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Disaggregating radar-derived rainfall measurements in East Azarbaijan, Iran, using a spatial random-cascade model

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Abstract The availability of spatial, high-resolution rainfall data is one of the most essential needs in the study of water resources. These data are extremely valuable in providing flood awareness for dense urban and industrial areas. The first part of this paper applies an optimization-based method to the calibration of radar data based on ground rainfall gauges. Then, the climatological Z-R relationship for the Sahand radar, located in the East Azarbaijan province of Iran, with the help of three adjacent rainfall stations, is obtained. The new climatological Z-R relationship with a power-law form shows acceptable statistical performance, making it suitable for radarrainfall estimation by the Sahand radar outputs. The second part of the study develops a new heterogeneous randomcascade model for spatially disaggregating the rainfall data resulting from the power-law model. This model is applied to the radar-rainfall image data to disaggregate rainfall data with coverage area of $512 \times 512 \text{ km}^2$ to a resolution of 32×32 km². Results show that the proposed model has a good ability to disaggregate rainfall data, which may lead to improvement in precipitation forecasting, and ultimately

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better water-resources management in this arid region, including Urmia Lake.

1 Introduction

Information about precipitation patterns influences the design and management of large and small (especially in urban areas) water-resources systems, ranging from continuous flow simulation and soil-erosion modeling to the evaluation of alternative policies for environmental impact assessments (e.g., Pui et al. 2012; Sivakumar and Sharma 2008). Short-term rainfall patterns are one of the most common ones required in hydrologic modeling. For instance, these patterns are used to assess urban stormwater, pollution transfer, and many other issues in environmental research.

There are different sources for rainfall measurements: The first and foremost is through ground-based gauge networks. While gauge observations are usually treated as the groundtruth data set, they are point measurements and cannot provide a global view of the precipitation field. Weather radar is another tool in measuring the rainfall, which can provide rainfall data in high spatial and temporal resolution. The first report to measure the rainfall by using radar was provided by Marshall and Palmer (1948). They suggested that the relationship between the reflectivity factor, i.e., Z, and the rainfall rate R have a power-law form of $Z = AR^b$ (Pedersen et al. 2010). The Z-R relationship can be estimated with two approaches: raindrop size distribution (RDSD) and optimization. For the first approach (RDSD), Z and R are calculated directly by using raindrop size-distribution data recorded by a disdrometer (Mapiam and Sriwongsitanon 2008). Satellite-based precipitation data sets, such as the Climate Prediction Center morphing technique (CMORPH; Joyce et al. 2004), Precipitation Estimation from Remotely Sensed Information

using Artificial Neural Networks (PERSIANN; Hsu et al. 1997; Sorooshian et al. 2000), and PERSIANN-Climate Data Record (PERSIANN-CDR, Ashouri et al. 2015) provide global and high-resolution precipitation data at a fine spatial resolution. These data sets are particularly useful for estimating precipitation in remote regions where gauge observations are not available.

In most developing countries, including Iran, the existing rain-gauge stations do not adequately record rainfall data in short-time intervals (less than 1 day). Even if such data sets exist, they often have limited record lengths, as well as gaps. Moreover, gauge-based observations are point measurements and are usually not available in remote regions. In order to model precipitation data for many hydrological applications in a more suitable manner, it is necessary to simulate the statistical properties of observed rainfall events, in both space and time, in real water resources systems, such as rivers or lake basins. For this purpose, a rainfall disaggregating model, which enables a large-scale rainfall model to be disaggregated to the smaller-response scales necessary for modeling hydrological catchments, is required (Licznar et al. 2011; Olsson 1998).

The statistical-moments function method is one of the models to disaggregate (Frisch and Parisi 1985; Over and Gupta 1996). Olsson (1998) developed a model to disaggregate daily-rainfall data in a rain-gauge station located in southern Sweden. Mouhous et al. (2001) studied daily-rainfall data in a rain-gauge station located in Nantes in France and attempted to disaggregate the data. Sivakumar and Sharma (2008) applied a random-cascade model to disaggregate daily-rainfall data. Licznar et al. (2011) tested six randomcascade models for generating synthetic rainfall-time series, with a special focus on the needs of urban hydrology, e.g., hydrodynamic simulations of urban-drainage systems. Pui et al. (2012) evaluated the performance of a number of daily to subdaily rainfall disaggregates, such as two versions of random-cascade models and the randomized Bartlett-Lewis model using continuous rainfall data at four cities in Australia.

