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A Dynamic Discrete/Continuous Choice Model for Forward-Looking Agents Owning One or More Vehicles

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Abstract During the last 40 years, a large number of studies have analyzed car holding and use behavior. Most of these ignore the dynamics of household and driver needs that very likely drive such decisions. Our work builds up on a disaggregate (compensatory) approach using revealed choices to address these dynamics. We develop a dynamic discrete/continuous choice model of car holding duration for forward-looking agents. We estimate this model using French panel survey data. Our findings indicate that a household's time preference is a crucial element in car use and holding decisions.

Keywords Forward-looking agents · Discrete/continuous choice modeling · Transportation demand

JEL Classification C35 · C41 · D12

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1 Introduction

During the last decade more than 80 % of French households chose to own motor vehicles, and the average holding duration of a car has increased.

The modeling of car demand and use plays a central role in many research fields, such as energy management, public transportation planning, auto industry policies, and air quality monitoring. It is thus of great importance to identify factors that drive household's decisions about its car fleet[1] composition and use in order to be more accurate in forecasting car demand.

Car demand is made of different dimensions: to cite only a few, how many cars to own and their makes/models/types, which fuel to use, and how much to use their fleet. As already highlighted by Train (1986), car ownership, fleet size and variety, fuel efficiency, and mileage are important considerations when managing transport-related policies that are in line with sustainable development.

Existing literature shows how important are some car attributes, such as purchase price, maintenance and use costs, and value on this choice. Increases in automobile market and fuel prices, contractions in household's budget constraint, and household socio-demographic and geographic characteristics (e.g. age and profession of household head, household size, urban size, dwelling type, transportation supply), as highlighted by the French Automobile Constructors Committee (CCFA 2006), are all important topics in car demand and use modelling.

The durable character of a car implies that agents may consider its purchase as an investment. Costs and benefits of owning a car fleet and using it must be considered over a period of time. As shown in Julliard (2007, Fig. 1), a higher initial expenditure may require a longer period of time to return on investment. But the cost of operating a car also increases with its age due to physical depreciation (Baron 2002). The agent may choose to intensively use a car in the short-run, or to use it less to obtain a longer holding duration, or to adapt mileage to variations of fuel prices, by reducing the use of the car when it increases (Julliard 2007; Graham and Glaister 2004). All these factors have to be accounted for.

A large number of studies have analyzed car holding and use behaviors. Existing literature may be split into several groups depending on the level of disaggregation, data type (describing real or hypothetical choices), existence of dynamics, use of supply/demand equilibrium, and choice dimensions. For our purpose, see for example, Hocherman et al. (1983), Mannering (1983a), Berkovec and Rust (1985), Hensher et al. (1990), Gilbert (1992), de Jong (1990, 1996) and more recently Bhat and Sen (2006). The authors use various data and techniques to analyze the dynamics of car ownership, holding duration and/or use at the disaggregate level.

Only few approaches consider fleet size, vehicle type, and mileage to travel as simultaneous choices, an important feature of car demand modeling. See Train, 1986 who proposed a Generalized Extreme Value (GEV) model considering the unordered aspect of these choices.

[1] With a slight abuse of definition, we will use the term *fleet* in this paper to indicate the stock of cars that an household owns.

Also, as pointed out by de Jong and Kitamura (2009), the majority of these studies consider the problem from a static point of view, ignoring the dynamics that drives such decisions. To our knowledge, with rare exceptions (see also Rust 1987; Xu 2011), even when all demand dimensions are considered as simultaneous, agents are myopic. In the rare case dynamics is explicitly considered, it is almost always backward-looking.

Even if the backward-looking agent assumption might be consistent, car is a durable good: it does not quickly wear out and it yields utility over time. We argue that when decisions on car holding and use are taken, one must in some way consider the future decline of the vehicle.

We here suppose that an agent anticipates the future evolution of his environment. Households choose to scrap or keep a vehicle by anticipation of its future use, fuel price evolution, future changes in household structure, and so on. This paper formulates a discrete/continuous finite-horizon optimal stopping problem under uncertainty to describe the behavior of a household as it regards car holding duration and mileage driven. A household accounts for the uncertain evolution of fuel prices and income. In the present approach, the decision to keep or dispose of a car in the fleet will be taken conditional on household and vehicle characteristics, choices made for other vehicles and principal user's characteristics. We combine a discrete/continuous model and Rust (1987) dynamic forward-looking specification. To our knowledge, it has never been done. Application to car ownership and use is even less common (see again Xu 2011).

As the approach is finite-horizon, it is solved by backward induction. The parameters of the structural model are estimated using a Nested Fixed Point Algorithm.

The model is demonstrated by drawing data from the French "Parc Auto" (Car Fleet) panel survey. We focus on the population of households observed from 2002 to 2008.

The outline of the article is as follows: The model is developed in Sect. 2, discussing the different parts constituting the approach as well as how the parameters of the primitives are estimated. Data are then presented in Sect. 3. The results are debated in Sect. 4. The last section draws conclusions and defines a roadmap for future research.

2 Model

2.1 Framework

Our specification is based on the six assumptions that define what is known as the "dynamic programming − conditional logit" model of Rust (1987): additive separability of utility functions, unobserved state variables are iid with cdf F, conditional independence of future observed state variables with current unobserved state variables, conditional independence of pay-off variables with current unobserved state variables, unobserved state variables are extreme value type 1 identically and independently distributed, discrete support of observed state variables. This framework is the simplest for estimation: it is computationally manageable but it comes at the cost of rather restrictive assumptions on the interpretation of the model.

Household i simultaneously determine the optimal ownership duration $\bar{t}_j \in \{1, \ldots, T\}$ and mileage $\mathbf{m}_{j,.} = \left(m_{j,1}, \ldots, m_{j,t}\right)$ for every cars j it owns. We assume that these decisions, car disposal or car keeping (d) and use (m), are made at the beginning of each period t, conditional on the household's state and environment. The problem is an optimal "use and stop" problem. When vehicle j is kept, the household chooses a mileage m_{ijt}. If it disposes of the vehicle, the decision process stops. The household has no more decision to make on the driven mileage for this specific vehicle. The decision of whether or not to replace it is here exogeneous[2] and the relation between the choice to dispose and the purchase decision is not explicitly estimated.

