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Market-based valuation of transmission network expansion. A heuristic application in GB

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ABSTRACT

Transmission investments are currently needed to meet an increasing electricity demand, to address security of supply concerns, and to reach carbon emissions targets. A key issue when assessing the benefits from an expanded grid concerns the valuation of the uncertain cash flows that result from the expansion. We develop a valuation model which combines optimization techniques, Monte Carlo simulation over the expansion project lifetime, and market data from futures contracts on commodities. The model allows for random failures in generation and transmission infrastructure. Uncertainty stems also from nodal loads, fuel prices, allowance prices, wind generation, and hydro generation. Thus the model accounts for the stochastic dynamics on both the demand side and the supply side. To demonstrate the model by example, we consider a simplified network with two nodes. It is intended to broadly resemble the power generation sectors in England/Wales and Scotland. We then focus on the proposed Western HVDC subsea link. We simulate the whole distribution of effects on system costs, carbon emissions, and unserved load.

Keywords: electricity transmission, network expansion, uncertainty, load, coal, natural gas, renewables, carbon allowances, stochastic processes, futures markets, optimal power flow, Monte Carlo.

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1 Introduction

The demand for electricity in the European Union is expected to keep rising in the foreseeable future albeit rather moderately; ENTSOe [30]. In this regard, the EU aims to increase the use of renewable energy sources (wind, solar, biomass, etc.) to 20 % of total energy consumption by 2020 (around 8.5 % in 2010). But the most suitable sites turn out to be located in remote areas. This fact raises the need to lay new transmission lines to deliver the electricity at load centers; Macilwain [43].[1]

Transmission expansion is critical to address not only growing demand, but also some other immediate and strategic concerns. In addition to reducing hours of curtailed demand it can: (a) reinforce competition among participants in the electricity market, (b) enable more efficient operation of generating units (with savings in fuel costs and emission allowances spared) and lower electricity bills, (c) hedge against climate and physical uncertainties, and (d) clear the way to suitable yet remote resources; Buygi et al. [14].

End users are concerned about the price they pay for electricity and the availability and quality of the service delivered. Power producers are focused on the difference between the unit price of electricity and the cost to producing that unit. And transmission system operators arrange operations and planning in such a way that the overall system performs as desired now and in the future.

As the above suggests, any transmission project (or the lack of it) impacts a number of different stakeholders. Consequently, decision makers must assess not only strict efficiency issues but also distributive ones. As a previous step before considering the stakeholders claims, though, all the agents should be interested in an accurate description of what is at stake, in other words, of the potential benefits of the project. It is here where this paper tries to make a contribution.[2]

Every new project comes with an anticipated bill,[3] but this is clearly only half of the picture. Projects afford benefits in a variety of forms. Unfortunately, translating these potential gains into monetary units is far from trivial. This may explain to some extent why investments to expand (or upgrade) the transmission grid have not kept pace with those in new generation over the last years. Indeed, the existing transmission (and distribution) networks face a growing problem of ageing across Europe (and also the United States); IPCC [37].

Any progress should thus be welcome if only because it would somehow allow a discussion about the cost-effectiveness of a given project. Further, to the extent that the assessment of the gains draws on observable market prices and sound financial principles, we get closer to a value-based approach with its ensuing impact on the stakeholders' incentives

[1]Up to 35,000 km of new transmission lines (25,700 of them AC and 9,600 DC) are required in the EU by 2020; ENTSOe [30]. Several thousand kilometers of upgraded connectors are also envisaged.

[2]We thus deal with 'transmission planning' as explained in Wu et al. [60]. This includes a technical assessment (reliability, feasibility, etc.) and an economic assessment. It falls in the realm of the transmission planning authority (e.g. the ISO). 'Transmission investment', instead, considers candidates for transmission expansion and is addressed by the entity that decides whether to invest in a particular transmission project or not.

[3]The investment costs of transmission projects to be completed in the EU within the period 2010 to 2014 were anticipated to range between 23 and 28 billion euros; ENTSOe [30].

schemes. Not less importantly, the expected returns on the project might attract the interest of the private sector thus mobilizing much needed capital toward this critical area.[4]

From a methodological point of view, most of the economics literature on transmission investment completely ignores reliability and related issues; Joskow [38]. On the other hand, most of the engineering literature is very detailed as far as the system description and optimal operation are concerned. Risk is approached through a number of measures as the expected unserved energy, expected generation/load curtailment, duration, frequency, and the like. Thus, there is an asymmetry in the literature regarding the treatment of physical uncertainties and economic uncertainties. As the electricity market deregulation moves forward this difference in emphasis becomes ever less tenable; Wu et al. [60]. We aim at a more balanced approach where uncertainty from market forces coexists more on a par with that stemming from physical infrastructures. Further, when it comes to market uncertainties, we try to exploit the information embedded in observed market prices (as long as these are available/reliable). This differs from usual practice which typically takes account of economic uncertainty through different *ad hoc* values or expert/official forecasts.

Of course, uncertainty about the future impacts the rate at which future cash flows must be discounted to the present. Our approach sidesteps the debate concerning the appropriate discount rate. There are sensible arguments in favor of discounting at the private investor's weighted average cost of capital. Still others argue that high voltage transmission has come to be a public good in that no agent (whether consumer or producer) can be excluded from its benefits;[5] one should accordingly use a social discount rate. Needless to say, minor differences in the discount rate can make a big difference in the present value of a decades-long stream of cash flows. In our approach, futures markets play a major role. In addition to their informational role, the use of futures prices allows discount at the risk-free interest rate (as shown in Finance textbooks).

Our valuation approach considers the whole useful life assumed for the expansion project, in contrast to related papers that usually perform economic dispatch on an hourly (or shorter) basis, with a time horizon extending over one (or a few) year(s); see Foley et al. [32]. This lets us capture better the dynamic relationship between generation and transmission assets (there can be delays between the date in which a given line becomes active and power stations seize its potential). Unfortunately, our long-term simulation comes at the cost of framing the optimization problem on a longer time (for example, a week instead of an hour).[6] We allow random infrastructure failures. We run a number of simulations for key physical and economic variables. Optimization takes place under each of these potential settings; the objective function to be minimized involves electricity generation costs and the value of lost

[4]For example, the ability to determine the savings from avoided generation costs could signal the way to some funding sources (Kindler [40]) or allow implementation of the `beneficiaries pay' principle (Buijs et al. [11]).

[5]New lines certainly benefit many parties. But this may be a mixed blessing; since expansion costs are nonlinear, incentives toward free riding can easily arise; Bushnell and Stoft [13]. The incentives to co-operate and undertake beneficial investments further depend on the business models adopted in different electricity markets (Wu et al. [60]) and on network tariff architectures (Sakhrani and Parsons [53]).

[6]There would be no further problem in using our model for a yearly period on an hourly basis (8,760 steps) apart from the increase in the time required for computation.

load. Out of these simulations, we derive the cumulative distribution function of effects over major variables and/or determine several metrics (not only averages).

The main distinctive features of our model are therefore:

a) load, renewable generation, fuel prices, and emission allowance prices are stochastic;
b) we use market data to infer the future behavior of economic variables;
c) by using futures prices we set the ground for risk-neutral valuation;
d) we consider the performance of the system over the project lifetime;
e) we include projections of generation investments and retirements;
f) we simulate the whole distribution of effects of the expansion project.

To demonstrate how the model works we undertake a heuristic application. In particular, we consider a specific, simplified grid which broadly represents the case of Great Britain. The two nodes stand for England/Wales and Scotland, respectively. Right now these countries are linked through a 2,200 MW transmission corridor which already runs at almost full capacity with the real danger of becoming a bottleneck. Starting from this base case scenario, we focus on a particular expansion recently proposed by major stakeholders; we address that investment in a now-or-never context. Note that both jurisdictions are covered by the EU Emissions Trading Scheme (ETS), so their electricity generators operate under binding greenhouse gas (GHG) emission constraints. The generation and transmission infrastructure is optimally managed by changing input fuel and electricity output as required.

The paper first provides a quick overview of the related literature with a special focus on papers that address investment under uncertainty. We then stress the main characteristics of our valuation methodology and describe the model which explicitly takes into account both physical and economic uncertainties. To ease the way for the heuristic application we develop a two-node circuit; the model can be naturally extended (with the appropriate changes) to a more general setting with an arbitrary number of nodes and loop flows. Several variables are assumed to follow a stochastic dynamics in continuous time. Nonetheless, econometric estimation from observed data and subsequent Monte Carlo simulation call for a version of the model in discrete time; we develop these operational issues to some extent in Appendix A. Our heuristic application of the model provides some background on the GB electricity sector and the proposed expansion to improve the transmission flow from north to south. Then we analyze the optimal behavior of the system under two different network configurations (without and with the expansion project) in three scenarios (the base case and two others with altered demand and supply). In each scenario, we compare the system behavior of the current network with that of the expanded network. By focusing on several system attributes we can assess the cost effectiveness of the proposed interconnection.

2 Literature review

Following Kirschen and Strbac [41], there are basically two ways to increase the capacity of a network. One involves upgrading the transmission capacity of the existing network, the other consists of building new lines.

Major transmission expansions entail both economic and strategic benefits. The first category encompasses the increased reliability, efficiency, and incremental competition that expansions bring about. Benefits from load and fuel diversity, environmental improvements, and insurance against contingencies fall within the second category; CERTS [16]. These are hard to measure and harder still to translate into monetary units: "*The ISOs are challenged when asked to develop a business case justifying a market economics project and lack the necessary market models to adequately forecast and 'prove' their need*"; EPG [28].

A number of methodologies have been advanced to assess the benefits from network expansions; Bresesti et al. [10]. Some are strong in accounting for the strategic behavior by power generators. Others distinguish the expansion's impact on consumer surplus from that on producer surplus. The extent to which uncertainty is taken into account varies from one methodology to other. The most sophisticated ones combine an optimization model with Monte Carlo simulation in a limited fashion; the former relates to market supply and demand and their impact on (re-)dispatch, while the latter determines the status of different network elements. One limitation, however, is that the simulation period is usually taken to be one year. A longer time frame would permit consideration of the interplay between generation and transmission along with their changes. In our model we account for market structure issues through the profit margin of the electricity price-setting (or ‘marginal’) technology.

