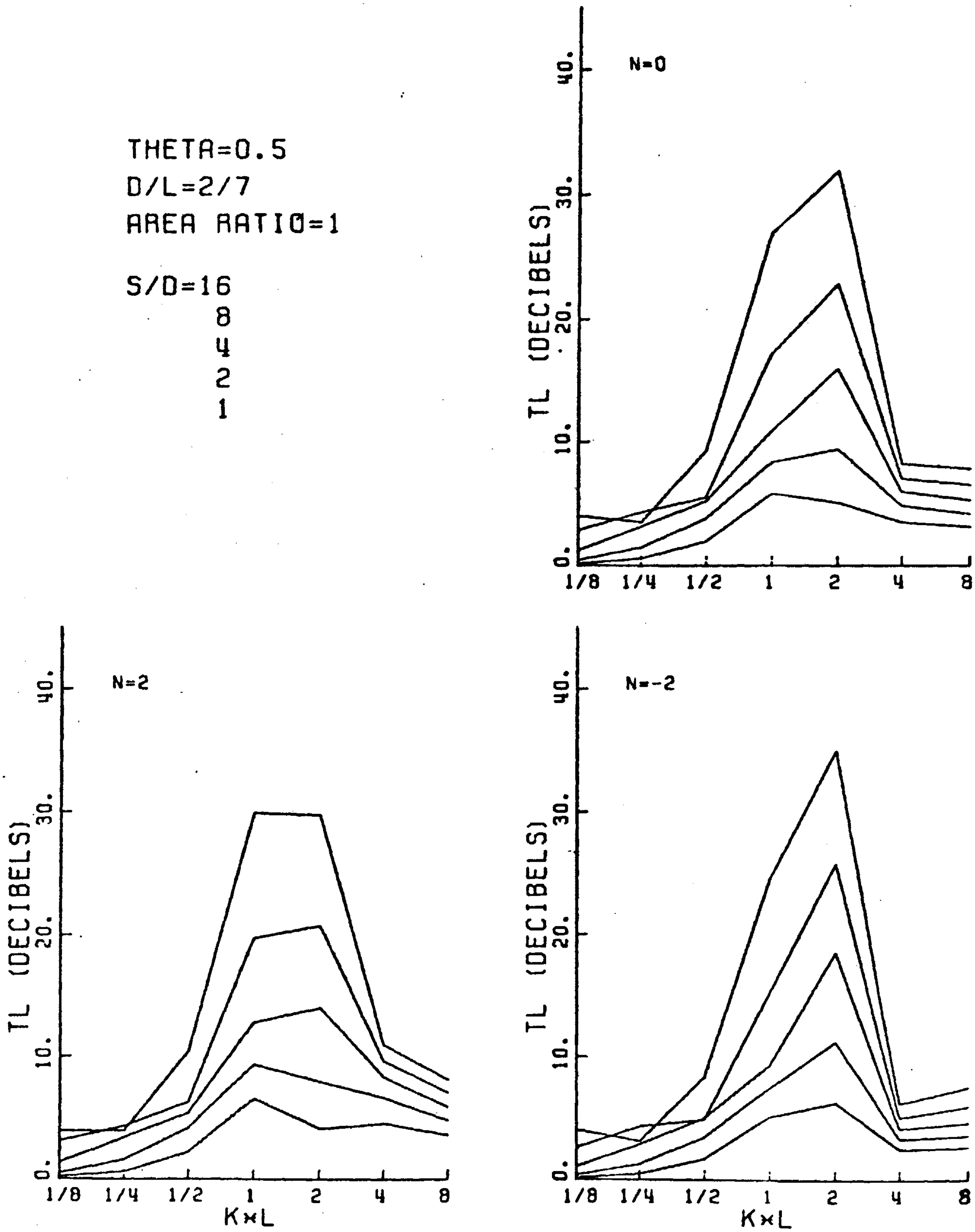

THETA=0.5
D/L=2/7
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K×L

Figure 3.2

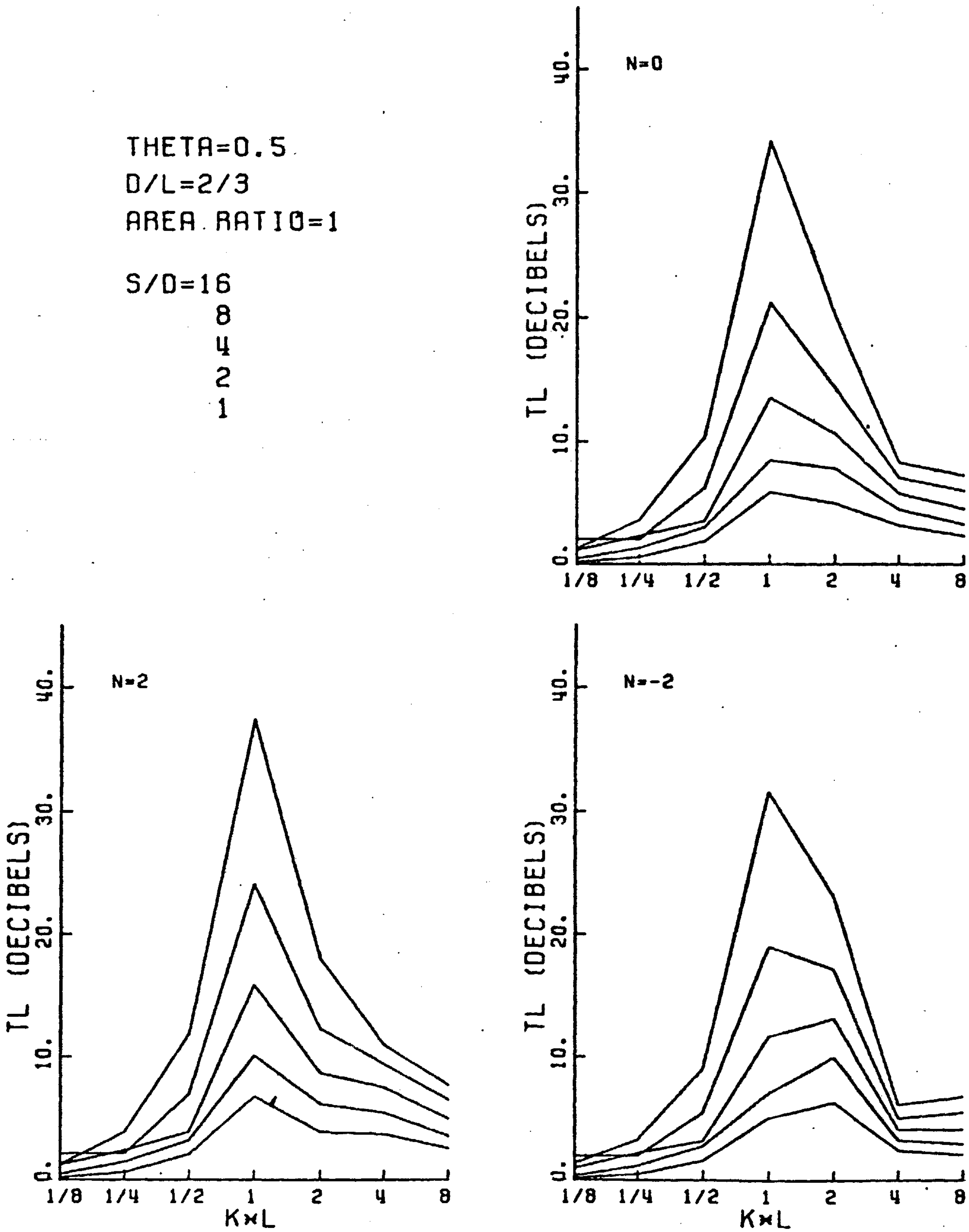

THETA=0.5
D/L=2/3
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8 1/4 1/2 1 2 4 8
K*L

Figure 3.3

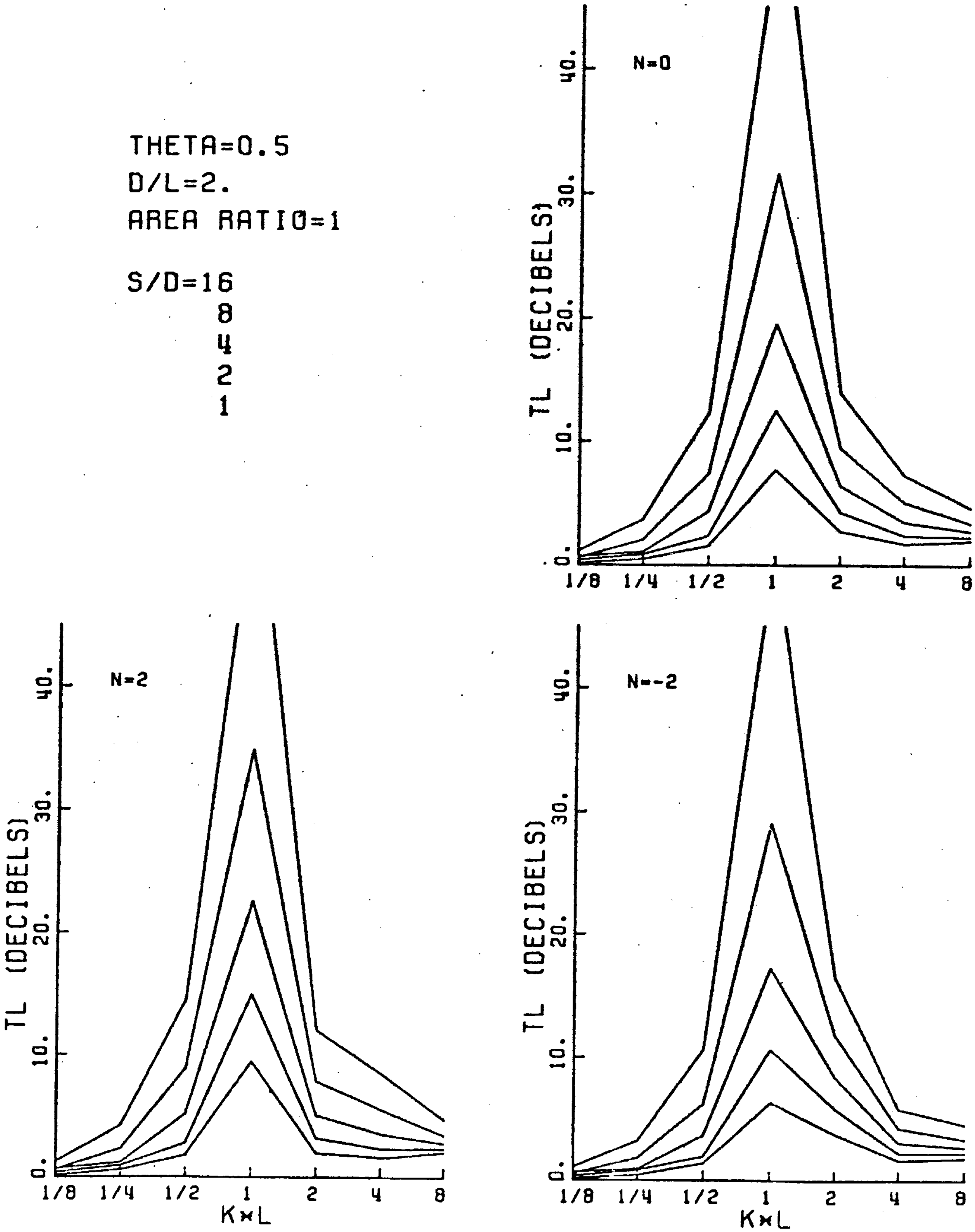

THETA=0.5
D/L=2.
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.4

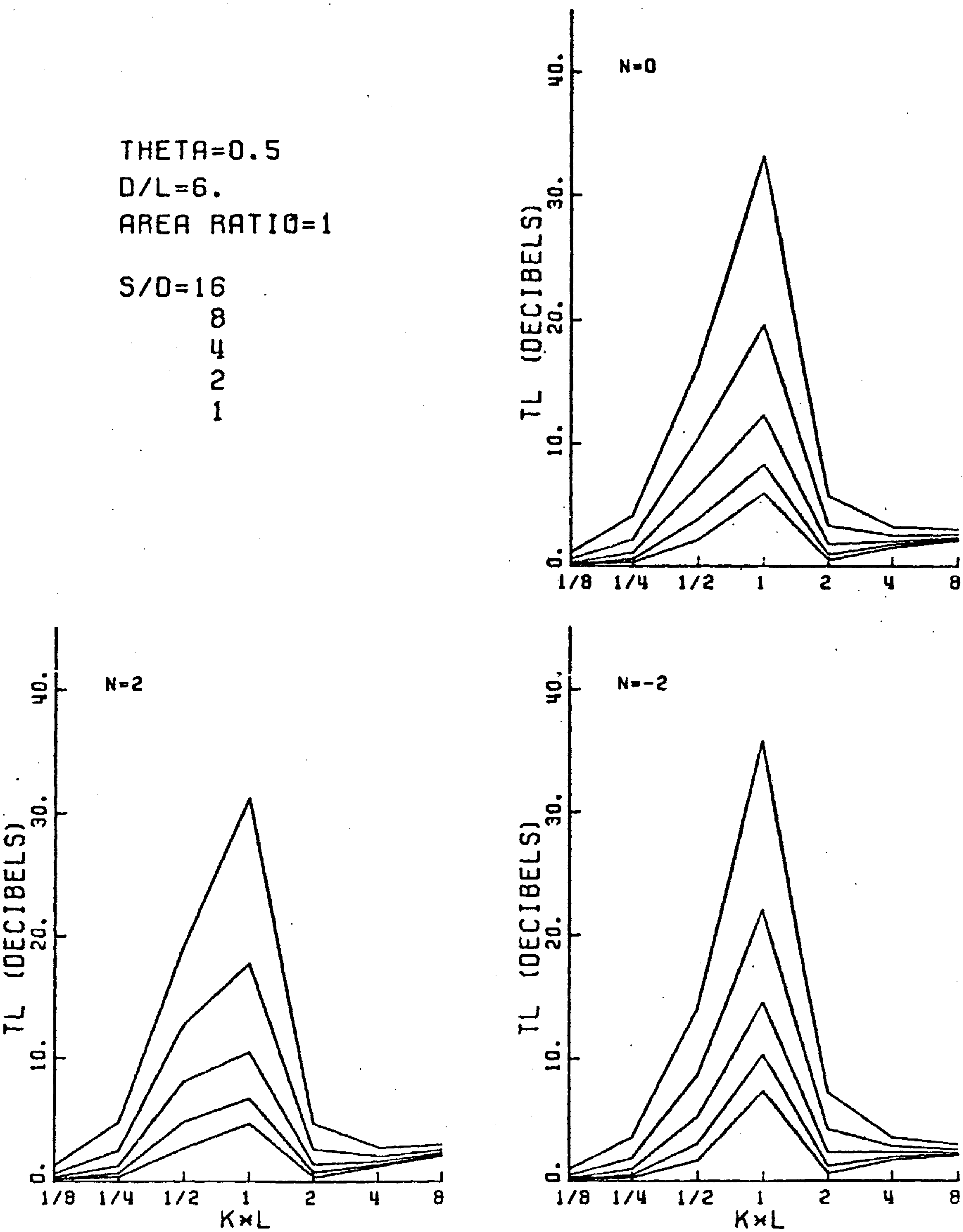

THETA=0.5
D/L=6.
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.5

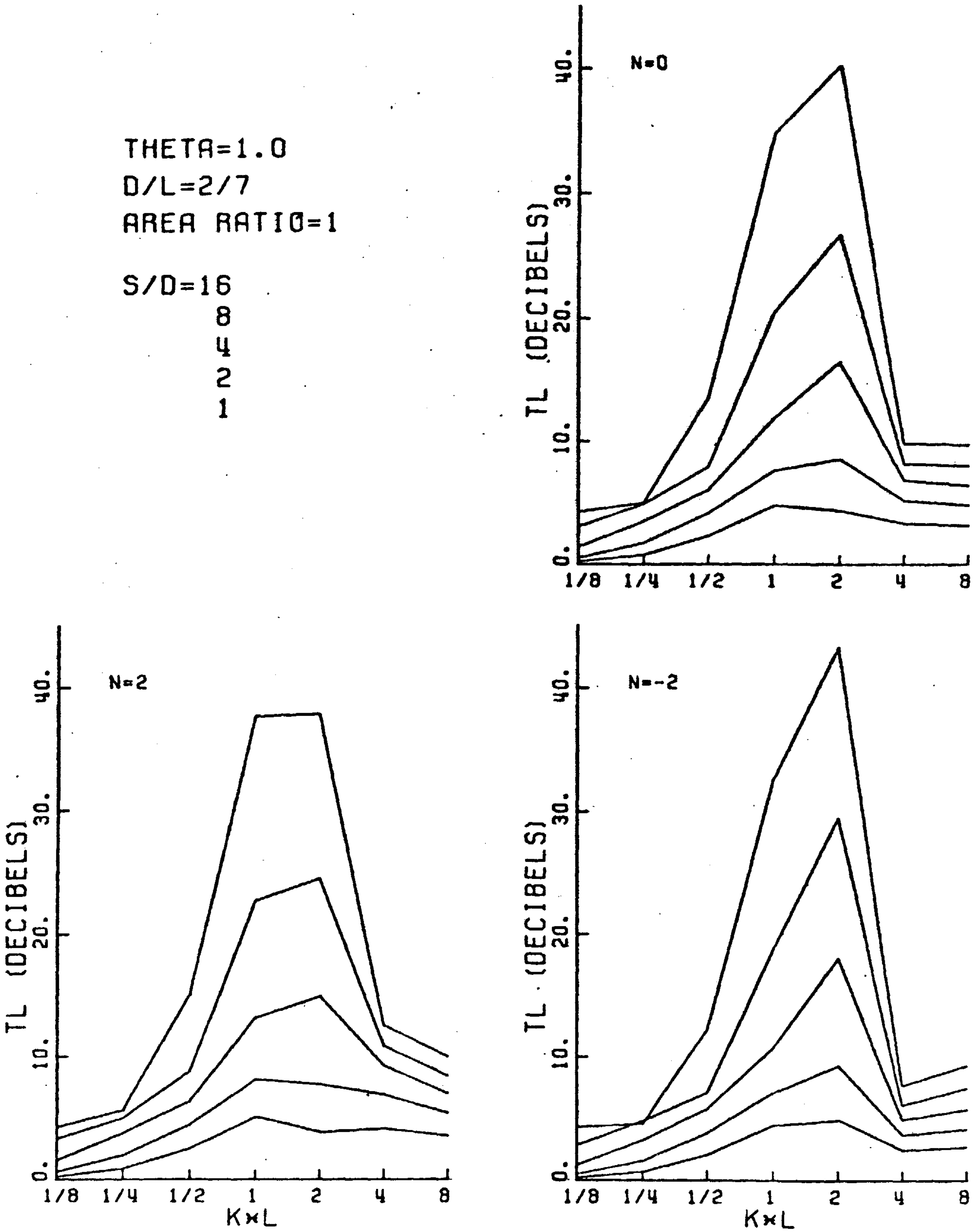

THETA=1.0
D/L=2/7
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.6

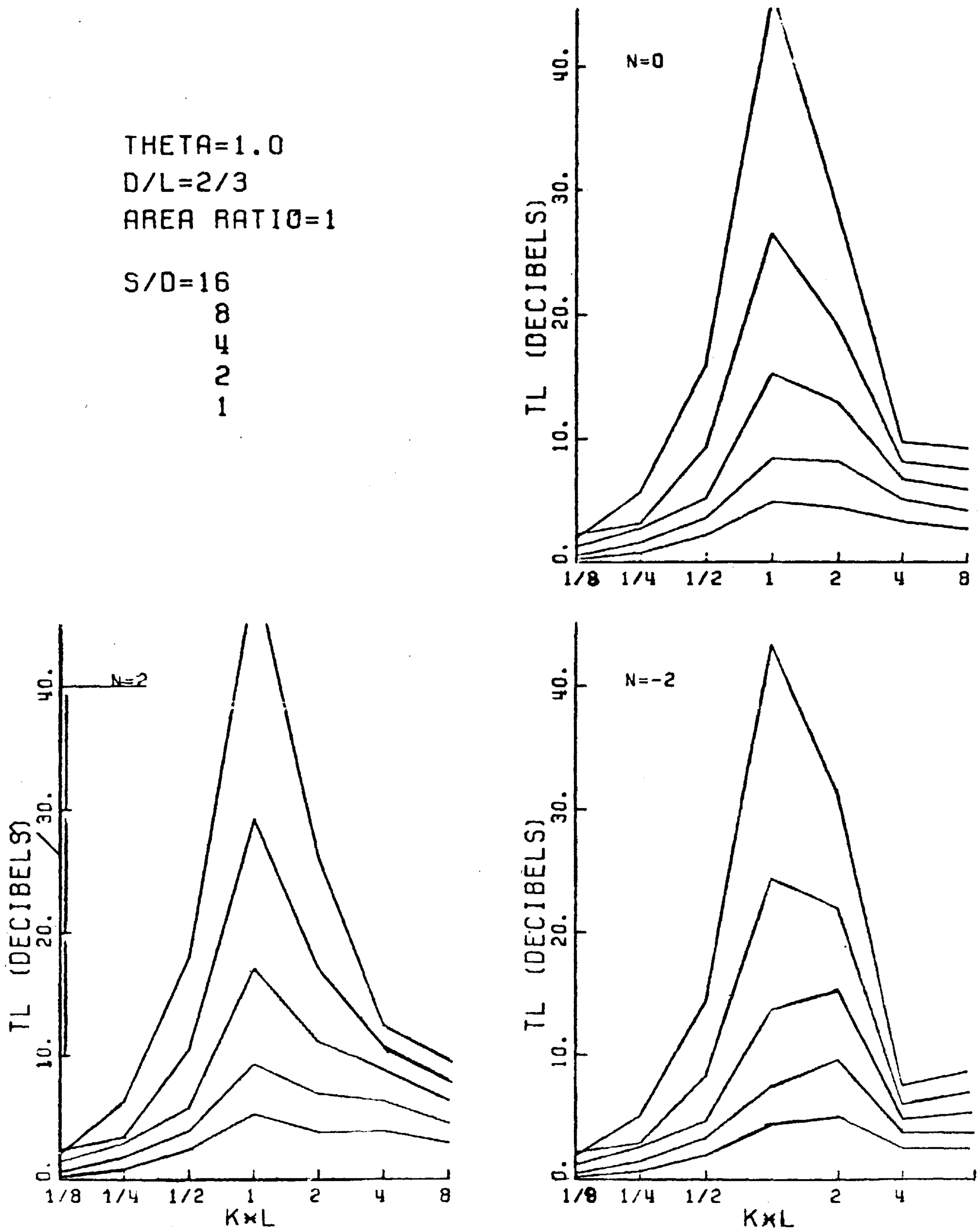

THETA=1.0
D/L=2/3
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
K*L
1/8
1/4
1/2
1
2
4
8
0.
10.
20.
30.
40.

Figure 3.7

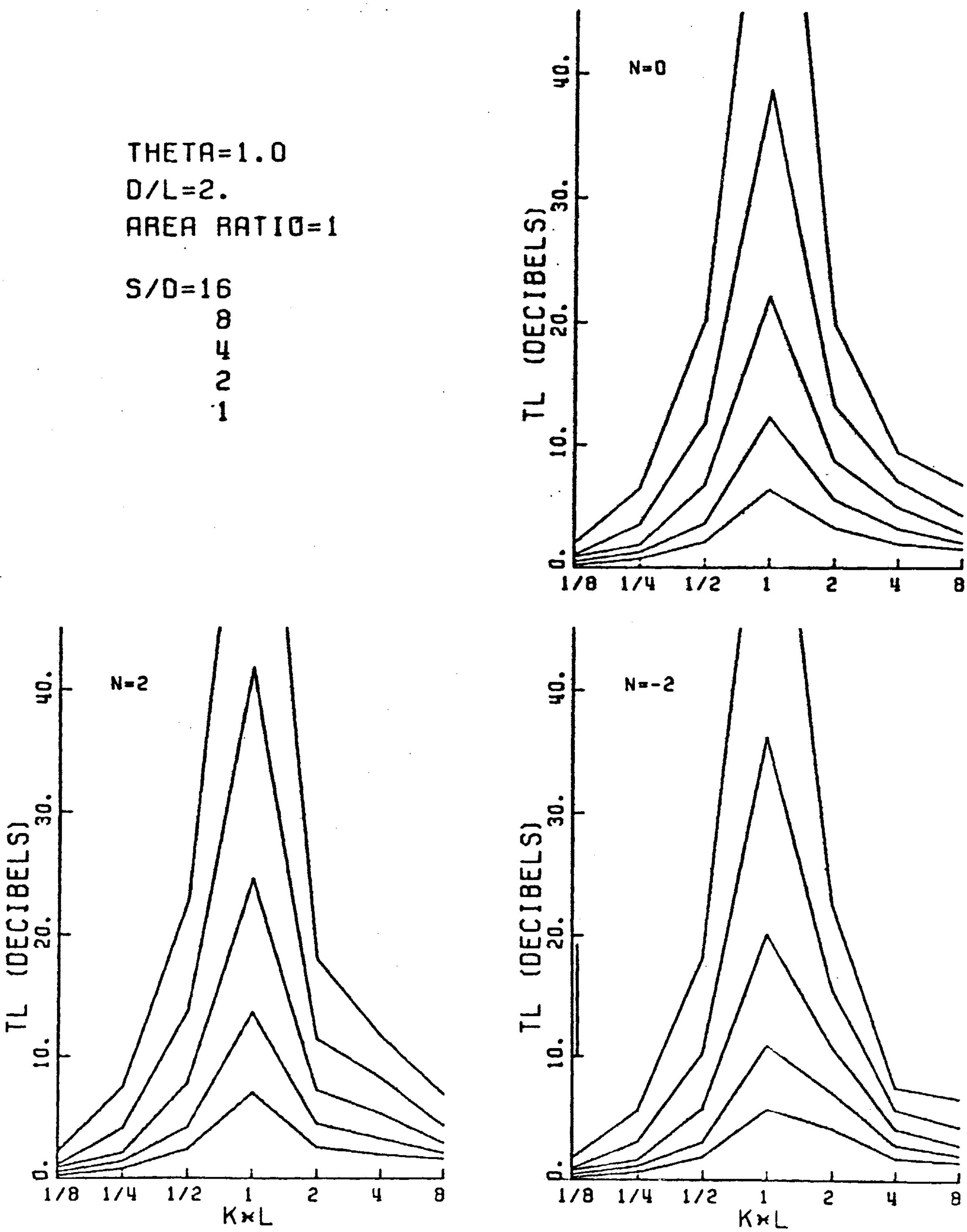

THETA=1.0
D/L=2.
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.8

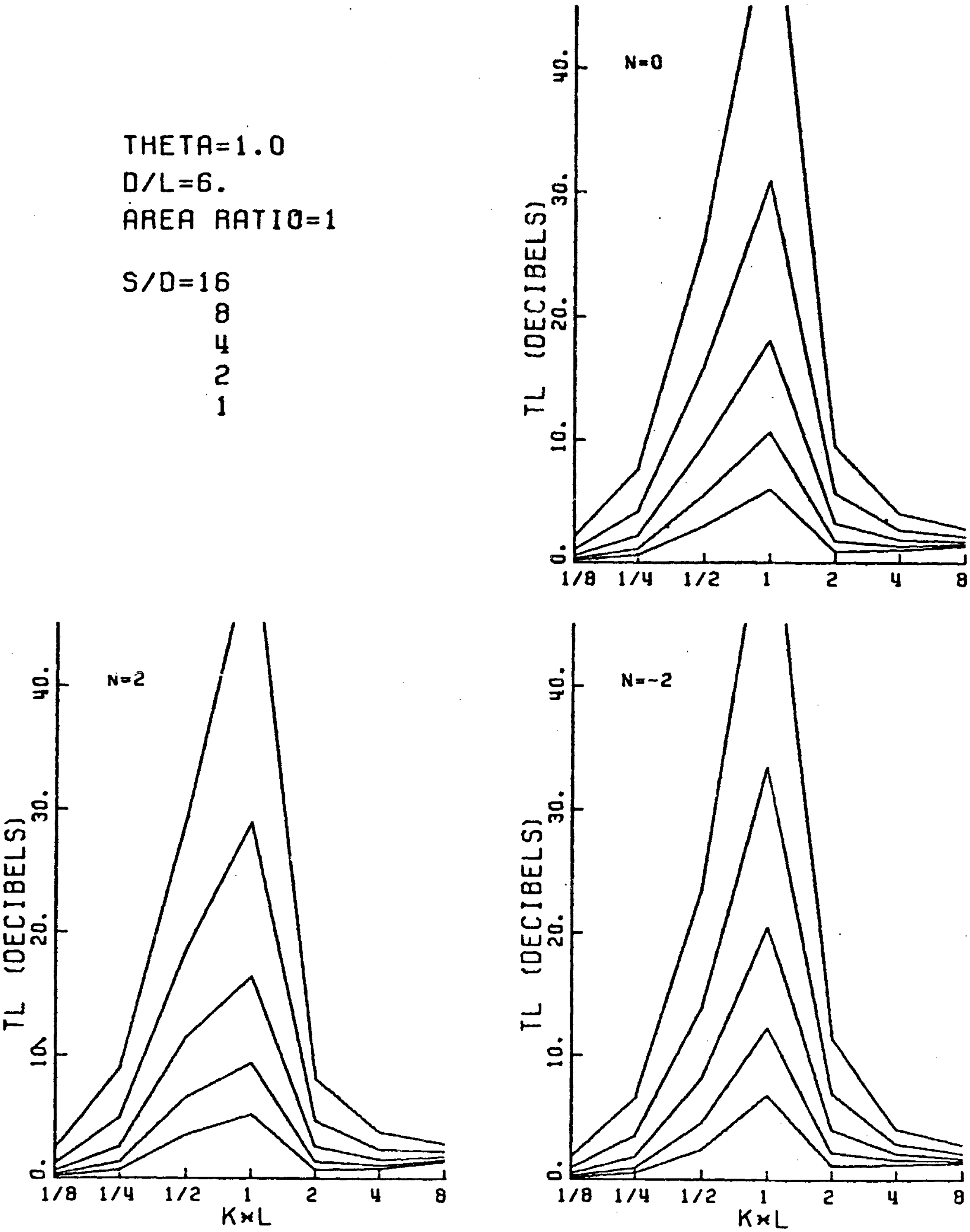

THETA=1.0
D/L=6.
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.9

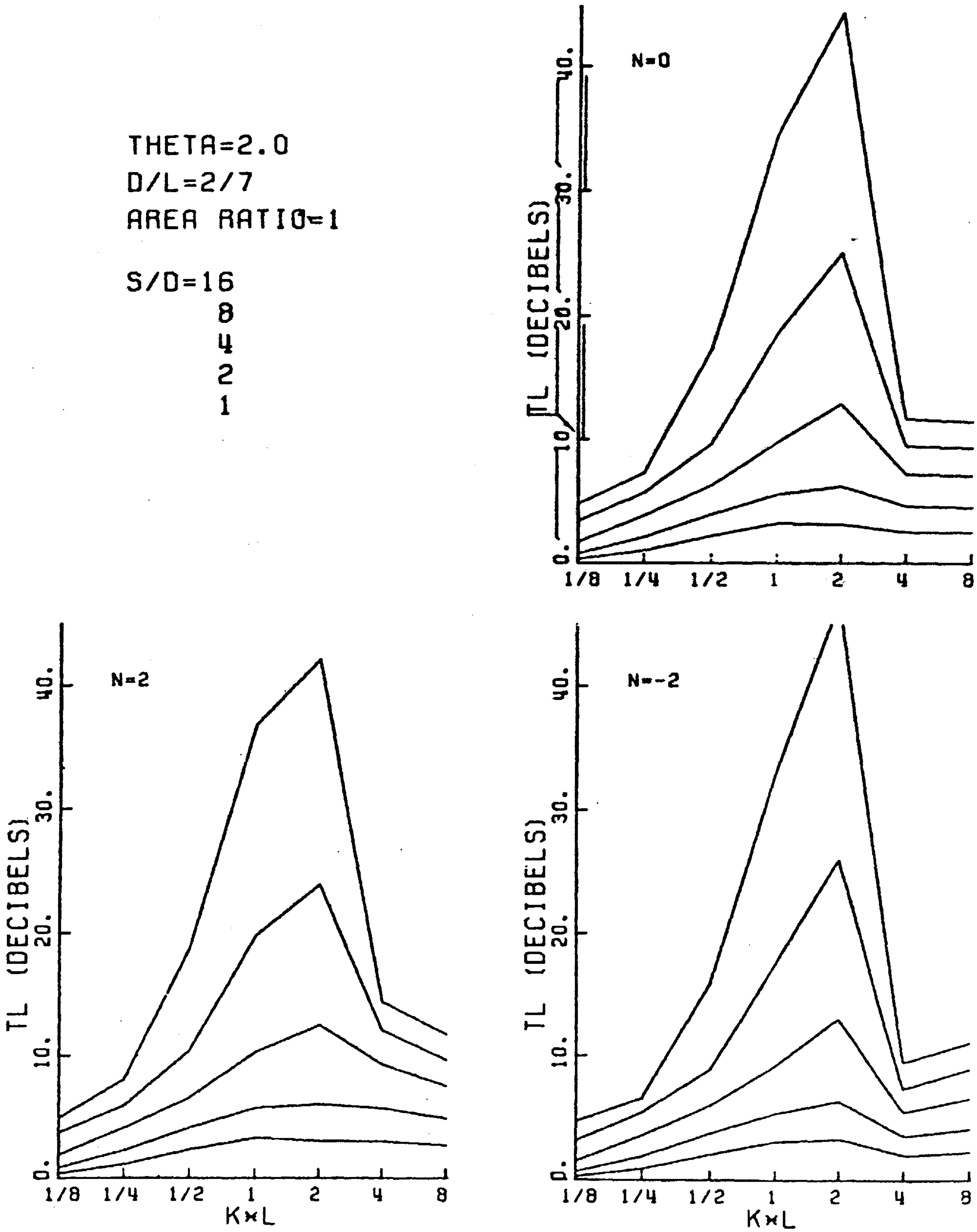

THETA=2.0
D/L=2/7
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8 1/4 1/2 1 2 4 8
K*L

Figure 3.10

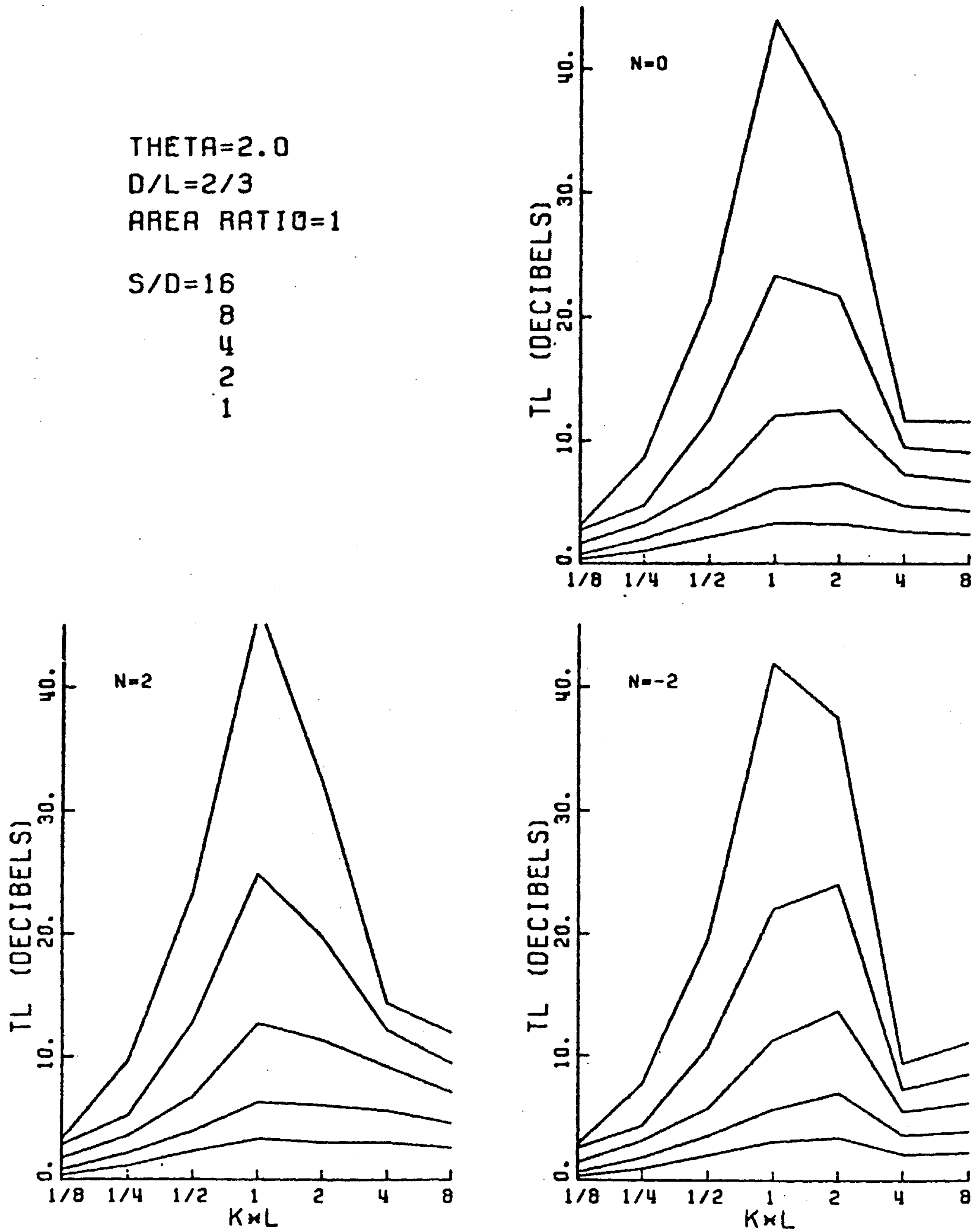
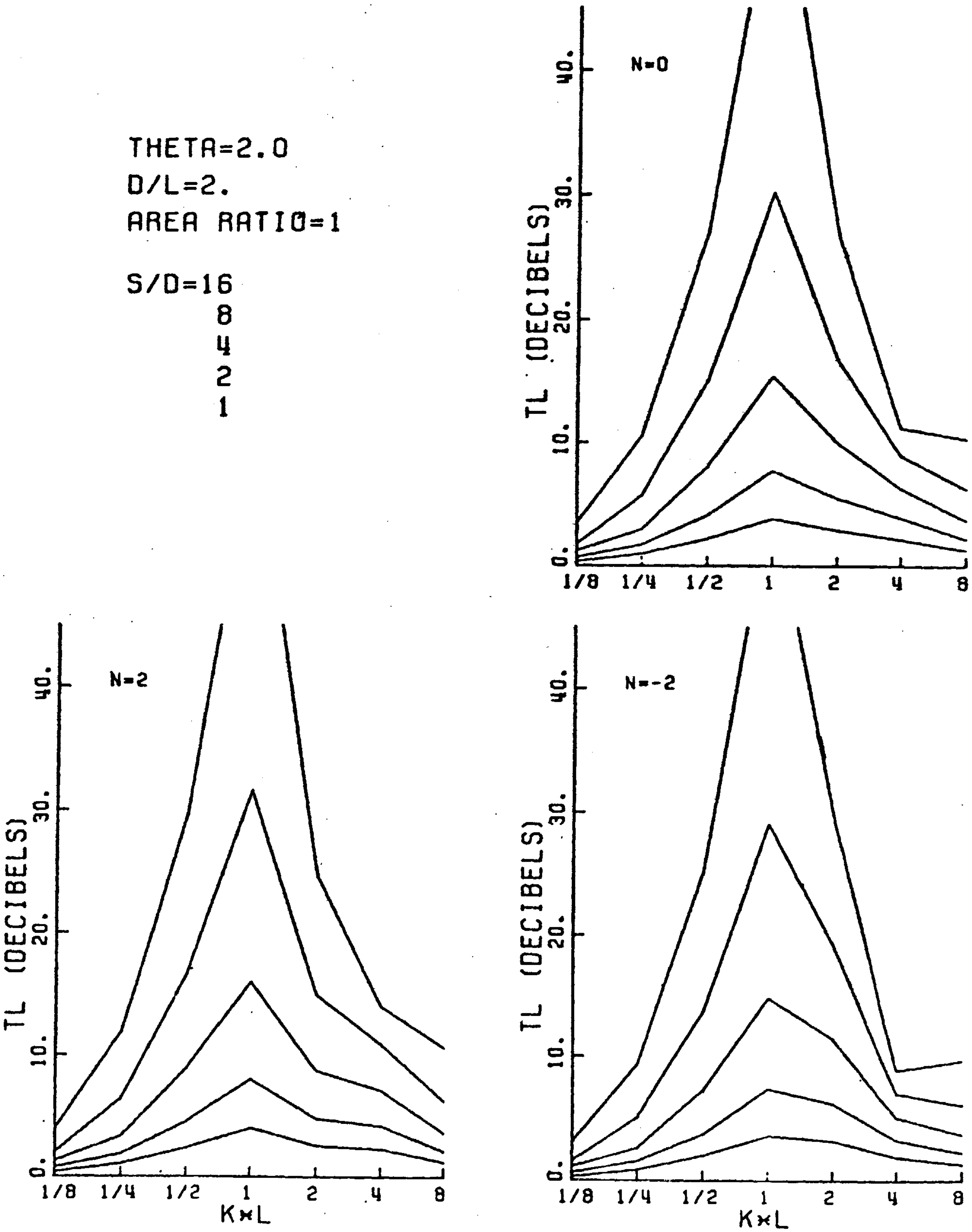
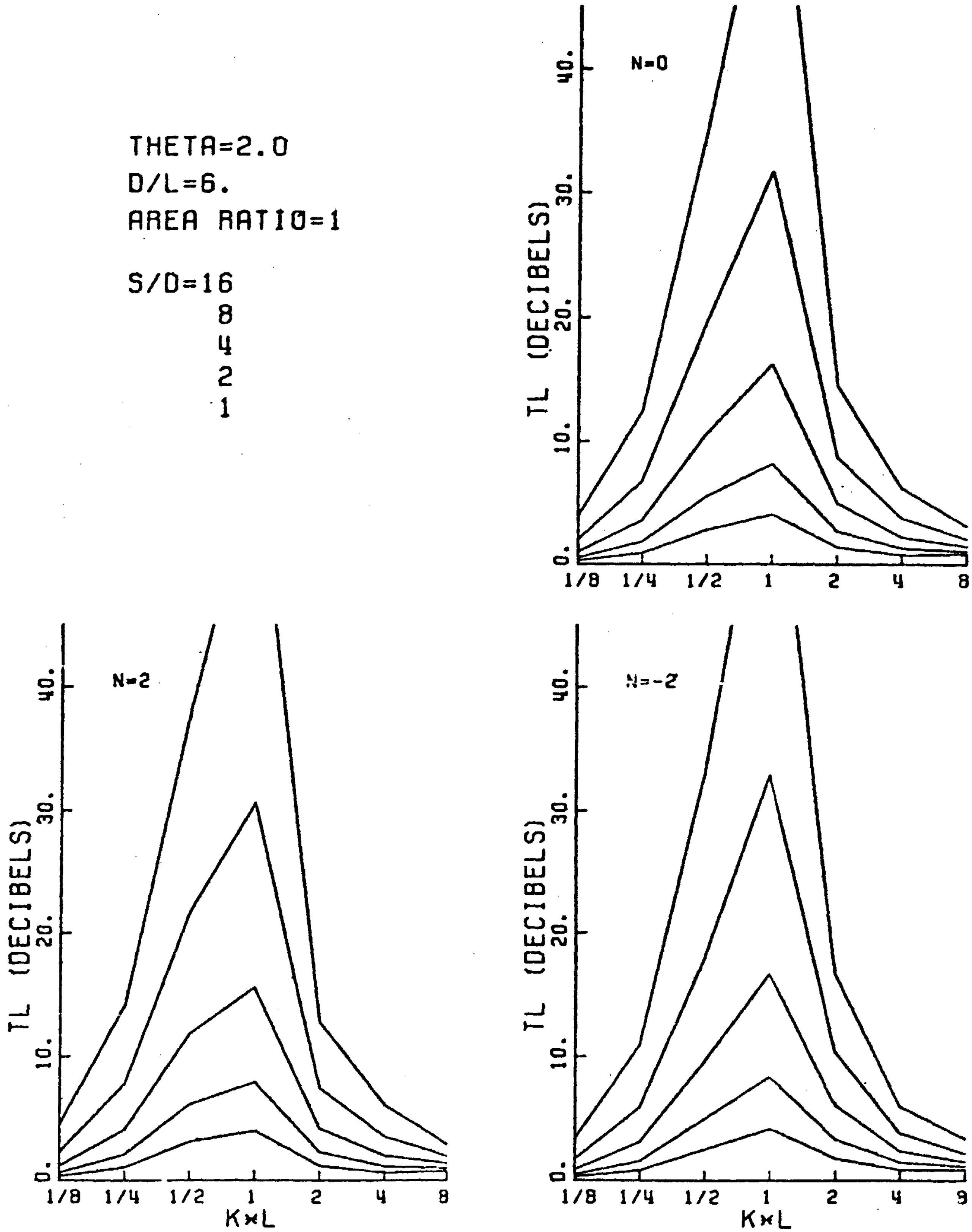

THETA=2.0
D/L=2/3
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.11



THETA=2.0
D/L=2.
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8 1/4 1/2 1 2 4 8
K*L

Figure 3.12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D/L=6.
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8 1/4 1/2 1 2 4 8
K*L

Figure 3.13

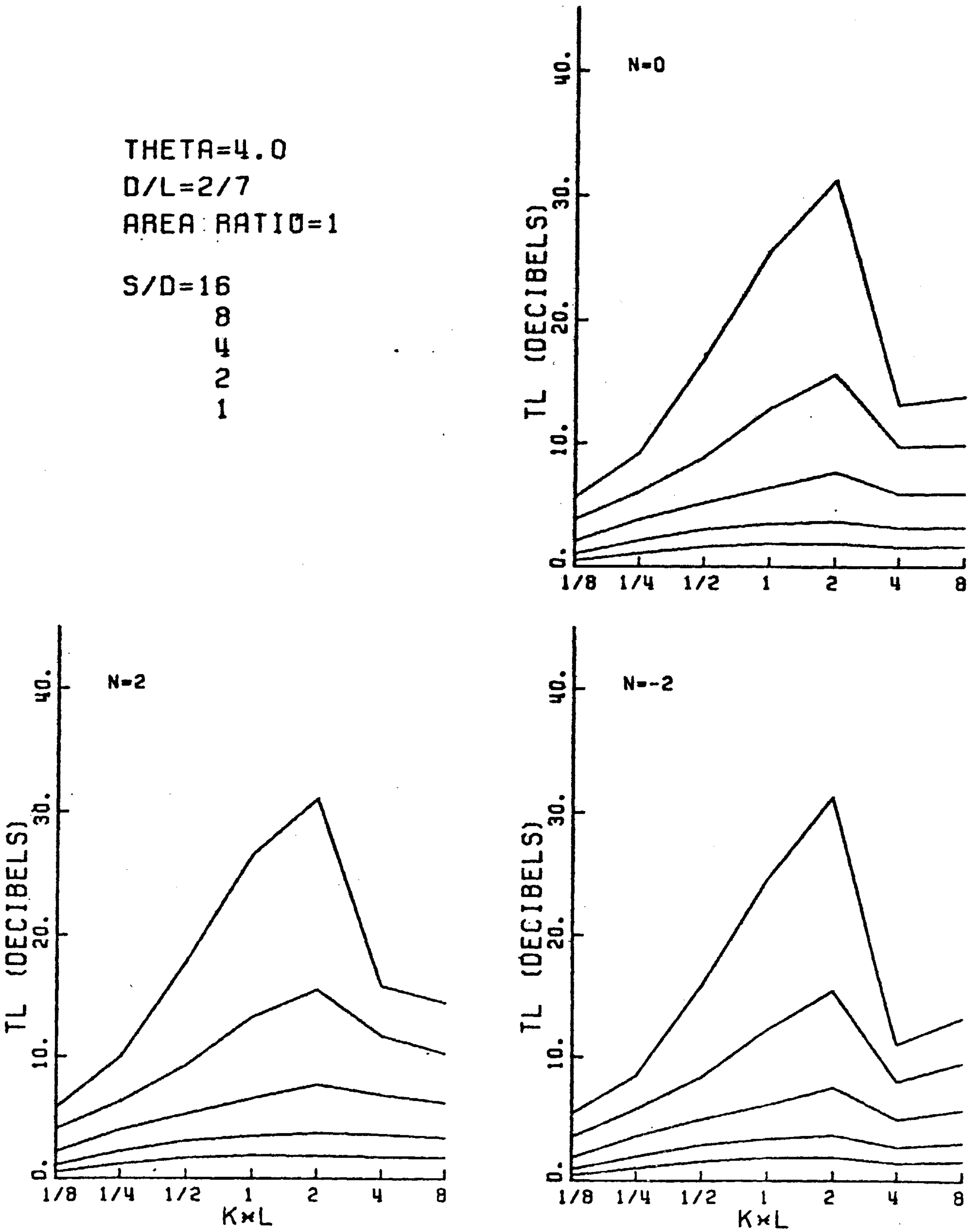

THETA=4.0
D/L=2/7
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.14

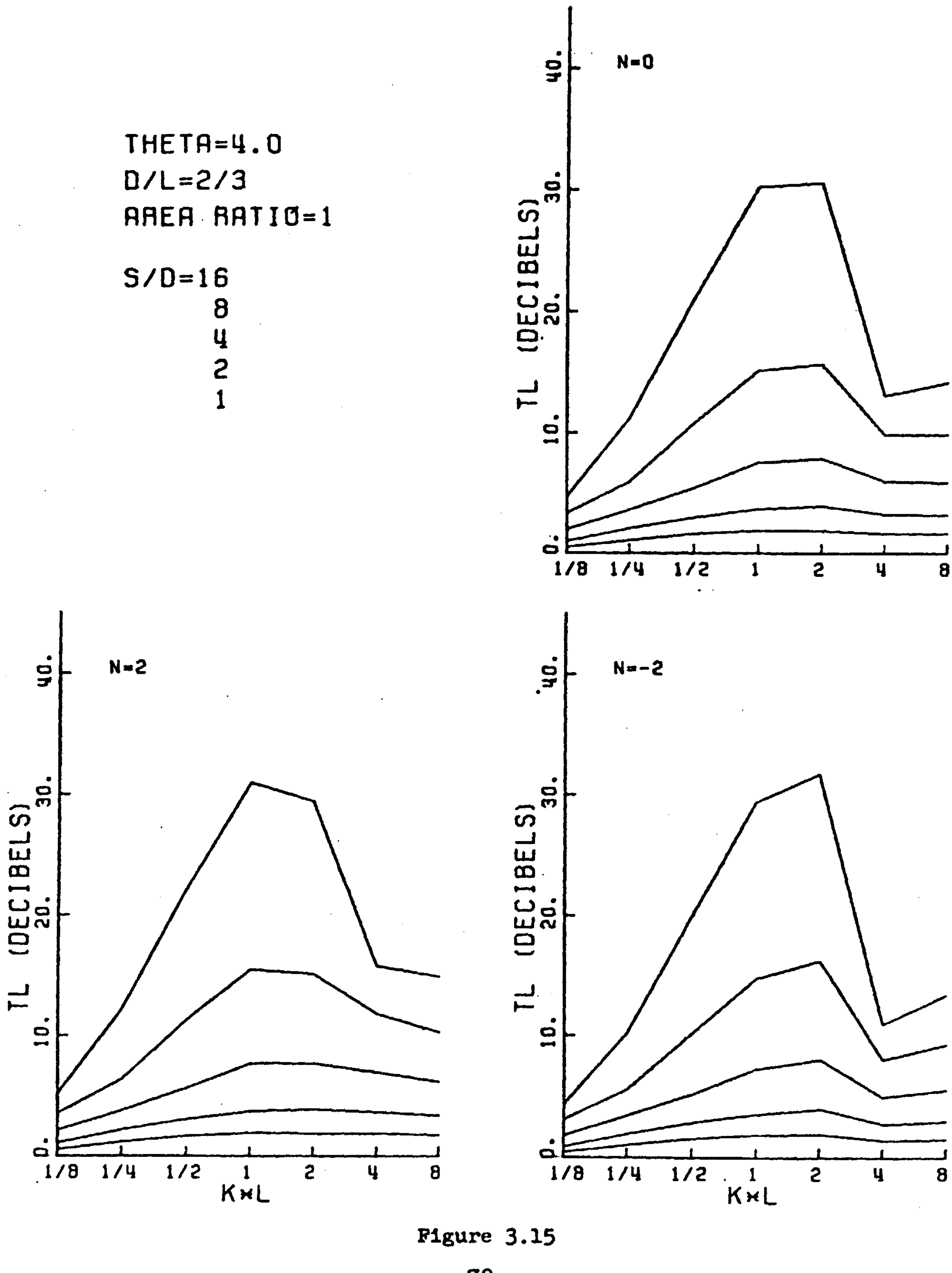

THETA=4.0
D/L=2/3
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.15

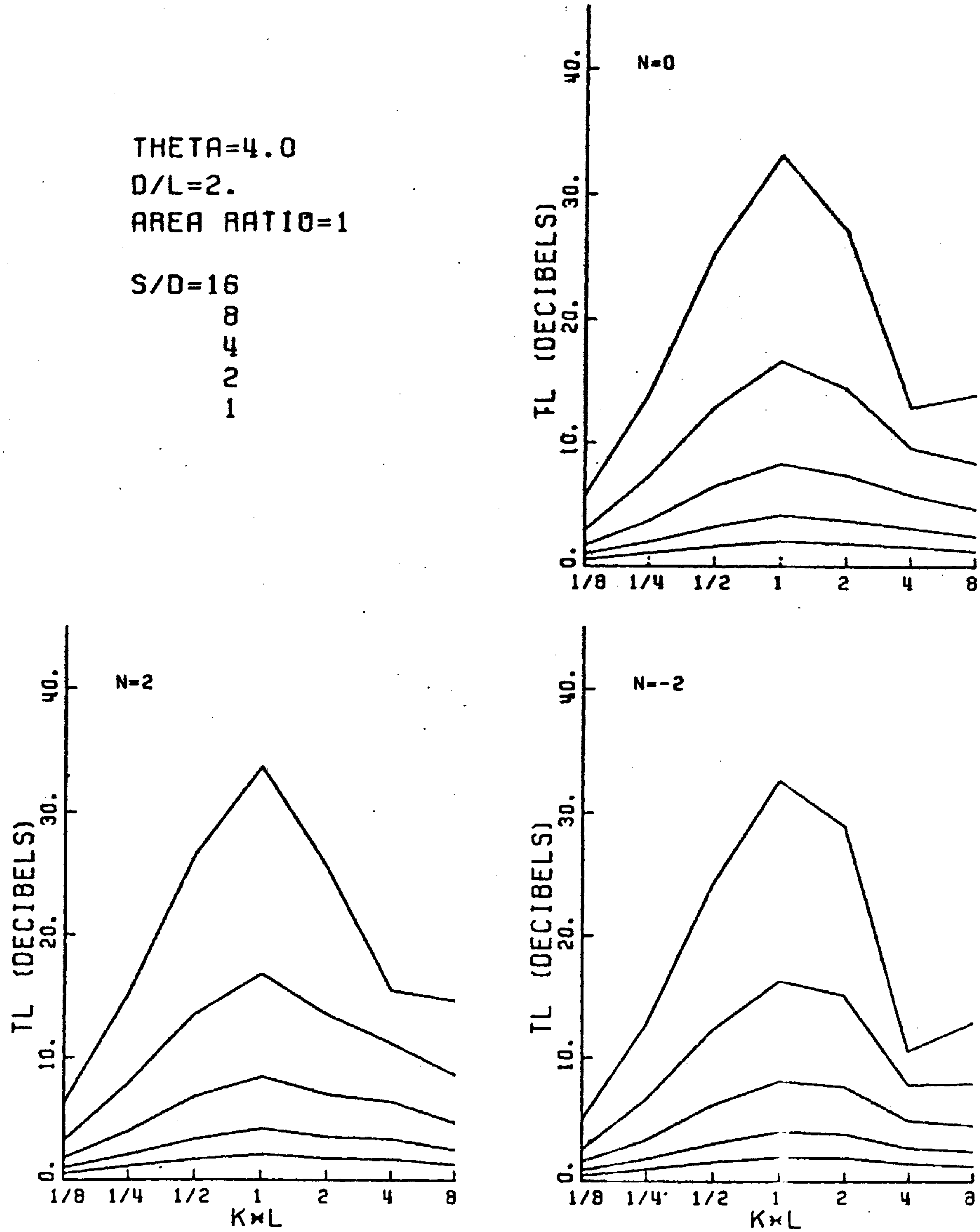

THETA=4.0
D/L=2.
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.16

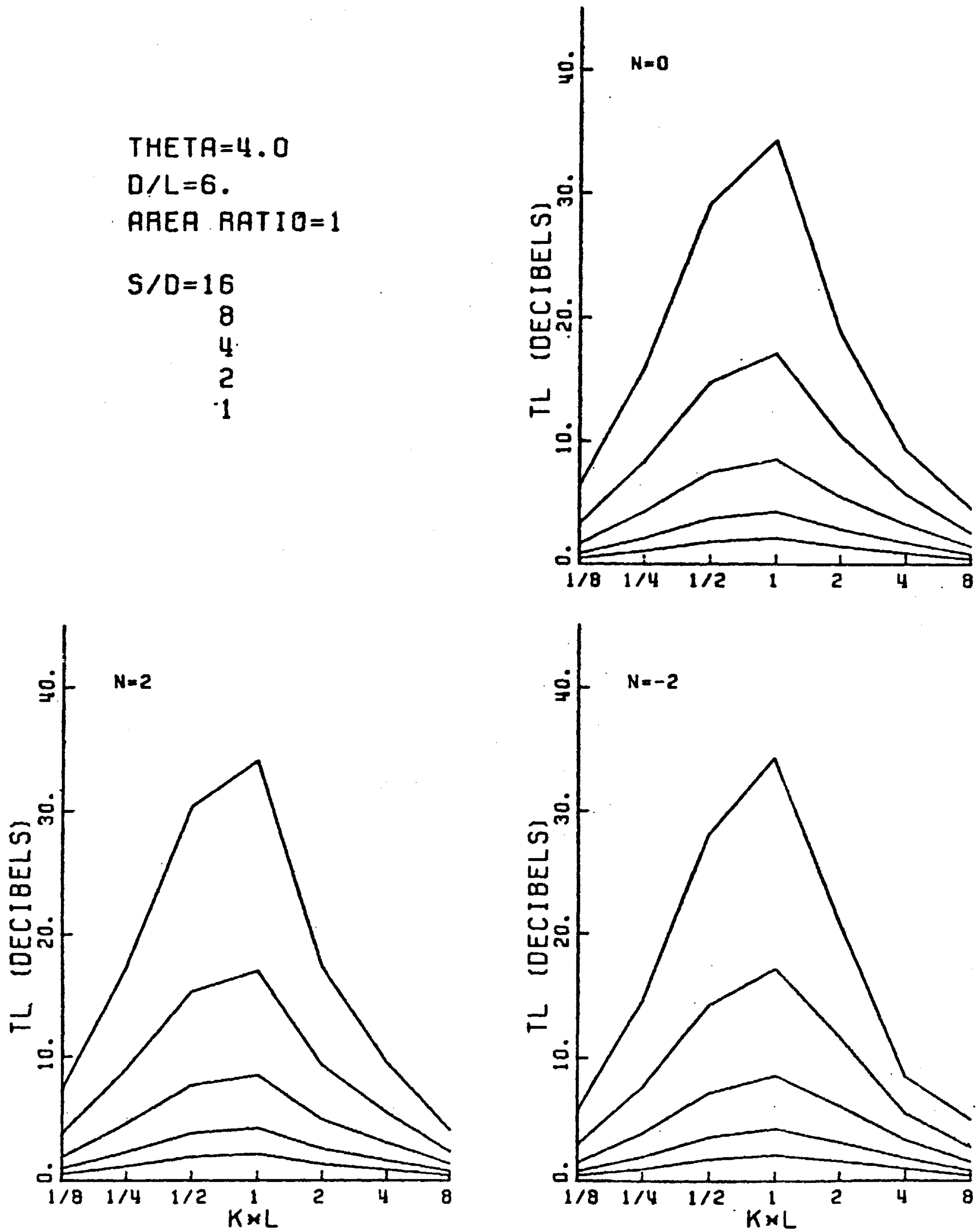

THETA=4.0
D/L=6.
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.17

Figures 3.18-3.49. Octave band TL vs kL in a rectangular duct lined with a porous liner. The format is the same as in Figures 3.2-3.17 except that four more values of θ are included here. (For definition of θ, see Eq. 2.11.)