For multiple vehicle households, we assume uniform income repartition between cars. Choices concerning a multiple car fleet are not simultaneously estimated in this work. There are no explicit substitution effects between cars. We also do not consider modeling of make/model/type choices.

The vector of state variables is labeled $\mathbf{z}_{ijt}$. It is defined as two subsets, $\mathbf{z}_{ijt} = \left\{\mathbf{x}_{ijt}, \boldsymbol{\epsilon}_{ijt}\right\}$. One is about state variables that are observed by the econometrician, $\mathbf{x}_{ijt}$. The other is a vector of unobserved state variables $\epsilon_{D,ijt}$,$\epsilon_{K,ijt}$ that are assumed to be independently and identically distributed with cumulative distribution function $\exp\left(-\exp\left(-\epsilon_{d,ijt}\right)\right)$. It is also assumed that the unobserved state variables enter the utility functions as additive shocks.

Households' preferences over possible sequences of state variables can be represented by a time separable discounted utility function $\sum_{t=1}^{T} \gamma^t u\left(\mathbf{z}_{ijt}, d\left(\mathbf{z}_{ijt}\right), m\left(\mathbf{z}_{ijt}\right)\right)$ where γ is the discounting factor and $u\left(\mathbf{z}_{ijt}, d\left(\mathbf{z}_{ijt}\right), m\left(\mathbf{z}_{ijt}\right)\right)$ is the per-period utility function.

The sequence of decisions is taken to maximize the expected utility with respect to the distribution of the sequences of state variables:

$$\max_{\bar{t}_i} \left\{ \mathbb{E}_{\mathbf{z}_{i,\cdot}} \left(\sum_{t=1}^{T_i} \gamma^t u\left(\mathbf{z}_{ijt}, d_{ijt}, m_{ijt}\right) | \mathbf{z}_{ij0} \right) \right\} \tag{1}$$

where $\mathbf{z}_{ij0}$ is a set of initial conditions.

The intertemporal optimization problem can be formulated as a sequential decision problem whose solution is the same when using its Bellman representation (see for instance Bellman 1957; Rust 1996; Bertsekas 2000). Let $V(\mathbf{z}_{ijt})$ denote the maximum value of all utility streams to household i knowing its state variables $\mathbf{z}_{ijt}$. $V(\mathbf{z}_{ijt})$ is defined recursively by the solution to the following Bellman equation:

$$V\left(\mathbf{z}_{ijt}\right) = \max\left\{v_D\left(\mathbf{x}_{ijt}\right) + \epsilon_{D,ijt}, v_K\left(\mathbf{x}_{ijt}\right) + \gamma \mathbb{E}_{\mathbf{z}_{ijt+1}}\left[V\left(\mathbf{z}_{ijt+1}\right) | \mathbf{z}_{ijt}\right] + \epsilon_{K,ijt}\right\}. \tag{2}$$

The optimal demand for mileage $m^\star\left(\mathbf{z}_{ijt}\right)$ and the decision to keep or dispose of the car, $d^\star\left(\mathbf{z}_{ijt}\right)$, are the arguments that maximize equation 2. The decisions at period t affect the evolution of future values of the state variables, but the household faces uncertainty about these future values. The beliefs of household i about sequences of states are modeled by a Markov transition distribution function $G_z\left(\mathbf{z}_{ijt+1}|\mathbf{z}_{ijt}, d_{ijt}, m_{ijt}\right)$ where, by convention, $d_{ijt} = d\left(\mathbf{z}_{ijt}\right)$ and $m_{ijt} = m\left(\mathbf{z}_{ijt}\right)$.

[2] If a new car enter in the household's fleet this is accounted in the fleet size and a new decision process starts for this new vehicle.

Considering a vehicle's value as the amount a household invests every time it decides to continue ownership one period more, as proposed by Bento et al. (2009), the utility for household i to keep vehicle j for period t can be defined as follows:

$$v_K\left(\mathbf{z}_{ijt}\right) = \beta_0 - exp\left(-\beta_1\left(y_{ijt} - v_{jt}\right)\right) + exp\left(-\beta_2 o_{jt} + \sum_{d=1}^{D} \mu_d t_{djt}\right) \tag{3}$$

where y_{ijt} is the amount of available income used for vehicle j during period t, and v_{jt} is the vehicle's scrap value at date t modeled as a geometrically decreasing function of ownership duration a_{ijt} and cumulative use k_{ijt}.

$$v_{jt} = \frac{P_j}{1 + a_{ijt} * k_{ijt}} \tag{4}$$

Purchase price P_j belongs to the set of initial conditions and represents a vehicle's scrap value at the purchase date.

$o_{jt} = c_j * p_{jt}$ is vehicle operating cost, computed as a function of vehicle consumption c_j (initial condition) and annual fuel price p_{jt} defined as

$$p_{ijt} = \mathbb{I}\,(\text{fuel type of } i \text{ is petrol})\; p_{\text{petrol},t} + \mathbb{I}\,(\text{fuel type of } i \text{ is diesel})\; p_{\text{diesel},t}, \tag{5}$$

although different households owning cars with the same fuel type are faced with the same market price. This simplifying assumption is used because the available data on fuel prices are national annual averages over the considered period. These prices are actually different at several locations within the French metropolitan territory.

t_{djt} are dummy variables representing age and employment status of principal user of j for period t.

- t_{1j}: principal user less than 40 years old and employed
- t_{2j}: principal user less than 40 years old and unemployed
- t_{3j}: principal user age $\in$ [40; 60[and employed
- t_{4j}: principal user age $\in$ [40; 60[and unemployed

As postulated by Mannering (1983b), principal user characteristics have an important effect on car use choice. As we focus on decisions made for each car separately, and not on the total fleet's choice, only these characteristics are included in the estimation process. Household structure is totally modeled by its income level.

