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Analogies Between Complex Systems and Phases of Matter

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Abstract. *The behavior of a complex system in a changing environment is strongly affected by the system's architecture. We present an analogy between the major phases of matter (solid, liquid, gas) and three major generic architectures of complex systems: tree structures, layered structures and grid networks. This analogy is realized using a graph-based formalism, with nodes and edges in a given configuration. Solid materials are akin to tree structures, especially when we consider that most solids actually have cracks. Solids with cracks between their components can be modeled by nodes (representing each component) and their interconnection, leading to a tree structured hierarchy. Gases made up of molecules can be modeled by nodes (the molecules) with local interconnections representing nearby molecules in space, thus forming a grid network. Liquids can form layers as in a mixture of oil and water. We represent this by connections that are densely horizontal within layers as well as sparsely vertical between layers.*

A key issue for complex systems is the ease by which they may be changed, which we call the system's flexibility. Our definition of flexibility indicates that tree structures, like solids, are relatively inflexible and that grid networks, like gases, are extremely flexible, possibly leading to loss of control and chaotic behavior. Like liquids, layered systems are intermediate in flexibility and controllability. Solids, even with cracks, are relatively difficult to modify, whereas gases change internal form so quickly that they can only be constrained; not controlled. Liquids are intermediate in their ability to change form internally. Just as heating solids can lead to liquids, and heating liquids can result in gases, we shall present transformations in the interconnection structure of systems, analogous to heating, that change tree structures into layered ones and layered structures into networks.

**Keywords.** Complex systems, system architectures, layered structures, phases of matter, flexibility

1 Introduction

A system may exhibit complex behavior for many reasons. In some cases, complexity arises from the environment. For example, an ant, which may be said to exhibit a relatively simple internal structure, may leave complex tracks in a hilly domain (Simon, 1996). On the other hand, a complex internal structure can enable a system to exhibit a multiplicity of useful behaviors under different circumstances. For example, humans have manifold distinct behaviors and are indeed internally quite complex. The

relationship between internal structure and system behavior has not traditionally been well understood. Therefore, we focus on how a system's architecture influences its behavior, or how the way in which it is internally structured impacts upon its ability to exhibit different behaviors.

Systems with complex internal structures do not always enable change; indeed, a very complex internal structure can make it difficult to make additional changes or adjustments. Thus a goal of systems design is to develop system architectures that permit ready adaptation to environmental changes without becoming so internally complex that such adaptations can no longer be pursued effectively. Such adaptations in the internal structure of complex engineered or human systems are usually the result of an internal structure that enables flexibility in behavior.

We model a system's architecture as a set of nodes, with edges representing flows (e.g., of information) between them. A node might be a person in an organization, a terminal in an electric or electronic network, or any other element of the system that can be isolated. Similarly, edges can represent frequent communication between individuals (as in a social network), the transmission of packets or power within a network, or any other means by which one node can induce a change in its neighbor. A system is more flexible if information can take several paths from a source node to a destination node. Furthermore, many, if not most, systems that we aim to control exhibit hierarchical structure, with the top node in the hierarchy designating the controller and the bottom-most nodes designating what we would like to control. Consequently, we define a measure of flexibility for a system as the total number of paths in it, starting at a top node and ending at a bottom node. Cycles or loops in the system are counted just once. We note that different phases of matter permit different rates of internal change in response to changes in the environment.
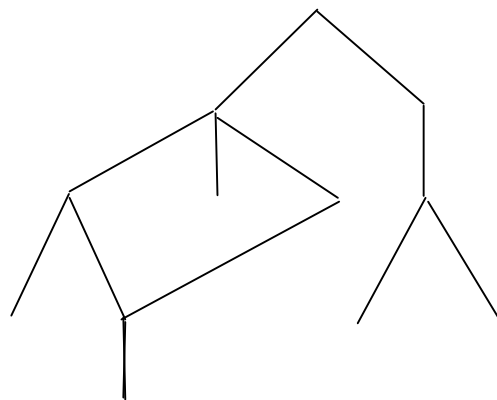
Fig. 1. An example of a system architecture. A nearly pure tree structure with one non-standard edge

We consider three phases of matter – solids, liquids and gases. Just as there are three fundamental phases of matter, we contend that there are three generic architectures for engineering systems – namely tree structures, layered structures and networks (Moses, 2010). A key reason why we are interested in the analogy to phases of matter is that it gives an argument for our emphasis on the three generic architectures for large scale systems. There is little argument about the importance of the three major phases of matter. The analogies we shall consider are summarized in the table below.