Another approach, which has recently become popular, is the statistical-spatial approach that disaggregates the rainfall data to smaller spatial levels. This model can be used to disaggregate rainfall from global climate models (GCMs) to subgrid levels. Groppelli et al. (2010) used the stochastic space random-cascade (SSRC) approach to disaggregate rainfall from a GCM for an Italian Alpine watershed. In Japan, Pathirana and Herath (2002) proposed a new spatial randomcascade model to generate disaggregate rainfall data. None of these models has been applied (based on our knowledge) for northwestern Iran; furthermore, because of the recent climate change and droughts in that area, water-resources management is a major concern (Hassanzadeh et al. 2012). The study area, i.e., the Urmia Lake region, is in a critical situation. The water level of the lake is rapidly desiccating due to human activities and the changes in the climate system. Therefore, the reliable high-resolution precipitation data requires scientists (including hydrologists, climatologists, and waterresources planners and managers) as well as decision makers to better prepare and plan for the restoration of this lake.

In this work, we attempt to span the gap between meteorological and hydrological model scales by using a simple, multifractal disaggregating framework, which maintains the observed statistical properties of the rainfall at different spatial scales. One important aspect of this approach is its ability to serve as a practical tool to calculate the low-scale data required in water-resource systems studies. Then, the study uses the data from ground gauges and radar data. Briefly, a new climatological *Z-R* relationship is provided for the Sahand radar outputs. In addition, a new spatially heterogeneous randomcascade model (rarely developed) will be developed to disaggregate the rainfall data. In this model, the effect of seasons also will be considered in developing the random-cascade model. In upcoming studies, the results will be compared with the data obtained from satellites.

This paper is organized as follows. In Section 2, a short description of the study area is presented. In Section 3, the model structure and methodology of each part of the study, including the calibration method of a climatological Z-R relationship and the spatial disaggregation method, are discussed. In Section 4, the results of these two parts of the study are presented. Conclusions and findings and future research directions are presented in Section 5.

2 Case study

The study region is a mountainous area located in northwestern Iran (Fig. 1). The area is important mostly because of Urmia Lake, whose water level is currently shrinking (Hassanzadeh et al. 2012). This is because of human activities and climate-altering trends; hence, the precipitation studies in this region are critical for restoration of the lake. The Sahand radar is a C-band Doppler located near the Sahand Mountains (Fig. 2). In this section, the space random-cascade model, which was developed by Pathirana and Herath (2002) for heterogeneous regions, is used to disaggregate rainfall for the East Azarbaijan Province. The Sahand daily (or 6-h) radar-reflectivity data, in the form of images with a size of 512 rows \times 512 columns and a resolution of 1 \times 1 km², has a scan coverage from 35° N-40° N latitude and 43° E-49° E longitude (Fig. 3). The radar rainfall data used for this study are 6 and 24-h rainfall from 2009 to 2013. These images are controlled by gauged rainfall (calibration of a climatological Z-R relationship for the Sahand station) before they are applied to the space random-cascade model. In this study, 6hourly rainfall data observed in the rain-gauge stations in Sahand and the adjacent stations of Ahar and Tabriz, Iran,



Fig. 1 Location of the study region (East Azarbaijan Province, Iran)

are used. Details of the selected rainfall stations for this study are presented in Table 1.

3 Methodology

To achieve high-spatial resolution rainfall data for the East Azarbaijan Province, the following two steps were completed: The first part is the calibration of the climatological *Z-R* relationship for the Sahand radar. The second part describes the spatial random-cascade model.

3.1 Z-R relationship for the Sahand radar

In this step of the study, the most suitable climatologically Z-*R* relationship of the Sahand radar for 6-h radar rainfall is estimated by a calibration technique based on optimization, i.e., the least-squares method. Chumchean (2004) suggested that the optimal values of *A* and *b* (in Eq. 1) are within the ranges of 31-500 and 1.1-1.9, respectively. Because Fields et al.



Fig. 2 The Sahand radar near Tabriz City, Iran

(2004) recommended that only the multiplicative term A needs to be adjusted to minimize the errors, the value of b was fixed as 1.6. The new parameter A can then be calculated as follows:

$$A_1 = A_0 / m^b \tag{1}$$

Where A_1 is a new multiplicative term, which is considered in the Z-R relationship, m is the gradient of regression line between the observed radar rainfall and the gauge rainfall obtained from the standard Z-R relationship ($Z = 200R^{1.6}$), and A_0 is the initial parameter of A, which is set to 200. The quality of the estimation procedure is evaluated by mean error (ME), mean absolute error (MAE), root mean-square error (RMSE), and bias (B).