Assuming that ownership decisions for other vehicles are known by household i when it makes the decision for vehicle j, we can calculate available income y_{ijt} as

$$y_{ijt} = Y_{it} / S_{K,it} \tag{6}$$

where Y_{it} is annual household income and $S_{K,it}$ is fleet size obtained from keeping vehicle j. Household income is observed as a categorical variable. Y_{it} is the center value of a household's income class.

There is nothing to discuss regarding the evolution of the ownership duration variable. It is a deterministic state variable that is defined as:

$$a_{ijt} = a_{ijt-1} + 1. \tag{7}$$

The cumulative use at the beginning of a period t is a deterministic function of the chosen mileage at the beginning of period $t-1$, which was optimally chosen as a function of the other state variables.

$$k_{ijt} = k_{ijt-1} + m_{ijt-1}. \tag{8}$$

In the present approach, vehicle j optimal use $m^{\star}_{ijt} = VMT_{K,u}$ is estimated using Roy's Identity:

$$VMT_K = -\frac{\delta v_K/\delta o_{jt}}{\delta v_K/\delta Y_{it}} \tag{9}$$

Under our definition of utility v_K, Marshallian demand for mileage is

$$m^{\star}_{ijt} = \alpha * exp\left(\beta_1\left(\frac{Y_{it}}{T_{K,it}} - \frac{P_j}{1 + a_{ijt} * k_{ijt}}\right) - \beta_2\left(c_j * p_{jt}\right) + \sum_{u=1}^{4}\mu_u t_u\right) \tag{10}$$

The probability for vehicle j to have cumulative use k_{ijt+1} knowing k_{ijt} is the probability for household i to chose mileage $m^{\star}_{ijt}$ for period t:

$$P\left(m^{\star}_{ijt} \mid \mathbf{z}_{ijt}\right) = \frac{1}{(2\pi)^{1/2}\sigma_i} exp\left(-\frac{1}{2}\left(\frac{m^{\star}_{ijt} - m_{ijt}}{\sigma_i}\right)^2\right) \tag{11}$$

As stated by Aguirregabiria and Mira (2010), assumptions on $G_{\mathbf{z}}$ are key modeling decisions in the econometrics of dynamic discrete structural models. The form of the estimation criteria (e.g. log-likelihood function) and choice of the appropriate solution and estimation methods crucially depend on these assumptions. The joint Markov transition distribution of the state variables is defined as the product of marginal distributions.

Fleet size and principal user optimal sequences of choice are assumed to be known by household. It therefore does not face uncertainty about them, and probability transitions are not estimated.

Households are assumed to be sensitive to fuel price evolution stage-by-stage. Expectations about this evolution are then based on a discrete representation of their respective processes. Sensitivity remains the same in between some lower and upper bound values. Our modelling point of view about the evolution of fuel prices is extremely simple. We assume that their dynamics are not stationary (at the second order) over time. $\forall g \in$ {petrol, diesel},

$$p_{g,t} = p_{g,t-1} + \epsilon_{g,t} \tag{12}$$

where $\epsilon_{g,t} \stackrel{iid}{\to} \mathcal{N}\left(0, \sigma_g^2\right)$. Then let $\left(\bar{p}_{g,0}, \ldots, \bar{p}_{g,R}\right)$ be a sequence of predetermined values over the real line. It serves as a bound of contiguous intervals that models the different stages of fuel price g. Let also $\tilde{p}_{g,r} = (\bar{p}_{g,r} + \bar{p}_{g,r+1})/2$ be the centers of these intervals (up to some convention as it regards the first and the last of these intervals). They will model some "representative" values. Then:

$$\Pr\left(p_{g,t} = \tilde{p}_{g,r} | p_{g,t-1} = \tilde{p}_{g,l}, d_{ijt-1} = 0; \sigma_g\right) \Phi\left(\frac{\bar{p}_{g,r} - \tilde{p}_{g,l}}{\sigma_g}\right) - \Phi\left(\frac{\bar{p}_{g,r-1} - \tilde{p}_{g,l}}{\sigma_g}\right) \tag{13}$$

where Φ is the cumulative distribution function of a standard normal distribution. Nevertheless, such an approach neither solves the problem of choosing the right number of discrete intervals to obtain an acceptable representation of the Markov transition probability nor gives a clue about whether choosing the center of the interval as the representative value is correct. In this application, intervals have a range equal to 1 cent of €.

The last observed state variable is household income. The interviewed households have to declare, for each participation year, their annual income class. The income variable is grouped in 13 classes. Stewart (1983), Terza (1985) and Bhat (1994) point out that it is better to consider an interval regression model when income is reported as a discrete variable.

Let $y^\star$ be the latent continuous variable (not observed), y the variable grouped in J classes (observed) and μ_{j-1} , μ_j $j = 1, \ldots, J$ the logarithm[3] of class j bounds. Then:

$$y = j \leftrightarrow \mu_{j-1} < ln(y^\star) < \mu_j$$

Assuming that $ln\left(y^\star\right) = \beta' x + \epsilon, \quad \epsilon \rightsquigarrow Logistic\left(0, \sigma\right)$,[4] the probability to observe the income class j knowing the household's characteristics x writes as:

$$P(y = j \mid x) = \left[F\left(\frac{\mu_j - \beta' x}{\sigma}\right) - F\left(\frac{\mu_{j-1} - \beta' x}{\sigma}\right)\right] \tag{14}$$

where $F(x) = \frac{1}{1+exp(-x)}$ is the logistic function. It is an *Ordered Logit Model with Known Bounds*. By convention, the extreme bounds are defined as $\mu_0 = -\infty$ and $\mu_J = \infty$. One obtains:

$$P(y = 1 \mid x) = F\left(\frac{\mu_1 - \beta' x}{\sigma}\right)$$

and

$$P(y = J \mid x) = 1 - F\left(\frac{\mu_J - \beta' x}{\sigma}\right)$$

[3] It is known that the normality assumption is not adapted for income.

[4] We acknowledge that the hypothesis of normal errors, proposed in the literature, is more credible for the considered problem. However, as probit results are generally proportioned to the logit ones and the latter interpretation is easier than the former, the errors here are considered as logistic.

