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Point-coupling Models from Mesonic Hypermassive Limit and Mean-field Approaches

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Abstract In this work, we show how nonlinear point-coupling models, described by a Lagrangian density containing only terms up to fourth order in the fermion condensate $(\bar{\psi}\psi)$, are derived from a modified meson-exchange nonlinear Walecka model. We present two methods of derivation, namely the hypermassive meson limit within a functional integral approach and the mean-field approximation, in which equations of state at zero temperature of the nonlinear point-coupling models are directly obtained.

Keywords Point-coupling models · Walecka models · Mean-field approach · Functional integral method

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1 Introduction

The point-coupling interaction problem was first addressed in the early thirties by L. H. Thomas [1] while investigating the range of the two-nucleon force. As a side remark, he observed that when the range of the two-body force vanishes at fixed two-body binding energy, the binding energy of the quantum three-body state goes to minus infinity. Decades later, a new, apparently unrelated three-body effect was proposed by Efimov [2]. When a quantum two-body system has a zero-energy bound state, then the three-body system will have an infinite number of bound states with an accumulation point at the common two and three-body threshold. Both the Efimov and the Thomas effects are universal, since the associated three-body wave functions have long tails in the classically forbidden region outside the range of the potential. In a unified momentum space description, based on ideas of Amado and Noble [3], it has been claimed that these two apparently different effects are related to the same singular structure of the kernel of the Faddeev equation [4]. In appropriate units, on the other hand, the presence of one of these effects implies the other [4, 5]. The Thomas–Efimov effect explains very well some few-body correlations [6] and is conjectured to be behind the Coester band [7] for different nuclear matter models [8].

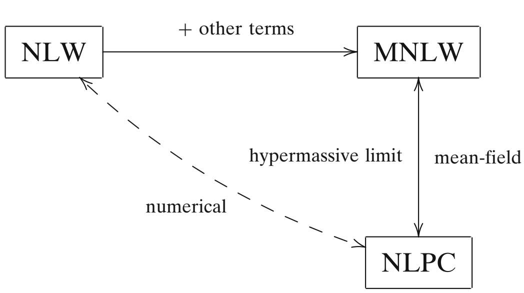
Since relativistic hadronic point-coupling models, which have been used in the description of infinite nuclear matter, as well as of finite nuclei [9], can be viewed as a connection between the well-established finite-range relativistic models and the Skyrme models [9], a better understanding of their structure is of interest, given that an important theoretical challenge is to construct a universal effective nuclear density functional [10].

In this work, we deal with a specific nonlinear point-coupling model (NLPC) described by a fermionic Lagrangian density with interaction terms in third and fourth powers of the scalar density operator, which we have used in ref. [11] in a comparative study with the standard nonlinear Walecka model(s), and in ref. [12], in which we took its nonrelativistic limit to obtain a generalized Skyrme energy density functional.

We focus on the derivation of the NLPC model from finite-range models in two different ways. First, we present the infinitely massive meson limit within the formal point of view of the integral functional approach. This method clearly shows the equivalence of the usual Walecka model to the linear point-coupling one. By contrast, nonlinear Walecka (NLW) models are formally inequivalent to NLPC ones. We therefore pose the question of how NLPC models can be derived if one insists on obtaining them from a meson-exchange. To answer this question, we construct a modified nonlinear Walecka (MNLW) model in which the limit of infinite meson masses leads exactly to the Lagrangian density of the NLPC model. This MNLW model includes third and fourth powers of the scalar meson field, along with lower powers of the fields coupled to the fermionic scalar density operator. Preliminary results on this hypermassive meson limit have been presented in ref. [13].

The traditional mean-field approximation, performed with a few physical requirements, is an alternative way to construct the NLPC models from the MNLW ones. In this case we show that the equations of state (EOS) of the MNLW models, relating energy, density, and pressure, are exactly the same as the NLPC ones.

The following diagram summarizes the study of the NLW, MNLW, and NLPC models.



The numerical equivalences between NLW and NLPC models have been analyzed in ref. [11], and the different connections among MNLW and NLPC models will be analytically studied in this work, which extends the following aspects of our previous study:

- We use the hypermassive meson limit in the functional integral model to derive the point-coupling models from the modified NLW models, rigorously and in detail. This approach shows how the linear PC models can be obtained from the Walecka one(s), and in the same way, how the MNLW model(s) generates the NLPC one(s).
- We also use the mean-field approach in the no-sea approximation to construct the equations of state of infinite nuclear matter for the MNLW model. In this approach, we show that these EOS are exactly the same as those for the NLPC model.

Our paper is organized as follows. In Section 2, by using a functional integral formalism, we derive the linear point-coupling model from the Walecka one. The same study is extended to obtain the NLPC models from the MNLW ones. In Section 3, we explicitly derive the equations of state of the MNLW model. Finally, the main conclusions are summarized.

2 Hypermassive Meson Limit

2.1 Linear Point-coupling Model from the Walecka One

We start with the Walecka model Lagrangian density given by [14]:

$$\mathcal{L}_W = \bar{\psi}(i\gamma^\mu\partial_\mu - M)\psi + \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{1}{2}m_s^2\phi^2 - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}m_V^2 V_\mu V^\mu - g_s\bar{\psi}\phi\psi - g_V\bar{\psi}\gamma^\mu V_\mu\psi \tag{1}$$

where $F^{\mu\nu} \equiv \partial^\mu V^\nu - \partial^\nu V^\mu$.

In this Lagrangian, ψ, ϕ, and V^μ are the nucleon, scalar, and vector fields, respectively and M, m_s, and m_v refer to the bare nucleon and the mesonic σ and ω masses, respectively.

The fermionic, scalar, and vector fields constitute an irreducible set of generators $\{\psi, \bar{\psi}, \phi, V_\mu\}$ of the intrinsic local algebra of fields of the model, $\mathbf{A}$. The polynomial algebra of intrinsic fields allows for the construction of the net of Wightman functions. From the Wightman functions of the polynomial algebra of intrinsic fields, the physical Hilbert space is reconstructed, $\mathbf{H} = \mathbf{A}\,|0\rangle$, thus defining the physical content of the model [15]. Under this setting, an equivalence between models should be understood as an assertion on the isomorphism of their physical Hilbert spaces. This kind of equivalence is extremely rare. It is believed to occur in the context of duality transformations and is witnessed in the context of two-dimensional field theory as in the bosonization phenomenon [16]. In establishing this isomorphism, the operator and functional integral methods are usually complementary tools [17].