Regarding environmental benefits, most up to date studies have dealt with the potential impact of environmental constraints either on generation expansion or generator maintenance scheduling, but not on transmission. Merely computing the total emissions in a given region over a certain period is a first step. This can be improved by internalizing the cost of pollutant emissions in the dispatch algorithm. But this is typically undertaken using some pre-determined allowance price (assumed to be correct on average at best). Kazerooni and Mutale [39], instead, approximate carbon allowance price by means of a Weibull distribution based on past data. They then introduce the carbon constraint as a changing operation cost in the objective function of transmission network planning. In this respect, we adopt a geometric Brownian motion (GBM) for carbon price and estimate the underlying parameters from futures contracts on emission allowances.

On the other hand, a few papers have adopted the Real Options approach (Dixit and Pindyck [27], Trigeorgis [57]) in transmission expansion planning, both at the theoretical and applied levels; see Hedman et al. [34] and the references therein. Kurlinski [42] assessed users' willingness to pay for a transmission upgrade in a simplified electric system. Every contingency in a system can be assigned a probability of occurring each time period. Many other aspects of the system that affect the system cost for each state at each time period are stochastic as well (e.g. fuel prices, electricity price, carbon price, and when and where generators will enter or exit the market in the future). For clarity, though, only stochastic (future) load is explored in detail.

Blanco et al. [3] considered a test system consisting of three areas, represented by three nodes, linked by three transmission lines. They considered a thermal generation system with two generators using fossil fuel as primary energy source. They assessed a potential investment in a new transmission line between two of the nodes as an alternative to enhancing the capacity of an existing line using an electronics device to increase its flexibility and efficiency.

Blanco et al. [4] assessed two network upgrades to strengthen German-Dutch interconnections: an interconnection project, and installing a device which can be relocated if necessary. The demand growth rate is assumed stochastic, and three wind situations are considered. Nuclear fuel prices are assumed constant over the time horizon; fossil fuel prices follow mean-reverting stochastic processes. There is no mention of cross-correlations between these prices; simulating price paths as if they were independent seems unlikely to provide an accurate quantitative result. Correlation has been shown to play a major role in the assessment of energy projects; Roques et al. [52]. Besides, if fossil fuel prices are taken as major drivers of their use, it would be sensible to include a model for the allowance price, since both countries are subject to the EU ETS and the carbon content of fossil fuels is different. The same applies to Blanco et al. [5]. Both works are, nonetheless, interesting contributions in terms of the collection of options considered and the detailed description of technologies and procedures.

Garcia et al. [33] used the original IEEE 24-bus reliability test system without two particular lines, and then consider three investment alternatives: adding only one line or the other or adding both together. The aim is to maximize social welfare subject to the network constraints. The prices of oil, nuclear fuel, and coal are assumed to follow (correlated) mean-reverting processes. Load and installed generation capacity are unknown; their annual growth rates follow a generalized Wiener process. Their main contribution is to estimate the net present value of the investment as a function of the investment timing.

3 Methodology

We propose a model for the value of a network expansion. The value of any particular expansion depends on factors that change over time, e.g. network topology, market structure, fuel and electricity prices, energy policy, environmental and climate policies, etc. Our valuation model rests on solving an optimization problem. At any time it minimizes the total costs of electricity generation and delivery; in this sense it draws on Bohn et al. [8]. A distinctive feature of our model is that the optimization process is subject to the behavior of the stochastic variables (e.g. nodal loads, fuel prices); thus we deal with a problem of stochastic optimal control. We allow for the possibility that a fraction of the demand is unserved, but this has a non-negligible cost. Regarding market power or strategic bidding by power generators, it is possible to use a mark-up or profit margin estimated from historical data.[7]

The model allows for random failures in generation and transmission infrastructure. Uncertainty stems also from nodal loads, wind generation, and hydro generation. They are assumed to follow a particular stochastic process with suitable properties (e.g. seasonality or stationarity). These processes can be estimated from official statistics. Stochastic processes similarly govern the economic sources of uncertainty (fossil fuel prices and allowance prices). For estimation purposes, the ideal market data are composed of futures prices; this is important because (assuming the required liquidity/maturity are met) they enable us to estimate parameter values in a risk-neutral setting.

Note that the standard explanation for the role of futures markets is that they help to spread and hence reduce risks, and to motivate the collection and dissemination of information relevant to the planning of consumption and production. As for the latter, the numerical estimates of the underlying parameters can then be used to simulate random paths of all risk factors (apart from infrastructures). Concerning the former, through the ability to construct a

[7] This approach is used by the California Independent System Operator; see CERTS [16]. It is clearly a compromise between adopting some *ad hoc* variable cost adders/modifiers and resorting to complex game theoretic models.

riskless hedge (involving the futures contract and the underlying commodity), risk can be effectively " squeezed out" of the problem, so that investors' attitudes toward risk do not matter (Trigeorgis [57]). Therefore, for valuation purposes, we can conveniently pretend to be in a risk-neutral world, where expected cash flows (weighted by the risk-neutral probabilities) can be appropriately discounted at the risk-free rate. Thus, by using readily available futures commodity prices and volatilities, and discounting at the riskless rate (itself also taken preferably from market data), we can derive the present value of cumulative costs/savings in a relatively simple manner.

At this point, it is possible to assess the performance of the whole system (before and after expansion) according to several metrics, e.g. generation costs, unserved load, carbon emissions, etc. Comparing the performance without and with the transmission expansion sheds light on the potential gains from that expansion. These potential gains can then be finally checked against the anticipated cost of the expansion (thus enabling a cost-benefit analysis).

Our model considers that the decision to invest in the expansion is now-or-never. In other words, it does not include any option to delay or otherwise alter the project.[8] It therefore does not address the question of the optimal time to expand. There are other ways to relieve congestion in addition to grid expansions (see for instance Ilić et al. [35] or Buijs et al. [12]), but they fall beyond the scope of this paper. And some transmission 'expansions' can indeed increase congestion (Blumsack et al. [6], Bushnell and Stoft [13]), but they are more the exception than the rule. We ignore inflation and efficiency targets at this stage. We abstract from access-pricing problems for new generators.

The model lends itself to assess the potential substitution between transmission and generation expansion, and indeed other potential alternatives to the expansion at hand. Thus, it not only provides an estimate of the "value" of the expansion in a given context; it can help uncover the "best value" of the expansion (possibly under a different generation mix, network topology, or load behavior thus allowing to address demand-side management issues).

4 The model

A transmission system expansion (or upgrade) increases reliability by decreasing both the amount and probability of unserved load in the event of a transmission line or generator failure. Transmission investments can reduce congestion costs,[9] and thus make consumers less vulnerable to the exercise of market power; Sauma and Oren [54]. Network expansions also allow operation of power plants at higher efficiency levels.

We aim to value the positive impacts of investments in network expansion through the

[8] Actual practice sometimes adopts this approach. For example, the New England system undertakes market efficiency upgrades (investments designed to reduce bulk system-wide costs) where the net present value of the reduction in system costs exceeds the net present value of the transmission investment; Sakhrani and Parsons [53]. The NPV criterion omits any consideration of the option to delay; this amounts to assuming a now-or-never context.

[9] In the electrical engineering sense, a line is 'congested' when the flow of power is equal to the line's thermal capacity, as determined by various engineering standards. As Sauma and Oren [54] point out, transmission congestion increases the risk of blackouts, reduces the ability to import power from remote cheap generators (thus raising the cost of energy), and impedes competition and trade (hampering the substitution effect).

increase in reliability, the reduction in congestion and operation costs, and the abatement of CO2 emissions. The model comprises two stages, namely simulation and optimization. The optimization model minimizes an objective function subject to constraints. The objective function considers two kinds of system costs: those of electricity generation costs and of unserved or lost load. The constraints can be split into two blocks concerning the physical and economic environment. The optimization provides, for each node and time, the level of generation from each technology, load, served load, and transmission levels along with aggregate generation costs, carbon emissions, allowance costs, and transmission losses. We consider a 20-year time horizon, the assumed lifetime of the expansion project. Over this period the network topology changes naturally as new stations start operation while others are decommissioned. Each year is decomposed into 60 timesteps (five per month); i.e. the relevant period for the optimization problem is 1/60 year.

The optimization model is nested in Monte Carlo simulation. A single run determines the operation state of both generation and transmission infrastructures over $60 \times 20 = 1{,}200$ consecutive time steps. The same holds for the value of stochastic nodal loads, wind- and hydro-based generation, fossil fuel prices, and carbon price. Under each setting, the optimization problem is solved: depending on the circumstances in place, generation is optimally dispatched subject to the network topology. Therefore, one simulation run involves 1,200 optimizations. We repeat the sampling procedure 750 times (so we solve 900,000 optimization problems). We thus come up with 750 time profiles of each variable of interest. Needless to say, if simulations are to be realistic then we must previously get numerical estimates of the underlying parameters from official statistics, market data, and the like. Our model can thus assess the benefits from transmission expansions at both nodal and aggregate levels, for both consumers and generators.

In our context, electricity prices do not reflect the primary source of uncertainty. Instead, we model more fundamental variables as stochastic. These are external processes (physical as well as economic) whose effects on the supply of, and demand for, electricity are reasonably well understood; see Skantze and Ilić [56]. Major features of our model are:

a) We assume stochastic behavior for the price of coal, natural gas, and carbon emission allowances. The unit price of each of these commodities is assumed the same for all generators irrespective of their physical location. Nuclear, wind, and hydro generation are assumed to bid at zero electricity price.

b) Each component of the network (generation unit, transmission line) has a probability of being out of service at any time (a so-called 'system contingency', especially damaging when infrastructure fails when demand is high because of weather conditions). In other words, the units potentially in operation are determined stochastically.

c) Given commodity prices and components availability, the model determines the supply curve of generators, ordered from lower to higher bid price of electricity.[10] Thus at each time it derives a stochastic realization of the 'merit order' or supply curve.

d) At any time there is a stochastic demand at each node. The loads can be correlated

[10]Each generator is assumed to submit its bid to a common pool that sets a single price for the relevant period. Absent strategic behavior by generators, and to operate for as long as possible, the bids account for short-term marginal costs.