As bounds μ_j are known, β and σ are identifiable and can be estimated. The log-likelihood associated to the model defines as

$$LL = \sum_i \sum_j I(y_i = j) ln\left(P(y_i = j \mid x_i)\right) \tag{15}$$

where $I(y_i = j) = 1$ if household i income is observed in class j and 0 otherwise.

We also assume that the income follows a simple autoregressive process. The income class in period t is defined as a function of only the income class observed in period $t-1$:

$$\ln\left(y^{\star}_{i,t}\right) = \beta_0 + \sum_{j=1}^{J-1} \beta_j \mathbb{I}\left(y_{i,t-1} = j\right) + \epsilon_{y,i,t} \tag{16}$$

A final assumption is that disposing of a car is an absorbing decision. Thus, once the household disposes of the car, all the transition probabilities are set equal to 0.

2.2 A Short Discussion About Assumptions

We agree that our assumptions are restrictive and might be subject to criticism.

We treat decisions on each vehicle separately and the purchase behavior is not modeled. We do not account for substitution effects between owned vehicles. The choice to ignore the replacement decision, and so to see ownership as a stopping problem, is mostly due to lack of appropriate data. It is not possible to relate which disposed vehicle is replaced by which purchased vehicle in our sample. We thus ignore existence of trading costs and we agree that it is a shortcoming of our approach.

We also consider only principal user characteristics, assuming that income fully characterize the household's status. Needs related to non driving members are left out of the estimation.

Another point is the credibility of the forward-looking behavior assumption. A recent study (Turrentine and Kurani 2007) conducted on 56 American household shows that households do not explicitly consider car use in terms of investment. However, as previously discussed, the durable character of a car implies a decline of its value and efficiency that is well-known by the household. Assuming that an agent takes into account the future impact of his current car usage seems correct.

Our model is a starting point for a more comprehensive way to study automobile equipment choices. The above limits are left aside for future research.

2.3 Estimation

The estimation of the structural parameters

$$\boldsymbol{\lambda} = \left\{\alpha, \boldsymbol{\beta}, \boldsymbol{\theta}, \boldsymbol{\delta}, \omega_m, \sigma_{\text{petrol}}, \sigma_{\text{diesel}}, \tau_y\right\}$$

is based on the sequences of choices (keep/dispose and mileages), state variables and initial conditions observed for each vehicle j of sample's households.

According to Magnac and Thesmar (2002), the discount factor γ is "poorly" identified. As a result, even though econometric theory would allow estimating it as an additional parameter, it often leads to serious numerical problems when utility functions are linear in parameters. To this end, it is considered as fixed, and we estimate the model for different values.

The estimation process is composed of two steps.
In the first step, parameters of the transition probability distributions are estimated. The probabilities for mileage, fuel price and income to have the observed values are obtained by Eqs. 11, 13, 14 in the former subsection.
In the second step, we estimate the parameters of the dynamic programming problem given the transition probability distributions.

Let $\underline{t}_{ij}$ be the first date that vehicle j is observed in household i during the considered period and $\bar{t}_{ij}$ be the date household i disposed of the car. As the panel is not cylindrical, it is necessary to define it.

The likelihood function of the observed sample may be written as

$$\sum_{i=1}^{n} \ln \ell \left(\boldsymbol{\lambda}, \gamma | \mathbf{d}_{i,\cdot}, \mathbf{m}_{i,\cdot}, \mathbf{y}_{i,\cdot}, \mathbf{p}_{\cdot}\right) \tag{17}$$

where

$$\begin{aligned} &\ell \left(\boldsymbol{\lambda}, \gamma | \mathbf{d}_{i,\cdot}, \mathbf{m}_{i,\cdot}, \mathbf{y}_{i,\cdot}, \mathbf{p}_{\cdot}\right) \\ &\quad = \Pr\left(d_{i,\bar{t}_i} = 1 | y_{i,\bar{t}_i}, a_{i,\bar{t}_i}, k_{i,\bar{t}_i}, p_{i,\bar{t}_i}, \mathbf{x}_{ij0}; \alpha, \boldsymbol{\beta}\right) \\ &\quad \textstyle\prod_{t=\underline{t}_i}^{\bar{t}_i - 1} \Pr\left(d_{ijt} = 0 | m_{ijt}, y_{ijt}, a_{ijt}, k_{ijt}, p_{ijt}, \mathbf{x}_{ij0}; \boldsymbol{\lambda}, \gamma\right) \\ &\quad \textstyle\prod_{t=\underline{t}_i}^{\bar{t}_i - 1} \Pr\left(m_{ijt} | y_{ijt}, k_{ijt}, p_{ijt}, d_{ijt-1} = 0, \mathbf{x}_{ij0}; \boldsymbol{\theta}, \omega_m\right) \\ &\quad \textstyle\prod_{t=\underline{t}_i}^{\bar{t}_i} \Pr\left(p_{ijt} | p_{ijt-1}, d_{ijt-1} = 0, \mathbf{x}_{ij0}; \sigma_{\text{petrol}}, \sigma_{\text{diesel}}\right) \\ &\quad \textstyle\prod_{t=\underline{t}_i}^{\bar{t}_i} \Pr\left(y_{ijt} | y_{ijt-1}, d_{ijt-1} = 0, \mathbf{x}_{ij0}; \boldsymbol{\beta}, \tau_y\right) \end{aligned} \tag{18}$$

Under our assumptions one obtains as choice probabilities, assuming that households are utility maximizers,

$$\Pr\left(d_{ijt} = 1 | y_{ijt}, a_{ijt}, k_{ijt}, p_t, \mathbf{x}_{ij0}; \alpha, \boldsymbol{\beta}\right) = \frac{1}{1 + \exp\left(v_K\left(\mathbf{x}_{ijt}\right)\right)}, \tag{19}$$

and

$$\begin{aligned} &\Pr\left(d_{ijt} = 0 | y_{ijt}, a_{ijt}, k_{ijt}, p_t, \mathbf{x}_{ij0}; \boldsymbol{\lambda}, \gamma\right) \\ &\quad = \frac{\exp\left(v_K\left(\mathbf{x}_{ijt}\right) + \gamma \mathbb{E}_{\mathbf{z}_{ijt+1}}\left[V\left(\mathbf{z}_{ijt+1}\right) | \mathbf{z}_{ijt}\right]\right)}{\exp\left(v_K\left(\mathbf{x}_{ijt}\right) + \gamma \mathbb{E}_{\mathbf{z}_{ijt+1}}\left[V\left(\mathbf{z}_{ijt+1}\right) | \mathbf{z}_{ijt}\right]\right) + \exp\left(v_D\left(\mathbf{x}_{ijt}\right)\right)}, \end{aligned} \tag{20}$$