Our concern here, however, is with much less stringent equivalences. We shall, first, derive an equivalence between the usual Walecka model to the linear point-coupling model with terms of second order in the fermionic density and vector current (fourth order in the fermion fields). This is not an equivalence between the Hilbert spaces but of the physical content between the models amenable to mean-field procedures. The mean-field procedures start from discarding the kinetic terms for the scalar and vector fields, which is related to the infinite limit of the masses of the bosons. The derivation of this equivalence will be provided here using functional integral methods. In this section, we discuss this treatment in detail, in order to explain ideas with a view to applying them to interaction Lagrangian models, which contain higher powers of the mesonic fields. This will help us distinguish valid procedures from ones that cannot be applied to this more complex models.

We start out by constructing the generating functionals within the functional integral formalism from which the correlation functions will be obtained. We will then use this formalism to connect the Walecka model to the linear point-coupling model.

For the Walecka model, the generating functional is given by the equality:

$$W[J, A_\mu, \eta, \bar{\eta}] = N \int [D\psi]\left[D\bar{\psi}\right][DV^\mu][D\phi]\, \mathrm{e}^{iS_S} \quad (2)$$

with

$$N^{-1} = \int \mathrm{d}^4x\, \mathrm{e}^{iS}, \quad (3)$$

$$S = \int \mathrm{d}^4x\, \mathcal{L}_W \quad \text{and} \quad (4)$$

$$S_S = \int \mathrm{d}^4x\, \left[\mathcal{L}_W + A_\mu(x)V^\mu(x) + J(x)\phi(x) + \bar{\eta}(x)\psi(x) + \eta(x)\bar{\psi}(x)\right] \quad (5)$$

where $A_\mu(x)$, $J(x)$, $\eta(x)$, and $\bar{\eta}(x)$ are the sources for the vectorial (V^μ), scalar (ϕ), and spinorial ($\bar{\psi}$ and ψ) fields, respectively. S_S and S are the actions with and without the source terms. Here, we define $V'^\mu \equiv m_V V^\mu$, $\phi' \equiv m_s\phi$, $G'_s \equiv g_s/m_s$, $G'_V \equiv g_V/m_V$, $A'_\mu \equiv A_\mu/m_V$, and $J' \equiv J/m_s$. With these definitions we can write

$$\mathcal{L}_W = \mathcal{L}'_W + \frac{1}{2m_s^2}\partial^\mu\phi'\partial_\mu\phi' - \frac{1}{4m_V^2}F'^{\mu\nu}F'_{\mu\nu} \equiv \mathcal{L}'_W + U\left(\phi', V'^\mu\right) \quad (6)$$

where

$$F'^{\mu\nu} = \partial^\mu V'^\nu - \partial^\nu V'^\mu \quad \text{and} \quad (7)$$

$$\mathcal{L}'_W = \bar{\psi}\left(i\gamma^\mu\partial_\mu - M\right)\psi - \frac{1}{2}\phi'^2 + \frac{1}{2}V'_\mu V'^\mu - G'_s\bar{\psi}\phi'\psi - G'_V\bar{\psi}\gamma^\mu V'_\mu\psi\,. \quad (8)$$

Now, the generating functional Eq. (2) may be written in the following form:

$$W\left[J', A'_\mu, \eta, \bar{\eta}\right] = N\int[D\psi]\left[D\bar{\psi}\right][DV'^\mu]\left[D\phi'\right] \times \exp\left\{i\left[\int \mathrm{d}^4x\, U\left(\phi', V'^\mu\right) + S'_S\right]\right\} \quad (9)$$

where

$$S'_S = \int \mathrm{d}^4x\, \left[\mathcal{L}'_W + A'_\mu(x)V'^\mu(x) + J'(x)\phi'(x) + \bar{\eta}(x)\psi(x) + \eta(x)\bar{\psi}(x)\right]\,. \quad (10)$$

In order to make contact with mean-field methods, we consider the limit in which the mesonic masses become very large, so that terms involving $1/m_s^2$ and $1/m_V^2$ can be treated perturbatively in a generating functional expansion. It is in this perspective that we identify the fields in $U(\phi', V'^\mu)$ with the respective functional derivatives,

$$\begin{aligned} U(\phi', V'^\mu) &= \frac{1}{2m_s^2}\partial^\mu\phi'\partial_\mu\phi' - \frac{1}{4m_V^2}F'^{\mu\nu}F'_{\mu\nu} \\ &= \frac{1}{2m_s^2}\left(\partial^\mu\frac{\delta}{\delta J'}\right)^2 - \frac{1}{4m_V^2}\left(\partial^\nu\frac{\delta}{\delta A'_\mu} - \partial^\mu\frac{\delta}{\delta A'_\nu}\right)^2 \\ &= U\left(\frac{\delta}{\delta J'}, \frac{\delta}{\delta A'_\mu}\right), \end{aligned} \quad (11)$$

which allows us to write Eq. (9) as follows:

$$W\left[J', A'_\mu, \eta, \bar{\eta}\right] = N\, \mathrm{e}^{i\left[\int \mathrm{d}^4x\, U\left(\frac{\delta}{\delta J'}, \frac{\delta}{\delta A'_\mu}\right)\right]} \times \int[D\psi]\left[D\bar{\psi}\right]\left[DV'^\mu\right]\left[D\phi'\right]\, \mathrm{e}^{iS'_S}. \quad (12)$$

Up to now, we have not changed the physical content of the Walecka model but merely rewritten the generating functional in a form suited to a nonstandard expansion. We consider in the following the zeroth-order term of the expansion of the generating functional with the approximation

$$e^{i\left[\int d^4x\, U\left(\frac{\delta}{\delta J'},\frac{\delta}{\delta A'_\mu}\right)\right]} \simeq 1, \tag{13}$$

so that

$$W_{MF}\left[J', A'_\mu, \eta, \bar{\eta}\right] = N\int [D\psi]\left[D\bar{\psi}\right]\left[DV'^{\mu}\right]\left[D\phi'\right]\, e^{iS'_S} \tag{14}$$

in which the kinetic terms associated to the mesonic fields are neglected, and W_{MF} refers to the generating functional associated to mean-field treatment.