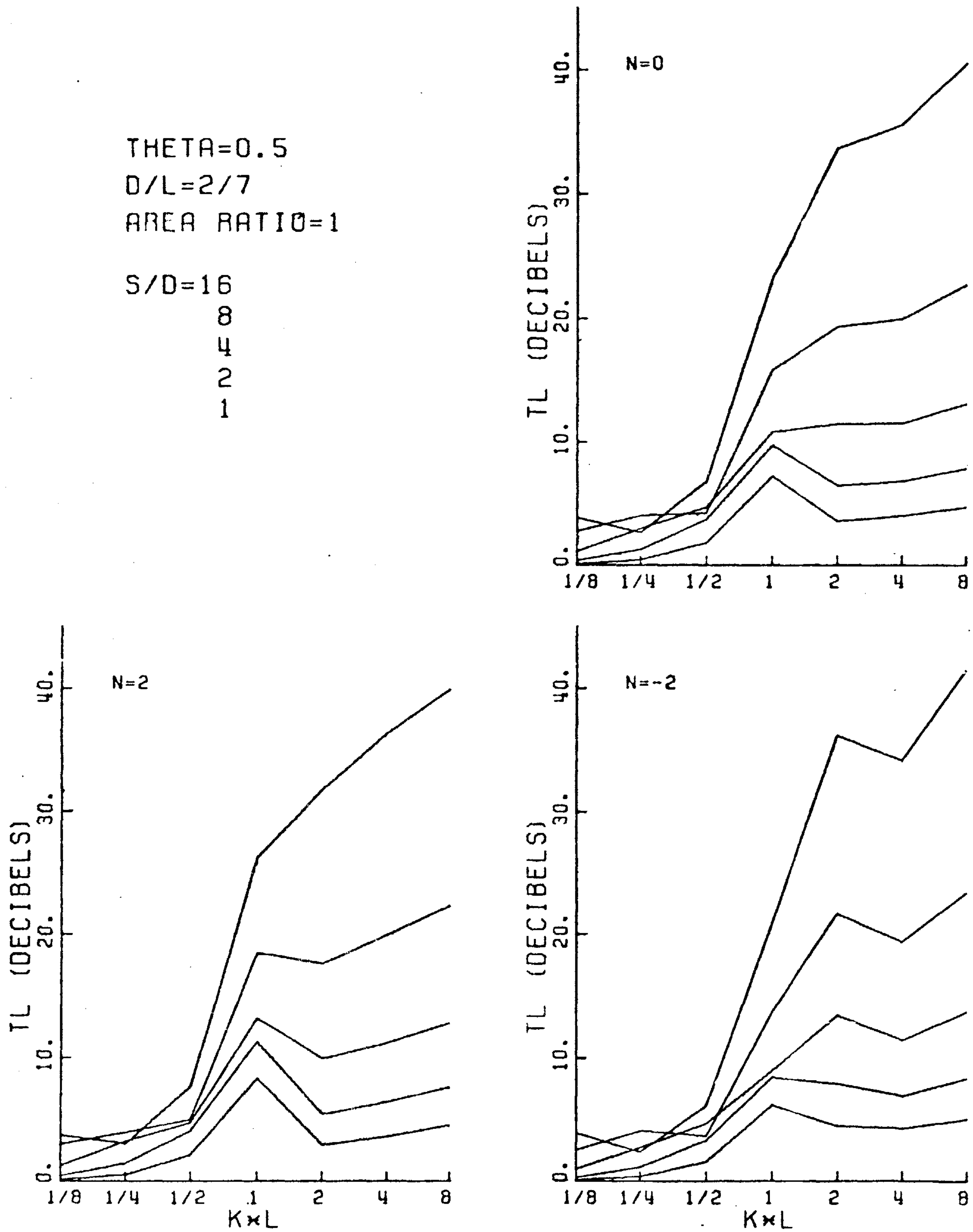

THETA=0.5
D/L=2/7
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.18

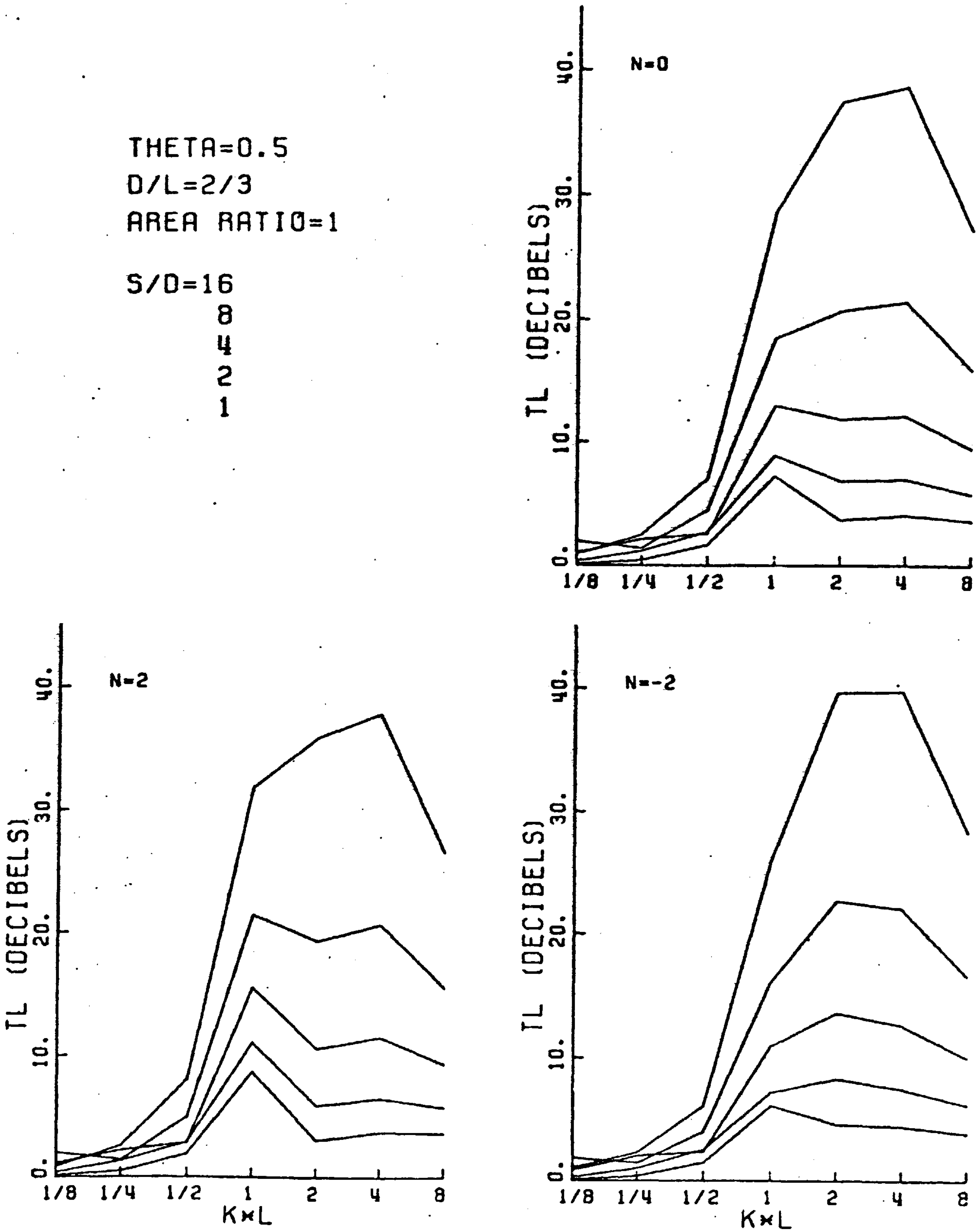

THETA=0.5
D/L=2/3
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.19

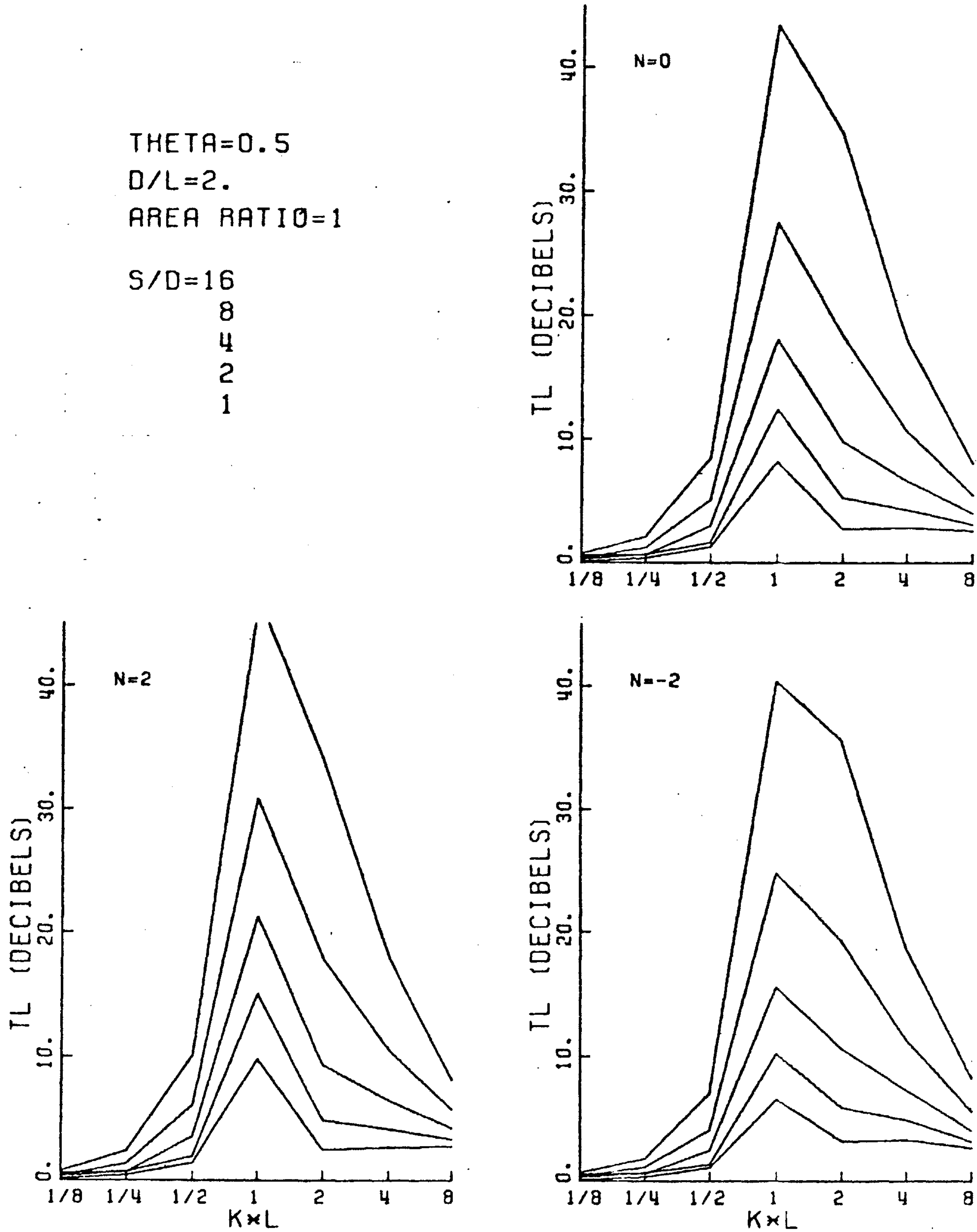

THETA=0.5
D/L=2.
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.20

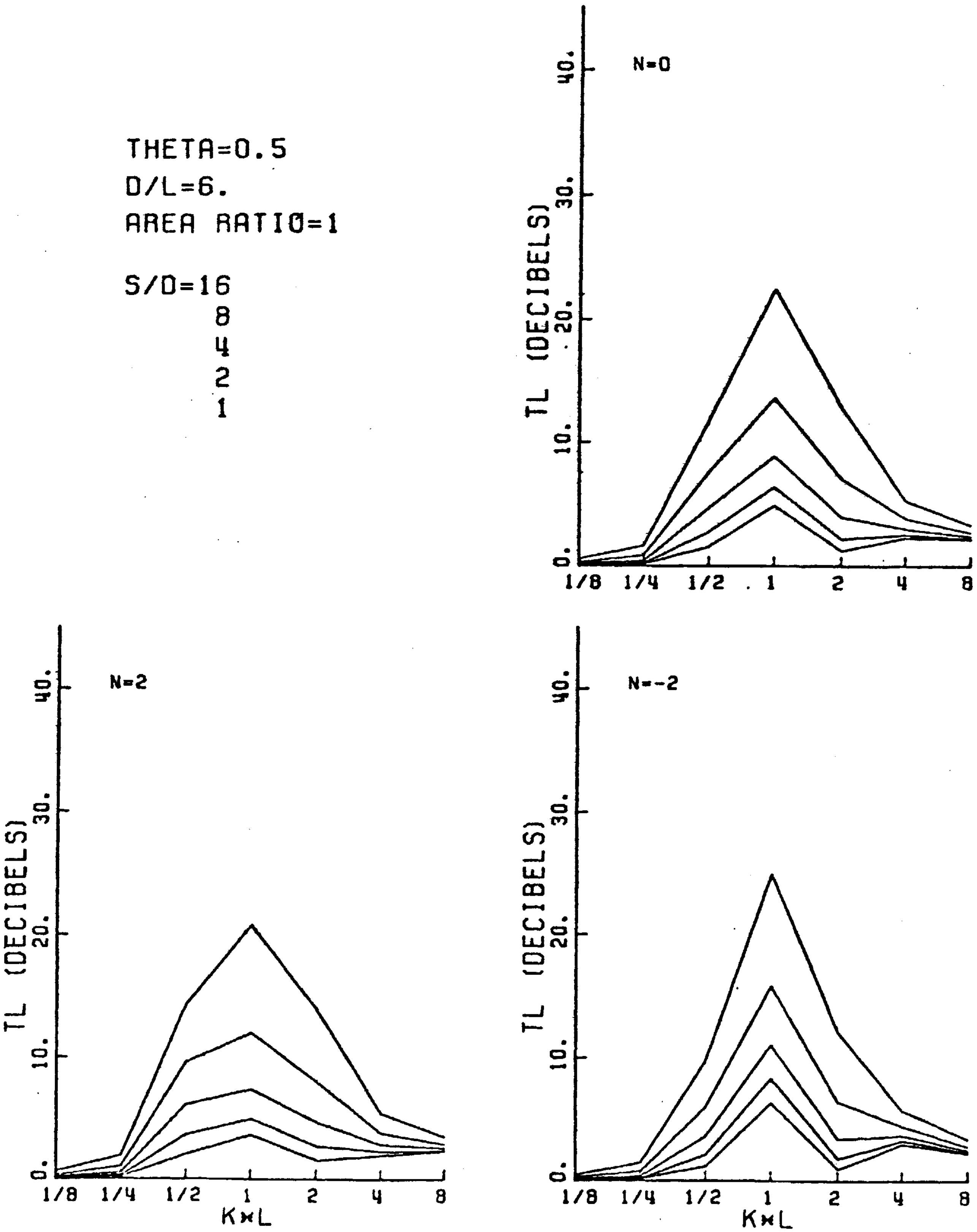

THETA=0.5
D/L=6.
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.21

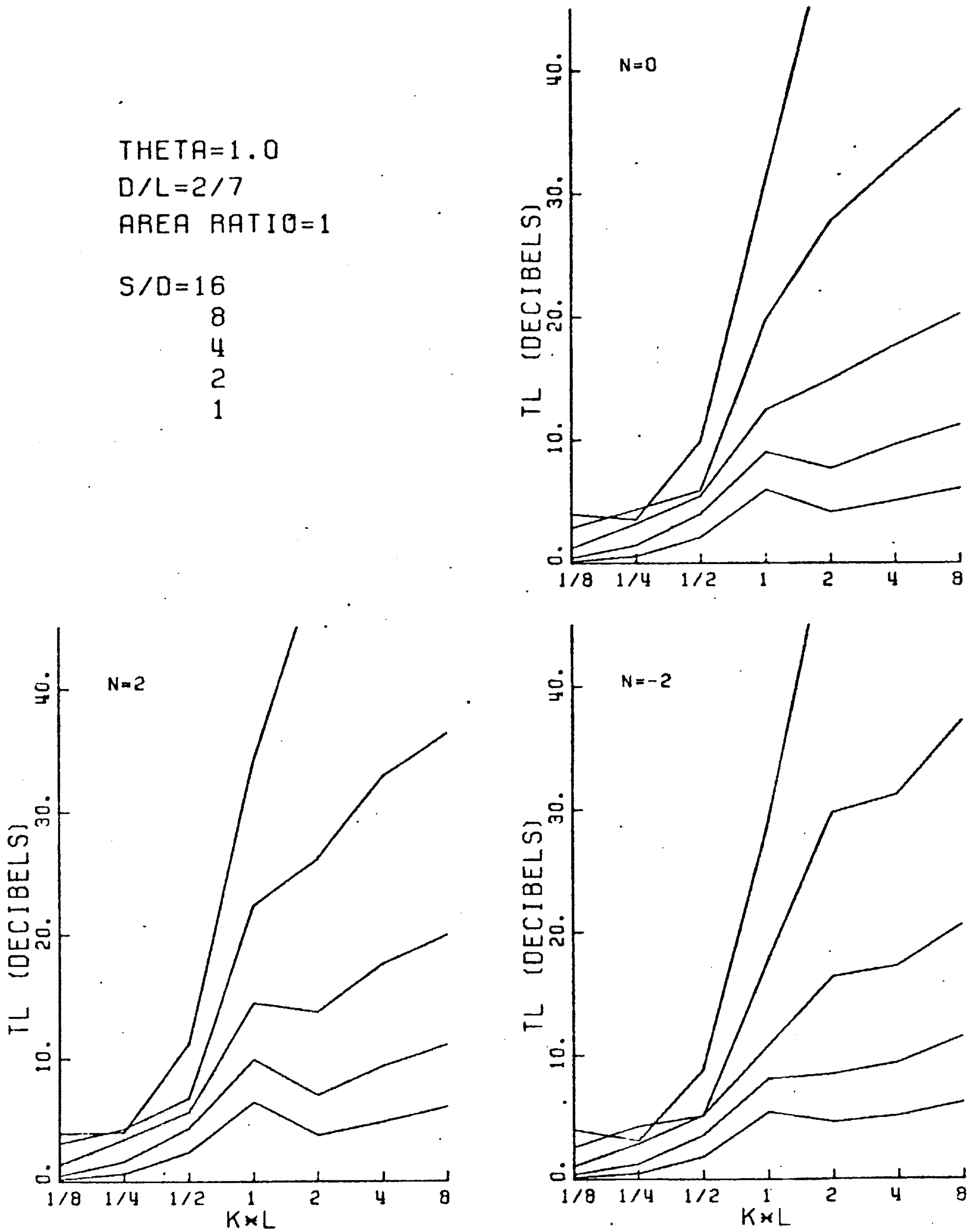

THETA=1.0
D/L=2/7
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.22

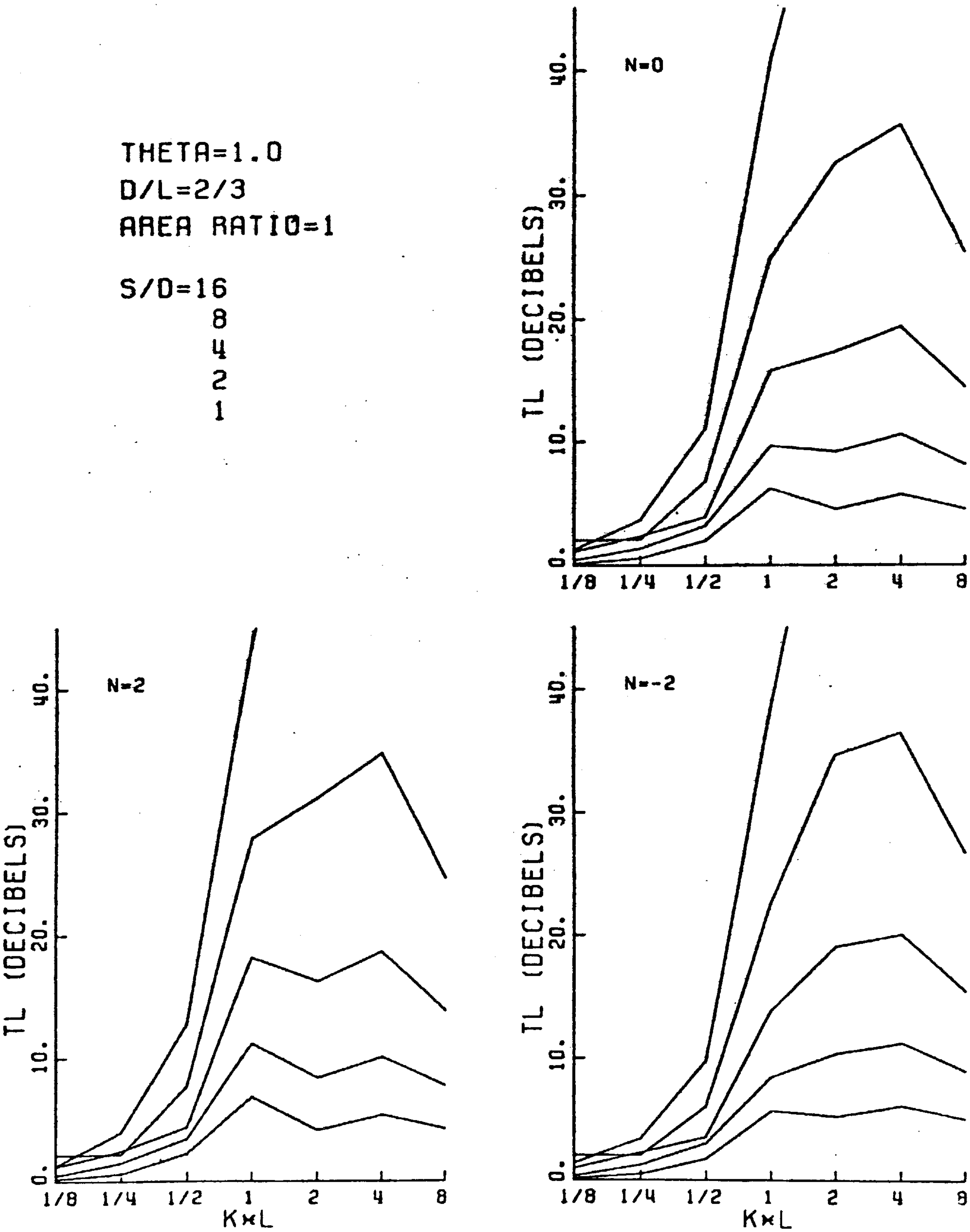

THETA=1.0
D/L=2/3
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.23

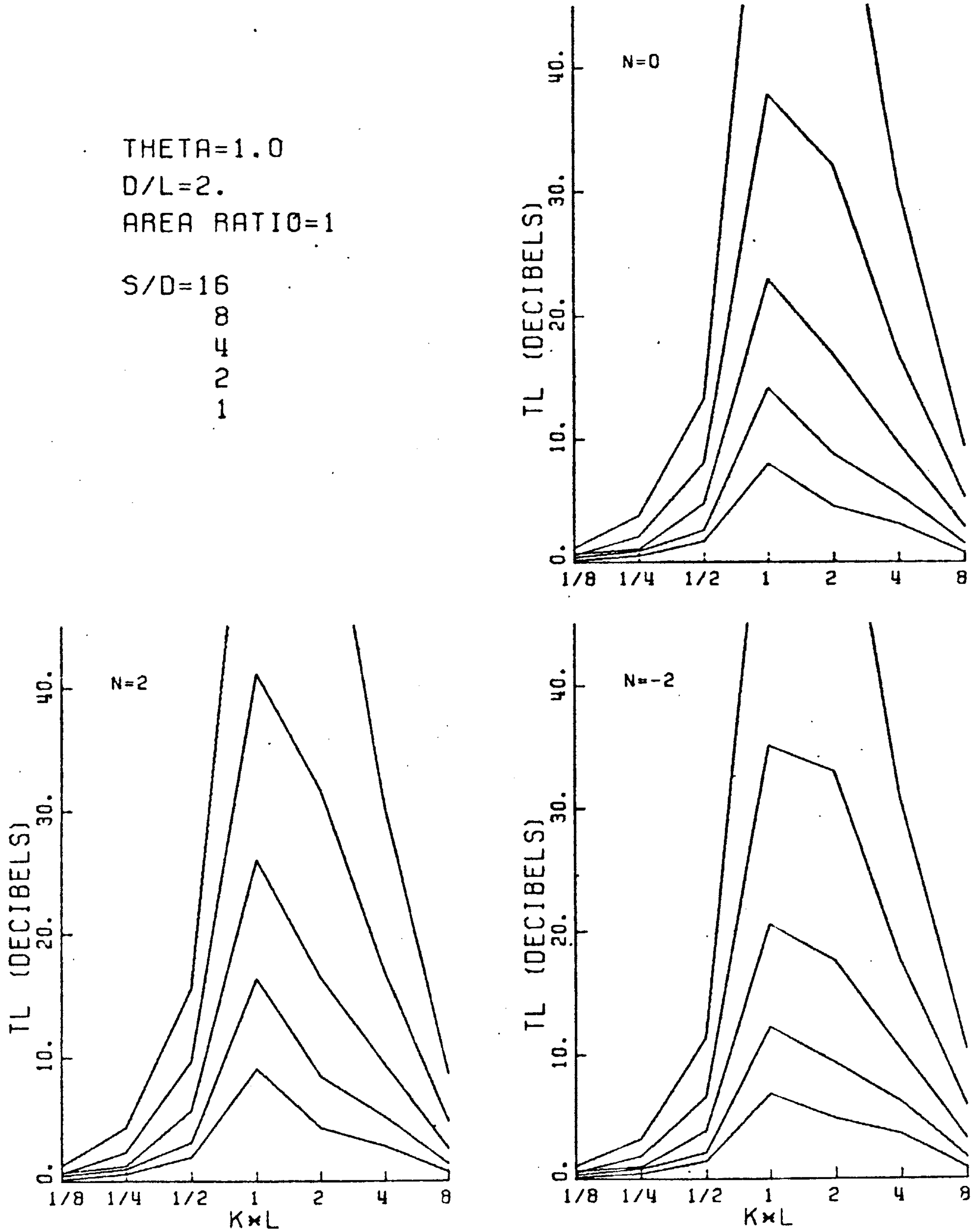

THETA=1.0
D/L=2.
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.24

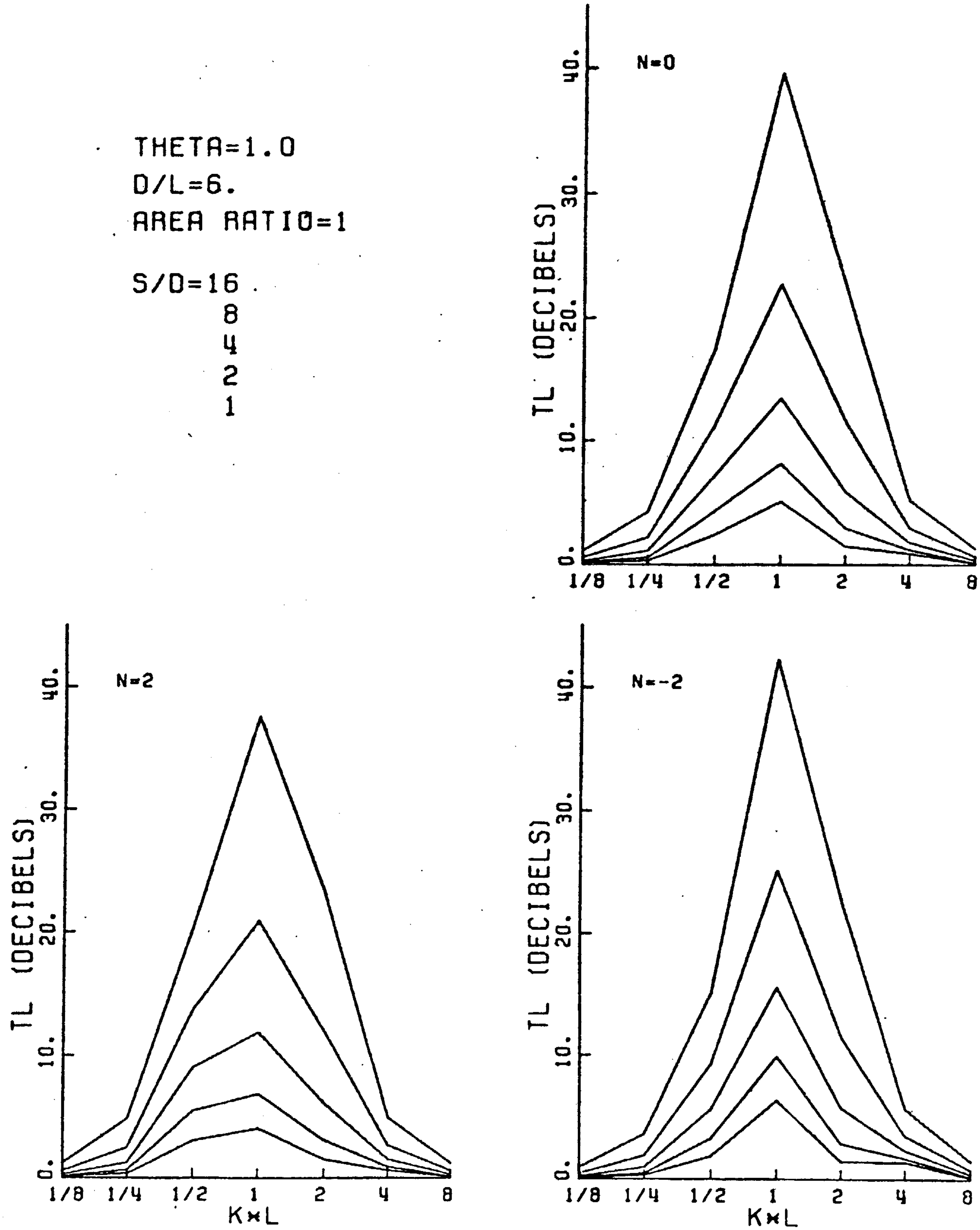

THETA=1.0
D/L=6.
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.25

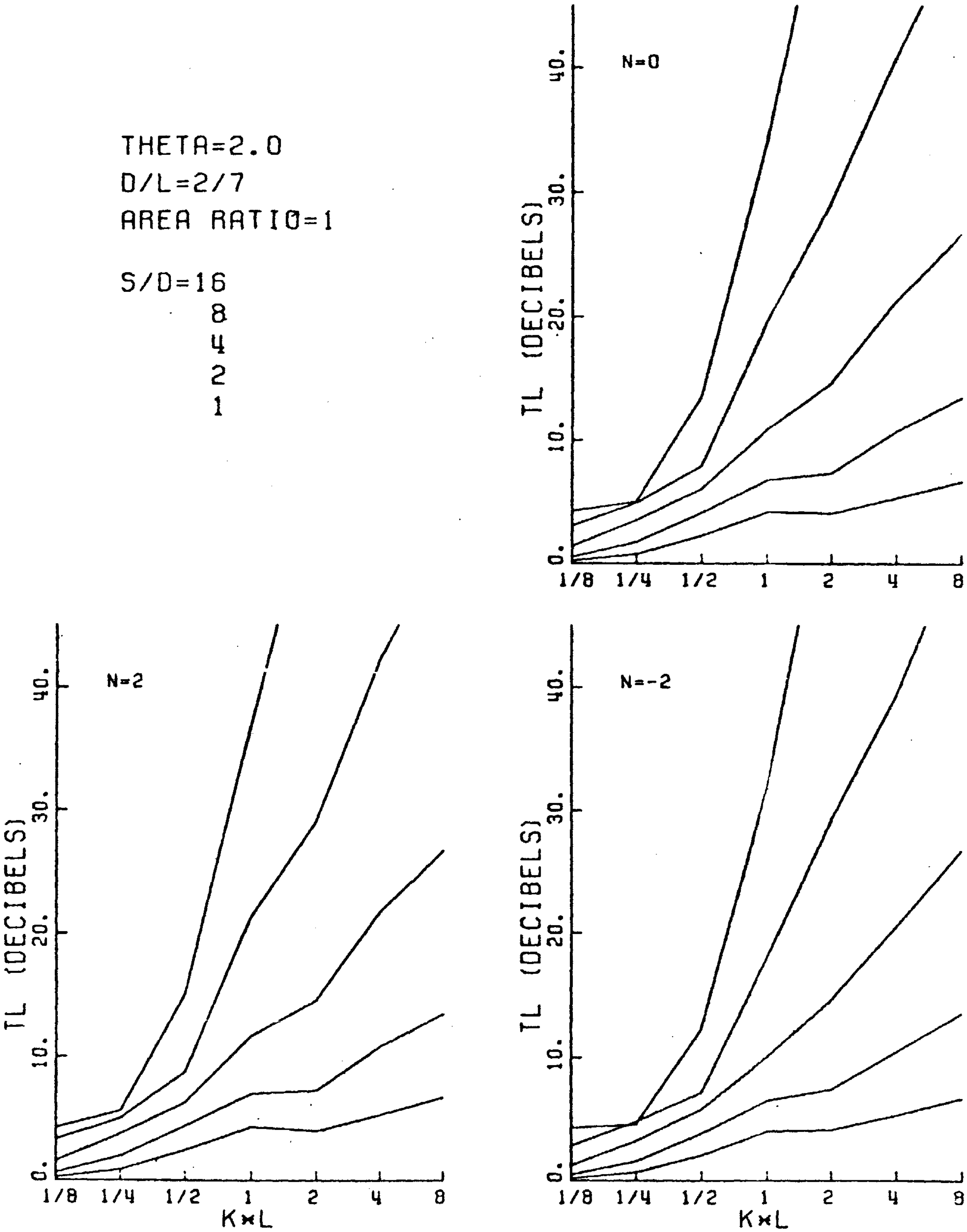

THETA=2.0
D/L=2/7
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.26

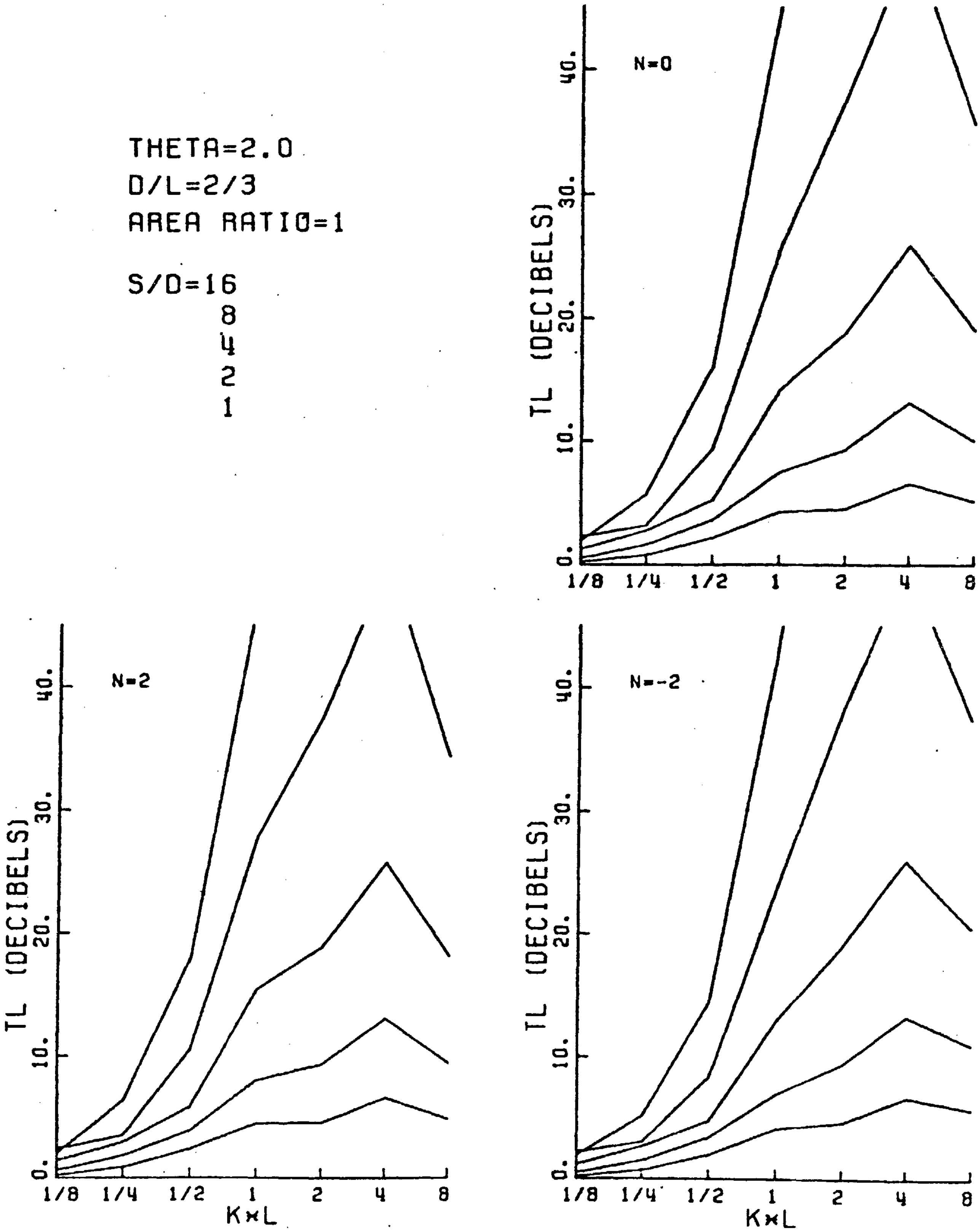
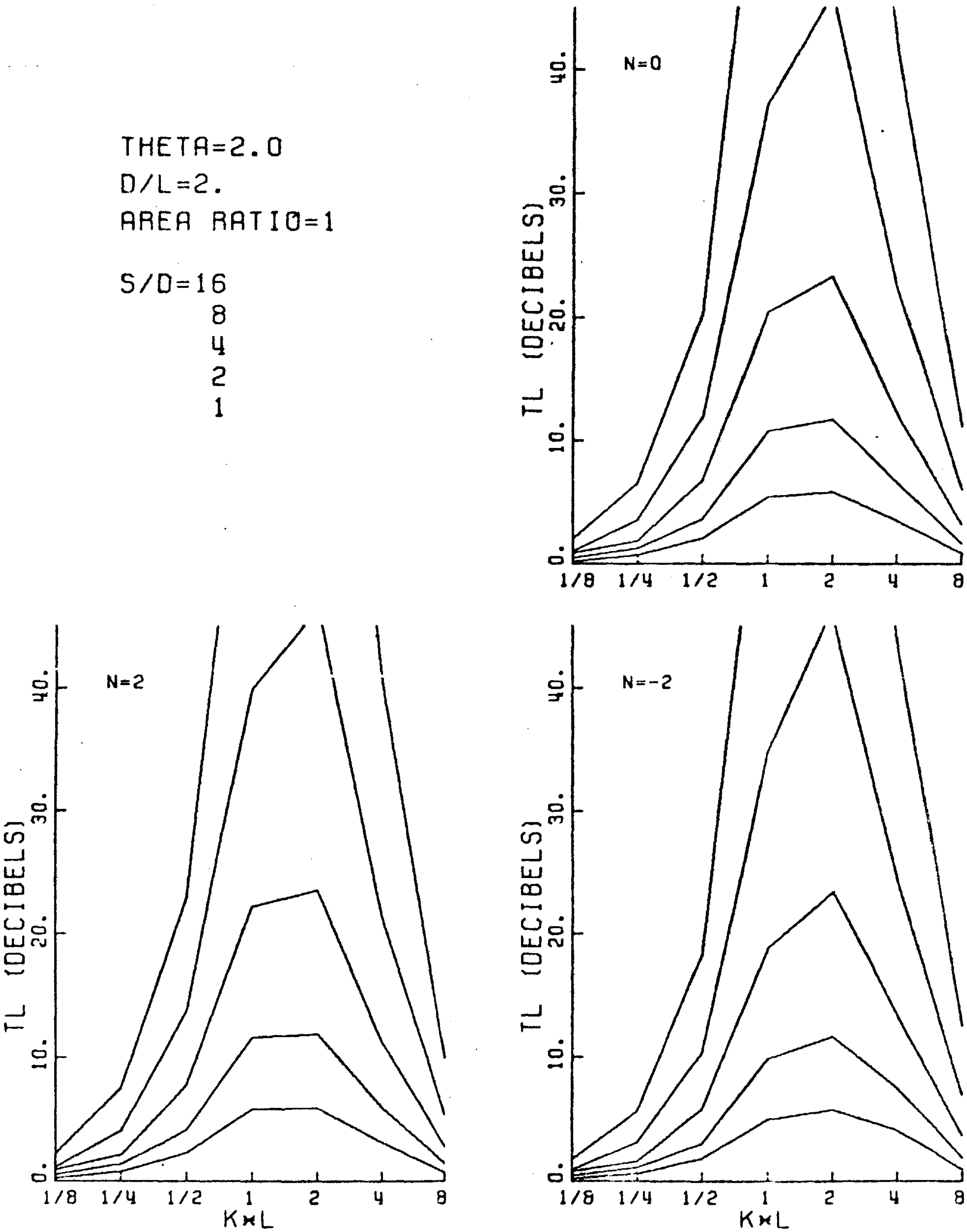

THETA=2.0
D/L=2/3
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.27



THETA=2.0
D/L=2.
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.28

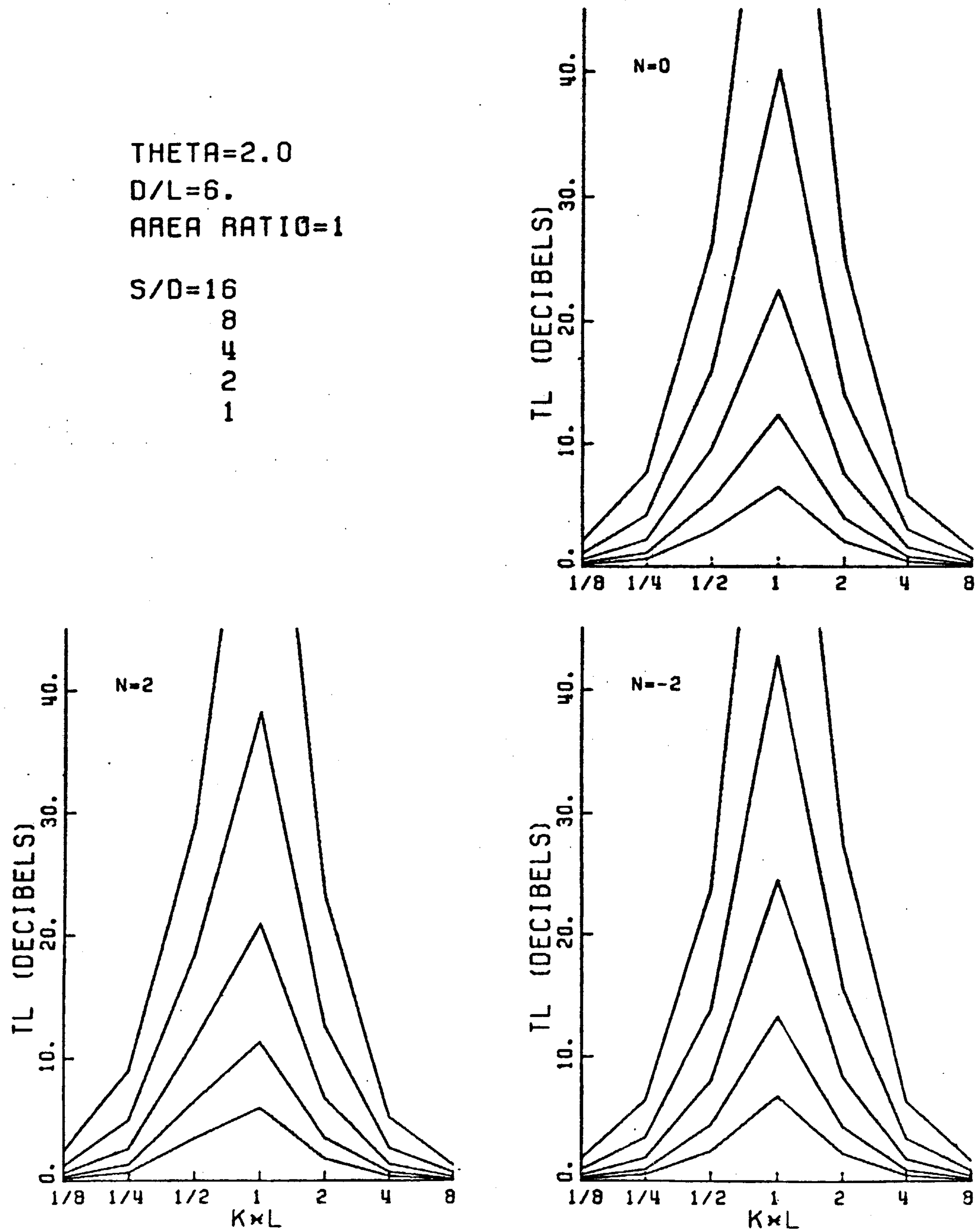

THETA=2.0
D/L=6.
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.29

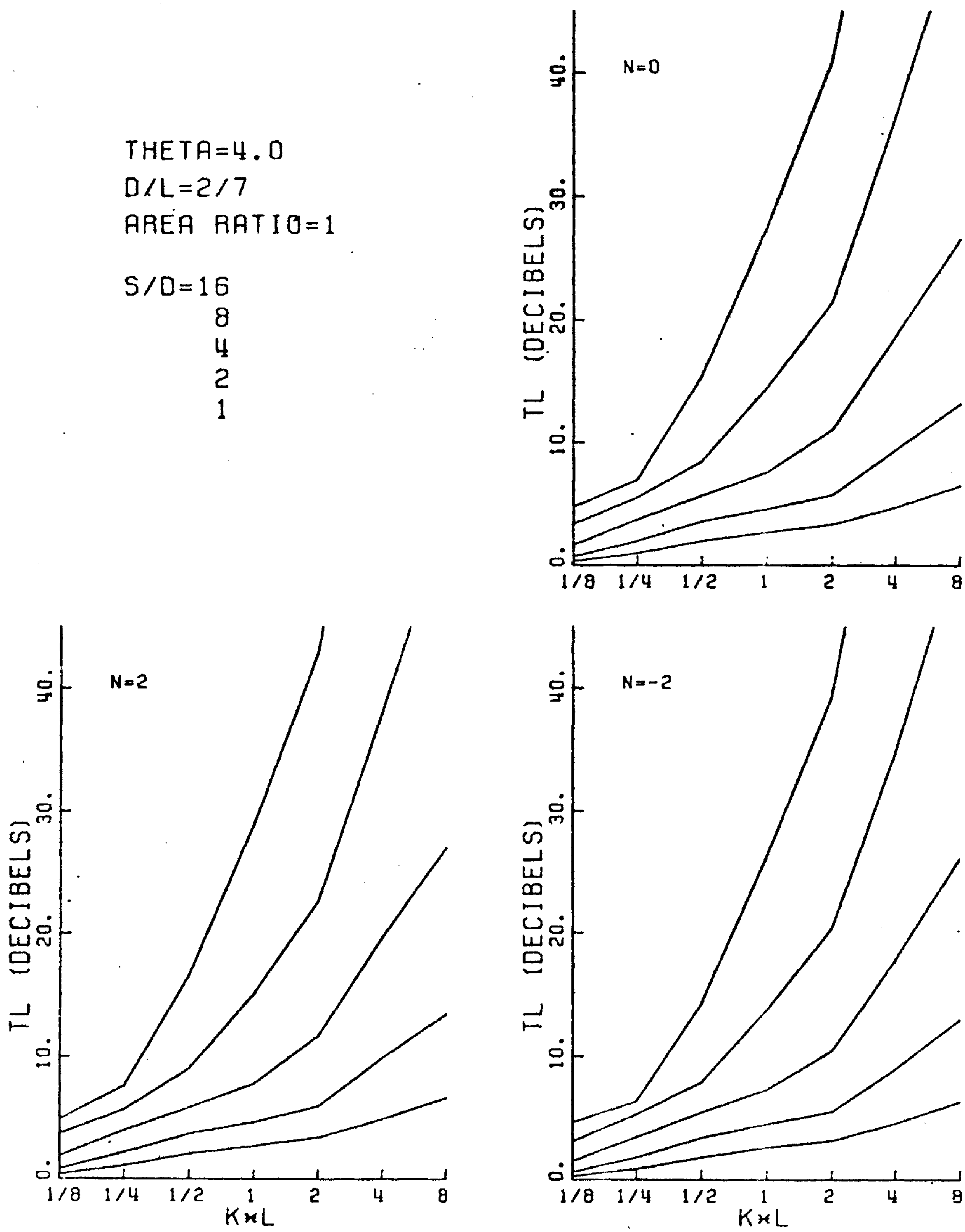

THETA=4.0
D/L=2/7
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.30

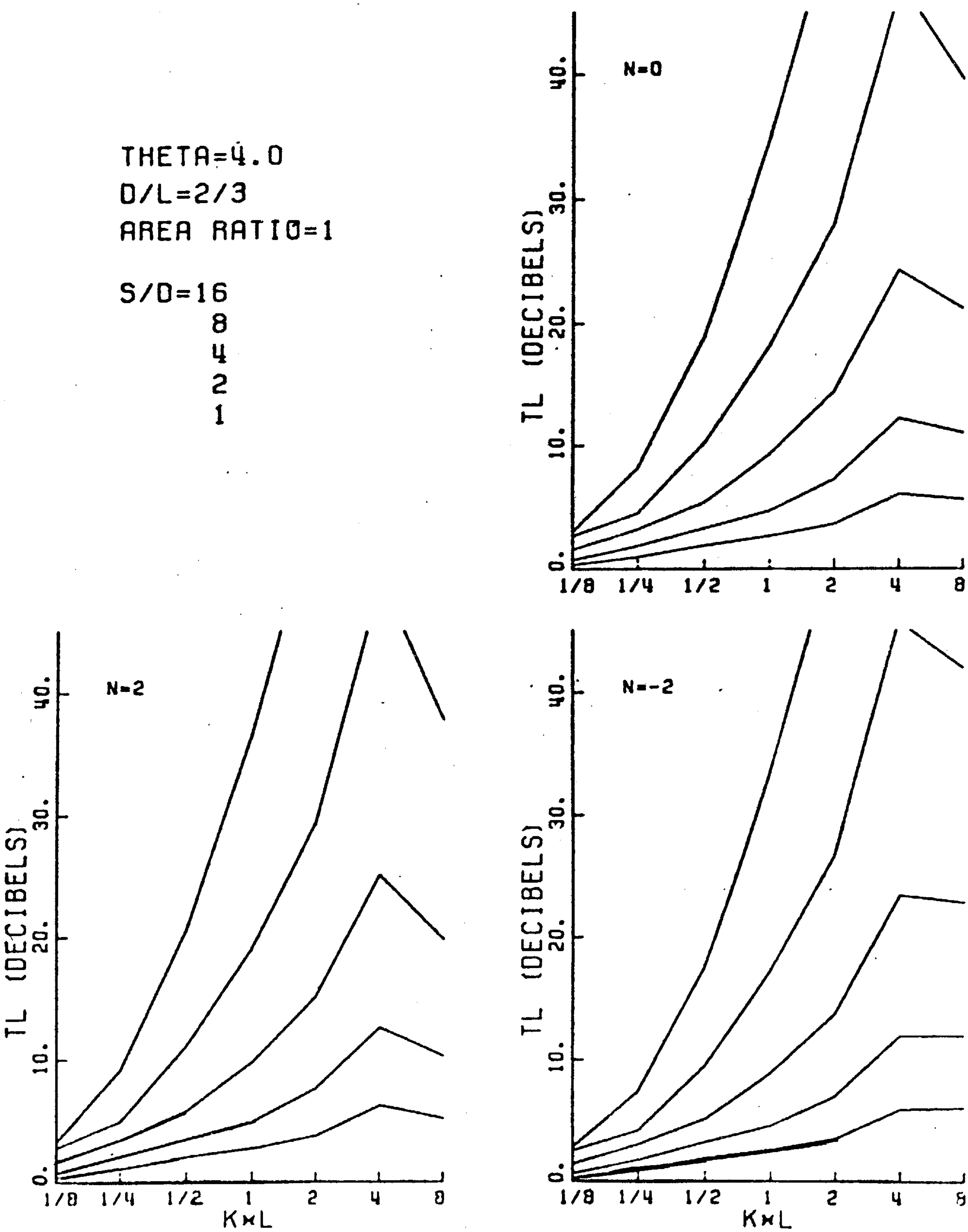
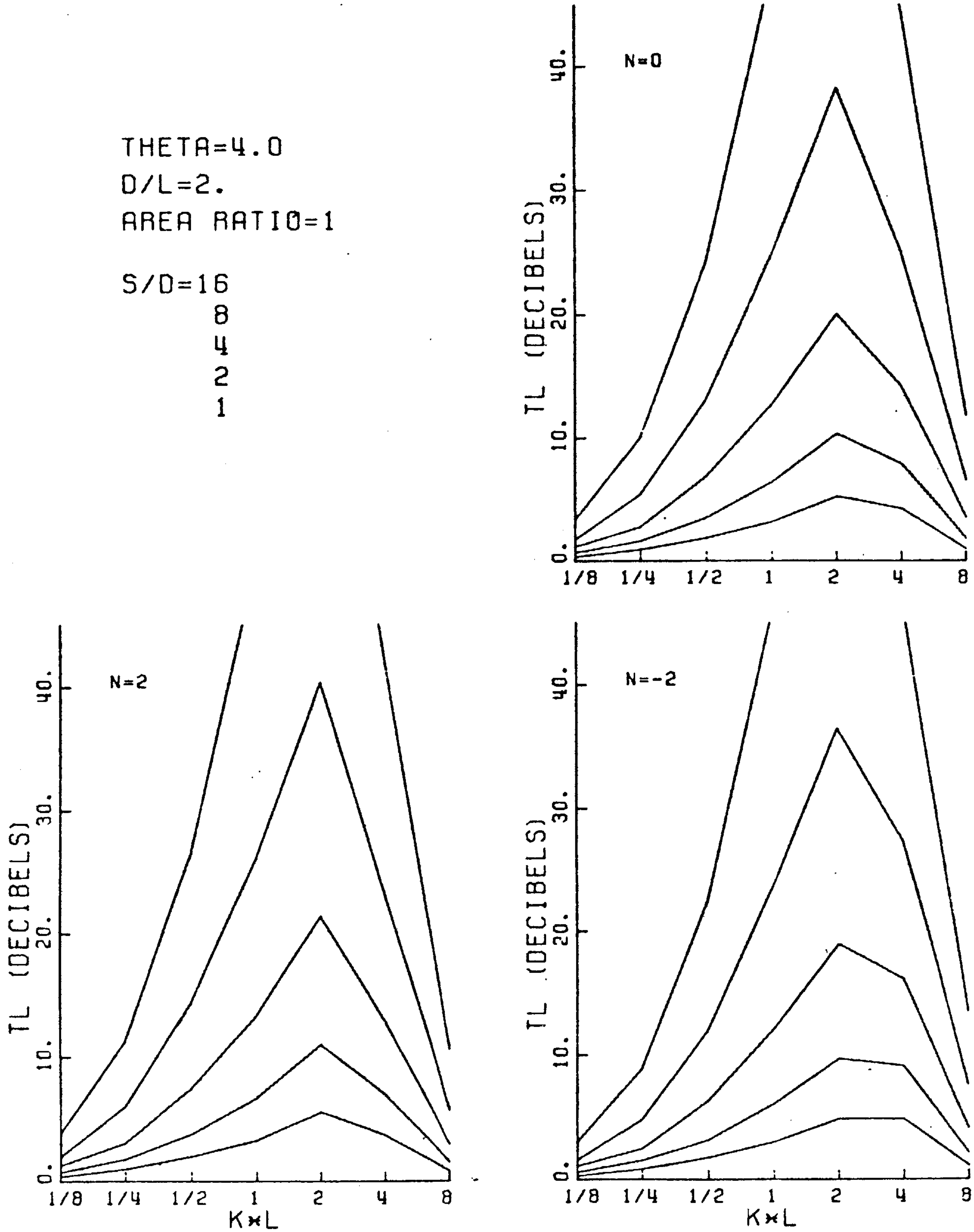

THETA=4.0
D/L=2/3
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.31



THETA=4.0
D/L=2.
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.32

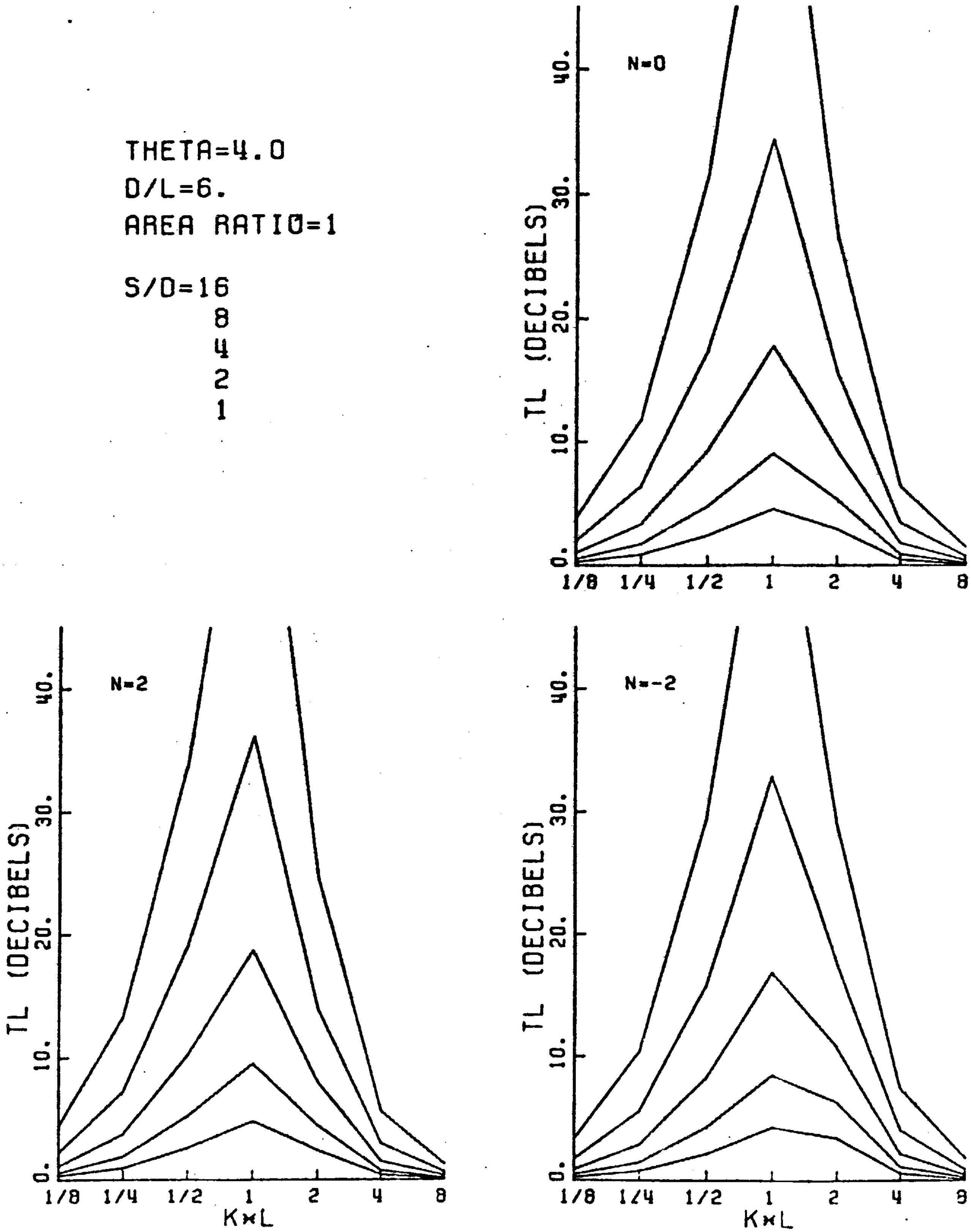

THETA=4.0
D/L=6.
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.33

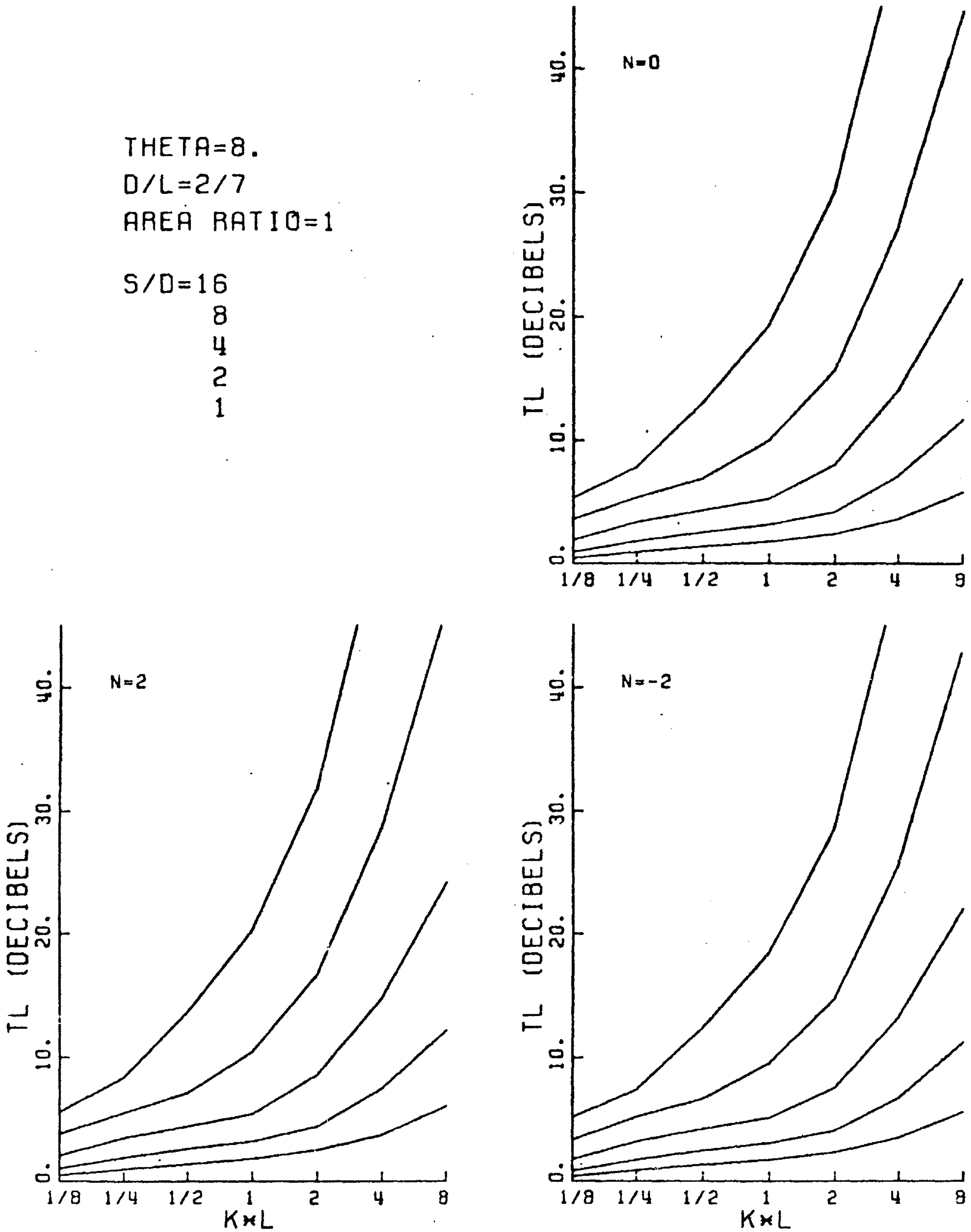

THETA=8.
D/L=2/7
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.34

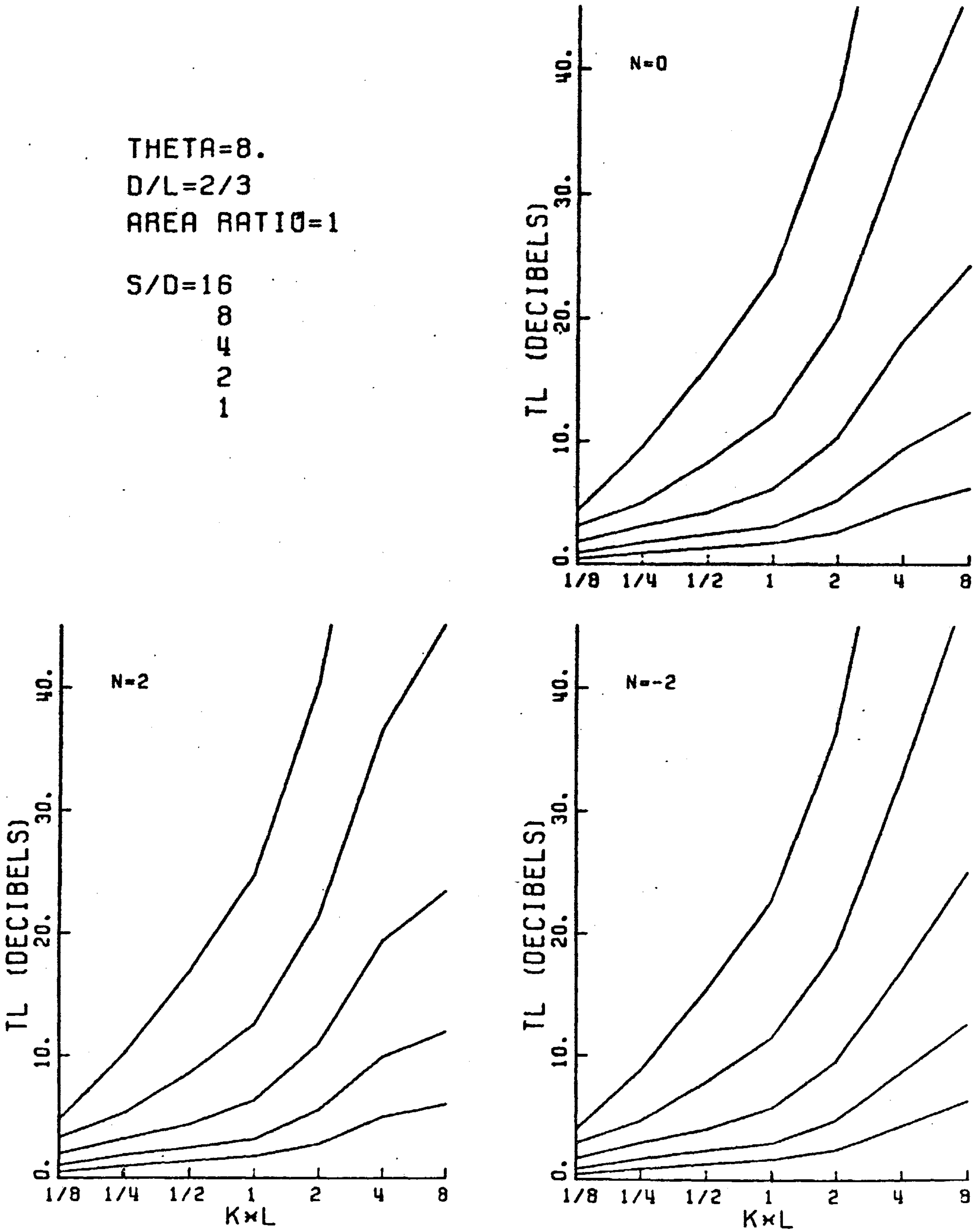

THETA=8.
D/L=2/3
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.35

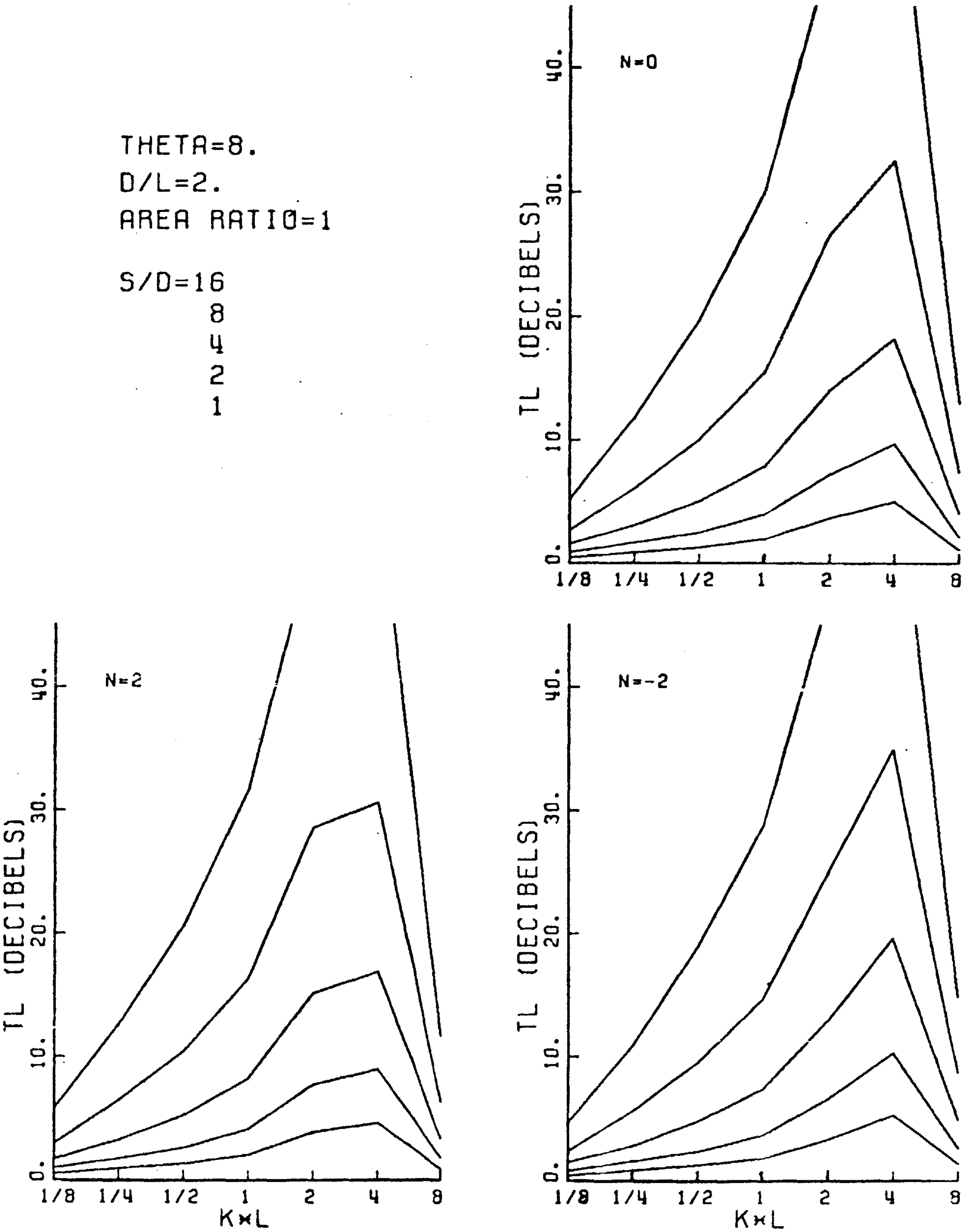

THETA=8.
D/L=2.
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.36

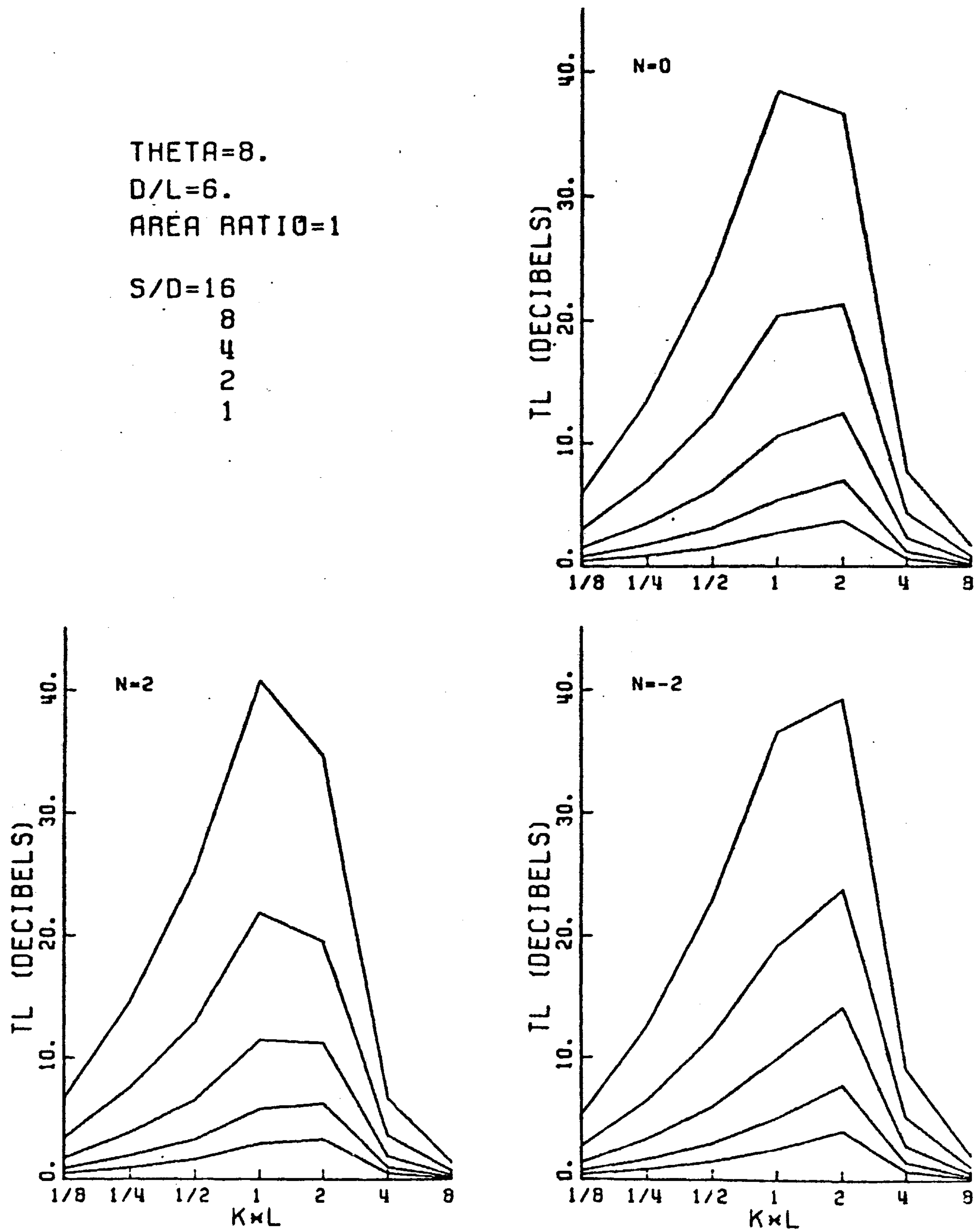

THETA=8.
D/L=6.
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.37

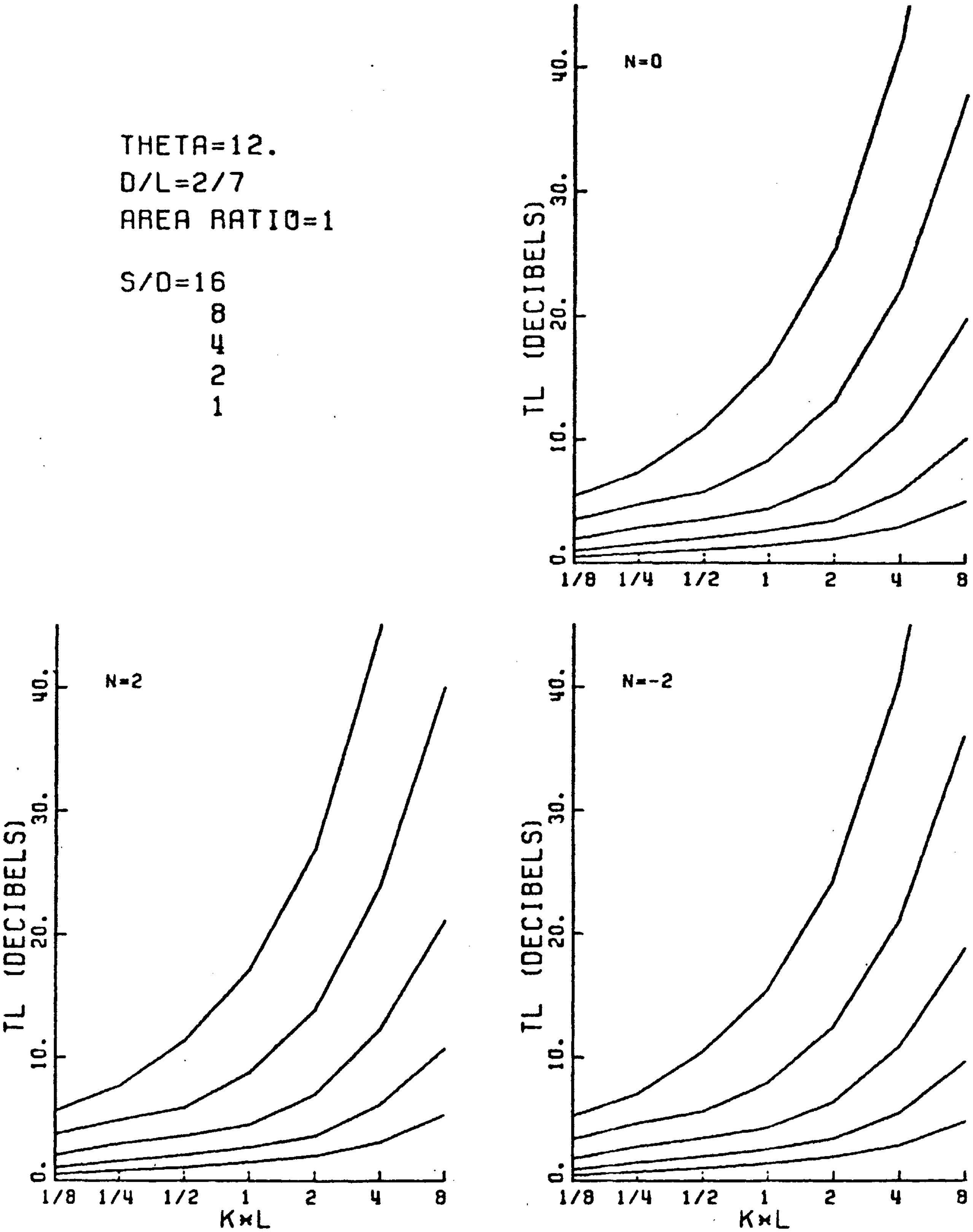

THETA=12.
D/L=2/7
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.38

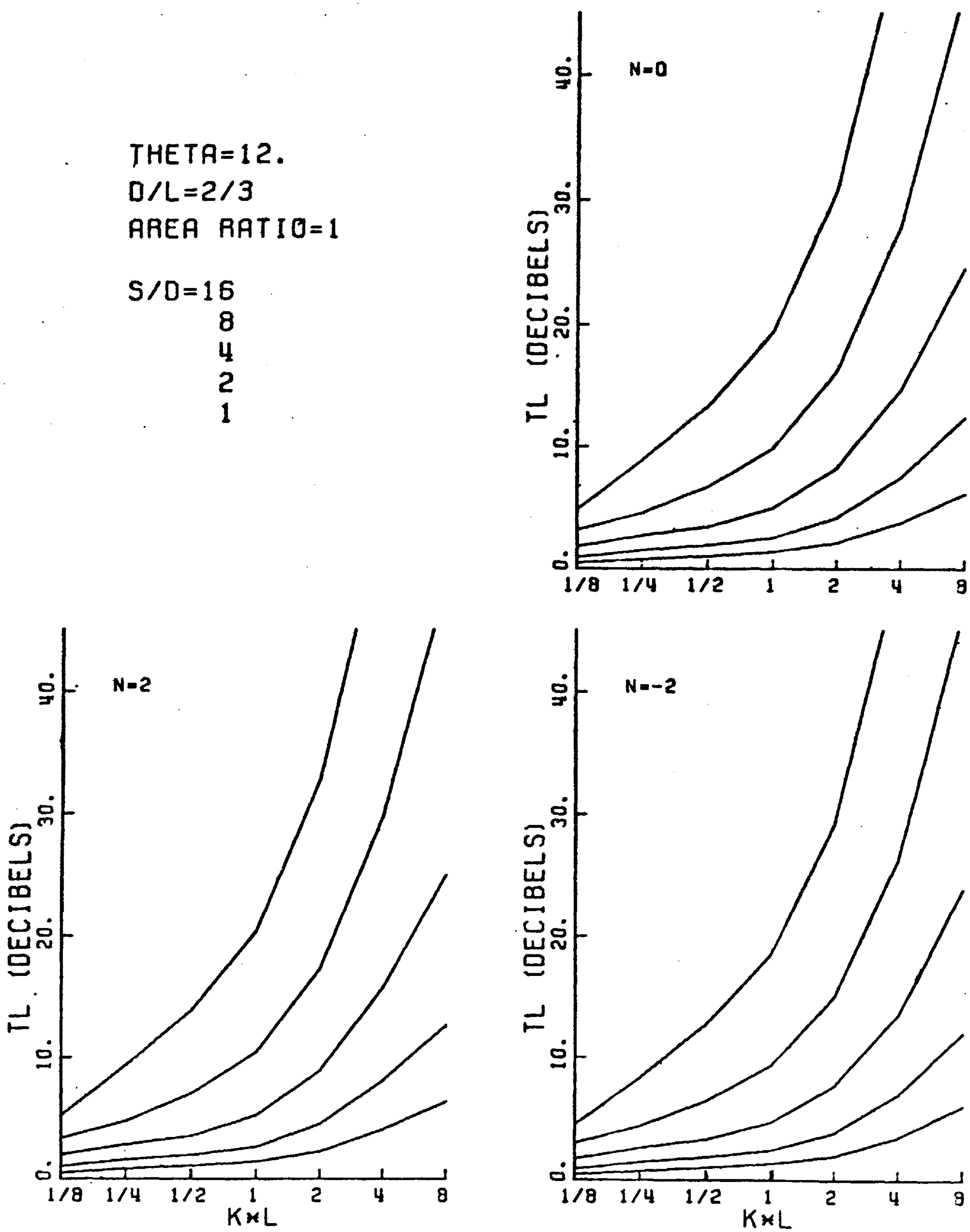

THETA=12.
D/L=2/3
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.39

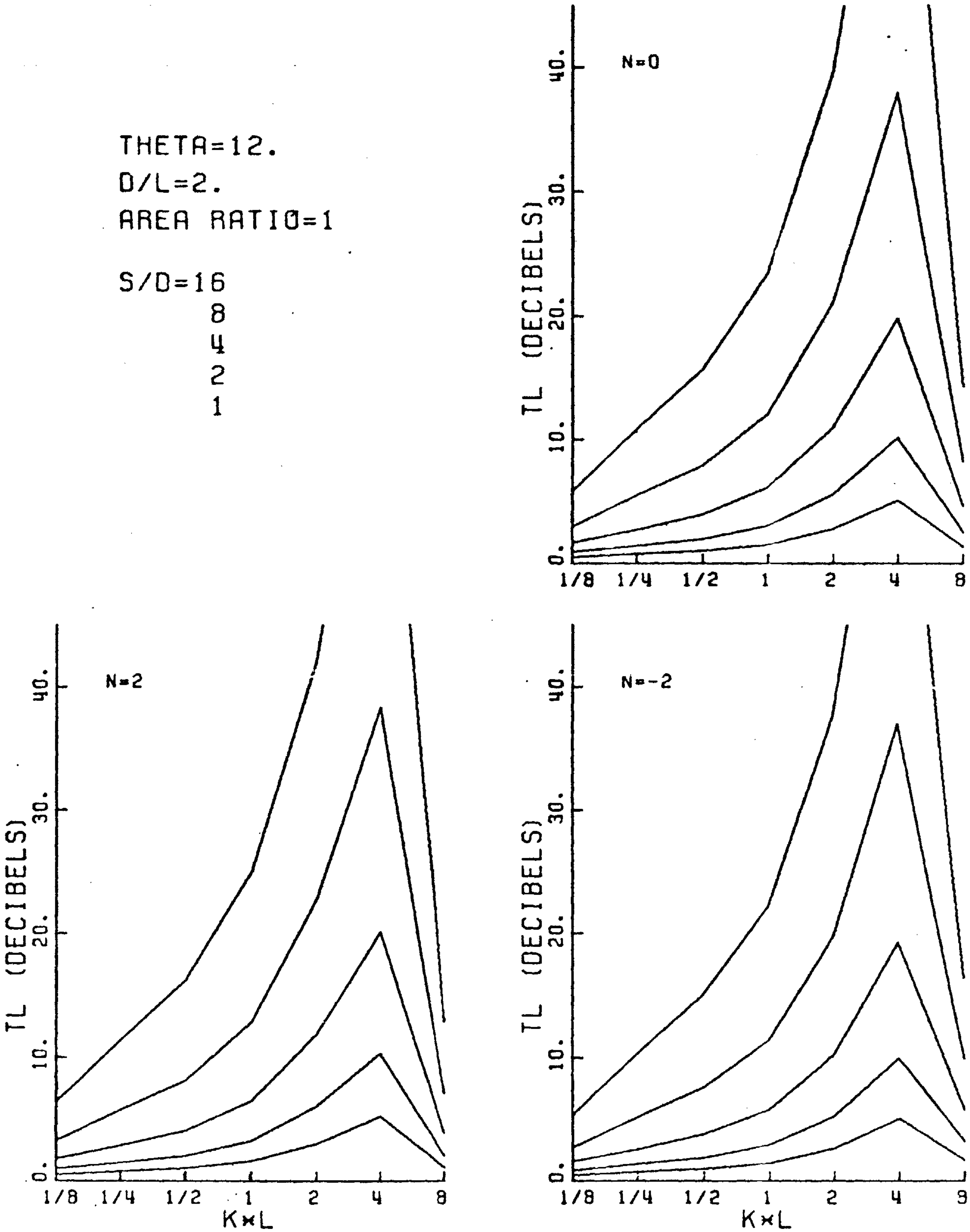

THETA=12.
D/L=2.
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.40

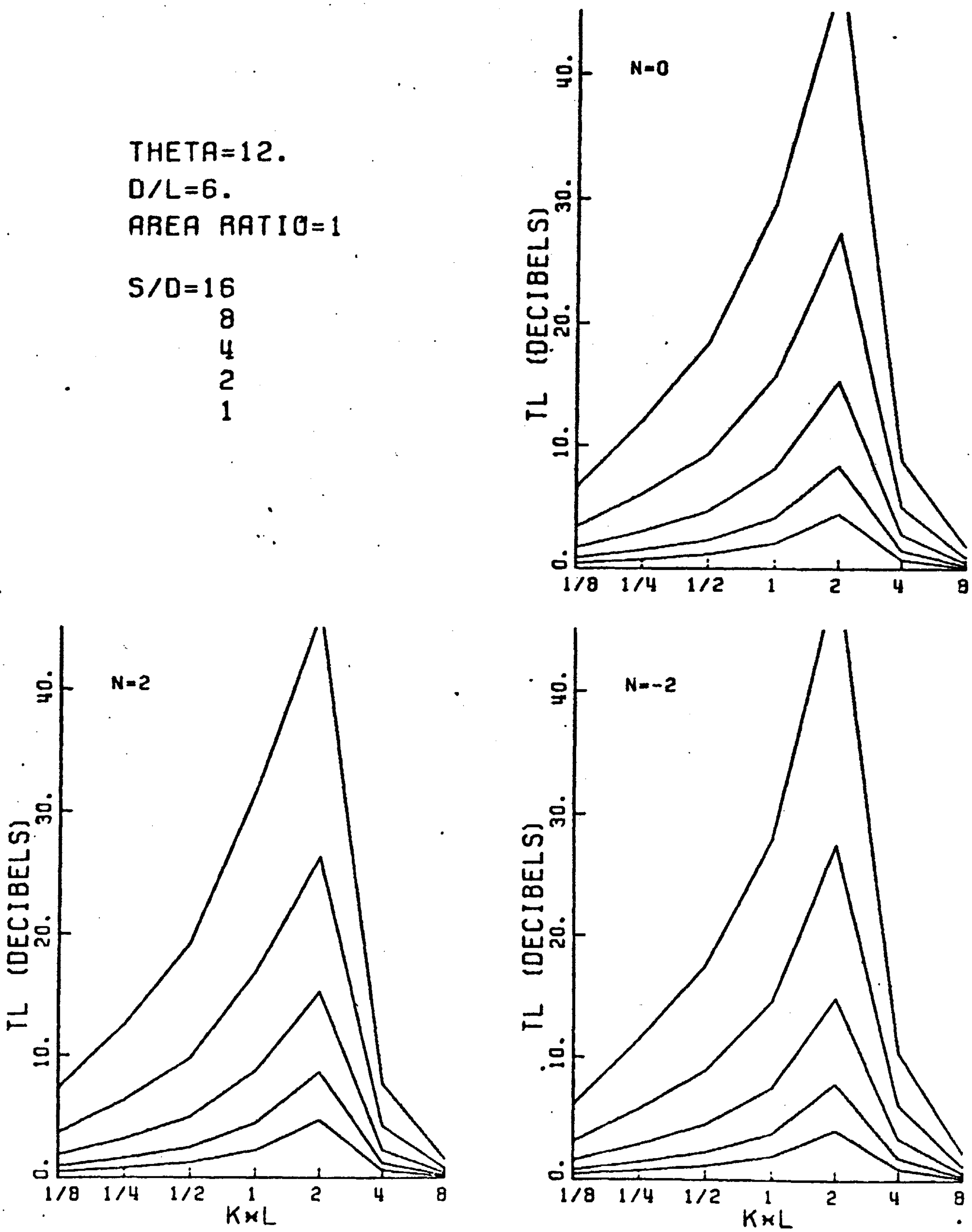

THETA=12.
D/L=6.
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.41

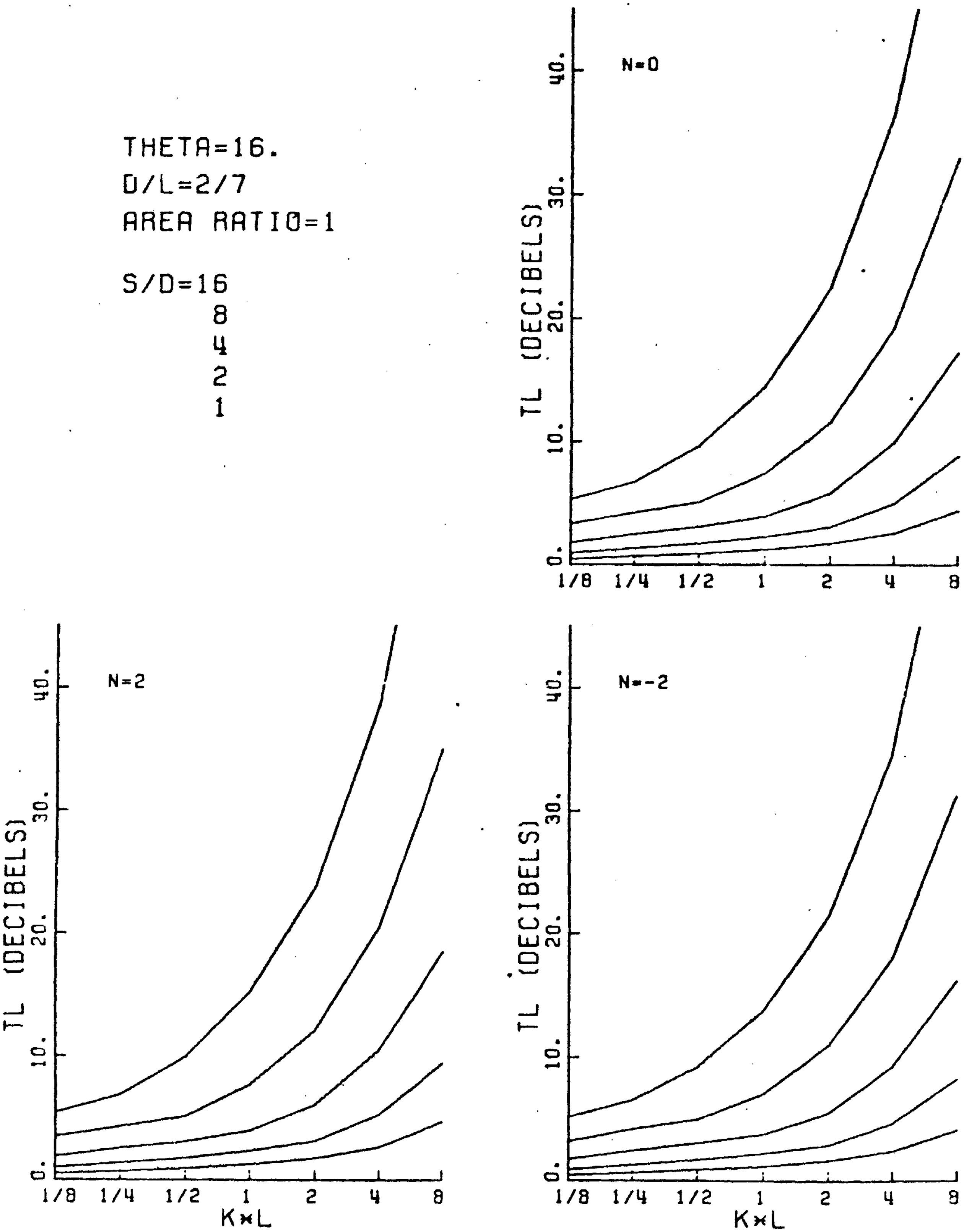

THETA=16.
D/L=2/7
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.42

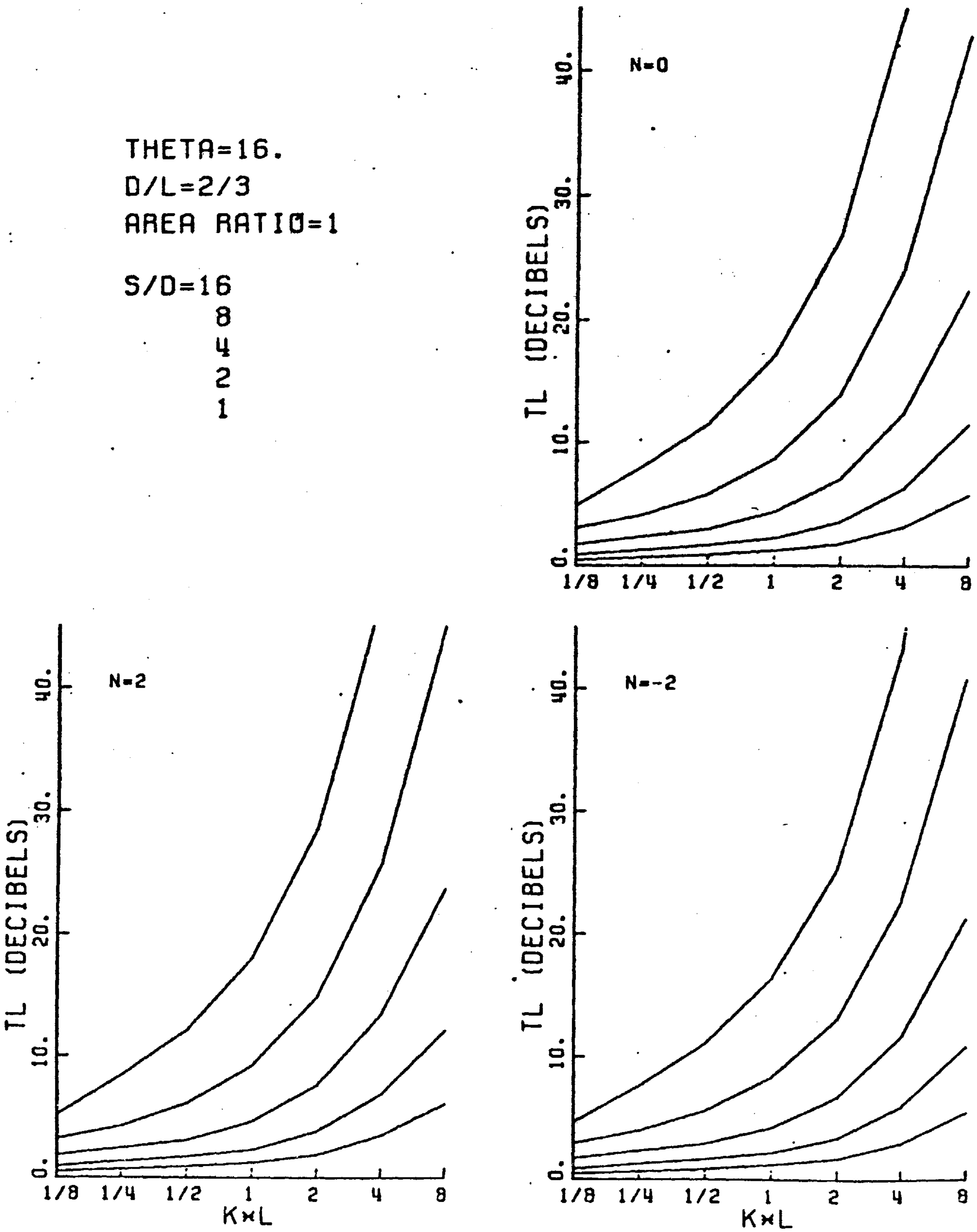

THETA=16.
D/L=2/3
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.43

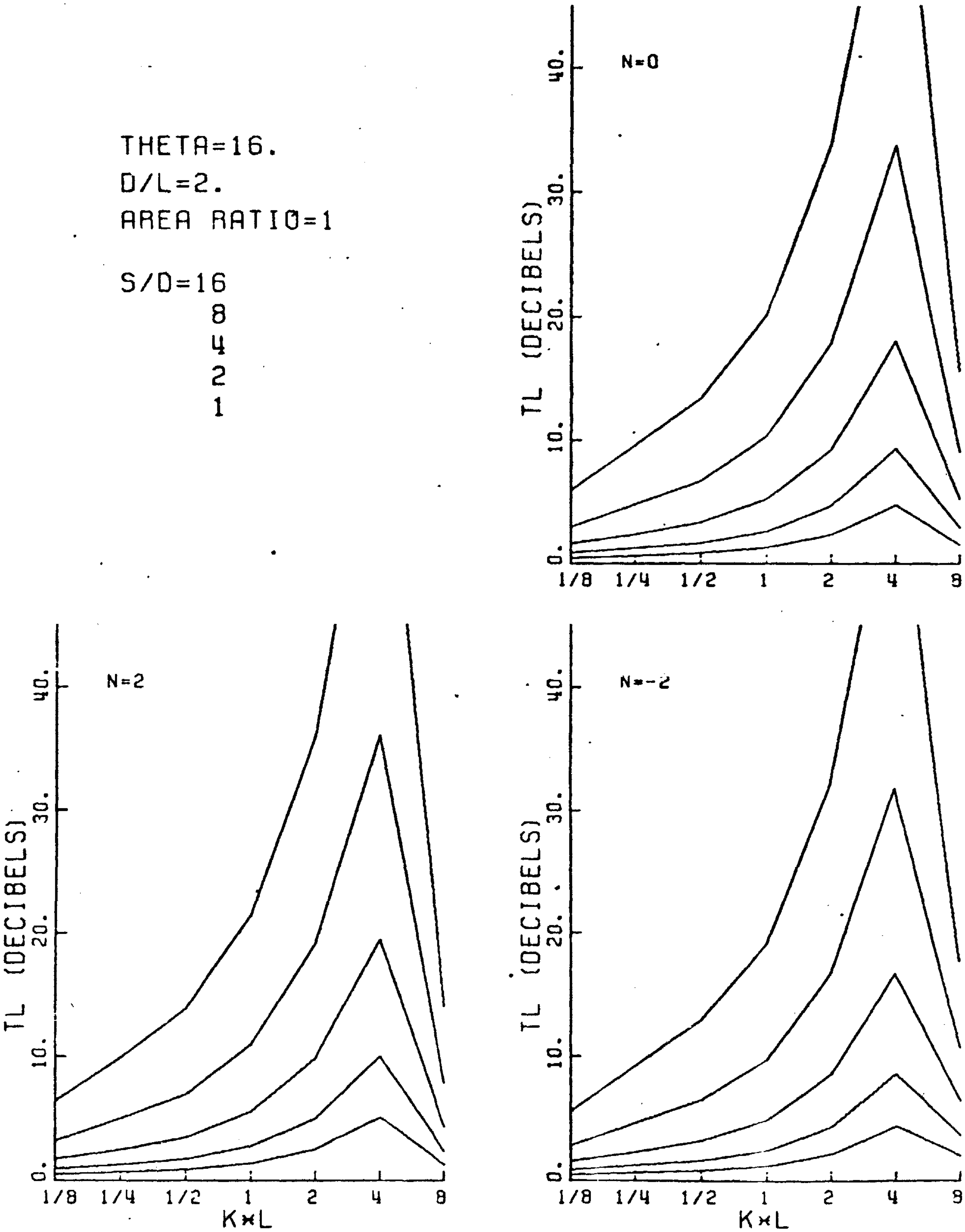

THETA=16.
D/L=2.
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.44

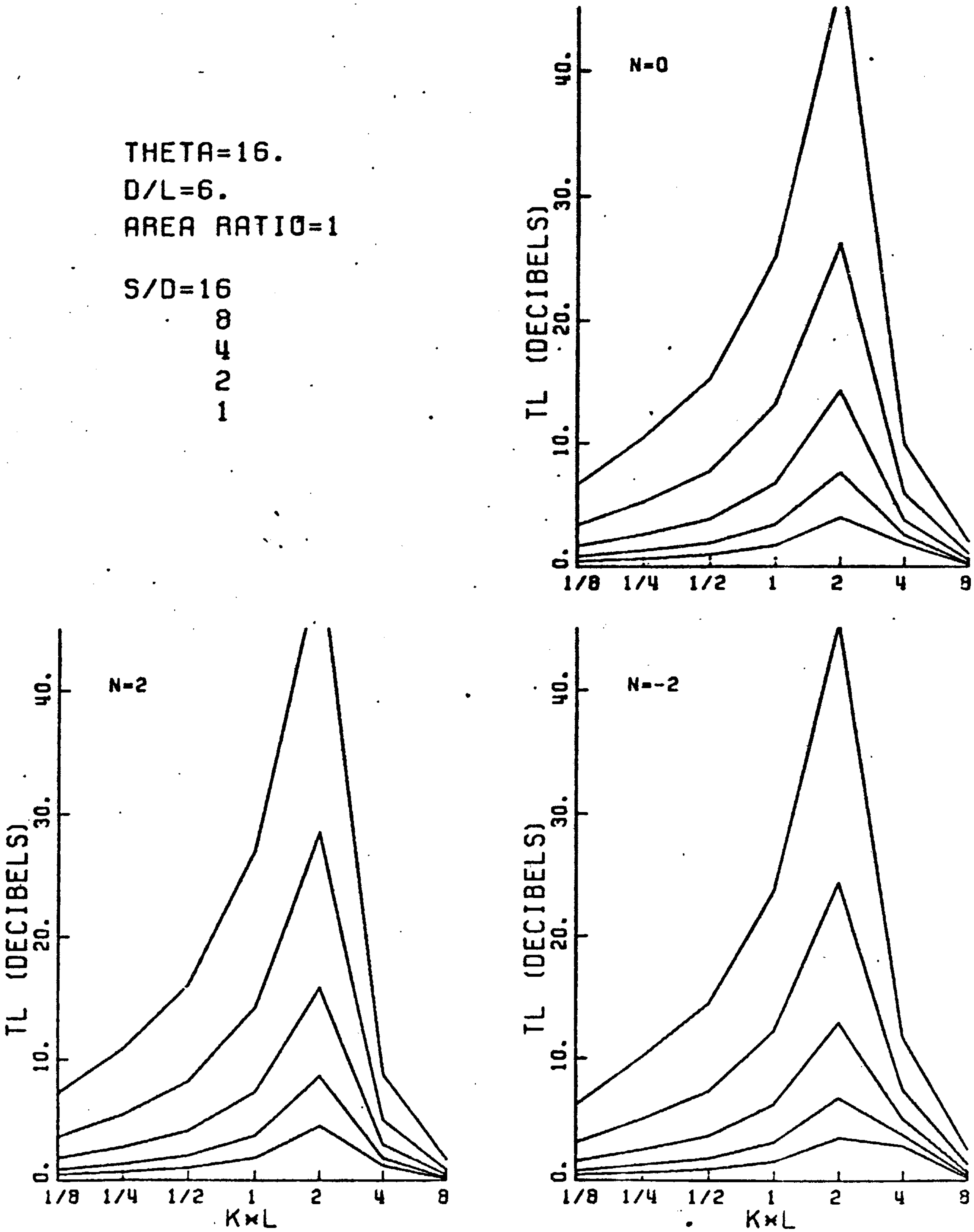

THETA=16.
D/L=6.
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8 1/4 1/2 1 2 4 8
K*L