3.2 Spatial random-cascade model

The main objective of the spatial random-cascade model is to validate and evaluate the proposed model's ability to distribute a single amount of rainfall (large-scale forcing) into the constituent spatial grids. A schematic of this model is shown in Fig. 4. The proposed model was tested for both observed and synthetic rainfall data for the East Azarbaijan Province from 2009 to 2013. To reach the total rainfall of the scanned area, the observed data of the $1 \times 1 \text{ km}^2$ pixels are firstly aggregated to 512×512 km². Then, the parameters of the randomcascade model are estimated based on the disaggregation of aggregated radar data from 512×512 km² to 32×32 km². The main idea behind this aggregation/disaggregation process is to find the correct parameters of the random-cascade model which can be later used to disaggregate observations from other sources of rainfall such as gauge observations or satellite-derived products, given the fact that each of these data sources are different with different error and uncertainty characteristics and structures.



Fig. 3 Observed rainfall image for the Sahand radar (event on November 17, 2009)

The mathematical notation of the spatial random-cascade model is (Pathirana and Herath 2002):

$$R_{i,j} = M_{i,j}G_{i,j}, \qquad \qquad M_{i,j} = \begin{cases} 0 & forG_{i,j} = 0 \\ R_{i,j}/G_{i,j} & \text{otherwise} \end{cases}$$
(2)

Where $R_{i,j}$ is the rainfall on pixel (i,j), $G_{i,j}$ is the component of the rainfall, which is constant over time (months or seasons), and M produces a uniform field at large accumulation. Hence, M is a candidate for multifractal modeling (which was used to describe spatial rainfall and is spatially homogeneous in the statistical sense). The β -lognormal model, which was proposed by Over and Gupta (1996), is considered in this work due to its simplicity, wide range of application, and its ability to explicitly consider arid zones with extended dry spells without any rainfall. The branching number b is given by:

$$b = N_{i+1} / N_i \tag{3}$$

Where N_i is the total number of pixels at the *i*th cascade level. To calculate the rainfall in one of the four subintervals,

the interval rainfall R should be multiplied by the cascade weight W (Pathirana and Herath 2002; Licznar et al. 2011). This multiplicative operation, known as a fine-grating process, is repeated again and again to successively finer-cascade levels. At each cascade level, each pixel is divided into b equal parts by the cascade weight W (as in Eq. 4). The two cascade models of Eq. 4 could be combined to propose a b-lognormal model given in Eq. 5 (Over and Gupta 1996).

$$P(W = 0) = 1 - b^{-\beta}, P(W = b^{\beta}) = b^{-\beta}$$
(4)

$$P(W=0) = 1 - b^{-\beta}, P\left(W = b^{\beta - \sigma^2 \frac{\log[b]}{2} + \sigma X}\right) = b^{-\beta}$$
(5)

Where β and σ^2 are model parameters and X is a standard normal variable. The disaggregation of rainfall by this model involves two steps: (1) calibration (identification of scaling behavior in the rainfall) and (2) generation of synthetic data. These steps are described in detail below.

Station name	Latitude	Longitude	Time period (start/end year, dd/mm/yy
Tabriz	38°05′	46°17′	01/01/1992-31/03/2009
Sahand	37°56′	46°07′	15/04/2008-18/04/2009
Ahar	38°26′	47°04′	01/01/1992-31/03/2007
	Station name Tabriz Sahand Ahar	Station nameLatitudeTabriz38°05'Sahand37°56'Ahar38°26'	Station nameLatitudeLongitudeTabriz38°05'46°17'Sahand37°56'46°07'Ahar38°26'47°04'





3.2.1 Calibration

Figure 5 shows the calibration part of the disaggregation model. As shown, radar-rainfall images are used for finding the model parameters.