We now examine the structural properties of the model. Originally, the intrinsic algebra of fields was generated by the irreducible set of fields $\mathbf{S_1} = \{\psi, \bar{\psi}, \phi, V^\mu\}$. Now, in the zeroth-order approximation, since the mesonic kinetic terms have been suppressed, the equations of motion will allow us to express the mesonic fields in terms of the fermion densities explicitly in this case. The algebra of fields has been converted into a reducible algebra, and the irreducible algebra of fields is now constructed from the fermionic fields $\mathbf{S_2} = \{\bar{\psi}, \psi\}$, only. This seems to be a mathematical counterpart to the spirit of the mean-field treatment. Of course the mesonic fields are still in play in the dynamics of the model, since they are still coupled to the fermionic fields, but we have lost control on the independent degrees of freedom associated to the mesonic fields. The physical picture associated with this mathematical aspect will be discussed at the end of this subsection and of Section 2.2.

We proceed to the last step of the process, expressing the mesonic fields in terms of the fermion degrees and expressing the dynamics solely in terms of fermion fields. To do so in the functional integral formalism, we decouple the mesonic fields reducing them to auxiliary fields devoid of physical content. To decouple them, we will proceed with a transformation of the mesonic fields to the auxiliary ones. Before this, since we are not interested in analyzing the mesonic correlation functions, we will set $J'(x) = A'_\mu(x) = 0$ in Eq. (10). With this condition and the following identities,

$$-\frac{1}{2}\phi'^2 - G'_s\bar{\psi}\phi'\psi = -\frac{1}{2}\left(\phi' + G'_s\bar{\psi}\psi\right)^2 + \frac{1}{2}{G'_s}^2\left(\bar{\psi}\psi\right)^2 \tag{15}$$

$$\frac{1}{2}V'^{\mu}V'_\mu - G'_V\bar{\psi}\gamma^\mu V'_\mu\psi = \frac{1}{2}\left(V'^{\mu} - G'_V\bar{\psi}\gamma^\mu\psi\right)^2 - \frac{1}{2}{G'_V}^2\left(\bar{\psi}\gamma^\mu\psi\right)^2, \tag{16}$$

Eq. (10) may be rewritten as follows:

$$S'_S = \int d^4x\left[\bar{\psi}\left(i\gamma^\mu\partial_\mu - M\right)\psi - \frac{1}{2}\left(\phi' + G'_s\bar{\psi}\psi\right)^2 + \frac{1}{2}{G'_s}^2\left(\bar{\psi}\psi\right)^2 + \frac{1}{2}\left(V'^{\mu} - G'_V\bar{\psi}\gamma^\mu\psi\right)^2 - \frac{1}{2}{G'_V}^2\left(\bar{\psi}\gamma^\mu\psi\right)^2 + \bar{\eta}(x)\psi(x) + \eta(x)\bar{\psi}(x)\right]. \tag{17}$$

We now change variables, from ϕ' and V'^{μ} to λ and R^μ:

$$\lambda = \phi' + G'_s\bar{\psi}\psi \tag{18}$$

$$R^\mu = V'^{\mu} - G'_V\bar{\psi}\gamma^\mu\psi\,. \tag{19}$$

With this the generating functional, Eq. (14) becomes

$$W[\eta,\bar{\eta}] = N\int [D\lambda]\, e^{-i\int d^4x\,\frac{\lambda^2}{2}} \times \int [DR^\mu]\, e^{i\int d^4x \frac{1}{2}R^\mu R_\mu}\int [D\psi]\left[D\bar{\psi}\right] e^{iS''_S} \tag{20}$$

$$= \mathcal{N}\int [D\psi]\left[D\bar{\psi}\right]\, e^{iS''_S}, \tag{21}$$

where

$$\mathcal{N}^{-1} = \int [D\psi]\left[D\bar{\psi}\right] e^{iS''} \tag{22}$$

$$S'' = \int d^4x\left[\bar{\psi}(i\gamma^\mu\partial_\mu - M)\psi + \frac{1}{2}{G'_s}^2\left(\bar{\psi}\psi\right)^2 - \frac{1}{2}{G'_V}^2\left(\bar{\psi}\gamma^\mu\psi\right)^2\right] \quad \text{and} \tag{23}$$

$$S''_S = \int d^4x\left[\bar{\psi}\left(i\gamma^\mu\partial_\mu - M\right)\psi + \frac{1}{2}{G'_s}^2\left(\bar{\psi}\psi\right)^2 - \frac{1}{2}{G'_V}^2\left(\bar{\psi}\gamma^\mu\psi\right)^2 + \bar{\eta}\psi + \eta\bar{\psi}\right]. \tag{24}$$

The Lagrangian density associated to Eqs. (23)–(24) is now simply given by

$$\mathcal{L}_{\text{PC}} = \bar{\psi}\left(i\gamma^\mu\partial_\mu - M\right)\psi + \frac{1}{2}{G'_s}^2\left(\bar{\psi}\psi\right)^2 - \frac{1}{2}{G'_V}^2\left(\bar{\psi}\gamma^\mu\psi\right)^2, \tag{25}$$

devoid of mesonic fields.

The decoupling procedure is just the integration off of the mesonic fields leading to contributions in the Lagrangian density, Eq. (25), that are quadratic in the fermion density and fermion vector current. Indeed Eq. (20) expresses the auxiliary character of the mesonic fields. Their contribution, in the zeroth-order approximation, is resumed in the quadratic terms, and no true quanta can be assigned to the mesons in this approximation. We have thus rigorously derived the equivalence between the linear point-coupling model and the zero-order approximation to the Walecka model. This amounts to an equivalence between the fermion correlation functions, and Wightman functions of the models. A reconstruction of the Hilbert space of the zero-order Walecka and linear point-coupling models will lead to an isomorphism of their Hilbert spaces. On the other hand, this implies a rigorous derivation of the equivalence between the mean-field Walecka model and the linear point-coupling one. It means that both models will lead to the same equations of state, as it was pointed out in [18]. From the physical point of view, it should be said that the meson degrees of freedom are not excited in the infinite mass limit. This is the physical mechanism that leads to the isomorphism of the Hilbert spaces of the mentioned models.