Figure 3.45

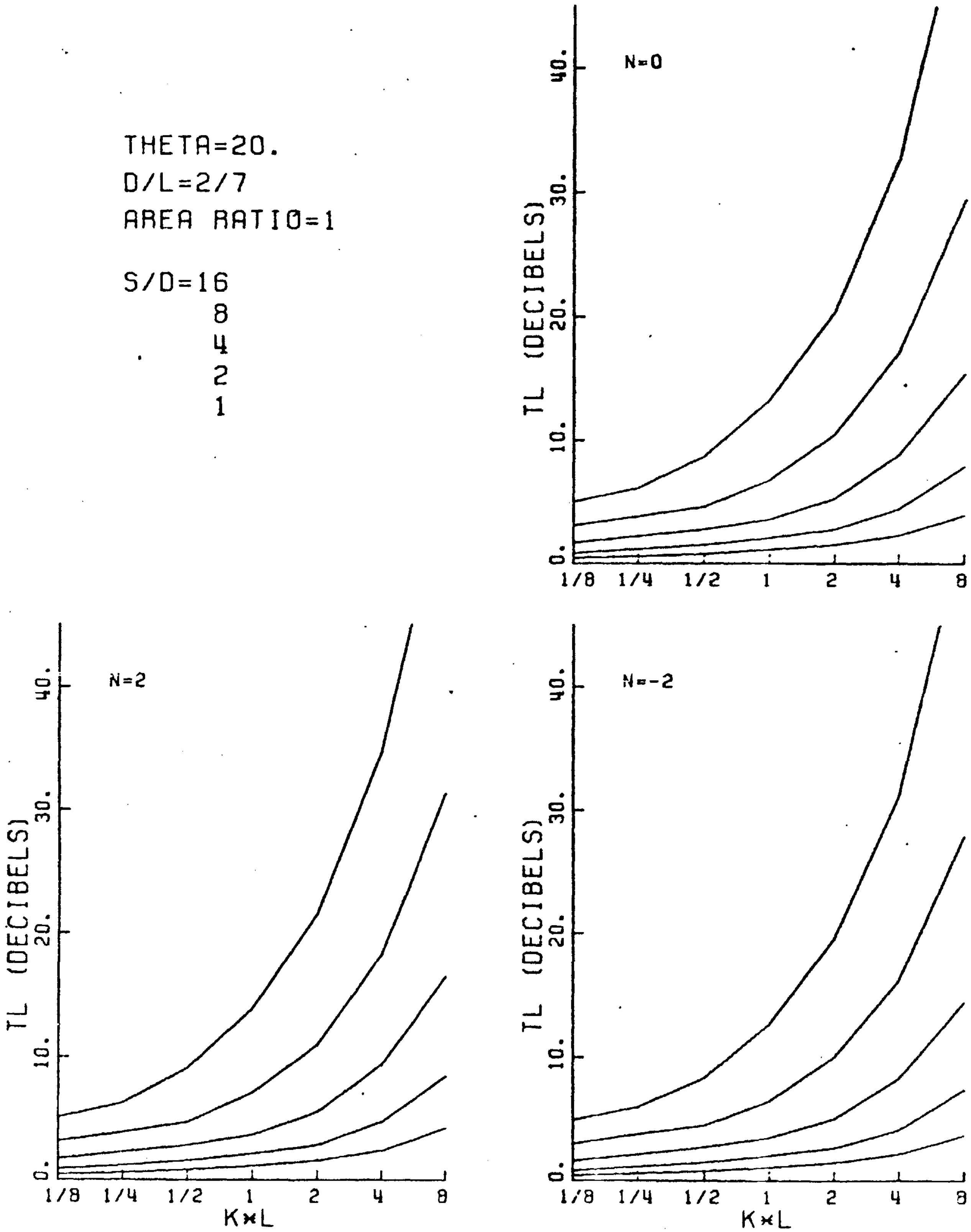
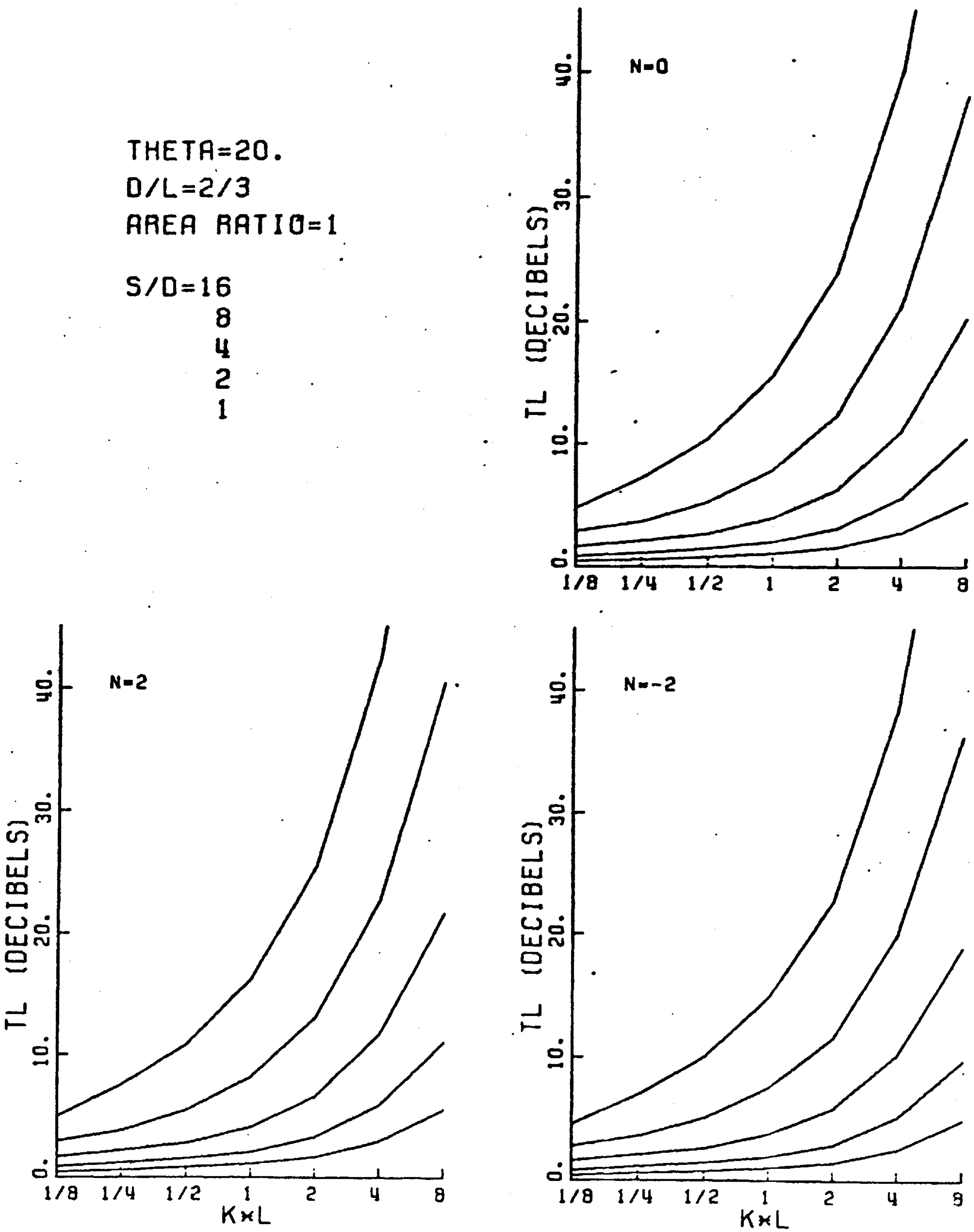
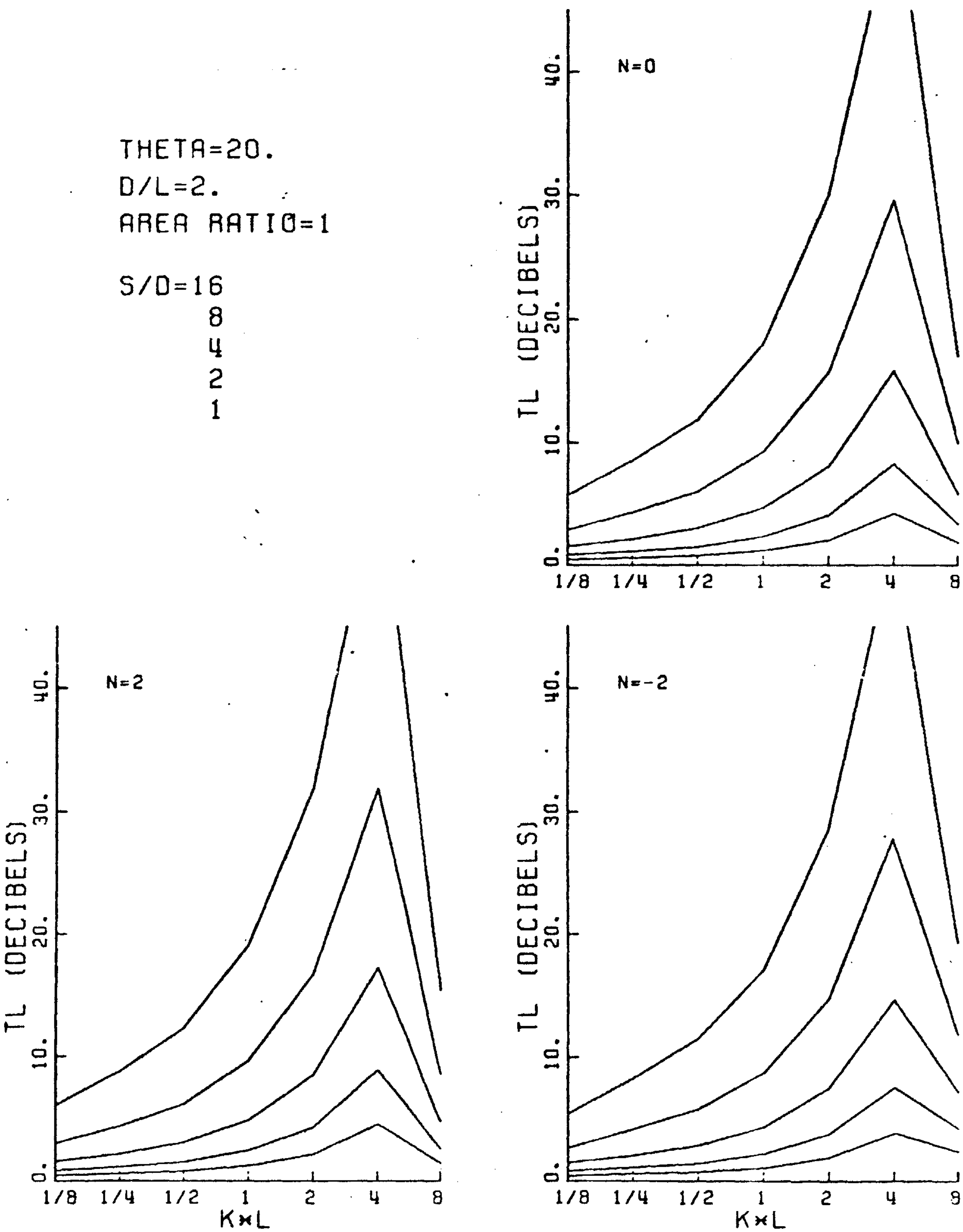

THETA=20.
D/L=2/7
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.46



THETA=20.
D/L=2/3
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.47



THETA=20.
D/L=2.
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.48

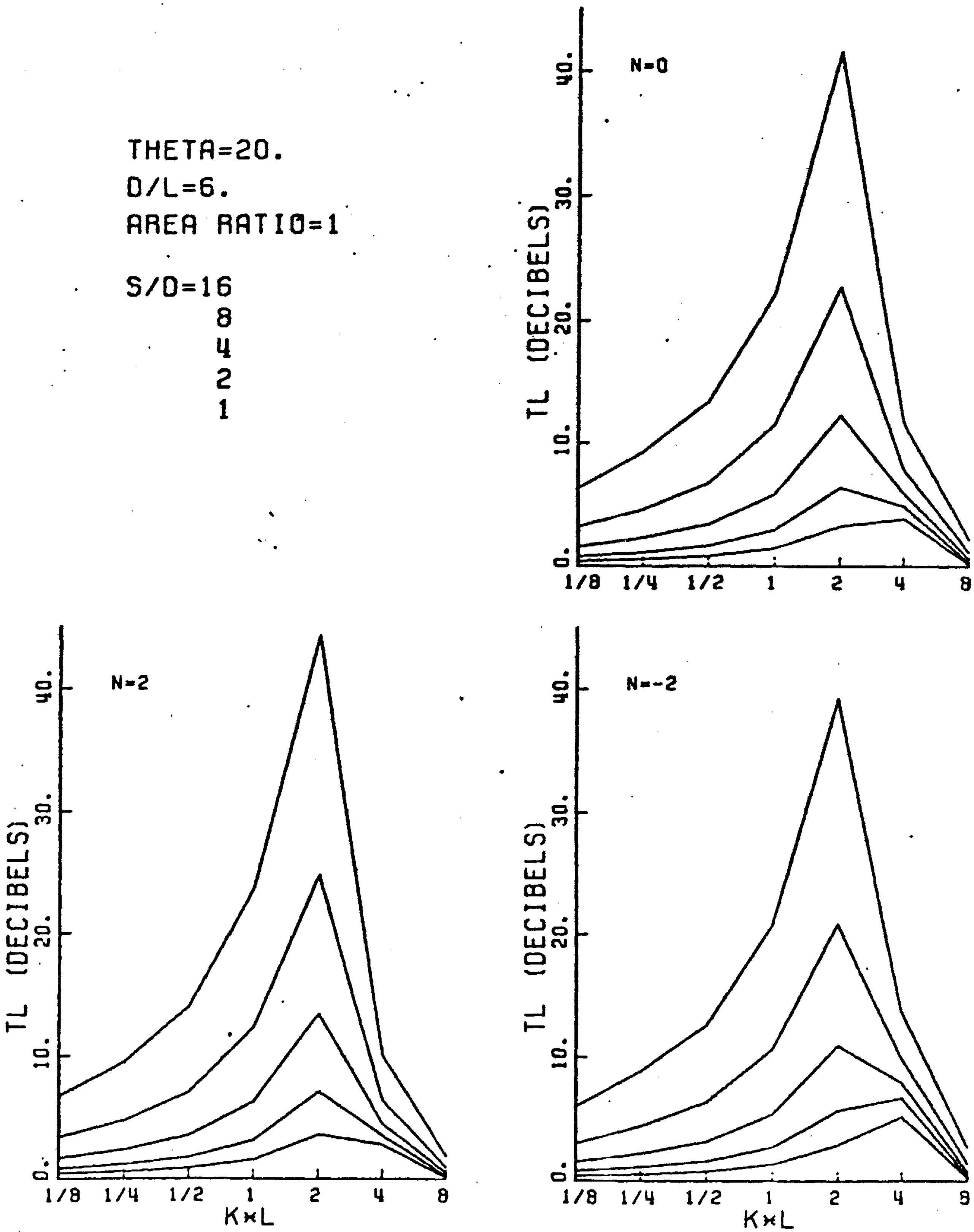

THETA=20.
D/L=6.
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
K*L
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8

Figure 3.49

Figures 3.50-3.65. Octave band TL vs kL in circular ducts lined with a resistive screen type resonator liner. The format is the same as in Figures 3.2-3.17.

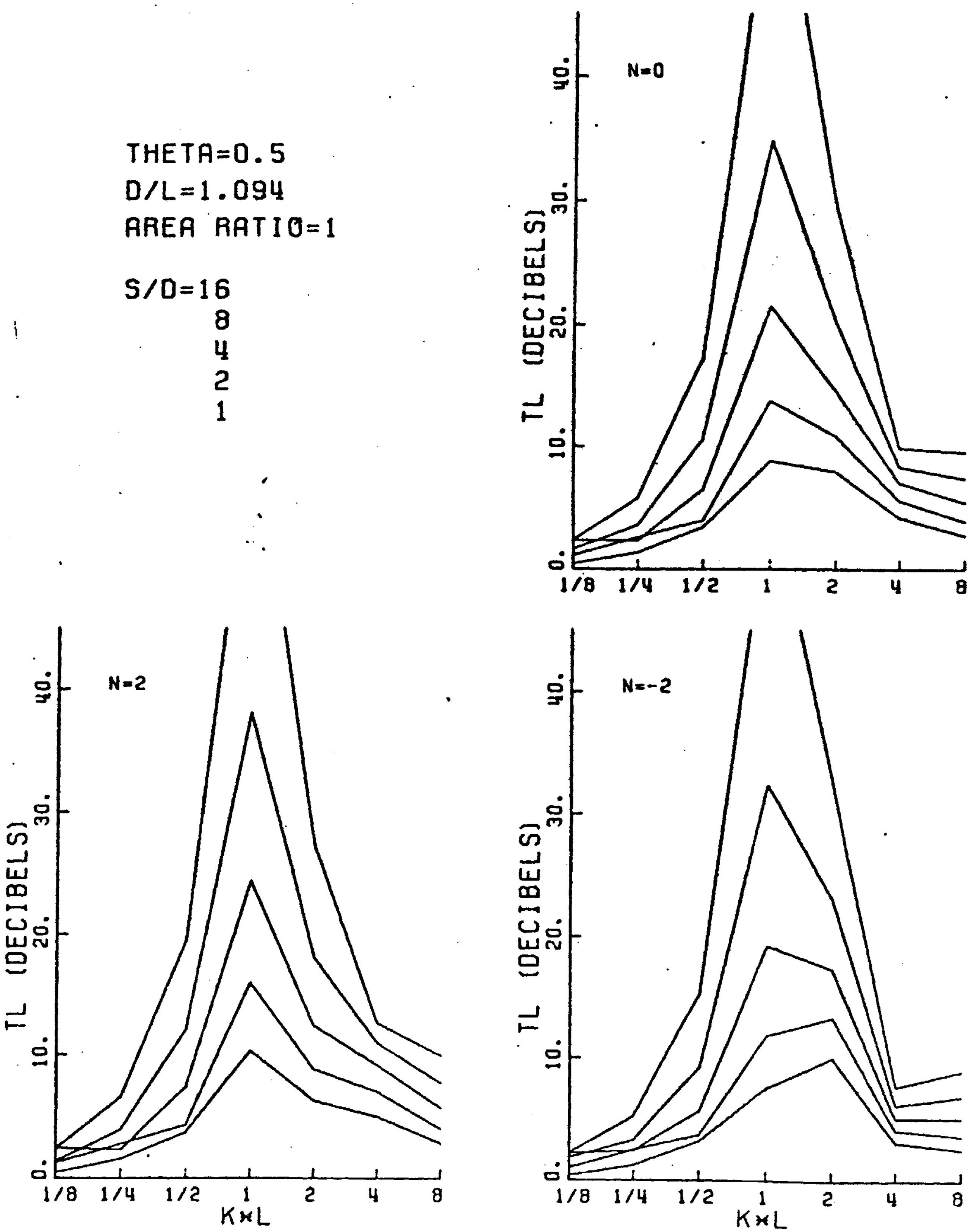

THETA=0.5
D/L=1.094
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.50

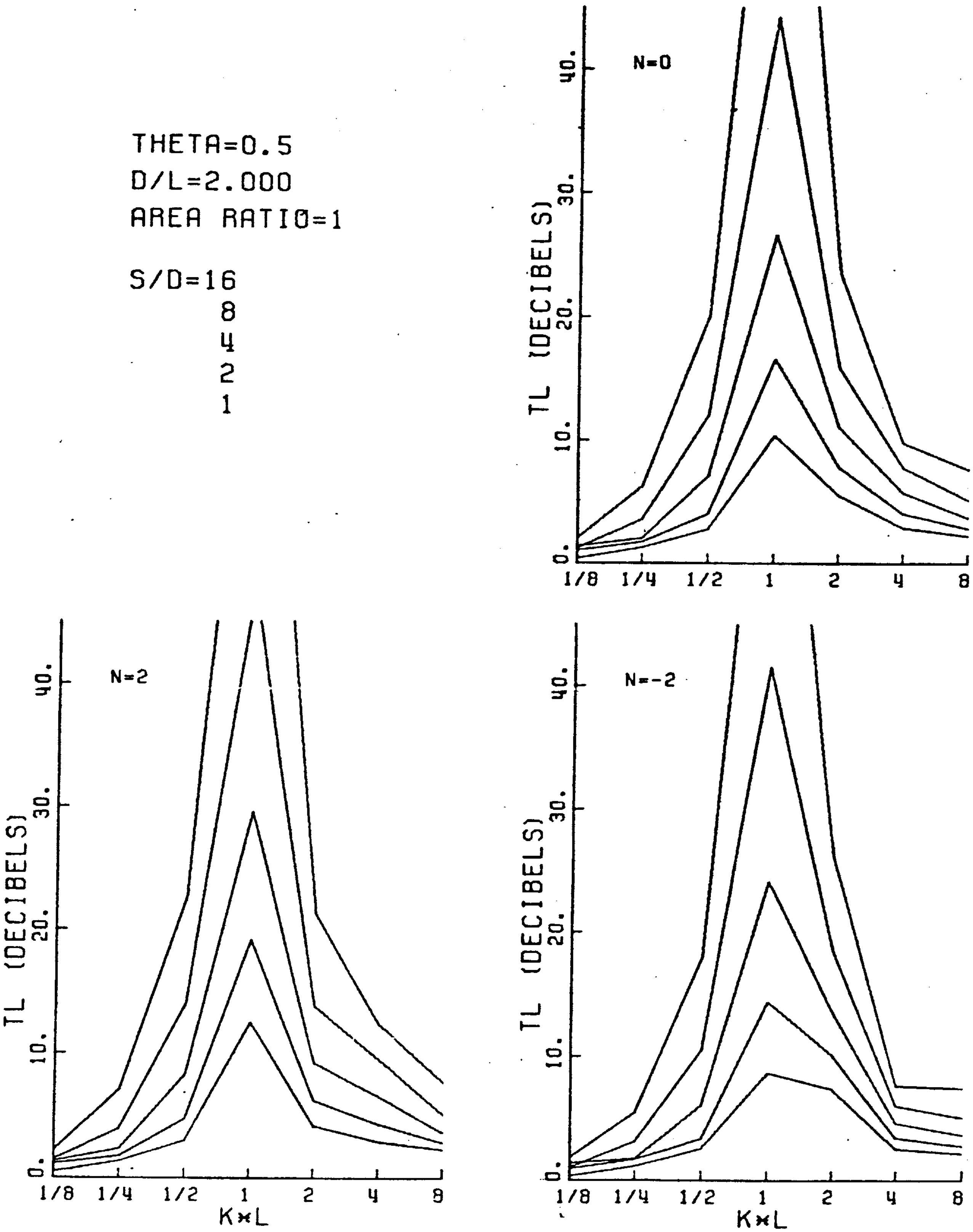

THETA=0.5
D/L=2.000
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.51

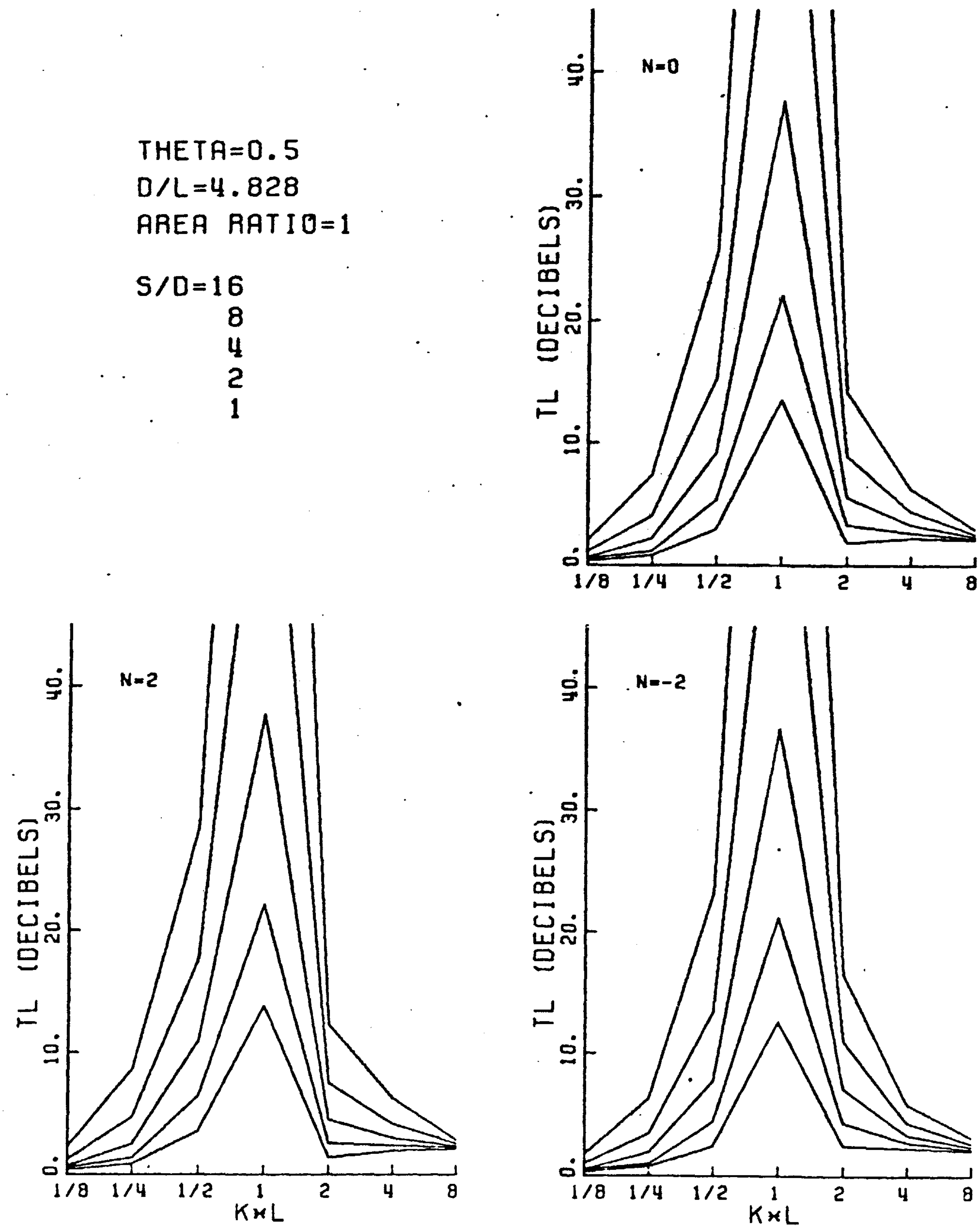

THETA=0.5
D/L=4.828
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.52

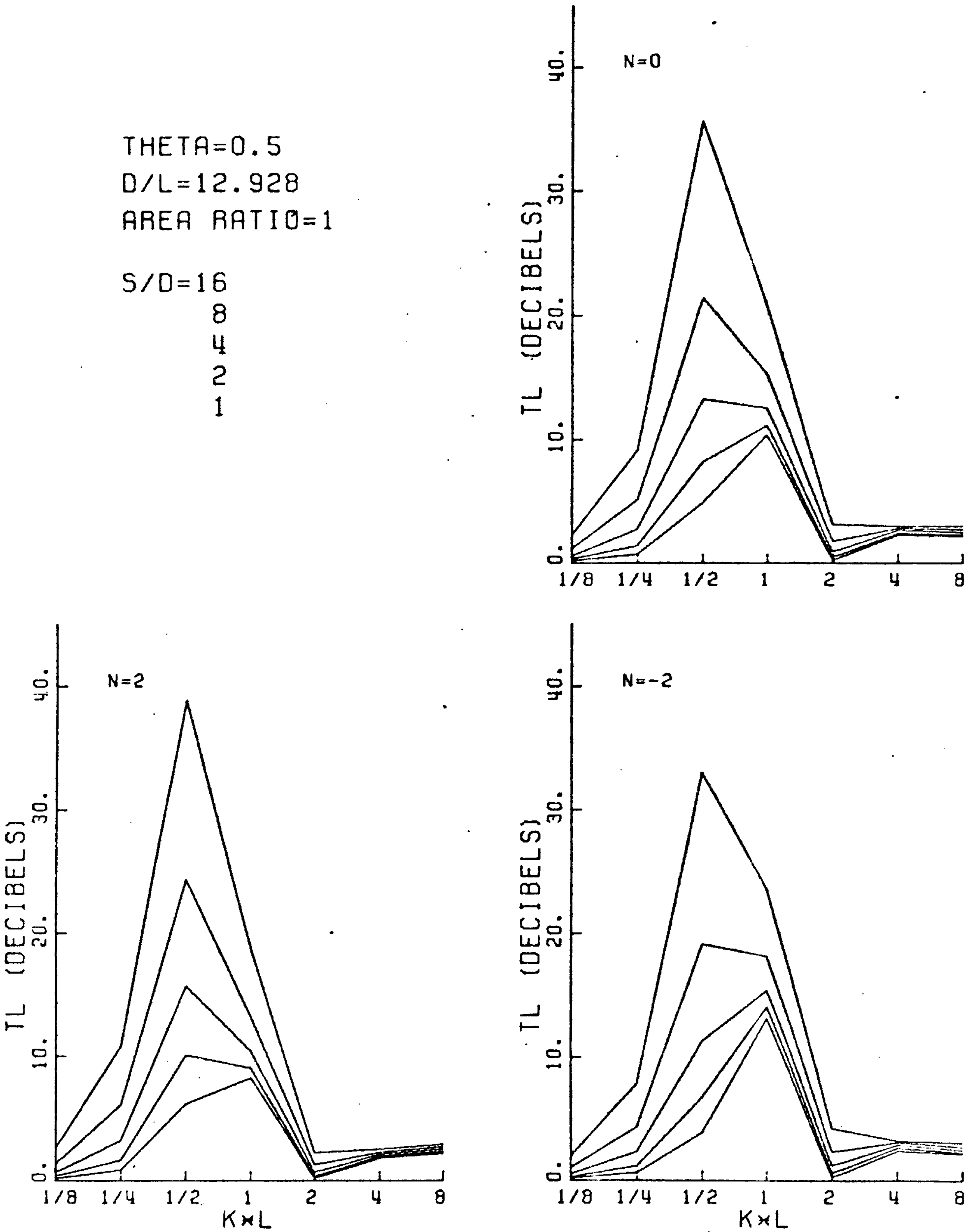

THETA=0.5
D/L=12.928
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.53

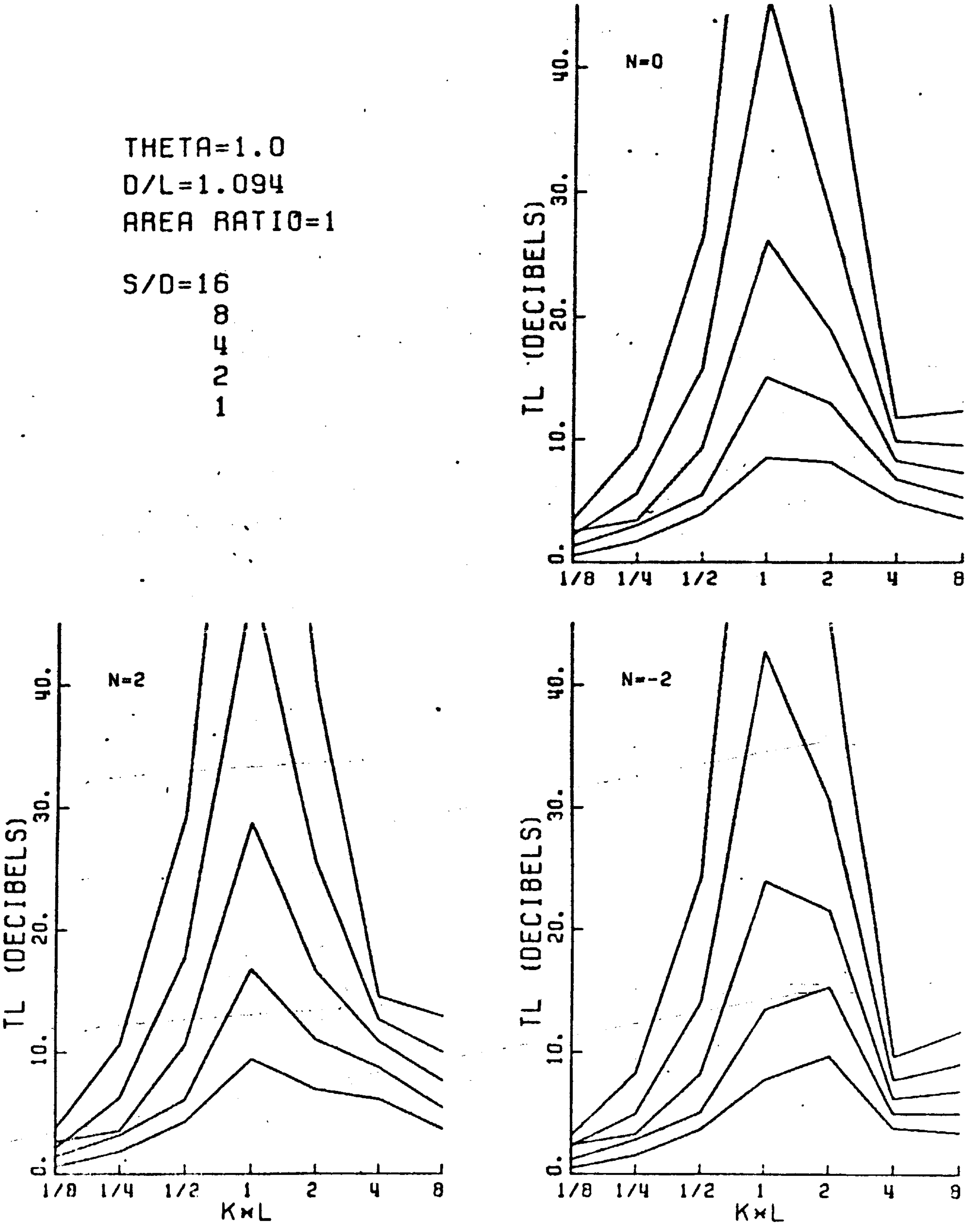

THETA=1.0
D/L=1.094
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.54

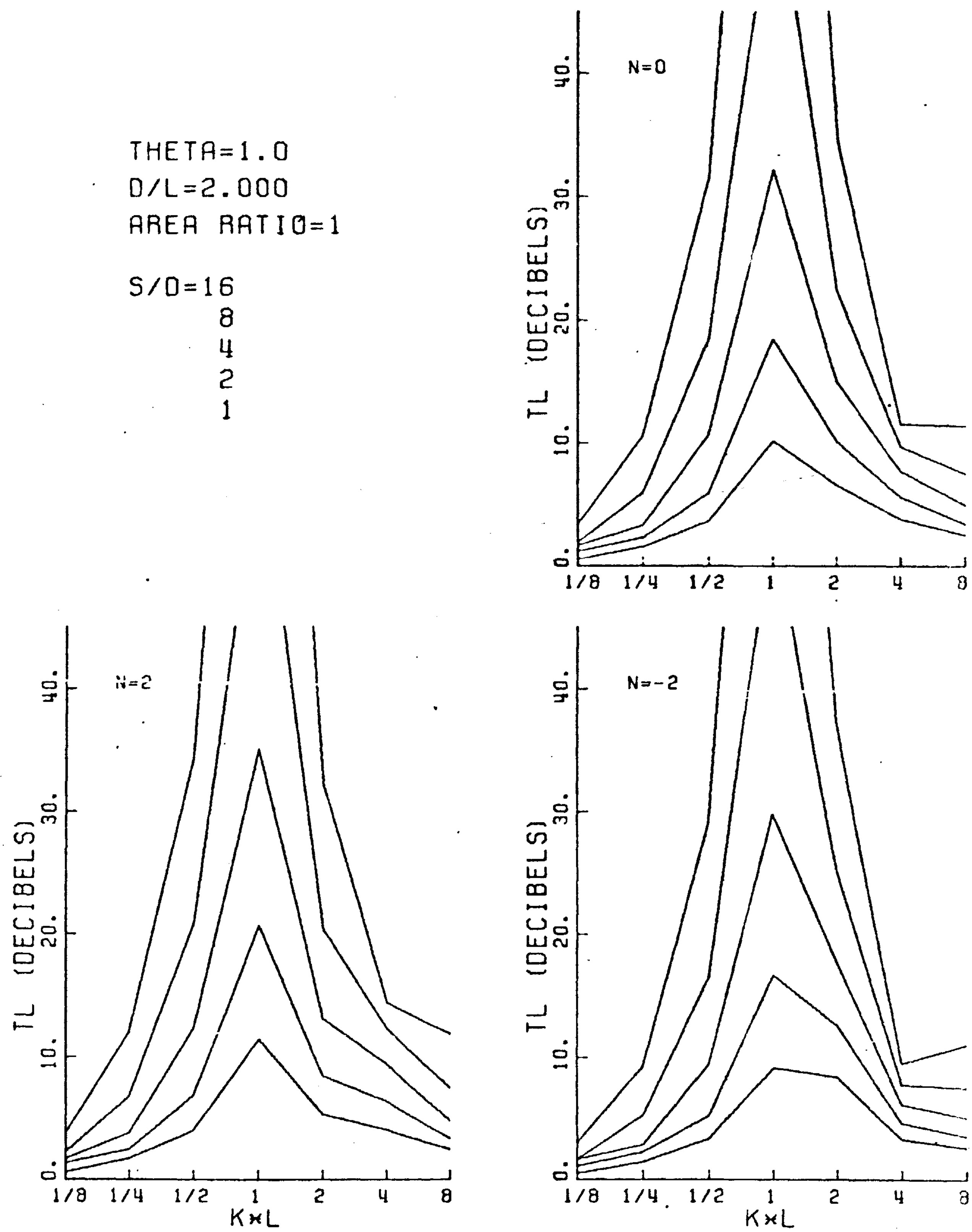

THETA=1.0
D/L=2.000
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
K*L
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8

Figure 3.55

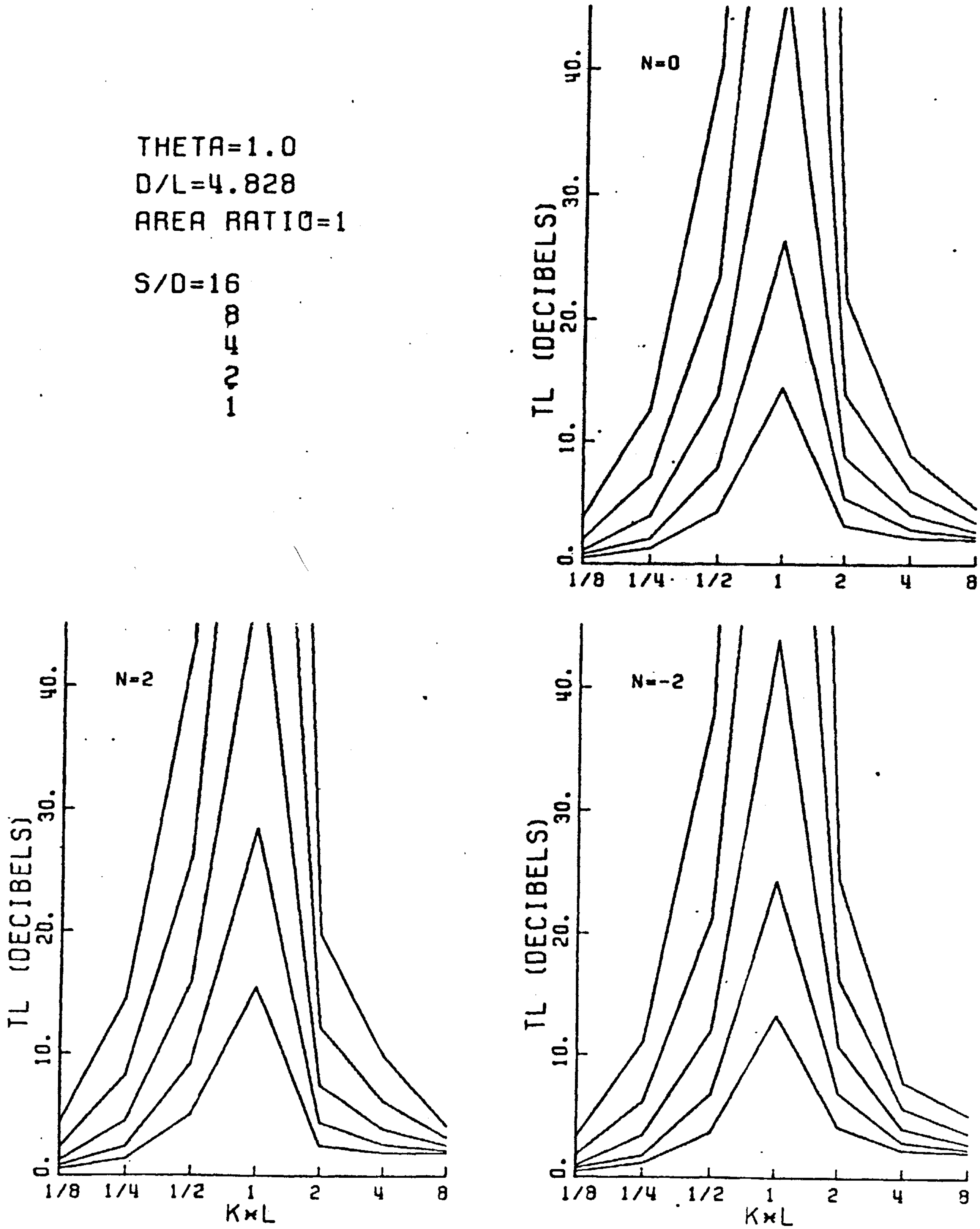

THETA=1.0
D/L=4.828
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.56

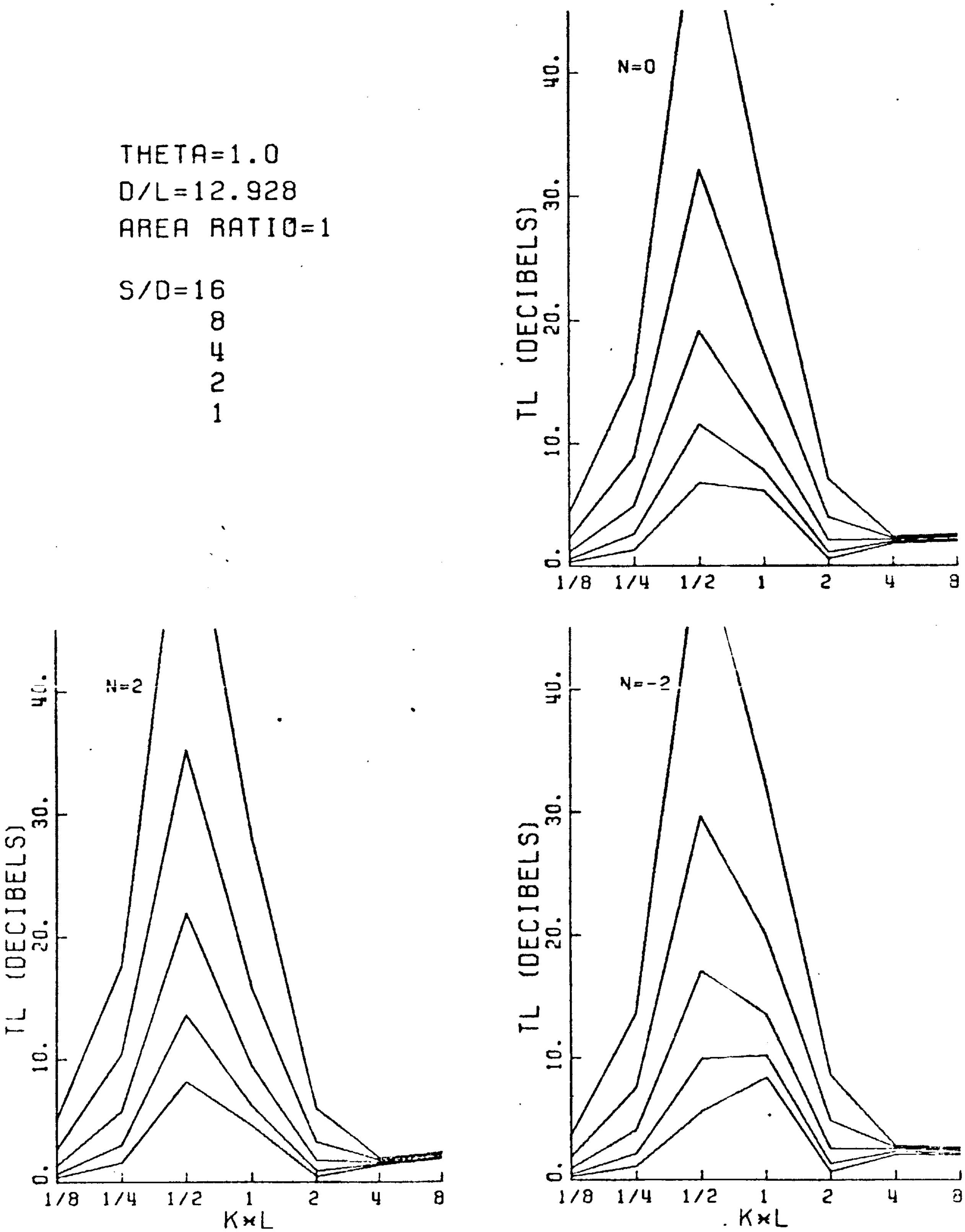

THETA=1.0
D/L=12.928
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.57

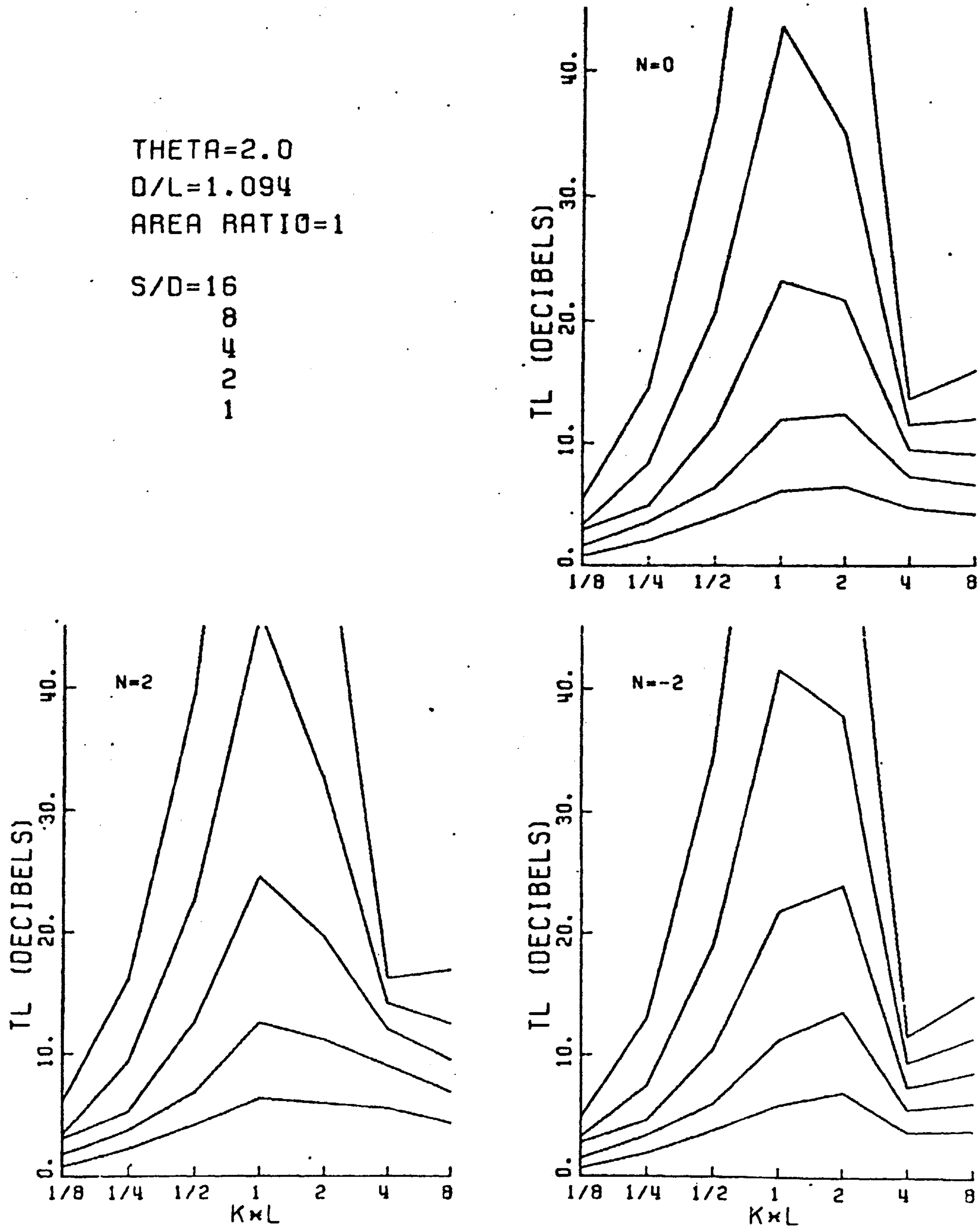

THETA=2.0
D/L=1.094
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.58

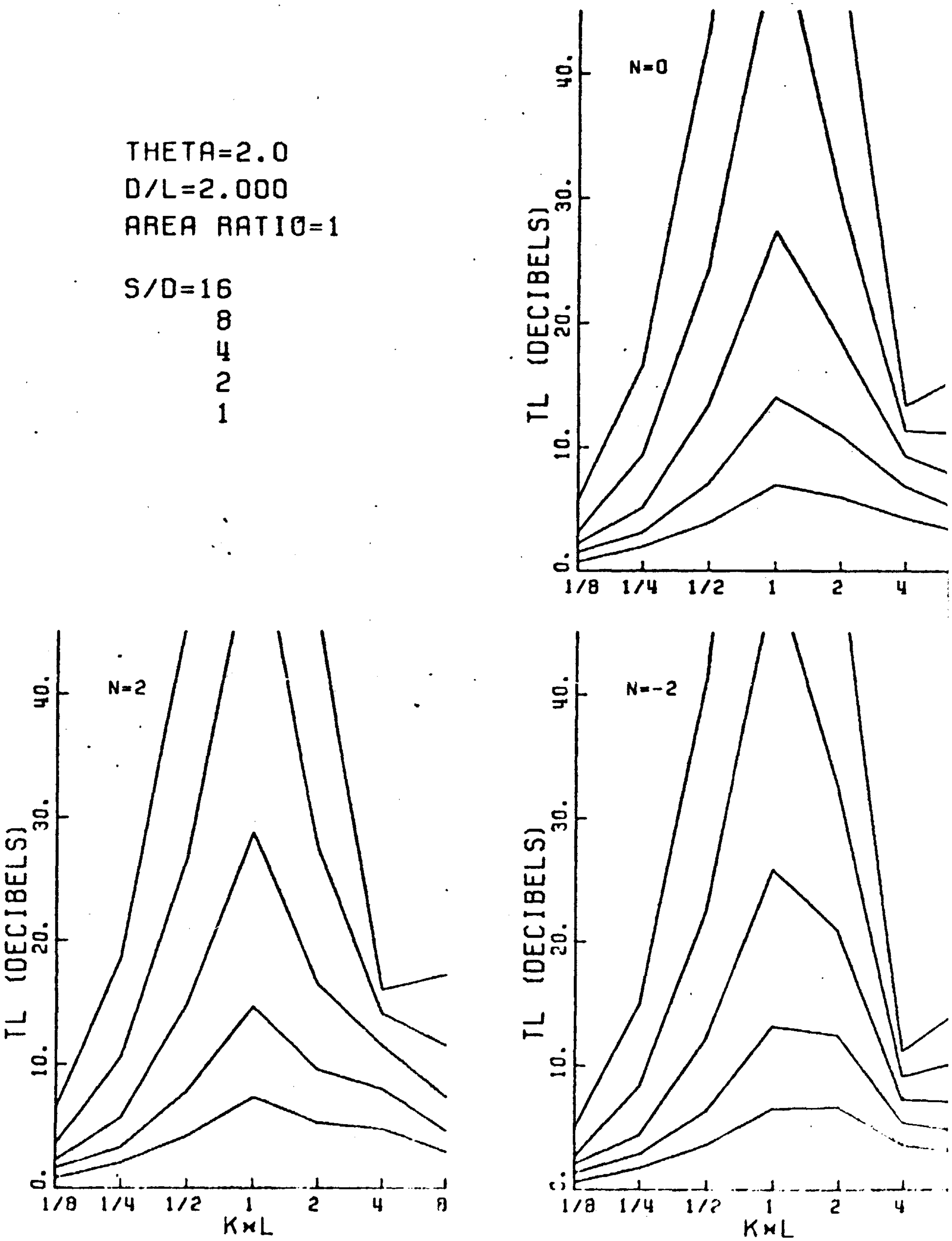

THETA=2.0
D/L=2.000
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.59

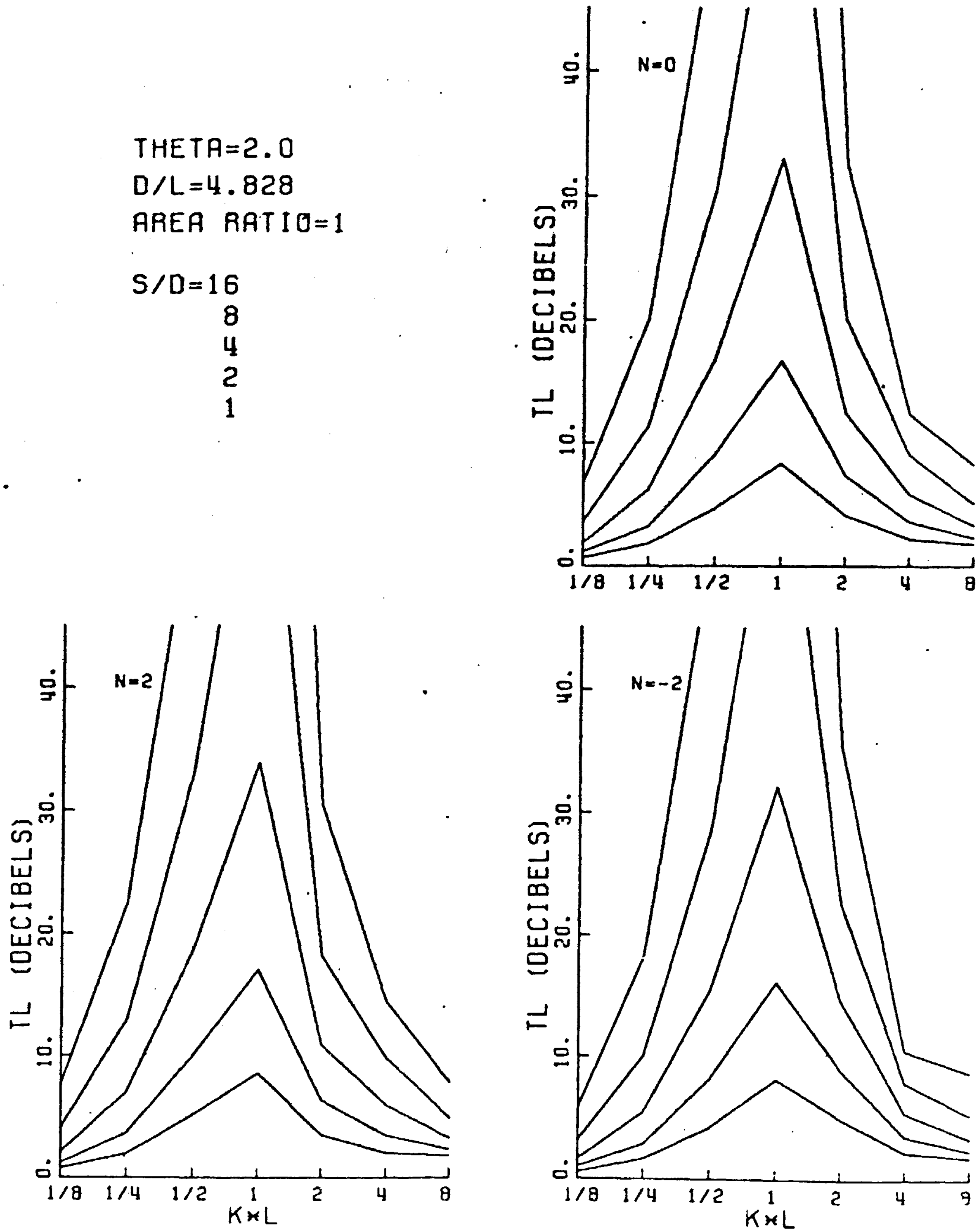

THETA=2.0
D/L=4.828
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
9
K*L

Figure 3.60

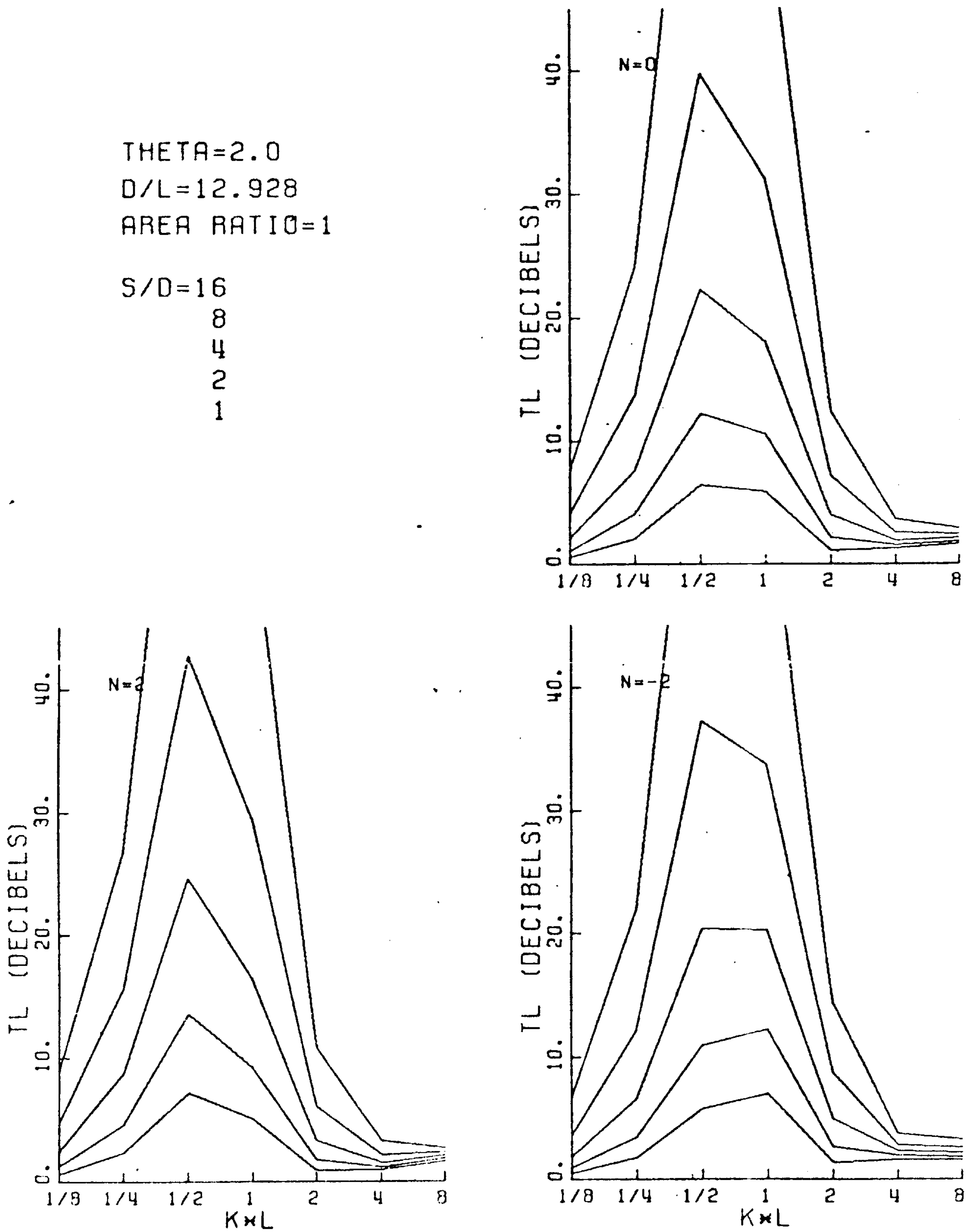

THETA=2.0
D/L=12.928
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8 1/4 1/2 1 2 4 8
K*L

Figure 3.61

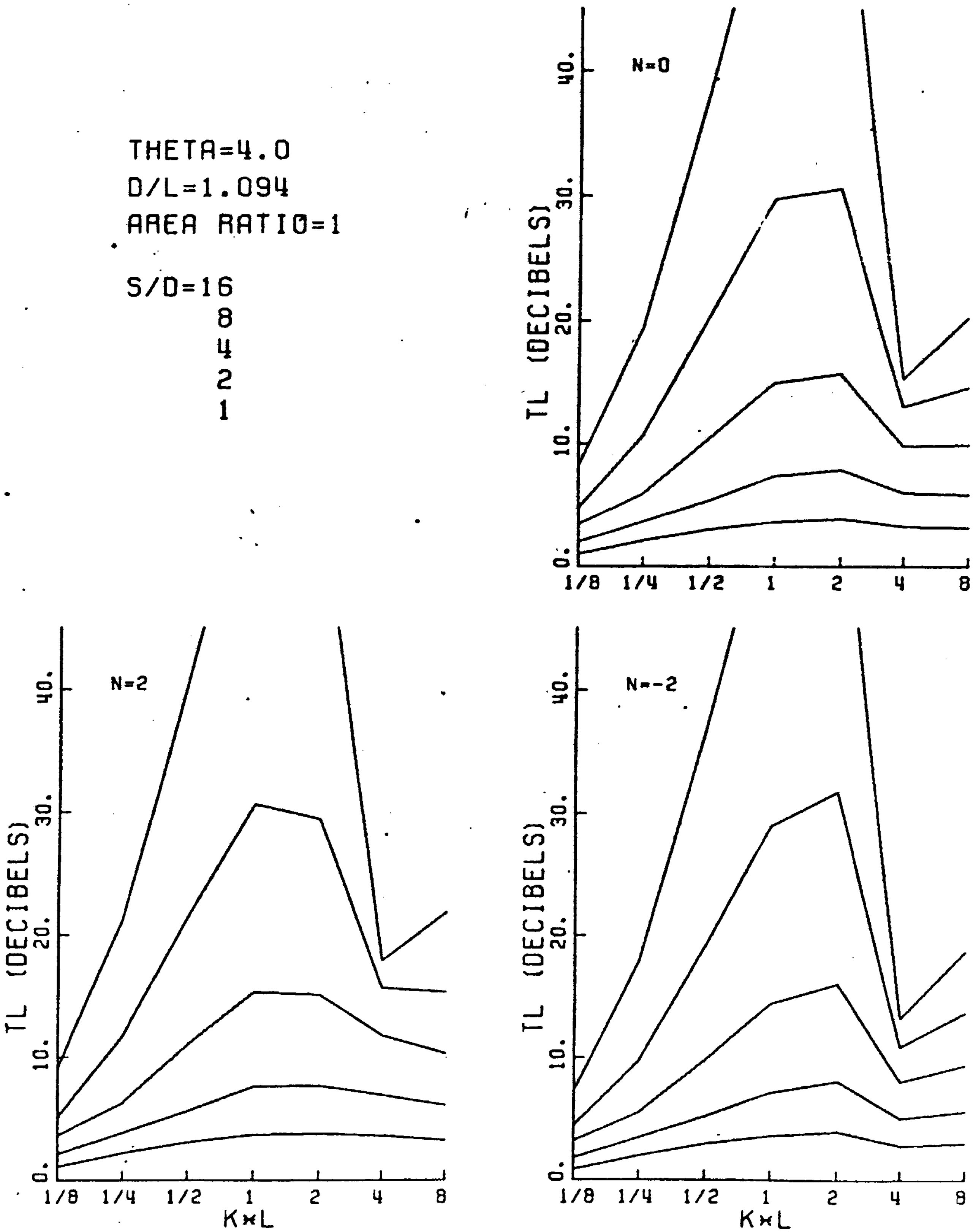
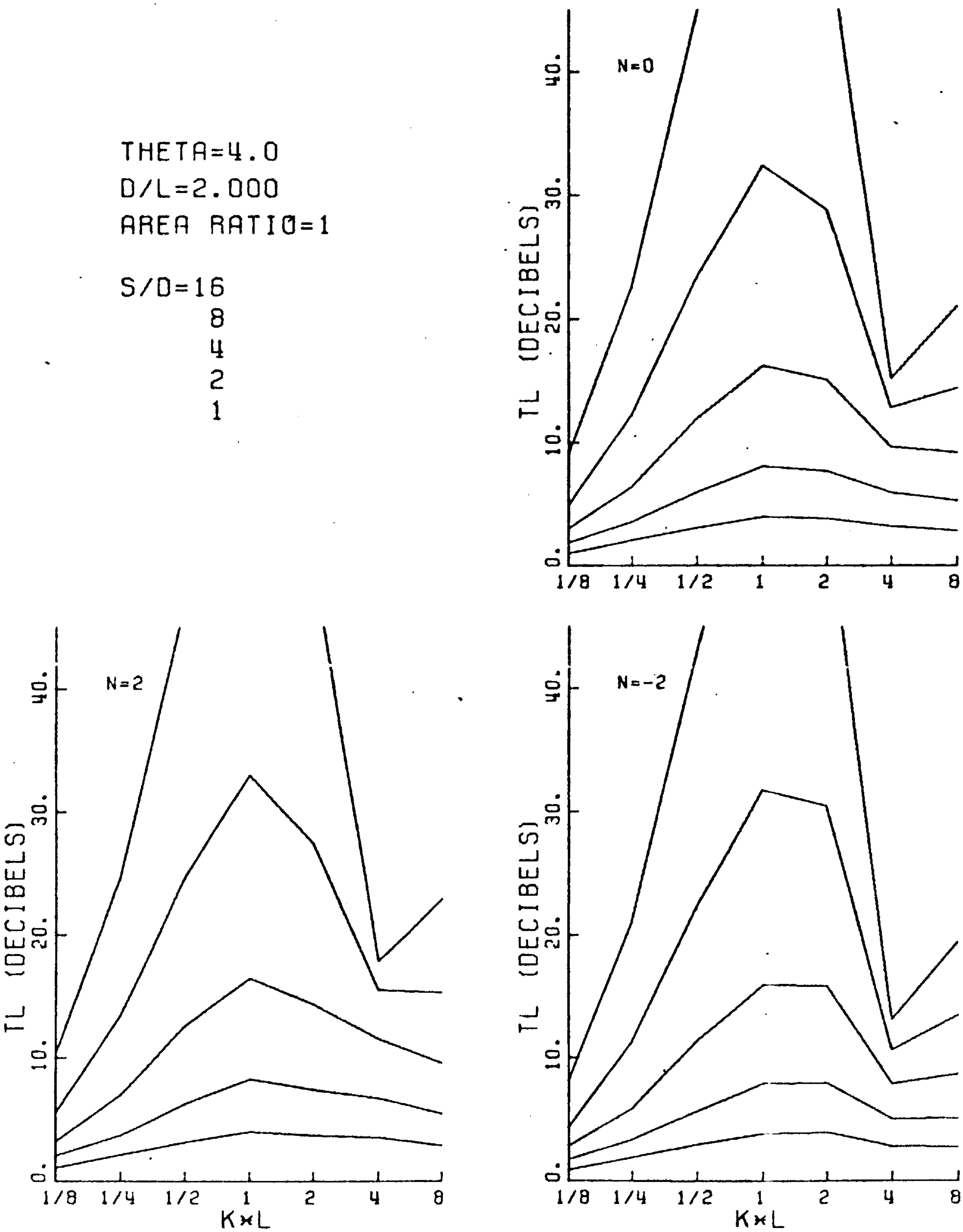

THETA=4.0
D/L=1.094
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.62



THETA=4.0
D/L=2.000
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.63

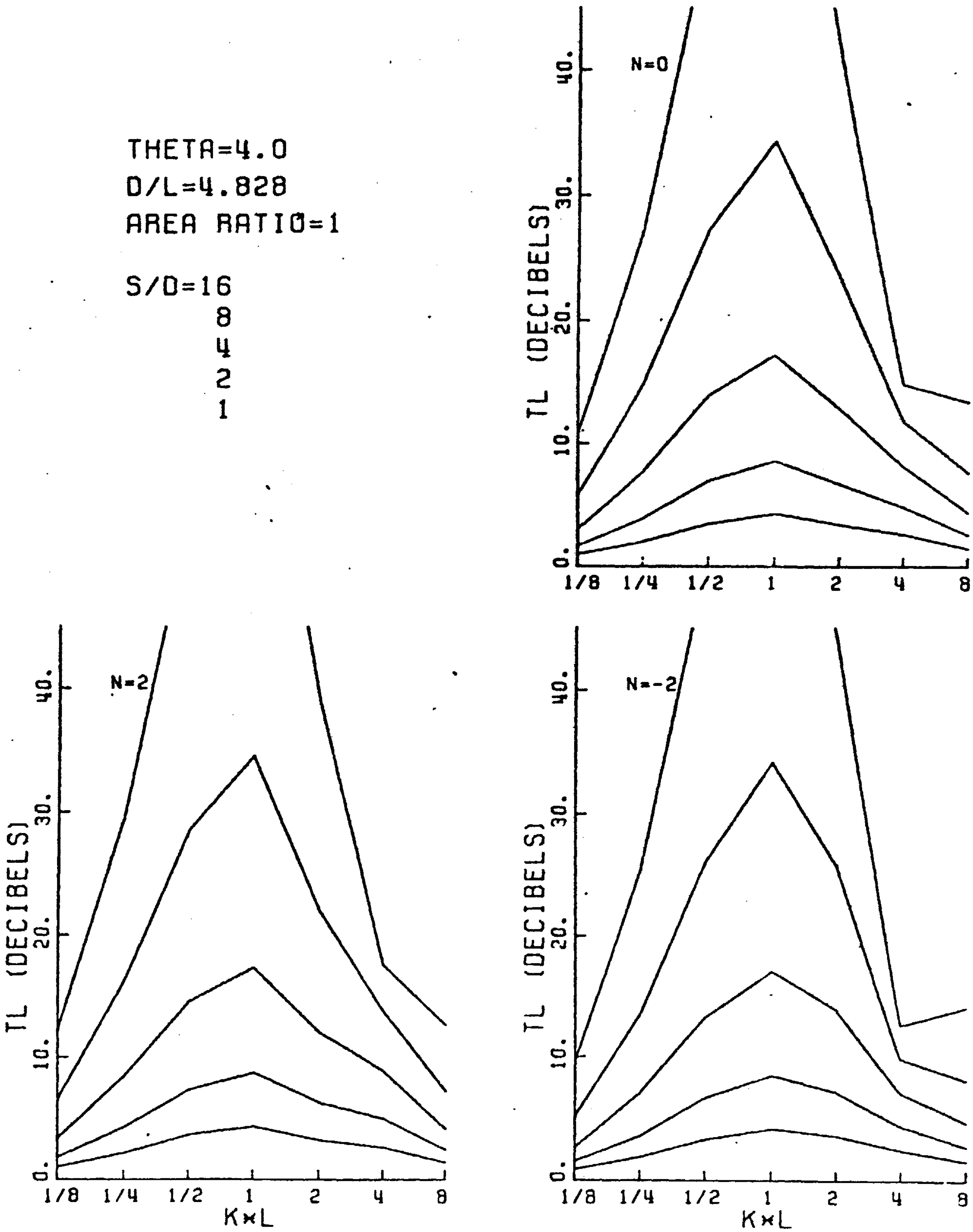

THETA=4.0
D/L=4.828
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.64

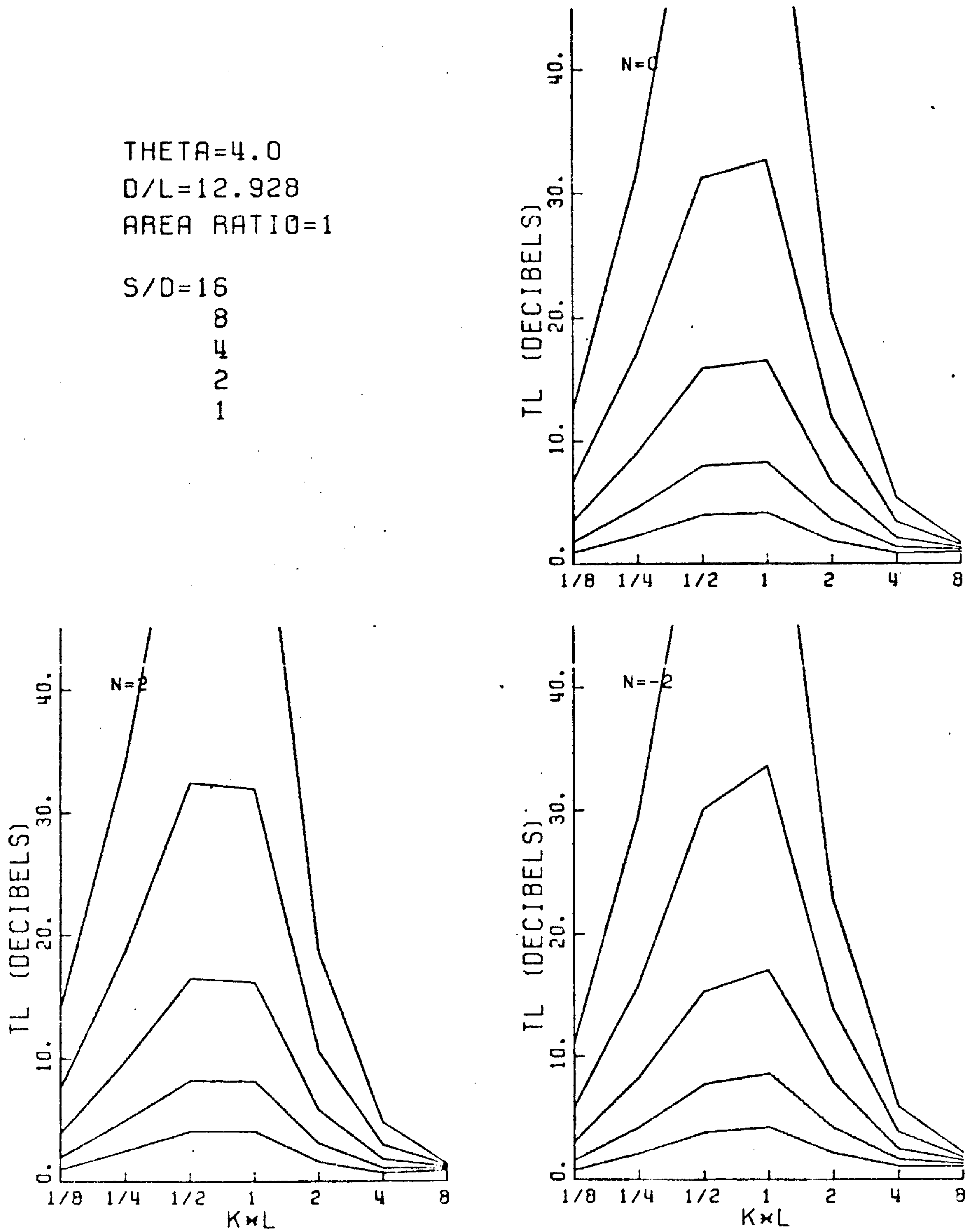

THETA=4.0
D/L=12.928
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.65

Figures 3.66-3.97. Octave band TL vs kL for a circular duct lined with a porous liner. The format is the same as in Figures 3.50-3.65.