The scaling behavior of the statistical moment, i.e., $M(\lambda, q)$, is computed as follows:

$$M(\lambda, q) = \sum_{i} \left[R_{i,\lambda} \right]^{q} = \lambda^{\tau(q)}$$
(6)

Over and Gupta (1996) also proposed the concept of a Mandelbrot-Kahane-Peyriere (MKP) function (Mandelbrot 1982) to estimate model parameters for the β -lognormal model as follows:

$$\chi(q) = (\beta - 1)(q - 1) + \left(\frac{\sigma^2 \ln(b)}{2}\right) / (q^2 - q)$$
(7)

The optimization procedure for the function given in Eq. (7) is performed using the least-squares method to find the model parameters (β , σ^2), where *b* is the branch number and *d* is equal to 2 in cascade models (Pathirana and Herath 2002).

3.2.2 Generation

 $A_{i,j} = \frac{1}{n} \sum_{k=1}^{n} R_{i,j,k}$ (8)

determined as follows:

In the generation part of the model, the disaggregating process proceeds from the largest spatial scan area, 512×512 km² to 32×32 km² (Fig. 6). In this station, the installed radar shows a

$$G_{i,j} = TA_{i,j} / \sum_{i,j} A_{i,j}$$
⁽⁹⁾



Fig. 5 A schematic of the calibration part of the model

very high performance in recording the images. Based on the characteristics of the manufacturer, the radar's pictures allow for converting their data to a resolution of $1 \times 1 \text{ km}^2$; however, in order to obtain more certain outputs, we will provide the outputs up to the scale of $32 \times 32 \text{ km}^2$. Then, in other radar studies, the quality of the observed images should be checked before disaggregating.

It is necessary to point out that disaggregating coarse resolution observations based on spatial cascade models differ from downscaling of the GCM simulations, where the downscaling ratios are on the order of around 3. In the disaggregation studies using spatiotemporal cascade models, however, a higher number is possible. For instance, Deidda et al. (1999); Sharma et al. (2007), and Kang and Ramirez (2010) used random-cascade models to disaggregate the rainfall data with high ratios of conversion.

conversion. Eq. (2) expresses the space random-cascade model mathematically. Filtering the observed rainfall fields to obtain the Mfields is the most important step in model construction. In order to obtain field G for a given month, the averages of all snapshots for that month were taken at the pixel level, which is



Where *n* is the number of snapshots with the value (i,j) and the parameter T is the total number of pixels in a snapshot. The field $G_{i,j}$ is normalized based on the long-term averages and is used to represent the spatial heterogeneity. When first using the model for disaggregation, the β -model is the cascade generator that determines that the probability of wet (non-zero) and dry (zero) rainfall periods. Then, only for non-zero results, *W* is drawn from the following distribution:

$$W = b^{\beta - \sigma^2 \frac{\log(b)}{2} + \sigma \mathbf{X}} \tag{10}$$

The field M is statistically homogeneous in space, and the following relationship is used to modify M to represent the spatial heterogeneity:

$$R_{i,j} = AM_{i,j}G_{i,j} / \sum_{i,j} \left(M_{i,j}G_{i,j} \right)$$
(11)

Where A is the large-scale forcing parameter.

4 Results

4.1 Calibration of the Z-R relationship

An adjusted A value of 105 is computed using Eq. (1) with the m value (the gradient of the regression line between the simulated-radar rainfall and the gauge rainfall). Figure 7 shows the plot of gauge rainfall, the default-radar method $(Z = 200R^{1.6})$, our local calibration method $(Z = 77.43R^{1.284})$, and the Fields et al. (2004) method ($Z = 105R^{1.6}$).

The results show that the local Z-R relationship $(Z = 77.43R^{1.284})$ can improve the accuracy of the radar rainfall compared to the application of traditional $Z = 200R^{1.6}$





Fig. 8 Scaling properties of the statistical moment, $M(\lambda,q)$, for the Sahand radar event (October 3, 2010)

 Table 2
 Comparison of the statistical measures obtained from the
 modified Z-R relationship

Statistical measures	Default radar equation	Fields et al., (2004) method	New equation by using optimization
Mean error(mm)	-0.29	0.16	0.10
Mean absolute error(mm)	0.56	0.47	0.44
Root mean-square error(mm)	0.74	0.61	0.61

default-radar method

od ($Z = 105R^{1.6}$)

 $(Z = 200R^{1.6})$, our local calibra-

tion method ($Z = 77.43R^{1.284}$),



Fig. 9 Estimation of the scaling exponent, $\tau(q)$, for the Sahand radar event (December 31, 2010)



(Fouladi Osgouei and Zarghami 2014). During the calibration process, parameters (A, b) are adjusted to minimize the four well-known statistical error measures. The results showed that $Z = 77.43R^{1.284}$ is more suitable for radar-rainfall prediction by the Sahand radar (Table 2).