In order to evaluate the log-likelihood function for particular values of $\boldsymbol{\lambda}$ and γ, the dynamic programming problem needs to be solved exactly or its solution approximated in some way. Under the assumptions, one obtains

$$\mathbb{E}_{\mathbf{z}_{ijt}}\left[V\left(\mathbf{z}_{ijt}\right)|\mathbf{z}_{ijt-1}\right]$$
$$= \sum\nolimits_{q=1}^{Q}\sum\nolimits_{r=1}^{R}\left\{\begin{array}{l}\Pr\left(y_{ijt}|y_{ijt-1},d_{ijt-1}=0,\mathbf{x}_{ij0};\boldsymbol{\beta},\tau_y\right)\\ \times\Pr\left(p_{ijt}|p_{ijt-1},d_{ijt-1}=0,\mathbf{x}_{ij0};\sigma_{\text{petrol}},\sigma_{\text{diesel}}\right)\\ \times\ln\left(\exp\left(v_K\left(\mathbf{x}_{ijt}\right)+\gamma\mathbb{E}_{\mathbf{z}_{ijt+1}}\left[V\left(\mathbf{z}_{ijt+1}\right)|\mathbf{z}_{ijt}\right]\right)+\exp\left(v_D\left(\mathbf{x}_{ijt}\right)\right)\right)\end{array}\right\} \tag{21}$$

The next step can be broken into two parts. The inner step consists of evaluating equation 21 for the current value of $\boldsymbol{\lambda}, \gamma$. The outer step consists of finding a new value of $\boldsymbol{\lambda}, \gamma$ by iterating over the partial log-likelihood function that regards choice probabilities using a non-linear maximization algorithm. These two steps are repeated until convergence.
As the problem is a finite horizon, backward induction is used. Estimation starts from the last date of observation and the terminal condition $\mathbb{E}_{\mathbf{z}_{ijt+1}}\left[V\left(\mathbf{z}_{ijt+1}\right)|\mathbf{z}_{ijt}\right]$ if $d_t = 1$ (see Rust 1987, 1996; Aguirregabiria and Mira 2010, for details about the procedure).

3 Data

3.1 Statistical Sources

Data are drawn from the French "Parc Auto" panel survey. The survey is nationally representative, beginning in 1983. It is the best available database to study the dynamics of the demand for car equipment and use.

Annual rounds of a self-reported survey are designed by SOFRES, with IFSTTAR scientific council. Data are collected at the beginning of each year. They describe the previous year.

The database is hierarchical. The top level reports household information (i.e. socio-economic level, demographic structure, habitat zone, fleet size). At the bottom level, vehicles' characteristics are detailed. Only three vehicles are described when the car fleet size is larger. In the present approach, we restrict our analysis to a simple subset of variables.

The subsample of the population of motorized households that were observed over the period 2002−2008 is used for estimation. It is composed of 693 households, 437 of which own 637 petrol-powered or diesel-powered vehicles and scrap it during the observation period.

Data are completed by drawing a time series of fuel prices in France over the time period in the "DIREM" database.

3.2 Descriptive Statistics

We consider, first of all, sample means of observed decisions, reported in Table 1 for each year.

As we observe use and disposal of cars on a fixed period, it is natural to have attrition in our sample. However, in this case, we consider that sample attrition occurs

Table 1 Observed decisions, sample averages

Label	Frequencies and means					
	2003	2004	2005	2006	2007	2008
Car disposal[a]	18.41	36.04	55.69	71.61	87.36	100
Age of car (years)	8.36	8.04	8.26	8.01	8.95	8.83
Holding duration (years)	6.03	5.98	6.39	5.98	6.44	6.70
Mileage[b]	101,295	90,764	106,677	104,747	101,193	110,439
Use[c]	74,187	67,580	83,219	75,171	74,322	79,729
Sample size	1,012	1,006	1,014	983	991	962

Data source PARC AUTO
[a] Fraction of cars disposed during the period
[b] Cumulative mileage of the car in km
[c] Cumulative mileage driven with the car by current household in km

Table 2 Whole period descriptive statistics, sample averages

Label	Petrol	Diesel
Age when bought (years)	2.42	1.81
Holding duration (years)	7.06	5.20
Cumulated mileage at date of disposal (km)	84,782	115,289
Cumulated use at date of disposal (km)	68,866	84,739
Average fuel consumption (l/100 km)	7.6	6.6

Data source PARC AUTO

randomly such that the underlying causes of attrition are independent of the survey responses being studied. We know that it is rarely the case in panel data, but we argue that the reasons causing a household to exit the panel are not directly linked to the car's disposal choice[5] that we study here. Therefore, sample attrition would not pose any challenges in estimation.

A distinction is made here between general vehicle characteristics (age of car and mileage) and those related only to the household's use of the vehicle (holding duration and use).

The sample means of observed decisions are reported in Table 2 for the whole period, distinguishing by fuel type.

Some important differences in observed decisions can be highlighted by separately analyzing petrol-powered and diesel-powered vehicles. On average, the former are bought at an older age, held longer, driven fewer kilometres and consume less fuel on average than the latter.

As mentioned previously, we use average annual fuel prices in metropolitan France. Fuel price is given to households. They are price takers and they anticipate evolution of such prices. Looking at the available values over the considered period (Fig. 1), one

[5] We suggest that it is not completely true. When a single-vehicle household chooses to scrap its sole vehicle, it effectively chooses to end its motorization. Thus, the household can consider that its inclusion in the panel is no longer useful for the survey's purpose and exit, thereby causing non-random attrition.