Table 1. Analogies between phases of matter and generic system architectures.

Solids	Tree structures
Liquids	Layered structures
Gases	Grid networks

2 Solids – Tree Structures

Some physical solids, such as poly-crystals, are extremely regular composites of atoms or molecules. These could be represented with a single top node in a tree structure that points to all bottom nodes, representing the atoms and molecules in the poly-crystal. For such a tree with only two levels, the number of connections from the top node in such a structure is likely to be huge. Often, poly-crystals exhibit structure at multiple levels. For example, most solids have some defects, and such solids can be represented with several different nodes, each of which decomposes into several branches.
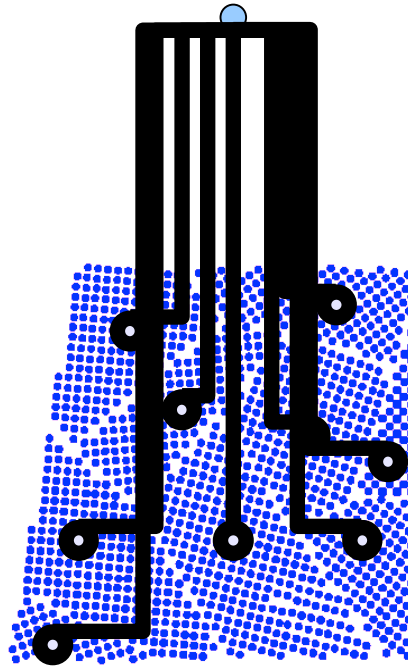

Fig. 2. A tree structure for a solid poly-crystal. Each bottom node (small blue dot) represents the smallest chunk in the system (e.g., a molecule). Each middle node (medium size black dot) represents a crystalline chunk with no defects. Finally, the top node (large blue dot) represents the entire poly-crystal.

Bottom nodes of such structures represent highly regular combinations of atoms or molecules. If each defect defines a "chunk" in a poly-crystal, this can enable us to create a tree structure with multiple levels, but with no lateral or horizontal connections – each "chunk" in the poly-crystal is self-contained. In either case, we have a tree structure that represents the entire solid, but which has relatively little flexibility.

One characteristic most solids have in common with tree structured systems and human organizations is the relative difficulty by which changes can be made in the solid's structure. Tree structured systems tend to be difficult to change, in comparison with other generic architectures (Moses, 2010). Changing a tree structure typically requires adding or removing sub-trees. Likewise, most solids are usually relatively difficult to change, usually requiring the fracturing of the solid material.

3 Fluids - Layered Structures

If one wants to make changes in structure at a greater rate than is normally possible in a solid, one can heat it. Many solids will melt at a sufficiently high temperature, creating a liquid. Such a phase change has an analog in the generic architectures of a system, namely a change from a tree structure to a layered structure. Layered systems differ from tree structures in that there is an emphasis on horizontal connections in addition to the hierarchical (top-down or bottom-up) interconnections. Pure tree structures do not possess horizontal connections at all. Systems with horizontal interconnections can introduce more changes or exhibit more behaviors than systems that have no such interconnections. An example of a layered human organization is a law firm, with three layers consisting of senior partners, junior partners, and associates. Individuals within a layer work as a team such that if one person is not available to address a concern from the environment (e.g., a change in requirements from a client) the other partners will be able to address this concern sufficiently well. Senior partners may get together to elect new senior partners from the ranks of the junior partners when the elevated junior partner has demonstrated the ability to function as a senior partner does.
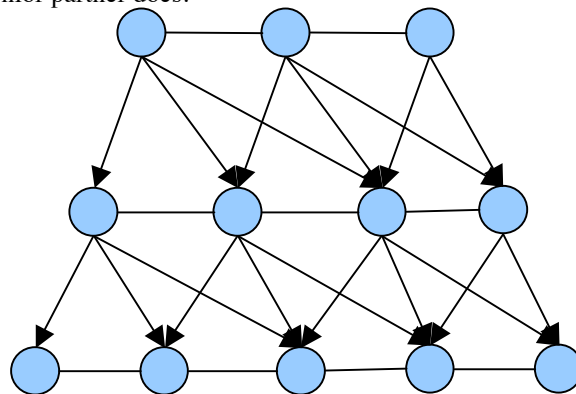

Fig. 3.1. A layered system with three layers.