(for example, because they are affected by common factors such as outdoor temperature or sun light). The model takes account of seasonal behavior.

e) The optimal power flow (OPF) algorithm dispatches generation assets in merit (least-cost) order subject to the physical constraints of the electric network. The economic dispatch problem is to find output for each available generator so as to minimize total (system) costs while meeting all of the loads plus line losses.

f) At every time demand and supply must be balanced, and the Laws of Physics must apply in the network. We adopt the DC load flow model, so we do not model the impact of reactive power on the system.

In principle, any network system considered in the electric engineering literature with deterministic load and fuel prices can accommodate uncertain load and fuel prices as we model them. At each simulation run, nodal loads and input/output prices are set at a given level. Cost minimization is then undertaken subject to physical and technical constraints. Optimization should work equally well whatever the number of nodes. However in our example, we introduce a transmission model comprising just two nodes. This way our (deterministic and stochastic) equations apply directly to the case considered later to demonstrate the model (thus avoiding any redundancy).

As a general rule, uncertainty pervades different layers of our model and thus shows up in the time behavior of a number of variables. A complete characterization of uncertainty involves the outcomes of uncertain processes, the associated probabilities, and their impacts on system performance. Accordingly, we first focus on physical variables like nodal loads or generation levels. Along the way, we describe the technical infrastructures in terms of their random availability or delivered electricity and deal with economic issues like the value of lost load and stochastic generation costs. The optimization problem faced by the system operator encapsulates it all.

4.1 Physical environment

Load. Stochastic load at each node is assumed inelastic in that a given amount is demanded regardless of the price of electricity. In our model D^i is the net demand for electricity from consumers at node i, with $i = \{1,2\}$. As it turns out, there is a power technology that effectively consumes electricity, namely pumped storage.[11] Its contribution at node i, P^i, has a negative sign. Therefore, the gross demand d at node i is the sum of the realizations of two different stochastic processes:

$$d^i = D^i + P^i.$$

Depending on the infrastructure available, load can be fully served or not. The electricity actually served at node i is denoted by s^i.

Future demand is uncertain. It varies by hour of the day, day of the week, and season. In addition to these deterministic drivers, there are unpredictable weather-related components. We assume that the deseasonalised load at each node i evolves over time according to the

[11] More electricity is consumed in pumping the water up to the reservoir than is generated when the water later runs down through the turbines. This is why, for electricity supplied, pumped storage is usually a negative figure.

following Inhomogeneous geometric Brownian motion (IGBM):

$$dD_t^1 = k_1(L^1 - D_t^1)dt + \sigma_1 D_t^1 dV_t^1,$$
$$dD_t^2 = k_2(L^2 - D_t^2)dt + \sigma_2 D_t^2 dV_t^2,$$
$$dV_t^1 dV_t^2 = \rho_{12} dt.$$

Each nodal load, D^i, is assumed to show mean reversion. L^i is the long-term equilibrium level toward which the present deseasonalized load tends. k_i is the speed of reversion towards that "normal" level. It can be computed as $k_i = \ln 2 / t_{1/2}^i$, where $t_{1/2}^i$ is the expected half-life for (deseasonalized) load at node i, i.e. the time required for the gap between D_0^i and L^i to halve. The instantaneous volatility of this load is denoted by σ_i. dV_t^i is the increment to a standard Wiener process; it is normally distributed with mean zero and variance dt. The correlation coefficient between nodal loads D^1 and D^2 is ρ_{12}.

Generation capacity. In principle, every node can host an array of generation technologies, conventional and unconventional, renewable and non-renewable. S stands for a given particular power station, and its actual electricity generation is denoted by x with an upper bound $\bar{x}$.

The coal (c), natural gas (g), and nuclear (n) fuel technologies in our model are prone to failure. We adopt a set of binary (Bernoulli) random variables for the possibility of any one contingency. We thus assume that each station S at node i and of type $\{c, g, n\}$ is in service for a fraction Λ of the year (and 1-Λ out of service because of failures and maintenance works):

$$S_c^i = \begin{Bmatrix} 0, \text{'off'state with probability } 1 - \Lambda_c dt \\ 1, \text{'on' state with probability } \Lambda_c dt \end{Bmatrix},$$
$$S_g^i = \begin{Bmatrix} 0, \text{'off'state with probability } 1 - \Lambda_g dt \\ 1, \text{'on' state with probability } \Lambda_g dt \end{Bmatrix},$$
$$S_n^i = \begin{Bmatrix} 0, \text{'off'state with probability } 1 - \Lambda_n dt \\ 1, \text{'on' state with probability } \Lambda_n dt \end{Bmatrix}.$$

Here $c = \{,..., \bar{C}\}$ stands for coal plants, irrespective of whether they are operative or not. Note that $\bar{C}$ is not fixed; it can change over time due to openings or closures on a planned schedule (our model takes this into account). Similarly, $g = \{,..., \bar{G}\}$ and $n = \{,..., \bar{N}\}$ refer to gas and nuclear plants.[12]

For any fleet of power plants there is a specific number of possible cases depending on the number of stations involved. For instance, assume the number of coal plants at a node is $\bar{C} =$ 4. There would be five possible cases, namely that none, one, two, three, and four plants are active or operative; each of these would correspond to a different value of the coal activity

[12] Ojeda et al. [50] consider different (time-varying) unavailability rates for coal, gas, and nuclear technologies. They also adopt a more sophisticated approach that uses random times to failure and to repair; see also Blanco et al. [3].

parameter: $a_c \in \{0, 0.25, 0.50, 0.75, 1\}$. The probability of each of these cases is markedly different. In general, for coal we have: $a_c \in \{0, \frac{1}{\overline{C}}, \frac{2}{\overline{C}}, ..., \frac{\overline{C}-1}{\overline{C}}, 1\}$, and similarly for the other 'fallible' technologies $\{g, n\}$ at any node.

We do not consider that wind (w), natural-flow or hydro (h), and pumped storage (p) stations can be 'off'. A prominent characteristic of these renewable resources is intermittence. This fact drives a sizeable wedge between installed capacity and metered electricity; it can be measured through the load factor. It seems reasonable to think that the usual pattern of failures in $\{w, h, p\}$ stations is already subsumed in the time series of electricity produced or consumed. All the intermittences for whatever reasons are modeled through the stochastic behavior of the load factor. The theoretical model assumed is an IGBM:

$$dW_t = k_W (W_m - W_t)dt + \sigma_W W_t dY_t^W.$$
$$dH_t = k_H (H_m - H_t)dt + \sigma_H H_t dY_t^H;$$
$$dP_t = k_P (P_m - P_t)dt + \sigma_P P_t dY_t^P.$$

The standard notation for reversion speed, long-term value, and volatility holds (wind: k_W, W_m, and σ_W; hydro: k_H, H_m, and σ_H; pumped storage: k_P, P_m, and σ_P).

Generation from wind, natural flow and pumped storage stations shows seasonality. Our simulations assume a seasonal behavior for renewable electricity, so the seasonality in each load factor must be previously identified (from the historical time series).

For any node i we can define the activity vector $a^i \equiv \{a_c^i, a_g^i, a_n^i, 1, 1, 1\}$ across all its technologies $f = \{c, g, n, w, h, p\}$. Aggregate output electricity at node i, denoted x^i, comprises generation from all its energy sources $f = \{c, g, n, w, h, p\}$:

$$x^i \equiv \sum_f x_f^i = x_c^i + x_g^i + x_n^i + x_w^i + x_h^i + x_p^i.$$

The maximum power that can be generated at a given time (t) by the coal plants sited at node i is $a_c^i \overline{x}_c^i$. Therefore, the aggregate output electricity at any node is bounded from above. And the ability of a node to meet load at other nodes depends on the transmission capacity of the lines connecting them.

<u>Transmission capacity</u>. Every line $L = \{1, ..., \overline{L}\}$ in our network has an exogenously given transmission capacity. Thus, the line between any two nodes i and j (with $j = \{1, 2\}$ but $i \neq j$), denoted L^{ij}, has a (notional) capacity l^{ij} (in MW). We assume that the availability rate of any transmission line is also lower than one:

$$L^{ij} = \begin{Bmatrix} 0, \text{'off' state with probability } 1 - \Lambda_L dt \\ 1, \text{'on' state with probability } \Lambda_L dt \end{Bmatrix}.$$

The active/inactive state of this line is given by the parameter $b^{ij} \in \{0, 1\}$.

A percentage p of each megawatt-hour transmitted is 'lost' in that only a fraction reaches the opposite end. This parameter sets an upper bound on the power that node i can contribute to node j, or the other way round. Thus L may become a bottleneck during episodes of excess demand or low supply. Transmission losses along the line L^{ij} amount to the maximum of three quantities: a known percentage of the surplus electricity in node i (exported to j), the same percentage of the surplus electricity in node j (exported to i), and zero:

$$m^{ij} = max\left[p(x^i - s^i); p(x^j - s^j); 0 \right]$$

Overall, depending on whether each (generation or transmission) asset is 'on' or 'off', at any time there can be $2^{(\overline{C}+\overline{G}+\overline{N}+\overline{L})}$ possible states. When we consider the potential availability of an additional transmission line $\overline{L'}$, the number of possible states rises to $2^{(\overline{C}+\overline{G}+\overline{N}+\overline{L}+\overline{L'})}$.