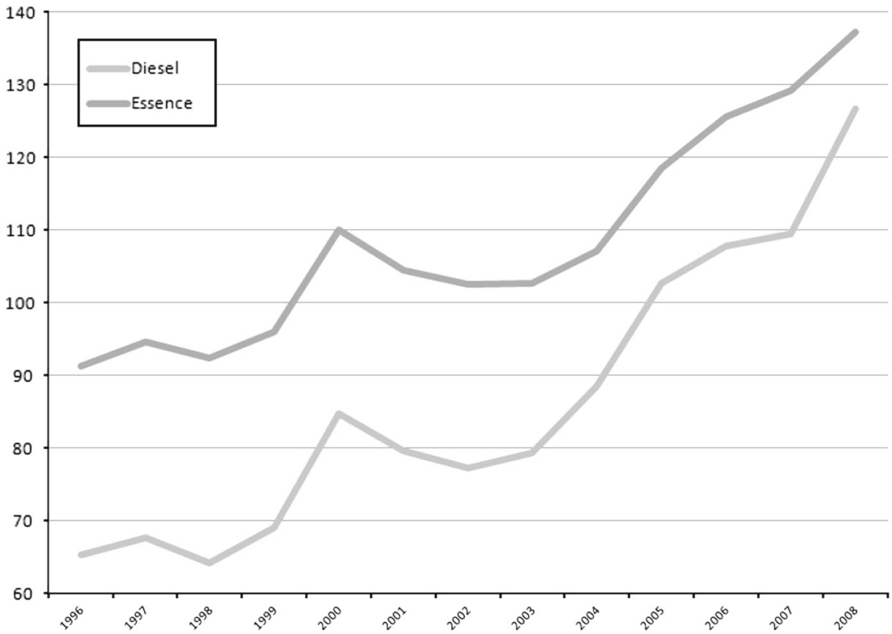

Fig. 1 Average annual fuel prices per litre and by type

observes a sharp rise both in petrol and diesel's prices. The latter rose at a higher rate than the former since 2002. The trends observed before this date suggest that a first order stationary Markov model does not correctly approximates the evolution of fuel prices before 2002.

As stated earlier, the income was collected in 13 classes. Though we observe only income classes, not actual levels, information is available on how these classes are defined. We finally group income into 9 classes. Even though the size of the observed population of households diminishes from year to year, the relative proportions do not differ significantly.

4 Results

We first estimate fuel price, performing the standard regression technique, based on the time series presented in Fig. 1, to test for non-stationarity of fuel price dynamics. Though we have little information on fuel prices and a small samples, we do not reject the baseline assumption (Table 3). We compute transition matrices for fuel prices varying from 0€ to 4€ at the cent level. For the sake of clarity, we do not report the two 401 × 401 matrices.[6]

We acknowledge that choosing to use such a small sample (2000/2008 observations) is not a negligible issue and that, due to this fact, the estimates used to compute the

[6] The reader may contact the authors whenever he/she desires to obtain any of the estimated transition matrices regarding fuel prices.

Table 3 Estimates: fuel prices

Label	Estimate
Variance σ^2_{petrol}	0.026
Variance σ^2_{diesel}	0.037

Data source DIREM

Table 4 Estimates: income

Label	Estimate	Std. dev.	T-stat.
Intercept	21.9575	0.01012	259.65
Previous inc. class is [15.2; 19.1[K€	2.0909	0.01340	18.67
Previous inc. class is [19.1; 22.9[K€	3.7148	0.01335	33.32
Previous inc. class is [22.9; 26.7[K€	4.9304	0.01399	42.17
Previous inc. class is [26.7; 30.5[K€	6.0385	0.01467	49.26
Previous inc. class is [30.5; 38.1[K€	7.4954	0.01332	67.35
Previous inc. class is [38.1; 45.7[K€	9.1370	0.01484	73.71
Previous inc. class is [45.7; 61[K€	10.8784	0.01622	80.25
Previous inc. is ≥61 K€	13.1990	0.02522	62.64
σ	8.3549	0.000450	31.82
Log-Lik at convergence	-5761		
Number of observations[a]	4, 122		
Log-lik. H0[b]	-18868		

Data source PARC AUTO

Std. Dev. robust standard deviation, *T-stat.* robust Student statistic, *inc.* income, computed in thousands of €, *Log-lik* value of the partial log-likelihood function

[a] Used data: each available observation for considered households (see Sect. 3.1)

[b] Trivial log-likelihood

discrete transition probability distributions may be criticized. However, employing a longer annual time series for fuel prices would instead create additional problems as the evolution of fuel prices results from various causes in recent history. Therefore, doing so may likely be irrelevant because of structural changes in formation of fuel prices over the past few decades that are not taken into account with such a simple assumption on their evolution.

Results concerning income are reported in Table 4. One can expect that the probability of belonging to a higher class of income at the observation date t would be greater the richer household i was at date $t-1$. The fact that the effect of the previous income class on the current one increases with the higher classes confirms our expectation.

As mentioned in Sect. 2 (see Eqs. 14 and 16), the probability for a household in income class j at date $t-1$ to be in class i for the following period t is computed as

$$P(i_t|j_{t-1}) = \frac{1}{1+exp\left(-\sigma\mu_i+\beta_0+\beta_j\right)} - \frac{1}{1+exp\left(-\sigma\mu_{i-1}+\beta_0+\beta_j\right)} \quad (22)$$

where $\beta_1 = 0$ (i.e. reference class).