2.2 NLPC Model from the MNLW Ones

In the previous section, we have shown that the point-coupling model was obtained from the Walecka one in the limit of hypermassive mesons. To improve the agreement with experimental data, for finite nuclei [19] and with the bulk properties of infinite nuclear matter, the well-known nonlinear Walecka model [20] adds cubic and quartic scalar self-couplings to the Walecka model:

$$\mathcal{L}_{\rm NLW} = \mathcal{L}_W - \frac{A}{3}\phi^3 - \frac{B}{4}\phi^4. \tag{26}$$

Indeed, there is a family of acceptable NLW models which differ in respect to how the A and B free parameters are chosen to fit different sets of experimental nuclear data [21]. As we have pointed out before, higher order point-coupling models involving $(\bar\psi\psi)^3$ and $(\bar\psi\psi)^4$ (NLPC) have also been successfully applied to finite nuclei [9].

Still at the finite-range level, i.e., finite meson masses, different kinds of Walecka-type models such as variants of NLW ones [22, 23], with density-dependent coupling constants [24], and the linear chiral model [25], were also used in the description of nuclear frameworks. The NLW models derived from a quark model perspective can be found in ref. [26]. In particular, the authors show that the Walecka model is the limit of infinite quark mass, in which the quark dynamics freezes.

The question we pose in this section is whether a NLW model, Eq. (26), leads to a nonlinear point-coupling model in the limit of infinite meson masses, which includes cubic and quartic self-fermionic terms:

$$\mathcal{L}_{\rm NLPC} = \bar\psi(i\gamma^\mu\partial_\mu - M)\psi + \frac{1}{2}{G'_s}^2\left(\bar\psi\psi\right)^2 - \frac{1}{2}{G'_V}^2\left(\bar\psi\gamma^\mu\psi\right)^2 + \frac{A'}{3}\left(\bar\psi\psi\right)^3 + \frac{B'}{4}\left(\bar\psi\psi\right)^4. \tag{27}$$

The answer is negative. Indeed, reproducing the procedure of the last section with Eq. (17), instead of Eq. (14), integrating away the vectorial field, we obtain the following result:

$$W_{MF-NL}[\eta,\bar\eta] = N\int[D\psi]\left[D\bar\psi\right]\left[D\phi'\right]\, e^{iS'_{S-NL}} \tag{28}$$

with

$$S'_{S-NL} = \int d^4x\Big[\bar\psi\left(i\gamma^\mu\partial_\mu - M\right)\psi - \frac{1}{2}\left(\phi' + G'_s\bar\psi\psi\right)^2 + \frac{1}{2}{G'_s}^2\left(\bar\psi\psi\right)^2 - \frac{A}{3}\phi'^3 - \frac{B}{4}\phi'^4 - \frac{1}{2}{G'_V}^2 \times\left(\bar\psi\gamma^\mu\psi\right)^2 + \bar\eta(x)\psi(x) + \eta(x)\bar\psi(x)\Big]. \tag{29}$$

Now the functional integral for the field ϕ' can no longer be explicitly performed, and all we can say is that it gives rise to an unknown functional of $\bar\psi\psi$. With this, the identification between NLW and NLPC fails. That is, we cannot assert the formal equivalence between NLW and NLPC even at the restricted sense of zero-order expansion in the kinetic terms. An approach connecting the NLW and NLPC models has been nicely developed in ref. [27] where the authors use an expansion in the meson propagators treating the nonlinearity in the ϕ field by an iterative process.

In order to gain a deeper understanding on how to obtain the NLPC model, Eq. (27), in the meson hypermassive limit, we consider here a modification of $\mathcal{L}_{\rm NLW}$ that includes second and third powers of the scalar meson field coupled to the appropriate powers of the fermion scalar density, which allows us to decouple the scalar meson field when the scalar mass goes to infinity.

We consider then the modified nonlinear Lagrangian:

$$\mathcal{L}_{\text{MNLW}} = \mathcal{L}_W + \mathcal{L}_3 + \mathcal{L}_4 \tag{30}$$

where

$$\mathcal{L}_3 = -\frac{A'}{3}\left[\left(\frac{m_s^2}{g_s}\right)^3\phi^3 + 3\left(\frac{m_s^2}{g_s}\right)^2\phi^2\bar{\psi}\psi + 3\frac{m_s^2}{g_s}\phi\left(\bar{\psi}\psi\right)^2\right] \text{ and} \tag{31}$$

$$\mathcal{L}_4 = -\frac{B'}{4}\left[\left(\frac{m_s^2}{g_s}\right)^4\phi^4 + 4\left(\frac{m_s^2}{g_s}\right)^3\phi^3\bar{\psi}\psi + 6\left(\frac{m_s^2}{g_s}\right)^2\phi^2\left(\bar{\psi}\psi\right)^2 + 4\frac{m_s^2}{g_s}\phi\left(\bar{\psi}\psi\right)^3\right] \tag{32}$$

The generating functional for this Lagrangian is given by Eq. (2) with $\mathcal{L}_W$ substituted for $\mathcal{L}_{\text{MNLW}}$.