4.2 Economic environment

Demand-side costs. The overall unmet load is computed as:

$$\sum_{i=1}^{2} d^i - \sum_{i=1}^{2} s^i.$$

In our model, unserved or lost load has value. Thus we have implicit rationing costs. Following Blumsack et al. [6], we make two simplifying assumptions: all consumers in the network have an identical and constant value of lost load per unit, $VOLL$, for any level of electricity use. Thus demand-side costs equal the above difference times $VOLL$.

Supply-side costs. A major driver of stations' short-term marginal costs is fuel cost (in addition to emissions cost). We assume that wind, hydro, and nuclear stations bid a price of zero (Valeri [59]); that pumped storage takes electricity from the network at the bottom of the price range; and that the prices of coal (C), natural gas (G), and the CO2 allowance (A) evolve stochastically over time.

According to IPCC [36], a plant burning natural gas has an emissions factor of 56.1 $kgCO_2/GJ$. Since under 100% efficiency conditions 3.6 GJ would be consumed per megawatt-hour, we get the (carbon) emission intensity I_G of the gas-fired power plant:

$$I_G = \frac{0.20196}{H_G} \frac{tCO_2}{MWh}$$

where H_G denotes its thermal efficiency. Similarly, a plant burning bituminous coal has an emission factor of 94.6 $kgCO_2/GJ$ under 100% efficiency conditions; so that its (carbon) emission intensity I_C is

$$I_C = \frac{0.34056}{H_C} \frac{tCO_2}{MWh}$$

where H_C denotes its thermal efficiency.

Note that, in a deregulated electricity market, economic costs include both explicit input (fuel) and output (emissions) costs, and a margin to get a 'reasonable' profit for the generation units. Its size (here assumed constant) crucially depends on the 'marginal' technology that sets the electricity price, and the scope for market power and/or strategic behavior by generators.

Generation costs comprise the (bid-based) costs incurred by all power technologies $f = \{c, g, n, w, h, p\}$. Since wind, hydro, and nuclear generators are assumed to bid a zero electricity price, these sources will be fully dispatched whenever available as long as load surpasses their availability and there are no congestion problems: $x_w^i = \overline{x}_w^i$, $x_h^i = \overline{x}_h^i$, $x_n^i = \overline{x}_n^i$.

As for the other technologies, let

$$x_c \equiv \sum_{i=1}^{2} x_c^i ; x_g \equiv \sum_{i=1}^{2} x_g^i ; x_p \equiv \sum_{i=1}^{2} x_p^i ,$$

denote the aggregate coal-, gas- and pumped storage-based generation, respectively. Noting that pumped storage stations tend to adjust their operation to the time when electricity prices are at the higher end, even above natural gas turbines,[13] we assume their 'cost' function is a multiple of that of gas turbines, in our case, 1.10. Thus total generation costs are:

$$c(x_1, x_2) = x_c(M_m + \frac{C + 0.34056A}{H_C}) +$$
$$+ x_g(M_m + \frac{G + 0.20196A}{H_G}) + x_p 1.1(M_m + \frac{0.20196A}{H_G}).$$

C and G denote the price (in €$/MWh$) of coal and natural gas, respectively, while A stands for the price (in €$/tCO_2$) of a carbon emission allowance. In electricity markets where natural gas-fired stations are the usual marginal technology, the fixed margin M_m will be the 'average' or long-term clean spark spread. When coal-fired plants or pumped storage stations are the marginal plants, we assume that they earn the same margin.

We assume that natural gas prices display a seasonal pattern, but that coal and carbon do not. The long-term prices of natural gas and coal are described by the following IGBM stochastic processes in a risk-neutral world:

$$dG_t = df_G(t) + [k_G G_m - (k_G + \lambda_G)(G_t - f_G(t))]dt + \sigma_G (G_t - f_G(t)) dZ_t^G .$$
$$dC_t = [k_C (C_m - C_t) - \lambda_C C_t] dt + \sigma_C C_t dZ_t^C .$$

Carbon prices are assumed to follow a standard geometric Brownian motion (GBM); Çetin and Verschuere [17]. Analytically:

$$dA_t = (\alpha - \lambda_A) A_t dt + \sigma_A A_t dZ_t^A .$$

Both G and C are assumed to show mean reversion. G_m and C_m denote the long-term equilibrium levels toward which current (deseasonalized) gas and coal prices tend in the long run. $f_G(t)$ is a deterministic function that captures the effect of seasonality in gas prices. In general the function is defined by $f(t) = \gamma \cos(2\pi(t + \varphi))$, with the time t measured in years and the angle in radians; when $f(t = -\varphi) = \gamma$ the seasonal maximum value is reached. k_G and k_C are the reversion speeds towards the " normal" gas and coal prices. They can be computed as $k_G = \ln 2 / t_{1/2}^G$, where $t_{1/2}^G$ is the expected half-life for (deseasonalized) natural gas, i.e. the time required for the gap between $[G_0 - f_G(0)]$ and G_m to halve; similarly $k_C = \ln 2 / t_{1/2}^C$. Regarding the price of the emission allowance, the parameter α stands for the instantaneous drift rate of carbon price. σ_G , σ_C and σ_A are the instantaneous volatility of natural gas, coal and carbon allowance. λ_G, λ_C and λ_A denote the market price of risk for gas, coal, and allowance prices. dZ_t^G , dZ_t^C and dZ_t^A are the increments to standard Wiener processes. They

[13] This is consistent with observed patterns in the Spanish electricity market; see Federico and Vives [31]. Of course, the situation in other markets can be different.

are normally distributed with mean zero and variance dt; besides:

$$dZ_t^G dZ_t^C = \rho_{GC} dt; dZ_t^G dZ_t^A = \rho_{GA} dt; dZ_t^C dZ_t^A = \rho_{CA} dt.$$

From the above stochastic differential equation for the price of natural gas under risk neutrality it is possible to derive the time-0 expectation of natural gas price at time t. One can then resort to equilibrium arguments to show that this expectation (under the risk-neutral probability measure) is equivalent to the futures price of natural gas for delivery at t. Therefore, we have a theoretical model for the futures price with any desired maturity. We estimate the parameters in this stochastic model using daily prices and non-linear least-squares regression. Upon estimation of the parameters we can simulate the behavior of natural gas price any number of times. The same approach holds for the other two commodities, namely coal and carbon allowance. Note that futures market players take the delivery of the underlying commodity for granted. Therefore, no risk premium shows up in futures prices and these can be discounted at the risk-free rate. (This is equivalent to simulating the behavior in the physical, real world, where the risk premium impinges on spot prices, and then discounting cash flows at a rate commensurate with risk).

Economic dispatch. We assume that the system operator dispatches generating resources to minimize the total costs of generation and unserved energy ($VOLL$) across all the nodes. The aim is to find an optimal vector of power generated at all the nodes $\{x^1, x^2\}$ and power consumed at all the nodes $\{s^1, s^2\}$ that minimizes system costs at any time:[14]

$$\min_{\{x_c^1, x_g^1, x_p^1, x_c^2, x_g^2, x_p^2, s^1, s^2\}} \{c(x_c^1, x_g^1, x_p^1, x_c^2, x_g^2, x_p^2) + (d^1 + d^2 - s^1 - s^2) \times VOLL\}$$

s.t.

$$0 \le x_f^1 \le a_f^1 \bar{x}_f^1; 0 \le x_f^2 \le a_f^2 \bar{x}_f^2; f = \{c, g, n, w, h, p\};$$

$$0 \le s^1 \le d^1; 0 \le s^2 \le d^2;$$

$$\sum_f x_f^1 + \sum_f x_f^2 = s^1 + s^2 + m^{12};$$

$$\sum_f x_f^1 - s^1 \le b^{12} l^{12}; \sum_f x_f^2 - s^2 \le b^{12} l^{12};$$

$$dD = a(D,t)dt + b(D,t)dV; D = \{D^1, D^2\};$$

$$dR = a(R,t)dt + b(R,t)dY; R = \{W, H, P\};$$

$$dX = a(X,t)dt + b(X,t)dZ; X = \{C, G, A\}$$

The first four restrictions set the environment as determined by the operation state of the physical assets. The components of the power system are subject to limits. Actual

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[14] In addtion to carbon dioxide emissions there can be others (SO_2, NO_x, ..) which are valued. If so, the objective function could include another term. This would yield a different solution to the minimization problem. Other environmental impacts could encompass land use, erosion risk, threats to wildlife, and so on.

generation levels cannot surpass generation capacity constraints; the power delivered at each node is lower than or equal to the amount demanded (it is possible that some load is not met when cost is minimized); power balance requires that the electricity generated be equal to consumption plus transmission losses; surplus generation at each node must be lower than the transmission capacity of the lines departing from that node.

The last three restrictions are the stochastic differential equations. Local demands at the two nodes $\{D^1, D^2\}$ have different initial values and evolve seasonally and stochastically over time according to an Ito process. The load factor of renewable, intermittent wind- and hydro-based generation stations $\{W, H, P\}$ is governed by a stochastic process. Similarly, the price of each commodity (coal, natural gas, and emission allowance) follows another Ito process. The increments to standard Wiener process dV, dY and dZ differ. dZ also differs for each commodity $\{C, G, A\}$ along with the terms $a(X,t)$ and $b(X,t)$.

Note that the above model can be stated in discrete time for estimation and simulation purposes. See Appendix A.

5 A heuristic application in Great Britain

In GB the highest gap between generation and demand takes place around London, whose demand approaches one tenth of the system peak. There is also a small gap in the south west. The remaining areas export their surplus production. Boundary 6 is of particular importance as the primary importing border from Scotland to England.

Electricity demand is naturally related to price. In a deregulated electricity market, at any time it is necessary to produce it at the lowest possible cost, which includes fuel costs, allowance costs, and a profit margin. According to Roques et al. [52], daily quarter-ahead forward prices for base-load electricity and natural gas in the U.K. market from 2001 to August 2005 exhibited a correlation factor of 89%; and the correlation between electricity and carbon prices from the start of trading in October 2004 until September 2005 stood at 73%. As shown in National Grid [47], both peak and baseload electricity prices more or less track natural gas prices at National Balancing Point (which does not happen with coal or oil, for instance). This is relevant when we deal with the profit margin included in generation costs; see Appendix B.