Table 5 Income: transition probability

Actual income	Previous income								
	1	2	3	4	5	6	7	8	9
1	**0.6854**	0.2121	0.0504	0.0155	0.0052	0.0012	0.0002	4.1E-5	4.0E-6
2	0.2509	**0.4326**	0.2131	0.0804	0.0287	0.0069	0.0013	0.0002	2.3E-5
3	0.0490	0.2473	**0.3562**	0.2298	0.1037	0.0277	0.0056	0.0010	0.0001
4	0.0106	0.0755	0.2349	**0.3096**	0.2276	0.0823	0.0182	0.0033	0.0003
5	0.0028	0.0215	0.0924	0.2058	**0.2710**	0.1713	0.0478	0.0091	0.0009
6	0.0011	0.0092	0.0444	0.1303	0.2820	**0.4338**	0.2629	0.0679	0.0073
7	0.0002	0.0013	0.0067	0.0222	0.0627	0.1995	**0.3621**	0.2070	0.0297
8	4.2E-5	0.0003	0.0017	0.0058	0.0174	0.0698	0.2645	**0.5306**	0.2695
9	4.2E-6	3.4E-5	0.0002	0.0006	0.0017	0.0074	0.0373	0.1809	**0.6922**

Bold values show probabilities to observe a household perceiving an income in the same class that previous year
Data source PARC AUTO

Looking at the 9 × 9 estimated transition matrix (Table 5), one observes that the greatest probability is for a household to remain in the same class from a year to the following, whatever the previous class. This probability ranges from 0.27 (corresponding to the central class 5) to 0.69 (observed for the two extremes classes).

The probability of moving to the lower or upper income classes is always inferior to 0.3 and remains considerable only when a household transitions through less than two income classes. Moreover, households characterized by an income lower than 38.1 K€ in the previous year have a higher probability of moving down, while one observes the opposite for the others.

We first estimate the model in a static framework, aiming to test our hypothesis on the aforementioned factors (Table 6).

The effects of unemployed principal user are not significant. We can then test the hypothesis that $\mu_2 = \mu_4$.

Let the principal user typology be now defined as:

- t_{1j} : principal user less than 40 years old and employed
- t_{2j} : principal user age $\in$ [40; 60[and employed
- t_{3j} : principal user less than 60 years old and unemployed

Thus, we estimate the model again.

The estimation results, reported in Table 7, fail to reject the hypothesis of independence of principal user age for unemployed users.

We then estimate the latter in dynamic framework.

As previously argued, multiple hypotheses can be constructed about a household's behavior facing uncertainty. The discount rate γ represents this concept. $\gamma = 0$ indicates that the household is myopic, e.g. it has an infinite preference for the present. When $\gamma = 1$, the household has no preference for the present: only the future is taken into account. When γ takes any value in between 0 and 1, the household trades off both the present and future.

Table 6 Estimates: static model

Label	Estimate	Std. dev.	T-stat.
Keep utility intercept (β_0)	0.2963	0.06820	4.35
VMT intercept (α)	0.6985	0.1630	4.29
Vehicle value (β_1)	0.02234	0.002393	9.33
Operating cost (β_2)	0.1893	0.03529	5.36
Principal user type 1[a] (μ_1)	0.4005	0.08359	4.79
Principal user type 2 (μ_2)	−0.4491	0.2726	−1.65
Principal user type 3 (μ_3)	0.3113	0.07605	4.09
Principal user type 4 (μ_4)	−0.6457	0.3341	−1.93
σ	0.9167	0.01784	51.39
Log-Lik at convergence	−2913.05		
AIC	5, 844.1		
Number of observations[b]: 1, 944			

Data source PARC AUTO
Std. Dev. robust standard deviation, *T-stat.* robust Student statistic, *Log-lik* value of the partial log-likelihood function, *AIC* Log Akike Information Criterion (small si better)
[a] Definition in Sect. 2.1
[b] Used data: select panel's observation (see Sect. 3.1)

Table 7 Estimates: unemployed users' effect test

Label	Estimate	Std. dev.	T-stat.
Keep Utility Intercept (β_0)	0.2981	0.06808	4.38
VMT Intercept (α)	0.7065	0.1635	4.32
Vehicle value (β_1)	0.02227	0.002385	9.34
Operating cost (β_2)	0.1904	0.03517	5.41
Principal user type 1 (μ_1)	0.3995	0.08357	4.78
Principal user type 2 (μ_2)	0.3104	0.07604	4.08
Principal user type 3 (μ_3)	−0.5416	0.2202	−2.46
σ	0.9164	0.01781	51.46
Log-Lik at convergence	−2913.15		
AIC	5, 842.3		
Number of observations[a]: 1, 944			

Data source PARC AUTO
Std. Dev. robust standard deviation, *T-stat.* robust Student statistic, *Log-lik.* value of the partial log-likelihood function, *AIC* Log Akike Information Criterion (small si better)
[a] Used data: select panel's observation (see Sect. 3.1)

We estimate the model using three different discount factors. Since it is not plausible that a household considers only the future, the model is estimated for $\gamma = 0.99$ as a limiting case. For both present and future preferences, we define $\gamma = 0.5$, while the myopic behavior is tested with $\gamma = 0$.

Results, reported in Table 8, are in line with our expectation.

Table 8 Estimates: probability to keep

	Estimate	Std. dev.	T-stat.	Estimate	Std. dev.	T-stat.	Estimate	Std. dev.	T-stat.
Keep Utility Intercept (β_0)	0.8997	0.0879	10.24	0.9409	0.0692	13.59	0.7244	0.0690	10.49
VMT Intercept (α)	0.5039	0.0787	6.40	0.7521	0.1147	6.55	0.8946	0.1326	6.74
Vehicle value (β_1)	0.0208	0.0024	8.56	0.0212	0.0027	7.97	0.0208	0.0028	7.34
Operating cost (β_2)[a]	0.1287	0.0234	5.49	0.2198	0.0274	8.02	0.2581	0.0288	8.95
Principal user type 1 (μ_1)[b]	0.6437	0.1014	6.35	0.7796	0.1163	6.70	0.8199	0.1231	6.66
Principal user type 2 (μ_2)[c]	0.5111	0.1035	4.95	0.7045	0.1183	5.95	0.7578	0.1241	6.11
Principal user type 3 (μ_3)[d]	-0.4497	0.2731	−1.65	−0.1099	0.2523	−0.43	0.1046	0.2230	0.47
σ	0.8252	0.0058	141.94	0.8386	0.0059	140.99	0.8459	0.0060	139.82
Discount Factor γ	0			0.5			0.99		
Log-lik. at convergence	-2818.56			−2573.66			−2373.76		
Number of observations[e]: 1, 944									

Data source PARC AUTO

Std. Dev. standard deviation, *T-stat.* robust Student statistic, *Log-lik.* value of the partial log-likelihood function

[a] REMEMBER: β_2 enter negatively in 3 and 10

[b] Less than 40 years old and employed

[c] Age $\in$ [40; 60[and employed

[d] Less than 60 years old and unemployed

[e] Used data: select panel's observation (see Sect. 3.1)

When we suppose agents to be myopic, we observe that a decrease in the vehicle scrap value has a double effect. For the same available income, the car's decline implies a decrease in the utility to keep it, but it also causes an increase in the demand for mileage. The same trend can be highlighted for a household income effect when the scrap value remains the same.