Again, the definitions for V'^{μ}, ϕ', G'_s, G'_V, A'_{μ}, and J', turn Eq. (30) into the following:

$$\begin{aligned}\mathcal{L}_{\text{MNLW}} &= \bar{\psi}(i\gamma^{\mu}\partial_{\mu} - M)\psi + \frac{1}{2m_s^2}\partial^{\mu}\phi'\partial_{\mu}\phi' - \frac{1}{2}\phi'^2 \\ &\quad -\frac{1}{4m_V^2}F'^{\mu\nu}F'_{\mu\nu} + \frac{1}{2}V'_{\mu}V'^{\mu} - G'_s\bar{\psi}\phi'\psi \\ &\quad -G'_V\bar{\psi}\gamma^{\mu}V'_{\mu}\psi + \mathcal{L}'_3 + \mathcal{L}'_4 \qquad (33)\\ &\equiv \mathcal{L}'_{\text{MNLW}} + \mathcal{L}'_3 + \mathcal{L}'_4 + U\left(\phi', V'^{\mu}\right) \qquad (34)\end{aligned}$$

where

$$\mathcal{L}'_{\text{MNLW}} = \bar{\psi}(i\gamma^{\mu}\partial_{\mu} - M)\psi - \frac{1}{2}\phi'^2 + \frac{1}{2}V'_{\mu}V'^{\mu} - G'_s\bar{\psi}\phi'\psi - G'_V\bar{\psi}\gamma^{\mu}V'_{\mu}\psi\,, \tag{35}$$

$$\mathcal{L}'_3 = -\frac{A'}{3}\left[\frac{1}{G_s'^3}\phi'^3 + \frac{3}{G_s'^2}\phi'^2\bar{\psi}\psi + \frac{3}{G'_s}\phi'\left(\bar{\psi}\psi\right)^2\right], \tag{36}$$

$$\mathcal{L}'_4 = -\frac{B'}{4}\left[\frac{1}{G_s'^4}\phi'^4 + \frac{4}{G_s'^3}\phi'^3\bar{\psi}\psi + \frac{6}{G_s'^2}\phi'^2\left(\bar{\psi}\psi\right)^2 + \frac{4}{G'_s}\phi'\left(\bar{\psi}\psi\right)^3\right] \tag{37}$$

with $U(\phi', V'^{\mu})$ given by Eq. (11). Therefore, the generating functional can be rewritten:

$$W[J', A'_{\mu}, \eta, \bar{\eta}] = N\int[D\psi]\left[D\bar{\psi}\right][DV'^{\mu}]\left[D\phi'\right] \times \times\, e^{i\left[\int d^4x\, U(\phi', V'^{\mu}) + S'_S\right]} \tag{38}$$

where

$$S'_S = \int d^4x\big[\mathcal{L}'_{\text{MNLW}} + \mathcal{L}'_3 + \mathcal{L}'_4 + A'_{\mu}(x)V'^{\mu}(x) + J'(x)\phi'(x) + \bar{\eta}(x)\psi(x) + \eta(x)\bar{\psi}(x)\big]\,. \tag{39}$$

Again, $U(\phi', V'^{\mu})$ will be treated perturbatively in the generating functional. The zeroth-order approximation, Eq. (13), leads to

$$W[J', A'_{\mu}, \eta, \bar{\eta}] = N\int[D\psi][D\bar{\psi}][DV'^{\mu}][D\phi']\, e^{iS'_S}\,. \tag{40}$$

As previously, we will discard the control on the mesonic correlation functions by taking $J'(x) = A'_{\mu}(x) = 0$ in Eq. (39). Now, along with Eqs. (15)–(16), we will use the following set of identities:

$$\begin{aligned}&-\frac{A'}{3}\left[\frac{1}{G_s'^3}\phi'^3 + \frac{3}{G_s'^2}\phi'^2\bar{\psi}\psi + \frac{3}{G'_s}\phi'\left(\bar{\psi}\psi\right)^2\right] \\ &\quad = -\frac{A'}{3G_s'^3}\left(\phi' + G'_s\bar{\psi}\psi\right)^3 + \frac{A'}{3}\left(\bar{\psi}\psi\right)^3,\end{aligned} \tag{41}$$

$$\begin{aligned}&-\frac{B'}{4}\left[\frac{1}{G_s'^4}\phi'^4 + \frac{4}{G_s'^3}\phi'^3\bar{\psi}\psi + \frac{6}{G_s'^2}\phi'^2\left(\bar{\psi}\psi\right)^2 + \frac{4}{G'_s}\phi'\left(\bar{\psi}\psi\right)^3\right] \\ &\quad = -\frac{B'}{4G_s'^4}\left(\phi' + G'_s\bar{\psi}\psi\right)^4 + \frac{B'}{4}\left(\bar{\psi}\psi\right)^4,\end{aligned} \tag{42}$$

which allows us to rewrite Eq. (39) as follows:

$$\begin{aligned}S'_S = \int d^4x\,\Big[&\bar{\psi}(i\gamma^{\mu}\partial_{\mu} - M)\psi - \frac{1}{2}\left(\phi' + G'_s\bar{\psi}\psi\right)^2 \\ &+ \frac{1}{2}G_s'^2\left(\bar{\psi}\psi\right)^2 + \frac{1}{2}\left(V'^{\mu} - G'_V\bar{\psi}\gamma^{\mu}\psi\right)^2 \\ &- \frac{1}{2}G_V'^2\left(\bar{\psi}\gamma^{\mu}\psi\right)^2 - \frac{A'}{3G_s'^3}\left(\phi' + G'_s\bar{\psi}\psi\right)^3 \\ &+ \frac{A'}{3}\left(\bar{\psi}\psi\right)^3 - \frac{B'}{4G_s'^4}\left(\phi' + G'_s\bar{\psi}\psi\right)^4 \\ &+ \frac{B'}{4}\left(\bar{\psi}\psi\right)^4 + \bar{\eta}(x)\psi(x) + \eta(x)\bar{\psi}(x)\Big]\,.\end{aligned} \tag{43}$$

If we once again define the change of fields leading to the auxiliary fields,

$$\lambda = \phi' + G'_s\bar{\psi}\psi \tag{44}$$

$$R^{\mu} = V'^{\mu} - G'_V\bar{\psi}\gamma^{\mu}\psi, \tag{45}$$

we will have the following forms for the mesonic field integrals in Eq. (38):

$$\int [D\phi'] \exp\left\{-i\int \mathrm{d}^4x \left[\frac{1}{2}\left(\phi'+G'_s\bar{\psi}\psi\right)^2 + \frac{A'}{3G_s'^3}\left(\phi'+G'_s\bar{\psi}\psi\right)^3 + \frac{B'}{4G_s'^4}\left(\phi'+G'_s\bar{\psi}\psi\right)^4\right]\right\} = \int [D\lambda]\, \mathrm{e}^{-i\int \mathrm{d}^4x\left[\frac{1}{2}\lambda^2+\frac{A'}{3G_s'^3}\lambda^3+\frac{B'}{4G_s'^4}\lambda^4\right]} \tag{46}$$

and

$$\int [DV'^{\mu}]\, \mathrm{e}^{i\int \mathrm{d}^4x\, \frac{1}{2}(V'^{\mu}-G'_V\bar{\psi}\gamma^{\mu}\psi)^2} = \int [DR^{\mu}]\, \mathrm{e}^{i\int \mathrm{d}^4x\, \frac{1}{2}R^{\mu}R_{\mu}}. \tag{47}$$