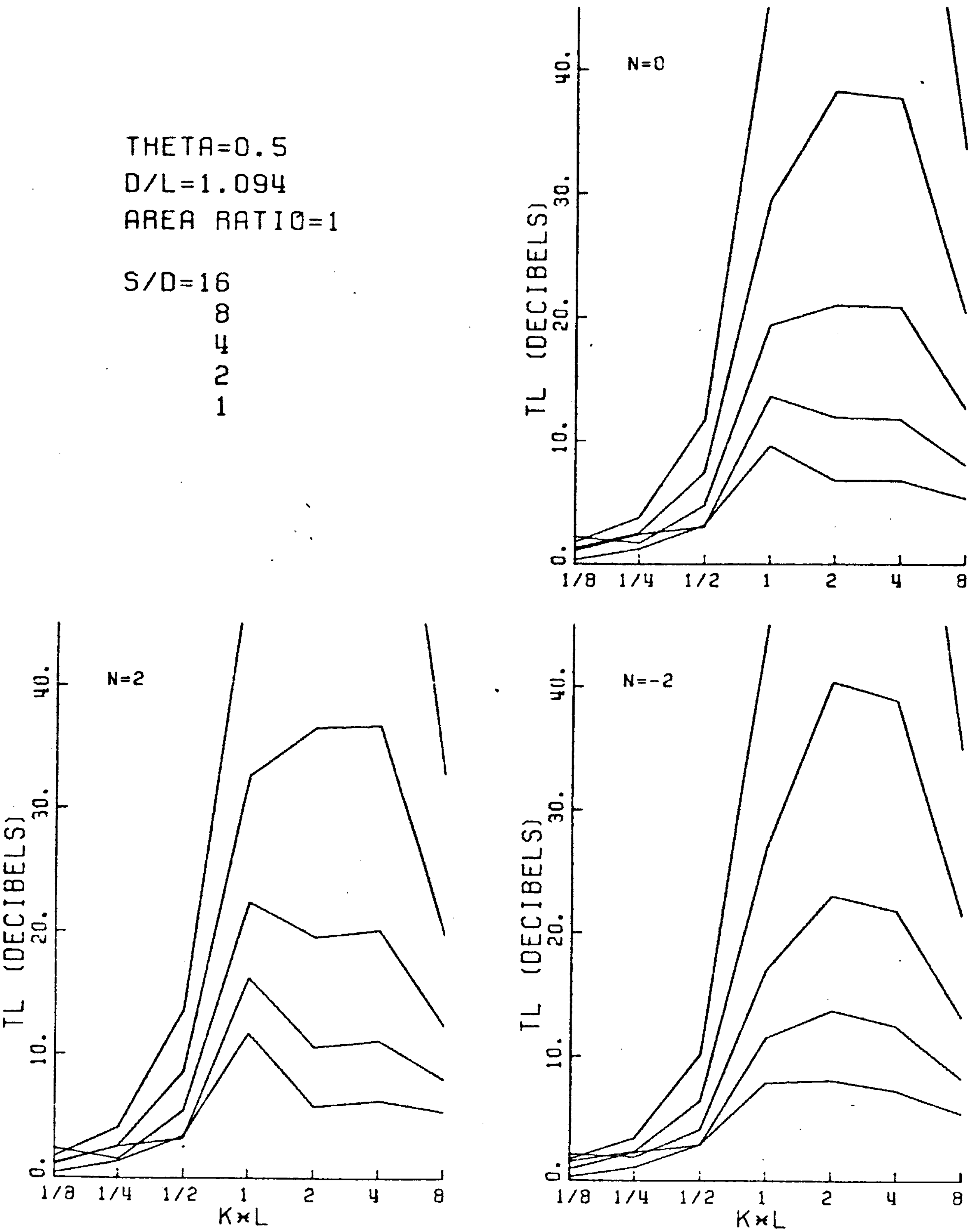

THETA=0.5
D/L=1.094
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.66

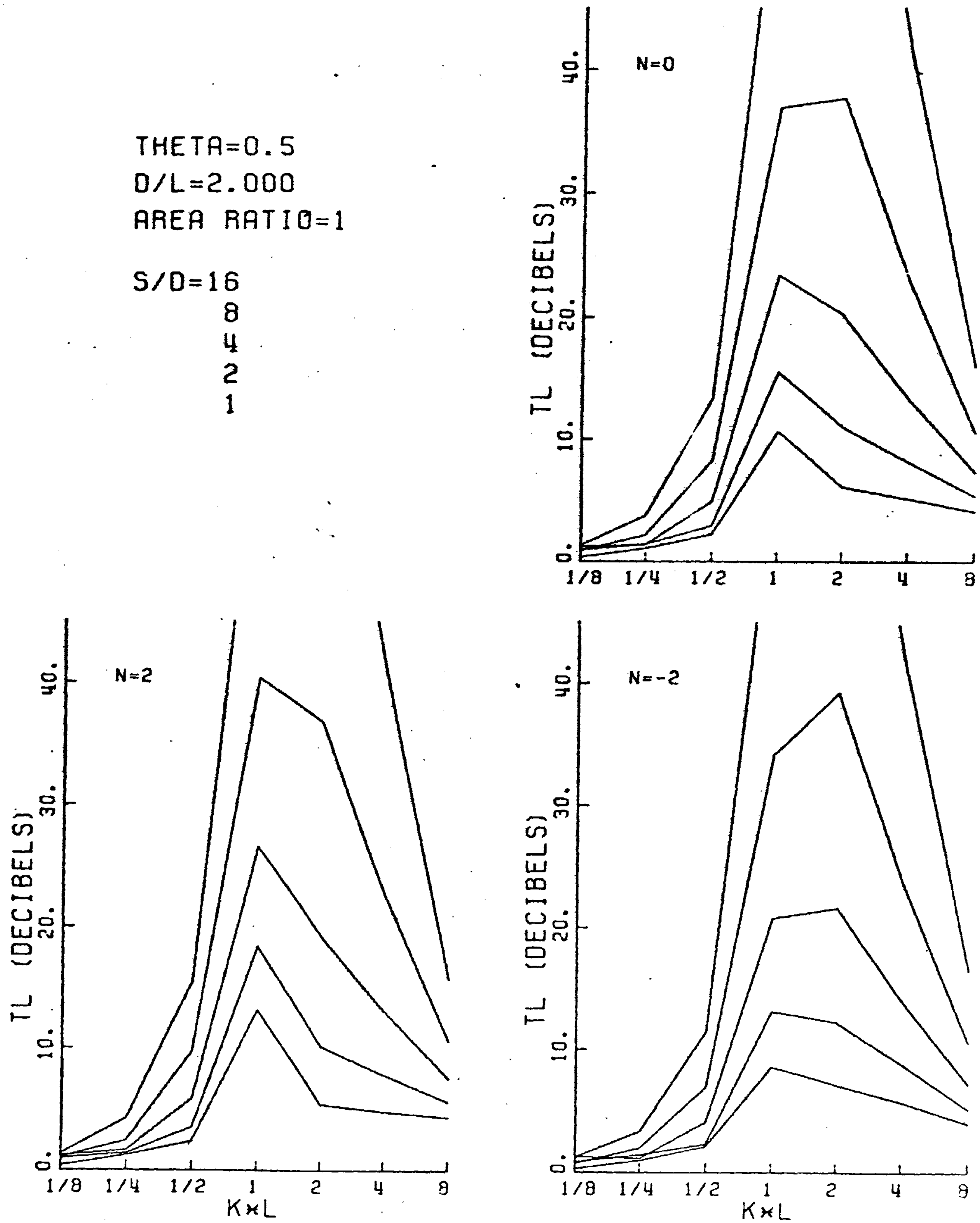

THETA=0.5
D/L=2.000
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.67

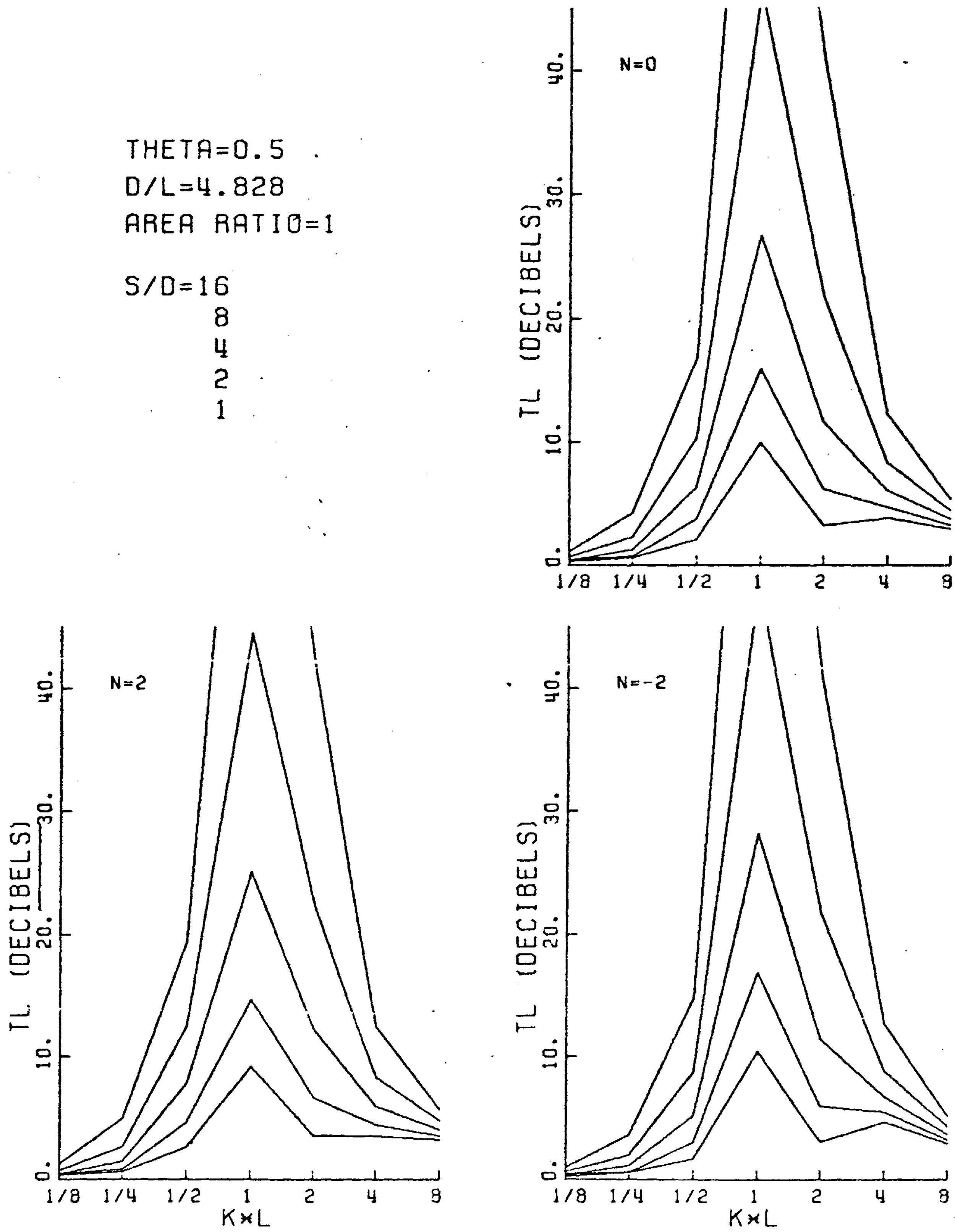

THETA=0.5
D/L=4.828
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
K*L
1/8 1/4 1/2 1 2 4 8
0. 10. 20. 30. 40.

Figure 3.68

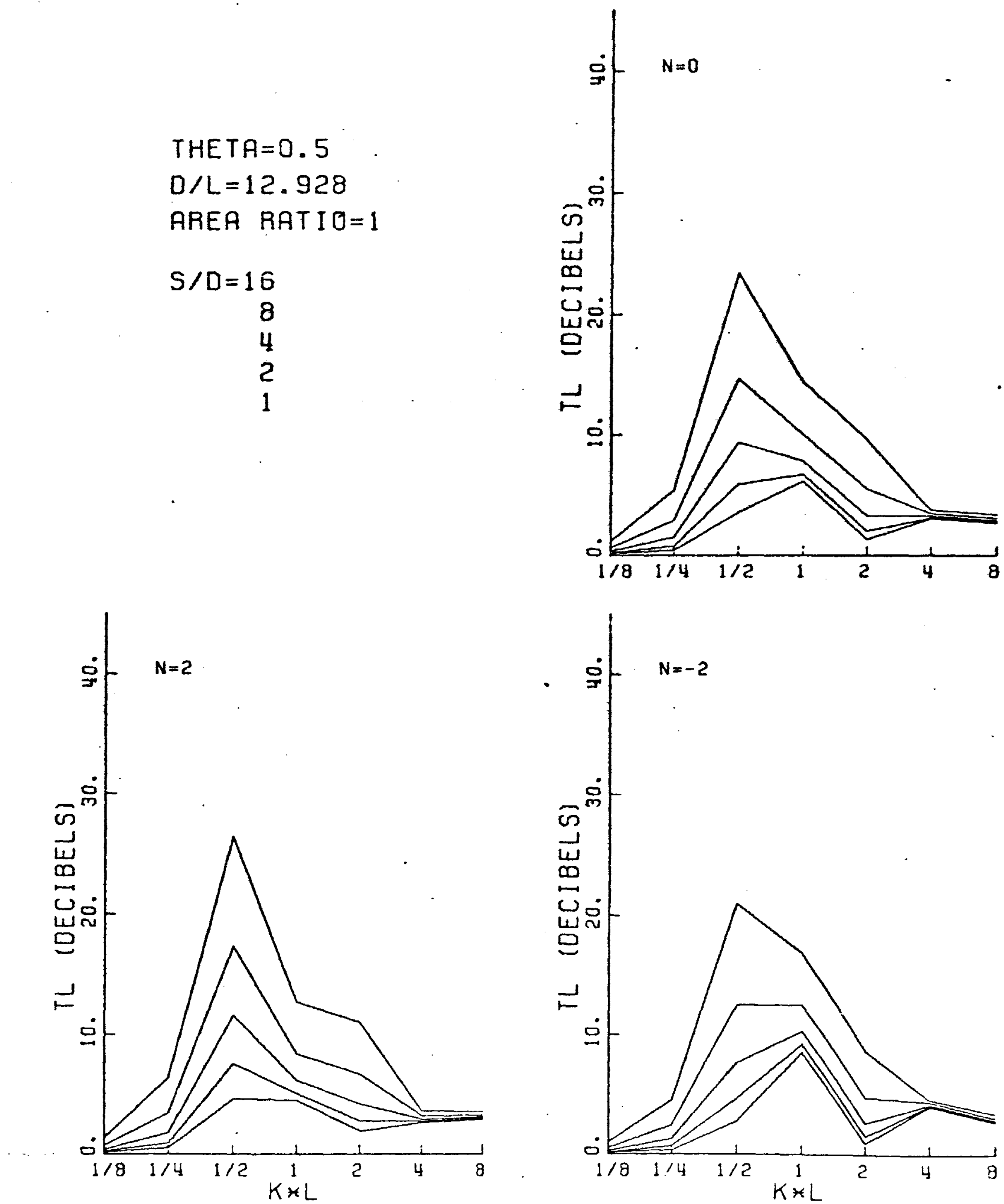

THETA=0.5
D/L=12.928
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.69

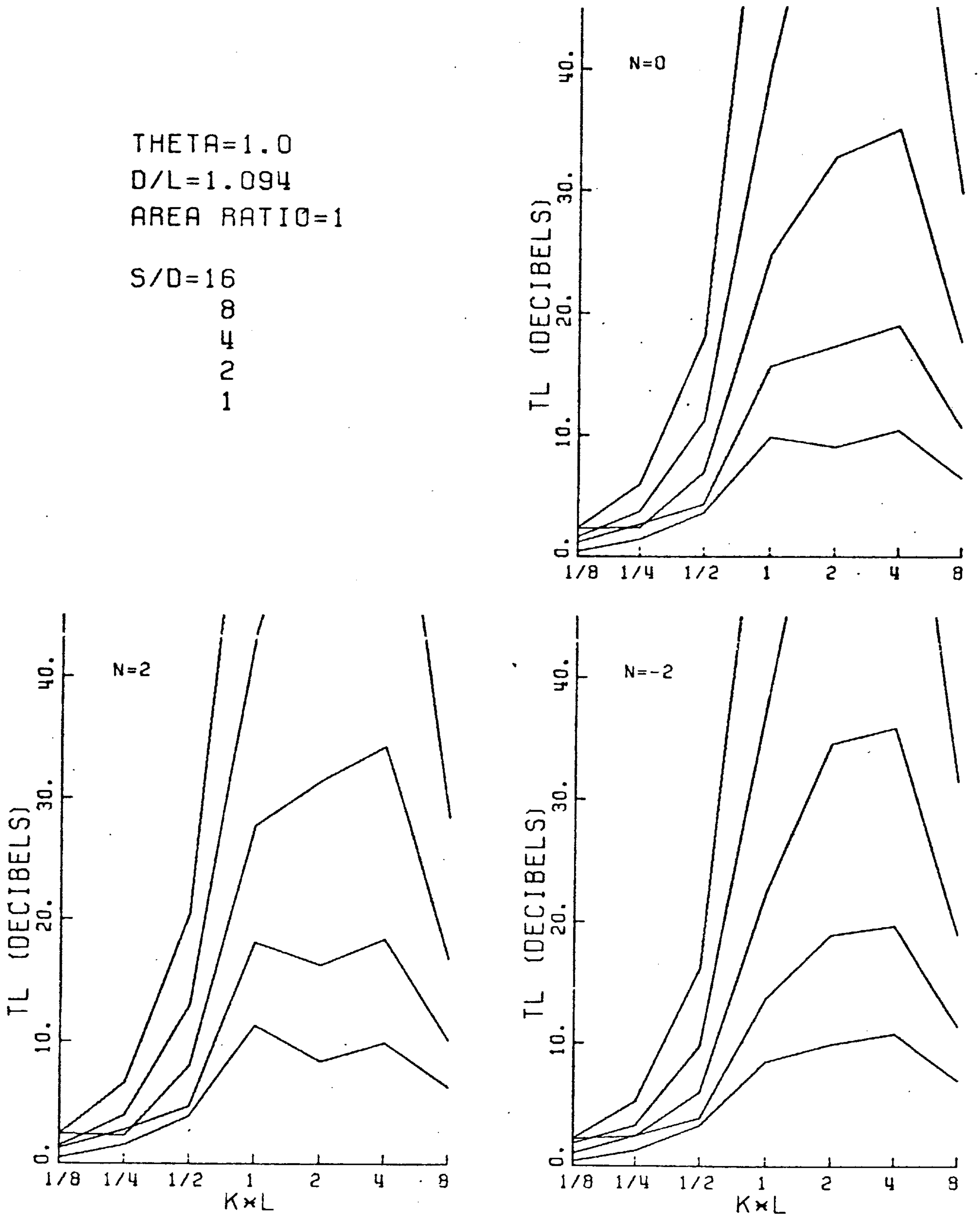

THETA=1.0
D/L=1.094
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8 1/4 1/2 1 2 4 8
K*L

Figure 3.70

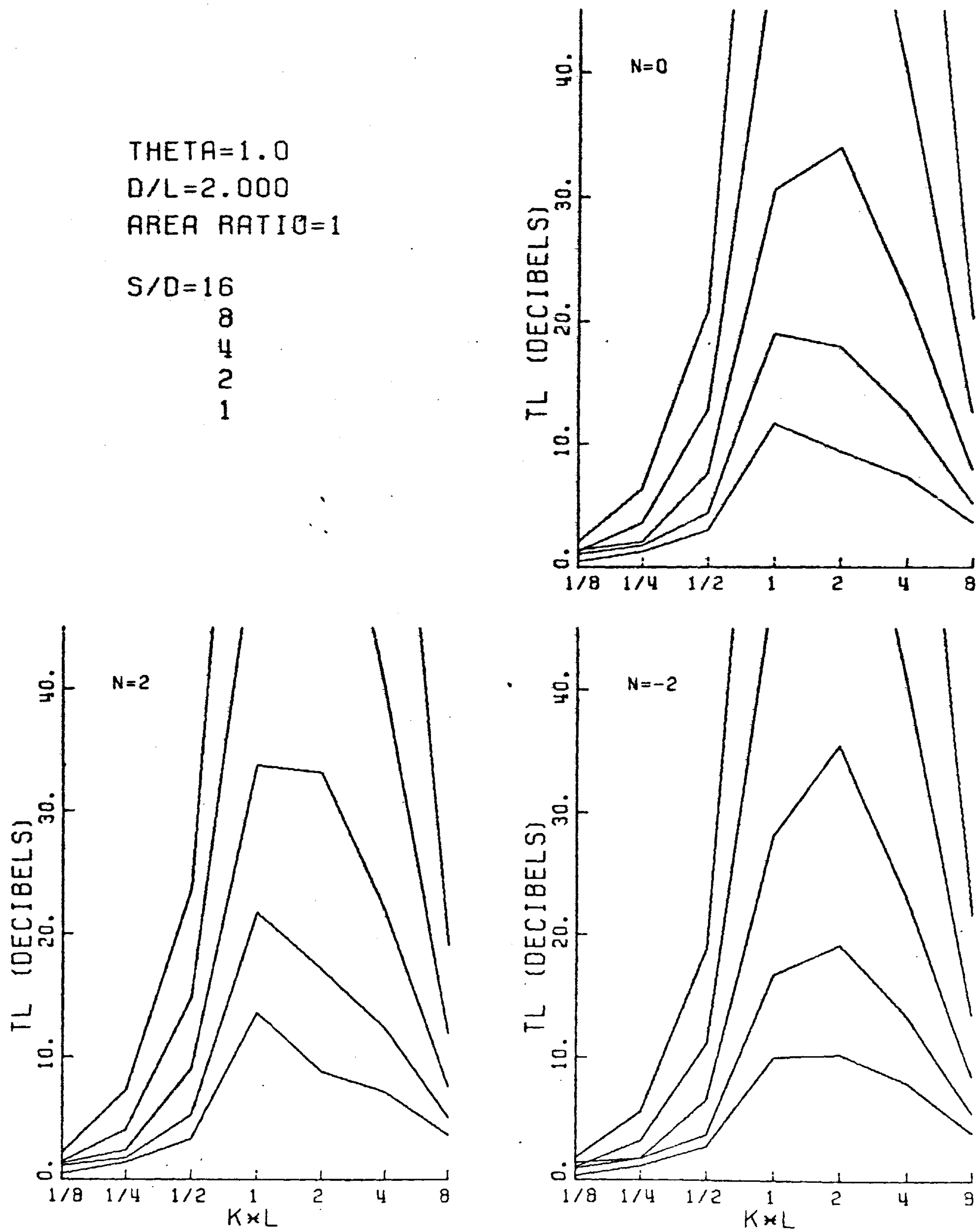

THETA=1.0
D/L=2.000
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
9
K*L

Figure 3.71

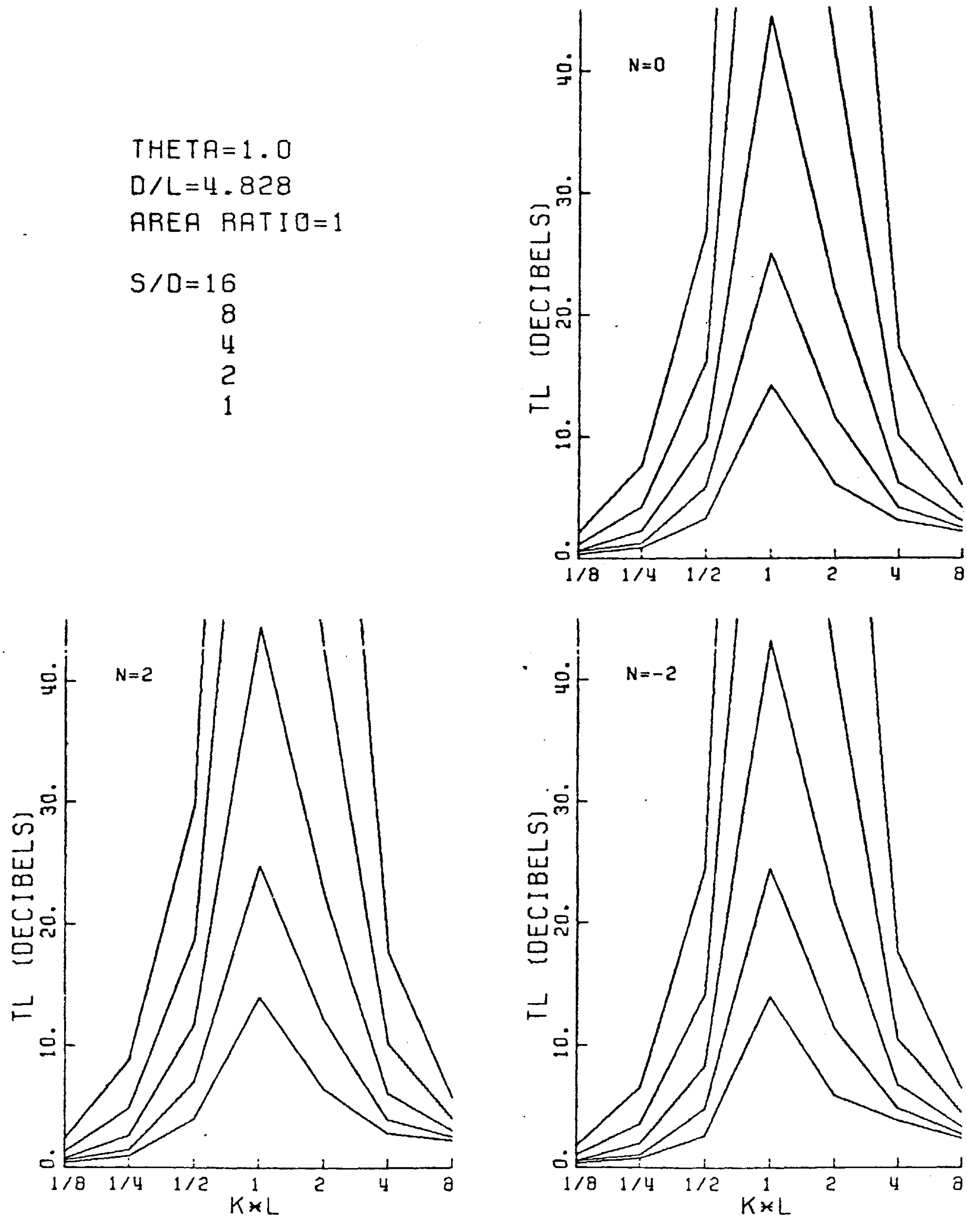

THETA=1.0
D/L=4.828
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
K*L
1/8 1/4 1/2 1 2 4 8
0. 10. 20. 30. 40.

Figure 3.72

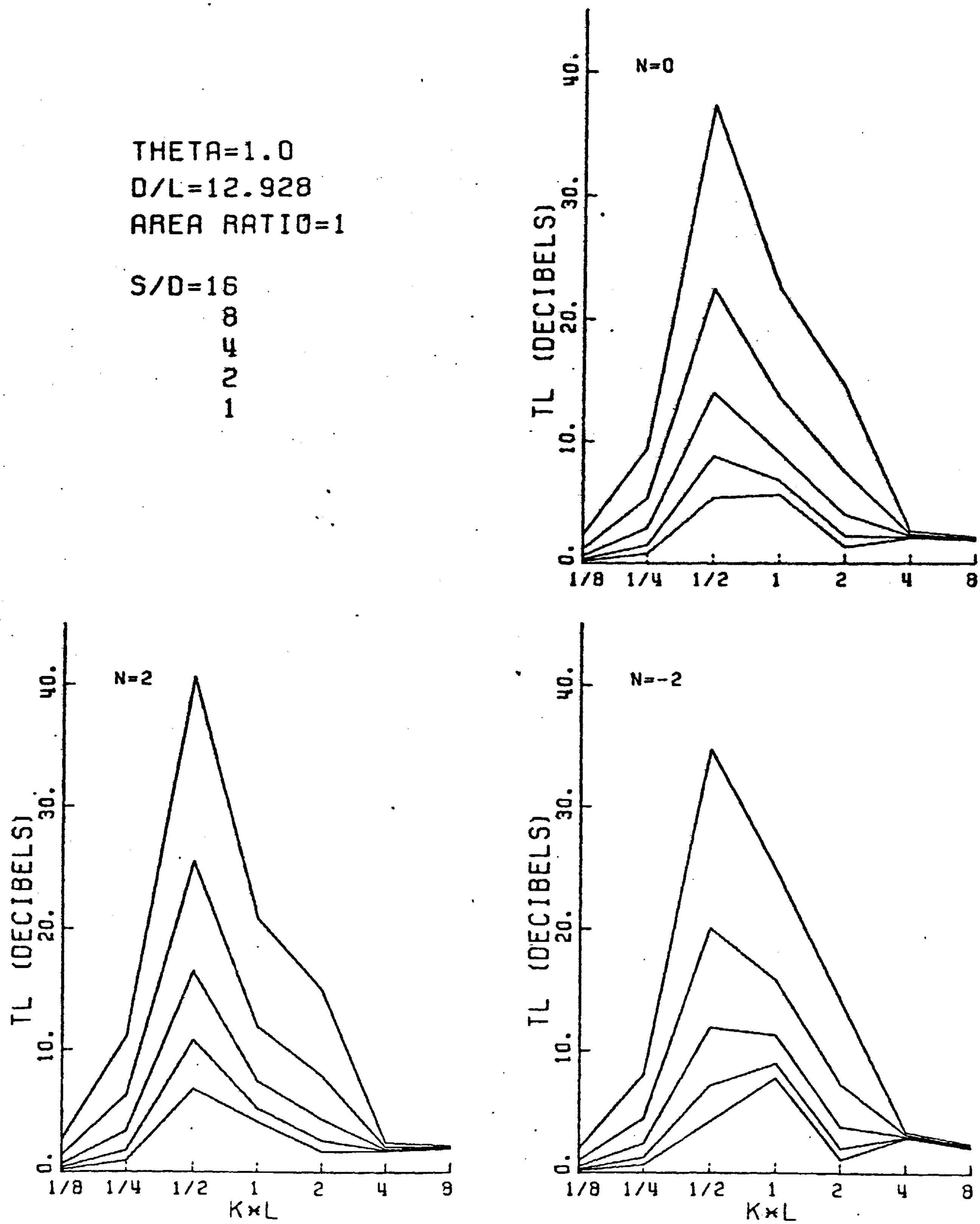
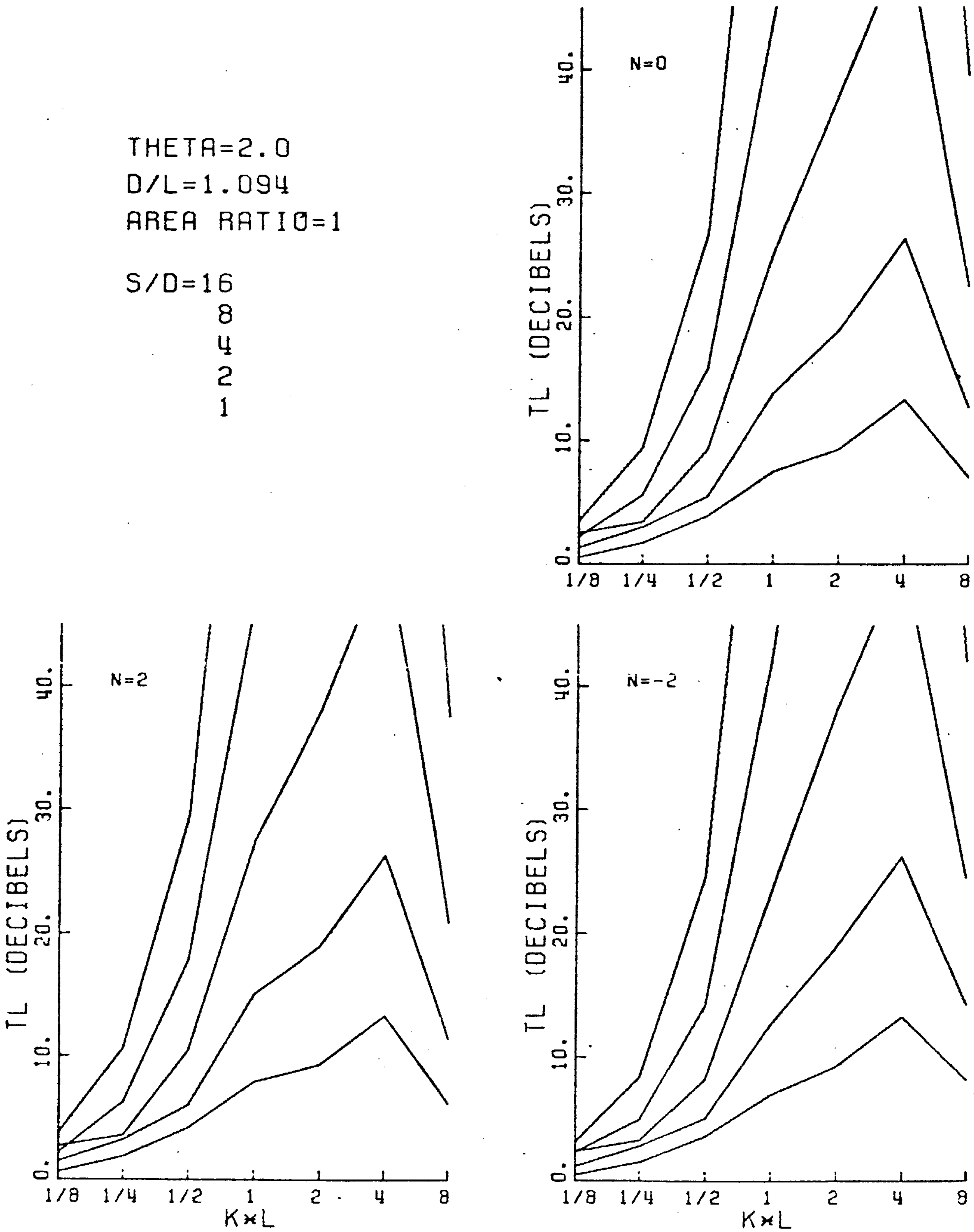

THETA=1.0
D/L=12.928
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.73



THETA=2.0
D/L=1.094
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0. 10. 20. 30. 40.
1/8 1/4 1/2 1 2 4 8
K*L

Figure 3.74

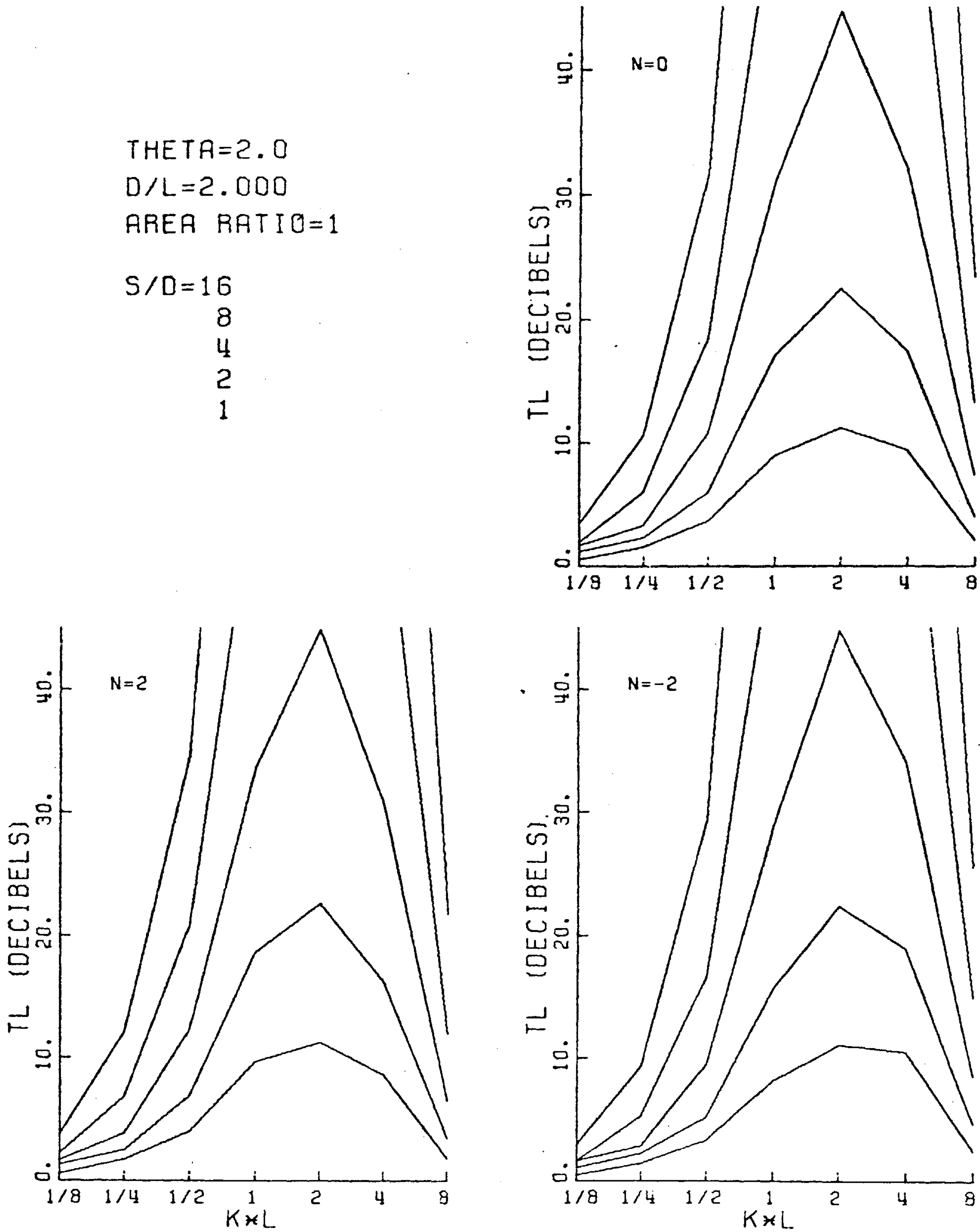

THETA=2.0
D/L=2.000
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.75

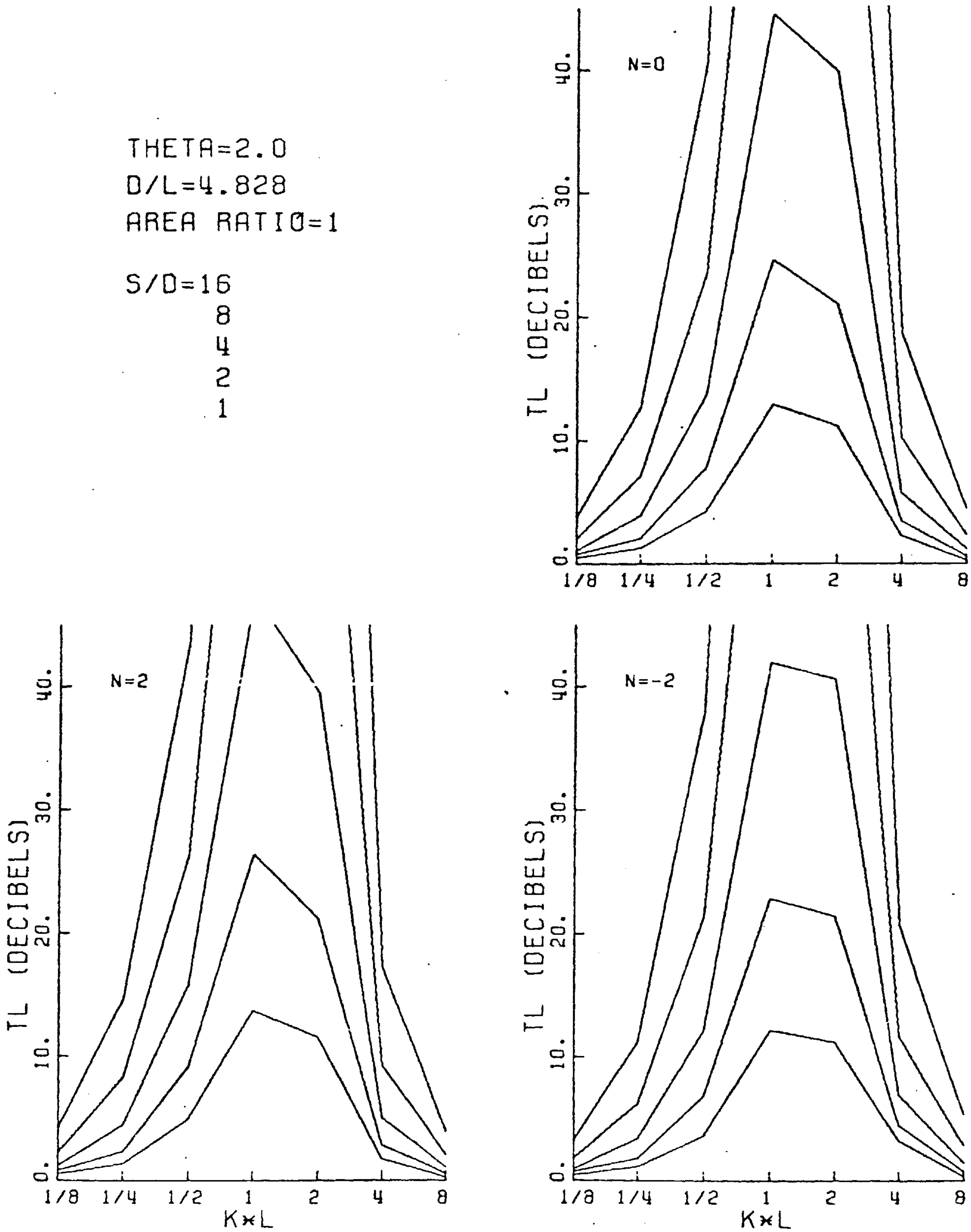

THETA=2.0
D/L=4.828
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.76

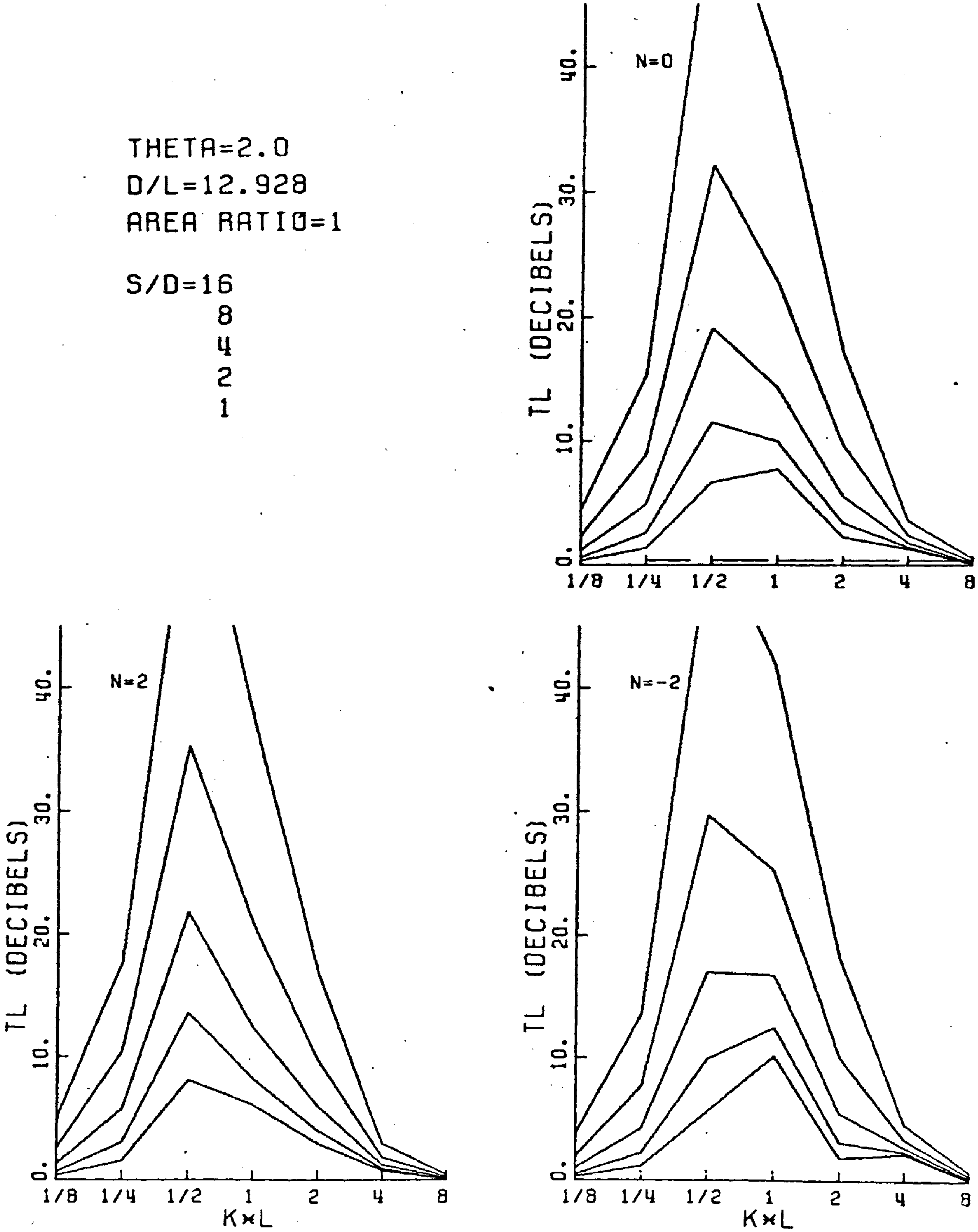

THETA=2.0
D/L=12.928
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.77

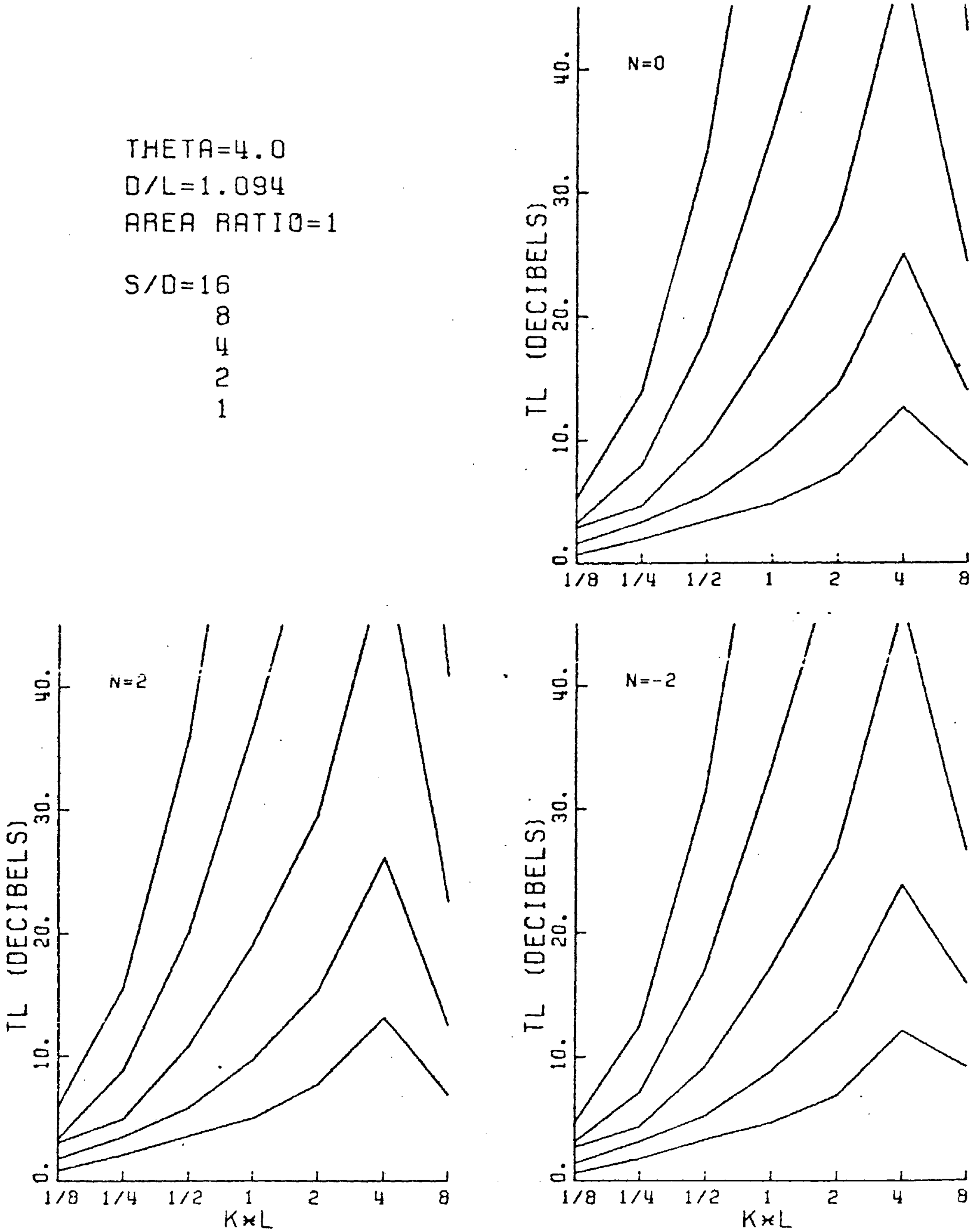

THETA=4.0
D/L=1.094
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.78

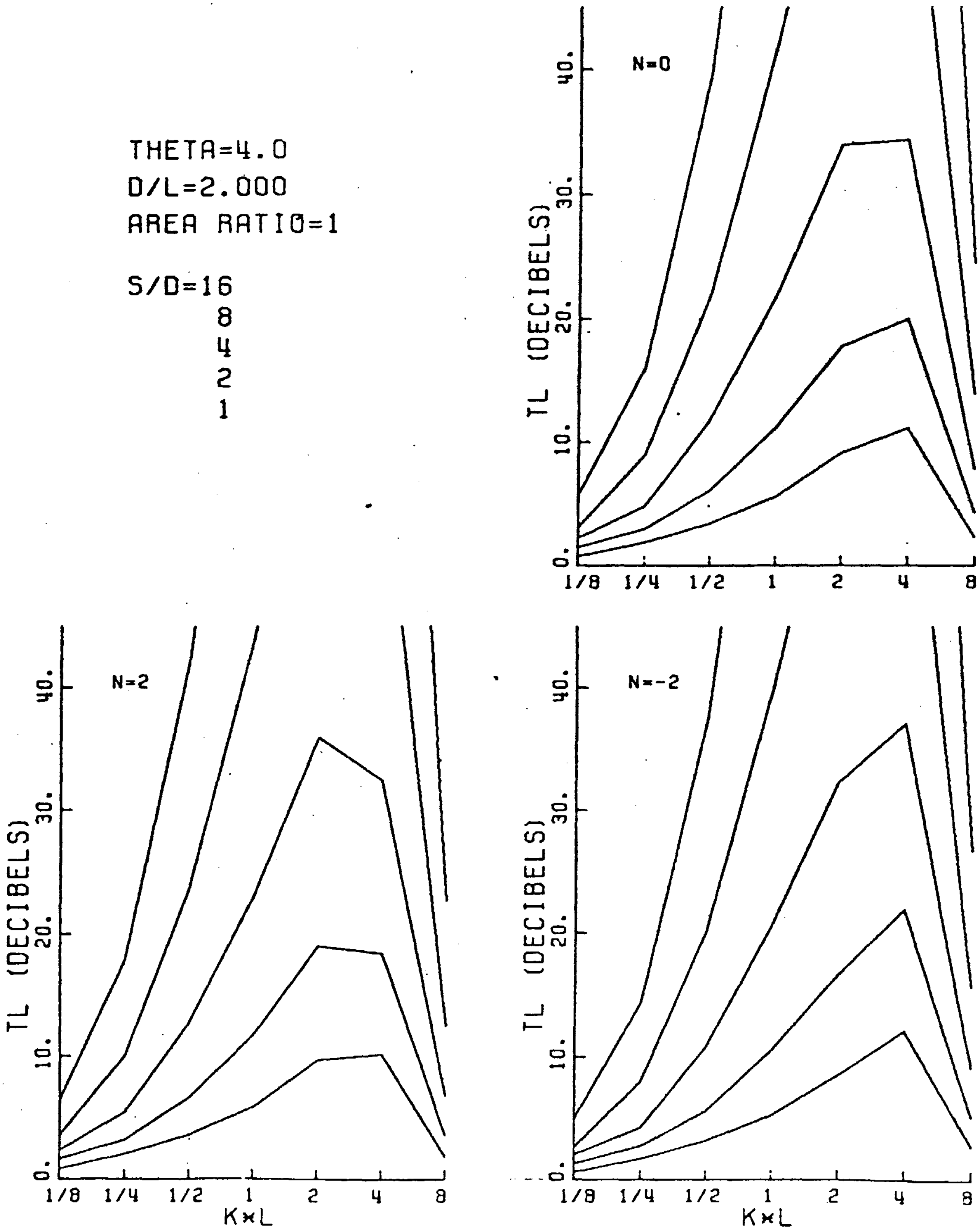

THETA=4.0
D/L=2.000
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.79

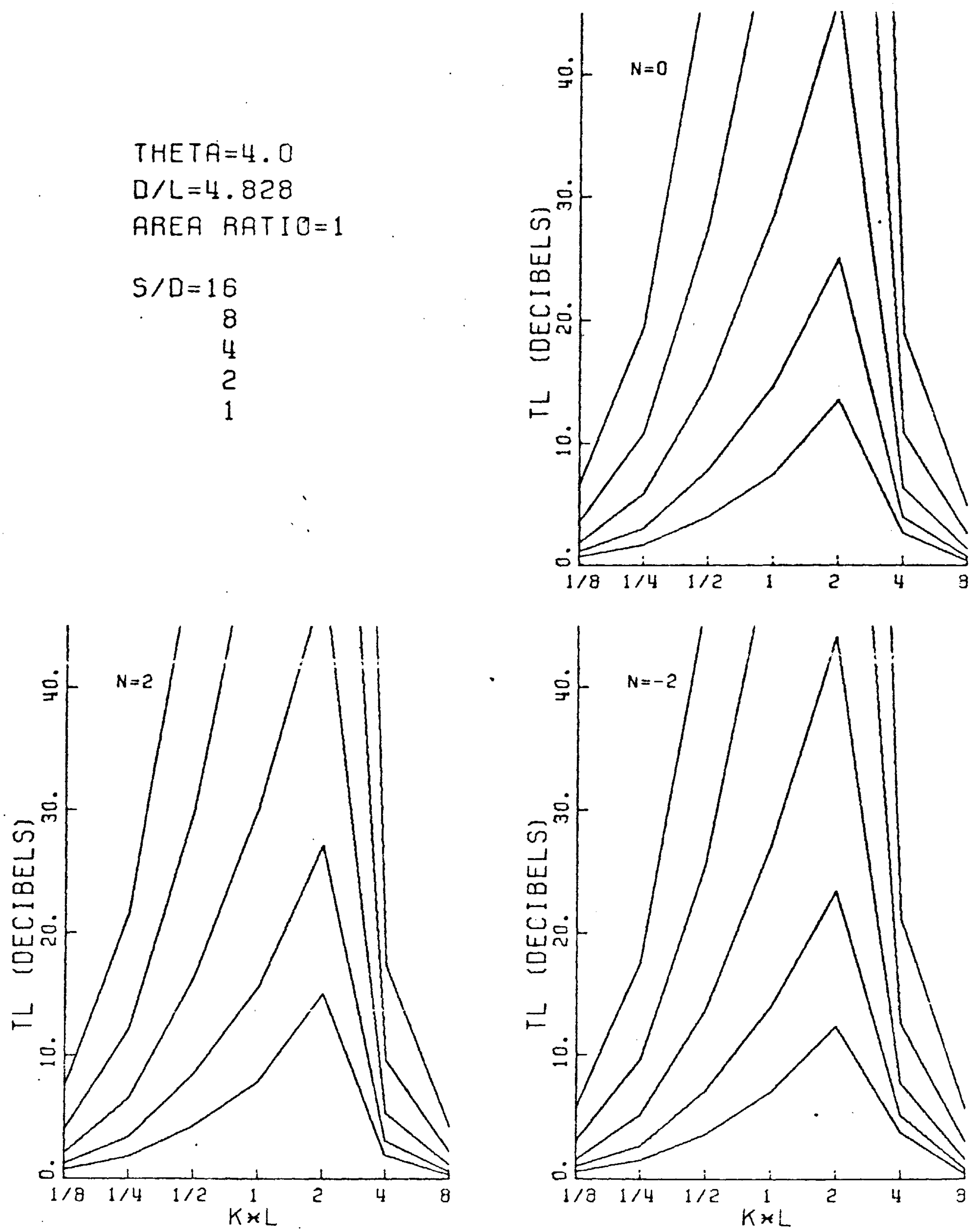

THETA=4.0
D/L=4.828
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.80

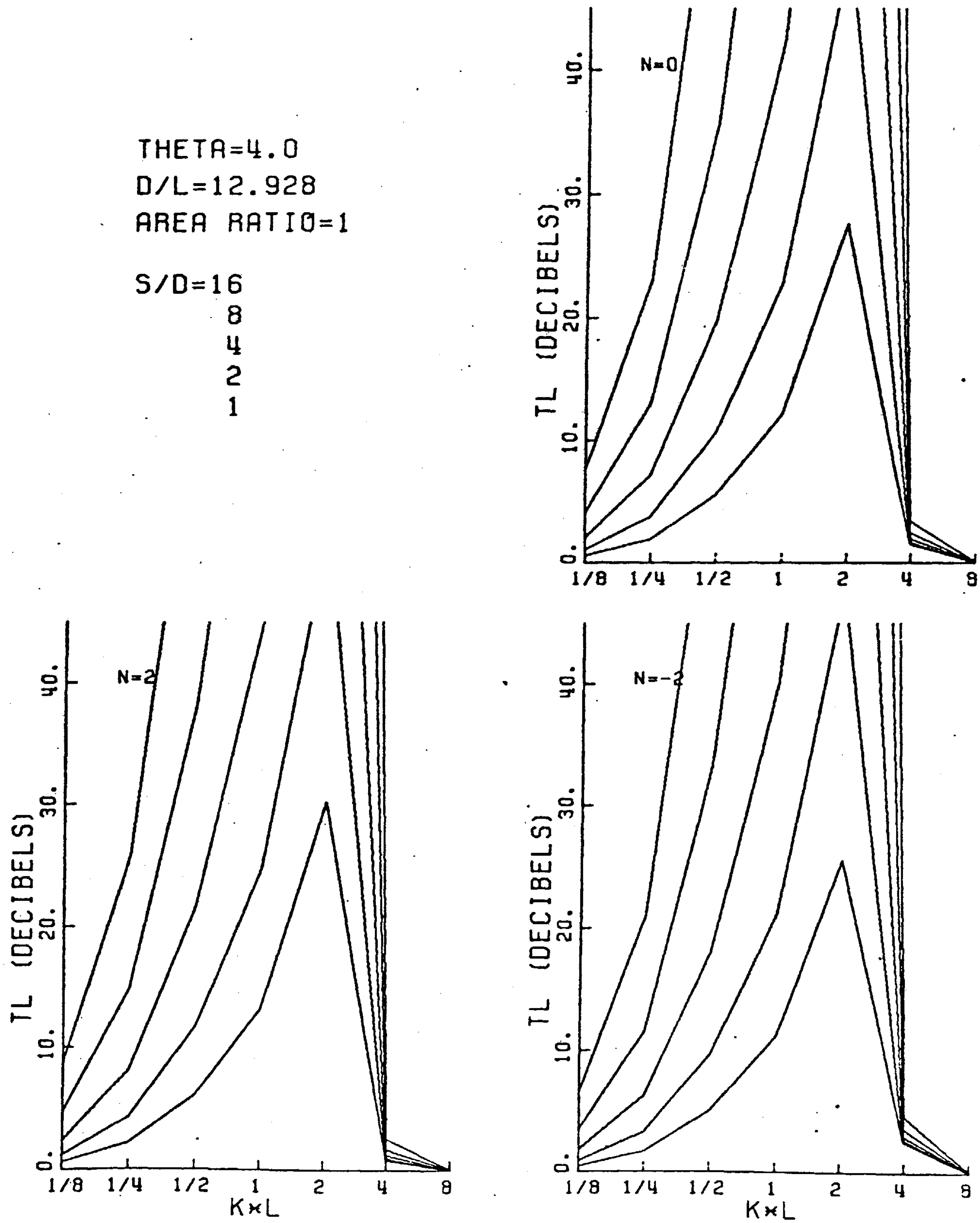

THETA=4.0
D/L=12.928
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.81

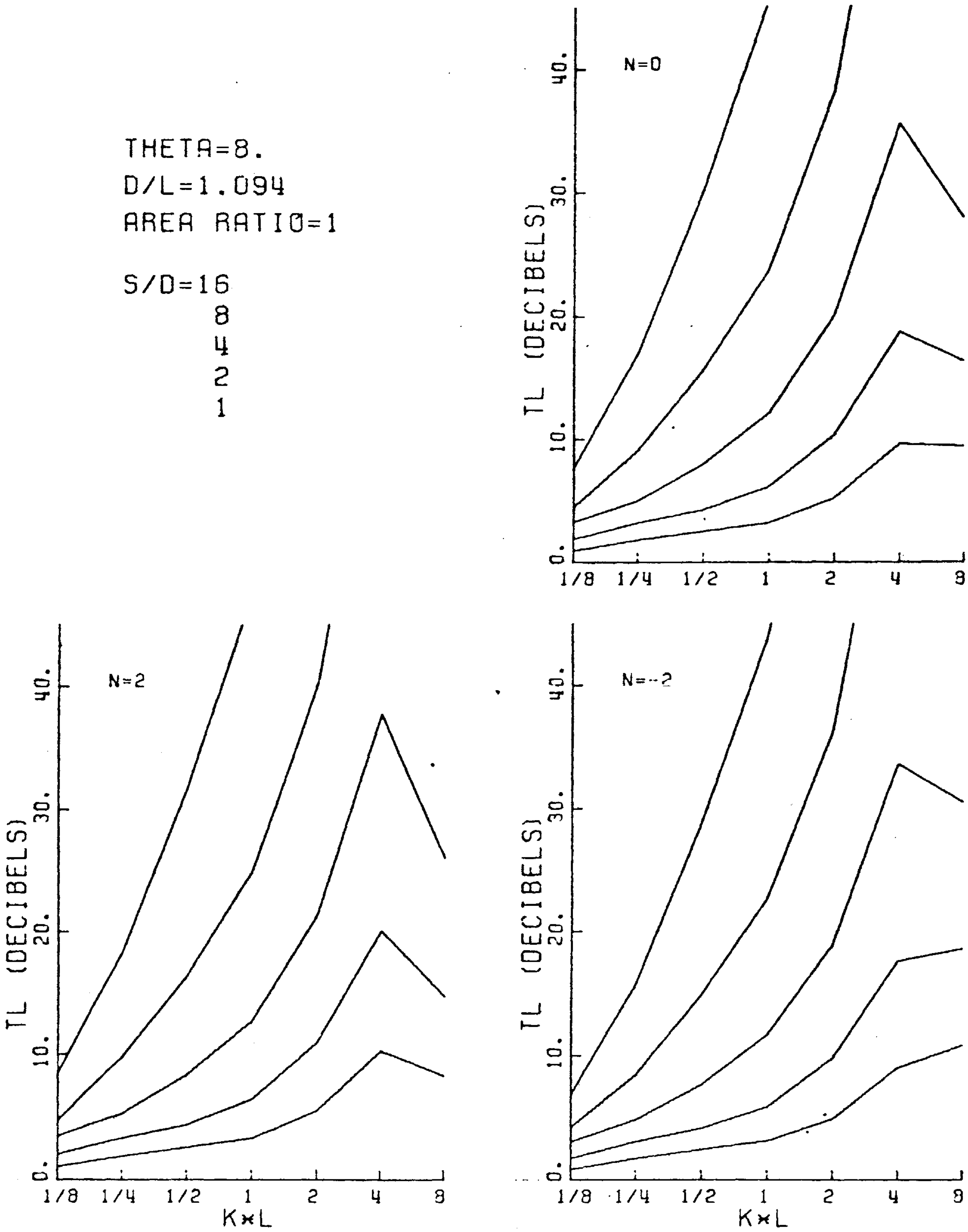
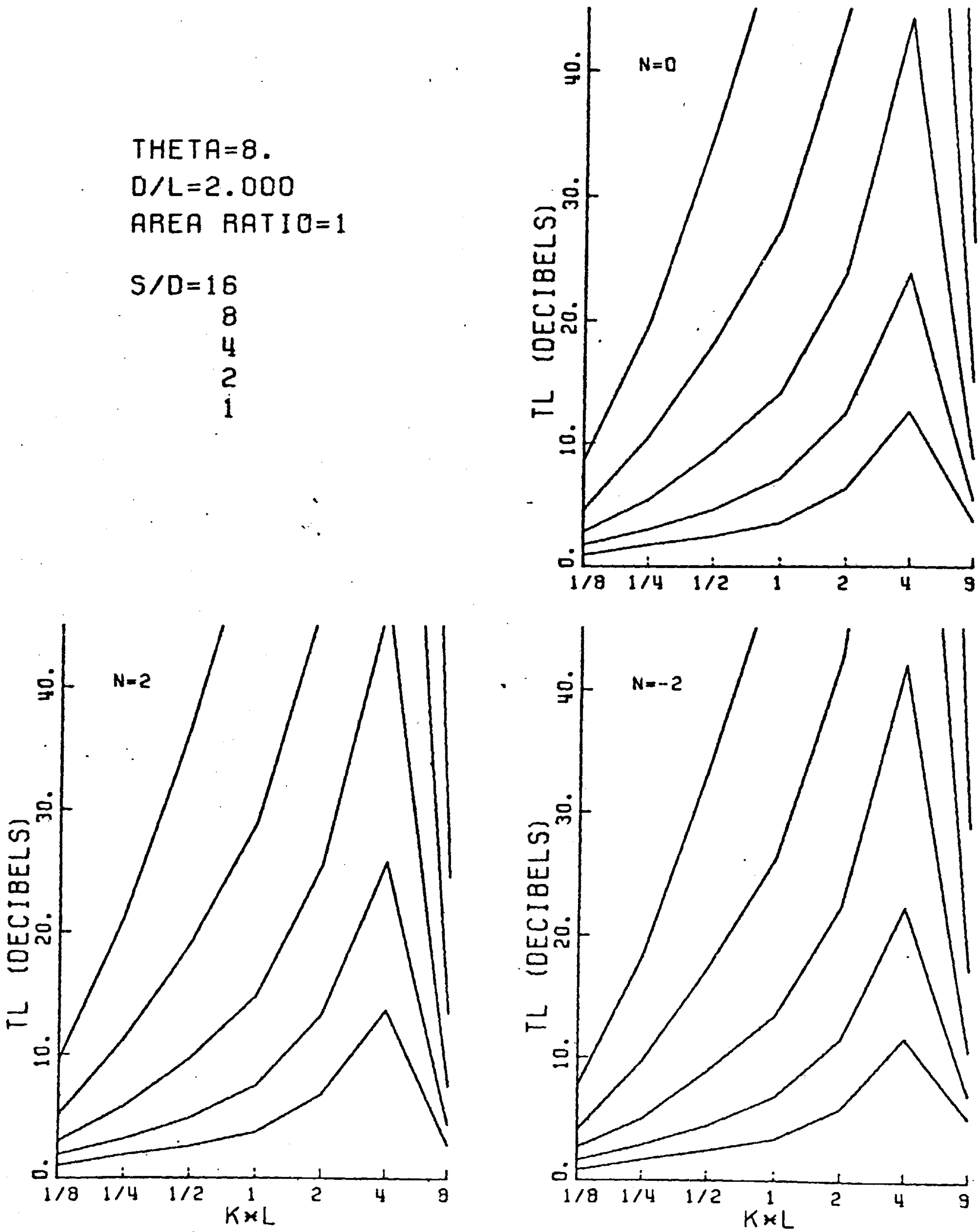

THETA=8.
D/L=1.094
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.82



THETA=8.
D/L=2.000
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
K*L
1/8 1/4 1/2 1 2 4 8
0. 10. 20. 30. 40.