Looking into the future, National Grid [48] expects zero growth for England/Wales and Scotland from 2011/12 to 2017/18, and Scottish Power Distribution [55] suggests a mild growth of 0.1 %. This information may be particularly useful in case historical data fail to provide a reliable estimate for the future or there are important developments to take place that do not show up in past data. Our base case analysis assumes that electricity demand shows mean reversion over time with a null rate of growth.

The UK has two key environmental targets relating to renewables and GHG emissions; National Grid [48]. It aims to achieve them and become a hub for green power imports, exports, and transits. Ofgem (the UK Office of the Gas and Electricity Markets) and UK Government invited the Energy Networks Strategy Group (ENSG) to set out alternatives that demonstrate how these goals can be reached. Its study, ENSG [29], proposes a number of transmission network reinforcements across GB.

Recent and forecast growth in generation in Scotland is significant, partly due to the high volume of new renewable sources. An extensive reinforcement program is already being undertaken. Yet even with these investments the future "planned transfer" from Scotland to England still exceeds the expected capability of that transmission boundary; National Grid [48]. Some of the generating capacity might need to be constrained and, consequently, may be regarded as "sterilized". This would affect both generators in terms of potentially uneconomic operation,[15] and consumers in importing centers.

Among the proposals for the Scotland-England interface, we consider the Western subsea High Voltage Direct Current (HVDC) Link, a 400-km 1.8 GW subsea link between Hunterston and Deeside.[16] It would provide additional capacity between Scotland and England and across the upper North of England. ENSG [29] estimated the total cost of the reinforcement at ￡760M (€860M at 2011 exchange rates).

In January 2011 National Grid and Scottish Power Transmission revealed plans for this link; Platss [51]. A joint venture company, NGET/SPT Upgrades Ltd., has been set up to manage the project. What are the expected benefits?

5.1 The basic setup

To assess this transmission investment as intuitively as possible we consider a simple two-node network topology that is given and fixed. Node #1 represents England/Wales (E) and node #2 stands for Scotland (S). For simplicity, we abstract from the whole network within each country: we concentrate all Scottish stations and loads into one node, and similarly for England/Wales into the other. This implies that we neglect 'internal' congestion and focus only on the 'external' one. We also abstract from the particular arrangements of the British wholesale electricity market (Valeri [59]), which does not operate as a pool, but is based on voluntary bilateral agreements and consequently does not have a unique price. In addition, we leave aside several minor technologies (e.g. biomass, oil + AGT, or tidal) thus reducing the number of fuel sources.

Load. UK official statistics take 'Electricity available' as the starting point for sales of electricity to consumers in Scotland and England/Wales . This amount reflects the contribution from all stations including pumped storage P. Therefore, the gross demand d at each node $i=\{E,S\}$ is computed as:

$$d^E = D^E + P^E; d^S = D^S + P^S.$$

Below we estimate separate load functions for England/Wales and Scotland with their corresponding seasonalities and cross correlation. Our base case assumes a zero growth rate in both nodes. We performed a sensitivity analysis assuming 2 % growth in both nodes.

Generation capacity. The upper half of Table 1 shows the generation mix by fuel source in both nodes as of 2010.[17] "Coal" includes large and medium unit coal, large and medium unit

[15] This applies both to wind stations (which cannot pocket all the potential revenues) and thermal plants (which have to ramp up and down more frequently thus decreasing their efficiency and increasing their costs).

[16] The official web site can be accessed at: http://westernhvdclink.co.uk/

[17] 'Transmission entry capacity' (TEC) is a Connection and Use of System Code term that defines a generator's maximum allowed export capacity onto the transmission system. All companies whose prime purpose is the generation of electricity are included under the heading 'Major power producers' (MPPs); they account for more than

coal + AGT, and small unit coal. Based on DECC [23], the thermal efficiency of coal-fired stations is 36.4 %. "CCGT" stands for combined cycle gas turbines; their thermal efficiency is 46.7 %. Nuclear stations have a thermal efficiency of 39 %. "Wind" denotes both offshore and onshore wind. "Hydro" refers to stations that generate electricity by flowing water through turbines from sources naturally replenished through rainfall. "Pumped storage" stations use off-peak electricity to pump water to a reservoir. They then release water to generate electricity at times of peak demand. These are not considered to be renewable sources; DECC [22].

Table 1. Generation mix 2010; National Grid [46], DECC [23].

TEC (MW)	Coal	CCGT	Nuclear	Wind	Hydro	Pmp.S.	TOTAL
England/Wales	25,490	26,044	8,605	800	140	2,004	63,083
Scotland	3,386	1,547	2,289	1,992	1,129	740	11,083
MPP stations							
England/Wales	20	77	8	36	5	2	148
Scotland	2	2	2	35	74	2	117

The lower half of Table 1 shows the number of power stations owned or operated by MPPs classified by location and type of fuel. These stations have different age, capacity, thermal efficiency, and so on. Our model does not take these details into account. It assumes a fleet of identical average plants for each technology in each country every year. It does this by simply dividing the installed capacities shown in the upper part by the number of stations of each type in the lower part. For example, the resulting average capacity for coal plants is close to 1,500 MW and above 1,000 MW for nuclear.

The number and type of power stations is expected to change significantly in the years ahead. National Grid [48] and National Grid [49] provide information on the generation projects by power technology and geographic location. This includes proposed new generation for which an appropriate Bilateral Agreement is in place. There are different project status: scoping, awaiting consents, consents approved, under construction, and built. There is no guarantee that all the projects will complete. The time horizon in the official report extends until 2025. There is no new contracted generation in England /Wales after 2025, in the north of Scotland after 2020, and for south and central Scotland after 2019. Our simulations cover the twenty years from 2011 to 2030. To this end we keep constant the official statistics for 2017 over the next thirteen years; see Table 2. We also performed a sensitivity analysis assuming the nuclear capacity is cut in half from the start.

Maintenance and other works make plants unavailable from time to time. We assume that natural gas plants are available 85 % of the time; nuclear plants 75 %; and coal plants 55 %, which is their observed rate.[18] As for renewable sources, all the stations are active in principle

90 % of total electricity generation. Large scale hydro, large scale wind, and some biofuels fall within this category. Most generators of electricity from renewable sources are "Other generators" because of their comparatively small size, even though their main activity is electricity generation.

[18]Since the start of the EU Large Combustion Plant Directive (LCPD) UK coal plants have operated 55 % on average. As of 1 January 2008 the LCPD requires large electricity generators to meet more stringent air quality standards or to close by the end of 2015 or after 20,000 hours of operation if they "opt out" of this obligation. In the UK, the capacity of stations that thus will close by December 2015 amounts to 12 GW of coal and oil capacity;

but are intermittent. The time series of their metered output accounts for their active/inactive state and load factor in a unified form. We use these data to estimate the underlying parameters of wind generation, pumped storage and hydro generation; see Appendix C.

Table 2. Forecast evolution of generation capacity (MW) by country and technology.							
	2011	*2012*	*2013*	*2014*	*2015*	*2016*	*2017*
Coal	25,490	25,490	25,490	25,490	25,490	18,378	18,378
CCGT	28,259	29,009	30,369	34,920	36,440	40,005	40,475
Nuclear	8,605	8,605	8,605	8,605	8,605	8,605	10,275
Wind	2,209	3,159	4,497	5,986	8,763	11,271	13,376
Hydro	140	140	140	140	140	140	140
Pumped S,	2,004	2,004	2,004	2,004	2,004	2,004	2,004
ENGL./WAL.	*66,707*	*68,407*	*71,105*	*77,145*	*81,442*	*80,403*	*84,648*
Coal	3,386	3,386	3,386	3,386	3,386	2,284	2,284
CCGT	1,547	1,547	1,547	1,547	1,547	1,547	1,547
Nuclear	2,289	2,289	2,289	2,289	2,289	2,289	2,289
Wind	3,030	4,289	5,269	7,517	7,517	7,517	12,655
Hydro	1,129	1,129	1,129	1,129	1,129	1,129	1,129
Pumped S,	740	740	740	740	740	740	740
SCOTLAND	*12,121*	*13,380*	*14,360*	*16,608*	*16,608*	*15,506*	*20,644*

Transmission capacity. A transmission link connects both nodes. This is the existing 2,200 MW transmission corridor (comprising two lines) between north and south. It already operates at almost full capacity while new renewable energy projects come on-line in the north. Sizeable transfers of electricity from north to south are anticipated; National Grid [48]. However, the required transfer capability significantly exceeds the current capability, indicating a strong need for reinforcement to avoid potential congestion. The western subsea 1,800 MW link would provide this reinforcement, in addition to other existing programs.

The ability of the north to serve load at the south depends on the transmission capacity of the currently available line L^1 which connects both nodes. It has a transmission capacity l_1^{12} and comprises two cables each assumed to be 1,100 MW. Since either can be available or not we have $b_1^{12} \in \{0, 0.5, 1\}$. The probability of a cable going 'off' is 1 per thousand.

A percentage $p = 7$ % of each megawatt-hour transmitted is 'lost'; DECC [23]. Thus the north can effectively contribute $(1-0.07)\times 2{,}200 =$ 2,046 MW at most. Transmission losses over this line m^{12} are computed as Section 3.1 explains. When assessing the potential gains from this investment project, we do not consider the time required for building it. We assume that the link has a useful life of 20 years.[19]

National Grid [48]. The exact timing of these closures is a commercial matter for plant owners (taking into account other factors, e.g. the state of repair of the plants or expectations about policy changes).