Typically, individuals at different layers within such a system examine a problem at different levels of abstraction. For example, lower-level officers in military organizations solve tactical problems, mid-level officers solve operational problems, and high-level officers (generals) solve strategic problems. In mathematics different layers represent different levels of abstraction. In contrast with pure tree structures where each node except the top node has exactly one parent node, nodes in mathematical layers can have many parent nodes.
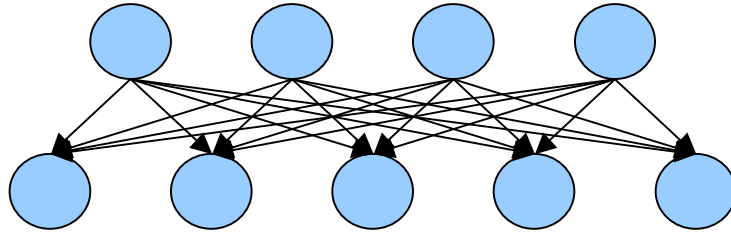

Fig. 3.2. A Two-Layered System in mathematics. Each layer represents a different level of abstraction.

Layered human and engineering systems are layered largely due to horizontal interconnections among their nodes. Furthermore, the number of layers corresponding to the number of levels in a tree structure will be lower after a phase change.
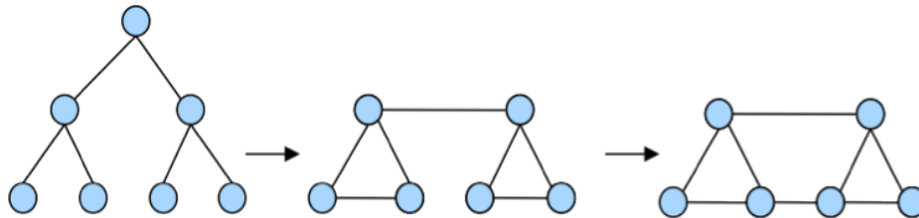
Figure 3.3. Transitions from a tree structure to a partially layered hybrid structure to a fully layered structure.

The field of fluid mechanics actually has a concept called 'boundary layers', which shares some properties with what we call layers here. In fluid dynamics, a boundary layer has a high viscosity compared to a free fluid stream, creating two different regions of relatively uniform density. Within each fluid layer, molecules move at roughly the same rate. Changes within one layer (e.g., due to turbulence) are relatively independent of what may change in another layer. Since fluids deform under sheer (lateral) forces, each molecule can change the relative position of other molecules within its layer relatively easily, leading to multiple horizontal connections. Across layers, molecules move at different rates. For example, the more viscous boundary layers move less easily does than the less viscous free stream. Whereas fluid layers are contiguous, nodal layers are boundaries between two regimes. Thus Figure 3.4 has two layers when viewed as an organizational structure, and one layer when viewed as a fluid (the bottom of the structure may be a solid wall, and the top of the structure may be a free stream, with a boundary layer in between).
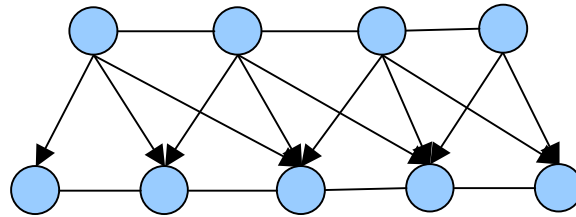
Fig. 3.4. A Layered System with Two Layers as Organizational Structure and One Layer as Fluid

The Atlantic Ocean has been described as having five fluid layers or zones (e.g., Sea and Sky, 2012), corresponding to areas of different density due to water pressure, and therefore different capacity to support life. Five fluid layers are equivalent to six nodal layers – one for each interface.