Figure 3.83

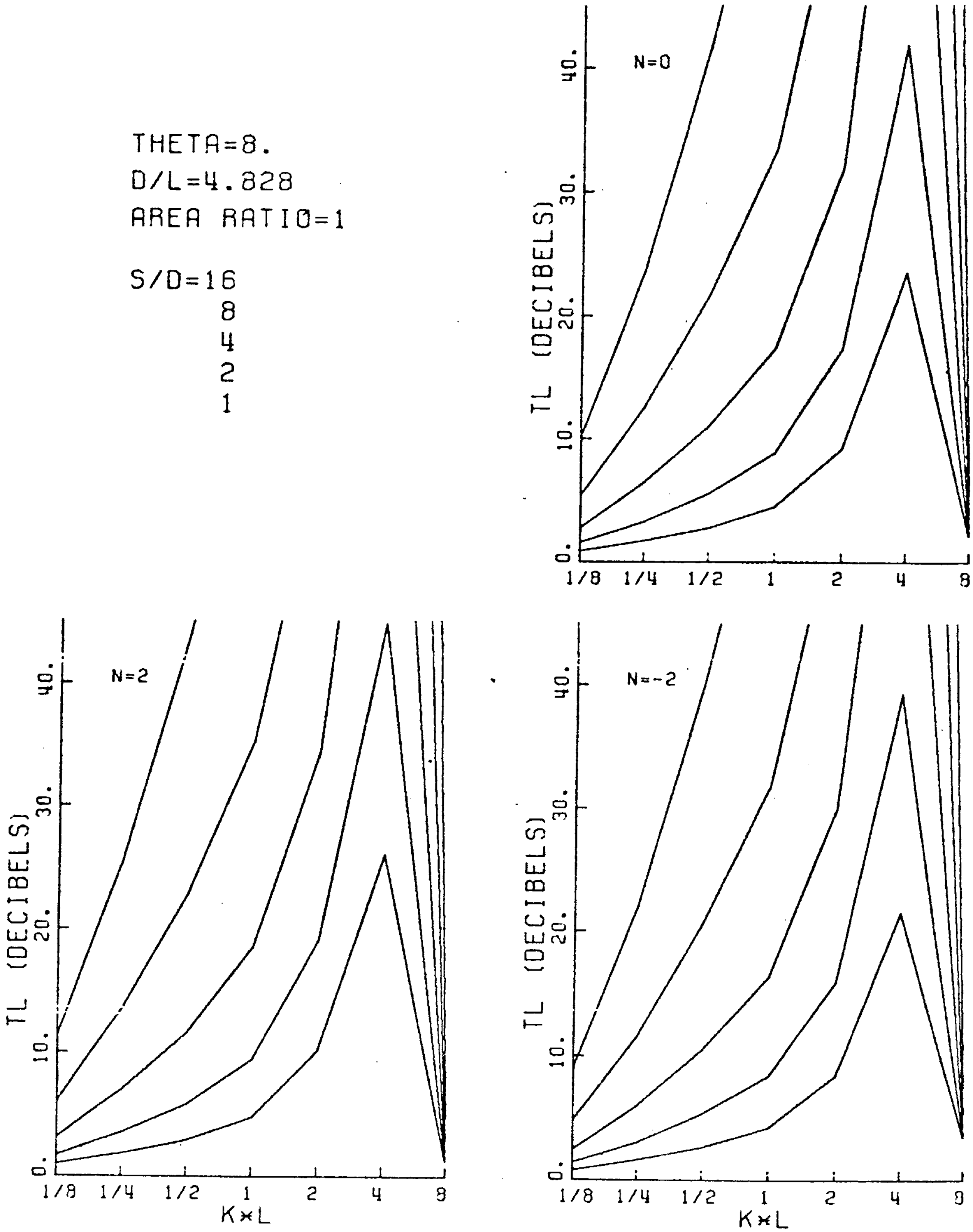

THETA=8.
D/L=4.828
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.84

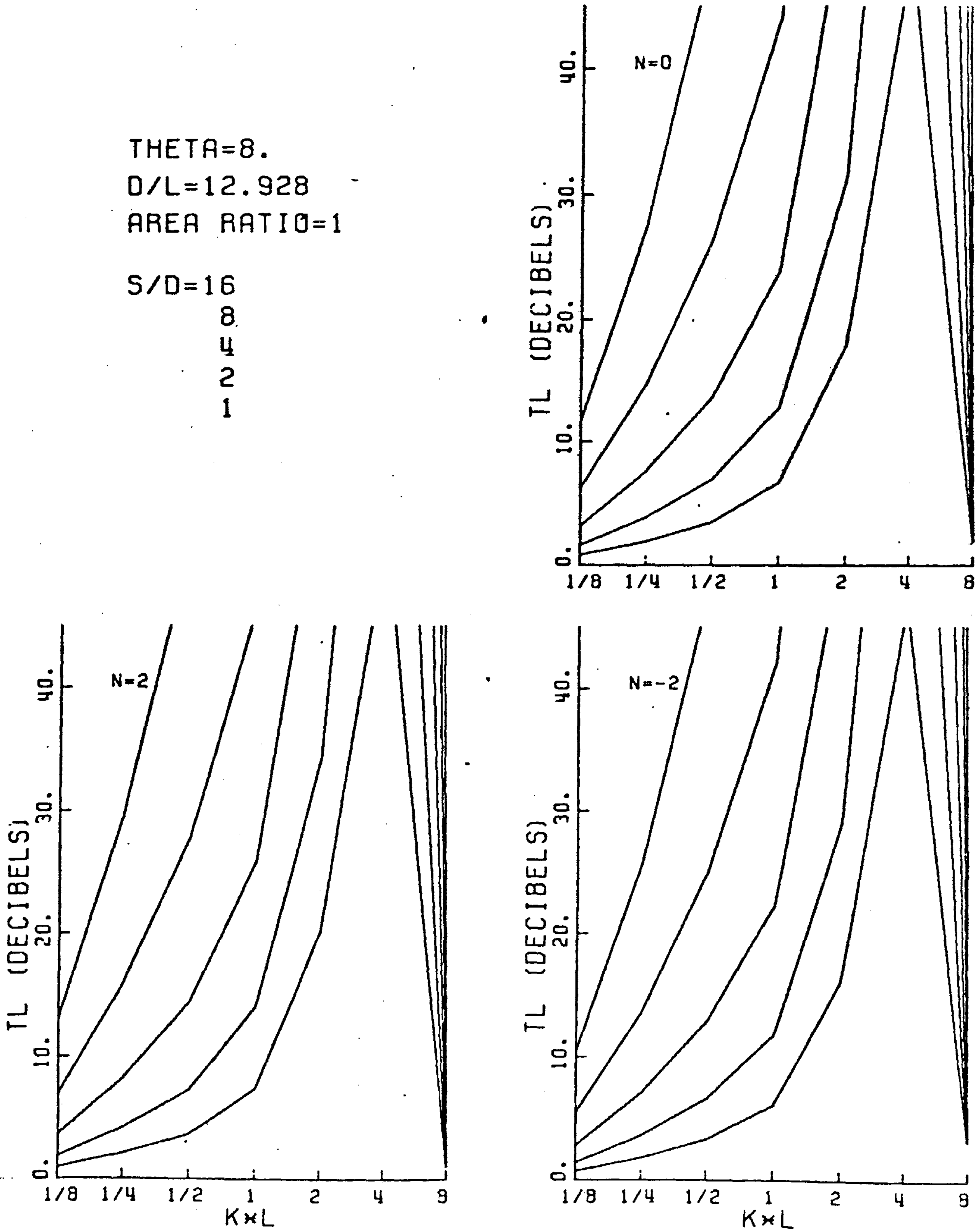

THETA=8.
D/L=12.928
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
K*L
1/8 1/4 1/2 1 2 4 8
0. 10. 20. 30. 40.

Figure 3.85

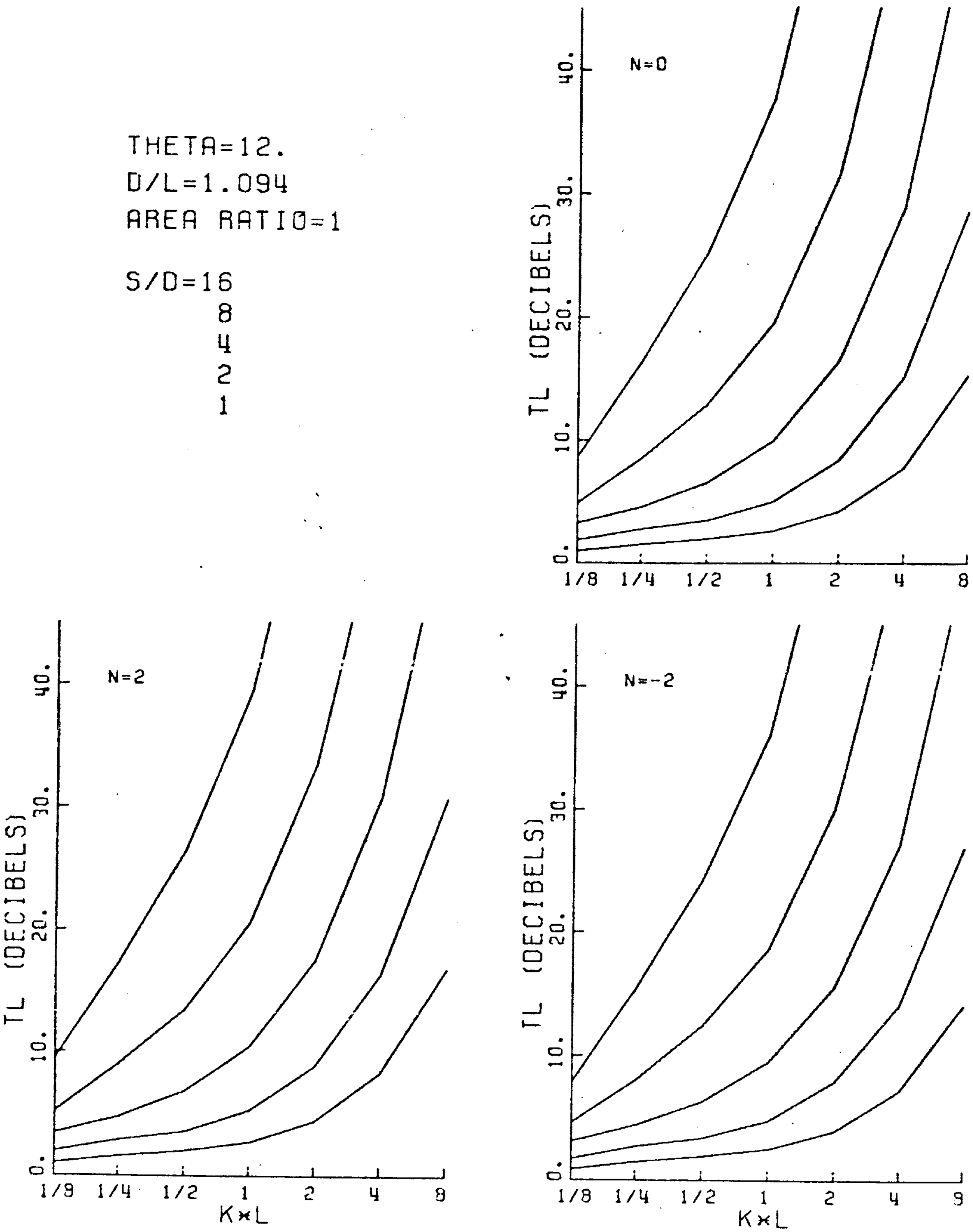

THETA=12.
D/L=1.094
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
K*L
1/8 1/4 1/2 1 2 4 8
0. 10. 20. 30. 40.

Figure 3.86

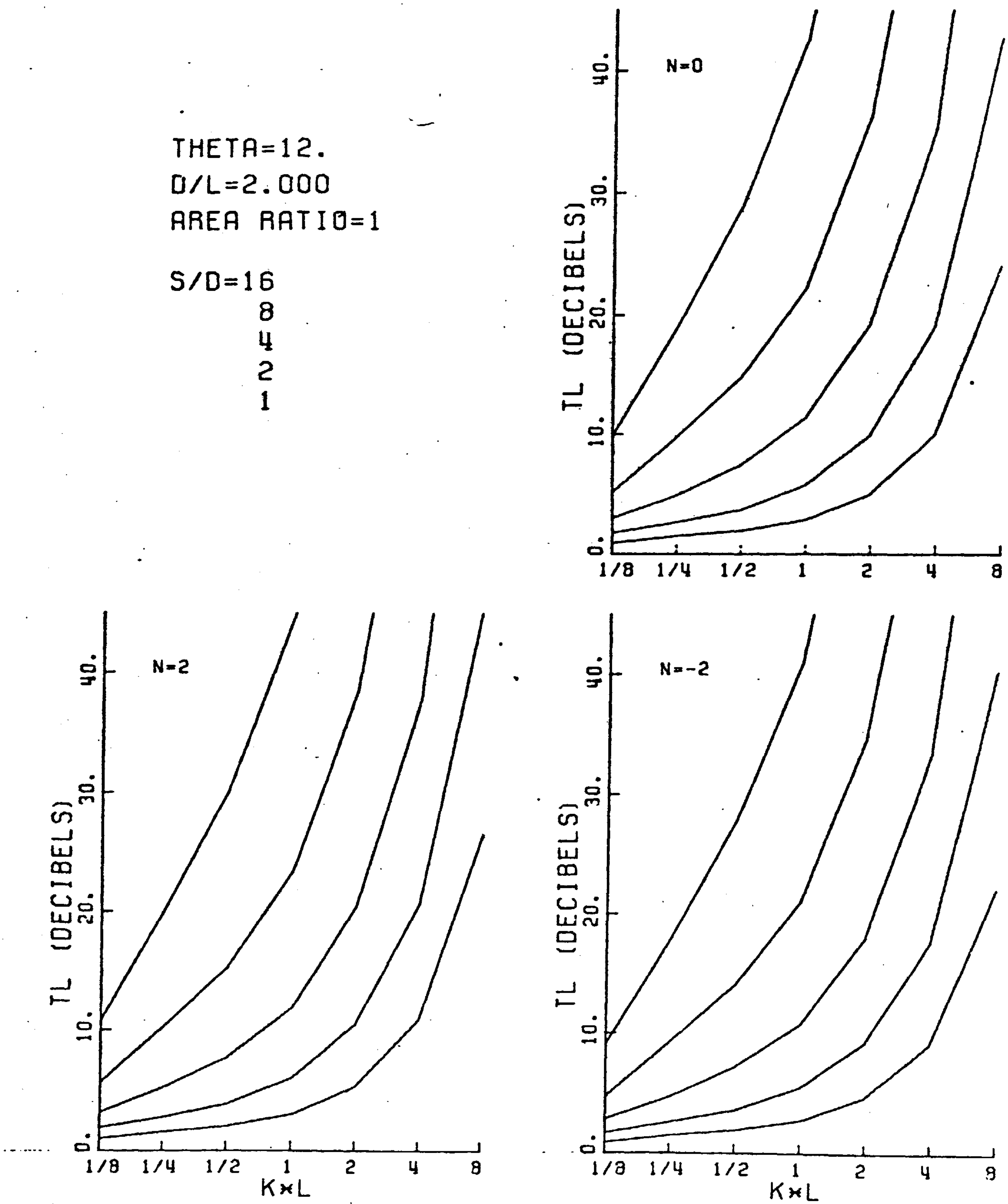

THETA=12.
D/L=2.000
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.87

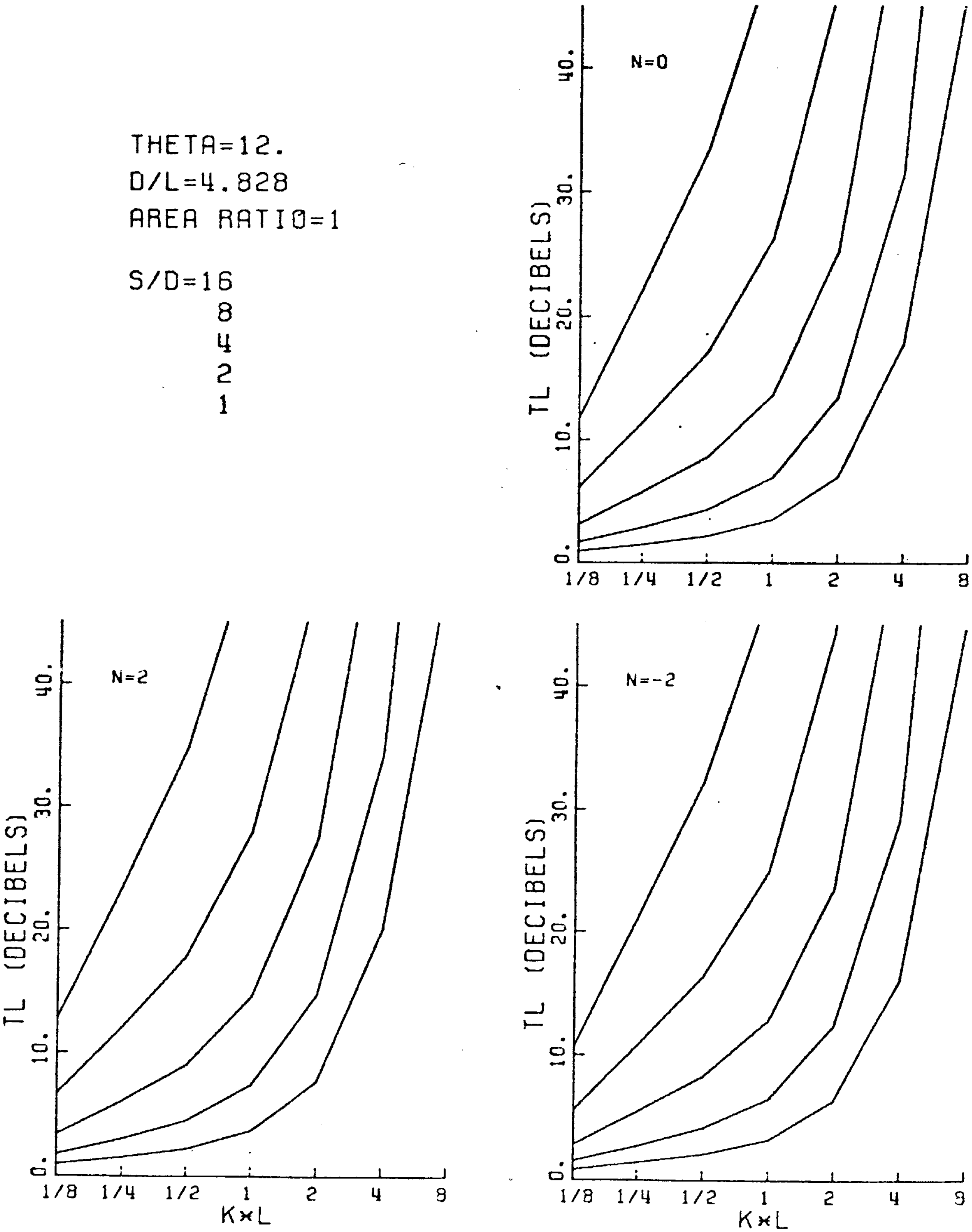

THETA=12.
D/L=4.828
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
K*L
1/8 1/4 1/2 1 2 4 8
0. 10. 20. 30. 40.

Figure 3.88

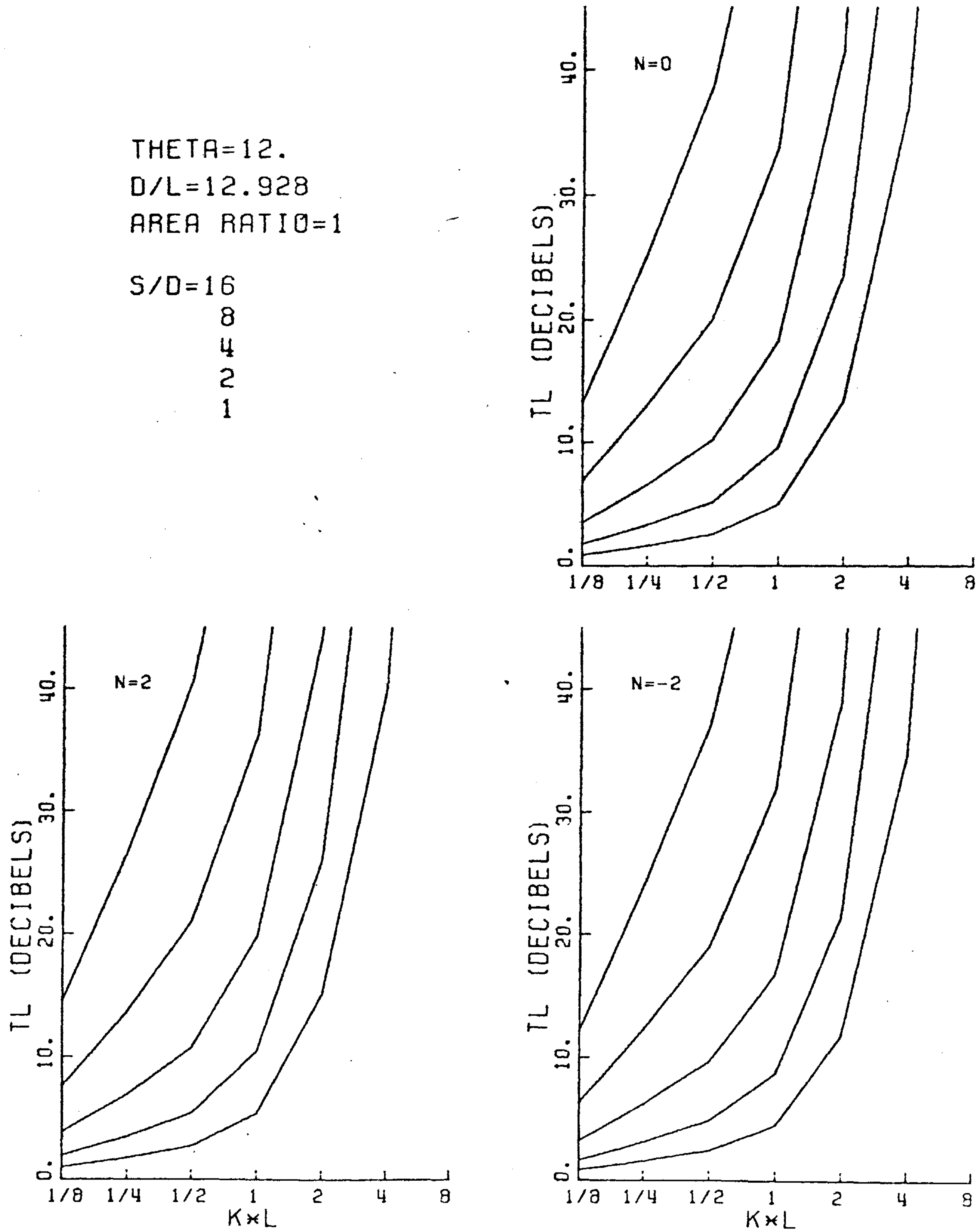

THETA=12.
D/L=12.928
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.89

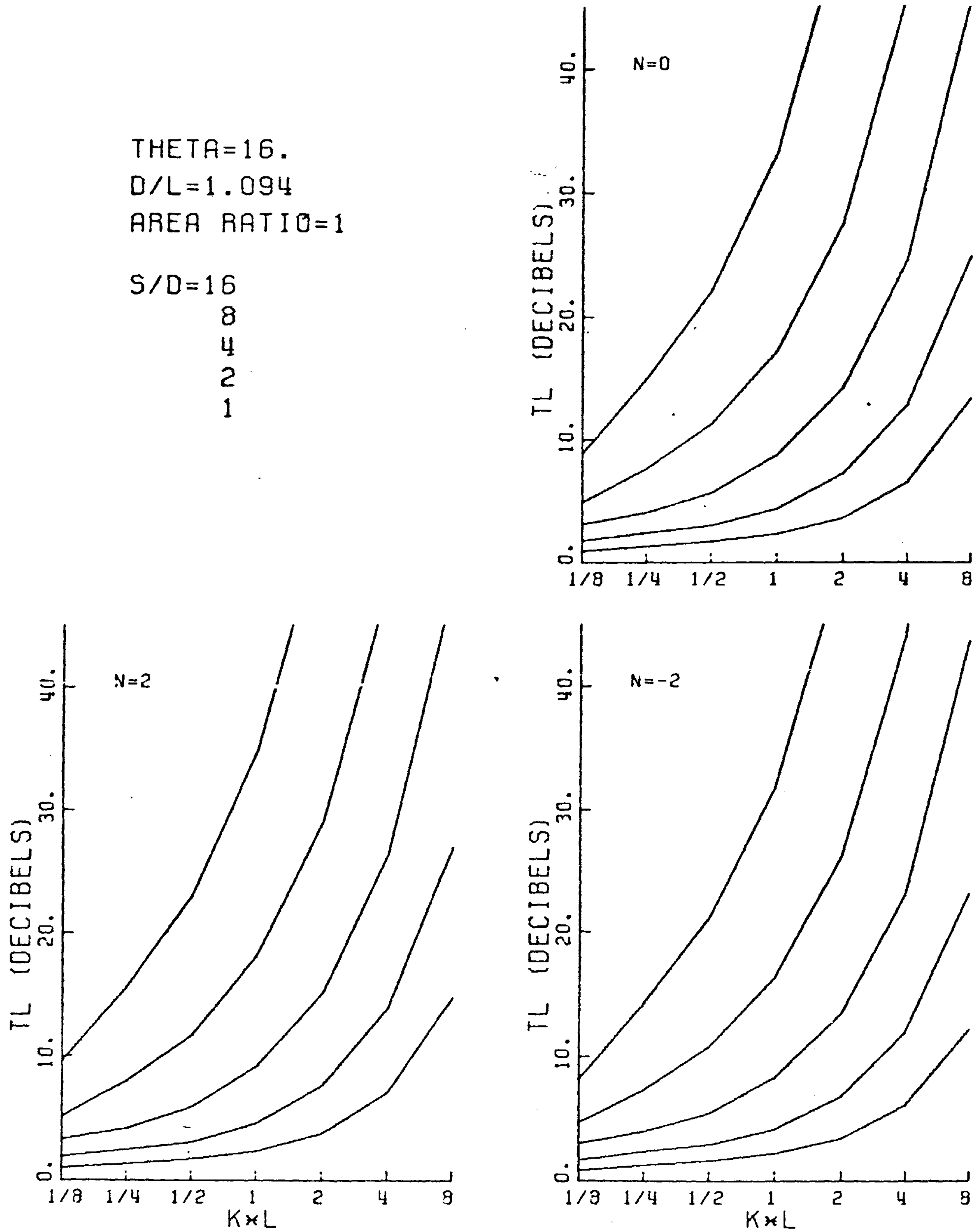

THETA=16.
D/L=1.094
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.90

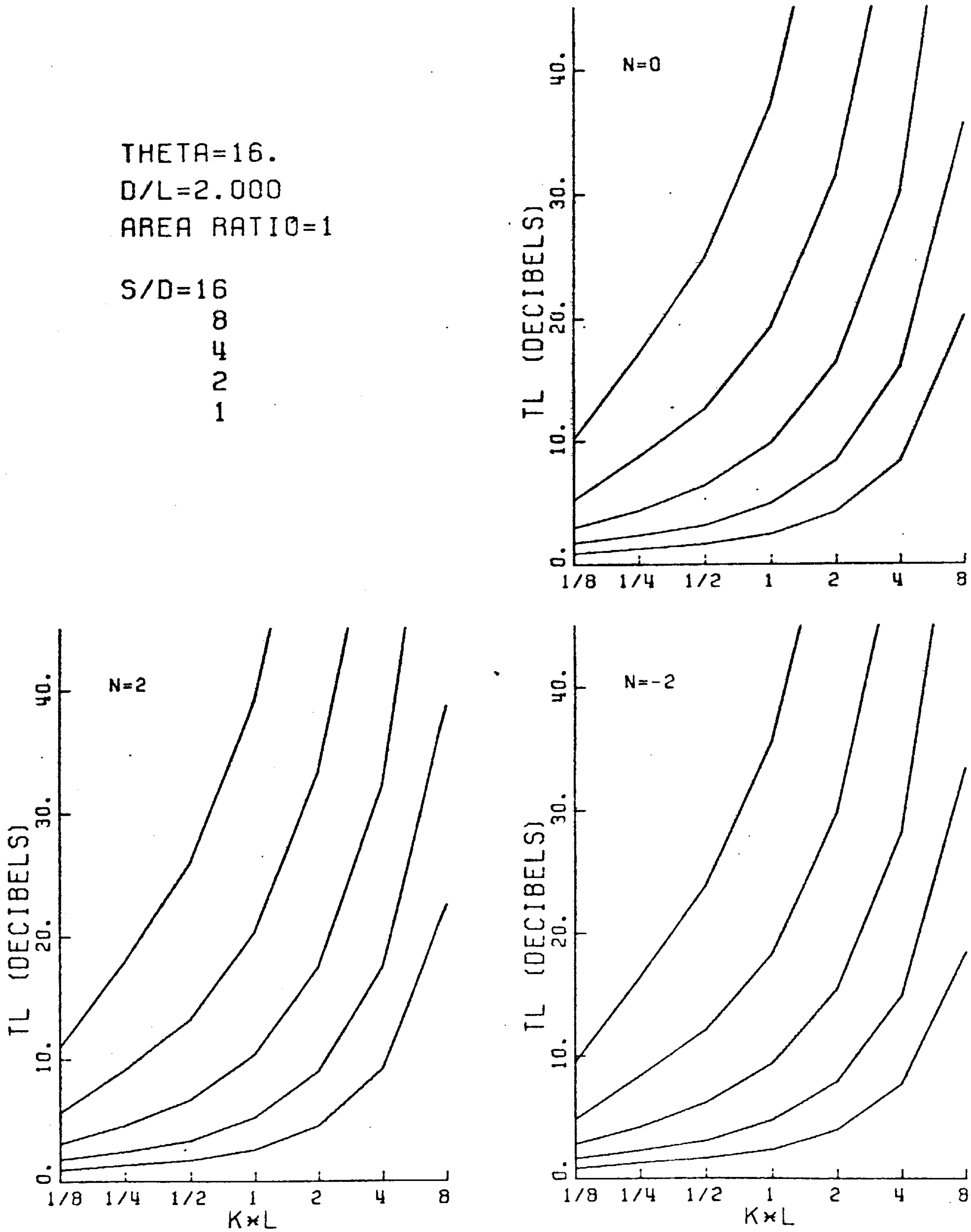

THETA=16.
D/L=2.000
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.91

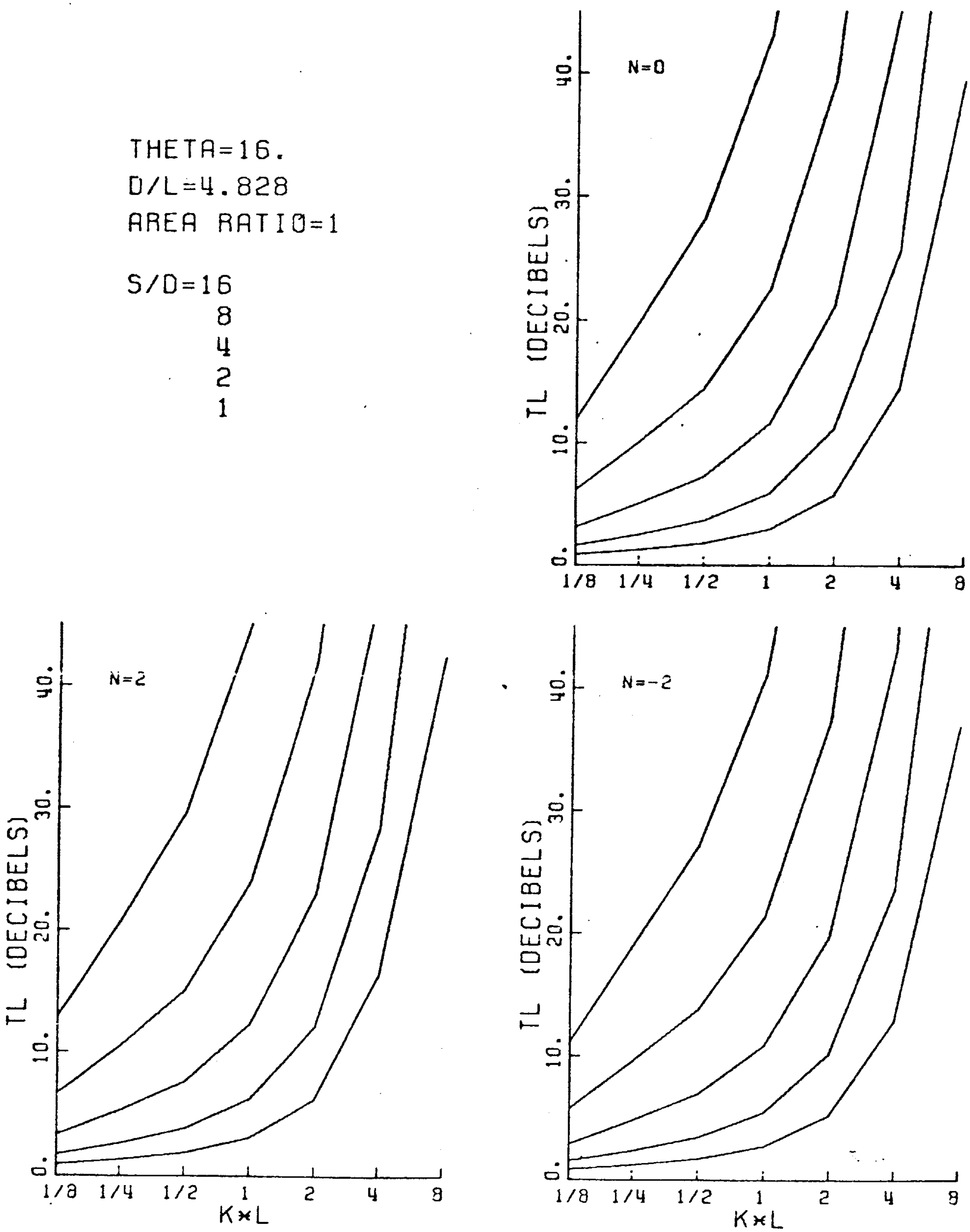
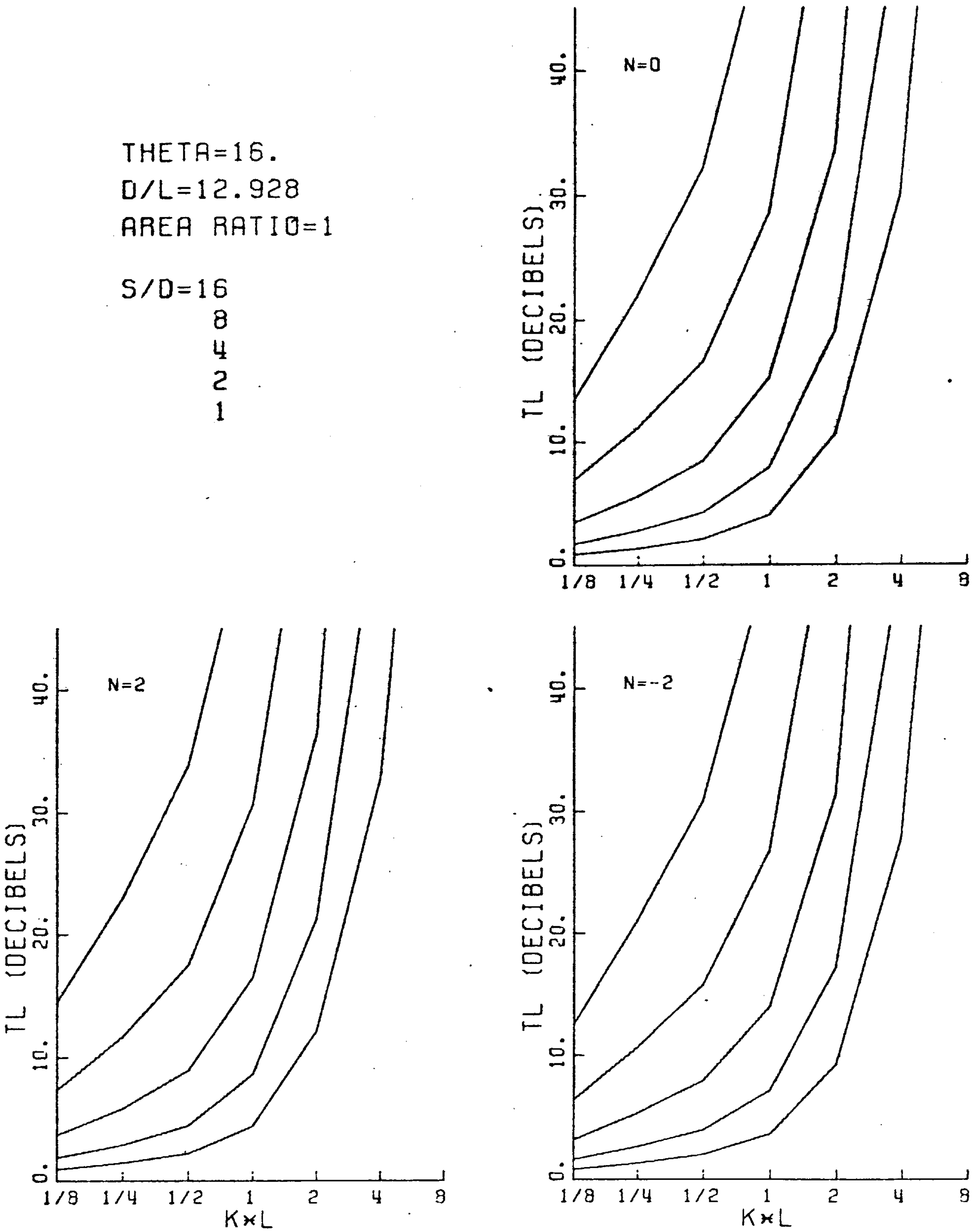

THETA=16.
D/L=4.828
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.92



THETA=16.
D/L=12.928
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.93

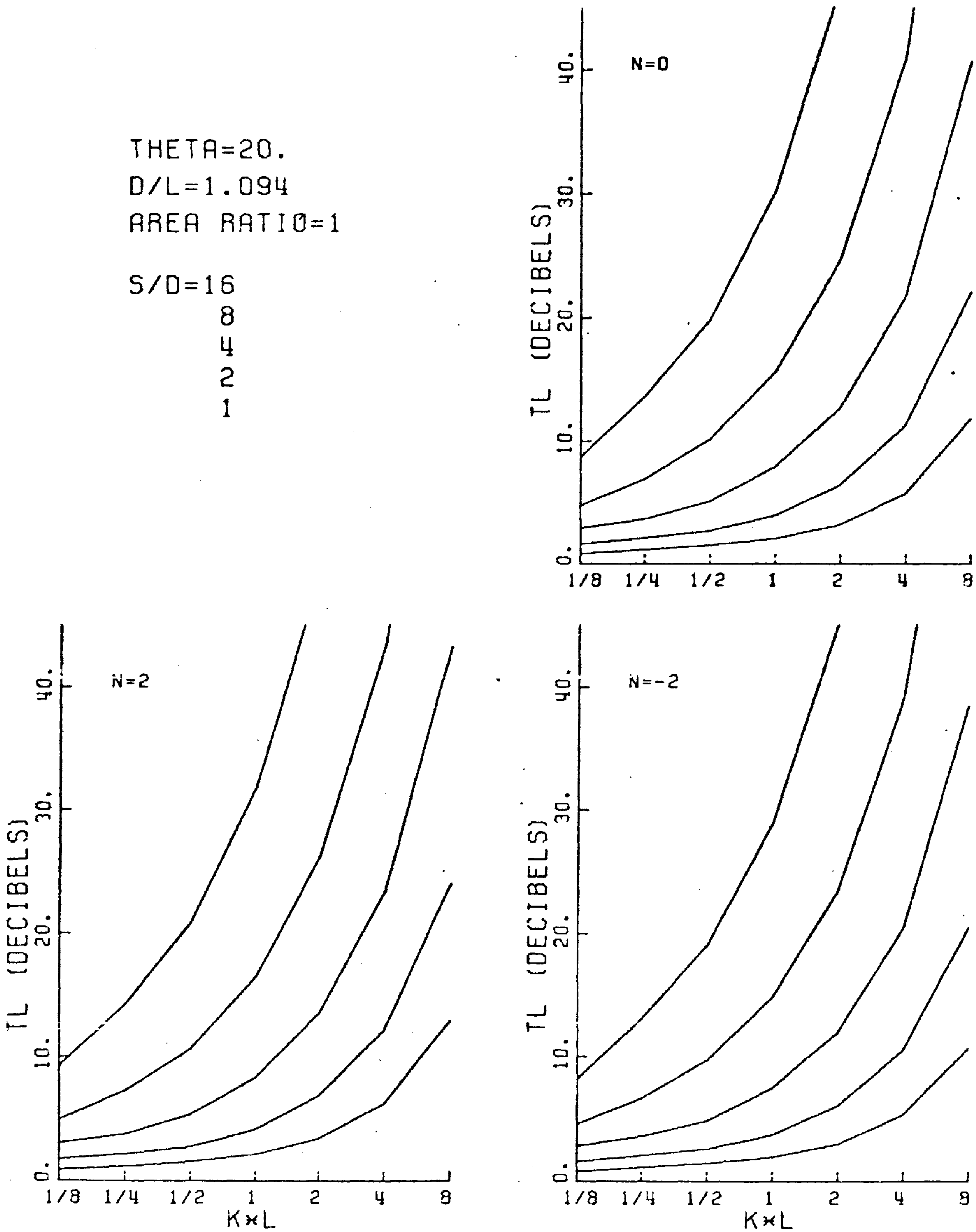
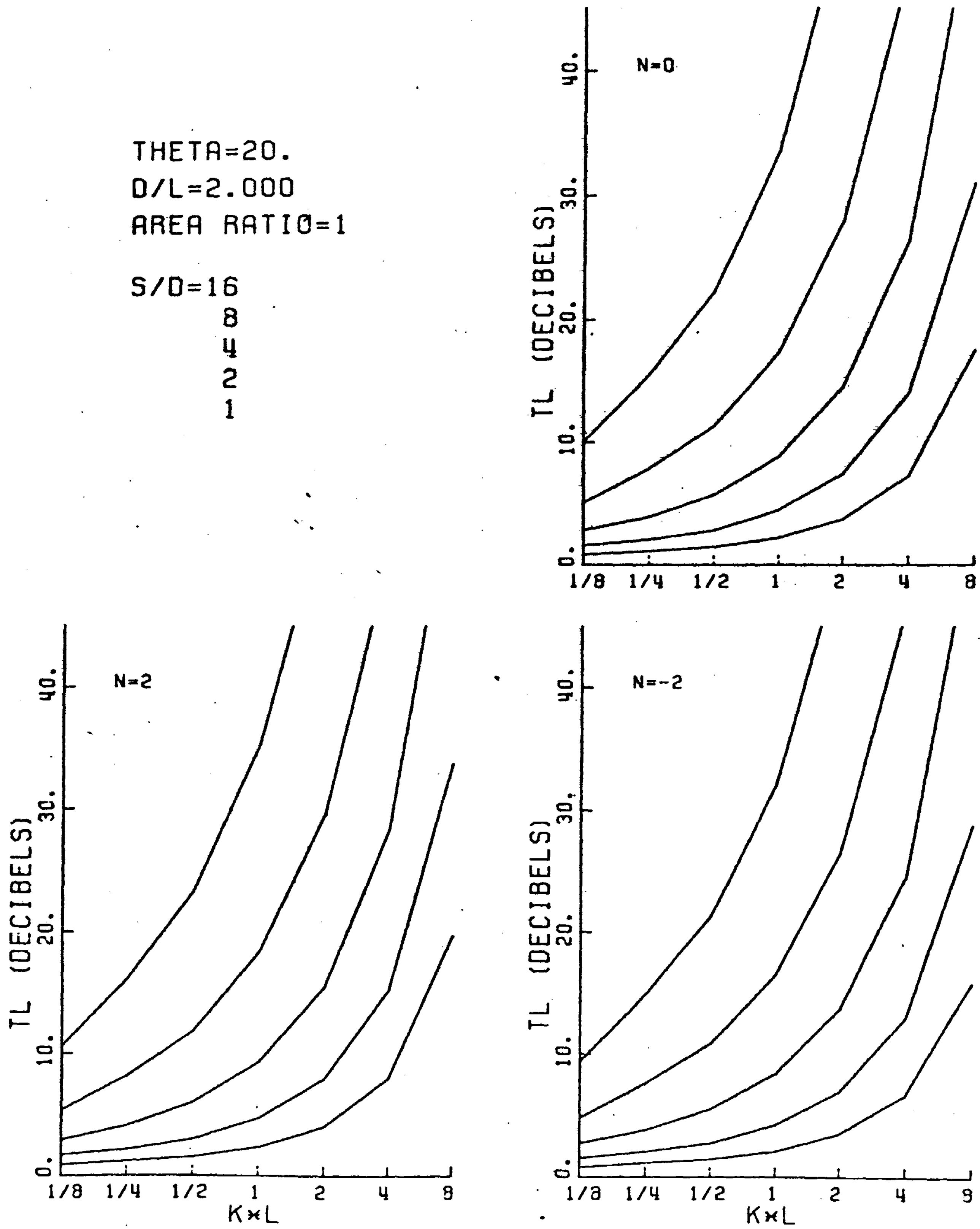

THETA=20.
D/L=1.094
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8 1/4 1/2 1 2 4 8
K*L

Figure 3.94



THETA=20.
D/L=2.000
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.95

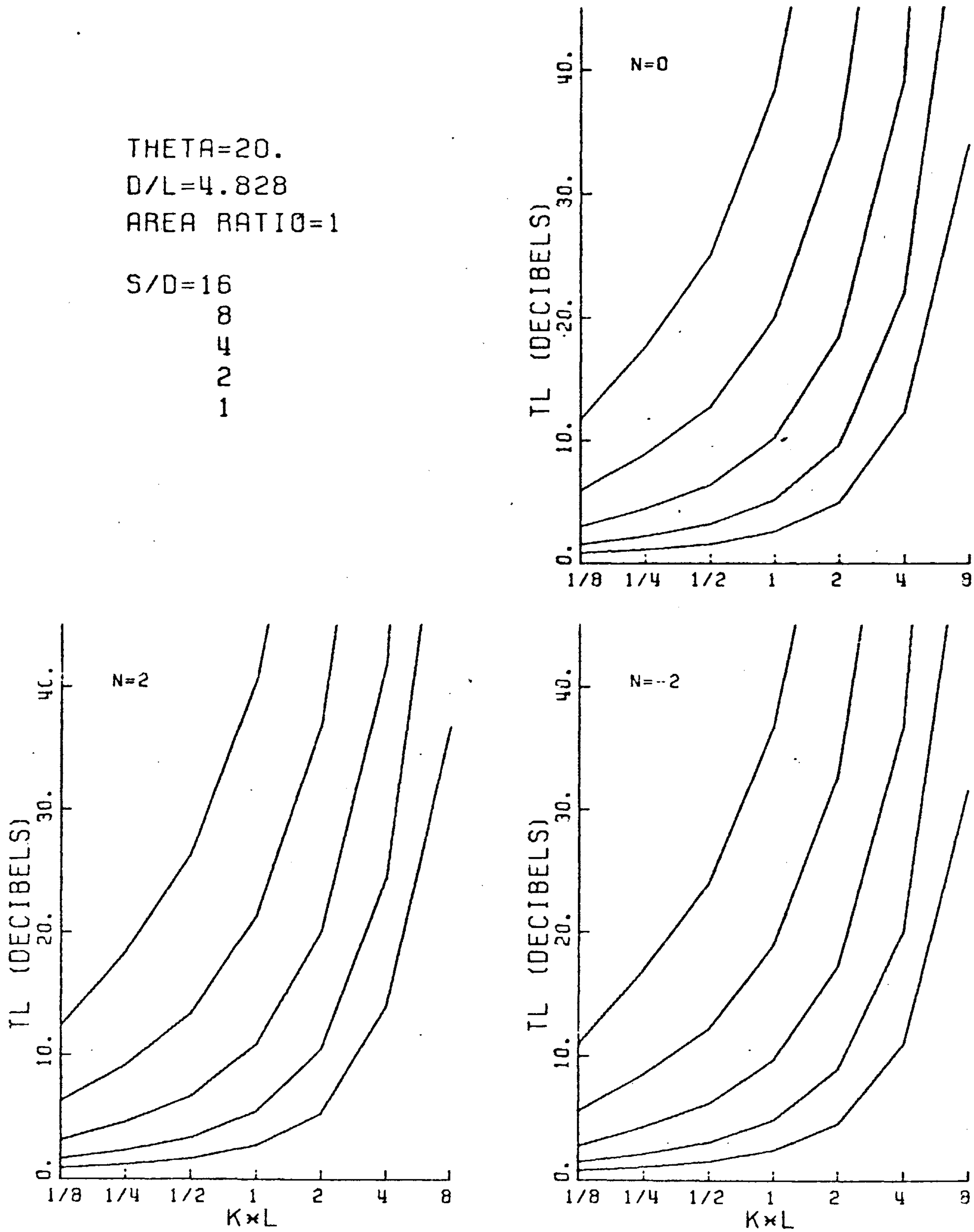

THETA=20.
D/L=4.828
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.96

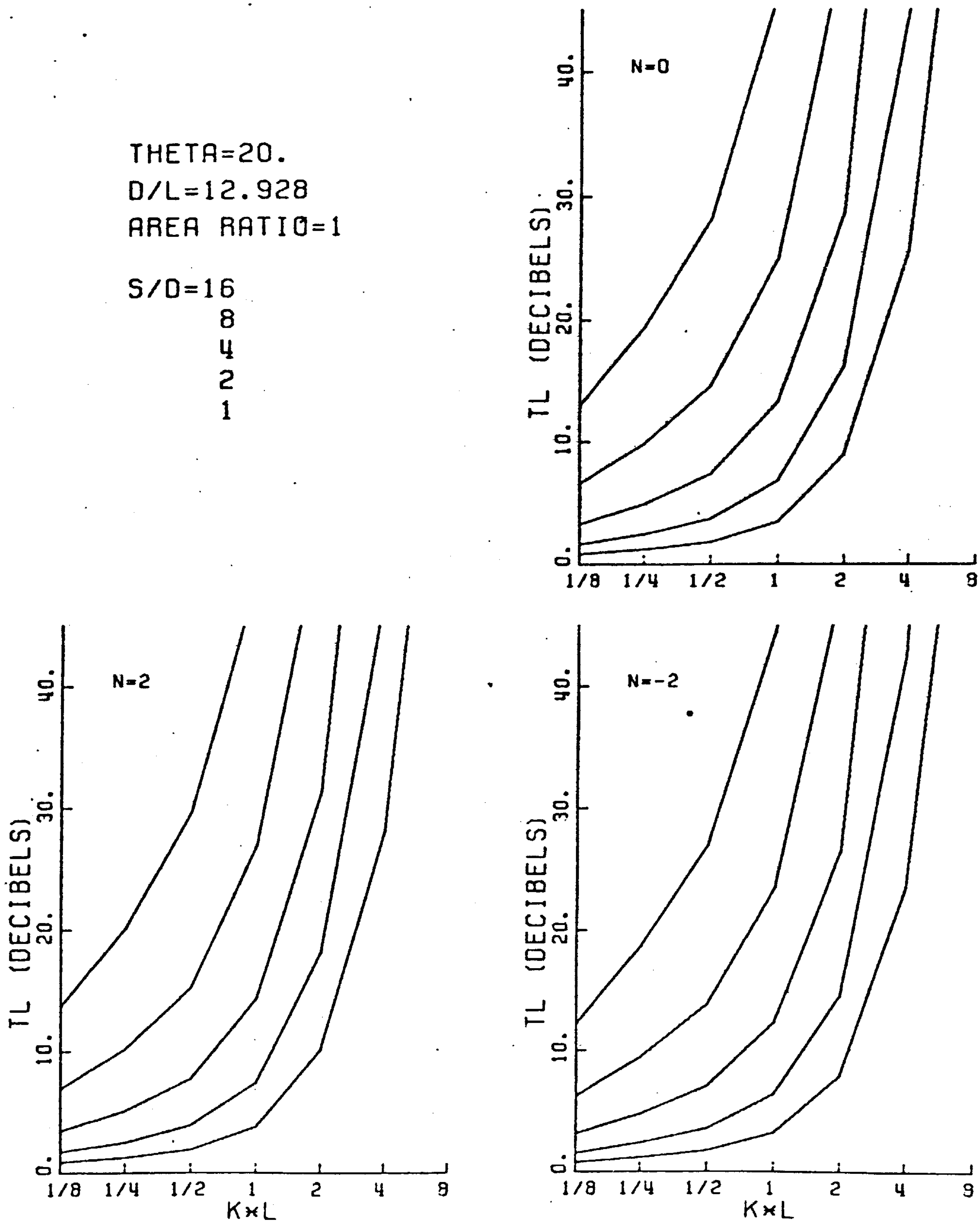

THETA=20.
D/L=12.928
AREA RATIO=1
S/D=16
8
4
2
1
N=0
N=2
N=-2
TL (DECIBELS)
0.
10.
20.
30.
40.
1/8
1/4
1/2
1
2
4
8
K*L

Figure 3.97

3.3 The Influence of the Duct Length on the Octave Band Transmission Loss

One of the most interesting results of the study of the octave band transmission loss is the influence of the length of the duct liner, which is illustrated by the results in Figures 3.98-3.121. For a pure tone the TL is proportional to the length, if we neglect the reflections from the ends of the liner. For an octave band, or any other finite frequency range, the influence of the length is much more complicated. The reason is that as the wave advances through the duct, the spectrum shape will change with position in such a way that the frequency range with low attenuation will be more and more emphasized, the farther the wave travels. This change in the spectrum has the effect of making the contribution to the octave band TL by a liner element smaller and smaller, the farther into the duct the liner element is located. As a result, the octave band TL becomes practically independent of the length S, for sufficiently large values of S, in frequency regions where the attenuation constant $Im(k_z)$ depends strongly on frequency. For a resonator type liner this behavior is found at frequencies both below and above the fundamental resonance of the liner.

Figures 3.98-3.101. Octave band TL vs the duct length parameter S/D for rectangular ducts lined with a resistive screen type resonator liner. Each figure corresponds to a different value of D/L. In each figure, four frames are shown, corresponding to θ = 0.5, 1, 2, and 4. In each frame, four curves are plotted, corresponding to four different values of kL as indicated by symbols.

AREA RATIO=1.0 D/L=2/7

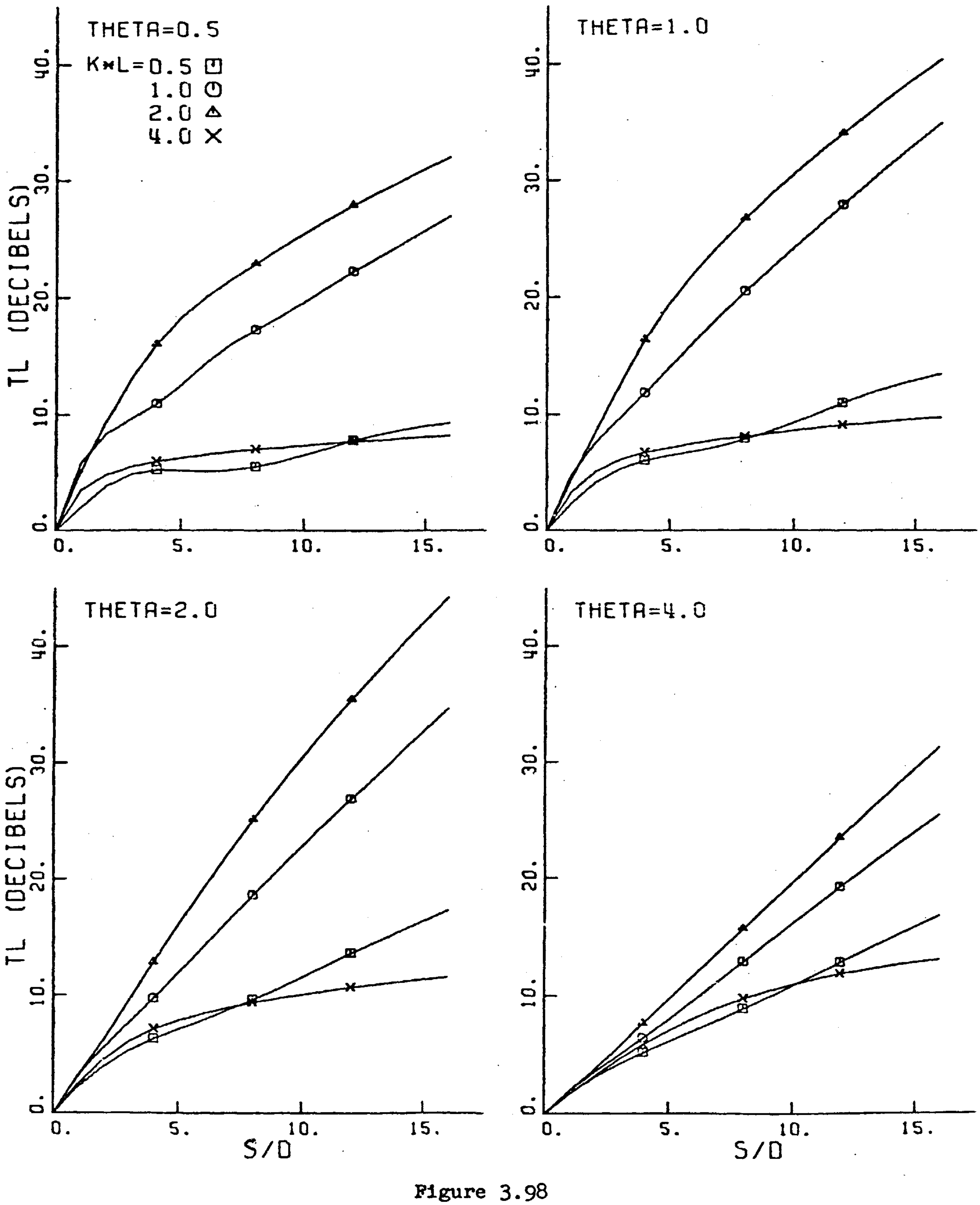


Figure 3.98

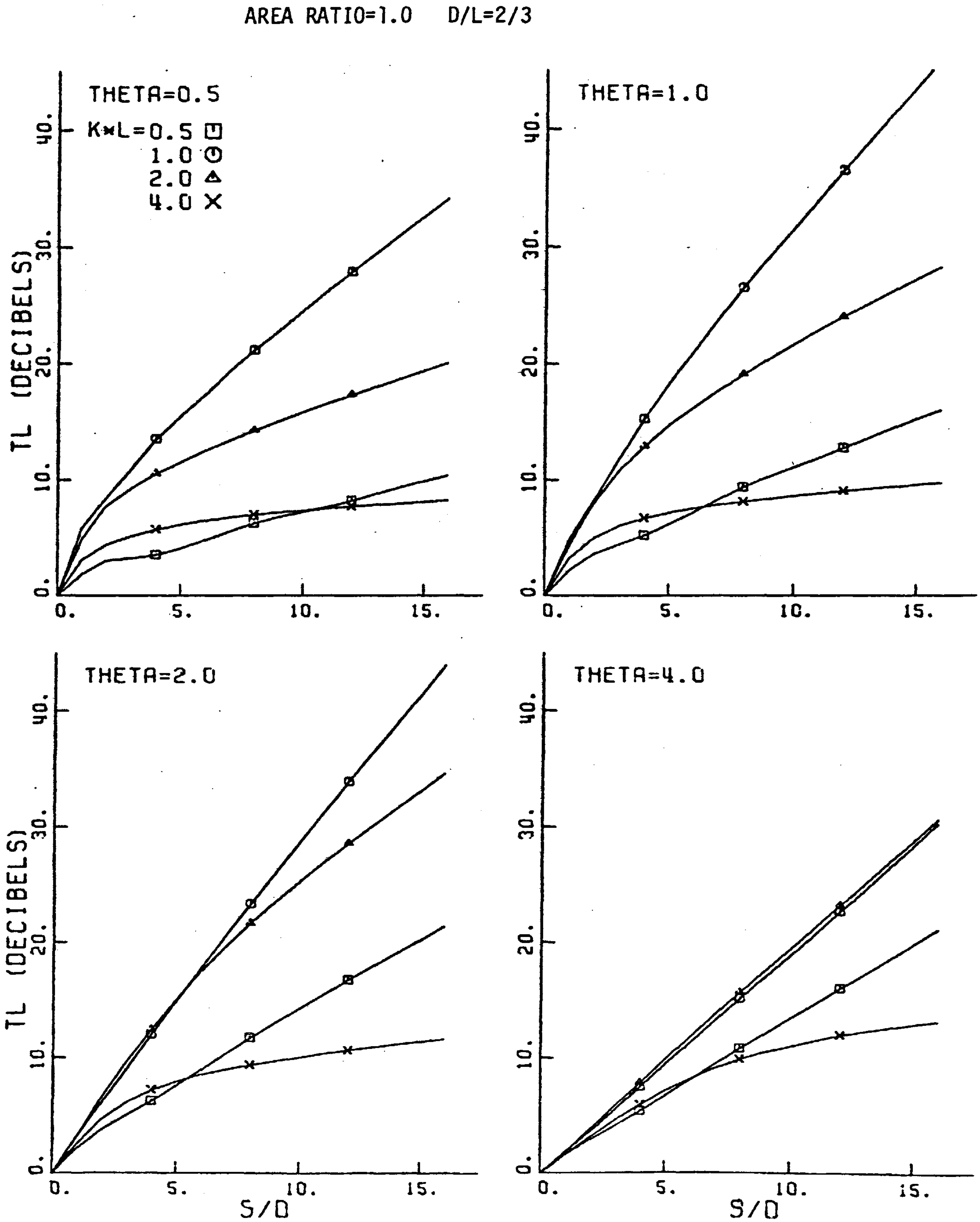

AREA RATIO=1.0 D/L=2/3
THETA=0.5
K*L=0.5
1.0
2.0
4.0
THETA=1.0
THETA=2.0
THETA=4.0
TL (DECIBELS)
S/D
0.
5.
10.
15.
20.
30.
40.

Figure 3.99

AREA RATIO=1.0   D/L=2.0

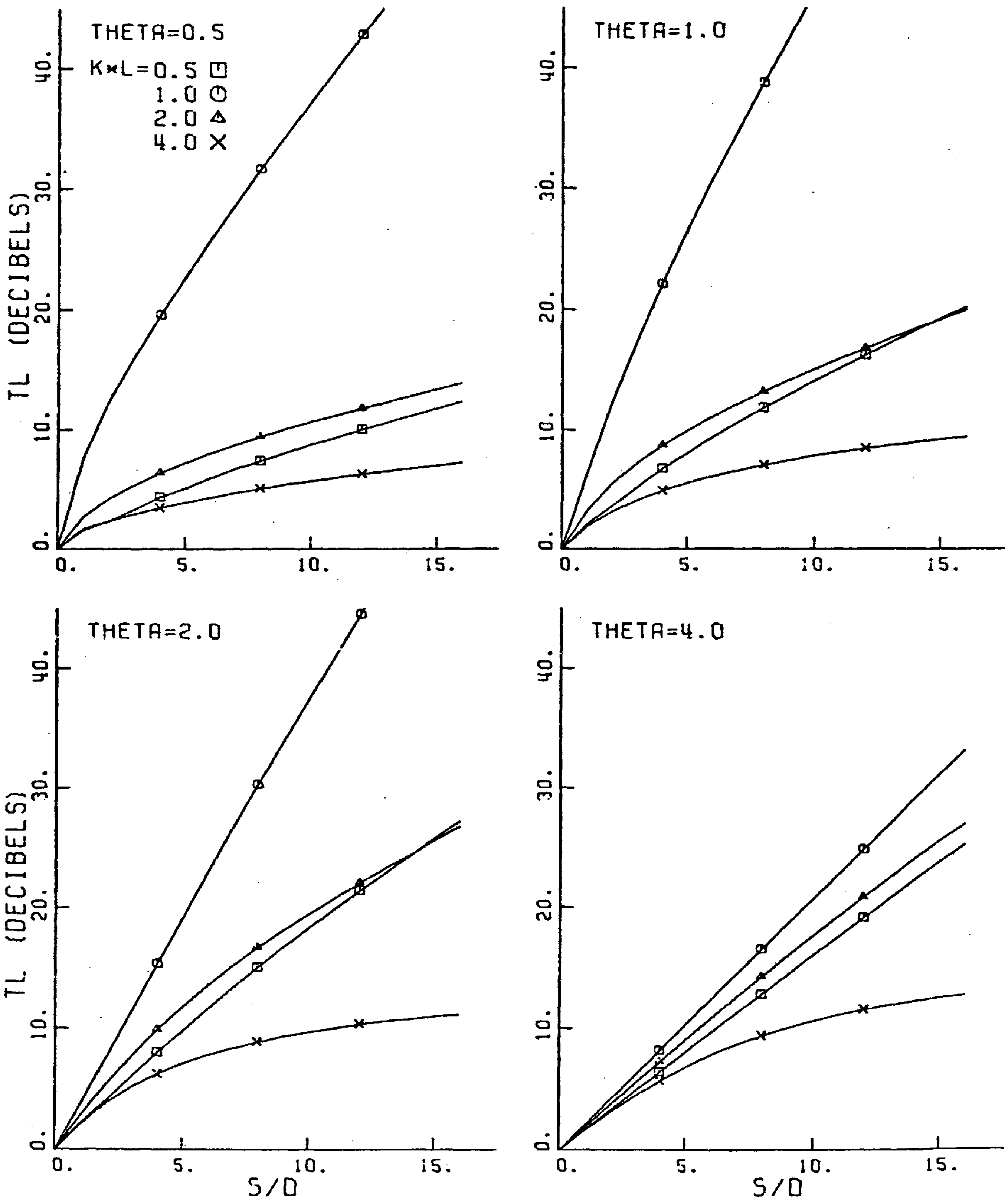


Figure 3.100

AREA RATIO=1.0   D/L=6.0

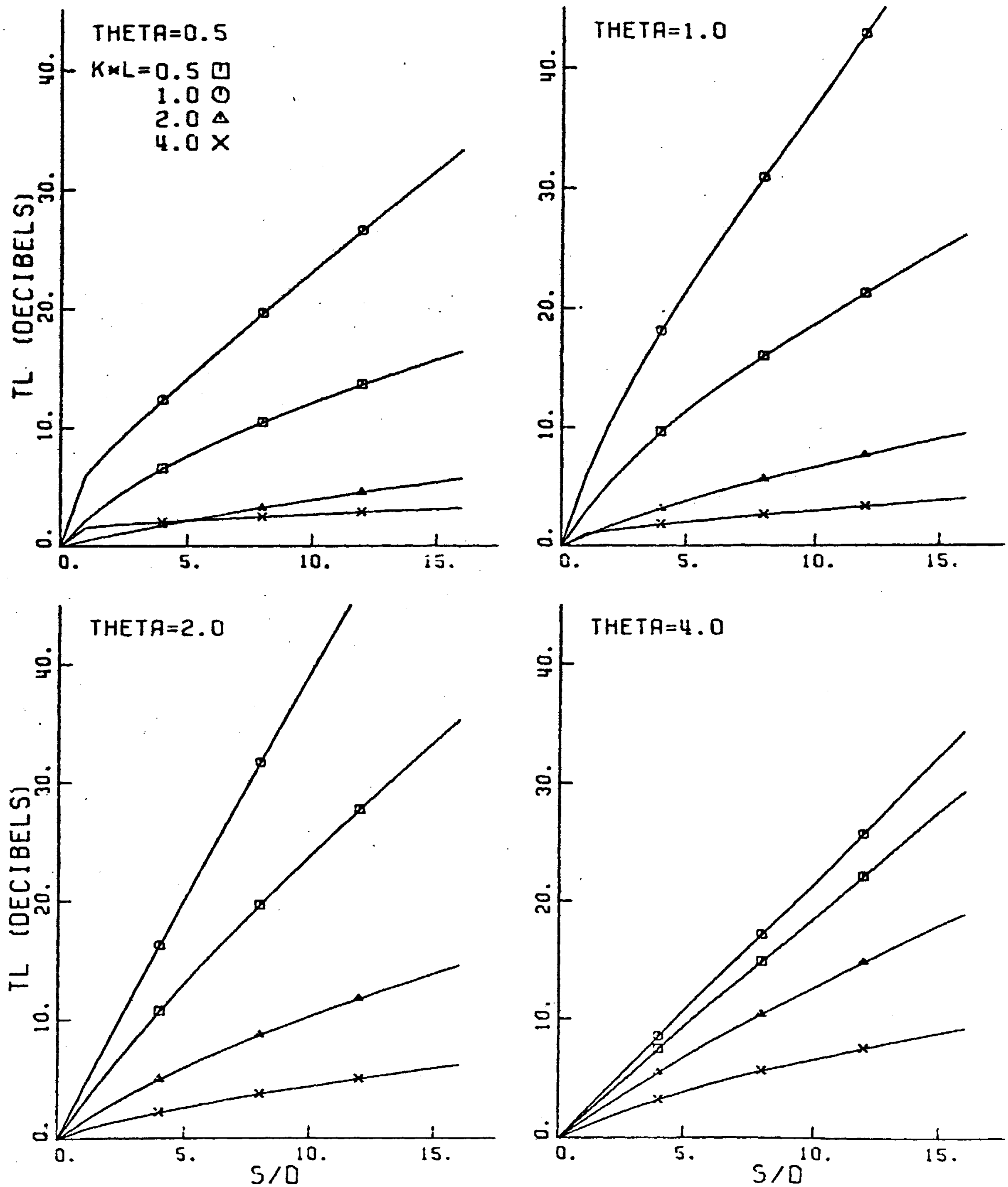


Figure 3.101

Figures 3.102-3.109. Octave band TL vs S/D for a rectangular duct lined with a porous liner. The format is the same as in Figures 3.98-3.101 except that four more values of Θ are considered here. (For definition of Θ, see Eq. 2.11.)