4.2 Spatial disaggregation model

In this part, the statistical moment $M(\lambda,q)$ (based on Eq. 6) is calculated for different values of q and scales, λ . The scaling exponent, $\tau(q)$, can be evaluated as the slope of log $[M(\lambda,q)]$ versus log $[\lambda]$ curve for various values of q (Fig. 8). The shape of the $\tau(q)$ curves shown in Fig. 9 shows the scaling properties of rainfall fields.

This disaggregation method depends mainly on the parameter values (β , σ^2). In this step, the seasonal values of these two parameters are estimated using Eq. (7) and the leastsquares method (Table 3). Figures 10 and 11 show two examples of the disaggregating rainfall of the East Azarbaijan area from a study area of 512 × 512 km² to lower scales. The space random-cascade model is then applied from the study area of 512 × 512 km² to a scale of 32 × 32 km².

These two parameters are estimated for the years from 2010 to 2013 using 6 and 24-h observed rainfall data. From Table 3, we can conclude that a constant

 Table 3
 Estimated parameters of the spatially heterogeneous randomcascade model for the Sahand radar (scale of 6-h)

Seasons	β	σ^2	Mean rainfall (mm)	Date
Spring	0.10	0.123	1.37	23 April 2011, 10:00 a.m.
Spring	0.10	0.134	1.82	18 May 2011, 10:00 p.m.
Fall	0.10	0.134	0.61	03 October 2010, 04:00 p.m
Summer	0.10	0.161	1.17	24 July 2011, 02:00 p.m
Winter	0.10	0.122	0.54	30 January 2011, 06:00 p.m

Fig. 10 Disaggregated rainfall (mm) for the East Azarbaijan from a study area of 512×512 km² to a scale of 32×32 km² (April 26, 2011). The *x*-axis represents the longitude, and the *y*-axis represents the latitude

value of 0.1 is suitable for β . In addition, the σ^2 ranges from 0.12–0.16 and is sensitive to the average seasonal rainfall.

The results of about 100 synthetic images were compared statistically with the 6-h rainfall images originally observed. First, the exceedance probability curve of rainfall at the selected point is estimated, and then the simulated output from the disaggregation model is compared with the observed rainfall fields (mostly 6-h and some 24-h) using quantile-quantile (Q-Q) plots. Exceedance probability curves for the observed and disaggregated rainfall at the Sahand and Tabriz stations are then calculated. Figures 12 and 13 show the exceedance probability curve and the Q-Q plot for the Sahand and Tabriz stations, respectively. The spatial heterogeneity model is then calibrated for 2009–2013.

5 Conclusions

As smaller catchments become increasingly urbanized, there is a growing need for utility of high spatial and temporal-resolution rainfall data. In many regions with sparse rain-gauge networks, weather radar provides an opportunity to create high-resolution data useful for hydrological applications if the data can be disaggregated successfully. The main objective of this paper was to develop and apply a disaggregating model that accounts for the local-scale spatial variability of rainfall. We introduced a calibration procedure in which parameters were adjusted to minimize four statistical measures (mean error, mean absolute error, root mean-square error, and bias); we then compared this procedure to existing radar-calibration



Fig. 11 Disaggregated rainfall (mm) for the East Azarbaijan from a study area of $512 \times 512 \text{ km}^2$ to a scale of $64 \times 64 \text{ km}^2$ (May 18, 2011). Left: two-dimensional view; right: the same image in three-dimensional view. The x-axis represents the longitude, and the *v*-axis represents the latitude

techniques. In the second part of this study, we considered the suitability of a spatial random-cascade scaling-based approach for the spatial heterogeneity rainfall disaggregation. This approach was applied to rainfall data observed at the Sahand station in East Azarbaijan, Iran. The simulation successfully reproduced the exceedance probability curve represented by the Q-Q plot. The success of this approach encourages future research to determine if it can also be used for disaggregating satellite rainfall and global-climate models.

Observed Rainfall (mm)



Rainfall (mm)

Fig. 13 Comparison of rainfall simulation at the Tabriz station. Left: exceedance probabilities; right: the quantile-quantile plots.

Fig. 12 Comparison of rainfall

Left: exceedance probabilities;

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