The identities and translations above allow us to rewrite Eq. (40) as follows:

$$W[\eta,\bar{\eta}] = \mathcal{N}\int [D\psi]\left[D\bar{\psi}\right]\, \mathrm{e}^{iS''_S} \tag{48}$$

where

$$\mathcal{N}^{-1} = \int [D\psi]\left[D\bar{\psi}\right]\, \mathrm{e}^{iS''}, \tag{49}$$

$$S'' = \int \mathrm{d}^4x \left[\bar{\psi}(i\gamma^{\mu}\partial_{\mu} - M)\psi + \frac{1}{2}G_s'^2\left(\bar{\psi}\psi\right)^2 - \frac{1}{2}G_V'^2\left(\bar{\psi}\gamma^{\mu}\psi\right)^2 + \frac{A'}{3}\left(\bar{\psi}\psi\right)^3 + \frac{B'}{4}\left(\bar{\psi}\psi\right)^4\right] \tag{50}$$

$$S''_S = \int \mathrm{d}^4x \left[\bar{\psi}(i\gamma^{\mu}\partial_{\mu} - M)\psi + \frac{1}{2}G_s'^2\left(\bar{\psi}\psi\right)^2 - \frac{1}{2}G_V'^2\left(\bar{\psi}\gamma^{\mu}\psi\right)^2 + \frac{A'}{3}\left(\bar{\psi}\psi\right)^3 + \frac{B'}{4}\left(\bar{\psi}\psi\right)^4 + \bar{\eta}(x)\psi(x) + \eta(x)\bar{\psi}(x)\right]. \tag{51}$$

The Lagrangian density in Eqs. (50)–(51) describes the fermionic nonlinear point-coupling model we wanted. We have seen that the generating Lagrangian to obtain the NLPC model through the mesonic hypermassive limit is $\mathcal{L}_{\mathrm{MNLW}}$, given by Eq. (30), and not $\mathcal{L}_{\mathrm{NLW}}$ as it could naively be expected.

Let us consider the structural properties of the model. We are not asserting here an equivalence of NLPC and MNLW models. The Hilbert spaces of the two models are not isomorphic. But the zeroth-order expansion of the MNLW model has been exactly mapped onto the NLPC models. Once again from the viewpoint of structural analysis, the irreducible algebra of fields of MNLW composed of the polynomial algebra of the fermion and meson fields becomes reducible in the zeroth-order approximation. The mesonic fields turn out to be functions of the fermion fields, and the irreducible algebra is composed solely of the fermion field algebra. In the language of functional integrals, this is implemented by the decoupling of the auxiliary fields λ and R^{μ}. The equations of motion of the auxiliary fields bring about the functional relation between the original mesonic fields and the fermion bilinears. Contrary to the linear case treated in the preceding section, now the equations of motion do not demand $\lambda = 0$ and $R^{\mu} = 0$. The equations for the auxiliary fields includes, in principle, other roots besides the trivial ones. Actually, as we will discuss in the next section, the equations of state of the MNLW model in the mean-field approximation depend on the mean value of the auxiliary field λ and differ from those of the NLPC model only by the terms containing this field. However, the physical requirement of vanishing pressure at zero Fermi momentum is satisfied only by the trivial solution for the auxiliary field λ.

Consider now the renormalization properties of the models. The infinite mass expansion effectively changes the power-counting dimensions of the mesonic fields. Since their kinetic terms are discarded, there appear no inverse powers of the momenta in their propagators in the ultraviolet region. The result is that the $\mathcal{L}_{\mathrm{NLPC}}$ models are nonrenormalizable, while the $\mathcal{L}_{\mathrm{NLW}}$ are (power-counting) renormalizable. The physical reason for this change deserves emphasis: our approximation freezes the meson degrees of freedom that are necessary to render the Walecka model (power-counting) renormalizable.

3 Mean-field Approximation

We now follow an alternative procedure to derive the NLPC model from the meson-exchange MNLW one. Here, we adopt the largely used mean-field approach (MFA), instead of the infinite meson mass limit in the previous section. We also use the no-sea approximation, i.e., we consider only the valence Fermi states. We will show that the EOS for the MNLW and NLPC models are exactly the same.

For infinite nuclear matter, the energy density and pressure of the NLPC models are given by the following:

$$\mathcal{E} = \frac{1}{2}{G'_V}^2\rho^2 + \frac{1}{2}{G'_s}^2\rho_s^2 + \frac{2}{3}A'\rho_s^3 + \frac{3}{4}B'\rho_s^4 + \frac{\gamma}{2\pi^2}\int_0^{k_F} k^2\left(k^2 + M^{*2}\right)^{1/2}\mathrm{d}k \tag{52}$$

and

$$P = \frac{1}{2}{G'_V}^2\rho^2 - \frac{1}{2}{G'_s}^2\rho_s^2 - \frac{2}{3}A'\rho_s^3 - \frac{3}{4}B'\rho_s^4 + \frac{\gamma}{6\pi^2}\int_0^{k_F}\frac{k^4}{\left(k^2 + M^{*2}\right)^{1/2}}, \tag{53}$$

respectively, with the vector and the scalar density defined as

$$\rho = \frac{\gamma}{2\pi^2}\int_0^{k_F} k^2\mathrm{d}k \quad \text{and} \tag{54}$$

$$\rho_s = \frac{\gamma}{2\pi^2}\int_0^{k_F}\frac{M^*}{\left(k^2 + M^{*2}\right)^{1/2}}k^2\mathrm{d}k, \tag{55}$$

with k_F being the Fermi momentum, $\gamma = 4$ for symmetric nuclear matter, and $\gamma = 2$ for neutron matter. The nucleon effective mass reads