AREA RATIO=1.0   D/L=2/7

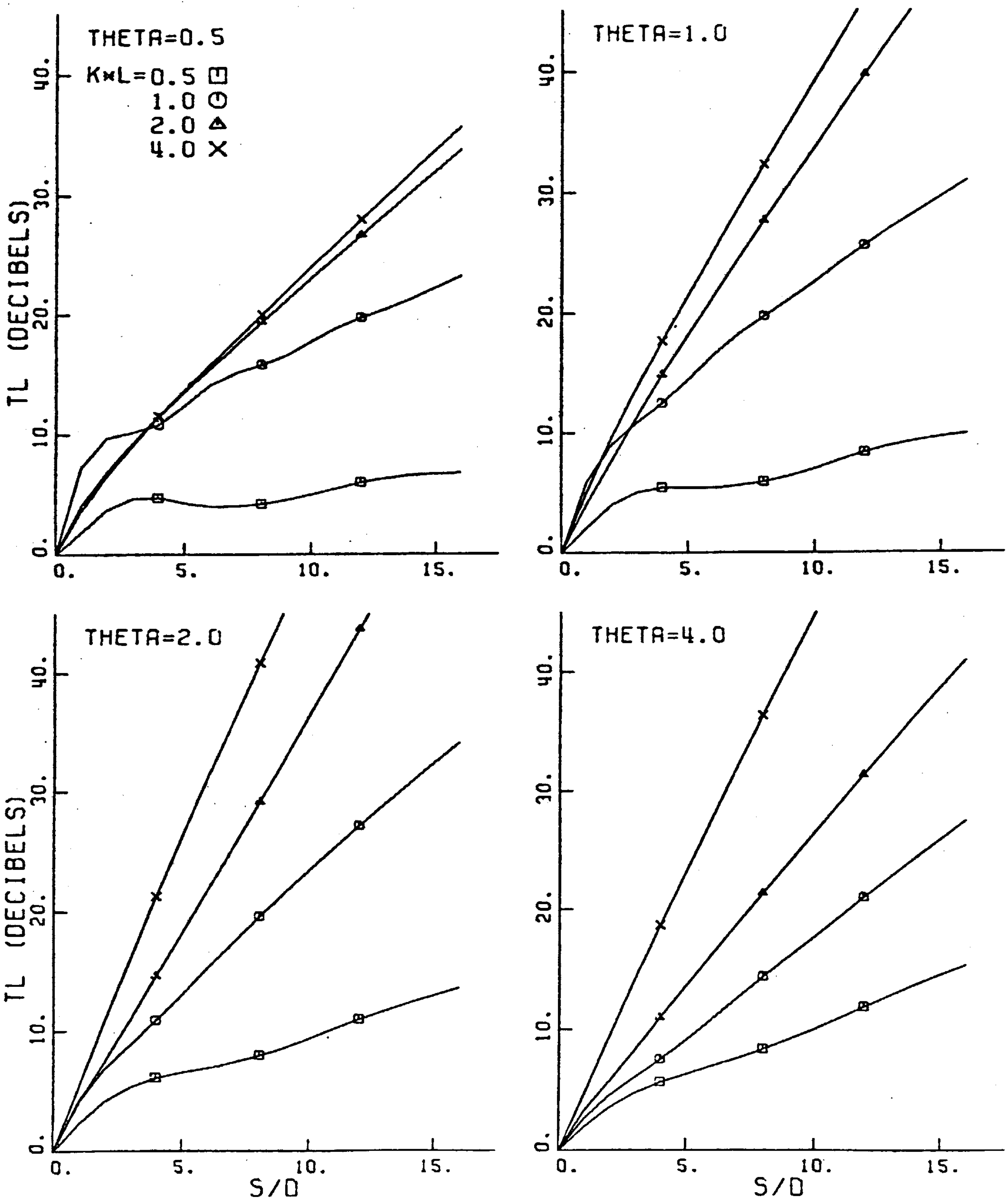


Figure 3.102

AREA RATIO=1.0   D/L=2/7

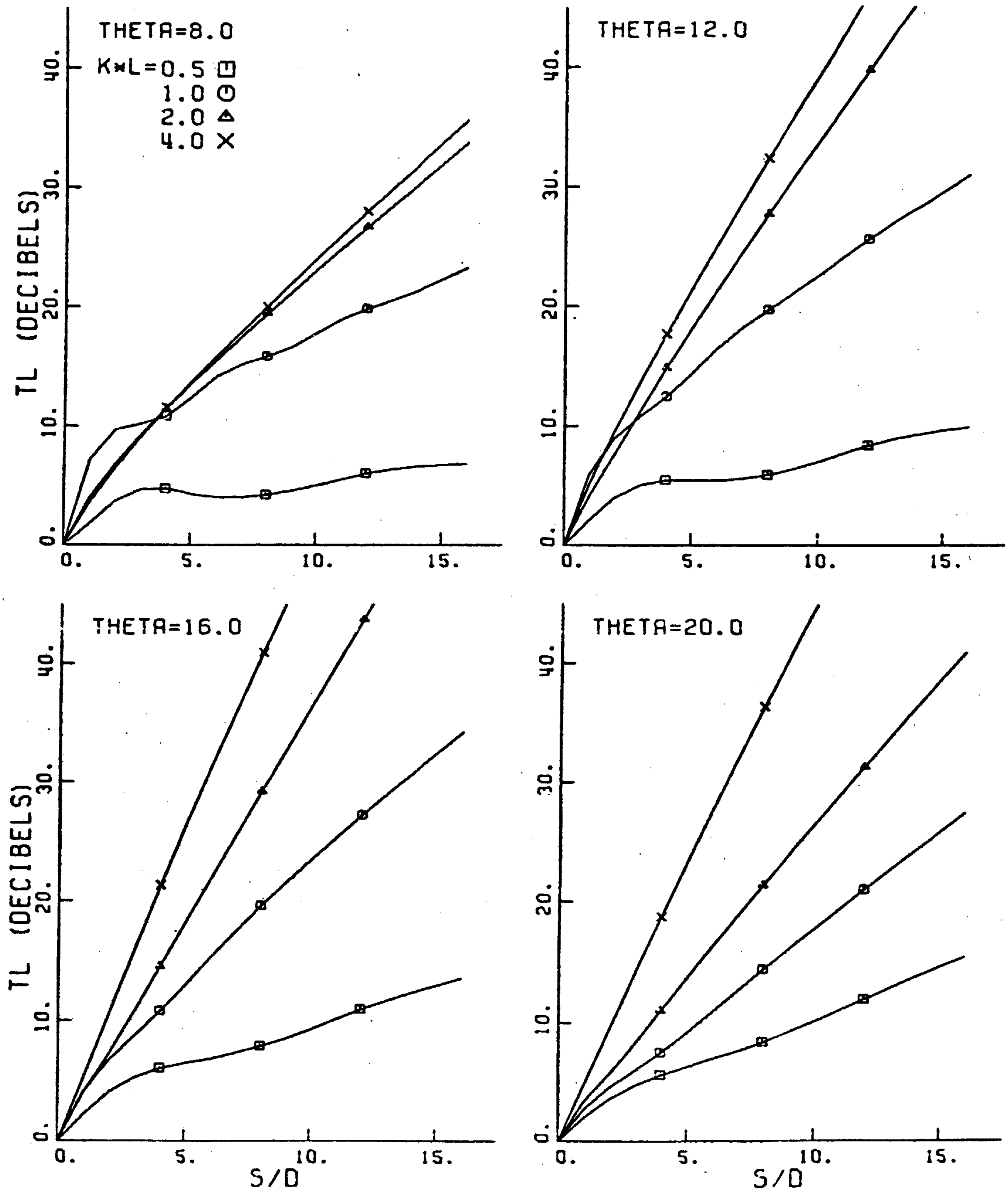


Figure 3.103

AREA RATIO=1.0   D/L=2/3

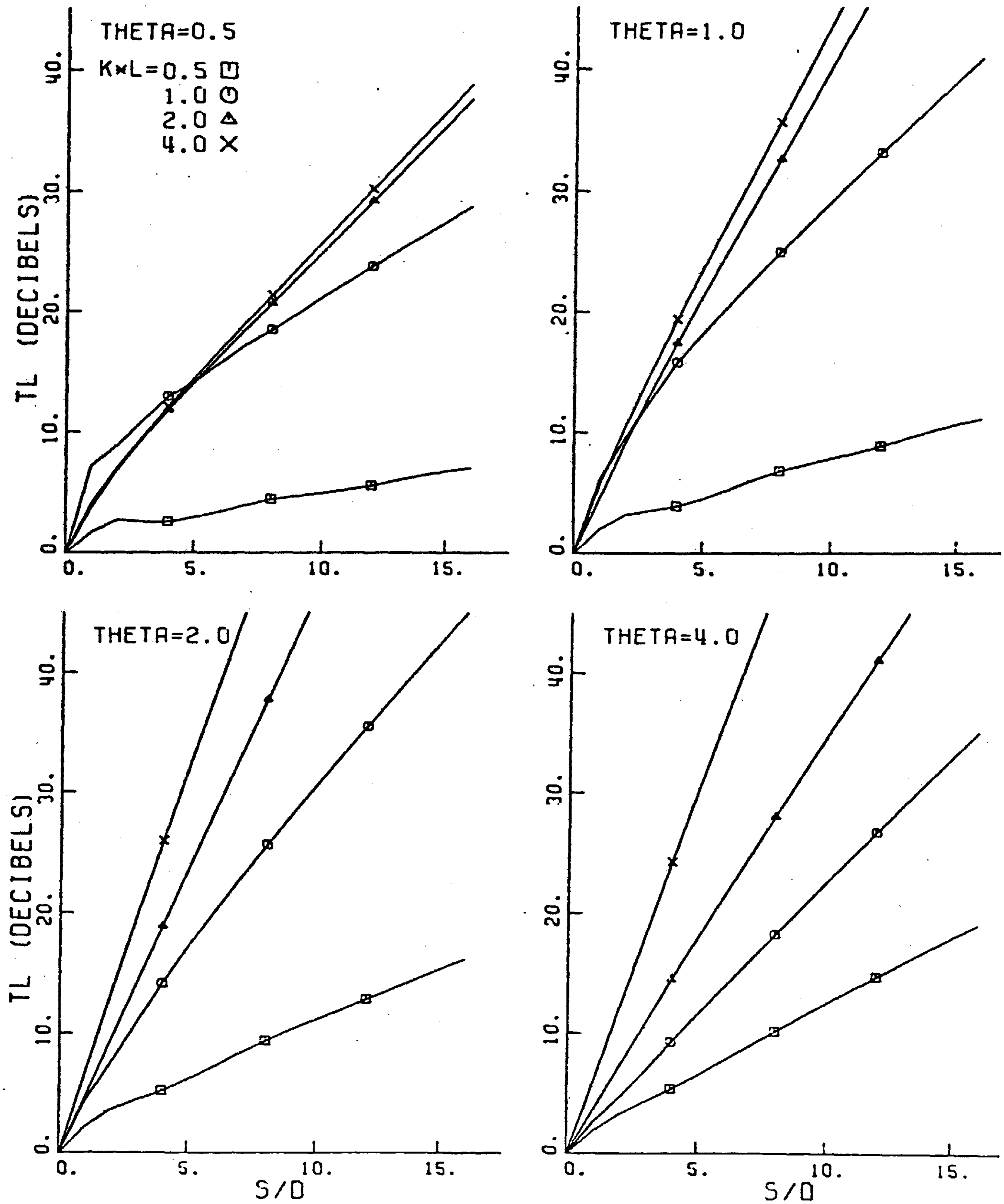


Figure 3.104

AREA RATIO=1.0   D/L=2/3

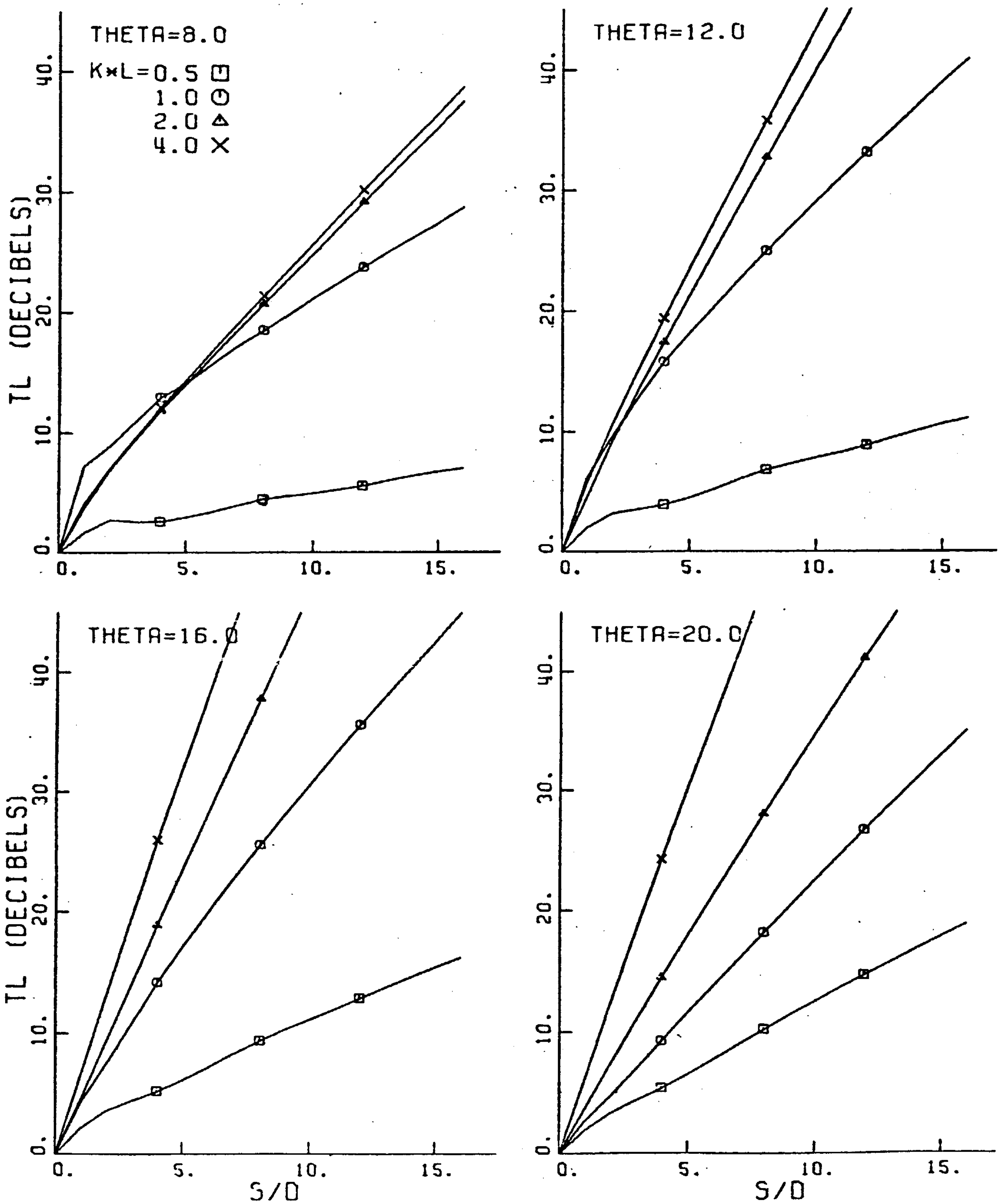


Figure 3.105

AREA RATIO=1.0 D/L=2.0

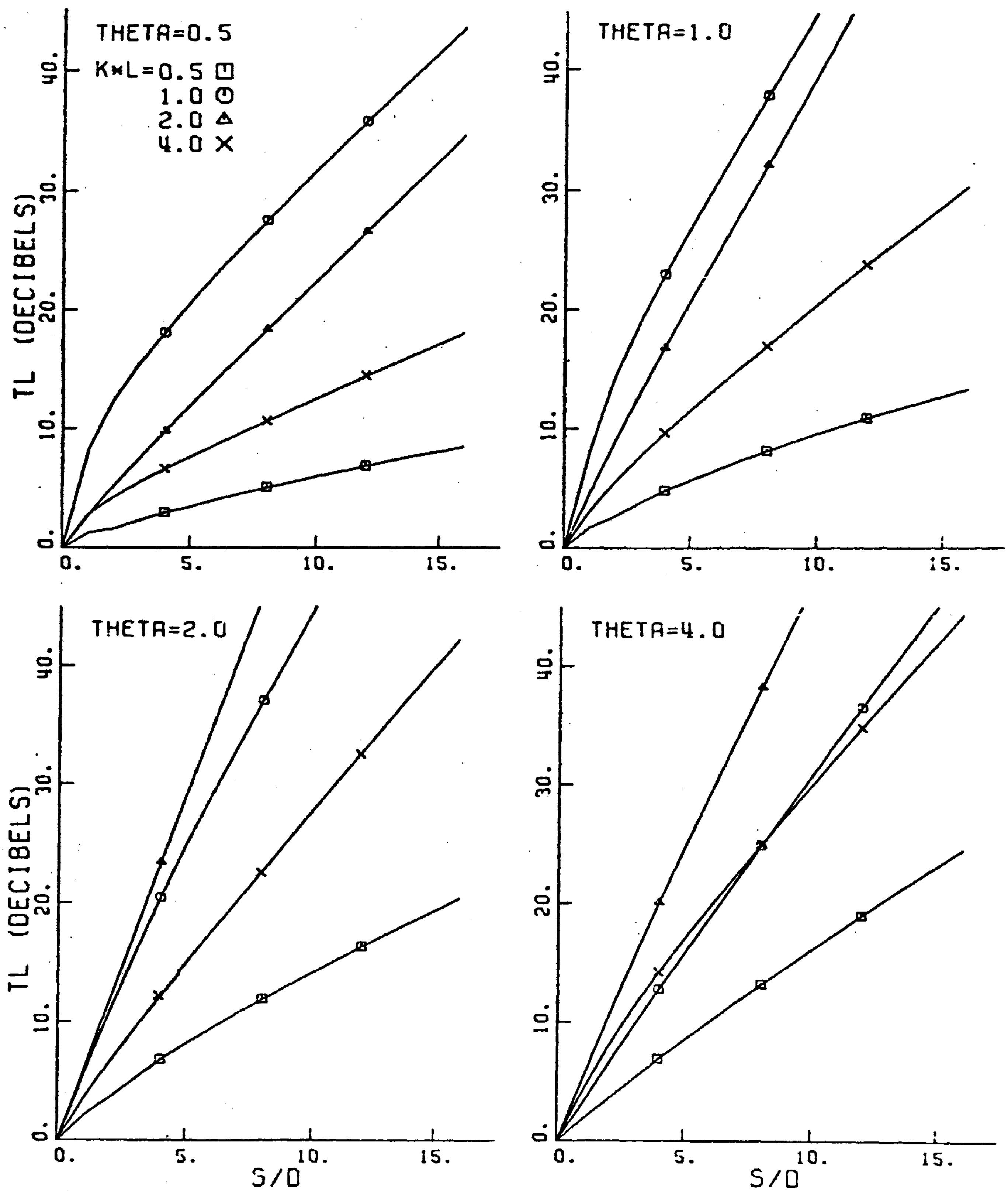


Figure 3.106

AREA RATIO=1.0 D/L=2.0

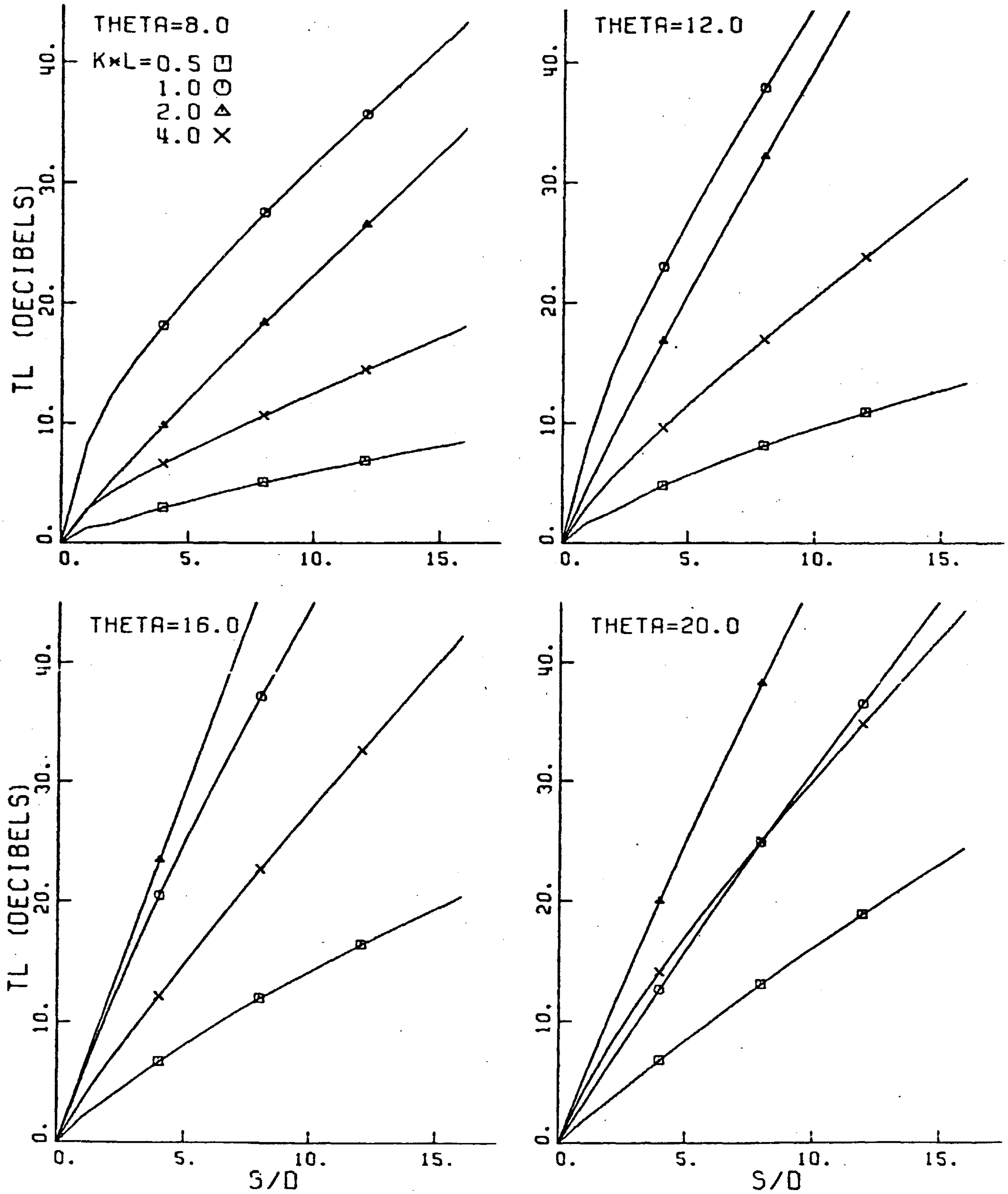


Figure 3.107

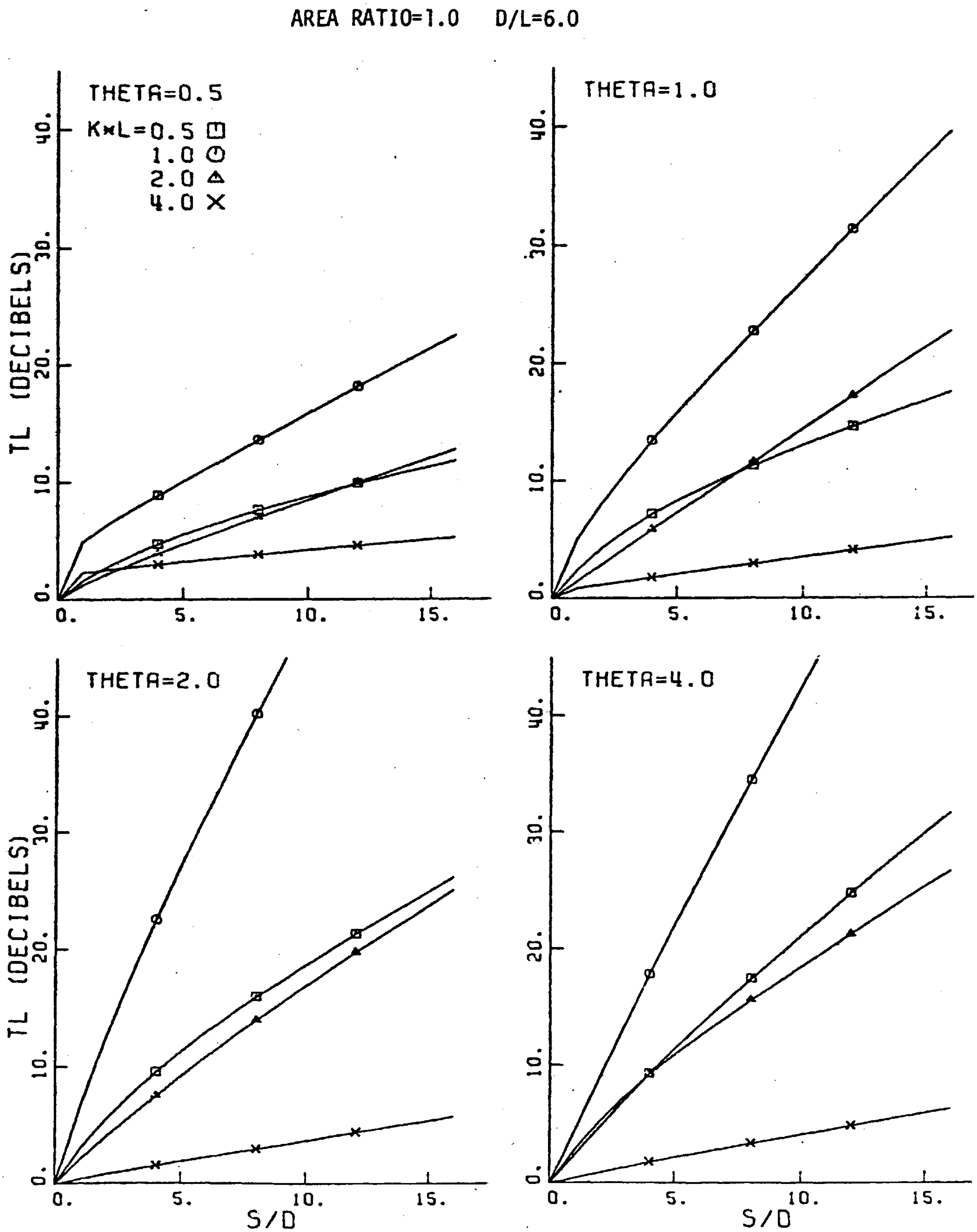

AREA RATIO=1.0 D/L=6.0
THETA=0.5
K*L=0.5
1.0
2.0
4.0
THETA=1.0
THETA=2.0
THETA=4.0
TL (DECIBELS)
S/D
0.
5.
10.
15.
20.
30.
40.

Figure 3.108

AREA RATIO=1.0 D/L=6.0

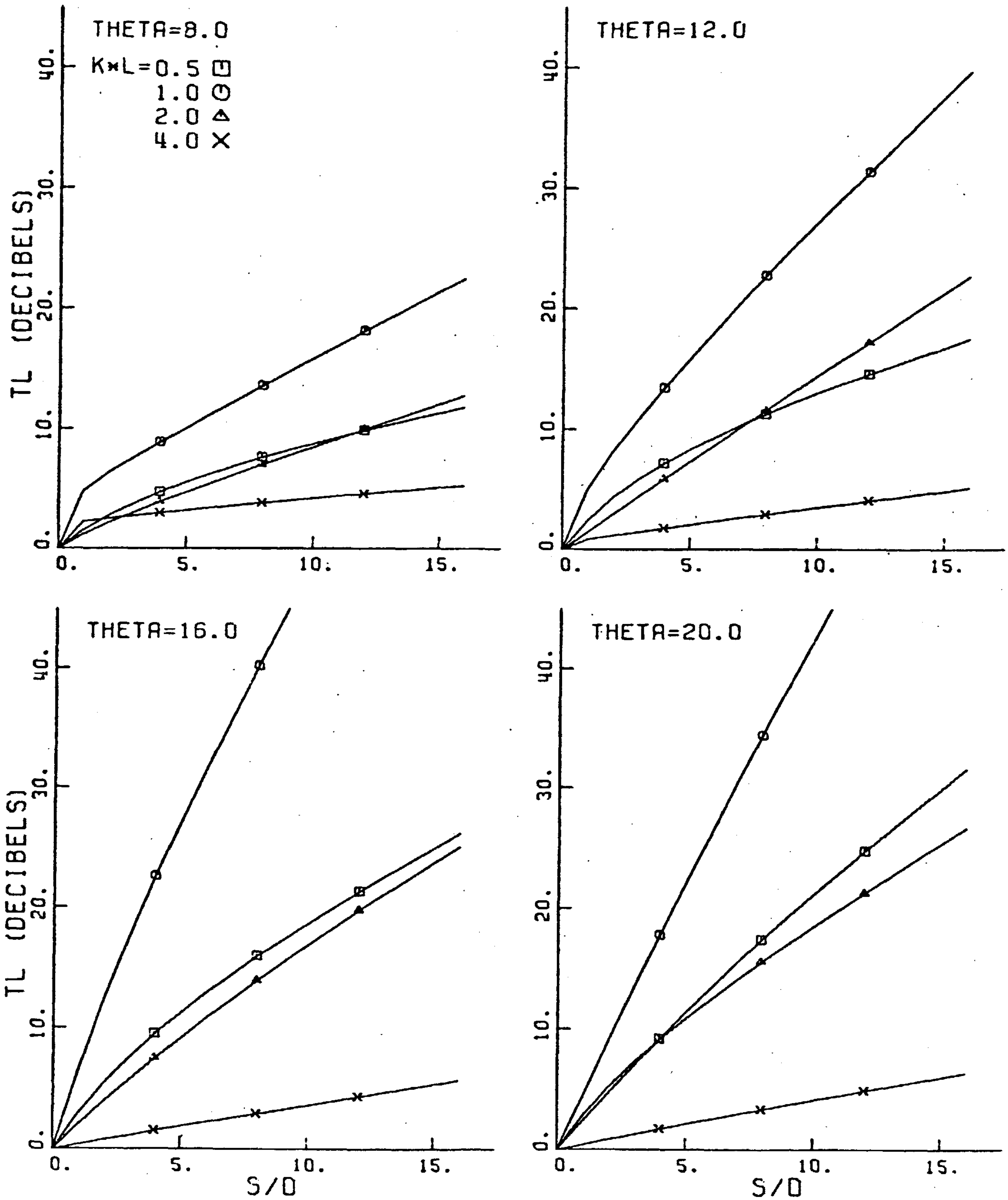


Figure 3.109

Figures 3.110-3.113. Octave band TL vs S/D for a circular duct lined with a resistive screen type resonator liner. The format is the same as Figures 3.98-3.101.

AREA RATIO=1.0   D/L=1.054

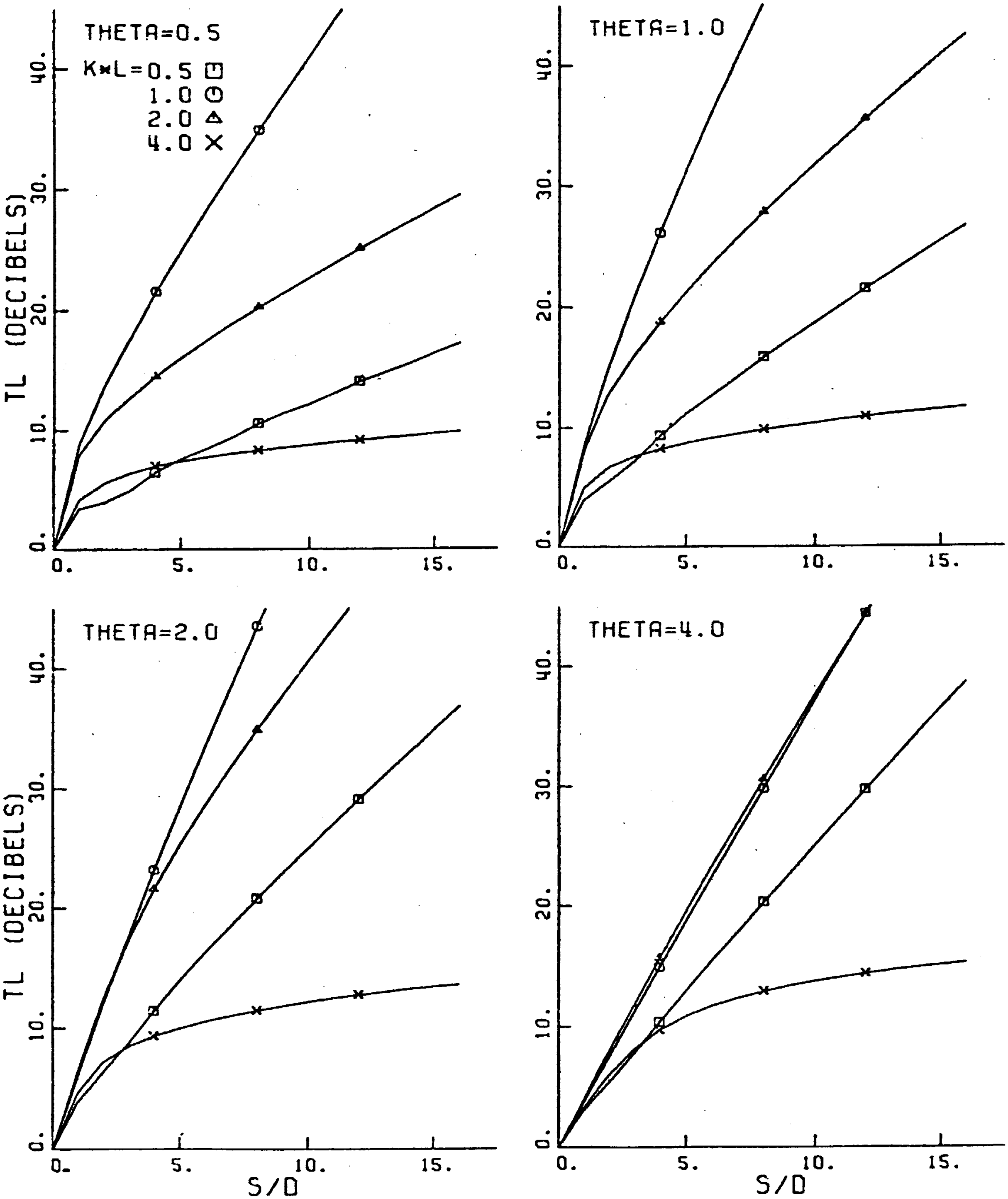


Figure 3.110

AREA RATIO=1.0 D/L=2.000

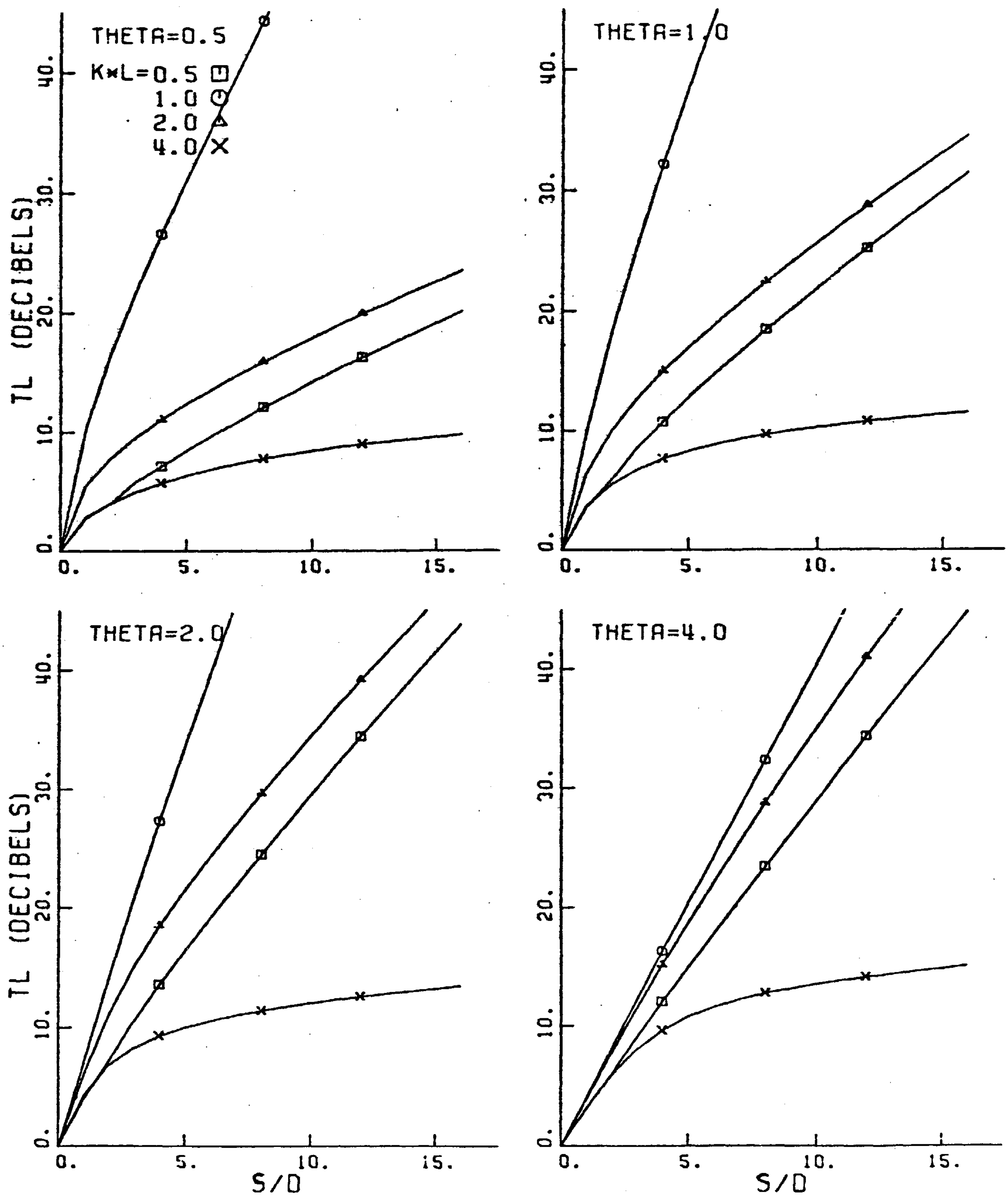


Figure 3.111

AREA RATIO=1.0   D/L=4.828

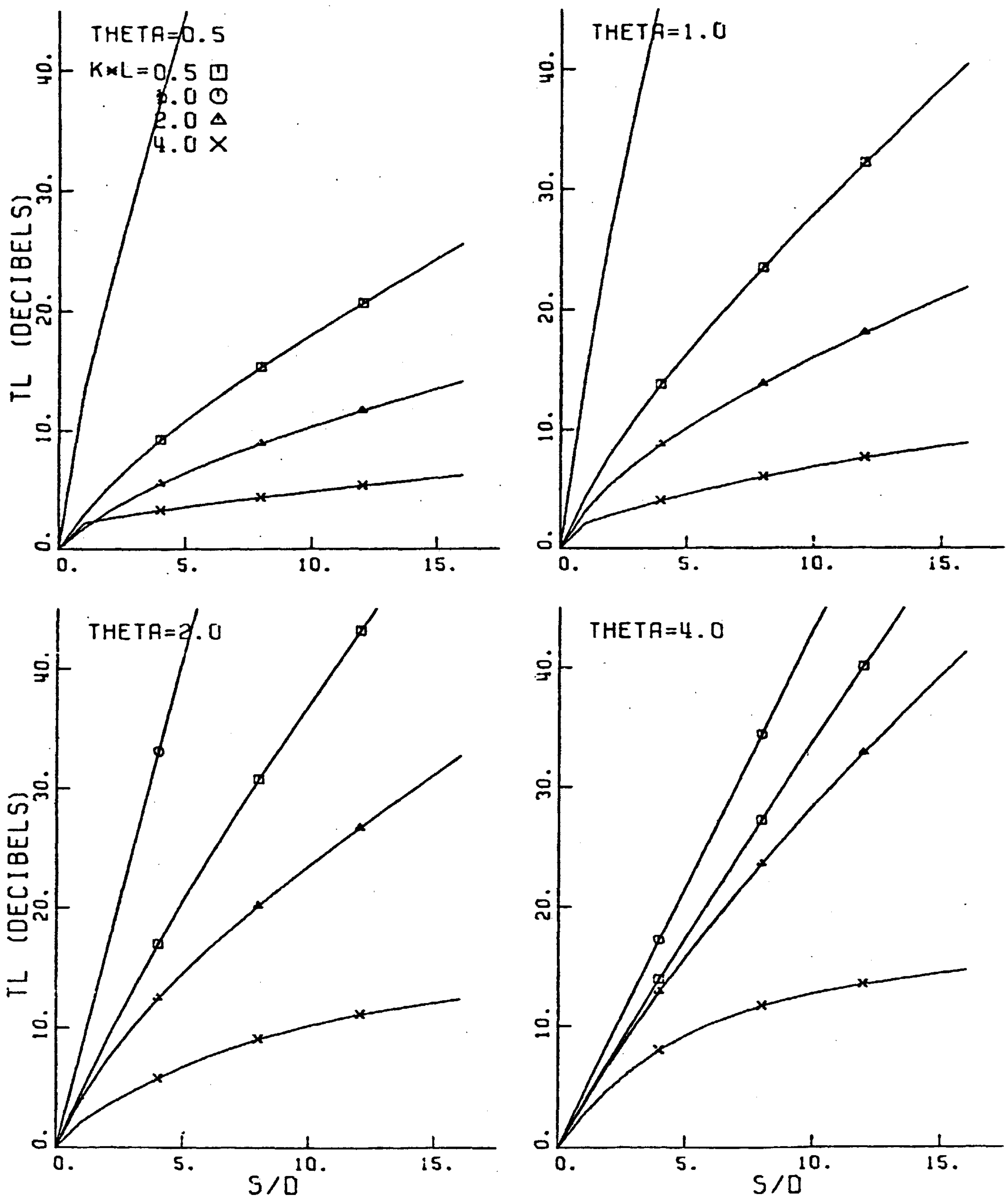


Figure 3.112

AREA RATIO=1.0   D/L=12.928

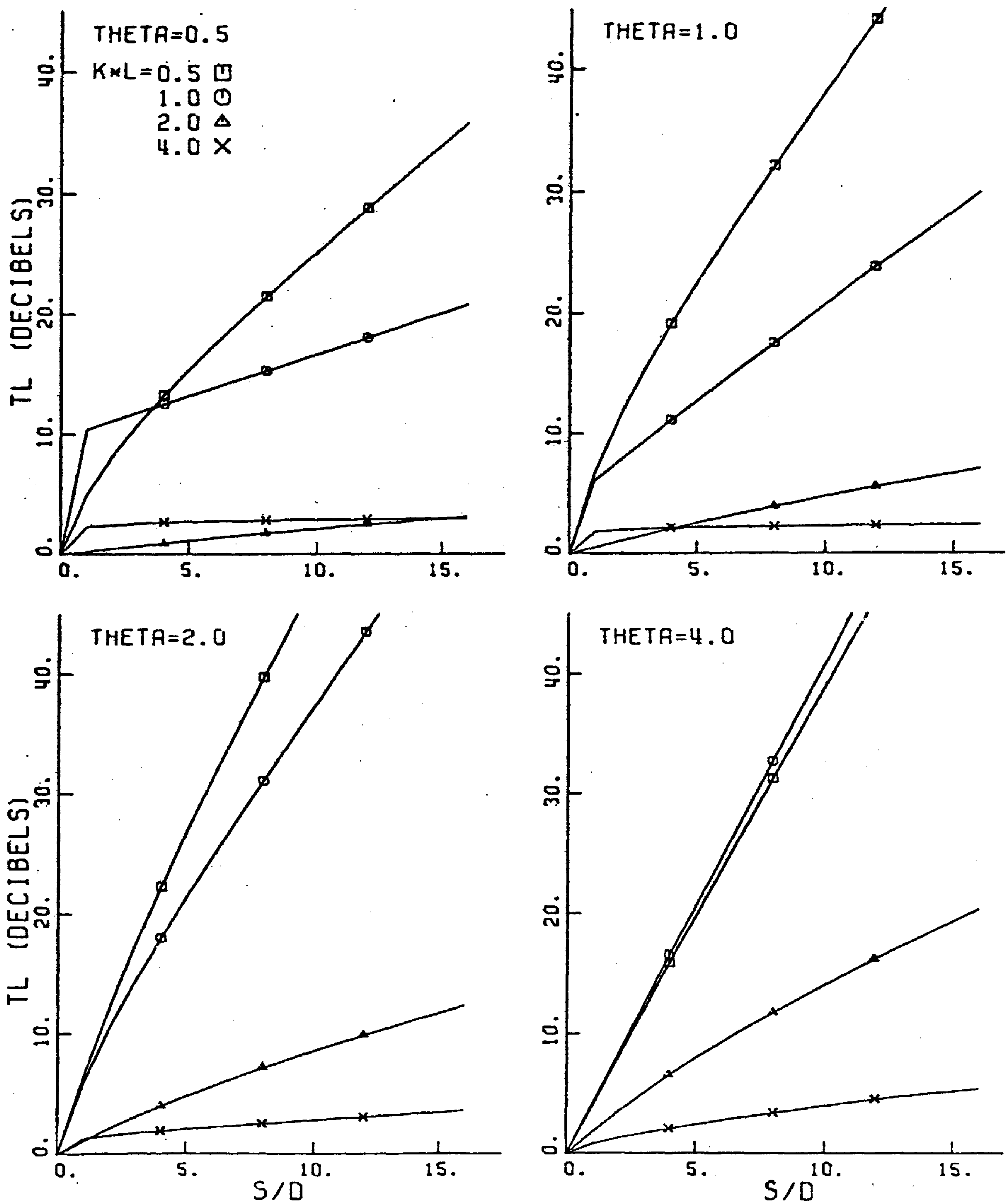


Figure 3.113

Figures 3.114-3.121. Octave band TL vs S/D for a circular duct lined with a porous liner. The format is the same as in Figures 3.110-3.113.

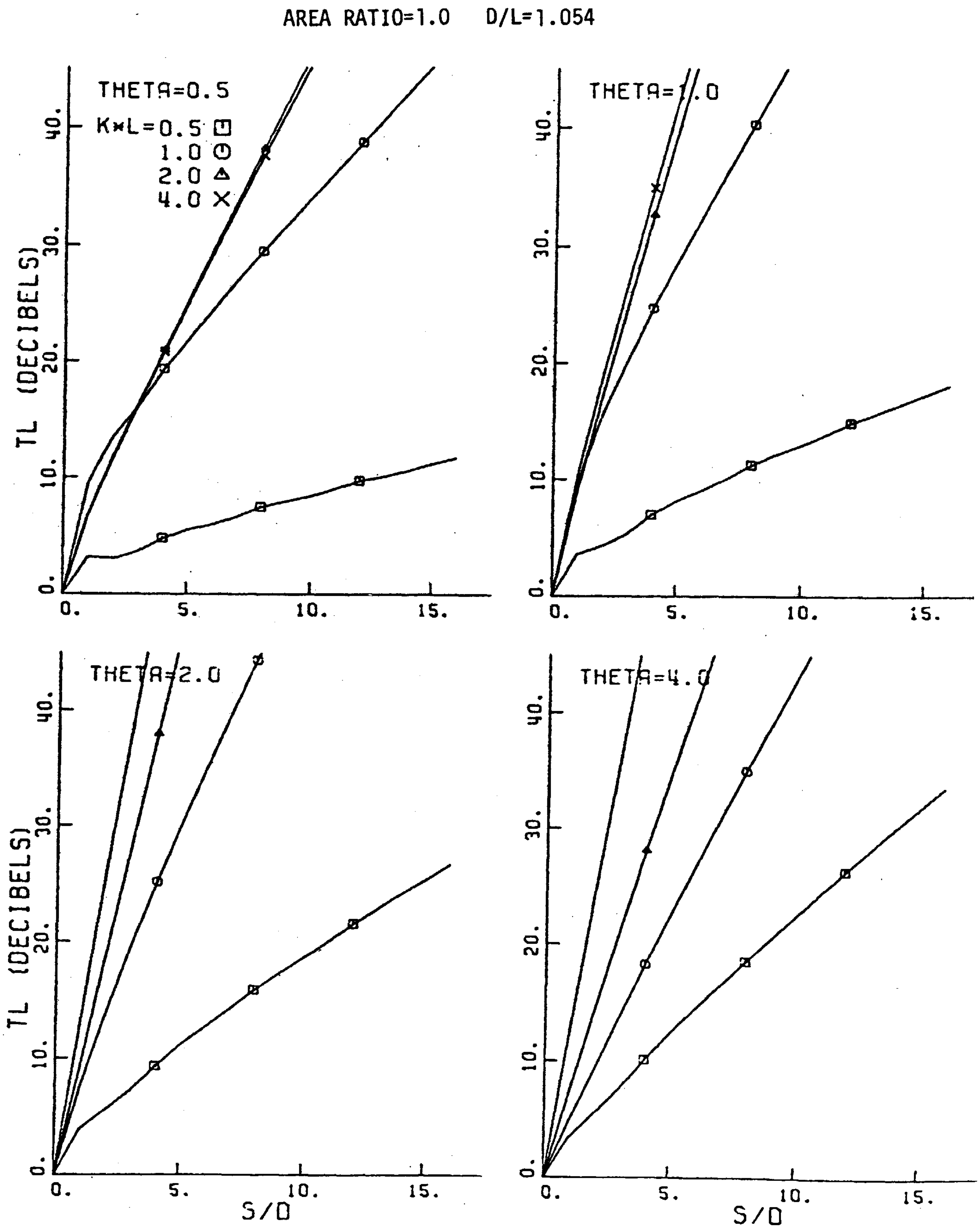

AREA RATIO=1.0 D/L=1.054
THETA=0.5
K*L=0.5
1.0
2.0
4.0
THETA=1.0
THETA=2.0
THETA=4.0
TL (DECIBELS)
S/D
0.
5.
10.
15.
20.
30.
40.

Figure 3.114

AREA RATIO=1.0   D/L=1.054

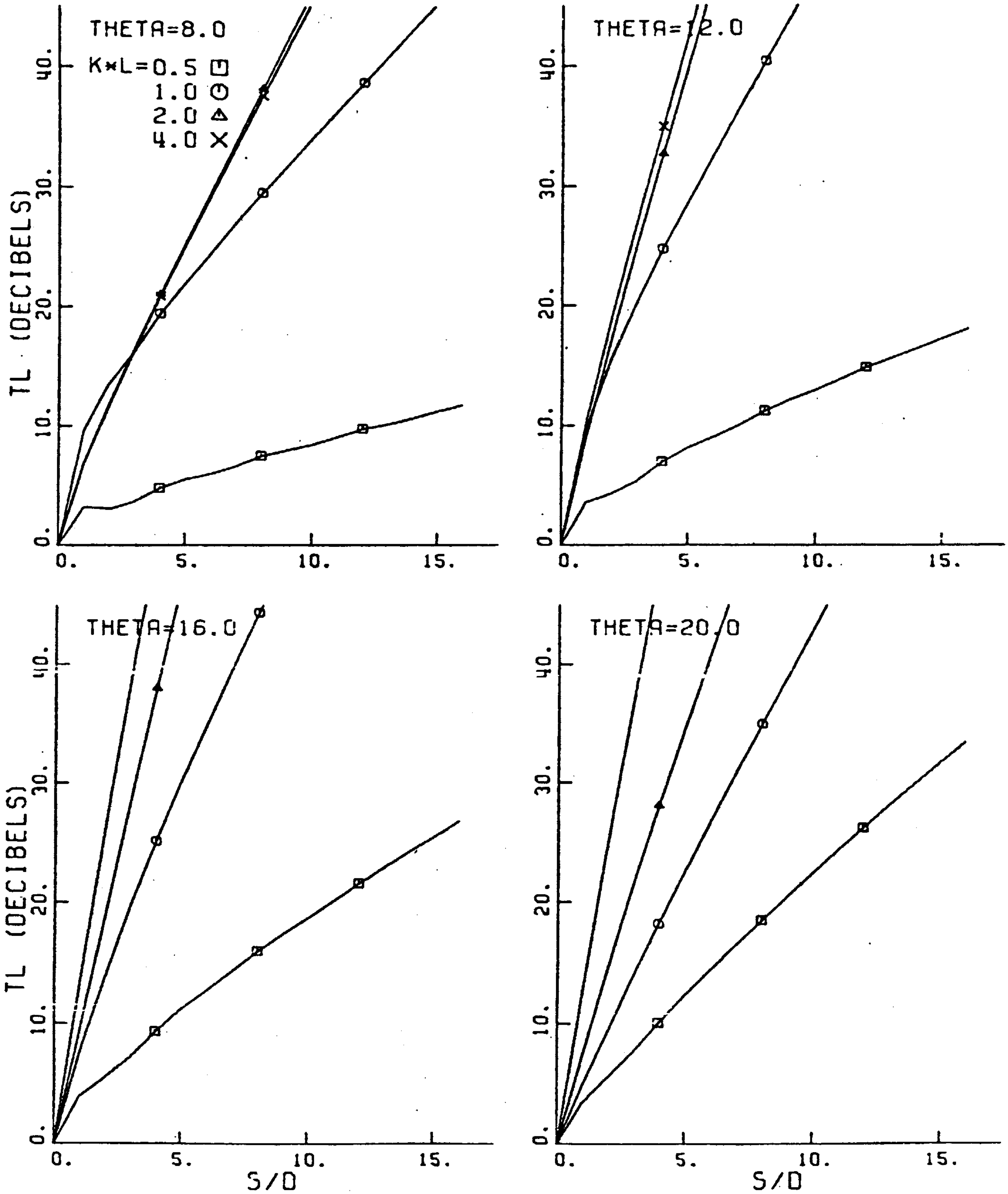


Figure 3.115

AREA RATIO=1.0 D/L=2.000

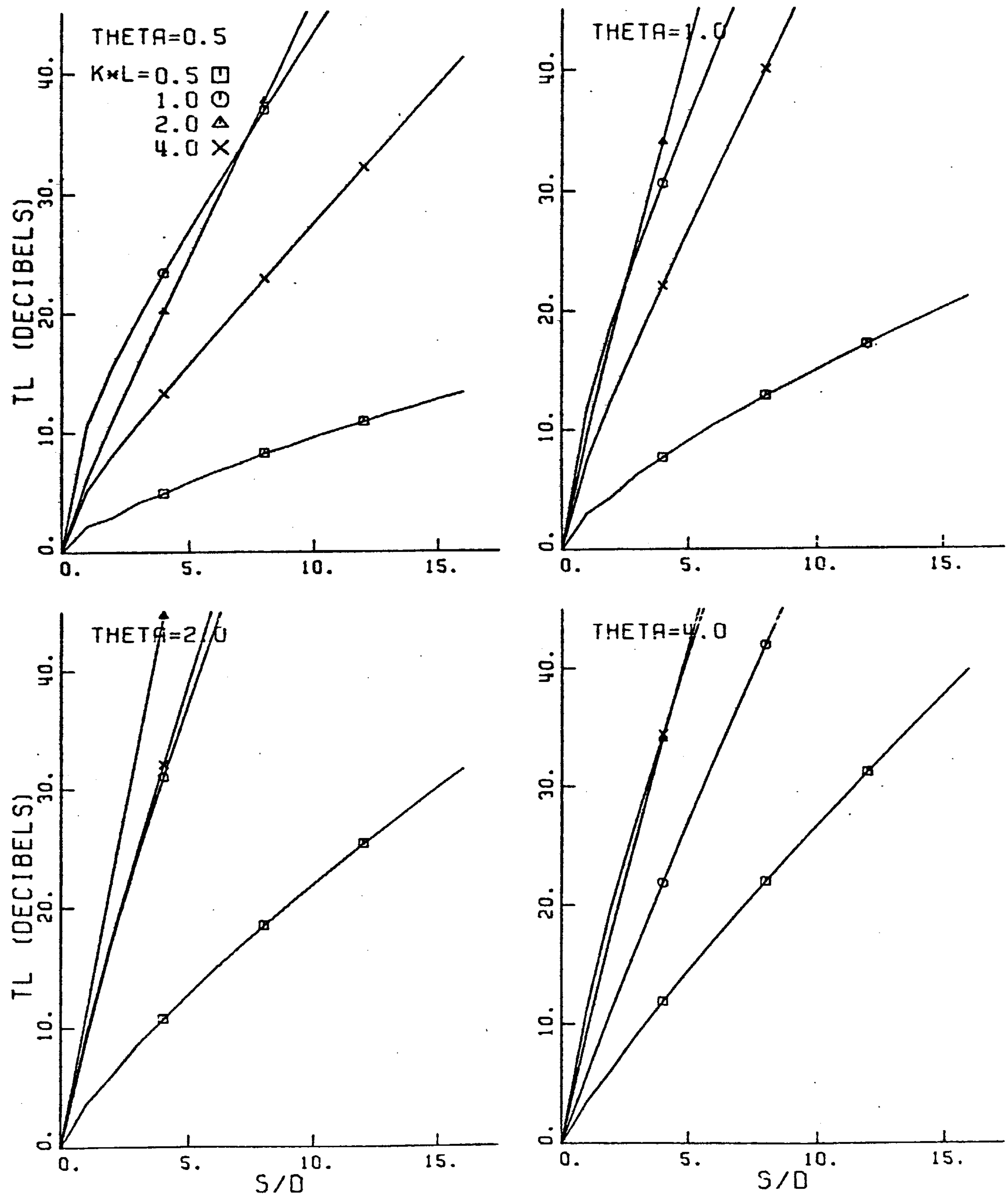


Figure 3.116

AREA RATIO=1.0   D/L=2.000

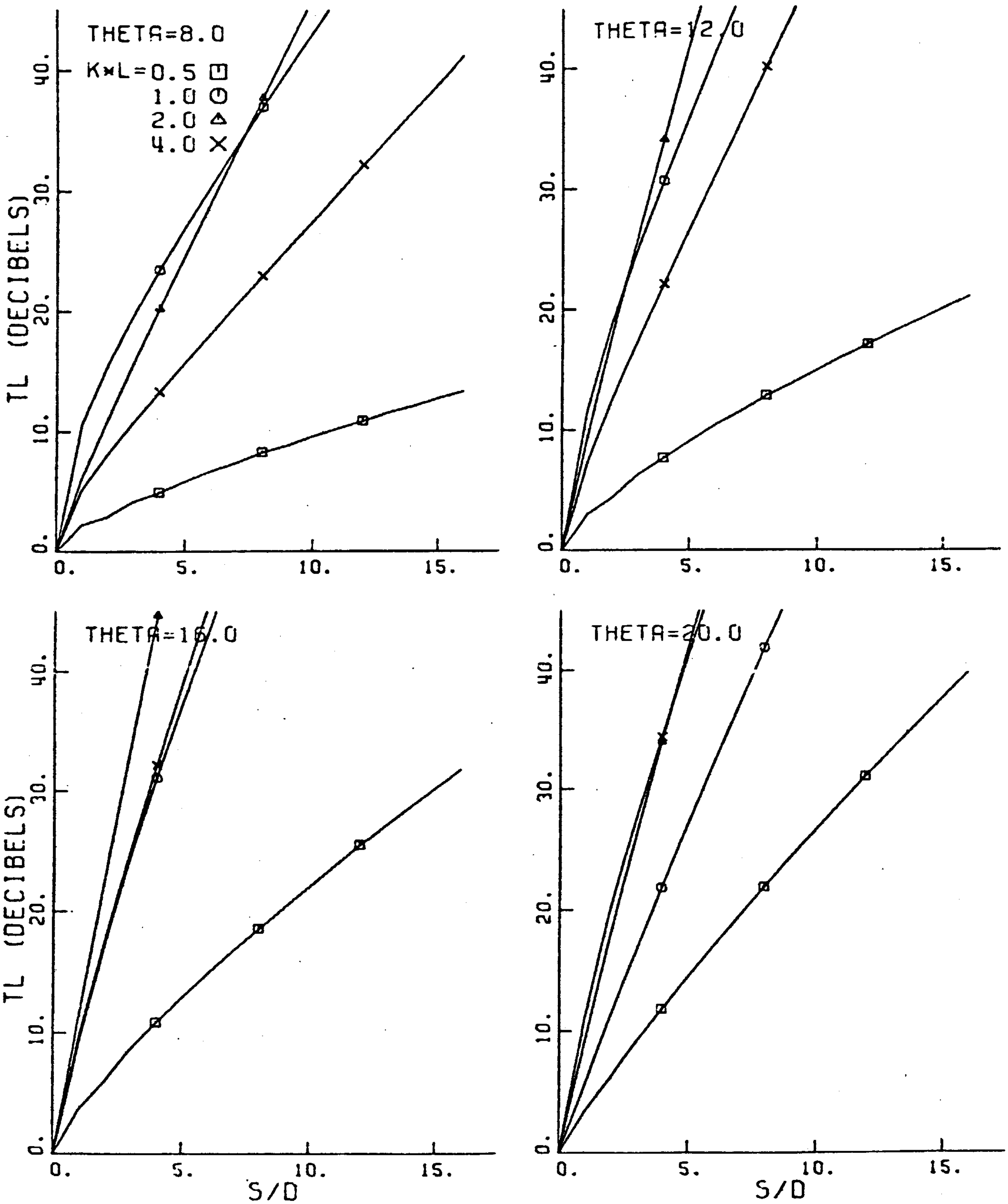
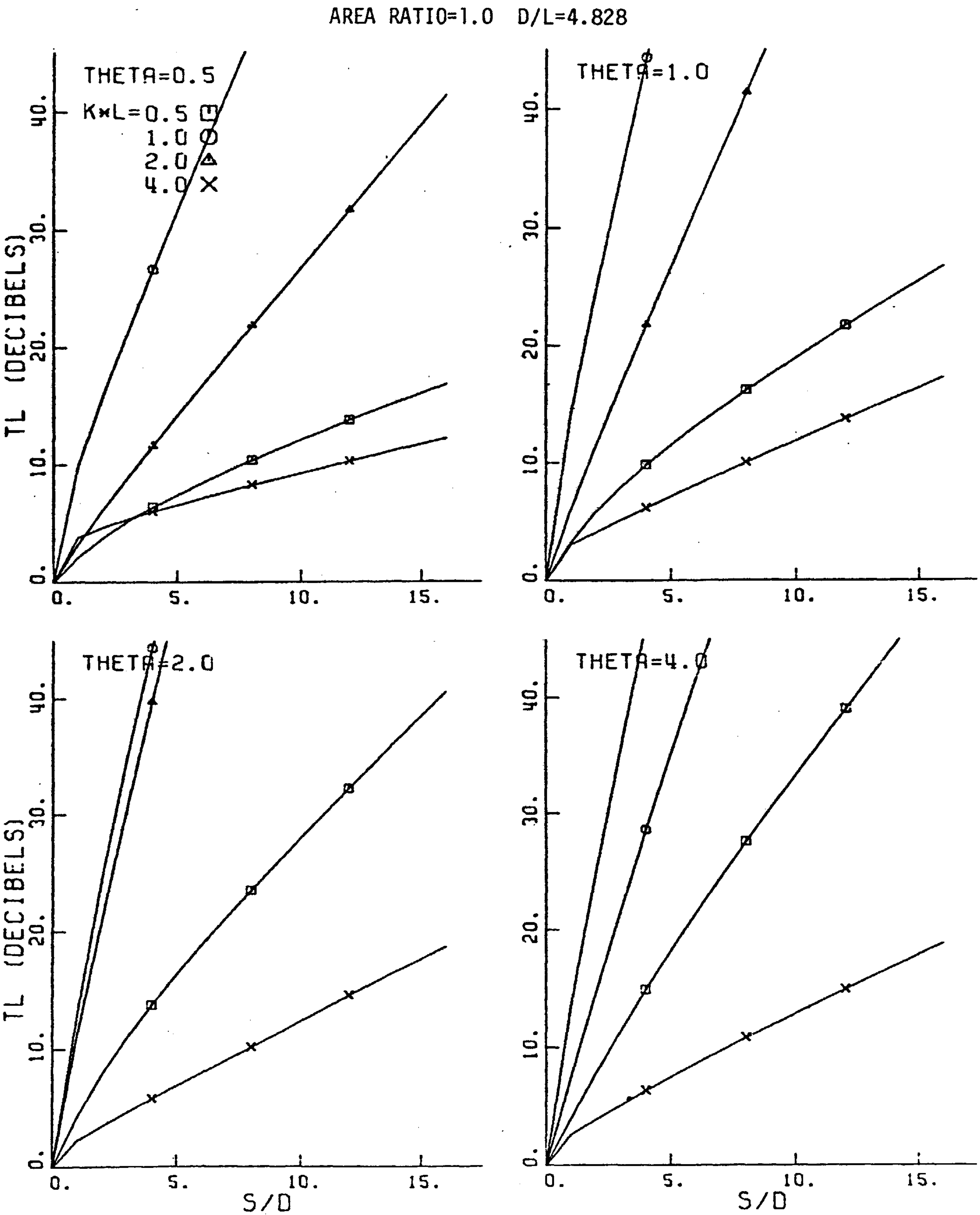


Figure 3.117



AREA RATIO=1.0 D/L=4.828
THETA=0.5
K*L=0.5
1.0
2.0
4.0
THETA=1.0
THETA=2.0
THETA=4.0
TL (DECIBELS)
S/D
0.
10.
20.
30.
40.
5.
15.