[19] This can seem a short period of time since the physical and economic life may well run from 30 to 50 years (CERTS [16]). Boyle et al. [9] and Ojeda et al. [50] also consider a time horizon of 20 years. In the PJM interconnection, the cost-effectiveness of an upgrade is estimated over a 10 year window. The Dutch electricity

Demand-side costs. Value is sacrificed whenever load is lost. We assume $VOLL =$ 2,500 £/MWh interrupted (see Newbery [45]), or 2,904.44 €/MWh (at current rates).

Supply-side costs. We assume thermal efficiencies of $H_c = 0.364$ and $H_g = 0.467$. We estimated the fixed margin M_m (or 'average' clean spark spread) as 6.56 €/MWh; the long-term average for UK natural gas-fired power plants. Any day we have futures prices of all contracts on natural gas with monthly, quarterly, seasonal (April-September and October-March), and yearly maturities on the European Energy Exchange (EEX, Leipzig). We collect these data over 231 days. Similarly for coal to be delivered in Amsterdam, Rotterdam, or Antwerp (so-called ARA coal). We also collect the prices of futures contracts on EU emission allowances traded on the Intercontinental Exchange (ICE; London). We undertake the valuation of future physical flows of commodities at a given time t, so valuation rests on the time-t futures curve. Thus, our model leaves room for the underlying parameters to change in value on a daily basis. The estimation procedure consists of two steps. In the first step, using the futures prices on each day and non-linear least-squares, we derive the curve that best fits futures prices on that day; this provides an estimate of the parameters in the (risk-neutral) stochastic model. Upon the calibration on each of the sample days, we compute the corresponding average values in the second step; we use them as reasonable estimates of future behavior. They enable us to run any number of realistic simulations for the three commodity prices.

Economic dispatch. The system operator aims to find an optimal vector of power generated (x_1, x_2) and consumed (s_1, s_2) at both nodes that minimizes the sum of (bid-based) generation costs and unserved demand costs subject to the restrictions stated above. The number of possible states of the system is $2^{(22+79+10+2)}$; a new transmission line will rise this amount to $2^{(22+79+10+4)}$. Note that these numbers change as new stations are connected to the network or old ones are disconnected.

5.2 Estimation of the underlying parameters

This subject is briefly sketched in Appendix B. This includes the description of the sample used in each case alongside numerical estimates of the underlying parameters. They refer to the dynamics of nodal loads, wind and hydro electricity, and commodity prices. Further details of the estimation process are available from the authors on request.

5.3 Monte Carlo simulation

Our aim is to assess the prospective benefits of the potential expansion of the transmission network. We check two possible configurations of the grid, namely the one-line L^1 transmission model and the two-line $L^1 + L^2$ model. We accomplish this using simulation.[20] We discount future cash-flows at the risk-free interest rate using risk-neutral parameters.

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regulator does not take costs and benefits after 25 years into account because the technical life of (underground) HVDC cables is uncertain and economic models become ever less detailed and reliable; de Nooij [21].

[20]Simulation was undertaken on a Dell Latitude D630 Intel Core 2 DUO CPU running at 2.39 GHz with 1.99 GBRAM. Fully developing two scenarios takes around 26 hours.

Initially only the line L^1 is available. We ran 750 simulations each consisting of 1,200 steps over 20 years (i.e. five steps per month). At each step the optimal dispatch problem is solved subject to the restrictions then in place; i.e. we solved 900,000 optimization problems that minimized the sum of the bid-based costs of electricity generation and the cost of unserved load, subject to linear and non-linear restrictions. The solution to each problem defines the levels of generation and the power effectively served. Hence we computed the bid-based production costs, the carbon emissions, and the allowance costs. We followed the same steps with the network expanded with line L^2. The comparison of the results under both scenarios L^1 and $L^1 + L^2$ defines the benefits of that expansion.

5.4 Main results and discussion

In principle, the only way to assure that a new transmission line would be in the public interest would be to run a system-wide OPF with and without the investment under a relevant range of conditions; Blumsack et al. [7]. We evaluate the costs and benefits of a given expansion of the grid through simulations.

The performance of the system can be assessed by metrics other than costs to generators; there are other attributes which can be of interest. Indeed, they are functions of options and uncertainties. Options could be whether to use a double circuit line of a given voltage or a simple circuit line of double voltage. As for the uncertainties, we have considered several sources but still others loom large, e.g. regulatory changes. The attributes represent stakeholders' objectives. Some of them may be difficult to measure, though this is not the case with those we focus on. Sometimes they may conflict each other, as in the trade-off between reducing fuel costs and reducing carbon emissions.

5.4.1 Base case: no load growth with only L1

Each of the 750 simulations gives whole paths of a number of variables. For example, we have 750 levels of cumulative coal generation in England/Wales from 2011 to 2030. Similarly for nodal loads and transmission levels.

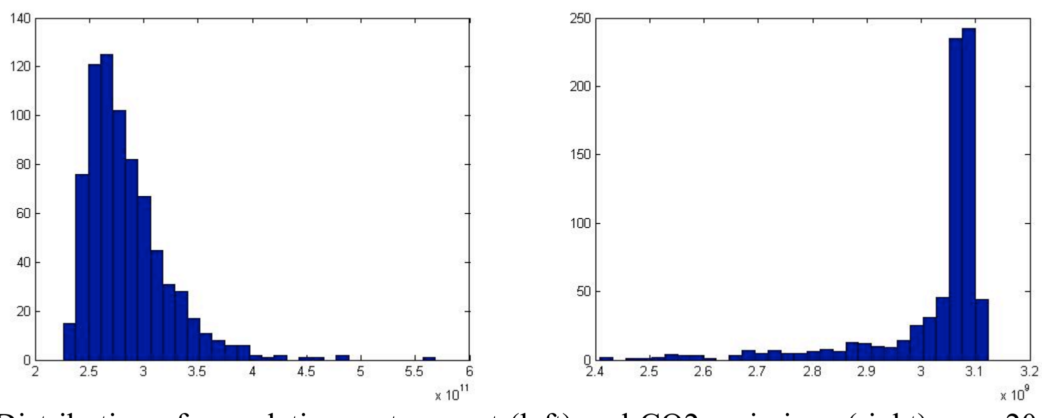

Figure 1. Distribution of cumulative system cost (left) and CO2 emissions (right) over 20 years with only L^1. Average cost: 284,745 M€; average emissions: 3,023 MtCO2.

Figure 1 shows frequency distributions for select aggregate GB variables before the expansion. The left part displays the cumulative results for system cost. It is skewed right: while total costs are extremely high in a few cases, most cases and the probability mass are concentrated on the left, lower costs. Contrarily, the distribution of cumulative emissions is skewed left; the most likely outcomes are on the upper range of possible values and the mean (3,023 MtCO2) is higher than the median. Section 5.4.5 provides information not only on average values but also on other metrics (which can be of interest to market players and regulatory agencies for decision making).

We computed the average of the 750 cumulative values for the above variables and others. Dividing these by the 20 years in our time horizon we obtained annual averages under the assumption of a null growth rate for both nodal loads. Table 3 compares these with the actual 2009 values in the bottom row.

Table 3. Average yearly generation (GWh): Only L^1 and no load growth.							
	Coal	CCGT	Nuclear	Wind	Hydro	Pmp.Sto.	TOTAL
E	92,110	135,824	72,172	23,708	205	4	324,027
S	6,385	679	17,041	22,703	3,478	1	49,211
GB	98,496	136,503	89,214	46,413	3,685	5	373,238
GB-09	*103,204*	*159,809*	*69,098*	*9,324*	*5,227*	*3,685*	*367,638*

Overall, the average generation predicted by the model over the next twenty years is very close to that actually observed in 2009 (just 1.5 % higher). This is consistent with an increasing capacity installed when the annual load remains more or less flat in the foreseeable future. By fuel type, coal generation falls below current levels (as implied by coal stations approaching closure and/or reducing production) whereas gas and nuclear generation remain high (nuclear stations in particular are now returning to normal from past outages for repairs and maintenance). Note that a number of new gas turbines are anticipated to be available in the future, but this is also true for wind stations, which belong to the zero-bid generation capacity. If average demand remains stable while wind generation increases by a factor of five (from 2009 levels) then decreased production must mostly come from gas plants (accompanied by pumped storage plants). This is probably further exacerbated by our assumption of 85 % availability of nuclear plants (which are also in the zero-bid generation category).[21] Also, future rises in the price of natural gas contribute to this technology losing ground. Since the model assumes that pumped storage stations bid 1.1 times above gas stations, this technology falls even more than natural gas. According to our results, this technology, which now plays a very limited role in power generation, would become insignificant.[22]

Table 4 displays similar average values for nodal, served, and unserved loads. However, we did not find official data on total and served loads by location. According to DECC [23], the UK Total Demand in 2009 was 378,714 GWh. Subtracting the sales of electricity in Northern

[21] The load factor of nuclear stations in 2009 was 65.4 %, almost fifteen percentage points below the peak in 1998, but as many points higher than in 2008, when there were many maintenance outages; DECC [23].

[22] Our approach to generation from pumped storage in GB is a bit crude and may miss some relevant characteristic. Mixed thermal and hydro systems are harder to model appropriately than pure thermal systems.

Ireland of 7,621 GWh we arrive at an estimate of 371,093 GWh for GB which is close to our figure of 372,328. At the same time, Total Supply in the UK was 378,524 GWh; hence the statistical difference between UK demand and supply is 190 GWh. We do not interpret this as the amount of unserved load, and do not compare it with our estimate of 25.81 GWh for GB. Indeed, the total estimated unsupplied electricity for 2009-2010 was merely 671.4 MWh. The lower part of Table 4 focuses on the remaining physical variables and others measured in monetary units. The former include transmission between both nodes, losses, and carbon emissions. The latter comprise generation costs, demand-side costs, and allowance costs.

Table 4. Average demand, transmission, and costs: no load growth and L^1.