We previously defined, in Eq. 4, the scrap value of a car as a geometrically decreasing function of its ownership duration and cumulative use. That is, when the cumulative mileage increases, scrap value decreases. The demand for mileage of a household is then an increasing function of the past mileage, through the cumulative mileage variable.

Considering income and scrap value simultaneously, we can note that a greater vehicle value (i.e. a large positive difference between income and scrap value) implies a lower utility from owning it. Such a result can be interpreted as the desire to own a "better" vehicle if the available income is large enough. As the latter is defined with regard to fleet size, this effect can be caused by a high household wealth level as well as a small fleet size. In other words, a household possessing a small number of cars is more tempted to scrap one for which the value is low.

When the vehicle is owned, a greater difference causes an increase in demand for mileage. This result indicates a tendency to use a vehicle more intensively when its scrap value is low relative to income, perhaps due to the high probability of scrapping it early.

According to results reported by Goodwin (1992), de Jong and Gunn (2001), Graham and Glaister (2002, 2004), the demand for mileage significantly decreases with average fuel expenditure. Moreover, the utility to keep a vehicle also decreases when operating costs increase.

Observing the principal user effect one can highlight that, ceteris paribus, an unemployed user possesses a decreased demand for mileage, while demand increases when the principal user is employed, whatever his or her age. This result can derive from the fact that the presence of a regular activity implies a greater quantity of trips than for unemployed users.

Moreover, looking at employed principal users, the results show that the utility of keeping the car, as well as demand for mileage, decreases conditional on age. Such an effect can be linked to an unobserved factor. We observe that, in fact, a household's head and spouse are more frequently declared as the principal users of a car fleet as opposed to adult sons and, furthermore, a younger head of household is more likely to include young children in the car use structure. This finding implies, for example, a higher travel need for these types of households. However, as we left household characteristics out of our approach, we cannot analyze this idea too deeply.

Estimation results for forward-looking agents show how the underlying behaviour that drives the choice to keep and use or to dispose of the car changes when considering a different discount rate γ.

Vehicle value is the only factor remaining almost the same when testing for different degrees of forward-looking behavior. That is, the effect of saved income is nearly stable whatever the discount factor. Results from unemployed principal users do not significantly impact owning decision and demand for milleage.

All of the other effects and their significance increase when accounting for a larger discount factor. Moreover, the gap between type 1 and 2 principal user effects decreases when the preference for the future increases.

There are potentially many underlying reasons behind these results, but we argue that deeper studies should be conducted and thus prefer to avoid speculation in discussing the results.

5 Conclusions

The demand for mileage is function of saved income, computed by difference between household's income and vehicle's scrap value, operating cost and principal user's characteristics.

Saved income effect principally drive household's decisions on car's holding duration and use. Vehicle's scrap value is function of the cumulated mileage, defined as a deterministic function of mileage. When future is considered by the household in taking, at date t, the decision to own or not a car (i.e. the agent is forward-looking), that then depend on the mileage chosen for this period through the computation of the expected maximum utility obtained from the car at the next date whenever it is kept one year more.

Our estimation results confirm the hypothesis that accounting for forward-looking behaviour greatly improves the understanding of the modelled decision processes.

The approach, however, needs to be refined in several aspects.

As stated, the optimal use of a car depends on its principal user characteristics. This choice of the principal user for a household's car is an important topic in mileage demand estimation. By using GEV as an inter-temporal choice model, one can circumvent this problem.

Another limit of this model is that the optimal use for a vehicle j in a multi-vehicle household depends on other vehicles' use but these are not estimated simultaneously. Mannering (1983b) proposes a simultaneous equation model to bypass this issue. Defining available income for a vehicle j as the difference between household income and other car use expenditure, as in the previous model, one can calculate mileage utility as being dependent on other cars' optimal mileage, in accordance with Mannering (1983b). Moreover, it can be of a great importance to combine keep and use decisions with purchase choices. This improvement would enable to account for the trading costs involved in the decision to sell a current car and replace it with a new one as well as for the effect of subsidies on durable goods markets (Adda and Cooper 2000).

More observed state variables, especially those related to household demographics and descriptions of the initial conditions, could be introduced in order to account for other possibles effects.

One can think that a more appropriate hypothesis for income transition probabilities estimation should be that the household's income class depend on his characteristics and that the latter follow a autoregressive process. Moreover, it's plausible to think that the household's income for a period t play a role on some event, as for example a new birth or the chief retirement. One have then the following equations system:

$$\begin{aligned} x_{i,t} &= \beta x_{i,t-1} + \gamma y_{i,\, t-1} \\ y_{i,t} &= \theta x_{i,t} \end{aligned} \tag{23}$$

where $x_{i,t}$ is a vector of household i characteristics for period t and $z_{i,t}$ his income class.

Because simultaneity, the income class is to be firstly estimated and the fitted class enter in the household's characteristics estimation.

Such an approach is at present in development and will be included in a further stage of our research.

Another significant improvement would involve a more realistic specification of the unobserved state variables, i.e. the problem of persistent unobserved heterogeneity may create unobserved correlations over time as well as the likely effect of additional time-varying unobserved state variables (like the possibility that the car is in a dilapidated state).

A last, but not least, possible improvement to this model would involve estimating different car usage. Our "Parc Auto" data enables us, for example, to distinguish between long travel mileage, work-travel mileage, and so on. This information would potentially allow for a joint estimation of car choice and trip behaviour in the future.

Combining all of these aspects would provide a better description of the underlying behaviours that drive choices of a household as it regards its demand for cars.

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