Figure 3.118

AREA RATIO=1.0   D/L=4.828

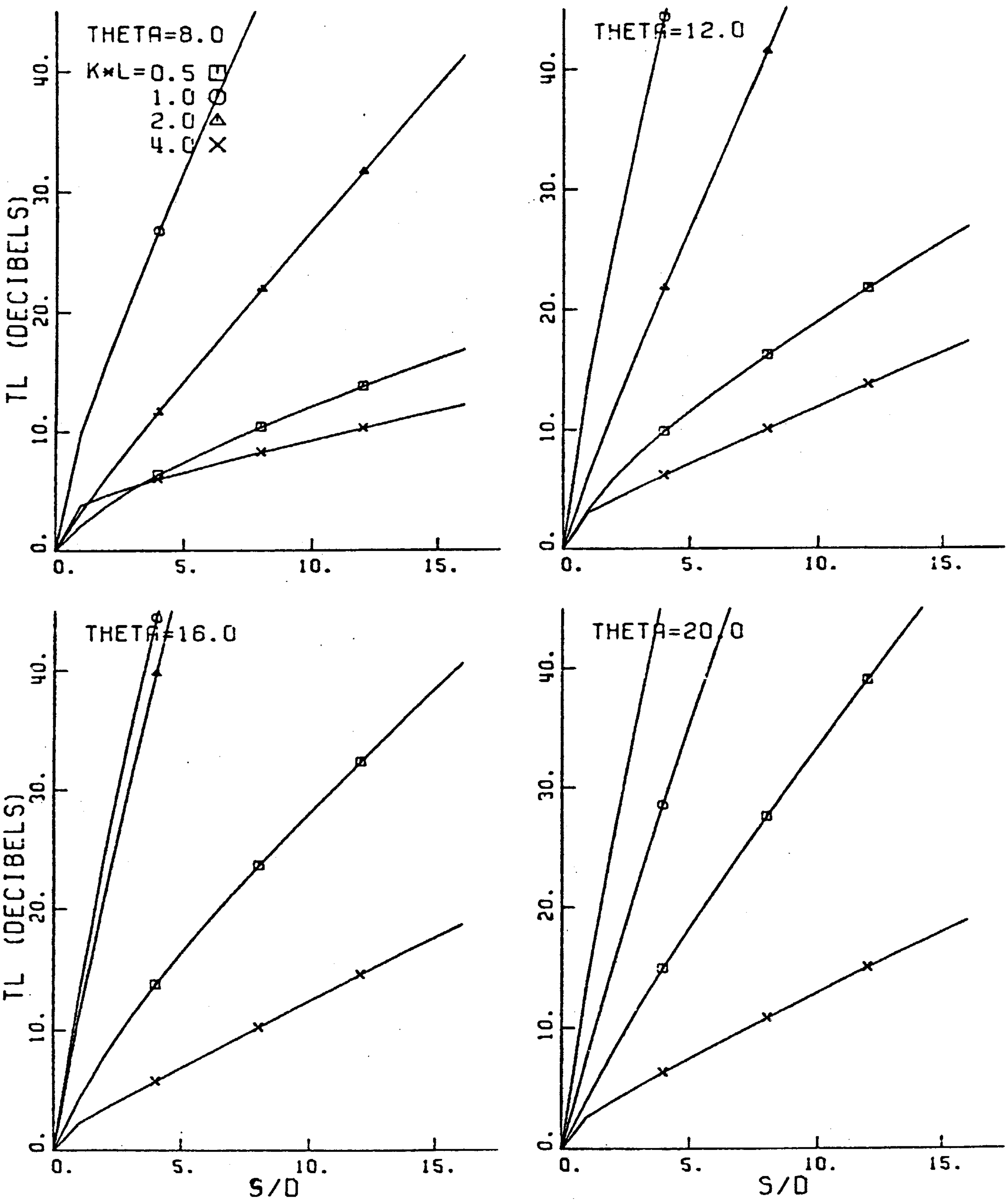


Figure 3.119

AREA RATIO=1.0   D/L=12.928

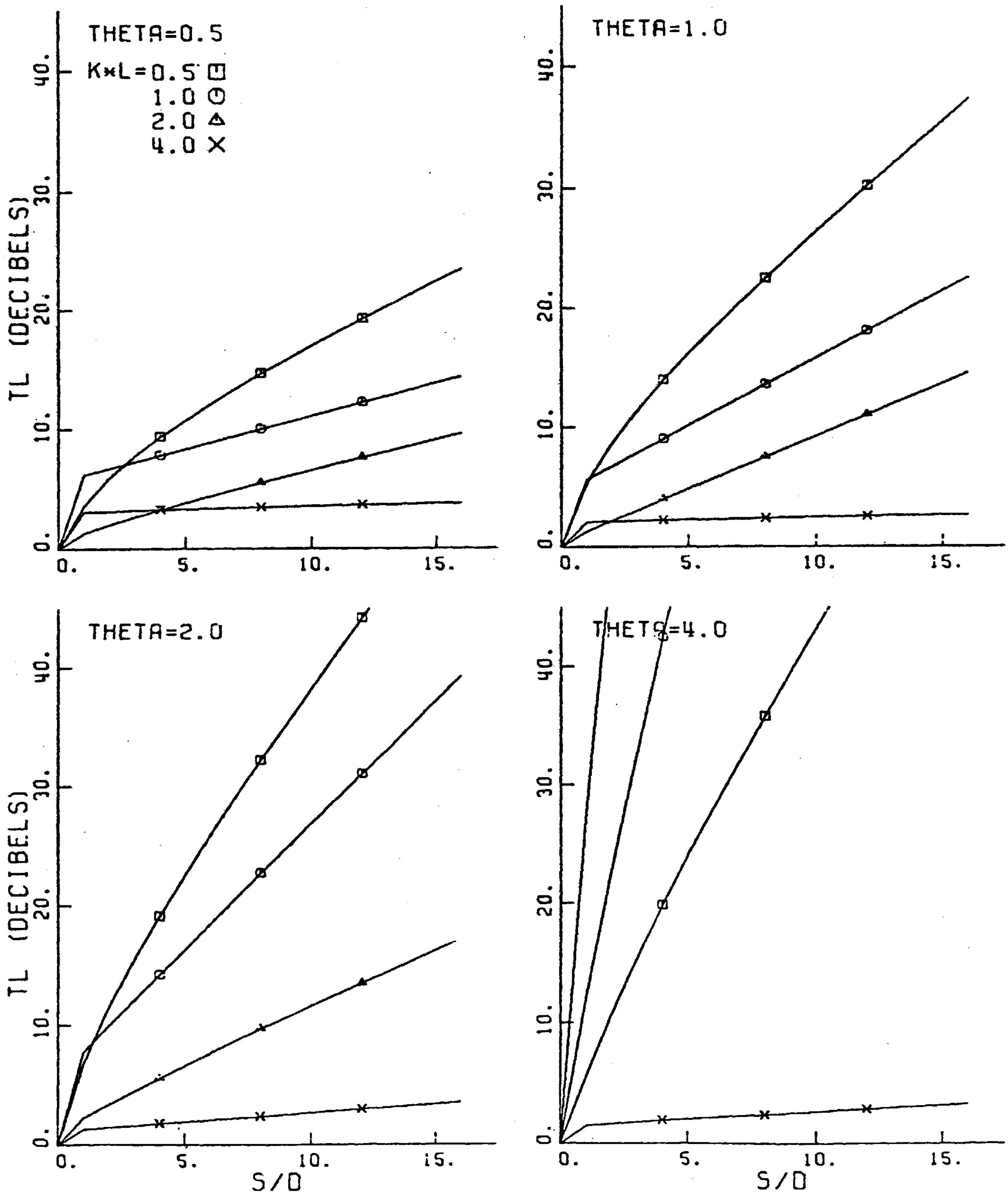


Figure 3.120

AREA RATIO=1.0   D/L=12.928

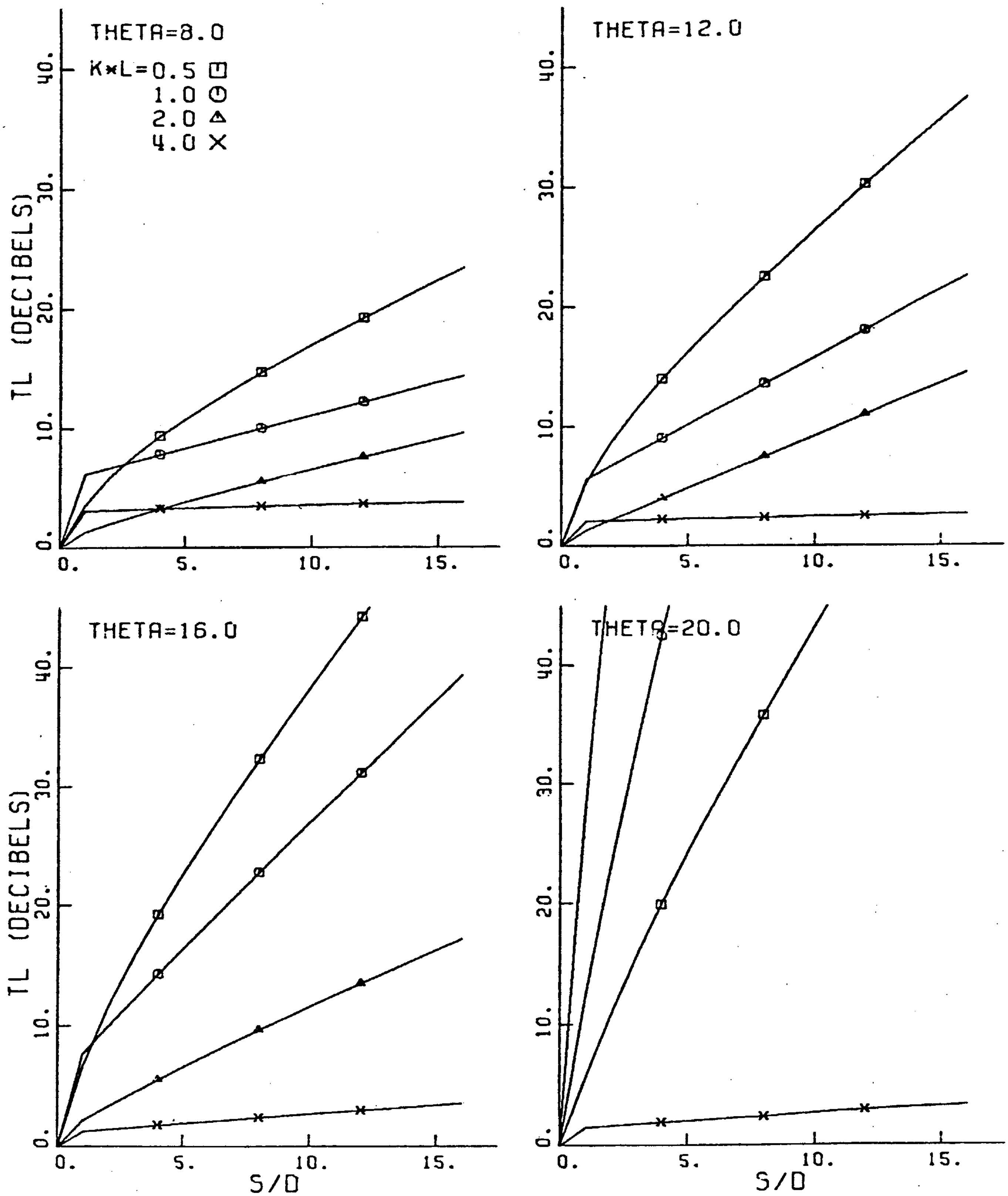


Figure 3.121

4. COMPUTER PROGRAMS

```
C  THIS PROGRAM COMPUTES COMPLEX WAVE CONSTANTS IN A LINED RECTANGULAR
C  DUCT BY USING A PROGRAM MADE BY 'ANTHONY GALAITSIS', AND THEN
C  OBTAINS OCTAVE BAND TRANSMISSION LOSS OF SOUND POWERS.
C  SOUND TRANSMISSION THROUGH LINED RECTANGULAR DUCT
C  WITH ONE WALL OR ADJACENT TWO WALLS LINED, FOR THE LATTER
C  COMPLEX WAVE VECTOR CKIA IS REPLACED BY NEW CKIA.
C  TRANSMISSION COEFFICIENT IS COMPUTED FOR 7 OCTAVE BAND.
C  INCIDENT WAVE IS A PULSE, WHICH IS FLAT, PARABOLIC OR INVERSELY
C  PROPORTIONAL TO SQUARE OF FREQUENCY.
C  USED HERE FOR AXIAL COMPONENT OF WAVE FUNCTION IS
C  A* COS(KZ*Z) + B*SIN(KZ*Z).  THIS IS BETTER THAN EXP(I*KZ*Z).
C  THIS PROGRAM CAN BE USED FOR SILENCER BY REPLACING CKR.
C  'D' IS A SIDE OF OPEN CROSS SECTION OF SILENCER
      IMPLICIT COMPLEX*8 (C)
      COMMON /CHART/R(25,23),HC(25),PHC(23)
      COMPLEX*8 R,EI,GROOT,GTAHGP
      COMPLEX*8 CKIA(2,4,4,114)
      REAL TT(4)/8.,12.,16.,20./
      REAL ARATIO(4)/.125,.25,.5,.75/
      REAL XRS(5)/1.,2.,4.,8.,16./,CABS,COS,COUNT(7),COTAN
      REAL V(7)/.0625,.125,.25,.5,1.,2.,4./
      REAL GN(113),      YO(17),YP(17),YN(17),ZO(17),ZP(17),ZN(17)
      REAL ATTO(7),ATTP(7),ATTN(7)
      REAL ATTD(2,2,4,4,5,3,7),YCOM(113)
      REAL GBR(4)/.142857,.333333,1.,3./
C     REAL GB(4)
  100 FORMAT (14F5.2)
  101 FORMAT  (10F8.3)
  102 FORMAT (10F8.5)
  103 FORMAT(8F10.5)
  202 FORMAT(1X,3I1,1X,7E17.7/)
  300 FORMAT('1        SOUND ATTENUATION IN SQUARE SILENCER, TWO OPPOSITE
     1 SIDE WALLS OF WHICH ARE LINED WITH POROUS MATERIAL.'/)
C 302 FORMAT('1        SOUND ATTENUATION IN SQUARE SILENCER, FOUR WALLS
C    1OF WHICH ARE LINED WITH RESONATOR.'/)
  304 FORMAT('1  SOUND ATTENUATION IN RECTANGULAR DUCT, TWO OPPOSITE
```

```
     1 SIDE WALLS OF WHICH ARE LINED WITH POROUS MATERIAL.'/)
 305 FORMAT('1  SOUND ATTENUATION IN RECTANGULAR DUCT, TWO OPPOSITE
     1 SIDE WALLS OF WHICH ARE LINED WITH RESONATOR.'/)
 303 FORMAT('1        SOUND ATTENUATION IN SQUARE SILENCER, TWO OPPOSITE
     1 SIDE WALLS OF WHICH ARE LINED WITH RESONATOR.'/)
 407 FORMAT(6X,7F17.8/)
 408 FORMAT(1X,' TRANSMISSION COEFFICIENT COMPUTED ON OCTAVE BANDS'/
     11X,' CENTER FREQUENCIES ARE EQUAL TO'/)
C 409 FORMAT(2X,' GAMMA=',F4.2/)
 415 FORMAT(10X,' T=',F4.2/)
 411 FORMAT(20X,' AREA RATIO=',F5.3,'   OR D/L=',F5.3/)
 412 FORMAT(30X,' LENGTH OF  LINING /D=',F5.2/)
 501 FORMAT(3I1,I3,2X,6E12.5)
 505 FORMAT(6I1,4X,7F10.4)
      EI  = CMPLX(0.,1.)
      PI  = 3.14159
      READ (5,100) (HC(I),I=1,25)
      READ (5,101) (PHC(J),J=1,23)
      READ (5,102)((R(I,J),I=1,25),J=1,23)
      READ(5,103) (GN(L),L=1,113)
C     GB(3)=1./(SQRT(2.)-1.)
C     GB(1)=1./(SQRT(8.)-1.)
C     GB(2)=1.
C     GB(4)=1./(2./SQRT(3.)-1.)
C     OBTAIN THE COMPLEX WAVEVECTOR FOR SOUND
C     PROPAGATING IN A LINED RECTANGULAR DUCT.
      DO 40 IJK=2,2
      DO 40 IJL=1,2
      IF(IJK.EQ.2.AND.IJL.EQ.1) WRITE(6,300)
      IF(IJK.EQ.2.AND.IJL.EQ.2) WRITE(6,304)
      IF(IJK.EQ.1.AND.IJL.EQ.1) WRITE(6,303)
      IF(IJK.EQ.1.AND.IJL.EQ.2) WRITE(6,305)
      GAMJ=1.5
      DO 50 K=1,4
      TTK=TT(K)
      DO 50 I=1,4
```

```
      GBI=GBR(I)                                                        CHOR0073
      DO 50 M=1,5                                                       CHOR0074
      XRSM=XRS(M)                                                       CHOR0075
      DO 55 LS=1,113                                                    CHOR0076
      GNL=GN(LS)                                                        CHOR0077
      GNB=GNL*GBI                                                       CHOR0078
      IF(M.NE.1.OR.IJL.NE.1) GO TO 2                                    CHOR0079
      IF(IJK.EQ.1) GO TO 15                                             CHOR0080
      RTG=SQRT(.5*GAMJ)                                                 CHOR0081
      SQRTN=SQRT(1.+(TTK/GNL)**2)                                       CHOR0082
      QPKR=RTG*SQRT(SQRTN+1.)                                           CHOR0083
      QPKI=RTG*SQRT(SQRTN-1.)                                           CHOR0084
      CQF=CMPLX(-QPKI,QPKR)                                             CHOR0085
      QFKR=QPKI*GNL                                                     CHOR0086
      QFKI=QPKR*GNL                                                     CHOR0087
      IF(QFKR.GT.30.) GO TO 10                                          CHOR0088
      CQFG=2.*GNL*CQF                                                   CHOR0089
      CEXQ=CEXP(CQFG)                                                   CHOR0090
      CTAN=(0.,-1.)*(CEXQ-(1.,0.))/(CEXQ+(1.,0.))                       CHOR0091
      CH=GNL*GBI*CTAN/(PI*CQF)                                          CHOR0092
      GO TO 11                                                          CHOR0093
   10 CTAN=(0.,1.)                                                      CHOR0094
      CH=GNL*GBI*CTAN/(PI*CQF)                                          CHOR0095
      GO TO 11                                                          CHOR0096
   15 CAZ=TTK+EI*COTAN(GNL)                                             CHOR0097
      CH=GNL*GBI/(PI*CAZ)                                               CHOR0098
   11 GROOT=GTAHGP(CH)                                                  CHOR0099
      XFACT=1.                                                          CHOR0100
      CKIA(IJK,K,I,LS)=CSQRT(GNB**2+(PI*GROOT)**2*XFACT)                CHOR0101
    2 CYKE=2.*XRSM*CKIA(IJK,K,I,LS)                                     CHOR0102
C     IF(IJK.NE.2) GO TO 17                                             CHOR0103
C     CKR=GBI*CKIA(LS)/GNL/(1.+GBI)**2                                  CHOR0104
C     GO TO 18                                                          CHOR0105
      IF(IJL.EQ.1) CKR=CKIA(IJK,K,I,LS)/GNL/(1.+GBI)                    CHOR0106
      IF(IJL.EQ.2) CKR=CKIA(IJK,K,I,LS)/GNB                             CHOR0107
      CKRV=(0.,.5)*(CKR+1./CKR)                                         CHOR0108
```

```
      IF(AIMAG(CYKE).GT.80.) GO TO 5
      CSUM=CCOS(CYKE)-CKRV*CSIN(CYKE)
      ASUM=CABS(CSUM)
      YCOM(LS)=1./ASUM**2
      GO TO 55
    5 YCOM(LS)=0.
   55 CONTINUE
      CKIA(IJK,2,I,114)=(0.,0.)
      DO 20 L=1,7
      HI=SQRT(2.)*V(L)
      H=.0625*HI
      LM=16*(L-1)
      DO 30 KL=1,17
      KLM=KL+LM
      YO(KL)=YCOM(KLM)/HI
      AKL=FLOAT(KL-1)
      YP(KL)=(HI+H*AKL)**2*YO(KL)
      YN(KL)=YO(KL)/(HI+H*AKL)**2
   30 CONTINUE
      CALL QSF(H,YO,ZO,17)
      ATTO(L)=ZO(17)
      ATTD(IJK,IJL,K,I,M,1,L)=-10.*ALOG10(ZO(17))
      CALL QSF(H,YP,ZP,17)
      ATTP(L)=3.*ZP(17)/HI**2/7.
      ATTD(IJK,IJL,K,I,M,2,L)=-10.*ALOG10(ATTP(L))
      CALL QSF(H,YN,ZN,17)
      ATTN(L)=2.*ZN(17)*HI**2
      ATTD(IJK,IJL,K,I,M,3,L)=-10.*ALOG10(ATTN(L))
   20 CONTINUE
   50 CONTINUE
      DO 54 LX=1,7
      COUNT(LX)=2.*V(LX)
   54 CONTINUE
      WRITE(6,408)
      WRITE(6,407) (COUNT(LX),LX=1,7)
      DO 56 KX=1,4
```

```
      KP=KX+4                                                           CHOR0145
      WRITE(6,415) TT(KX)                                               CHOR0146
      DO 56 IX=1,4                                                      CHOR0147
      TGB=2.*GBR(IX)                                                    CHOR0148
      WRITE(6,411) ARATIO(IX),TGB                                       CHOR0149
      DO 56 MX=1,5                                                      CHOR0150
      WRITE(6,412) XRS(MX)                                              CHOR0151
      WRITE(6,202) KP,IX,MX,(ATTD(IJK,IJL,KX,IX,MX,1,LZ),LZ=1,7)        CHOR0152
      WRITE(6,202) KP,IX,MX,(ATTD(IJK,IJL,KX,IX,MX,2,LZ),LZ=1,7)        CHOR0153
      WRITE(6,202) KP,IX,MX,(ATTD(IJK,IJL,KX,IX,MX,3,LZ),LZ=1,7)        CHOR0154
   56 CONTINUE                                                          CHOR0155
   40 CONTINUE                                                          CHOR0156
      DO 1001 IUVW=1,2                                                  CHOR0157
      DO 1003 KB=1,4                                                    CHOR0158
      KN=KB+4                                                           CHOR0159
      DO 1003 IB=1,4                                                    CHOR0160
      DO 1002 LAB=1,38                                                  CHOR0161
      LABI=3*(LAB-1)+1                                                  CHOR0162
      LABT=LABI+2                                                       CHOR0163
      WRITE(7,501) IUVW,KN,IB,LABI,(CKIA(IUVW,KB,IB,LSB),LSB=LABI,LABT) CHOR0164
 1002 CONTINUE                                                          CHOR0165
 1003 CONTINUE                                                          CHOR0166
 1001 CONTINUE                                                          CHOR0167
      DO 2001 IQRS=1,2                                                  CHOR0168
      DO 2002 IJLD=1,2                                                  CHOR0169
      DO 2002 NB=1,3                                                    CHOR0170
      DO 2002 KD=1,4                                                    CHOR0171
      KM=KD+4                                                           CHOR0172
      DO 2002 ID=1,4                                                    CHOR0173
      DO 2002 MD=1,5                                                    CHOR0174
      WRITE(7,505) IQRS,IJLD,NB,KM,ID,MD,(ATTD(IQRS,IJLD,KD,ID,MD,NB,   CHOR0175
     1LVD),LVD=1,7)                                                     CHOR0176
 2002 CONTINUE                                                          CHOR0177
 2001 CONTINUE                                                          CHOR0178
      STOP                                                              CHOR0179
      END                                                               CHOR0180
```

```
C     THIS PROGRAM COMPUTES COMPLEX WAVE VECTORS IN A LINED CIRCULAR
C     DUCT USING COMPLEX EIGENVALUES OBTAINED BY THE SUBROUTINE RTCHO,
C     THEN PERFORMS INTEGRATION TO OBTAIN OCTAVE BAND TRANSMISSION
C     LOSS OF SOUND POWERS.
      IMPLICIT COMPLEX*8 (C)
      COMMON /RIMIT/ZIN(64,4)
      COMMON /QENT/AAA,BBB
      COMPLEX ANSW(4),W(4),ZIN,CKIA(2,4,4,114)
      REAL TT(4)/8.,12.,16.,20./
      REAL ARATIO(4)/.125,.25,.5,.75/
      REAL XRS(5)/1.,2.,4.,8.,16./,CABS,COS,COUNT(7)
      REAL V(7)/.0625,.125,.25,.5,1.,2.,4./
      REAL GN(113),GB(4),YO(17),YP(17),YN(17),ZO(17),ZP(17),ZN(17)
      REAL ATTO(7),ATTP(7),ATTN(7)
      REAL ATTD(2,2,4,4,5,3,7),YCOM(113),COTAN
  100 FORMAT(8F10.5)
  120 FORMAT(8F10.5)
  110 FORMAT(6I10)
  202 FORMAT(1X,3I1,1X,7E17.7/)
  300 FORMAT('1  SOUND ATTENUATION IN  CIRCULAR SILENCER LINED WITH
     1RESONATOR.'/)
  301 FORMAT('1  SOUND ATTENUATION IN  CIRCULAR SILENCER LINED WITH
     1POROUS MATERIAL.'/)
  601 FORMAT('1  SOUND ATTENUATION IN CIRCULAR DUCT LINED WITH
     1RESONATOR.'/)
  602 FORMAT('1  SOUND ATTENUATION IN CIRCULAR DUCT LINED WITH
     1POROUS MATERIAL.'/)
  410 FORMAT(1X,4I2,I4,2E12.4,' NO ROOT...'/)
  404 FORMAT(3X,3I3,I4,5E15.4/)
  407 FORMAT(6X,7F17.8/)
  408 FORMAT(1X,' TRANSMISSION COEFFICIENT COMPUTED ON OCTAVE BANDS'/
     11X,' CENTER FREQUENCIES ARE EQUAL TO'/)
C 409 FORMAT(2X,' GAMMA=',F4.2/)
  415 FORMAT(10X,' T=',F5.2/)
  411 FORMAT(20X,' AREA RATIO=',F5.3,'   OR D/L=',F6.3/)
  412 FORMAT(30X,' LENGTH OF LINING/D=',F5.2/)
```

```
  501 FORMAT(3I1,I3,2X,6F12.5)                                         CHOC0037
  505 FORMAT(6I1,4X,7F10.4)                                            CHOC0038
      READ(5,100) ((ZIN(L,I),I=1,4),L=1,64)                            CHOC0039
      READ(5,120) (GN(L),L=1,113)                                      CHOC0040
      READ(5,110) NK,NGK,NI,NGI,NM,NGM                                 CHOC0041
      GB(1)=1./(SQRT(8.)-1.)                                           CHOC0042
      GB(2)=1.                                                         CHOC0043
      GB(3)=1./(SQRT(2.)-1.)                                           CHOC0044
      GB(4)=1./(2./SQRT(3.)-1.)                                        CHOC0045
      ATMX=EXP(160.)                                                   CHOC0046
      ATMIV=1./ATMX                                                    CHOC0047
      ATDMX=10.*ALOG10(ATMX)                                           CHOC0048
      DO 40 IJK=2,2                                                    CHOC0049
      DO 40 IJL=1,2                                                    CHOC0050
      IF(IJK.EQ.1.AND.IJL.EQ.1) WRITE(6,300)                           CHOC0051
      IF(IJK.EQ.1.AND.IJL.EQ.2) WRITE(6,601)                           CHOC0052
      IF(IJK.EQ.2.AND.IJL.EQ.1) WRITE(6,301)                           CHOC0053
      IF(IJK.EQ.2.AND.IJL.EQ.2) WRITE(6,602)                           CHOC0054
C     IF(IJK.EQ.1) NJ=1                                                CHOC0055
C     IF(IJK.EQ.2) NJ=2                                                CHOC0056
C     IF(IJK.EQ.1) NGJ=1                                               CHOC0057
C     IF(IJK.EQ.2) NGJ=3                                               CHOC0058
C     DO 50 J=NJ,NGJ                                                   CHOC0059
C     GAMJ=GAM(J)                                                      CHOC0060
      GAMJ=1.5                                                         CHOC0061
      DO 50 K=NK,NGK                                                   CHOC0062
      TTK=TT(K)                                                        CHOC0063
      DO 50 I=NI,NGI                                                   CHOC0064
      GBI=GB(I)                                                        CHOC0065
      DO 50 M=NM,NGM                                                   CHOC0066
      XRSM=XRS(M)                                                      CHOC0067
      DO 55 LS=1,113                                                   CHOC0068
      GNL=GN(LS)                                                       CHOC0069
      GNB=GNL*GBI                                                      CHOC0070
      IF(M.NE.1.OR.IJL.NE.1) GO TO 33                                  CHOC0071
      IF(IJK.EQ.2) GO TO 15                                            CHOC0072
```

```
      CTTK=-COTAN(GNL)+(0.,1.)*TTK                                      CHOC0073
      CAB=GNB/CTTK                                                      CHOC0074
      GO TO 16                                                          CHOC0075
   15 RTG=SQRT(.5*GAMJ)                                                 CHOC0076
      SQRTN=SQRT(1.+(TTK/GNL)**2)                                       CHOC0077
      QPKR=RTG*SQRT(SQRTN+1.)                                           CHOC0078
      QPKI=RTG*SQRT(SQRTN-1.)                                           CHOC0079
      CQHR=CMPLX(QPKR,QPKI)                                             CHOC0080
      QFKR=QPKI*GNL                                                     CHOC0081
      IF(QFKR.GT.30.) GO TO 10                                          CHOC0082
      CQF=CMPLX(-QPKI,QPKR)                                             CHOC0083
      CQFG=2.*GNL*CQF                                                   CHOC0084
      CEXQ=CEXP(CQFG)                                                   CHOC0085
      CTAN=(0.,-1.)*(CEXQ-(1.,0.))/(CEXQ+(1.,0.))                       CHOC0086
      GO TO 11                                                          CHOC0087
   10 CTAN=(0.,1.)                                                      CHOC0088
   11 CAB=-GNB*CTAN/CQHR                                                CHOC0089
   16 AAA=REAL(CAB)                                                     CHOC0090
      BBB=AIMAG(CAB)                                                    CHOC0091
      IF(BBB.EQ.0..AND.AAA.GT.1.E+4) GO TO 303                          CHOC0092
      GO TO 351                                                         CHOC0093
  303 AAA=10000.                                                        CHOC0094
      ANSW(1)=CMPLX(2.40483,0.)                                         CHOC0095
      W(1)=CMPLX(0.,0.)                                                 CHOC0096
      NQ7=1                                                             CHOC0097
      TERMQ=GNB**2-2.40483**2                                           CHOC0098
      IF(TERMQ) 304,305,305                                             CHOC0099
  304 REALK=0.                                                          CHOC0100
      AIMAK=SQRT(-TERMQ)                                                CHOC0101
      GO TO 22                                                          CHOC0102
  305 REALK=SQRT(TERMQ)                                                 CHOC0103
      AIMAK=0.                                                          CHOC0104
      GO TO 22                                                          CHOC0105
  302 AAA=GNBSN/PARET                                                   CHOC0106
  351 IF(ABS(AAA).LE.1.E-5) AAA=0.                                      CHOC0107
      IF(ABS(BBB).LE.1.E-5) BBB=0.                                      CHOC0108
```

```
      CALL RTCHO(NQ7,ANSW,W)                                    CHOC0109
      IF(NQ7.EQ.0) GO TO 60                                     CHOC0110
      XIRE=REAL(ANSW(1))                                        CHOC0111
      XIIM=AIMAG(ANSW(1))                                       CHOC0112
      CALL WAVEK(GNB,XIRE,XIIM,REALK,AIMAK)                     CHOC0113
   22 CKIA(IJK,K,I,LS)=CMPLX(REALK,AIMAK)                       CHOC0114
      AW=CABS(W(1))                                             CHOC0115
      IF(AW.LT..01) GO TO 33                                    CHOC0116
      WRITE(6,404) IJK,K,I,LS,AW,AAA,BBB,ANSW(1)                CHOC0117
   33 CYKE=CKIA(IJK,K,I,LS)*XRSM*2.                             CHOC0118
      IF(IJL.EQ.1) CKR=GBI*CKIA(IJK,K,I,LS)/GNL/(1.+GBI)**2     CHOC0119
      IF(IJL.EQ.2) CKR=CKIA(IJK,K,I,LS)/GNB                     CHOC0120
      CKRV=(0.,.5)*(CKR+1./CKR)                                 CHOC0121
      AMCK=AIMAG(CYKE)                                          CHOC0122
      IF(AMCK.GT.80.) GO TO 5                                   CHOC0123
      CSUM=CCOS(CYKE)-CKRV*CSIN(CYKE)                           CHOC0124
      ASUM=CABS(CSUM)                                           CHOC0125
      YCOM(LS)=1./ASUM**2*ATMX                                  CHOC0126
      GO TO 55                                                  CHOC0127
    5 IF(AMCK.GT.160.) GO TO 6                                  CHOC0128
      YCOM(LS)=4.*EXP(-2.*(AMCK-80.))/CABS(1.+CKRV)**2          CHOC0129
      GO TO 55                                                  CHOC0130
    6 YCOM(LS)=0.                                               CHOC0131
      GO TO 55                                                  CHOC0132
   60 WRITE(6,410) NQ7,IJK,K,I,LS,AAA,BBB                       CHOC0133
   55 CONTINUE                                                  CHOC0134
      CKIA(IJK,K,I,114)=(0.,0.)                                 CHOC0135
      DO 20 L=1,7                                               CHOC0136
      HI=SQRT(2.)*V(L)                                          CHOC0137
      H=.0625*HI                                                CHOC0138
      LM=16*(L-1)                                               CHOC0139
      DO 30 KL=1,17                                             CHOC0140
      KLM=KL+LM                                                 CHOC0141
      YO(KL)=YCOM(KLM)/HI                                       CHOC0142
      AKL=FLOAT(KL-1)                                           CHOC0143
      YP(KL)=(HI+H*AKL)**2*YO(KL)                               CHOC0144
```

```
      YN(KL)=YO(KL)/(HI+H*AKL)**2                                       CHOC0145
   30 CONTINUE                                                          CHOC0146
      CALL QSF(H,YO,ZO,17)                                              CHOC0147
      IF(ZO(17).LT.ATMIV) GO TO 21                                      CHOC0148
      ATTD(IJK,IJL,K,I,M,1,L)=-10.*ALOG10(ZO(17))+ATDMX                 CHOC0149
      GO TO 26                                                          CHOC0150
   21 ATTD(IJK,IJL,K,I,M,1,L)=2.*ATDMX                                  CHOC0151
   26 CALL QSF(H,YP,ZP,17)                                              CHOC0152
      ATTP(L)=3.*ZP(17)/HI**2/7.                                        CHOC0153
      IF(ATTP(L).LT.ATMIV) GO TO 23                                     CHOC0154
      ATTD(IJK,IJL,K,I,M,2,L)=-10.*ALOG10(ATTP(L))+ATDMX                CHOC0155
      GO TO 24                                                          CHOC0156
   23 ATTD(IJK,IJL,K,I,M,2,L)=2.*ATDMX                                  CHOC0157
   24 CALL QSF(H,YN,ZN,17)                                              CHOC0158
      ATTN(L)=2.*ZN(17)*HI**2                                           CHOC0159
      IF(ATTN(L).LT.ATMIV) GO TO 25                                     CHOC0160
      ATTD(IJK,IJL,K,I,M,3,L)=-10.*ALOG10(ATTN(L))+ATDMX                CHOC0161
      GO TO 20                                                          CHOC0162
   25 ATTD(IJK,IJL,K,I,M,3,L)=2.*ATDMX                                  CHOC0163
   20 CONTINUE                                                          CHOC0164
   50 CONTINUE                                                          CHOC0165
      DO 54 LX=1,7                                                      CHOC0166
      COUNT(LX)=2.*V(LX)                                                CHOC0167
   54 CONTINUE                                                          CHOC0168
      WRITE(6,408)                                                      CHOC0169
      WRITE(6,407) (COUNT(LX),LX=1,7)                                   CHOC0170
C     DO 56 JX=NJ,NGJ                                                   CHOC0171
C     WRITE(6,409)  GAM(JX)                                             CHOC0172
      DO 56 KX=NK,NGK                                                   CHOC0173
      KP=KX+4                                                           CHOC0174
      WRITE(6,415) TT(KX)                                               CHOC0175
      DO 56 IX=NI,NGI                                                   CHOC0176
      TGB=2.*GB(IX)                                                     CHOC0177
      WRITE(6,411) ARATIO(IX),TGB                                       CHOC0178
      DO 56 MX=NM,NGM                                                   CHOC0179
      WRITE(6,412) XRS(MX)                                              CHOC0180
```

```
C     IF(IJK.EQ.1) JY=JX
C     IF(IJK.EQ.2) JY=JX-1
      WRITE(6,202) KP,IX,MX,(ATTD(IJK,IJL,KX,IX,MX,1,LX),LX=1,7)
      WRITE(6,202) KP,IX,MX,(ATTD(IJK,IJL,KX,IX,MX,2,LX),LX=1,7)
      WRITE(6,202) KP,IX,MX,(ATTD(IJK,IJL,KX,IX,MX,3,LX),LX=1,7)
   56 CONTINUE
   40 CONTINUE
      DO 1001 IJKB=2,2
      DO 1001 KB=1,4
      KN=KB+4
      DO 1001 IB=1,4
      DO 1002 LAB=1,38
      LABI=3*(LAB-1)+1
      LABT=LABI+2
      WRITE(7,501) IJKB,KN,IB,LABI,(CKIA(IJKB,KB,IB,LSB),LSB=LABI,LABT)
 1002 CONTINUE
 1001 CONTINUE
      DO 2001 IJKD=2,2
      DO 2001 IJLD=1,2
      DO 2001 NB=1,3
      DO 2001 KD=1,4
      KM=KD+4
      DO 2001 ID=1,4
      DO 2001 MD=1,5
      WRITE(7,505) IJKD,IJLD,NB,KM,ID,MD,(ATTD(IJKD,IJLD,KD,ID,MD,NB,
     1LXD),LXD=1,7)
 2001 CONTINUE
      STOP
      END
      SUBROUTINE WAVEK(GNB,XIRE,XIIM,REALK,AIMAK)
C K IS SCALED BY 1/B, B IS RADIUS OF DUCT.
      TERMD=GNB**2-XIRE**2+XIIM**2
      TERCR=(2.*XIRE*XIIM)**2
      TERFS=SQRT(TERMD**2+TERCR)
      X01=ABS(.5*(TERFS+TERMD))
      Y01=ABS(.5*(TERFS-TERMD))
```

```
      REALK=SQRT(X01)                                                   CHOC0217
      AIMAK=SQRT(Y01)                                                   CHOC0218
      RETURN                                                            CHOC0219
      END                                                               CHOC0220
```

```
      SUBROUTINE RTCHO(NQ7,ANSW,W)                                                    RCH00001
C     SUBROUTINE RCHO OBTAINS FUNDAMENTAL EIGENVALUES FOR WAVES IN A                  RCH00002
C     LINED CIRCULAR DUCT.  IN OTHER WORD, IT OBTAINS FIRST ROOTS                     RCH00003
C     OF THE EQUATION,  X*J1(X)/J0(X)=CMPLX(AAA,BBB), WHERE J0(X)                     RCH00004
C     AND J1(X) ARE BESSEL FUNCTIONS OF THE ORDERS OF ZERE AND ONE                    RCH00005
C     WITH COMPLEX ARGUMENTS.                                                         RCH00006
C     RCHO CALLS BECHO AND YSQNK9.  BECHO CALLS COMJB.  COMJB AND                     RCH00007
C     YSQNK9 ARE SUBROUTINES REVISED FROM M.I.T. MATH LIBRARY                         RCH00008
C     ROUTINES.   IN CALLING PROGRAM COMMON STATEMENT SHOULD BE                       RCH00009
C     MADE TO SUPLY VALUES OF AAA AND BBB, AND ZIN(64,4).                             RCH00010
      IMPLICIT COMPLEX*8 (C)                                                 RCH00020 RCH00011
      COMPLEX*8 Z(4),ERROR,ANSW(4),RETERR,FUN,W(4),CMPLX                     RCH00030 RCH00012
      COMPLEX*8 ZIN,ZAB                                                      RCH00040 RCH00013
      COMMON /RIMIT/ZIN(64,4)                                                         RCH00014
      COMMON /QFNT/AAA,BBB                                                            RCH00015
      REAL*4 JR0,JI0,JR1,JI1,BJRE(900),BJIM(900),CABS,COS                    RCH00060 RCH00016
      EXTERNAL FUN,FCT,FCTI                                                  RCH00070 RCH00017
   91 FORMAT(1X,' ROOT IS NOT ACCURATE')                                     RCH00080 RCH00018
      AAQ=AAA**2                                                             RCH00090 RCH00019
      BBQ=BBB**2                                                             RCH00100 RCH00020
      ABQ=AAQ+BBQ                                                            RCH00110 RCH00021
      IF(ABQ.GT.4.) GO TO 44                                                 RCH00120 RCH00022
      CAB=CMPLX(AAA,BBB)                                                     RCH00130 RCH00023
      CABDP=CAB+(2.,0.)                                                      RCH00140 RCH00024
      CABFR=CAB+(4.,0.)                                                      RCH00150 RCH00025
      CRDF=CSQRT(CABDP**2-CABFR*CAB)                                         RCH00160 RCH00026
      CX=(2.,0.)*(CABDP-CRDF)/CABFR                                          RCH00170 RCH00027
      IF(CABS(CX).GE.1.) GO TO 44                                            RCH00180 RCH00028
      CZT=(2.,0.)*CSQRT(CX)                                                  RCH00190 RCH00029
      IF(AIMAG(CZT).GT.0.) CZT=-CZT                                          RCH00200 RCH00030
      DO 8601 LSD=1,20                                                       RCH00210 RCH00031
      XPT=REAL(CZT)                                                          RCH00220 RCH00032
      YPT=AIMAG(CZT)                                                         RCH00230 RCH00033
      CALL BECHO(XPT,YPT,ART,BIT,GJR0,GJI0,GJR1,GJI1)                        RCH00240 RCH00034
      WT1=ART-AAA                                                            RCH00250 RCH00035
      WT2=BIT-BBB                                                            RCH00260 RCH00036
```

```
      W(1)=CMPLX(WT1,WT2)
      CANSP=CZT
      IF(CABS(W(1)).LT..0005) GO TO 8511
      CJ1=CMPLX(GJR1,GJI1)
      CJO=CMPLX(GJRO,GJIO)
      CZJO=CZT*CJO
      CZJ1=CZT*CJ1
      CAJO=CAB*CJO
      CAJ1=CAB*CJ1
      CAZJ=CAB*CJ1/CZT
      C1=CZJO+CAJ1
      C2=(2.,0.)*(CZJ1-CAJO)
      CJDB=CJO-CZJ1+CAJO-CAZJ
      CROT=CSQRT(C1**2-C2*CJDB)
      CZDLT=(CROT-C1)/CJDB
      CZT=CZT+CZDLT
 8601 CONTINUE
      CANSP=CZT
      IF(CABS(W(1)).LT..005) GO TO 8511
   44 AOS=.666667+.333333*AAA
      IF(ABQ.GT.AOS) GO TO 31
      IF(BBB.NE.0.) GO TO 80
      IF(AAA.LT.0.) GO TO 45
      XAB=SQRT(8.*AAA/(4.+AAA))
      YAB=0.
      D=1.E-5
      NON=0
      CALL BESJ(XAB,NON,BJ,D,IER)
      BJO=BJ
      NON=1
      CALL BESJ(XAB,NON,BJ,D,IER)
      BJ1=BJ
      FRE=XAB*BJ1-AAA*BJO
      ANSW(1)=CMPLX(XAB,0.)
      W(1)=CMPLX(FRE,0.)
      NQ7=1
```

```
      GO TO 21
   45 XAB=0.
      YAB=SQRT(-8.*AAA/(4.+AAA))
      CALL IO(YAB,RIO)
      CALL IBI(YAB,RI)
      FRE=AAA*RIO+YAB*RI
      ANSW(1)=CMPLX(0.,-YAB)
      W(1)=CMPLX(FRE,0.)
      NQ7=1
      GO TO 21
   80 SQSD=4.*AAA+ABQ
      SQSF=SQRT(SQSD**2+16.*BBQ)
      SQDN=4.+2.*AAA+.25*ABQ
      XAB=SQRT((SQSF+SQSD)/SQDN)
      YAB=-SQRT((SQSF-SQSD)/SQDN)
      ANSW(1)=CMPLX(XAB,YAB)
      W(1)=FUN(ANSW(1))
      NQ7=1
      GO TO 21
   31 IF(BBB.NE.0.) GO TO 81
      IF(AAA) 82,83,83
   82 IF(AAA.LT.-.7055) GO TO 251
      XLI=-AAA+.57
      XRI=-AAA+.6
      GO TO 90
  251 IF(AAA.LT.-1.705) GO TO 252
      XLI=-AAA+.59
      XRI=-AAA+.61
      GO TO 90
  252 IF(AAA.LE.-2.) GO TO 8500
      XLI=-AAA+.5
      XRI=-AAA+.6
      GO TO 90
   90 EPS=1.E-4
      IEND=300
      CALL RTMI(YN,FI,FCTI,XLI,XRI,EPS,IEND,IER)
```

```
      ANSW(1)=CMPLX(0.,-YN)
      W(1)=CMPLX(FI,0.)
      NQ7=1
      IF(IER.NE.0) WRITE(6,91)
      GO TO 21
   83 IF(AAA.GT.1.0944) GO TO 161
      XLI=1.25
      XRI=1.31
      GO TO 69
  161 IF(AAA.GT.1.3385) GO TO 162
      XLI=1.29
      XRI=1.41
      GO TO 69
  162 IF(AAA.GT.1.6351) GO TO 163
      XLI=1.39
      XRI=1.51
      GO TO 69
  163 IF(AAA.GT.2.0023) GO TO 164
      XLI=1.49
      XRI=1.61
      GO TO 69
  164 IF(AAA.GT.2.4679) GO TO 131
      XLI=1.59
      XRI=1.71
      GO TO 69
  131 IF(AAA.GT.3.0788) GO TO 169
      XLI=1.69
      XRI=1.81
      GO TO 69
  169 IF(AAA.GT.3.9181) GO TO 132
      XLI=1.79
      XRI=1.91
      GO TO 69
  132 IF(AAA.GT.5.1519) GO TO 133
      XLI=1.89
      XRI=2.01
```

```
      GO TO 69
  133 IF(AAA.GT.7.1631) GO TO 134
      XLI=1.99
      XRI=2.11
      GO TO 69
  134 IF(AAA.GT.10.) GO TO 8100
      XLI=2.09
      XRI=2.21
   69 IEND=300
      EPS=1.E-4
      CALL RTMI(X,F,FCT,XLI,XRI,EPS,IEND,IER)
      ANSW(1)=CMPLX(X,C.)
      W(1)=CMPLX(F,0.)
      NQ7=1
      IF(IER.NE.0) WRITE(6,91)
      GO TO 21
   81 IF(AAA.LT.0.) GO TO 2001
      IF(ABQ.GE.99.) GO TO 8100
       IF(AAA.GT.0.66) GO TO 1001
      IF(BBB.LT.-1.) GO TO 1101
      L=1
      GO TO 1000
 1101 IF(BBB.LT.-1.475) GO TO 1102
      L=2
      GO TO 1000
 1102 IF(BBB.LT.-2.1) GO TO 1103
      L=3
      GO TO 1000
 1103 IF(BBB.LT.-3.5) GO TO 1104
      L=4
      GO TO 1000
 1104 IF(BBB.LT.-5.) GO TO 1121
      L=5
      GO TO 1000
 1121 L=6
      GO TO 1000
```

```
1001 IF(AAA.GT.1.) GO TO 1002                                   RCH01710  RCH00181
     IF(BBB.LT.-1.5) GO TO 1201                                 RCH01720  RCH00182
     IF(BBB.LT.-.4) GO TO 1211                                  RCH01730  RCH00183
     L=7                                                        RCH01740  RCH00184
     GO TO 1000                                                 RCH01750  RCH00185
1211 IF(BBB.LT.-.7) GO TO 1213                                  RCH01760  RCH00186
     L=8                                                        RCH01770  RCH00187
     GO TO 1000                                                 RCH01780  RCH00188
1213 IF(BBB.LT.-1.) GO TO 1212                                  RCH01790  RCH00189
     L=60                                                       RCH01800  RCH00190
     GO TO 1000                                                 RCH01810  RCH00191
1212 L=9                                                        RCH01820  RCH00192
     GO TO 1000                                                 RCH01830  RCH00193
1201 IF(BBB.LT.-2.4) GO TO 1202                                 RCH01840  RCH00194
     L=10                                                       RCH01850  RCH00195
     GO TO 1000                                                 RCH01860  RCH00196
1202 IF(BBB.LT.-4.) GO TO 1203                                  RCH01870  RCH00197
     L=11                                                       RCH01880  RCH00198
     GO TO 1000                                                 RCH01890  RCH00199
1203 IF(BBB.LT.-5.) GO TO 1204                                  RCH01900  RCH00200
     L=12                                                       RCH01910  RCH00201
     GO TO 1000                                                 RCH01920  RCH00202
1204 IF(BBB.LT.-7.) GO TO 1205                                  RCH01930  RCH00203
     L=62                                                       RCH01940  RCH00204
     GO TO 1000                                                 RCH01950  RCH00205
1205 L=63                                                       RCH01960  RCH00206
     GO TO 1000                                                 RCH01970  RCH00207
1002 IF(AAA.GT.1.7) GO TO 1003                                  RCH01980  RCH00208
     IF(BBB.LT.-.85) GO TO 1301                                 RCH01990  RCH00209
     L=13                                                       RCH02000  RCH00210
     GO TO 1000                                                 RCH02010  RCH00211
1301 IF(BBB.LT.-1.9) GO TO 1302                                 RCH02020  RCH00212
     L=14                                                       RCH02030  RCH00213
     GO TO 1000                                                 RCH02040  RCH00214
1302 IF(BBB.LT.-3.4) GO TO 1303                                 RCH02050  RCH00215
     L=15                                                       RCH02060  RCH00216
```

```
      GO TO 1000
 1303 IF(BBB.LT.-6.) GO TO 1304
      L=16
      GO TO 1000
 1304 L=17
      GO TO 1000
 1003 IF(AAA.GT.3.5) GO TO 1004
      IF(BBB.LT.-1.) GO TO 1401
      L=18
      GO TO 1000
 1401 IF(BBB.LT.-2.7) GO TO 1402
      L=19
      GO TO 1000
 1402 IF(BBB.LT.-6.) GO TO 1403
      L=20
      GO TO 1000
 1403 L=21
      GO TO 1000
 1004 IF(AAA.GT.8.) GO TO 1005
      IF(BBB.LT.-3.5) GO TO 1501
      L=22
      GO TO 1000
 1501 L=23
      GO TO 1000
 1005 L=24
      GO TO 1000
 2001 PHI=ATAN(-AAA/BBB)*180./3.14159
      IF(PHI.LT.-23.3) GO TO 3001
      IF(ABQ.GE.99.) GO TO 8100
      IF(BBB.LT.-.72) GO TO 2101
      L=25
      GO TO 1000
 2101 IF(BBB.LT.-1.005) GO TO 2201
      L=26
      GO TO 1000
 2201 IF(BBB.LT.-1.486) GO TO 2301
```

```
      L=27
      GO TO 1000
 2301 IF(BBB.LT.-1.9) GO TO 2401
      L=28
      GO TO 1000
 2401 IF(BBB.LT.-2.4) GO TO 2501
      IF(AAA.LT.-.6) GO TO 2402
      L=29
      GO TO 1000
 2402 L=30
      GO TO 1000
 2501 IF(BBB.LT.-3.) GO TO 2601
      IF(AAA.LT.-.6) GO TO 2502
      L=31
      GO TO 1000
 2502 IF(AAA.LT.-1.) GO TO 2503
      L=32
      GO TO 1000
 2503 IF(AAA.LT.-1.2) GO TO 2504
      L=33
      GO TO 1000
 2504 L=34
      GO TO 1000
 2601 IF(BBB.LT.-3.5) GO TO 2701
      IF(AAA.LT.-.5) GO TO 2602
      L=35
      GO TO 1000
 2602 IF(AAA.LT.-.8) GO TO 2603
      L=36
      GO TO 1000
 2603 IF(AAA.LT.-1.) GO TO 2604
      L=37
      GO TO 1000
 2604 L=38
      GO TO 1000
 2701 IF(BBB.LT.-5.) GO TO 2801
```

```
      IF(AAA.LT.-.8) GO TO 2702                                 RCH02790 RCH00289
      L=39                                                      RCH02800 RCH00290
      GO TO 1000                                                RCH02810 RCH00291
 2702 IF(BBB.LE.-4.) GO TO 2703                                 RCH02820 RCH00292
      L=40                                                      RCH02830 RCH00293
      GO TO 1000                                                RCH02840 RCH00294
 2703 L=61                                                      RCH02850 RCH00295
      GO TO 1000                                                RCH02860 RCH00296
 2801 L=41                                                      RCH02870 RCH00297
      GO TO 1000                                                RCH02880 RCH00298
 3001 IF(AAA.LE.-1.5) GO TO 8500                                RCH02890 RCH00299
      IF(AAA.GT.-1.5) GO TO 7001                                RCH02900 RCH00300
      IF(BBB.LT.-1.5) GO TO 6002                                RCH02910 RCH00301
      L=42                                                      RCH02920 RCH00302
      GO TO 2000                                                RCH02930 RCH00303
 6002 IF(BBB.LT.-2.5) GO TO 6003                                RCH02940 RCH00304
      L=43                                                      RCH02950 RCH00305
      GO TO 2000                                                RCH03980 RCH00306
 6003 IF(BBB.LT.-3.5) GO TO 6004                                RCH03990 RCH00307
      L=44                                                      RCH04000 RCH00308
      GO TO 2000                                                RCH04010 RCH00309
 6004 L=45                                                      RCH04020 RCH00310
      GO TO 2000                                                RCH04030 RCH00311
 7001 IF(BBB.LT.-.5) GO TO 7101                                 RCH04040 RCH00312
      IF(AAA.GT.-1.1) GO TO 7002                                RCH04050 RCH00313
      L=46                                                      RCH04060 RCH00314
      GO TO 2000                                                RCH04070 RCH00315
 7002 IF(AAA.GT.-.5) GO TO 7003                                 RCH04080 RCH00316
      L=47                                                      RCH04090 RCH00317
      GO TO 2000                                                RCH04100 RCH00318
 7003 L=48                                                      RCH04110 RCH00319
      GO TO 2000                                                RCH04120 RCH00320
 7101 IF(BBB.LT.-1.) GO TO 7201                                 RCH04130 RCH00321
      IF(AAA.GT.-1.1) GO TO 7102                                RCH04140 RCH00322
      L=49                                                      RCH04150 RCH00323
      GO TO 2000                                                RCH04160 RCH00324
```

```
7102 IF(AAA.GT.-.5) GO TO 7103
     L=50
     GO TO 2000
7103 L=51
     GO TO 2000
7201 IF(BBB.LT.-1.5) GO TO 7301
     IF(AAA.GT.-1.1) GO TO 7202
     L=52
     GO TO 2000
7202 L=53
     GO TO 2000
7301 IF(BBB.LT.-2.) GO TO 7401
     L=54
     GO TO 2000
7401 IF(BBB.LT.-2.5) GO TO 7501
     L=55
     GO TO 2000
7501 IF(BBB.LT.-2.7) GO TO 7601
     IF(AAA.GT.-1.3) GO TO 7502
     L=56
     GO TO 2000
7502 L=57
     GO TO 3000
7601 IF(AAA.GT.-1.4) GO TO 7602
     IF(BBB.LT.-2.94) GO TO 7611
     L=58
     GO TO 2000
7611 L=64
     GO TO 2000
7602 L=59
     GO TO 2000
1000 Z(1)=ZIN(L,1)
     Z(2)=ZIN(L,2)
     Z(3)=ZIN(L,3)
     Z(4)=ZIN(L,4)
     GO TO 9000
```

```
2000 ZAB=CMPLX(-BBB,AAA)
     Z(1)=ZAB-ZIN(L,1)
     Z(2)=ZAB-ZIN(L,2)
     Z(3)=ZAB-ZIN(L,3)
     Z(4)=ZAB-ZIN(L,4)
     GO TO 9000
3000 ZAB=CMPLX(-BBB,0.)
     Z(1)=ZAB-ZIN(L,1)
     Z(2)=ZAB-ZIN(L,2)
     Z(3)=ZAB-ZIN(L,3)
     Z(4)=ZAB-ZIN(L,4)
9000 M=2
     N=4
     NPRINT=0
     ERROR=(1.E-2,1.E-2)
     CALL YSQNK9(Z,N,ERROR,M,NPRINT,ANSW,W,RETERR,NO,FUN,NQ7)
     GO TO 21
8100 XO=2.40483
     CAB=CMPLX(AAA,BBB)
     CAHF=CAB+(.5,0.)
     DQN=XO**2+.25
     CQRT=CSQRT((1.,0.)+1.33333*DQN/CAHF**2)
     CANSP=XO*((1.,0.)-1.5*CAHF*(CQRT-(1.,0.))/DQN)
     DO 8505 LPS=1,20
     XP=REAL(CANSP)
     YP=AIMAG(CANSP)
     CALL BECHO(XP,YP,ARE,BIM,JRO,JIO,JR1,JI1)
     W1=ARE-AAA
     W2=BIM-BBB
     W(1)=CMPLX(W1,W2)
     IF(CABS(W(1)).LT..0005) GO TO 8511
     CIAP=CMPLX(JRO,JIO)/CMPLX(JR1,JI1)
     CZAJ=CANSP/CAB
     CMAPJ=CIAP*CZAJ
     CADM=1./CANSP+CZAJ
     CZMAP=1.+CMAPJ
```

```
      CQI=2.*CADM*(CIAP-CZAJ)/CZMAP**2
      CDELZ=CZMAP*((1.,0.)-CSQRT((1.,0.)-CQI))/CADM
      CANSP=CANSP+CDELZ
 8505 CONTINUE
      GO TO 8511
 8500 CDAB=(.5,0.)
      CAB=CMPLX(AAA,BBB)
      CEI=(0.,1.)
      DO 8501 KPS=1,20
      CZ=CEI*(CAB-CDAB)
      XP=REAL(CZ)
      YP=AIMAG(CZ)
      IF(XP.NE.0..AND.YP.NE.0.) GO TO 8503
      CALL BECHO(XP,YP,ARE,BIM,JR0,JI0,JR1,JI1)
      CALP=CMPLX(JR1,JI1)/CMPLX(JR0,JI0)
      GO TO 8504
 8503 ALPHA=0.
      BETA=0.
      N=1
      CALL COMJB(XP,YP,ALPHA,BETA,N,BJRE,BJIM)
      CALP=CMPLX(BJRE(2),BJIM(2))/CMPLX(BJRE(1),BJIM(1))
 8504 CALP2=CALP**2
      CPLP=CALP2+(1.,0.)
      CPLN=(1.,0.)-CALP2
      CALPP=CPLP/CALP
      CZAP=CZ*CALP
      CPLZ=CPLN/CZAP
      CQ=(CPLP+(.5,0.)*CPLZ)*((1.,0.)-CAB/CZAP)/CALPP**2
      CQRP=(1.,0.)-CSQRT((1.,0.)-(4.,0.)*CQ)
      CDNM=(2.,0.)*CPLP+CPLZ
      CANSP=CZ-CALPP*CQRP/CDNM
      XP=REAL(CANSP)
      YP=AIMAG(CANSP)
      CALL BECHO(XP,YP,ARE,BIM,JR0,JI0,JR1,JI1)
      W1=ARE-AAA
      W2=BIM-BBB
```

```
      W(1)=CMPLX(W1,W2)                                           RCH05250 RCH00433
      IF(CABS(W(1)).LT..0005) GO TO 8511                          RCH05260 RCH00434
      DA=ARE-YP                                                   RCH05270 RCH00435
      DB=BIM+XP                                                   RCH05280 RCH00436
      CDAB=CMPLX(DA,DB)                                           RCH05290 RCH00437
 8501 CONTINUE                                                    RCH05300 RCH00438
 8511 ANSW(1)=CANSP                                               RCH05310 RCH00439
      NQ7=1                                                       RCH05320 RCH00440
   21 RETURN                                                      RCH05330 RCH00441
      END                                                         RCH05340 RCH00442
      SUBROUTINE BECHO(XP,YP,ARE,BIM,JRO,JIO,JR1,JI1)                      RCH00443
      REAL*4 JRO,JIO,JR1,JI1,BJRE(900),BJIM(900)                           RCH00444
      IF(YP.NE.0.) GO TO 21                                                RCH00445
      D=1.E-6                                                              RCH00446
      NJ=0                                                                 RCH00447
      CALL BESJ(XP,NJ,JRO,D,IER)                                           RCH00448
      IF(JRO.LT.1.E-10.AND.JRO.GE.0.) JRO=1.E-10                           RCH00449
      IF(JRO.LT.0..AND.JRO.GT.-1.E-10) JRO=-1.E-10                         RCH00450
      NJ=1                                                                 RCH00451
      CALL BESJ(XP,NJ,JR1,D,IER)                                           RCH00452
      ARE=XP*JR1/JRO                                                       RCH00453
      BIM=0.                                                               RCH00454
      JIO=0.                                                               RCH00455
      JI1=0.                                                               RCH00456
      GO TO 50                                                             RCH00457
   21 IF(XP.NE.0.) GO TO 31                                                RCH00458
      YN=-YP                                                               RCH00459
      CALL IO(YN,JRO)                                                      RCH00460
      IF(JRO.LT.1.E-10.AND.JRO.GE.0.) JRO=1.E-10                           RCH00461
      IF(JRO.LT.0..AND.JRO.GT.-1.E-10) JRO=-1.E-10                         RCH00462
      CALL IBI(YN,JI1)                                                     RCH00463
      ARE=YP*JI1/JRO                                                       RCH00464
      JIO=0.                                                               RCH00465
      JR1=0.                                                               RCH00466
      BIM=0.                                                               RCH00467
      GO TO 50                                                             RCH00468
```

```
 31 ALPHA=0.                                                          RCH00469
    BETA=0.                                                           RCH00470
    N=1                                                               RCH00471
    CALL COMJB(XP,YP,ALPHA,BETA,N,BJRE,BJIM)                          RCH00472
    RD=BJRE(1)**2+BJIM(1)**2                                          RCH00473
    RA=BJRE(1)*BJRE(2)+BJIM(1)*BJIM(2)                                RCH00474
    RB=BJRE(1)*BJIM(2)-BJRE(2)*BJIM(1)                                RCH00475
    ARE=(XP*RA-YP*RB)/RD                                              RCH00476
    BIM=(XP*RB+YP*RA)/RD                                              RCH00477
    JR0=BJRE(1)                                                       RCH00478
    JI0=BJIM(1)                                                       RCH00479
    JR1=BJRE(2)                                                       RCH00480
    JI1=BJIM(2)                                                       RCH00481
 50 RETURN                                                            RCH00482
    END                                                               RCH00483
    FUNCTION FCT(X)                                                   RCH00484
    COMMON /QFNT/AAA,BBB                                              RCH00485
    D=1.E-5                                                           RCH00486
    NJ=0                                                              RCH00487
    CALL BESJ(X,NJ,BJR0,D,IER)                                        RCH00488
    NJ=1                                                              RCH00489
    CALL BESJ(X,NJ,BJR1,D,IER)                                        RCH00490
    FCT=X*BJR1/BJR0-AAA                                               RCH00491
    RETURN                                                            RCH00492
    END                                                               RCH00493
    FUNCTION FCTI(YN)                                                 RCH00494
    COMMON /QFNT/AAA,BBB                                              RCH00495
    CALL I0(YN,AJI0)                                                  RCH00496
    CALL I1I(YN,AJI1)                                                 RCH00497
    FCTI=AAA+YN*AJI1/AJI0                                             RCH00498
    RETURN                                                            RCH00499
    END                                                               RCH00500
    COMPLEX FUNCTION FUN(Z)                                           RCH00501
    COMPLEX*8 Z,CMPLX,ACM,BJ0,BJ1                                     RCH00502
    DIMENSION BJRE(100),BJIM(100)                                     RCH00503
    COMMON /QFNT/AAA,BBB                                              RCH00504
```

```
      AC=AAA
      BC=BBB
      X=REAL(Z)
      Y=AIMAG(Z)
      ALPHA=0.
      BETA=0.
      N=1
      CALL COMJB(X,Y,ALPHA,BETA,N,BJRE,BJIM)
      ACM=CMPLX(AC,BC)
      BJ0=CMPLX(BJRE(1),BJIM(1))
      BJ1=CMPLX(BJRE(2),BJIM(2))
      FUN=Z*BJ1-ACM*BJ0
      RETURN
      END
      SUBROUTINE IBI(X,RI)
      DIMENSION A(6),B(9)
      T=X/3.75
      IF(T-1.) 1,1,2
    1 A(1)=.8789059
      A(2)=.5149887
      A(3)=.1508493
      A(4)=.2658733E-1
      A(5)=.301532E-2
      A(6)=.32411E-3
      S=.5
      TS=T**2
      TE=1.
      DO 10 I=1,6
      TE=TE*TS
      S=S+A(I)*TE
   10 CONTINUE
      RI=S*X
      RETURN
    2 B(1)=.3989423
      B(2)=.3988024E-1
      B(3)=.362018E-2
```

```
      B(4)=.163801E-2                                                   RCH00541
      B(5)=.1031555E-1                                                  RCH00542
      B(6)=.2282967E-1                                                  RCH00543
      B(7)=.2895312E-1                                                  RCH00544
      B(8)=.1787654E-1                                                  RCH00545
      B(9)=.420059E-2                                                   RCH00546
      TI=1./T                                                           RCH00547
      TE=TI                                                             RCH00548
      SUM=B(1)-B(2)*TI                                                  RCH00549
      DO 20 I=3,9                                                       RCH00550
      TE=-TE*TI                                                         RCH00551
      SUM=SUM+B(I)*TE                                                   RCH00552
   20 CONTINUE                                                          RCH00553
      RI=SUM*EXP(X)/SQRT(X)                                             RCH00554
      RETURN                                                            RCH00555
      END                                                               RCH00556
```

```
      SUBROUTINE COMJB(X,Y,ALPHA,BETA,N,BJRE,BJIM)                     COMB0010 CMJB0001
      DIMENSION BJRE(900),BJIM(900)                                    COMB0580 CMJB0002
      CALL START(X,Y,N,K,R)                                            COMB0590 CMJB0003
      CALL JRECUR(X,Y,ALPHA,BETA,K,R,BJRE,BJIM)                        COMB0600 CMJB0004
      CALL JSUM(ALPHA,BETA,K,BJRE,BJIM,SUMRA,SUMIA)                    COMB0610 CMJB0005
      CALL FACTOR(X,Y,ALPHA,BETA,Q,R)                                  COMB0620 CMJB0006
      KSL=2                                                                     CMJB0007
      CALL JNORM(KSL,Q,R,SUMRA,SUMIA,BJRE,BJIM)                        COMB0630 CMJB0008
   15 IF(N-1)10,12,12                                                           CMJB0009
   10 IF (N)13,12,12                                                   COMB0720 CMJB0010
   13 CALL NEGN(X,Y,ALPHA,BETA,N,BJRE,BJIM)                                     CMJB0011
   12 RETURN                                                           COMB0760 CMJB0012
       END                                                             COMB0770 CMJB0013
       SUBROUTINE START(X,Y,N,K,R)                                     COMB0780 CMJB0014
CBES402  START SUBROUTINE                  PART 2 OF 16                COMB0790 CMJB0015
      SSQ=X**2+Y**2                                                    COMB0800 CMJB0016
      KTEN = SQRT(SSQ) + 20.0                                          COMB0810 CMJB0017
      NTEN = IABS(N) + 10                                              COMB0820 CMJB0018
      M = MAX0(KTEN,NTEN)/2                                            COMB0830 CMJB0019
      K=2*M+1                                                          COMB0840 CMJB0020
          R = K + 1                                                    COMB0850 CMJB0021
      RETURN                                                           COMB0860 CMJB0022
      END                                                              COMB0870 CMJB0023
      SUBROUTINE JRECUR(X,Y,ALPHA,BETA,K,R,BJRE,BJIM)                  COMB0880 CMJB0024
CBES403  JRECUR SUBROUTINE                 PART 3 OF 16                COMB0890 CMJB0025
      DIMENSION BJRE(900),BJIM(900)                                    COMB0900 CMJB0026
      RALPHA=R+ALPHA                                                   COMB0910 CMJB0027
      SSQ=X**2+Y**2                                                    COMB0920 CMJB0028
      BJRE(K+2)=0                                                      COMB0930 CMJB0029
      BJIM(K+2)=0                                                      COMB0940 CMJB0030
      BJRE(K+1)=1.0E-37                                                COMB0950 CMJB0031
      BJIM(K+1)=0.0                                                    COMB0960 CMJB0032
      DO4I=1,K                                                         COMB0970 CMJB0033
      L1=K+1-I                                                         COMB0980 CMJB0034
      RALPHA=RALPHA-1.0                                                COMB0990 CMJB0035
      A=((2.0*X*RALPHA)+(2.0*BETA*Y))/SSQ                              COMB1000 CMJB0036
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```
      B=((-2.0*Y*RALPHA)+(2.0*BETA*X))/SSQ                                COMB1010 CMJB0037
      IF(B.EQ.0.) GO TO 10                                                         CMJB0038
      TESTI=BJIM(L1+1)*1.E10                                                       CMJB0039
      TESTT=1.E-37/B                                                               CMJB0040
      TESTA=ABS(TESTI)                                                             CMJB0041
      TESTB=ABS(TESTT)                                                             CMJB0042
      IF(TESTA.LE.TESTB) GO TO 10                                                  CMJB0043
      BBJIM=B*BJIM(L1+1)                                                           CMJB0044
      GO TO 11                                                                     CMJB0045
   10 BBJIM=0.                                                                     CMJB0046
   11 BJRE(L1)=A*BJRE(L1+1)-BBJIM-BJRE(L1+2)                                      CMJB0047
 4    BJIM(L1)=(B*BJRE(L1+1))+(A*BJIM(L1+1))-BJIM(L1+2)                   COMB1030 CMJB0048
    5 RETURN                                                              COMB1040 CMJB0049
      END                                                                 COMB1050 CMJB0050
      SUBROUTINE JSUM(ALPHA,BETA,K,BJRE,BJIM,SUMRA,SUMIA)                 COMB1060 CMJB0051
CBES404  JSUM SUBROUTINE                   PART 4 OF 16                    COMB1070 CMJB0052
      DIMENSION BJRE(900),BJIM(900)                                       COMB1080 CMJB0053
 801  SUMRA=(BJRE(3)*(ALPHA+2.0))-(BJIM(3)*BETA)                          COMB1090 CMJB0054
      SUMIA=(BETA*BJRE(3))+((ALPHA+2.0)*BJIM(3))                          COMB1100 CMJB0055
      GRE=1.0                                                             COMB1110 CMJB0056
      GIM=0                                                               COMB1120 CMJB0057
      S=1.0                                                               COMB1130 CMJB0058
      DO6I=5,K,2                                                          COMB1140 CMJB0059
      S=S+1.0                                                             COMB1150 CMJB0060
      GREN=((GRE*(ALPHA+S-1.0))-(BETA*GIM))/S                             COMB1160 CMJB0061
      GIM=((GIM*(ALPHA+S-1.0))+(BETA*GRE))/S                              COMB1170 CMJBC062
      GRE=GREN                                                            COMB1180 CMJB0063
      ALPTS=ALPHA+2.0*S                                                   COMB1190 CMJB0064
      GJR=GRE*BJRE(I)                                                     COMB1200 CMJB0065
      GJI=GIM*BJIM(I)                                                     COMB1210 CMJB0066
      GJRI=GRE*BJIM(I)                                                    COMB1220 CMJB0067
      GJIR=GIM*BJRE(I)                                                    COMB1230 CMJB0068
      SUMRB=ALPTS*(GJR-GJI)-BETA*(GJIR+GJRI)+SUMRA                        COMB1240 CMJB0069
      SUMIB=ALPTS*(GJIR+GJRI)-BETA*(GJI-GJR)+SUMIA                        COMB1250 CMJB0070
      IF (SUMRA) 12,21,12                                                 COMB1260 CMJB0071
   12 IF (ABS ( (SUMRB/SUMRA) -1.0) -5.0E-8) 21,21,10                     COMB1270 CMJB0072
```

```
 21    IF(SUMIA)20,11,20                                                    COMB1280 CMJB0073
  20 IF (ABS( (SUMIB/SUMIA) -1.0)  -5.0E-8) 11,11,10                        COMB1290 CMJB0074
 10    SUMRA=SUMRB                                                          COMB1300 CMJB0075
 6     SUMIA=SUMIB                                                          COMB1310 CMJB0076
   11 RETURN                                                                COMB1320 CMJB0077
       END                                                                  COMB1330 CMJB0078
      SUBROUTINE FACTOR(X,Y,ALPHA,BETA,Q,R)                                 COMB1340 CMJB0079
CBES405   FACTOR SUBROUTINE                  PART 5 OF 16                   COMB1350 CMJB0080
      REAL*8 ZZ1,ZZ2,ZZ3,ZZ4,DBLE                                           COMB1360 CMJB0081
      CALL COMLOG(X,Y,A1,B1)                                                COMB1370 CMJB0082
      A2=ALPHA*A1-BETA*B1                                                   COMB1380 CMJB0083
      B2=BETA*A1+ALPHA*B1                                                   COMB1390 CMJB0084
      A2=-A2                                                                COMB1400 CMJB0085
      B2=-B2                                                                COMB1410 CMJB0086
      CALL COMEXP(A2,B2,A3,B3)                                              COMB1420 CMJB0087
      A4 = .693147 * ALPHA                                                  COMB1430 CMJB0088
      B4 = .693147 * BETA                                                   COMB1440 CMJB0089
      CALL COMEXP(A4,B4,A5,B5)                                              COMB1450 CMJB0090
      A6=A3*A5-B3*B5                                                        COMB1460 CMJB0091
      B6=B3*A5+A3*B5                                                        COMB1470 CMJB0092
      ZZZ=ALPHA+1.0                                                         COMB1480 CMJB0093
      ZZ1=DBLE(ZZZ)                                                         COMB1490 CMJB0094
      ZZ2=DBLE(BETA)                                                        COMB1500 CMJB0095
      CALL LOGAAM(ZZ1,ZZ2,ZZ3,ZZ4)                                          COMB1510 CMJB0096
      U=ZZ3                                                                 COMB1520 CMJB0097
      V=ZZ4                                                                 COMB1530 CMJB0098
      CALL COMEXP(U,V,A7,B7)                                                COMB1540 CMJB0099
   16 Q = A6*A7 - B6*B7                                                     COMB1550 CMJB0100
      R=B6*A7+A6*B7                                                         COMB1560 CMJB0101
      RETURN                                                                COMB1570 CMJB0102
       END                                                                  COMB1580 CMJB0103
      SUBROUTINE COMLOG(X,Y,A,B)                                            COMB1590 CMJB0104
CBES406   COMLOG SUBROUTINE                  PART 6 OF 16                   COMB1600 CMJB0105
C     COMPLEX LOGARITHM - BRANCH CUT ON NEGATIVE REAL AXIS                  COMB1610 CMJB0106
       PI = 3.141592                                                        COMB1620 CMJB0107
      A = .5*ALOG(X*X+Y*Y )                                                 COMB1630 CMJB0108
```

```
      IF(X)5,1,4
1     B=.5*PI
      IF(Y)2,3,8
2     B=-B
      GO TO 8
3     B=0.
      GO TO 8
    4 IF(ABS(Y*1.E50).LE.ABS(X)) GO TO 11
      IF(ABS(Y*1.E5).LE.ABS(X)) GO TO 15
      B=ATAN(Y/X)
      GO TO 8
   15 B=Y/X
      GO TO 8
   11 B=0.
      GO TO 8
    5 IF(ABS(Y*1.E50).LE.ABS(X)) GO TO 12
      IF(ABS(Y*1.E5).LE.ABS(X)) GO TO 16
      B=ATAN(Y/X)
      GO TO 13
   16 B=Y/X
      GO TO 13
   12 B=0.
   13 CONTINUE
      IF(Y)6,7,7
6     B=B-PI
      GO TO 8
7     B=B+PI
8     RETURN
       END
      SUBROUTINE COMEXP(X,Y,A,B)
CBES407  COMEXP SUBROUTINE                 PART 7 OF 16
      C= EXP(X)
      A = COS(Y)      *C
      B = C*SIN(Y)
      RETURN
       END
```

```
      SUBROUTINE JNORM(K,Q,R,SUMRA,SUMIA,BJRE,BJIM)             COMB1870 CMJB0145
CBES408   JNORM SUBROUTINE             PART 8 OF 16              COMB1880 CMJB0146
      DIMENSION BJRE(900),BJIM(900)                              COMB189  CMJB0147
      S=((SUMRA+BJRE(1))*Q)-((SUMIA+BJIM(1))*R)                  COMB1900 CMJB0148
      T=((SUMIA+BJIM(1))*Q)+((SUMRA+BJRE(1))*R)                  COMB1910 CMJB0149
      IF  (ABS(S) - ABS(T) ) 100,101,101                         COMB1920 CMJB0150
  101 IF(ABS(S).GT.1.E-65) GO TO 1001                                     CMJB0151
      WRITE(6,9600) S                                                     CMJB0152
 9600 FORMAT(1X,E16.8,' ROOT MAY NOT BE GOOD.')                           CMJB0153
      GO TO 14                                                            CMJB0154
 1001 TS=T/S                                                              CMJB0155
      TSSQ=S*(1.0+(TS**2))                                       COMB1940 CMJB0156
 12   DO13I=1,K                                                  COMB1950 CMJB0157
      IF(TS.EQ.0.) GO TO 20                                               CMJB0158
      BJIMI=ABS(BJIM(I))                                                  CMJB0159
      TSI=ABS(1.E-37/TS)                                                  CMJB0160
      IF(BJIMI.LE.TSI) GO TO 20                                           CMJB0161
      BJIMT=BJIM(I)*TS                                                    CMJB0162
      GO TO 21                                                            CMJB0163
   20 BJIMT=0.                                                            CMJB0164
   21 BJREH=BJRE(I)+BJIMT                                                 CMJB0165
      IF(ABS(BJREH).LE.ABS(TSSQ*1.E-40)) GO TO 30                         CMJB0166
      BJREN=BJREH/TSSQ                                                    CMJB0167
      GO TO 31                                                            CMJB0168
   30 BJREN=0.                                                            CMJB0169
   31 IF(TS.EQ.0.) GO TO 40                                               CMJB0170
      BJREI=ABS(BJRE(I))                                                  CMJB0171
      IF(BJREI.LE.TSI) GO TO 40                                           CMJB0172
      BJRET=BJRE(I)*TS                                                    CMJB0173
      GO TO 41                                                            CMJB0174
   40 BJRET=0.                                                            CMJB0175
   41 BJIMH=BJIM(I)-BJRET                                                 CMJB0176
      IF(ABS(BJIMH).LE.ABS(TSSQ*1.E-40)) GO TO 50                         CMJB0177
      BJIM(I)=BJIMH/TSSQ                                                  CMJB0178
      GO TO 13                                                            CMJB0179
   50 BJIM(I)=0.                                                          CMJB0180
```

```
   13 BJRE(I)=BJREN                                                   CMJB0181
      GO  TO  14                                             COMB1990 CMJB0182
  100 ST=S/T                                                 COMB2000 CMJB0183
      STSQ=T*((ST**2)+1.0)                                   COMB2010 CMJB0184
  102 DO103I=1,K                                             COMB2020 CMJB0185
      IF(ST.EQ.0.) GO TO 60                                           CMJB0186
      IF(ABS(BJRE(I)).LE.ABS(1.E-40/ST)) GO TO 60                     CMJB0187
      BJIMJ=BJRE(I)*ST                                                CMJB0188
      GO TO 61                                                        CMJB0189
   60 BJIMJ=0.                                                        CMJB0190
   61 BJIMQ=BJIMJ+BJIM(I)                                             CMJB0191
      IF(ABS(BJIMQ).LE.ABS(STSQ*1.E-37)) GO TO 70                     CMJB0192
      BJREN=BJIMQ/STSQ                                                CMJB0193
      GO TO 71                                                        CMJB0194
   70 BJREN=0.                                                        CMJB0195
   71 IF(ST.EQ.0.) GO TO 80                                           CMJB0196
      IF(ABS(BJIM(I)).LE.ABS(1.E-37/ST)) GO TO 80                     CMJB0197
      BJIMR=BJIM(I)*ST                                                CMJB0198
      GO TO 81                                                        CMJB0199
   80 BJIMR=0.                                                        CMJB0200
   81 BJIMU=BJIMR-BJRE(I)                                             CMJB0201
      IF(ABS(BJIMU).LE.ABS(STSQ*1.E-37)) GO TO 90                     CMJB0202
      BJIM(I)=BJIMU/STSQ                                              CMJB0203
      GO TO 103                                                       CMJB0204
   90 BJIM(I)=0.                                                      CMJB0205
  103 BJRE(I)=BJREN                                          COMB2050 CMJB0206
   14 RETURN                                                 COMB2060 CMJB0207
       END                                                   COMB2070 CMJB0208
      SUBROUTINE NEGN(X,Y,ALPHA,BETA,N,BJRE,BJIM)                     CMJB0209
CBES413  NEGN SUBROUTINE                PART 13 OF 16        COMB3080 CMJB0210
      DIMENSION BJRE(900),BJIM(900)                          COMB3090 CMJB0211
       L = IABS(N) + 1                                       COMB3100 CMJB0212
      SSQ=X**2+Y**2                                          COMB3110 CMJB0213
      TX=2.0*X                                               COMB3120 CMJB0214
      TY=2.0*Y                                               COMB3130 CMJB0215
      RALPHA=ALPHA                                           COMB3140 CMJB0216
```

```
      A=(TX*RALPHA+TY*BETA)/SSQ                                          COMB3150 CMJB0217
      B=(-TY*RALPHA+TX*BETA)/SSQ                                         COMB3160 CMJB0218
      BJRE(2)=A*BJRE(1)-B*BJIM(1)-BJRE(2)                                COMB3170 CMJB0219
      BJIM(2)=B*BJRE(1)+A*BJIM(1)-BJIM(2)                                COMB3180 CMJB0220
      DO 1 I=3,L                                                         COMB3210 CMJB0221
      RALPHA=RALPHA-1.0                                                  COMB3220 CMJB0222
      A=(TX*RALPHA+TY*BETA)/SSQ                                          COMB3230 CMJB0223
      B=(-TY*RALPHA+TX*BETA)/SSQ                                         COMB3240 CMJB0224
      BJRE(I)=A*BJRE(I-1)-B*BJIM(I-1)-BJRE(I-2)                          COMB3250 CMJB0225
      BJIM(I)=B*BJRE(I-1)+A*BJIM(I-1)-BJIM(I-2)                          COMB3260 CMJB0226
    1 CONTINUE                                                                    CMJB0227
      RETURN                                                             COMB3290 CMJB0228
       END                                                               COMB3300 CMJB0229
      SUBROUTINE LOGAAM(X,Y,U,V)                                                  CMJB0230
CLOGGAM  LOG OF THE GAMMA FUNCTION OF COMPLEX ARGUMENTS  FORTRAN II      LOGG0020 CMJB0231
      IMPLICIT REAL*8 (A-H,O-Z)                                          LOGG0030 CMJB0232
      DIMENSION H(7)                                                     LOGG0110 CMJB0233
      H(1)=2.26948897400                                                 LOGG0120 CMJB0234
      H(2)=1.51747364900                                                 LOGG0130 CMJB0235
      H(3)=1.01152306800                                                 LOGG0140 CMJB0236
      H(4)=5.256064690D-1                                                LOGG0150 CMJB0237
      H(5)=2.523809524D-1                                                LOGG0160 CMJB0238
      H(6)=3.333333333D-2                                                LOGG0170 CMJB0239
      H(7)=8.333333333D-2                                                LOGG0180 CMJB0240
      E2=1.57079632679D0                                                 LOGG0190 CMJB0241
      E8=3.14159265359D0                                                 LOGG0200 CMJB0242
      B1=0.0D0                                                           LOGG0210 CMJB0243
      B2=0.0D0                                                           LOGG0220 CMJB0244
      J=2                                                                LOGG0230 CMJB0245
      X2=X                                                               LOGG0240 CMJB0246
    4 IF(X)1,2,3                                                         LOGG0250 CMJB0247
    3 B6=DATAN(Y/X)                                                      LOGG0260 CMJB0248
      T=X**2                                                             LOGG0270 CMJB0249
    5 B7=Y**2+T                                                          LOGG0280 CMJB0250
C  REAL PART OF LOG                                                      LOGG0290 CMJB0251
      T1=0.5D0*DLOG(B7)                                                  LOGG0300 CMJB0252
```

```
      IF(X-2.0D0)7,7,6                                                  LOGG0310 CMJB0253
    7 B1=B1+B6                                                          LOGG0320 CMJB0254
      B2=B2+T1                                                          LOGG0330 CMJB0255
      X=X+1.0D0                                                         LOGG0340 CMJB0256
      J=1                                                               LOGG0350 CMJB0257
      GO TO 4                                                           LOGG0360 CMJB0258
    6 T3=-Y*B6+(T1*(X-.5D0)-X+9.189385332D-1)                           LOGG0370 CMJB0259
      T2=B6*(X-.5D0)+Y*T1-Y                                             LOGG0380 CMJB0260
      T4=X                                                              LOGG0390 CMJB0261
      T5=-Y                                                             LOGG0400 CMJB0262
      T1=B7                                                             LOGG0410 CMJB0263
      DO 8 I=1,7                                                        LOGG0420 CMJB0264
      T=H(I)/T1                                                         LOGG0430 CMJB0265
      T4=T*T4+X                                                         LOGG0440 CMJB0266
      T5=-(T*T5+Y)                                                      LOGG0450 CMJB0267
    8 T1=T4**2+T5**2                                                    LOGG0460 CMJB0268
      T3=T4-X+T3                                                        LOGG0470 CMJB0269
      T2=-T5-Y+T2                                                       LOGG0480 CMJB0270
      GO TO (9,10),J                                                    LOGG0490 CMJB0271
    9 T3=T3-B2                                                          LOGG0500 CMJB0272
      T2=T2-B1                                                          LOGG0510 CMJB0273
   10 IF(X2)11,12,12                                                    LOGG0520 CMJB0274
   12 U=T3                                                              LOGG0530 CMJB0275
      V=T2                                                              LOGG0540 CMJB0276
      X=X2                                                              LOGG0550 CMJB0277
      RETURN                                                            LOGG0560 CMJB0278
   11 U=T3-E4                                                           LOGG0570 CMJB0279
      V=T2-E5                                                           LOGG0580 CMJB0280
      X=X2                                                              LOGG0590 CMJB0281
      RETURN                                                            LOGG0600 CMJB0282
C     X IS ZERO                                                         LOGG0610 CMJB0283
    2 T=0.0D0                                                           LOGG0620 CMJB0284
      IF(Y)13,14,15                                                     LOGG0630 CMJB0285
   13 B6=-E2                                                            LOGG0640 CMJB0286
      GO TO 5                                                           LOGG0650 CMJB0287
   15 B6=E2                                                             LOGG0660 CMJB0288
```

```
      GO TO 5
C     X IS NEGATIVE
    1 E4=0.0D0
      E5=0.0D0
      IE6=0
   16 E4=E4+.5D0*(DLOG(X**2+Y**2))
      E5=E5+DATAN(Y/X)
      IE6=IE6+1
      X=X+1.0D0
      IF(X)16,17,17
   17 IF(MOD(IE6,2))18,4,18
   18 E5=E5+E8
      GO TO 4
   14 WRITE(6,19)X2,Y
   19 FORMAT(29H ATTEMPTED TO TAKE LOGGAM OF ,2HX=,F6.0,1X,2HY=,F6.0)
      RETURN
      END
```

```
      SUBROUTINE YSQNK9(Z,N,ERROR,M,NPRINT,ANSW,W,RUM,NO,FUN,NQ7)
      COMPLEX Z(10),Z1,Z2,DZDT,ANSW(4),CURVE,FUN,ERROR,RUM,RUM1,W(4)
      COMPLEX CMPLX
      COMMON /POLY/Z1,Z2
      EXTERNAL CURVE,FUN
      NJUMP=0
      DO 12 I=1,M
12    ANSW(I) = (0.,0.)
      NO = 0
      RUM = (0.,0.)
      IF (NPRINT .NE. 0) WRITE (6,10)
10    FORMAT ('   NPRINT IS NON-ZERO')
      DO 13 J=1,N
      Z1=Z(J)
      IF (J .EQ. N) GO TO 11
      Z2=Z(J+1)
      GO TO 15
11    Z2=Z(1)
15    CALL LSQNK(0.,1.,ERROR,M,NJUMP,NPRINT,ANSW,RUM1,NO1,CURVE,FUN)
      IF (REAL(RUM1) .GT. REAL(RUM)) RUM=CMPLX(REAL(RUM1),AIMAG(RUM))
      IF (AIMAG(RUM1) .GT. AIMAG(RUM))RUM = CMPLX(REAL(RUM),AIMAG(RUM1))
13    NO = NO + NO1
      CALL ROOT7(Z(1),ANSW,W,M,NJUMP)
      NQ7=NJUMP
      RETURN
      END
      COMPLEX FUNCTION CURVE(T,DZDT)
      COMPLEX DZDT,Z1,Z2
      COMMON /POLY/Z1,Z2
      DZDT=Z2-Z1
      CURVE=Z1+T*DZDT
      RETURN
      END
      SUBROUTINE ROOT7(ZA,ANSW,W,M,NJUMP)
      COMPLEX ANSW(4),RLINE(4),Z(4),W(4),ZA,A,B,C,D,E,DISC,FUN
      COMPLEX CSQRT
```

```
      EXTERNAL FUN                                                      LSQ21430 YSK90037
      DO 170 I=1,M                                                      LSQ21440 YSK90038
170   RLINE(I) = -ANSW(I)/(0.,6.283186) + (ZA**I)*NJUMP                 LSQ21450 YSK90039
      IF (M .LT. NJUMP) RETURN                                          LSQ21520 YSK90040
      IF (NJUMP .EQ. 1) GO TO 60                                        LSQ21530 YSK90041
      IF (NJUMP .EQ. 2) GO TO 5                                         LSQ21540 YSK90042
      IF (NJUMP .EQ. 3) GO TO 100                                       LSQ21550 YSK90043
      IF (NJUMP .EQ. 4) GO TO 110                                       LSQ21560 YSK90044
      RETURN                                                            LSQ21570 YSK90045
5     DISC=CSQRT(.25*RLINE(1)*RLINE(1)-.5*(RLINE(1)*RLINE(1)-RLINE(2))) LSQ21580 YSK90046
      Z(1)=0.5*RLINE(1)+DISC                                            LSQ21590 YSK90047
      Z(2)=RLINE(1)-Z(1)                                                LSQ21600 YSK90048
      GO TO 114                                                         LSQ21610 YSK90049
60    Z(1)=RLINE(1)                                                     LSQ21620 YSK90050
      GO TO 114                                                         LSQ21630 YSK90051
100   A = (0.,0.)                                                       LSQ21640 YSK90052
      B = (1.,0.)                                                       LSQ21650 YSK90053
      C = -RLINE(1)                                                     LSQ21660 YSK90054
      D = -.5*(-C*C+RLINE(2))                                           LSQ21670 YSK90055
      E = -.3333333*(-D*C+C*RLINE(2) + RLINE(3))                        LSQ21680 YSK90056
      GO TO 113                                                         LSQ21690 YSK90057
110   A = (1.,0.)                                                       LSQ21700 YSK90058
      B = -RLINE(1)                                                     LSQ21710 YSK90059
      C = -.5*(-B*B + RLINE(2))                                         LSQ21720 YSK90060
      D = -.3333333*(-C*B + B*RLINE(2) + RLINE(3))                      LSQ21730 YSK90061
      E = -.25*(-D*B+C*RLINE(2)+B*RLINE(3)+RLINE(4))                    LSQ21740 YSK90062
113   CALL ROOT4A(E,D,C,B,A,Z)                                          LSQ21750 YSK90063
114   DO 117 I=1,NJUMP                                                  LSQ21760 YSK90064
117   W(I) = FUN(Z(I))                                                  LSQ21770 YSK90065
      DO 120 I=1,NJUMP                                                  LSQ21820 YSK90066
120   ANSW(I) = Z(I)                                                    LSQ21830 YSK90067
      RETURN                                                            LSQ21840 YSK90068
      END                                                               LSQ21850 YSK90069
      SUBROUTINE ROOT4A(E,D,C,B,A,ZR)                                   LSQ21860 YSK90070
      IMPLICIT COMPLEX(A-H,O-Z)                                         LSQ21870 YSK90071
      COMPLEX*16 CDEXP                                                  LSQ21880 YSK90072
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```
      REAL*4 REAL                                                       LSQ21890 YSK90073
      REAL CA1,CA2,TESTR,TESTI,RT12R,CABS,TTT                           LSQ21900 YSK90074
      DIMENSION ZR(4),AQ(2),BQ(2),CQ(2)                                 LSQ21910 YSK90075
      P=-.5*C                                                           LSQ21920 YSK90076
      AE=A*F                                                            LSQ21930 YSK90077
      Q=.25*B*D-AE                                                      LSQ21940 YSK90078
      R=.5*AE*C-.125*(A*D*D+E*B*B)                                      LSQ21950 YSK90079
      PD3=P/3.                                                          LSQ21960 YSK90080
      S=(Q-P*PD3)/3.                                                    LSQ21970 YSK90081
      T=.5*(PD3*(2.*PD3*PD3-Q)+R)                                       LSQ21980 YSK90082
      RT=CSQRT(T*T+S*S*S)                                               LSQ21990 YSK90083
      A1=-T+RT                                                          LSQ22000 YSK90084
      A2=-T-RT                                                          LSQ22010 YSK90085
      CA1=CABS(A1)                                                      LSQ22020 YSK90086
      CA2=CABS(A2)                                                      LSQ22030 YSK90087
      U=-PD3                                                            LSQ22040 YSK90088
      IF(CA1.GE.CA2) GO TO 10                                           LSQ22050 YSK90089
      A1=A2                                                             LSQ22060 YSK90090
      CA1=CA2                                                           LSQ22070 YSK90091
10    IF(CA1.EQ.0.) GO TO 20                                            LSQ22080 YSK90092
      A3=CDEXP(.3333333333333333D0*CLOG(A1))                            LSQ22090 YSK90093
      U=U+A3-S/A3                                                       LSQ22100 YSK90094
20    BSQ=B*B                                                           LSQ22110 YSK90095
      UMC=U+P                                                           LSQ22120 YSK90096
      F=8.*A*UMC                                                        LSQ22130 YSK90097
      IF(.1*CABS(BSQ).GT.CABS(F)) GO TO 40                              LSQ22140 YSK90098
      RT1=CSQRT(BSQ+F)                                                  LSQ22150 YSK90099
      RT2=CSQRT(U*U-AE)                                                 LSQ22160 YSK90100
      TT=RT1*RT2                                                        LSQ22170 YSK90101
      IF(CABS(TT).LT.1.E-10) GO TO 35                                   LSQ22180 YSK90102
      TTT=REAL((A*D-B*U)/TT)                                            LSQ22190 YSK90103
      IF(TTT.GT.0.) GO TO 35                                            LSQ22200 YSK90104
      RT2=-RT2                                                          LSQ22210 YSK90105
35    AQ(1)=A                                                           LSQ22220 YSK90106
      BQ(1) = .5*(B-RT1)                                                LSQ22230 YSK90107
      CQ(1)=U+RT2                                                       LSQ22240 YSK90108
```

```
      GO TO 50
40    Z=.5*F/BSQ
      FAC=1.-(.5-(.5-(.625-(.875-(1.3125-2.0625*Z)*Z)*Z)*Z)*Z)*Z
      RT1=B*(1.+Z*FAC)
      RT2=(A*D-U*B)/RT1
      AQ(1)=1.
      BQ(1)=-2.*UMC*FAC/B
      CQ(1)=(D-2.*U*BQ(1))/RT1
50    AQ(2)=A
      BQ(2)=.5*(B+RT1)
      CQ(2)=U-RT2
      L=0
      DO 80 N=1,2
      BQSQ=BQ(N)*BQ(N)
      AQCQ=2.*AQ(N)*CQ(N)
      IF(.05*CABS(BQSQ).GE.CABS(AQCQ)) GO TO 60
      RTQ=CSQRT(BQSQ-2.*AQCQ)
      L=L+1
      AQ2=2.*AQ(N)
      ZR(L)=(-BQ(N)+RTQ)/AQ2
      L=L+1
      ZR(L)=(-BQ(N)-RTQ)/AQ2
      GO TO 80
60    L=L+1
      IF(1.E-20*CABS(CQ(N)).GE.CABS(BQ(N))) GO TO 65
      X=AQCQ/BQSQ
      ZR(L)=-CQ(N)*(1.+(.5+(.5+(.625+(.875+(1.3125+20625*X)*X)*X)*X)
     1  *X)*X)/BQ(N)
      L=L+1
      IF(1.E-20*CABS(BQ(N)).GE.CABS(AQ(N))) GO TO 70
      ZR(L)=-BQ(N)/AQ(N)-ZR(L-1)
      GO TO 80
65    ZR(L)=1.E30
      L=L+1
70    ZR(L)=1.E30
80    CONTINUE
```

```
      RETURN                                                            LSQ22610 YSK90145
      END                                                               LSQ22620 YSK90146
      SUBROUTINE LSQNK(A,BIG,ERROR,M,NJUMP,NPRINT,ANSW,RUM,NO,CURVE,FUN)LSQ22630 YSK90147
      DIMENSION X3ST(30),X5ST(30),PREDIF(30),PREDFI(30),X(5)           LSQ22940 YSK90148
      COMPLEX FX(5),ANSW(4),FIFTH,LF(5),LOGF(5),F(5,4)                  LSQ22950 YSK90149
      COMPLEX FUN,CURVE,DZDT(5),RUM,ERROR,W(5)                          LSQ22960 YSK90150
      COMPLEX LOG3ST(30),LOG5ST(30),DZ3ST(30),DZ5ST(30)                 LSQ22970 YSK90151
      COMPLEX DIFF(4),W3ST(30),W5ST(30)                                 LSQ22980 YSK90152
      COMPLEX EST1(4),EST2(4),EST(4)                                    LSQ22990 YSK90153
      COMPLEX*16 SUM(4),SIM(4)                                          LSQ23000 YSK90154
      DIMENSION NJMP(5)                                                 LSQ23010 YSK90155
      COMPLEX CLOG,CMPLX                                                LSQ23020 YSK90156
      LOGICAL PJUMP,SW1,SW2                                             LSQ23030 YSK90157
      COMMON /TOL/CEPSF,CEPSFI,EPMACH,CEPS,CEPSI,LEVTAG,                LSQ23040 YSK90158
     1LEVTGI,FACERR,FACERI,QCEPS,QCEPSI                                 LSQ23050 YSK90159
      COMMON /RND/LEV,X,XZERO                                           LSQ23060 YSK90160
      COMMON /LFC/LOGF,LF,DZDT,NJMP,PJUMP                               LSQ23070 YSK90161
      COMPLEX Z,Y                                                       LSQ23080 YSK90162
      AMAG(Z)=ABS(REAL(Z))+ABS(AIMAG(Z))                                LSQ23090 YSK90163
      EPMACH = 0.0000000000075                                          LSQ23100 YSK90164
C     ****  STAGE ONE ****                                              LSQ23110 YSK90165
C  INITIALIZE ALL QUANTITIES REQUIRED FOR CENTRAL CALCULATION (STAGE3). LSQ23120 YSK90166
      PJUMP = .FALSE.                                                   LSQ23130 YSK90167
      DO 10 I=1,M                                                       LSQ23140 YSK90168
      SUM(I) = (0.D0,0.D0)                                              LSQ23150 YSK90169
   10 SIM(I) = (0.D0,0.D0)                                              LSQ23160 YSK90170
      CEPSF=180.0*REAL(ERROR)/(BIG-A)                                   LSQ23170 YSK90171
      CEPSFI=180.0*AIMAG(ERROR)/(BIG-A)                                 LSQ23180 YSK90172
      CEPS = CEPSF                                                      LSQ23190 YSK90173
      CEPSI=CEPSFI                                                      LSQ23200 YSK90174
      ADIFF = 0.0                                                       LSQ23210 YSK90175
      ADIFFI=0.                                                         LSQ23220 YSK90176
      LEVTAG = -1                                                       LSQ23230 YSK90177
      LEVTGI=-1                                                         LSQ23240 YSK90178
      FACERR = 1.0                                                      LSQ23250 YSK90179
      FACERI=1.0                                                        LSQ23260 YSK90180
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```
      XZERO = A                                                         LSQ23270 YSK90181
      EFACT = 0.0                                                       LSQ23280 YSK90182
      EFACTI=0.                                                         LSQ23290 YSK90183
      NIM = 1                                                           LSQ23300 YSK90184
      LEV = 0                                                           LSQ23310 YSK90185
C  FIRST INTERVAL                                                       LSQ23320 YSK90186
      X(1) = A                                                          LSQ23330 YSK90187
      X(5) = BIG                                                        LSQ23340 YSK90188
      X(3) = 0.5*(A+BIG)                                                LSQ23350 YSK90189
      X(2) = 0.5*(X(1)+X(3))                                            LSQ23360 YSK90190
      X(4) = 0.5*(X(3)+X(5))                                            LSQ23370 YSK90191
      NJMP(1) = NJUMP                                                   LSQ23380 YSK90192
      DO 20 I=1,5                                                       LSQ23390 YSK90193
      W(I) = CURVE(X(I),DZDT(I))                                        LSQ23400 YSK90194
      FX(I) = FUN(W(I))                                                 LSQ23410 YSK90195
      Z = W(I)                                                          LSQ23420 YSK90196
      IF (AMAG(FX(I)).LT. 1.E-20) GO TO 1200                            LSQ23430 YSK90197
C  IF FUNCTION GOES TOO CLOSE TO THE ORIGIN, RETURN                     LSQ23440 YSK90198
      LOGF(I) = CLOG(FX(I))                                             LSQ23450 YSK90199
      CALL LFX(I)                                                       LSQ23460 YSK90200
      IF (NPRINT .NE. 2) GO TO 20                                       LSQ23470 YSK90201
      WRITE (6,302) W(I),FX(I)                                          LSQ23480 YSK90202
 20   F(I,1) = LF(I)                                                    LSQ23490 YSK90203
      NO = 5                                                            LSQ23500 YSK90204
C  COMPUTE VALUES OF INTEGRANDS                                         LSQ23510 YSK90205
      IF (M .EQ. 1) GO TO 101                                           LSQ23520 YSK90206
      DO 100 K=2,M                                                      LSQ23530 YSK90207
      DO 100 I=1,5                                                      LSQ23540 YSK90208
      Y = K*(W(I)**(K-1))                                               LSQ23550 YSK90209
 100  F(I,K) = Y*F(I,1)                                                 LSQ23560 YSK90210
C     ****  STAGE TWO  ****                                             LSQ23570 YSK90211
C  SET STARTING VALUES FOR TOLERANCES IN THE CASE THAT CEPSF OR CEPSFI=0LSQ23580 YSK90212
 101  CALL SETTOL(LF)                                                   LSQ23590 YSK90213
      GO TO 305                                                         LSQ23600 YSK90214
C  INITIALIZING COMPLETE                                                LSQ23610 YSK90215
C     ****  STAGE THREE  ****                                           LSQ23620 YSK90216
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C  CENTRAL CALCULATION.
 300  PJUMP = .FALSE.
 301  X(2) = 0.5*(X(1) + X(3))
      X(4) = 0.5*(X(3) + X(5))
      W(2) = CURVE(X(2),DZDT(2))
      FX(2) = FUN(W(2))
      Z = W(2)
      IF (AMAG(FX(2)) .LT. 1.E-20) GO TO 1200
      LOGF(2) = CLOG(FX(2))
      W(4) = CURVE(X(4),DZDT(4))
      FX(4) = FUN(W(4))
      Z = W(4)
      IF (AMAG(FX(4)) .LT.1.E-20) GO TO 1200
      LOGF(4) = CLOG(FX(4))
      NO = NO + 2
      DO 303 I=1,5
      CALL LFX(I)
      IF (NPRINT .NE. 2) GO TO 303
      WRITE (6,302) W(I),FX(I)
 302  FORMAT(' Z = ',2E12.4,' F(Z) = ',2E12.4)
 303  F(I,1) = LF(I)
      IF (M .EQ. 1) GO TO 305
      DO 304 K=2,M
      DO 304 I=1,5
      Y = K*(W(I)**(K-1))
 304  F(I,K) = Y*F(I,1)
C  COMPUTE SIMPSON'S RULE FOR WIDER MESH
 305  DO 306 I=1,M
 306  EST(I) = F(1,I) + F(5,I) + (4.0,0.0)*F(3,I)
C  COMPUTE SIMPSON'S RULE FOR FINER MESH
      DO 307 I=1,M
      EST1(I) = F(1,I) + (4.,0.)*F(2,I) + F(3,I)
 307  EST2(I) = F(3,I) + (4.,0.)*F(4,I) + F(5,I)
      ADIFF1 = ADIFF
      ADIFI1=ADIFFI
      DO 308 I=1,M
```