	Load (GWh)	Served (GWh)	Unserved (GWh)
England/Wales	336,518	336,494	23
Scotland	35,809	35,807	2
GB	372,328	372,302	25
$E \rightarrow S$ (GWh)	52	Generation cost (€)	10,960,062,832
$S \rightarrow E$ (GWh)	13,453	Emissions (tCO2)	151,185,723
Total Trans. (GWh)	13,506	Allowance costs (€)	3,277,205,703
Trans. Losses (GWh)	946	Total Cost (€)	14,237,268,535

We must interpret these averages cautiously. Figure 2 shows the frequency distribution of cumulative aggregate load over the 20 year period. It is relatively symmetric (so the average and the median will be close), but the profile is jagged. More simulation runs might provide a smooth profile.

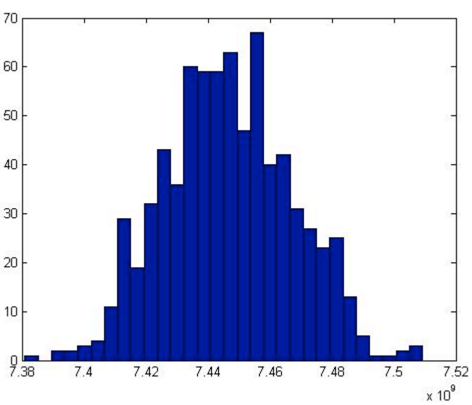

Figure 2. Distribution of cumulative GB load over 20 years with only L^1. Average: 7,446 TWh.

It is possible to derive an average electricity price as a by-product of the model. Note that in each optimization the operating technology with the highest cost sets the marginal price. So there are as many electricity prices as optimization problems. The average price turned out to be 68.16 €/MWh. Similarly, we estimated the implicit average carbon price: 21.67 €/tCO2. These figures may seem high by current standards; however, futures market prices suggest a long-run increase of the three commodity prices.

5.4.2 Base case: no load growth with L1 + L2

This scenario assumes that the proposed Western corridor is in place from the beginning of our time horizon; this means that there are two new 900 MW transmission lines in addition to the former two 1,100 MW lines. Again, nodal loads are expected to remain stable on average. For simplicity, we aggregated GB values for generation, loads, and emissions.

The distributions of cumulative system costs and CO2 emissions for this scenario are similar to those of the case with only L^1: they continue to be skewed. However, since these distributions are asymmetric, a deeper knowledge of their characteristics is of interest to market participants and/or oversight bodies.

Following de Neufville and Scholtes [20], we examined the cumulative distribution functions (or CDFs, sometimes referred to as "target curves"), which present a lot of information in a compact form and thus provide an effective way to compare alternative designs. (Section 5.5.5 provides a multidimensional overview according to several metrics beyond average values to enable a fair comparison of the two grid configurations.)

Figure 3 displays discounted system costs over the 20-year period (ranked from lower to higher) and their associated cumulative probabilities. The target curve with $L^1 + L^2$ (blue line) stays always above, that is, it stochastically dominates that for L^1 (red line). Thus the Western corridor entails a lower probability of surpassing any given level of total cost.

Figure 4 shows the CDF of yearly average values of CO2 emissions. The target curve of the expanded grid (blue line) is stochastically dominated by that of the single grid (red line). The addition of the new corridor increases the probability of any level of carbon emissions (presumably because the corridor serves a higher load).

Figure 5 displays a broad view of lost load (in MWh). Under both network configurations there is some probability of no lost load (leftmost part), but generally there is some level of lost load. The CDF with $L^1 + L^2$ (blue line) stochastically dominates that for L^1 (red line). Thus the addition of the new line reduces the probability that load will be unserved.

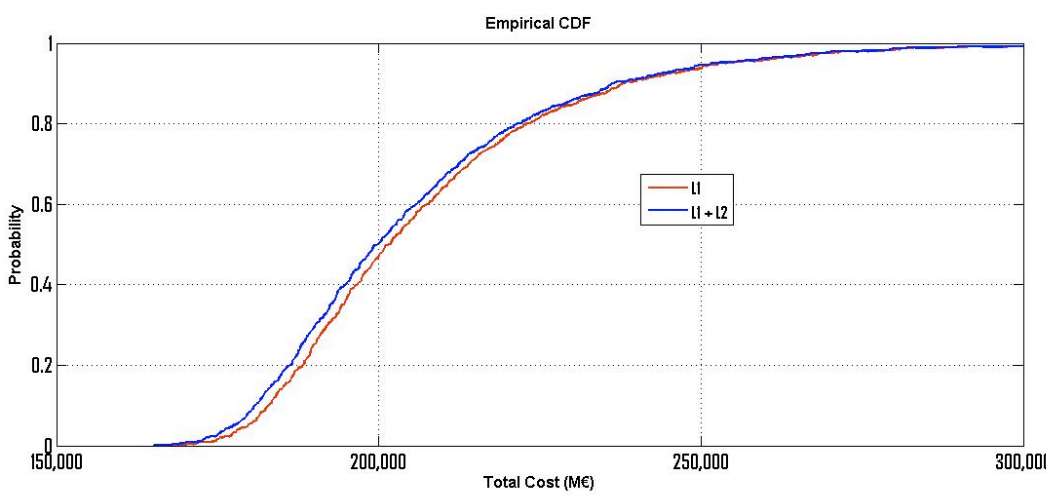

Figure 3. CDF of discounted system cost without and with L^2.

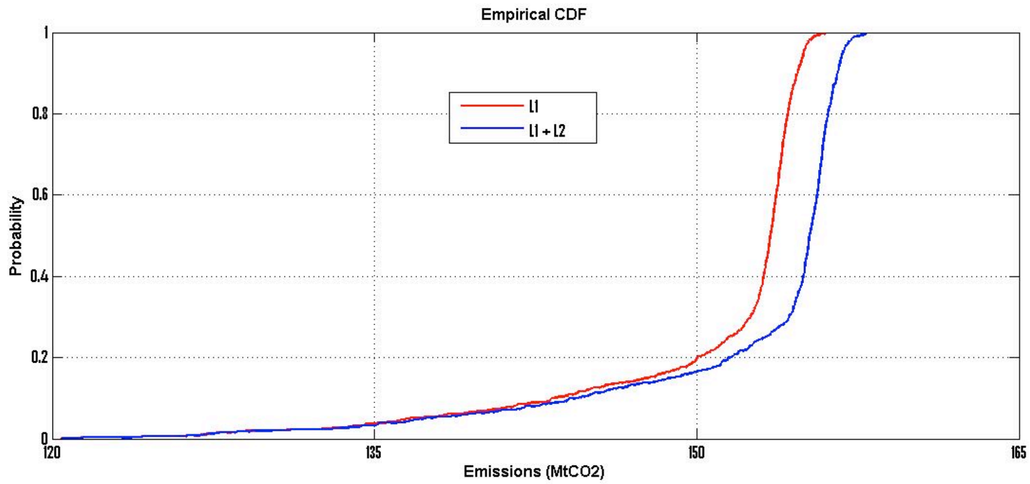

Figure 4. CDF of yearly carbon emissions without and with L^2.

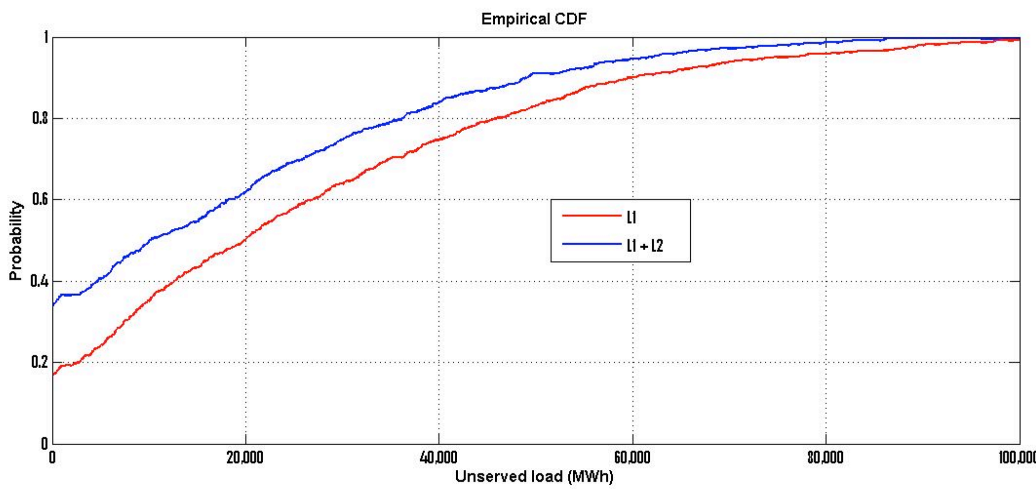

Figure 5. CDF of unserved load without and with L^2.

Table 5 compares the main average results with and without the new link. As zero-bid technologies $\{n, w, h\}$ run at full availability in any case, Table 5 shows that the Western link is most welcome for cheap coal stations to the detriment of expensive natural gas and pumped storage stations (left side), and that it entails a drop of almost 30 % in unserved load (right side). This is a first measurement of the improved physical reliability; Bresesti et al. [10]. We can translate it into economic terms: $25.81 - 18.34 =$ 7.47 GWh of interrupted power (valued at $VOLL$) has become served and paid. Using the average price of electricity, the improvement amounts to $7.47 \times 1{,}000 \times (2{,}904.44 - 68.16) =$ 21,187,011 euros. As expected, L^2 enhances transmission from north to south much more than in the opposite direction.

Table 5. Base case (no load growth) with L^1 and L^1+L^2.					
	L^1	L^1+L^2		L^1	L^1+L^2
Coal (GWh)	98,496	102,348	Load (GWh)	372,328	372,328
CCGT (GWh)	136,503	131,965	Unmet Load	25	18
Nuclear (GWh)	89,214	89,214	$E \rightarrow S$ (GWh)	52	52
Wind (GWh)	46,413	46,413	$S \rightarrow E$ (GWh)	13,454	18,275
Hydro (GWh)	3,685	3,685	Emiss. (MtCO2)	151	152
Pmp.Strg. (GWh)	5	5	Allowance Cost	3,277	3,306
Total Generation	373,239	373,580	Total Cost (M€)	14,237	14,087

As coal generation rises while gas generation falls, both carbon emissions and allowance costs increase. The total cost, however, decreases due to fuel substitution and greater scope for cheap Scottish exports. Though the cost reduction (-1.05 %) may not be significant, we develop the figures below for illustrative purposes. In our case, the average reduction amounts to $14{,}237 - 14{,}087 =$ 150 M€ per year (130 M£ at current rates). A back-of-the-envelope computation suggests that the project expenditure (860 M€ or 760 M£) would be recovered in six years (see Section 5.5.5).