$$M^* \equiv M - {G'_s}^2\rho_s - A'\rho_s^2 - B'\rho_s^3. \tag{56}$$

Let us now start to derive the MNLW equations of state by first rewriting its Lagrangian, Eq. (30), as follows:

$$\begin{aligned}\mathcal{L}_{\mathrm{MNLW}} = {} & \bar{\psi}\left(i\gamma^\mu\partial_\mu - M\right)\psi + \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} \\ & + \frac{1}{2}m_V^2 V_\mu V^\mu - g_V\bar{\psi}\gamma^\mu V_\mu\psi \\ & - \frac{1}{2}\left(m_s\phi + \frac{g_s}{m_s}\bar{\psi}\psi\right)^2 + \frac{g_s^2}{2m_s^2}\left(\bar{\psi}\psi\right)^2 \\ & - \frac{A'}{3}\left(\frac{m_s}{g_s}\phi + \bar{\psi}\psi\right)^3 + \frac{A'}{3}\left(\bar{\psi}\psi\right)^3 \\ & - \frac{B'}{4}\left(\frac{m_s^2}{g_s}\phi + \bar{\psi}\psi\right)^4 + \frac{B'}{4}\left(\bar{\psi}\psi\right)^4 .\end{aligned} \tag{57}$$

Given that a field translation does not alter the physical content of the model, we define

$$\lambda \equiv \frac{m_s^2}{g_s}\phi + \bar{\psi}\psi\,. \tag{58}$$

With this definition, the MNLW Lagrangian acquires the following form:

$$\begin{aligned}\mathcal{L}_{\mathrm{MNLW}} = {} & \bar{\psi}\left(i\gamma^\mu\partial_\mu - M\right)\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}m_V^2 V_\mu V^\mu \\ & - g_V\bar{\psi}\gamma^\mu V_\mu\psi - \frac{1}{2}{G'_s}^2\lambda^2 + \frac{1}{2}{G'_s}^2\left(\bar{\psi}\psi\right)^2 \\ & - \frac{A'}{3}\lambda^3 + \frac{A'}{3}\left(\bar{\psi}\psi\right)^3 - \frac{B'}{4}\lambda^4 + \frac{B'}{4}\left(\bar{\psi}\psi\right)^4 \\ & + \frac{G'_s}{2m_s^2}\left(\partial^\mu\lambda - \partial^\mu\bar{\psi}\psi\right)\left(\partial_\mu\lambda - \partial_\mu\bar{\psi}\psi\right),\end{aligned} \tag{59}$$

where $G'_s = g_s/m_s$.

In the MFA, the scalar and vector mesonic fields are replaced by their average values:

$$\lambda \to \langle\lambda\rangle \equiv \lambda \tag{60}$$

$$V^\mu \to \langle V^\mu\rangle \equiv \delta^{\mu 0}V^0\,. \tag{61}$$

Still in this approximation, we use the $\bar{\psi}\psi$ ground-state expectation value. We also assume the system to be spatially uniform, so that the derivative terms of λ and $\bar{\psi}\psi$ disappear. Therefore, Eq. (59) becomes

$$\begin{aligned}\mathcal{L}_{\mathrm{MNLW}}^{(\mathrm{MFA})} = {} & \bar{\psi}\left(i\gamma^\mu\partial_\mu - M\right)\psi + \frac{1}{2}m_V^2 V_0^2 - g_V\bar{\psi}\gamma^0 V_0\psi \\ & - \frac{1}{2}{G'_s}^2\lambda^2 + \frac{1}{2}{G'_s}^2\left(\bar{\psi}\psi\right)^2 - \frac{A'}{3}\lambda^3 + \frac{A'}{3}\left(\bar{\psi}\psi\right)^3 \\ & - \frac{B'}{4}\lambda^4 + \frac{B'}{4}\left(\bar{\psi}\psi\right)^4 .\end{aligned} \tag{62}$$

The independent fields of this theory may be taken as λ, $\bar{\psi}$, ψ, and V^0. From the Euler–Lagrange equations, one obtains the equations of motion for the fields:

$$\lambda({G'_s}^2 + A'\lambda + B'\lambda^2) = 0\,, \tag{63}$$

$$V_0 = \frac{g_V}{m_V^2}\bar{\psi}\gamma_0\psi \tag{64}$$

and

$$\left[i\gamma^\mu\partial_\mu - g_V\gamma^0 V_0 - \left(M - {G'_s}^2(\bar{\psi}\psi) - A'(\bar{\psi}\psi)^2 - B'(\bar{\psi}\psi)^3\right)\right]\psi = 0\,. \tag{65}$$

Now, substituting $\bar{\psi}\psi$ and $\bar{\psi}\gamma_\mu\psi$ by their mean values, we have that

$$V_0 = \frac{g_V}{m_V^2}\langle\bar{\psi}\gamma_0\psi\rangle = \frac{g_V}{m_V^2}\rho \tag{66}$$

and consequently, Eq. (65) may be rewritten as follows:

$$\left[i\gamma^\mu\partial_\mu - \gamma^0{G'_V}^2\rho - \left(M - {G'_s}^2\rho_s - A'\rho_s^2 - B'\rho_s^3\right)\right]\psi = 0, \tag{67}$$

where $G_V'^2 = g_V^2/m_V^2$, $G_s'^2 = g_s^2/m_s^2$, and $\rho_s = \langle\bar\psi\psi\rangle$. The above Dirac equation suggests the definition of the effective mass:

$$M^* = M - G_s'^2\rho_s - A'\rho_s^2 - B'\rho_s^3, \tag{68}$$

a result identical to Eq. (56).