Recall our definition of flexibility: the number of paths in the system from a top node to bottom nodes, with loops counted just once. Layered systems with three or more layers are more flexible than tree structured systems having a similar number of nodes. A pure tree with O(n) nodes will have O(n) paths at most. A layered system with O(n) nodes and k layers will have $O(n^{k-1})$ paths, a higher figure than for a tree structure, when k is greater than 2. Unlike solids, whose sub-components interface only at fixed locations, the relative mobility of fluid molecules within a layer, and the general ability of the fluid to change shape under sheer forces, means that there are many points of contact between two layers. This corresponds to the existence of several paths in an engineering system. The flexibility of a fluid is clearly larger than that a solid when the fluid container changes shape or heat is applied, yet fluids are largely controllable – indeed, mechanical engineers study the applications of fluids to hydraulic (liquid) and pneumatic (gas) control. Importantly, hydraulics are incompressible, and therefore transmit significant power with little loss of control, whereas pneumatics rely on compressible fluids (gasses) leading to some difficulties in control such as spring action.

4 Gases – Grid Networks

Networks can be used to model a wide variety of systems. Our emphasis is on modeling gas-like structures using two or three dimensional grid networks. Here the internal interconnection structure has each node connected to neighboring nodes. There is no clear hierarchy in such systems.
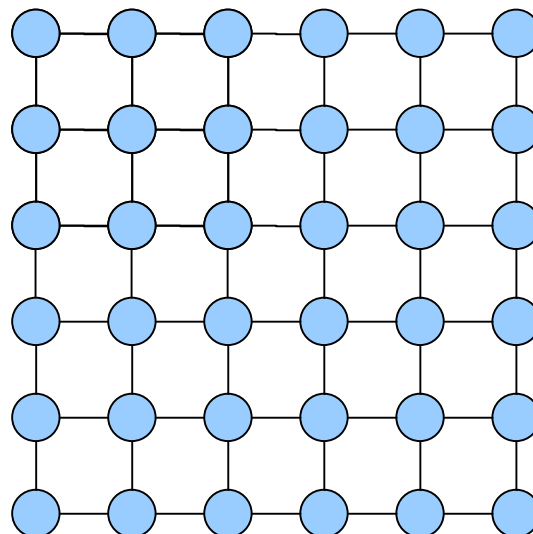
Fig. 4. A generic grid network in two dimensional space

If a node is connected to four neighbor nodes in a two dimensional space, say, then there are $O(4^n)$ paths in such a system, a huge increase over the flexibility measure of tree structures or solids as well as layered structures and fluids. The cost of such a large increase in flexibility is the relative lack of control over the behavior of such a system or gas. In some cases the system's or gas's behavior is chaotic and hard to predict. Examples of such chaotic behavior occur in hurricanes as well as stock markets. In contrast, tree structures or solids are relatively well controlled. Layered systems or fluids are intermediate in their controllability and also in their flexibility.

The transition from layered structures to grid-type networks can occur when local interconnections are added, but the hierarchy present in either layered or tree structured systems gets lost as a result. When heat is added to a gas then some local interconnections are severed and the number of possible interconnections (i.e., interactions between more distant nodes) increases. Given sufficient heat, chaos may ensue.

5 Related Work

S.A. Kauffman (1993) also uses an analogy of complex systems to solids, fluids and gas phases. His major interest is in the self-organization of systems rather than in changes made in those systems by outside designers. This design perspective is central to our work and may be said to differentiate an engineering approach from a natural science approach. Consequently, we are also interested in human organizations, an area which Kauffman seems not to address.

C.G. Langton (1992) also drew an analogy between complex systems and phases of matter. Langton's approach also uses computational models. His analogy to the liquid phase is a very thin slice of systems that are unusually complex. Layered systems in our approach are not so limited in extent nor are networked systems chaotic to the extent that Langton's analysis finds them to be.

The analogy of phases of matter to our generic architectures provides a reason why these architectures are so basic to large scale systems.

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