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 308  DIFF(I) = 2.*EST(I) - EST1(I) - EST2(I)                                LSQ23990 YSK90253
      IF (LEV .GE. 15) GO TO 800                                              LSQ24000 YSK90254
 309  ADIFF = AMAX1(ABS(REAL(DIFF(1))),ABS(REAL(DIFF(M))))                    LSQ24010 YSK90255
      ADIFFI = AMAX1(ABS(AIMAG(DIFF(1))),ABS(AIMAG(DIFF(M))))                 LSQ24020 YSK90256
      IF (NPRINT .EQ. 0) GO TO 311                                            LSQ24030 YSK90257
      WRITE (6,310) PJUMP,W(1),ADIFF,ADIFFI,NJMP(5),LEV                       LSQ24040 YSK90258
 310  FORMAT (' PJUMP = ',L1,3X,' Z(1) =',2E12.4,5X,'ADIFF =',E12.4,2X,'LSQ24050 YSK90259
     1 ADIFFI =',E12.4,5X,'NJUMP =',I3,3X,'LEVEL =',I2)                       LSQ24060 YSK90260
 311  IF (PJUMP) GO TO 500                                                    LSQ24070 YSK90261
      CRIT = ADIFF - CEPS                                                     LSQ24080 YSK90262
      CRITI=ADIFFI-CEPSI                                                      LSQ24090 YSK90263
C     ****  STAGE FOUR  ****                                                  LSQ24100 YSK90264
 400  SW1 = .FALSE.                                                           LSQ24110 YSK90265
      IF (CRIT .GT. 0.) GO TO 402                                             LSQ24120 YSK90266
 401  IF (LEV .GT. 0) GO TO 404                                               LSQ24130 YSK90267
 402  CALL ROUND(ADIFF,ADIFF1,EFACT,FACERR,CEPS,LEVTAG,QCEPS,&403,&404) LSQ24140 YSK90268
 403  SW1 = .TRUE.                                                            LSQ24150 YSK90269
 404  IF (CRITI .GT. 0.) GO TO 407                                            LSQ24160 YSK90270
 405  IF (LEV .LE. 0) GO TO 407                                               LSQ24170 YSK90271
 406  IF (SW1) GO TO 500                                                      LSQ24180 YSK90272
      GO TO 700                                                               LSQ24190 YSK90273
 407  CALL ROUND(ADIFFI,ADIFI1,EFACTI,FACERI,CEPSI,LEVTGI,QCEPSI,&500,        LSQ24200 YSK90274
     1&408)                                                                   LSQ24210 YSK90275
 408  IF (.NOT. SW1) GO TO 800                                                LSQ24220 YSK90276
500   CONTINUE                                                                LSQ24290 YSK90277
      NIM = 2*NIM                                                             LSQ24300 YSK90278
      LEV = LEV + 1                                                           LSQ24310 YSK90279
      X3ST(LEV) = X(4)                                                        LSQ24320 YSK90280
      X5ST(LEV) = X(5)                                                        LSQ24330 YSK90281
      W3ST(LEV) = W(4)                                                        LSQ24340 YSK90282
      W5ST(LEV) = W(5)                                                        LSQ24350 YSK90283
      LOG3ST(LEV) = LOGF(4)                                                   LSQ24360 YSK90284
      LOG5ST(LEV) = LOGF(5)                                                   LSQ24370 YSK90285
      DZ3ST(LEV) = DZDT(4)                                                    LSQ24380 YSK90286
      DZ5ST(LEV) = DZDT(5)                                                    LSQ24390 YSK90287
      PREDIF(LEV) = ADIFF                                                     LSQ24400 YSK90288
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      PREDFI(LEV)=ADIFFI
C     ****  STAGE SIX  ****
C   SET UP QUANTITIES FOR CENTRAL CALCULATION.
C   READY TO GO AHEAD AT LEVEL LOWER WITH LEFT HAND ELEMENTS
C   X1 AND FX1 ARE THE SAME AS BEFORE
      X(5) = X(3)
      X(3) = X(2)
      W(5) = W(3)
      W(3) = W(2)
      LOGF(5) = LOGF(3)
      LOGF(3) = LOGF(2)
      DZDT(5) = DZDT(3)
      DZDT(3) = DZDT(2)
      GO TO 300
C     ****  STAGE SEVEN  *****
700   CONTINUE
 705  SW2 = .FALSE.
      CALL CHECK(LEVTAG,CEPS,CEPST,CRIT,ADIFF,ADIFF1,QCEPS,EFACT,FACERR,
     1CEPSF,&715,&710)
 710  SW2 = .TRUE.
 715  CALL CHECK(LEVTGI,CEPSI,CEPSTI,CRITI,ADIFFI,ADIFI1,QCEPSI,EFACTI,
     1FACERI,CEPSFI,&725,&500)
 725  IF (SW2) GO TO 500
C     ****  STAGE EIGHT  ****
C   ACTUAL CONVERGENCE IN PREVIOUS INTERVAL.  INCREMENTS ADDED INTO
C   RUNNING SUMS
C   ADD INTO SUM AND SIM
 800  DO 801 I=1,M
 801  SUM(I) = SUM(I) + (EST1(I) + EST2(I))*(X(5) - X(1))
      IF (LEVTAG .GE. 0) GO TO 810
 804  IF (LEVTGI .GE. 0) GO TO 810
C   WE ADD INTO SIM ONLY IF WE ARE CLEAR OF ROUND OFF LEVEL.
 805  DO 806 I=1,M
 806  SIM(I) = SIM(I) + DIFF(I)*(X(5) - X(1))
810   CONTINUE
C     ****  STAGE NINE  ****
```

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C  SORT OUT WHICH LEVEL TO GO TO.  THIS INVOLVES NIM NUMBERING SYSTEM   LSQ24770 YSK90325
C  DESCRIBED BEFORE STAGE ONE.                                          LSQ24780 YSK90326
905   NUM = NIM/2                                                       LSQ24790 YSK90327
      NOM = NIM - 2*NUM                                                 LSQ24800 YSK90328
      IF (NOM .EQ. 0) GO TO 915                                         LSQ24810 YSK90329
910   NIM = NUM                                                         LSQ24820 YSK90330
      LEV = LEV - 1                                                     LSQ24830 YSK90331
      GO TO 905                                                         LSQ24840 YSK90332
915   NIM = NIM + 1                                                     LSQ24850 YSK90333
C  NEW LEVEL IS SET.  IF LEV = 0 WE HAVE FINISHED                       LSQ24860 YSK90334
      IF (LEV .LE. 0) GO TO 1100                                        LSQ24870 YSK90335
C    ****  STAGE TEN  ****                                              LSQ24880 YSK90336
C  SET UP QUANTITIES FOR CENTRAL CALCULATION.                           LSQ24890 YSK90337
1000  CONTINUE                                                          LSQ24900 YSK90338
      X(1) = X(5)                                                       LSQ24910 YSK90339
      X(3) = X3ST(LEV)                                                  LSQ24920 YSK90340
      X(5) = X5ST(LEV)                                                  LSQ24930 YSK90341
      W(1) = W(5)                                                       LSQ24940 YSK90342
      W(3) = W3ST(LEV)                                                  LSQ24950 YSK90343
      W(5) = W5ST(LEV)                                                  LSQ24960 YSK90344
      LOGF(1) = LOGF(5)                                                 LSQ24970 YSK90345
      LOGF(3) = LOG3ST(LEV)                                             LSQ24980 YSK90346
      LOGF(5) = LOG5ST(LEV)                                             LSQ24990 YSK90347
      DZDT(1) = DZDT(5)                                                 LSQ25000 YSK90348
      DZDT(3) = DZ3ST(LEV)                                              LSQ25010 YSK90349
      DZDT(5) = DZ5ST(LEV)                                              LSQ25020 YSK90350
      NJMP(1) = NJMP(5)                                                 LSQ25030 YSK90351
      ADIFF = PREDIF(LEV)                                               LSQ25040 YSK90352
      ADIFFI=PREDFI(LEV)                                                LSQ25050 YSK90353
      GO TO 300                                                         LSQ25060 YSK90354
C    ****  STAGE ELEVEN  ****                                           LSQ25070 YSK90355
C  CALCULATION NOW COMPLETE.  FINALIZE.                                 LSQ25080 YSK90356
1100  CONTINUE                                                          LSQ25090 YSK90357
      NJUMP = NJMP(5)                                                   LSQ25100 YSK90358
      EFACT = EFACT + CEPS *(BIG-XZERO)*FACERR                          LSQ25110 YSK90359
      EFACTI=EFACTI+CEPSI*(BIG-XZERO)*FACERI                            LSQ25120 YSK90360
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```
      RUM=CMPLX(EFACT/180.0,EFACTI/180.0)
      DO 1101 I=1,M
      FIFTH = -SIM(I)/(180.,0.)
 1101 ANSW(I) = ANSW(I) + SUM(I)/(12.,0.) + FIFTH
      RETURN
C  ERROR - VALUE OF FUN(W) BECOMES TOO SMALL
C  CONTOUR OF FUNCTION CURVE GOES TOO CLOSE TO A ZERO
 1200 WRITE (6,1201) Z
 1201 FORMAT (' CONTOUR OF FUNCTION CURVE FUN GOES TOO CLOSE TO A ZERO A
     1T Z = ',2E15.8)
      RETURN
C  END OF LSQNK
      END
      SUBROUTINE SETTOL(LF)
C  SET STARTING VALUES FOR THE TOLERANCES WHEN CEPSF OR CEPSFI = 0.
      COMMON /TOL/CEPSF,CEPSFI,EPMACH,CEPS,CEPSI,LEVTAG,
     1LEVTGI,FACERR,FACERI,QCEPS,QCEPSI
      COMPLEX LF(5)
C  SET STARTING VALUES FOR THE TOLERANCES IN THE CASE THAT CEPSF=0.
      IF (CEPSF .NE. 0) GO TO 240
 205  LEVTAG=0
      FACERR=15.0
C  SET REAL TOLERANCE
      IF (REAL(LF(1)) .EQ. 0.) GO TO 215
      CEPS = EPMACH*ABS(REAL(LF(1)))
      GO TO 240
 215  LFVTAG=3
      IF (REAL(LF(3)) .EQ. 0.) GO TO 225
      CEPS = EPMACH*ABS(REAL(LF(3)))
      GO TO 240
 225  IF (REAL(LF(5)) .EQ. 0.) GO TO 235
      CEPS = EPMACH*ABS(REAL(LF(5)))
      GO TO 240
 235  CEPS=EPMACH
C  SET IMAGINARY TOLERANCE
 240  IF (CEPSFI .NE. 0) GO TO 295
```

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 275  LEVTGI=0                                                                LSQ25490 YSK90397
      FACERI=15.                                                              LSQ25500 YSK90398
      IF (AIMAG(LF(1)) .EQ. 0.) GO TO 250                                     LSQ25510 YSK90399
      CEPSI = EPMACH * ABS(AIMAG(LF(1)))                                      LSQ25520 YSK90400
      GO TO 295                                                               LSQ25530 YSK90401
 250  LEVTGI=3                                                                LSQ25540 YSK90402
      IF (AIMAG(LF(3)) .EQ. 0.) GO TO 260                                     LSQ25550 YSK90403
      CEPSI = EPMACH*ABS(AIMAG(LF(3)))                                        LSQ25560 YSK90404
      GO TO 295                                                               LSQ25570 YSK90405
 260  IF (AIMAG(LF(5)) .EQ. 0.) GO TO 270                                     LSQ25580 YSK90406
 265  CEPSI = EPMACH*ABS(AIMAG(LF(5)))                                        LSQ25590 YSK90407
      GO TO 295                                                               LSQ25600 YSK90408
 270  CEPSI=EPMACH                                                            LSQ25610 YSK90409
 295  QCEPS=0.25*CEPS                                                         LSQ25620 YSK90410
      QCEPSI=0.25*CEPSI                                                       LSQ25630 YSK90411
      RETURN                                                                  LSQ25640 YSK90412
      END                                                                     LSQ25650 YSK90413
       SUBROUTINE ROUND(ADIFF,ADIFF1,EFACT,FACERR,CEPS,LEVTAG,QCEPS,*,*)LSQ25660 YSK90414
C   NO NATURAL CONVERGENCE.  A COMPLEX SEQUENCE OF INSTRUCTIONS               LSQ25670 YSK90415
C   FOLLOWS WHICH ASSIGNS CONVERGENCE AND / OR ALTERS TOLERANCE               LSQ25680 YSK90416
C   LEVEL  IN UPWARD DIRECTION IF THERE ARE INDICATIONS OF ROUND OFF          LSQ25690 YSK90417
C   ERROR.                                                                    LSQ25700 YSK90418
      DIMENSION X(5)                                                          LSQ25710 YSK90419
      COMMON /RND/LEV,X,XZERO                                                 LSQ25720 YSK90420
      IF (ADIFF1 .GT. ADIFF) GO TO 500                                        LSQ25730 YSK90421
C   IN A NORMAL RUN WITH NO ROUND OFF ERROR PROBLEM, ADIFF1 IS GREATER THLSQ25740 YSK90422
C   ADIFF AND THE REST OF STAGE FOUR IS OMITTED.                              LSQ25750 YSK90423
 410  IF (LEV .LT. 5) GO TO 500                                               LSQ25760 YSK90424
 415  EFACT = EFACT + CEPS * (X(1) - XZERO)*FACERR                            LSQ25770 YSK90425
      XZERO = X(1)                                                            LSQ25780 YSK90426
      FACERR = 15.0                                                           LSQ25790 YSK90427
C   THE REST OF STAGE FOUR DEALS WITH UPWARD ADJUSTMENT OF TOLERANCE (CEPLSQ25800 YSK90428
C   BECAUSE OF SUSPECTED ROUND OFF ERROR TROUBLE.                             LSQ25810 YSK90429
      IF (ADIFF .GT. 2.0*CEPS) GO TO 425                                      LSQ25820 YSK90430
C   SMALL JUMP IN CEPS. ASSIGN CONVERGENCE                                    LSQ25830 YSK90431
420   CEPS =   ADIFF                                                          LSQ25840 YSK90432
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      LEVTAG = 0                                                        LSQ25850 YSK90433
      QCEPS=0.25*CEPS                                                   LSQ25860 YSK90434
      RETURN 2                                                          LSQ25870 YSK90435
 425  IF (ADIFF1 .NE. ADIFF) GO TO 435                                  LSQ25880 YSK90436
C  LARGE JUMP IN CEPS                                                   LSQ25890 YSK90437
430   CEPS = ADIFF                                                      LSQ25900 YSK90438
      GO TO 445                                                         LSQ25910 YSK90439
C  FACTOR TWO JUMP IN CEPS                                              LSQ25920 YSK90440
435   CEPS = 2.0*CEPS                                                   LSQ25930 YSK90441
      IF (LEVTAG .GE. 3) GO TO 445                                      LSQ25940 YSK90442
440   LEVTAG = 2                                                        LSQ25950 YSK90443
445   QCEPS = 0.25*CEPS                                                 LSQ25960 YSK90444
 500  RETURN 1                                                          LSQ25970 YSK90445
      END                                                               LSQ25980 YSK90446
      SUBROUTINE CHECK(LEVTAG,CEPS,CEPST,CRIT,ADIFF,ADIFF1,QCEPS,EFACT, LSQ25990 YSK90447
     1FACERR,CEPSF,*,*)                                                 LSQ26000 YSK90448
C  NATURAL CONVERGENCE IN PREVIOUS INTERVAL. THE FOLLOWING COMPLEX SEQUELSQ26010 YSK90449
C  CHECKS PRIMARILY THAT TOLERANCE LEVEL IS NOT TOO HIGH. UNDER CERTAIN LSQ26020 YSK90450
C  CIRCUMSTANCES NON CONVERGENCE IS ASSIGNED AND / OR TOLERANCE LEVEL   LSQ26030 YSK90451
C  IS RE-SET.                                                           LSQ26040 YSK90452
      DIMENSION X(5)                                                    LSQ26050 YSK90453
      COMMON /RND/LEV,X,XZERO                                           LSQ26060 YSK90454
C  LEVTAG = -1 CEPS = CEPSF, ITS ORIGINAL VALUE.                        LSQ26070 YSK90455
C  LEVTAG =  0 CEPS IS GREATER THAN CEPSF. REGULAR SITUATION.           LSQ26080 YSK90456
C  LEVTAG = 2  CEPS IF GREATER THAN CEPSF. CEPS PREVIOUSLY ASKED FOR A BLSQ26090 YSK90457
C                   JUMP, BUT DID NOT GET ONE.                          LSQ26100 YSK90458
C  LEVTAG = 3  CEPS IS GREATER THAN CEPSF. CEPS PREVIOUSLY HAD A BIG JUMLSQ26110 YSK90459
 705  IF (LEVTAG .LT. 0) GO TO 800                                      LSQ26120 YSK90460
C  IN A NORMAL RUN WITH NO ROUND OFF ERROR PROBLEM, LEVTAG = -1 AND THE LSQ26130 YSK90461
C  REAT OF STAGE SEVEN IS OMITTED.                                      LSQ26140 YSK90462
710   CEPST = 15.0*CEPS                                                 LSQ26150 YSK90463
C  CEPST HERE IS FACERR*CURRENT VALUE OF CEPS                           LSQ26160 YSK90464
      IF (CRIT .GE. 0) GO TO 800                                        LSQ26170 YSK90465
715   IF(LEVTAG-2) 720, 740, 750                                        LSQ26180 YSK90466
C  LEVTAG = 0                                                           LSQ26190 YSK90467
 720  IF (ADIFF .LE. 0.) GO TO 800                                      LSQ26200 YSK90468
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 725  IF (ADIFF .GE. QCEPS) GO TO 800                                 LSQ26210 YSK90469
 730  IF (ADIFF .LE. CEPSF) GO TO 770                                 LSQ26220 YSK90470
735   LEVTAG = 0                                                      LSQ26230 YSK90471
      CEPS = ADIFF                                                    LSQ26240 YSK90472
      EFACT = EFACT + CEPST * (X(1) - XZERO)                          LSQ26250 YSK90473
      XZERO = X(1)                                                    LSQ26260 YSK90474
      QCEPS = 0.25*CEPS                                               LSQ26270 YSK90475
      RETURN 2                                                        LSQ26280 YSK90476
C  LEVTAG = 2                                                         LSQ26290 YSK90477
740   LEVTAG = 0                                                      LSQ26300 YSK90478
      IF (ADIFF .GT. 0.) GO TO 725                                    LSQ26310 YSK90479
      GO TO 765                                                       LSQ26320 YSK90480
C  LEVTAG = 3                                                         LSQ26330 YSK90481
750   LEVTAG = 0                                                      LSQ26340 YSK90482
      IF (ADIFF .GT. 0.) GO TO 730                                    LSQ26350 YSK90483
      GO TO 775                                                       LSQ26360 YSK90484
765   CEPS = ADIFF1                                                   LSQ26370 YSK90485
      GO TO 775                                                       LSQ26380 YSK90486
770   LEVTAG = -1                                                     LSQ26390 YSK90487
      FACERR = 1.0                                                    LSQ26400 YSK90488
      CEPS = CEPSF                                                    LSQ26410 YSK90489
 775  EFACT = EFACT + CEPST*(X(1) - XZERO)                            LSQ26420 YSK90490
      XZERO = X(1)                                                    LSQ26430 YSK90491
780   CONTINUE                                                        LSQ26440 YSK90492
      QCEPS = 0.25*CEPS                                               LSQ26450 YSK90493
 800  RETURN 1                                                        LSQ26460 YSK90494
      END                                                             LSQ26470 YSK90495
      SUBROUTINE LFX(N)                                               LSQ26480 YSK90496
      COMPLEX CMPLX                                                   LSQ26490 YSK90497
      COMMON /LFC/LOGF,LF,DZDT,NJMP,PJUMP                             LSQ26500 YSK90498
      COMPLEX LOGF(5),LF(5),DZDT(5),Z                                 LSQ26510 YSK90499
      LOGICAL PJUMP                                                   LSQ26520 YSK90500
      DIMENSION NJMP(5)                                               LSQ26530 YSK90501
      DATA PI/3.141593/                                               LSQ26540 YSK90502
      DATA PIO4/.7878982/                                             LSQ26550 YSK90503
      AMAG(Z) = ABS(REAL(Z)) + ABS(AIMAG(Z))                          LSQ26560 YSK90504
```

```
      IF (N .EQ. 1) GO TO 4                                              LSQ26570 YSK90505
      DARG = AIMAG(LOGF(N)) - AIMAG(LOGF(N-1))                           LSQ26580 YSK90506
      JUMP = 0                                                           LSQ26590 YSK90507
      IF (DARG .LE. -PI) JUMP = 1                                        LSQ26600 YSK90508
      IF (DARG .GT. PI) JUMP = -1                                        LSQ26610 YSK90509
      NJMP(N) = NJMP(N-1) + JUMP                                         LSQ26620 YSK90510
    4 LF(N) = CMPLX(REAL(LOGF(N)),AIMAG(LOGF(N)) + 6.283186 * NJMP(N))* LSQ26630 YSK90511
     1DZDT(N)                                                            LSQ26640 YSK90512
      IF (N .EQ. 1) RETURN                                               LSQ26650 YSK90513
      IF (ABS(AIMAG(LF(N)) - AIMAG(LF(N-1))) .GT. PIO4 * AMAG(DZDT(N))) LSQ26660 YSK90514
     1PJUMP = .TRUE.                                                     LSQ26670 YSK90515
      RETURN                                                             LSQ26680 YSK90516
      END                                                                LSQ26690 YSK90517
```

```
C    VALUES OF K*L.                                                                               KLGN0001
   0.08839   0.09391   0.09944   0.10496   0.11049   0.11601   0.12153   0.12706 KLGN0002
   0.13258   0.13811   0.14363   0.14916   0.15468   0.16020   0.16573   0.17125 KLGN0003
   0.17678   0.18783   0.19887   0.20992   0.22097   0.23202   0.24307   0.25412 KLGN0004
   0.26516   0.27621   0.28726   0.29831   0.30936   0.32041   0.33146   0.34250 KLGN0005
   0.35355   0.37565   0.39775   0.41984   0.44194   0.46404   0.48614   0.50823 KLGN0006
   0.53033   0.55243   0.57452   0.59662   0.61872   0.64082   0.66291   0.68501 KLGN0007
   0.70711   0.75130   0.79549   0.83969   0.88388   0.92808   0.97227   1.01647 KLGN0008
   1.06066   1.10485   1.14905   1.19324   1.23744   1.28163   1.32582   1.37002 KLGN0009
   1.41421   1.50260   1.59099   1.67938   1.76777   1.85615   1.94454   2.03293 KLGN0010
   2.12132   2.20971   2.29810   2.38648   2.47487   2.56326   2.65165   2.74004 KLGN0011
   2.82843   3.00520   3.18198   3.35876   3.53553   3.71231   3.88909   4.06586 KLGN0012
   4.24264   4.41942   4.59619   4.77297   4.94975   5.12652   5.30330   5.48008 KLGN0013
   5.65685   6.01041   6.36396   6.71751   7.07107   7.42462   7.77817   8.13173 KLGN0014
   8.48528   8.83883   9.19239   9.54594   9.89949  10.25305  10.60660  10.96015 KLGN0015
  11.31371                                                                        KLGN0016
```

```
C    ARRAY ELEMENTS OF ZIN(64,4)                                                         ZINA0001
 1.3      -.35     .9       -.35     .9       -.9      1.3      -.9       ZINA0002
 1.5      -.5      1.1      -.5      1.1      -1.      1.5      -1.       ZINA0003
 1.8      -.6      1.4      -.6      1.4      -1.      1.8      -1.       ZINA0004
 2.2      -.6      1.7      -.6      1.7      -1.      2.2      -1.       ZINA0005
 2.4      -.45     2.1      -.45     2.1      -.85     2.4      -.85      ZINA0006
 2.408    -.1      2.2      -.1      2.2      -.5      2.408    -.5       ZINA0007
 1.3      0.       1.       0.       1.       -.25     1.3      -.25      ZINA0008
 1.4      -.15     1.1      -.15     1.1      -.45     1.4      -.45      ZINA0009
 1.6      -.4      1.3      -.4      1.3      -.65     1.6      -.65      ZINA0010
 1.9      -.5      1.5      -.5      1.5      -.75     1.9      -.75      ZINA0011
 2.2      -.55     1.8      -.55     1.8      -.75     2.2      -.75      ZINA0012
 2.3      -.4      2.1      -.4      2.1      -.65     2.3      -.65      ZINA0013
 1.6      0.       1.2      0.       1.2      -.4      1.6      -.4       ZINA0014
 1.8      -.25     1.4      -.25     1.4      -.6      1.8      -.6       ZINA0015
 2.1      -.4      1.7      -.4      1.7      -.65     2.1      -.65      ZINA0016
 2.4      -.35     2.       -.35     2.       -.6      2.4      -.6       ZINA0017
 2.408    -.0001   2.2      -.0001   2.2      -.4      2.408    -.4       ZINA0018
 1.9      0.       1.5      0.       1.5      -.3      1.9      -.3       ZINA0019
 2.       -.1      1.6      -.1      1.6      -.5      2.       -.5       ZINA0020
 2.3      -.2      1.9      -.2      1.9      -.5      2.3      -.5       ZINA0021
 2.41     -.0001   2.2      -.0001   2.2      -.35     2.41     -.35      ZINA0022
 2.2      0.       1.8      0.       1.8      -.25     2.2      -.25      ZINA0023
 2.4      -.1      2.       -.1      2.       -.3      2.4      -.3       ZINA0024
 2.41     -.0001   2.1      -.0001   2.1      -.15     2.41     -.15      ZINA0025
 1.       -.8      .7       -.8      .7       -1.05    1.       -1.05     ZINA0026
 1.2      -.8      .8       -.8      .8       -1.3     1.2      -1.3      ZINA0027
 1.5      -.85     1.1      -.85     1.1      -1.4     1.5      -1.4      ZINA0028
 1.7      -.95     1.3      -.95     1.3      -1.5     1.7      -1.5      ZINA0029
 2.       -.9      1.6      -.9      1.6      -1.4     2.       -1.4      ZINA0030
 2.1      -1.2     1.6      -1.2     1.6      -1.6     2.1      -1.6      ZINA0031
 2.3      -.8      1.8      -.8      1.8      -1.3     2.3      -1.3      ZINA0032
 2.5      -1.      2.       -1.      2.       -1.5     2.5      -1.5      ZINA0033
 2.8      -1.1     2.1      -1.1     2.1      -1.6     2.8      -1.6      ZINA0034
 3.1      -1.2     2.5      -1.2     2.5      -1.6     3.1      -1.6      ZINA0035
 2.4      -.7      2.       -.7      2.       -1.      2.4      -1.       ZINA0036
```

2.5	-.85	2.2	-.85	2.2	-1.1	2.5	-1.1	ZINA0037
2.6	-.85	2.3	-.85	2.3	-1.2	2.6	-1.2	ZINA0038
3.	-.75	2.5	-.75	2.5	-1.3	3.	-1.3	ZINA0039
2.5	-.5	2.2	-.5	2.2	-.9	2.5	-.9	ZINA0040
2.8	-.6	2.4	-.6	2.4	-.9	2.8	-.9	ZINA0041
2.7	-.2	2.3	-.2	2.3	-.55	2.7	-.55	ZINA0042
0.	.55	.1	.55	.1	.61	0.	.61	ZINA0043
0.	.45	.15	.45	.15	.61	0.	.61	ZINA0044
-.15	.32	.15	.32	.15	.51	-.15	.51	ZINA0045
-.22	.46	0.	.46	0.	.66	-.22	.66	ZINA0046
0.	.59	.02	.59	.02	.63	0.	.63	ZINA0047
-.07	.56	.07	.56	.07	.65	-.07	.65	ZINA0048
-.16	.53	0.	.53	0.	.67	-.16	.67	ZINA0049
-.06	.6	.06	.6	.06	.66	-.06	.66	ZINA0050
-.07	.62	.07	.62	.07	.75	-.07	.75	ZINA0051
-.14	.61	.05	.61	.05	.77	-.14	.77	ZINA0052
-.12	.59	.12	.59	.12	.66	-.12	.66	ZINA0053
-.13	.63	.13	.63	.13	.8	-.13	.8	ZINA0054
.08	.54	.25	.54	.25	.77	.08	.77	ZINA0055
.135	.41	.32	.41	.32	.64	.135	.64	ZINA0056
.1	.27	.26	.27	.26	.45	.1	.45	ZINA0057
.14	1.5	.33	1.5	.33	1.7	.14	1.7	ZINA0058
-.01	.237	.182	.237	.182	.41	-.01	.41	ZINA0059
0.	.1	.27	.1	.27	.36	0.	.36	ZINA0060
1.5	-.3	1.2	-.3	1.2	-.55	1.5	-.55	ZINA0061
2.7	-.4	2.4	-.4	2.4	-.75	2.7	-.75	ZINA0062
2.4	-.3	2.2	-.3	2.2	-.5	2.4	-.5	ZINA0063
2.4	-.2	2.3	-.2	2.3	-.4	2.4	-.4	ZINA0064
-.2	.237	.05	.237	.05	.47	-.2	.47	ZINA0065

```
C     THIS IS A CALCOMP PLOTTING PROGRAM TO PLOT DISPERSION RELATION
C     OF ACOUSTIC WAVES IN A LINED DUCT.
      COMPLEX*8 CKR(114,4,4,2),CKP(114,4,8,2)
      DIMENSION AXT(19),AYT(28)
      DIMENSION GN(113),FG(113),RCRP(2,2),TIK(9),DTR(4),SDTR(4)
      DIMENSION RK(113),TS(113),BXT(7),BYT(9),AS(4)
      REAL DTC(4)/1.054,2.,4.828,12.928/
      REAL T(8)/.5,1.,2.,4.,8.,12.,16.,20./
      REAL BRT(3)/7.375,8.,8.625/
      REAL TG(9)/.301,.477,.602,.699,.778,.845,.903,.954,1./
      REAL YD(4)/5.3,5.1,4.9,4.7/
      DATA TIK/'.1  ','.2  ','.5  ','1.  ','2.  ','5.  ','10. ','20. ',
     1'50. '/
      DATA AS/'1/8 ','1/4 ','1/2 ','3/4 '/
      DATA SDTR/'2/7 ','2/3 ','2.0 ','6.0 '/
      DATA PCRP/'R-R ','C-R ','R-P ','C-P '/
  101 FORMAT(8X,6E12.5)
  102 FORMAT(8F10.5)
  201 FORMAT(8F10.5/)
      READ(5,101) ((((CKR(L,J,I,IR),L=1,114),J=1,4),I=1,4),IR=1,2)
      READ(5,101) ((((CKP(L,J,I,IR),L=1,114),J=1,4),I=1,8),IR=1,2)
      READ(5,102) (GN(L),L=1,113)
      DTR(1)=2./7.
      DTR(2)=2./3.
      DTR(3)=2.
      DTR(4)=6.
      RMG=1.079*2.5+3.
      AXT(1)=.25
      AYT(1)=1.5
      DO 5 KX=1,9
      AXT(KX+1)=.25+TG(KX)*2.5
      AXT(KX+10)=TG(KX)*2.5+2.75
      AYT(KX+1)=TG(KX)*1.75+1.5
      AYT(KX+10)=TG(KX)*1.75+3.25
      AYT(KX+19)=TG(KX)*1.75+5.
    5 CONTINUE
```

```
      DO 6 KF=1,113
      FG(KF)=ALOG10(GN(KF))*2.5+2.75
    6 CONTINUE
      WRITE(6,201) (FG(KF),KF=1,113,16)
      BXT(1)=AXT(1)-.05
      BXT(2)=AXT(2)-.05
      BXT(3)=AXT(5)-.05
      BXT(4)=AXT(10)-.05
      BXT(5)=AXT(11)-.05
      BXT(6)=AXT(14)-.05
      BXT(7)=AXT(19)-.05
      BYT(1)=AYT(1)-.05
      BYT(2)=AYT(2)-.05
      BYT(3)=AYT(5)-.05
      BYT(4)=AYT(10)-.05
      BYT(5)=AYT(11)-.05
      BYT(6)=AYT(14)-.05
      BYT(7)=AYT(19)-.05
      BYT(8)=AYT(20)-.05
      BYT(9)=AYT(23)-.05
      CALL PLOTS(IDUM,IDUM,11)
      CALL PLOT(3.,0.,-3)
      DO 50 IQ=1,2
      DO 50 IR=1,2
      SYB=RCRP(IR,IQ)
      DO 51 J=1,4
      DTJ=.5*DTR(J)
      IF(IR.EQ.2) DTJ=.5*DTC(J)
C   MAKE FRAME
      NI=1
      NT=4
   65 DO 7 IX=1,19
      IF(IX.EQ.1.OR.IX.EQ.5.OR.IX.EQ.10.OR.IX.EQ.14.OR.IX.EQ.19) GO TO
     171
      CALL SYMBOL(AXT(IX),1.05,.10,13,0.,-1)
      GO TO 7
```

```
71 CALL SYMBOL(AXT(IX),1.1,.2,13,0.,-1)                                CMP10073
 7 CONTINUE                                                            CMP10074
   DO 8 IX=1,27                                                        CMP10075
   AYTI=AYT(IX)                                                        CMP10076
   IF(IX.EQ.1.OR.IX.EQ.5.OR.IX.EQ.10.OR.IX.EQ.14.OR.IX.EQ.19.          CMP10077
  1OR.IX.EQ.23) GO TO 81                                               CMP10078
   BRMG=RMG-.05                                                        CMP10079
   CALL SYMBOL(BRMG,AYTI,.10,13,90.,-1)                                CMP10080
   GO TO 8                                                             CMP10081
81 BRMG=RMG-.1                                                         CMP10082
   CALL SYMBOL(BRMG,AYTI,.2,13,90.,-1)                                 CMP10083
 8 CONTINUE                                                            CMP10084
   CALL PLOT(RMG,6.75,3)                                               CMP10085
   CALL PLOT(RMG,1.,2)                                                 CMP10086
   CALL PLOT(0.,1.,2)                                                  CMP10087
   DO 9 IX=1,7                                                         CMP10088
   BXTI=BXT(IX)                                                        CMP10089
   CALL SYMBOL(BXTI,.85,.1,TIK(IX),0.,4)                               CMP10090
 9 CONTINUE                                                            CMP10091
   CALL SYMBOL(AXT(8),.65,.12,'K*L',0.,3)                              CMP10092
   DO 10 IX=1,27                                                       CMP10093
   AYTI=AYT(IX)                                                        CMP10094
   IF(IX.EQ.1.OR.IX.EQ.5.OR.IX.EQ.10.OR.IX.EQ.14.OR.IX.EQ.19.          CMP10095
  1OR.IX.EQ.23) GO TO 111                                              CMP10096
   CALL SYMBOL(.05,AYTI,.10,13,90.,-1)                                 CMP10097
   GO TO 10                                                            CMP10098
111 CALL SYMBOL(.1,AYTI,.2,13,90.,-1)                                  CMP10099
10 CONTINUE                                                            CMP10100
   CALL PLOT(0.,6.75,3)                                                CMP10101
   CALL PLOT(0.,1.,2)                                                  CMP10102
   DO 11 IX=1,9                                                        CMP10103
   BTIY=BYT(IX)                                                        CMP10104
   CALL SYMBOL(-.05,BTIY,.1,TIK(IX),90.,4)                             CMP10105
11 CONTINUE                                                            CMP10106
   CALL SYMBOL(-.23,3.,.12,'TL  (DECIBELS)',90.,14)                    CMP10107
   DO 12 IX=1,19                                                       CMP10108
```

```
      SZ=.2                                                             CMP10109
      IF(IX.EQ.1.OR.IX.EQ.5.OR.IX.EQ.10.OR.IX.EQ.14.OR.IX.EQ.19)        CMP10110
     1SZ=.4                                                             CMP10111
      CALL SYMBOL(AXT(IX),6.75,SZ,13,0.,-1)                             CMP10112
   12 CONTINUE                                                          CMP10113
      DO 13 IX=1,3                                                      CMP10114
      IF(IX.EQ.2) GO TO 113                                             CMP10115
      BRMG=RMG-.05                                                      CMP10116
      CALL SYMBOL(BRMG,BRT(IX),.10,13,90.,-1)                           CMP10117
      GO TO 13                                                          CMP10118
  113 BRMG=RMG-.1                                                       CMP10119
      CALL SYMBOL(BRMG,BRT(IX),.2,13,90.,-1)                            CMP10120
   13 CONTINUE                                                          CMP10121
      CALL PLOT(RMG,9.25,3)                                             CMP10122
      CALL PLOT(RMG,6.75,2)                                             CMP10123
      CALL PLOT(0.,6.75,2)                                              CMP10124
      DO 15 IX=1,3                                                      CMP10125
      IF(IX.EQ.2) GO TO 115                                             CMP10126
      CALL SYMBOL( .05,BRT(IX),.10,13,90.,-1)                           CMP10127
      GO TO 15                                                          CMP10128
  115 CALL SYMBOL(.1,BRT(IX),.2,13,90.,-1)                              CMP10129
   15 CONTINUE                                                          CMP10130
      CALL PLOT(0.,9.25,3)                                              CMP10131
      CALL PLOT(0.,6.75,2)                                              CMP10132
      IF(J.NE.4) GO TO 116                                              CMP10133
      CALL SYMBOL(-.05,6.7,.1,'.5',90.,2)                               CMP10134
      CALL SYMBOL(-.05,7.95,.1,'1.',90.,2)                              CMP10135
      CALL SYMBOL(-.05,9.10,.1,'1.5',90.,3)                             CMP10136
      GO TO 215                                                         CMP10137
  116 CALL SYMBOL(-.05,6.75,.1,'C.',90.,2)                              CMP10138
      IF(J.EQ.1.OR.(IR.EQ.1.AND.J.EQ.2)) GO TO 515                      CMP10139
      CALL SYMBOL(-.05,7.95,.1,'1.',90.,2)                              CMP10140
      CALL SYMBOL(-.05,9.2,.1,'2.',90.,2)                               CMP10141
      GO TO 215                                                         CMP10142
  515 CALL SYMBOL(-.05,7.95,.1,'2.',90.,2)                              CMP10143
      CALL SYMBOL(-.05,9.2,.1,'4.',90.,2)                               CMP10144
```

```
  215 CALL SYMBOL(-.23,7.6,.12,'REAL(KZ)/K',90.,10)
      DO 26 IX=1,19
      IF(IX.EQ.1.OR.IX.EQ.5.OR.IX.EQ.10.OR.IX.EQ.14.OR.IX.EQ.19)
     1GO TO 261
      CALL SYMBOL(AXT(IX),9.2,.10,13,0.,-1)
      GO TO 26
  261 CALL SYMBOL(AXT(IX),9.15,.2,13,0.,-1)
   26 CONTINUE
      CALL SYMBOL(5.5,9.3,.1,SYB,0.,4)
      CALL PLOT(RMG,9.25,3)
      CALL PLOT(0.,9.25,2)
      CALL SYMBOL(.25,6.,.12,'AREA RATIO=',0.,11)
      CALL SYMBOL(999.,999.,.12,AS(J),0.,3)
      CALL SYMBOL(.25,5.8,.12,'D/L=',0.,4)
      IF(IR.EQ.2) GO TO 17
      CALL SYMBOL(999.,999.,.12,SDTR(J),0.,3)
      GO TO 18
   17 CALL NUMBER(999.,999.,.12,DTC(J),0.,3)
   18 CALL SYMBOL(.25,5.6,.12,'S/D=1',0.,5)
      CALL SYMBOL(.25,5.3,.12,'THETA=',0.,6)
C   PLOT CURVES
      DO 52 I=NI,NT
      DO 16 IX=1,4
      YC=YD(IX)
      YS=YC+.06
      TT=T(IX)
      IF(I.GT.4) TT=T(IX+4)
      CALL NUMBER(.98,YC,.12,TT,0.,1)
      ITP=IX-1
      IF(IX.EQ.4) ITP=4
      CALL SYMBOL(1.6,YS,.12,ITP,0.,-1)
   16 CONTINUE
      DO 53 L=1,113
      GNL=GN(L)
      IF(J.EQ.1.OR.(IR.EQ.1.AND.J.EQ.2)) GNL=2.*GNL
      IF(IQ.EQ.2) GO TO 153
```

```
      RKR=REAL(CKR(L,J,I,IR))/DTJ
      IF(J.EQ.4) GO TO 154
      RK(L)=6.75
      IF(RKR.GT.0.) RK(L)=RKR*1.25/GNL+6.75
      GO TO 156
  154 RK(L)=RKR*2.5/GNL+5.5
      IF(RK(L).LT.6.75) RK(L)=6.75
  156 AMKR=AIMAG(CKR(L,J,I,IR))*17.372
      TS(L)=1.
      IF(AMKR.GT..0531) TS(L)=ALOG10(AMKR)*1.75+3.25
      GO TO 53
  153 RKP=REAL(CKP(L,J,I,IR))/DTJ
      RK(L)=6.75
      IF(RKP.GT.0.) RK(L)=RKP*1.25/GNL+6.75
      IF(J.EQ.4) GO TO 157
      GO TO 158
  157 RK(L)=RKP*2.5/GNL+5.5
      IF(RK(L).LT.6.75) RK(L)=6.75
  158 AMKP=AIMAG(CKP(L,J,I,IR))*17.372
      TS(L)=1.
      IF(AMKP.GT..0531) TS(L)=ALOG10(AMKP)*1.75+3.25
   53 CONTINUE
      NL=113
      ITEQ=I-1
      IF(I.EQ.4) ITEQ=4
      SZ=.07
      CALL ZDRW(FG,RK,TS,NL,SZ,ITEQ)
   52 CONTINUE
      CALL PLOT(9.,0.,-3)
      IF(IQ.EQ.1) GO TO 51
      IF(NI.EQ.5) GO TO 51
      NI=5
      NT=8
      GO TO 65
   51 CONTINUE
   50 CONTINUE
```

```
      CALL ENDPLT(3.,0.,999)
      STOP
      END
      SUBROUTINE ZDRW(FG,RK,TS,NL,SZ,ITEQ)
      REAL FG(NL),RK(NL),TS(NL)
      CALL PLOT(FG(1),RK(1),3)
      DO 50 L=2,NL
      FL=FG(L)
      RL=RK(L)
      IF(RK(L-1).GT.9.25) GO TO 51
      IF(RL.GT.9.25) GO TO 52
      CALL PLOT(FL,RL,2)
      GO TO 50
   52 CALL PLOT(FL,9.25,2)
      GO TO 50
   51 IF(RL.GT.9.25) GO TO 50
      CALL PLOT(FL,9.25,3)
   50 CONTINUE
      DO 60 LS=1,8
      LM=136-16*LS
      IF(LS.EQ.1) LM=110
      IF(LS.EQ.2) LM=102
      IF(RK(LM).GE.9.25) GO TO 60
      CALL SYMBOL(FG(LM),RK(LM),SZ,ITEQ,0.,-1)
   60 CONTINUE
      CALL PLOT(FG(1),TS(1),3)
      DO 30 L=2,NL
      FL=FG(L)
      TL=TS(L)
      IF(TS(L-1).GT.6.5) GO TO 31
      IF(TL.GT.6.5) GO TO 32
      CALL PLOT(FL,TL,2)
      GO TO 30
   32 CALL PLOT(FL,6.5,2)
      GO TO 30
   31 IF(TL.GT.6.5) GO TO 30
```

```
      CALL PLOT(FL,6.5,3)
   30 CONTINUE
      DO 40 LS=1,9
      LM=117-8*LS
      IF(LS.GT.5) LM=157-16*LS
      IF(TS(LM).GE.6.5) GO TO 41
      CALL SYMBOL(FG(LM),TS(LM),SZ,ITEQ,0.,-1)
      GO TO 40
   41 DO 42 IX=1,4
      LMR=LM+IX
      IF(TS(LMR).GE.6.5) GO TO 42
      TSR=TS(LMR)
      FGR=FG(LMR)
      GO TO 43
   42 CONTINUE
      GO TO 44
   43 CALL SYMBOL(FGR,TSR,SZ,ITEQ,0.,-1)
   44 DO 45 IX=1,4
      LML=LM-IX
      IF(TS(LML).GE.6.5) GO TO 45
      TSL=TS(LML)
      FGL=FG(LML)
      GO TO 46
   45 CONTINUE
      GO TO 40
   46 CALL SYMBOL(FGL,TSL,SZ,ITEQ,0.,-1)
   40 CONTINUE
      RETURN
      END
```

```
C     THIS IS A CALCOMP PLOTTING PROGRAM TO PLCT TRANSMISSION LOSS
C     VS. CENTER FREQUENCIES OF OCTAVE BANDS IN A LINED DUCT.
      DIMENSION BTA(12),BTB(12),BTI(3),AR(16),ARI(4),BIN(3)
      DIMENSION ATTD(2,2,2,3,4,5,7)
      DIMENSION ATX1(7),ATX2(7),ATX3(7),ATX4(7),ATX5(7),FREQ(7)
      REAL FREK(7)/0.,.5,1.,1.5,2.,2.5,3./
      DATA BTA/'D/L=','1.09','4   ','D/L=','2.00','0   ','D/L=','4.82',
     1'8   ','D/L=','12.9','28  '/
      DATA BTB/'D/L=','2/7 ','    ','D/L=','2/3 ','    ','D/L=','2.  ',
     1'    ','D/L=','6.  ','    '/
      DATA AR/'AREA',' RAT','IO=1','/8  ','AREA',' RAT','IO=1','/4  ',
     1'AREA',' RAT','IO=1','/2  ','AREA',' RAT','IO=3','/4  '/
      DATA BIN/'N=0 ','N=2 ','N=-2'/
  101 FORMAT(10X,7F10.4)
      READ(5,101) (((((((ATTD(ICR,IQ,IR,NR,I,M,L),L=1,7),M=1,5),I=1,4),
     1NR=1,3),IR=1,2),IQ=1,2),ICR=1,2)
      CALL PLOTS(IDUM,IDUM,11)
      DO 56 ICR=1,2
      DO 56 IQ=1,2
      DO 56 IR=1,2
      IF(ICR.EQ.2) GO TO 202
      IF(IQ.EQ.1.AND.IR.EQ.1) GO TO 91
      IF(IQ.EQ.1.AND.IR.EQ.2) GO TO 92
      IF(IQ.EQ.2.AND.IR.EQ.1) GO TO 93
      IF(IQ.EQ.2.AND.IR.EQ.2) GO TO 94
   91 CALL SYMBOL(0.,3.,.20,'TL IN CR-RES. SR',90.,16)
      GO TO 80
   92 CALL SYMBOL(0.,3.,.20,'TL IN CR-RES. DCT',90.,17)
      GO TO 80
   93 CALL SYMBOL(0.,3.,.20,'TL IN CR-PRS. SR',90.,16)
      GO TO 80
   94 CALL SYMBOL(0.,3.,.20,'TL IN CR-PRS. DCT',90.,17)
      GO TO 80
  202 IF(IQ.EQ.1.AND.IR.EQ.1) GO TO 96
      IF(IQ.EQ.1.AND.IR.EQ.2) GO TO 97
      IF(IQ.EQ.2.AND.IR.EQ.1) GO TO 56
```

```
      IF(IQ.EQ.2.AND.IR.EQ.2) GO TO 99
   96 CALL SYMBOL(0.,3.,.20,'TL IN RR-RES. SR',90.,16)
      GO TO 80
   97 CALL SYMBOL(0.,3.,.20,'TL IN RR-RES. DCT',90.,17)
      GO TO 80
   99 CALL SYMBOL(0.,3.,.20,'TL IN RR-PRS. DCT',90.,17)
   80 CALL PLOT(2.,0.,-3)
      DO 55 IX=1,4
      IXR=3*IX
      IXT=IXR-1
      IXI=IXT-1
      IF(ICR.EQ.2) GO TO 211
      BTI(1)=BTA(IXI)
      BTI(2)=BTA(IXT)
      BTI(3)=BTA(IXR)
      GO TO 210
  211 BTI(1)=BTB(IXI)
      BTI(2)=BTB(IXT)
      BTI(3)=BTB(IXR)
  210 IYF=4*IX
      IYR=IYF-1
      IYT=IYR-1
      IYI=IYT-1
      ARI(1)=AR(IYI)
      ARI(2)=AR(IYT)
      ARI(3)=AR(IYR)
      ARI(4)=AR(IYF)
      XA=.5
      YA=8.3
      SZA=.14
      CALL SYMBOL(XA,YA,SZA,'THETA=0.5',0.,9)
      YB=YA-.3
      CALL SYMBOL(XA,YB,SZA,BTI,0.,12)
      YC=YB-.3
      YD=YC-.5
      IF(IR.EQ.2) GO TO 73
```

```
      CALL SYMBOL(XA,YC,SZA,ARI,0.,16)
      GO TO 83
   73 CALL SYMBOL(XA,YC,SZA,'AREA RATIO=1',0.,12)
   83 CALL SYMBOL(XA,YD,SZA,'S/D=16',0.,6)
      XDD=XA+1.1
      YDD=YD+SZA/2.
      CALL SYMBOL(XDD,YDD,SZA,4,0.,-1)
      YE=YD-.25
      CALL SYMBOL(XA,YE,SZA,'     8',0.,6)
      YEE=YDD-.25
      CALL SYMBOL(XDD,YEE,SZA,3,0.,-1)
      YF=YE-.25
      CALL SYMBOL(XA,YF,SZA,'     4',0.,6)
      YFF=YEE-.25
      CALL SYMBOL(XDD,YFF,SZA,2,0.,-1)
      YG=YF-.25
      CALL SYMBOL(XA,YG,SZA,'     2',0.,6)
      YGG=YFF-.25
      CALL SYMBOL(XDD,YGG,SZA,1,0.,-1)
      YH=YG-.25
      CALL SYMBOL(XA,YH,SZA,'     1',0.,6)
      YHH=YGG-.25
      CALL SYMBOL(XDD,YHH,SZA,0,0.,-1)
      NB=2
      RFX=0.
      RFY=0.
  200 BJ=BIN(NB)
      RFNY=RFY+4.5
      CALL PLOT(RFX,RFNY,3)
      DO 40 L=1,7
      FREQ(L)=FREK(L)+RFX
      ATX1(L)=ATTD(ICR,IQ,IR,NB,IX,1,L)/10.+RFY
      ATX2(L)=ATTD(ICR,IQ,IR,NB,IX,2,L)/10.+RFY
      ATX3(L)=ATTD(ICR,IQ,IR,NB,IX,3,L)/10.+RFY
      ATX4(L)=ATTD(ICR,IQ,IR,NB,IX,4,L)/10.+RFY
      ATX5(L)=ATTD(ICR,IQ,IR,NB,IX,5,L)/10.+RFY
```

```
40 CONTINUE                                          CMP20109
   BMY=4.7+RFY                                       CMP20110
   DO 20 NX=3,7                                      CMP20111
   IF(ATX5(NX).GT.BMY) GO TO 61                      CMP20112
20 CONTINUE                                          CMP20113
   NI=1                                              CMP20114
   GO TO 31                                          CMP20115
61 DO 21 NX=3,7                                      CMP20116
   IF(ATX4(NX).GT.BMY) GO TO 62                      CMP20117
21 CONTINUE                                          CMP20118
   NI=2                                              CMP20119
   GO TO 31                                          CMP20120
62 DO 22 NX=3,7                                      CMP20121
   IF(ATX3(NX).GT.BMY) GO TO 63                      CMP20122
22 CONTINUE                                          CMP20123
   NI=3                                              CMP20124
   GO TO 31                                          CMP20125
63 DO 23 NX=3,7                                      CMP20126
   IF(ATX2(NX).GT.BMY) GO TO 64                      CMP20127
23 CONTINUE                                          CMP20128
   NI=4                                              CMP20129
   GO TO 31                                          CMP20130
64 DO 24 NX=3,7                                      CMP20131
   IF(ATX1(NX).GT.BMY) GO TO 33                      CMP20132
24 CONTINUE                                          CMP20133
   NI=5                                              CMP20134
31 NG=7                                              CMP20135
   SIZ=.08                                           CMP20136
   CALL PLOT(RFX,RFY,2)                              CMP20137
   YXA=RFX-.08                                       CMP20138
   CALL SYMBOL(YXA,RFY,.1,'0.',90.,2)                CMP20139
   YYB=RFY+.9                                        CMP20140
   CALL SYMBOL(YXA,YYB,0.1,'10.',90.,3)              CMP20141
   YYC=RFY+1.9                                       CMP20142
   CALL SYMBOL(YXA,YYC,0.1,'20.',90.,3)              CMP20143
   YYD=RFY+2.9                                       CMP20144
```

```
      CALL SYMBOL(YXA,YYD,0.1,'30.',90.,3)
      YYE=RFY+3.9
      CALL SYMBOL(YXA,YYE,0.1,'40.',90.,3)
      YXF=RFX+0.4
      YYF=RFY+4.
      CALL SYMBOL(YXF,YYF,.10,BJ,0.,4)
      YNT=RFY+4.
      YXTIC=RFX+.04
      DO 41 J=1,4
      CALL SYMBOL(YXTIC,YNT,.08,13,90.,-1)
   41 YNT=YNT-1.
      YDBX=RFX-.25
      YDBY=RFY+1.2
      CALL SYMBOL(YDBX,YDBY,.14,'TL (DECIBELS)',90.,13)
      CALL PLOT(RFX,RFY,3)
      RAX=RFX+3.
      CALL PLOT(RAX,RFY,2)
      XINT=RFX+3.
      XYTIC=RFY+.04
      DO 30 I=1,6
      CALL SYMBOL(XINT,XYTIC,.08,13,0.0,-1)
   30 XINT=XINT-.5
      XZA=RFX-.1
      YZA=RFY-.18
      XZB=RFX+.35
      XZC=RFX+.85
      XZD=RFX+1.45
      XZE=RFX+1.95
      XZF=RFX+2.45
      XZG=RFX+2.95
      CALL SYMBOL(XZA,YZA,.1,'1/8',0.,3)
      CALL SYMBOL(XZB,YZA,.1,'1/4',0.,3)
      CALL SYMBOL(XZC,YZA,.1,'1/2',0.,3)
      CALL SYMBOL(XZD,YZA,.1,'1',0.,1)
      CALL SYMBOL(XZE,YZA,.1,'2',0.,1)
      CALL SYMBOL(XZF,YZA,.1,'4',0.,1)
```

```
      CALL SYMBOL(XZG,YZA,.1,'8',0.,1)                                  CMP20181
      XKL=RFX+1.3                                                       CMP20182
      YKL=RFY-.38                                                       CMP20183
      IF(NB.EQ.1) GO TO 16                                              CMP20184
      CALL SYMBOL(XKL,YKL,.14,'K*L',0.,3)                               CMP20185
   16 GO TO (15,14,13,12,11),NI                                         CMP20186
   15 ITEQ=4                                                            CMP20187
      CALL ZDRAW(FREQ,ATX5,NG,SIZ,ITEQ)                                 CMP20188
   14 ITEQ=3                                                            CMP20189
      CALL ZDRAW(FREQ,ATX4,NG,SIZ,ITEQ)                                 CMP20190
   13 ITEQ=2                                                            CMP20191
      CALL ZDRAW(FREQ,ATX3,NG,SIZ,ITEQ)                                 CMP20192
   12 ITEQ=1                                                            CMP20193
      CALL ZDRAW(FREQ,ATX2,NG,SIZ,ITEQ)                                 CMP20194
   11 ITEQ=0                                                            CMP20195
      CALL ZDRAW(FREQ,ATX1,NG,SIZ,ITEQ)                                 CMP20196
   33 IF(NB.NE.2) GO TO 32                                              CMP20197
      NB=1                                                              CMP20198
      RFX=4.                                                            CMP20199
      RFY=5.                                                            CMP20200
      GO TO 200                                                         CMP20201
   32 IF(NB.EQ.3) GO TO 45                                              CMP20202
      NB=3                                                              CMP20203
      RFX=4.                                                            CMP20204
      RFY=0.                                                            CMP20205
      GO TO 200                                                         CMP20206
   45 CALL PLOT(11.,0.,-3)                                              CMP20207
   55 CONTINUE                                                          CMP20208
   50 CONTINUE                                                          CMP20209
   56 CONTINUE                                                          CMP20210
      CALL ENDPLT(3.,0.,999)                                            CMP20211
      STOP                                                              CMP20212
      END                                                               CMP20213
      SUBROUTINE ZDRAW(AF,ATY,N,SIZE,ITEQ)                              CMP20214
      REAL AF(N),ATY(N)                                                 CMP20215
      CALL PLOT(AF(1),ATY(1),3)                                         CMP20216
```

```
      DO 50 I=2,N                                                       CMP20217
      CALL PLOT(AF(I),ATY(I),2)                                         CMP20218
   50 CONTINUE                                                          CMP20219
      DO 51 J=1,N                                                       CMP20220
      JJ=N-J+1                                                          CMP20221
      CALL SYMBOL(AF(JJ),ATY(JJ),SIZE,ITEQ,0.,-1)                       CMP20222
   51 CONTINUE                                                          CMP20223
      RETURN                                                            CMP20224
      END                                                               CMP20225
```

```
C     THIS IS A CALCOMP PLOTTING PROGRAM TO PLOT TRANSMISSION LOSS
C     VS. LENGTH OF THE LINER.
      IMPLICIT COMPLEX*8 (C)
      COMMON YSF(17)
      REAL CABS,COS,GN(113),YO(17),ZO(17),GBC(4),V(4)/.25,.5,1.,2./
      REAL GBR(4)/.142857,.333333,1.,3./,XCO(3)/1.,2.,3./
      REAL YCO(4)/1.,2.,3.,4./,XCK(4)/0.,.95,1.9,2.9/
      REAL OX(4)/0.,4.,0.,4./
      REAL OY(4)/5.,5.,0.,0./
      REAL YCK(5)/0.,.9,1.9,2.9,3.9/
      REAL DTC(4)/1.054,2.,4.828,12.928/
      REAL T(8)/.5,1.,2.,4.,8.,12.,16.,20./
      DIMENSION XTK(4),YTK(8),AS(4),DSTR(4),RPCP(2,2),DKL(4)
      DATA XTK/'0.  ','5.  ','10. ','15. '/
      DATA YTK/'0.  ','10. ','20. ','30. ','40. '/
      DATA AS/'1/8 ','1/4 ','1/2 ','3/4 '/
      DATA DSTR/'2/7 ','2/3 ','2.0 ','6.0 '/
      DATA RPCP/'R-R ','C-R ','R-P ','C-P '/
      DATA DKL/'0.5 ','1.0 ','2.0 ','4.0 '/
      DIMENSION CKIA(114,4,8,2,2)
  101 FORMAT(10X,I5)
  100 FORMAT(8X,6E12.5)
  102 FORMAT(8F10.5)
      READ(5,100) ((((CKIA(L,J,I,IR,1),L=1,114),J=1,4),I=1,4),IR=1,2)
      READ(5,100) ((((CKIA(L,J,I,IR,2),L=1,114),J=1,4),I=1,8),IR=1,2)
      READ(5,102) (GN(L),L=1,113)
      READ(5,101) NCK
      WRITE(6,101) NCK
      IF(NCK.NE.12345) GO TO 1000
      RTT=SQRT(2.)
      GBC(1)=1./(2.*RTT-1.)
      GBC(2)=1.
      GBC(3)=1./(RTT-1.)
      GBC(4)=1./(2./SQRT(3.)-1.)
      ATMX=EXP(160.)
      ATMIV=1./ATMX
```

```
      ATDMX=ALOG10(ATMX)
C   PLOTTINGS
      CALL PLOTS(IDUM,IDUM,11)
      CALL PLOT(3.,0.,-3)
      DO 50 IQ=1,2
      DO 50 IR=1,2
      SYB=RPCP(IR,IQ)
      DO 50 IP=1,2
      DO 51 J=1,4
      IF(IR.EQ.1) GBJ=GBR(J)
      IF(IR.EQ.2) GBJ=GBC(J)
      NI=1
   11 CALL SYMBOL(1.5,9.85,.12,'AREA RATIO=',0.,11)
      IF(IP.EQ.2) GO TO 21
      CALL SYMBOL(999.,999.,.12,AS(J),0.,4)
      GO TO 22
   21 CALL SYMBOL(999.,999.,.12,'1.0',0.,3)
   22 CALL SYMBOL(999.,999.,.12,'    D/L=',0.,7)
      IF(IR.EQ.2) GO TO 3
      CALL SYMBOL(999.,999.,.12,DSTR(J),0.,4)
      GO TO 4
    3 CALL NUMBER(999.,999.,.12,DTC(J),0.,3)
    4 CALL SYMBOL(7.2,10.05,.1,SYB,0.,4)
      DO 52 I=1,4
      ORX=OX(I)
      ORY=OY(I)
      IT=I
      IF(NI.EQ.2) IT=I+4
C   X-TIC MARK
      DO 521 IX=1,3
      XT=ORX+XCO(IX)
      YT=ORY+.05
      CALL SYMBOL(XT,YT,.1,13,0.,-1)
  521 CONTINUE
C   DRAW X-AXIS LINE
      XF=ORX+3.5
```

```
      CALL PLOT(XF,ORY,3)
      CALL PLOT(ORX,ORY,2)
C    NUMBER X-AXIS
      DO 522 IX=1,4
      XCKX=ORX+XCK(IX)
      XCKY=ORY-.18
      XTKX=XTK(IX)
      CALL SYMBOL(XCKX,XCKY,.1,XTKX,0.,4)
  522 CONTINUE
      IF(I.EQ.1.OR.I.EQ.2) GO TO 7
      XRSD=ORX+1.55
      YRSD=ORY-.38
      CALL SYMBOL(XRSD,YRSD,.14,'S/D',0.,3)
C    Y-AXIS TIC MARKS
    7 DO 523 IX=1,4
      XT=ORX+.05
      YT=ORY+YCO(IX)
      CALL SYMBOL(XT,YT,.1,13,90.,-1)
  523 CONTINUE
C    DRAW Y-AXIS LINE
      YF=ORY+4.5
      CALL PLOT(ORX,YF,3)
      CALL PLOT(ORX,ORY,2)
C    NUMBER Y-AXIS
      DO 524 IX=1,5
      YCKX=ORX-.08
      YCKY=ORY+YCK(IX)
      YTKY=YTK(IX)
      CALL SYMBOL(YCKX,YCKY,.1,YTKY,90.,4)
  524 CONTINUE
      IF(I.EQ.2.OR.I.EQ.4) GO TO 5
      XTL=ORX-.24
      YTL=ORY+1.25
      CALL SYMBOL(XTL,YTL,.14,'TL (DECIBELS)',90.,13)
    5 XET=ORX+.25
      YET=ORY+4.25
```

```
      CALL SYMBOL(XET,YET,.12,'THETA=',0.,6)
      CALL NUMBER(999.,999.,.12,T(IT),0.,1)
      IF(I.NE.1) GO TO 6
      YKL=ORY+3.95
      CALL SYMBOL(XET,YKL,.12,'K*L=',0.,4)
      XFT=ORX+.75
      XSBT=ORX+1.25
      YFT=4.13+ORY
      DO 525 IX=1,4
      ITP=IX-1
      IF(IX.EQ.4) ITP=4
      YFT=YFT-.2
      YSBT=YFT+.06
      CALL SYMBOL(XFT,YFT,.12,DKL(IX),0.,4)
      CALL SYMBOL(XSBT,YSBT,.12,ITP,0.,-1)
  525 CONTINUE
    6 DO 53 KL=1,4
      HI=RTT*V(KL)
      H=.0625*HI
      KNS=16*(KL+1)
      DO 531 LD=1,17
      XRS=FLOAT(LD-1)
      DO 532 KQ=1,17
      AKQ=FLOAT(KQ-1)
      KN=KNS+KQ
      GNL=GN(KN)
      GNB=GNL*GBJ
      CKIB=CKIA(KN,J,I,IR,IQ)
      CYKE=CKIB*XRS*2.
      IF(IR.EQ.2) GO TO 201
      IF(IP.EQ.1) CKR=CKIB/GNL/(1.+GBJ)
      IF(IP.EQ.2) CKR=CKIB/GNB
      GO TO 202
  201 IF(IP.EQ.1) CKR=GBJ*CKIB/GNL/(1.+GBJ)**2
      IF(IP.EQ.2) CKR=CKIB/GNB
  202 CKRV=(0.,.5)*(CKR+1./CKR)
```

```
      AMCK=AIMAG(CYKE)                                          CMP30145
      IF(AMCK.GT.80.) GO TO 203                                 CMP30146
      CSUM=CCOS(CYKE)-CKRV*CSIN(CYKE)                           CMP30147
      ASUM=CABS(CSUM)                                           CMP30148
      YCOM=1./ASUM**2*ATMX                                      CMP30149
      GO TO 204                                                 CMP30150
  203 IF(AMCK.GT.160.) GO TO 205                                CMP30151
      YCOM=4.*EXP(-2.*(AMCK-80.))/CABS(1.+CKRV)**2              CMP30152
      GO TO 204                                                 CMP30153
  205 YCOM=0.                                                   CMP30154
  204 YO(KQ)=YCOM/HI                                            CMP30155
  532 CONTINUE                                                  CMP30156
      CALL QSF(H,YO,ZO,17)                                      CMP30157
      ZQ=ZO(17)                                                 CMP30158
      IF(ZQ.LT.ATMIV) GO TO 301                                 CMP30159
      YSF(LD)=-ALOG10(ZQ)+ATDMX                                 CMP30160
      GO TO 531                                                 CMP30161
  301 YSF(LD)=2.*ATDMX                                          CMP30162
  531 CONTINUE                                                  CMP30163
      ITQ=KL-1                                                  CMP30164
      IF(KL.EQ.4) ITQ=4                                         CMP30165
      CALL ZDRW(ORX,ORY,.07,ITQ)                                CMP30166
   53 CONTINUE                                                  CMP30167
   52 CONTINUE                                                  CMP30168
      CALL PLOT(11.,0.,-3)                                      CMP30169
      IF(IQ.EQ.1) GO TO 51                                      CMP30170
      IF(NI.EQ.2) GO TO 51                                      CMP30171
      NI=2                                                      CMP30172
      GO TO 11                                                  CMP30173
   51 CONTINUE                                                  CMP30174
   50 CONTINUE                                                  CMP30175
      CALL ENDPLT(3.,0.,999)                                    CMP30176
 1000 STOP                                                      CMP30177
      END                                                       CMP30178
      SUBROUTINE ZDRW(OX,OY,SZ,ITQ)                             CMP30179
      COMMON YSF(17)                                            CMP30180
```

```
      PMT=OY+4.5                                                1800 CMP30181
      YSI=YSF(1)+OY                                             1810 CMP30182
      CALL PLOT(OX,YSI,3)                                       1820 CMP30183
      DO 50 I=1,16                                              1830 CMP30184
      XS=OX+.2*FLOAT(I)                                         1840 CMP30185
      YS=OY+YSF(I+1)                                            1850 CMP30186
      YSN=OY+YSF(I)                                             1860 CMP30187
      IF(YSN.GT.PMT) GO TO 51                                   1870 CMP30188
      IF(YS.GT.PMT) GO TO 52                                    1880 CMP30189
      CALL PLOT(XS,YS,2)                                        1890 CMP30190
      GO TO 50                                                  1900 CMP30191
   52 XS=XS-.2*(YS-PMT)/(YS-YSN)                                1910 CMP30192
      CALL PLOT(XS,PMT,2)                                       1920 CMP30193
      GO TO 50                                                  1930 CMP30194
   51 IF(YS.GT.PMT) GO TO 50                                    1940 CMP30195
      XS=XS-.2*(PMT-YS)/(YSN-YS)                                1950 CMP30196
      CALL PLOT(XS,PMT,3)                                       1960 CMP30197
   50 CONTINUE                                                  1970 CMP30198
      DO 60 IG=1,3                                              1980 CMP30199
      XS=3.2-.8*FLOAT(IG)+OX                                         CMP30200
      IS=17-4*IG                                                2000 CMP30201
      YS=YSF(IS)+OY                                                  CMP30202
      IF(YS.GT.PMT) GO TO 60                                    2020 CMP30203
      CALL SYMBOL(XS,YS,SZ,ITQ,0.,-1)                           2030 CMP30204
   60 CONTINUE                                                  2040 CMP30205
      RETURN                                                    2050 CMP30206
      END                                                            CMP30207
```

APPENDIX

ESTIMATION OF ERROR

The calculations in this report are based on the assumption that the incident wave is plane and that the fundamental mode is dominant in the lined duct element. This appendix is to clarify this question by an estimate of the error involved.

We shall consider only a rectangular duct with two opposite walls lined (see Figures 2.1 or 2.6). Since the field amplitude of an acoustic mode in a lined duct varies across the duct, an incident plane wave will excite all the symmetric duct modes in the lined duct element.

A harmonic time-dependent pressure field in the lined duct is written as

$$p(y,z,t) = \sum_m A_m(\omega)\cos(k_{my}y)e^{i(k_{mz}z-\omega t)} . \qquad (A.1)$$

(See Section 2.1 for notations.)

At the entrance of the lined duct we have $z = 0$ and it follows from Eq. (A.1) that

$$A_m(\omega) = \frac{1}{2bC_m}\int_{-b}^{b} p(y,0)\cos(k_{my}y)dy, \qquad (A.2)$$

where

$$C_m = \frac{1}{2}\left[1 + \frac{\sin(2k_{my}b)}{2k_{my}b}\right]. \qquad (A.3)$$

If p is assumed to be independent of y at $z = 0$, we get

$$A_m(\omega) = \frac{2\ \sin(k_{my}b)/(k_{my}b)}{1 + \frac{\sin(2k_{my}b)}{2k_{my}b}} \qquad (A.4)$$

The fundamental mode corresponds to a range of values of $(k_{my}b)$ between 0 and $\pi/2$. In this range the fundamental mode amplitude varies between the extreme values $A_1 = 1$ and $A_1 \cong 4/\pi$, the latter corresponding to a boundary impedance equal to zero. The boundaries considered in this report always have an impedance which is generally greater than 0 (normalized) in magnitude, the variation of the plane wave amplitude (with frequency) will be even smaller, and the error involved can be considered unimportant for the purpose of the present report.

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