The mean-field equations of state will come from the energy–momentum tensor:

$$\begin{aligned}T_{\mu\nu}^{(\mathrm{MFA})} =& -g_{\mu\nu}\Big[\bar\psi(i\gamma^\alpha\partial_\alpha - g_V\gamma^0 V_0 - M)\psi + \frac{1}{2}m_V^2V_0^2 \\ &-\frac{1}{2}G_s'^2\lambda^2 + \frac{1}{2}G_s'^2(\bar\psi\psi)^2 - \frac{A'}{3}\lambda^3 \\ &+\frac{A'}{3}(\bar\psi\psi)^3 - \frac{B'}{4}\lambda^4 + \frac{B'}{4}(\bar\psi\psi)^4\Big] \\ &+ i\bar\psi\gamma_\mu\partial_\nu\psi \\ =& -g_{\mu\nu}\Big[-\frac{1}{2}G_s'^2(\bar\psi\psi)^2 - \frac{2A'}{3}(\bar\psi\psi)^3 - \frac{3B'}{4}(\bar\psi\psi)^4 \\ &+\frac{1}{2}m_V^2V_0^2 - \frac{1}{2}G_s'^2\lambda^2 - \frac{A'}{3}\lambda^3 - \frac{B'}{4}\lambda^4\Big] \\ &+ i\bar\psi\gamma_\mu\partial_\nu\psi\,. \end{aligned}\tag{69}$$

The density energy is obtained from

$$\begin{aligned}\mathcal{E} &= \left\langle T_{00}^{(\mathrm{MFA})}\right\rangle \\ &= \frac{1}{2}G_s'^2\rho_s^2 + \frac{2A'}{3}\rho_s^3 + \frac{3B'}{4}\rho_s^4 - \frac{1}{2}G_V'^2\rho^2 + \frac{1}{2}G_s'^2\lambda^2 \\ &\quad+\frac{A'}{3}\lambda^3 + \frac{B'}{4}\lambda^4 + i\left\langle\bar\psi\gamma_0\partial_0\psi\right\rangle,\end{aligned}\tag{70}$$

where we have used Eq. (66).

The quantity $i\langle\bar\psi\gamma_0\partial_0\psi\rangle$ is found from the dispersion relation $k_0 = g_V V_0 + (k^2 + M^{*2})^{1/2} = G_V'^2\rho + (k^2 + M^{*2})^{1/2}$, where k_0 is the fourth energy–momentum component. This leads to

$$\begin{aligned}\mathcal{E} =& \frac{1}{2}G_V'^2\rho^2 + \frac{1}{2}G_s'^2\rho_s^2 + \frac{2A'}{3}\rho_s^3 + \frac{3B'}{4}\rho_s^4 + \frac{1}{2}G_s'^2\lambda^2 \\ &+\frac{A'}{3}\lambda^3 + \frac{B'}{4}\lambda^4 + \frac{\gamma}{2\pi^2}\int_0^{k_F}\left(k^2 + M^{*2}\right)^{1/2}k^2\mathrm{d}k\,.\end{aligned}\tag{71}$$

The pressure is obtained from the expression

$$\begin{aligned}P &= \frac{1}{3}\langle T_{ii}\rangle \\ &= \frac{1}{2}G_V'^2\rho^2 - \frac{1}{2}G_s'^2\rho_s^2 - \frac{2A'}{3}\rho_s^3 - \frac{3B'}{4}\rho_s^4 - \frac{1}{2}G_s'^2\lambda^2 \\ &\quad-\frac{A'}{3}\lambda^3 - \frac{B'}{4}\lambda^4 + \frac{1}{3}i\left\langle\bar\psi\gamma_i\partial_i\psi\right\rangle.\end{aligned}\tag{72}$$

By extracting $i\langle\bar\psi\gamma_i\partial_i\psi\rangle$ from the the Dirac equation, we can write the pressure as follows:

$$\begin{aligned}P =& \frac{1}{2}G_V'^2\rho^2 - \frac{1}{2}G_s'^2\rho_s^2 - \frac{2A'}{3}\rho_s^3 - \frac{3B'}{4}\rho_s^4 - \frac{1}{2}G_s'^2\lambda^2 \\ &-\frac{A'}{3}\lambda^3 - \frac{B'}{4}\lambda^4 + \frac{\gamma}{6\pi^2}\int_0^{k_F}\frac{k^4}{(k^2 + M^{*2})^{1/2}}\mathrm{d}k\,.\end{aligned}\tag{73}$$

The auxiliary λ field is decoupled from the fermionic sector. Its contribution to both the pressure and to the energy can be dropped in view of the physical requirement that the pressure goes to zero when k_F vanishes, implying that only the trivial solution, $\lambda = 0$, of Eq. (63) should be kept. The energy density and the pressure become

$$\begin{aligned}\mathcal{E} =& \frac{1}{2}G_V'^2\rho^2 + \frac{1}{2}G_s'^2\rho_s^2 + \frac{2}{3}A'\rho_s^3 + \frac{3}{4}B'\rho_s^4 \\ &+\frac{\gamma}{2\pi^2}\int_0^{k_F}\left(k^2 + M^{*2}\right)^{1/2}k^2\mathrm{d}k\end{aligned}\tag{74}$$

and

$$\begin{aligned}P =& \frac{1}{2}G_V'^2\rho^2 - \frac{1}{2}G_s'^2\rho_s^2 - \frac{2}{3}A'\rho_s^3 - \frac{3}{4}B'\rho_s^4 \\ &+\frac{\gamma}{6\pi^2}\int_0^{k_F}\frac{k^4}{(k^2 + M^{*2})^{1/2}}\mathrm{d}k.\end{aligned}\tag{75}$$

The above equations for the MNLW model are identical to Eqs. (52) and (53) for the NLPC models, which goes to show that the former can be also obtained from the mean-field approximation at the level of the EOS instead of the Lagrangian density framework, illustrated by the hypermassive limit in Section 2.

4 Conclusion

The hypermassive meson limit of the usual NLW models fails to yield baryonic NLPC models. We have shown that in order to obtain NLPC models, the NLW models must be modified already at the level of the Lagrangian density. In this work, we have derived the point-coupling models from a modified NLW by using the hypermassive meson limit in the functional integral method. From this approach, we have shown how the linear PC models can be obtained from the Walecka ones and in the same way, how the MNLW model generates the NLPC one. This relation between the MNLW and NLPC is described by the equivalences of the physical content of these models encoded in their irreducible algebra of fields in the infinite meson masses limit. In addition, from the no-sea approximation, we have also constructed the equations of state of the MNLW model. Even in this alternative, (i.e., without taking the hypermassive meson limit) the EOS are identical to the NLPC EOS. The same NLPC models are therefore obtained, whether we treat the MNLW model from the meson hypermassive limit or directly from the mean-field approach.

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