It is possible to dig deeper. Following Blumsack et al. [6], we define the congestion cost as the difference in system cost to serve identical demand profiles in a system with and without a given line. Since our definition of total costs rests on two main pillars (supply- and demand-side costs), we can subtract the latter (unserved load times $VOLL$) from total costs to derive an (average) estimate of the former (generation costs). When only line L^1 is available we get 14,162 M€, while under L^1+L^2 we have 14,034 M€. Thus congestion benefits from the expansion amount to 129 M€. Note that the sum of reliability benefits (21 M€) and congestion benefits (129 M€) closely approaches the reduction in total costs (150 M€) of expansion.

5.5 Sensitivity analysis

Up to now we focused on the most likely scenario with no growth in load. We now move to more extreme scenarios. We first check the robustness of the above results to changes in a major driver of transmission expansions, namely load growth. We assume that both nodal loads grow at 2 % annually. Second, we consider a change in supply: an assumed 50 % cut in nuclear capacity.

5.5.1 Load growth of 2 % and only L1

If load grows, generation will follow pace. This applies totally to non-zero-bid generation; zero-bidding power stations operate at higher rates in this scenario, with average values closer to their maximum capacities. See Table 6. By assumption, coal plants have an availability of 55 % while for gas plants this rate is 85 %. Therefore, once the former reach their limit the remaining new loads must be served by the latter (together with pumped storage).

Table 6. Stable and growing loads with L^1.					
	0	2%		0	2%
Coal (GWh)	98,496	102,317	Load (GWh)	372,328	456,956
CCGT (GWh)	136,503	215,854	Unmet Load	25	317
Nuclear (GWh)	89,214	89,214	$E \to S$ (GWh)	52	181
Wind (GWh)	46,413	46,413	$S \to E$ (GWh)	13,454	10,204
Hydro (GWh)	3,685	3,685	Emiss. (MtCO2)	151	189
Pmp.Strg. (GWh)	5	34	Allowance Cost	3,277	4,355
Total Generation	373,239	457,361	Total Cost (M€)	14,237	21,031

Since the transmission infrastructure remains the same, the increase in nodal loads gives rise to a noticeable rise in unserved load. Interestingly, the exports from Scotland fall by almost 25 %. Higher local demand in Scotland must be met; this leaves less scope for exports to the south. Besides, unlike matching local consumption, transmission is subject to losses. Meanwhile, exports from south to north more than triple (despite being modest at any rate). Obviously the increase in fossil generation goes hand in hand with a higher bill from fuels consumed. It also entails more carbon emissions, and higher allowance costs.

5.5.2 Load growth of 2 % and L1 + L2

Overall electricity generation remains stable with the new line L^2 in place. Yet the scope for favorable trades improves. There is a small substitution of cheap generation (coal) for expensive generation (natural gas and pumped storage); see Table 7. This effect entails a minor increase in carbon emissions. Unserved load, however, is cut by almost 17 % as Scotland seizes the new corridor to increase its exports by some 20 % to the south.

Since carbon emissions go up, allowance costs necessarily rise. Nonetheless, this rise (22 M€) is more than offset by the ten times greater fall in generation costs. Overall, the total costs to running the system decrease by 195 M€ (some 170 M£ at current rates). Thus under this scenario, crude computation suggests that the project expenditure would be recovered in four and a half years. As in the base case, it is possible in principle to break down the reduction in system costs into reliability benefits and congestion benefits.

Table 7. Loads growing at 2 % with L^1 and $L^1 + L^2$.					
	L^1	$L^1 + L^2$		L^1	$L^1 + L^2$
Coal (GWh)	102,317	104,536	Load (GWh)	456,956	456,956
CCGT (GWh)	215,854	213,699	Unmet Load	317	263
Nuclear (GWh)	89,214	89,214	$E \to S$ (GWh)	181	182
Wind (GWh)	46,413	46,413	$S \to E$ (GWh)	10,204	12,542
Hydro (GWh)	3,685	3,685	Emiss. (MtCO2)	189	190
Pmp.Strg. (GWh)	34	33	Allowance Cost	4,355	4,377
Total Generation	457,361	457,577	Total Cost (M€)	21,031	20,836

5.5.3 No load growth with half the nuclear fleet and only L1

In this case we assume that the nuclear fleet is cut in half from the start (2011) in both countries and that the 1,670 MW of new nuclear capacity expected to be added in 2017 in the base case are now discarded (see Table 2). Table 8 shows that total generation is almost flat, which is consistent with the assumption of no growth in electricity loads. Nuclear generation falls a bit more than 50%, also as expected. The other zero-bidding technologies contribute the same as before. Therefore, coal and gas plants must fill the gap. This raises carbon dioxide emissions and allowance costs by almost 15 %. The main change, though, concerns total cost, which increases by more than 25 %.

A 50 % reduction in nuclear capacity across both countries leaves the relative proportions roughly unchanged, with England/Wales hosting some 80 % of the GB nuclear fleet and Scotland hosting the remaining 20 %. Nonetheless, the concentration of fossil technologies (which are the main beneficiaries of the nuclear dip) in England/Wales with respect to Scotland is much higher than four to one. This means that the first are able to cope with the problem better than the second. Thus, transmission from south to north increases fourfold while in opposite direction it falls by one fourth. Overall, the system performs worse in that unserved load increases almost by a factor of twelve across both countries; Scotland is more severely affected than England/Wales since in the north unmet load grows seventeen-fold.

Table 8. No load growth under full and half nuclear capacity with only L^1.

	Full fleet	Half fleet		Full fleet	Half fleet
Coal (GWh)	98,496	101,792	Load (GWh)	372,328	372,328
CCGT (GWh)	136,503	180,433	Unmet Load	25	306
Nuclear (GWh)	89,214	40,554	$E \to S$ (GWh)	52	203
Wind (GWh)	46,413	46,413	$S \to E$ (GWh)	13,454	9,829
Hydro (GWh)	3,685	3,685	Emiss. (MtCO2)	151	173
Pmp.Strg. (GWh)	5	22	Allowance Cost	3,277	3,802
Total Generation	373,239	372,720	Total Cost (M€)	14,237	18,252

5.5.4 No load growth with half the nuclear fleet and L1 + L2

As Table 9 shows, again there seems to be no major change in total generation because of the expansion; after all, there is no growth in expected load. Unserved load falls slightly (by 4%) after the expansion. While electricity flowing north from the south remains more or less the same, flows from north to south rise by almost 25 %. These changes allow the system to meet demand with a minor substitution of coal for natural gas which entails a minor increase in carbon emissions and allowance costs. System costs now fall by 81M€ annually on average. While in the base case the pay-back period was around six years, under the nuclear gap it is about twice as long. As before, in principle it is possible to break down the reduction in system costs into reliability benefits and congestion benefits.

Table 9. No load growth under half nuclear capacity with L^1 and L^1+L^2.					
	L^1	L^1+L^2		L^1	L^1+L^2
Coal (GWh)	101,792	103,938	Load (GWh)	372,328	372,328
CCGT (GWh)	180,433	178,272	Unmet Load	306	294
Nuclear (GWh)	40,554	40,564	$E \rightarrow S$ (GWh)	203	205
Wind (GWh)	46,413	46,413	$S \rightarrow E$ (GWh)	9,829	12,084
Hydro (GWh)	3,685	3,685	Emiss. (MtCO2)	173	174
Pmp.Strg. (GWh)	22	22	Allowance Cost	3,802	3,823
Total Generation	372,720	372,890	Total Cost (M€)	18,252	18,171

5.5.5 Summing up

Once we recognize that future contexts and outcomes are uncertain, a complete project evaluation must consider several factors to characterize the underlying distribution of the variable of interest in some way; de Neufville and Scholtes [20]. Table 10 provides statistics on the distribution across scenarios of three key variables: the yearly unserved load (GWh), discounted total fuel costs (M€), and carbon emissions (MtCO2). Regarding lost load, the average value is lowest in the base case, both when only L^1 is available (25.81) and with L^1+L^2 (18.34). In the three scenarios the values after the expansion are lower than those before. Note, though, that in 10 % of cases the unserved load will rise above a substantial level. For example, in the base case scenario, the amount unserved will exceed two times the average in one out of ten occasions. Concerning fuel costs, again the average value under L^1 and L^1+L^2 is lowest in the base case. Expansion in this case would entail discounted savings around 1,730 M€, roughly twice the project cost. In the other two scenarios, though, savings would be just one third of the former amount. As for carbon emissions, they are consistently higher after the expansion than before whatever the statistical factor we focus on.

Table 10. Statistics of the distribution of effects for key factors across scenarios.							
	L^1						
Unmet Load	Mean	Median	Std.Dev.	Min.	Max.	10 %	90 %
Base case	25.81	19.94	501	0	153.15	0	59.56
2% load	317.24	306.34	2,216	81.66	746.32	181.19	458.11
1/2 nuke	306.58	296.50	2,013	67.60	718.15	183.55	438.66
Fuel Costs							
Base case	207,392	201,661	25,041	167,159	385,414	182,975	237,902
2% load	284,523	275,476	35,254	231,690	569,030	251,363	326,484
1/2 nuke	252,898	245,937	29,441	206,688	469,385	224,374	288,824
CO2 Emiss.							
Base case	151.18	153.43	117.32	120.41	156.14	143.35	154.67
2% load	189.07	190.89	101.39	160.75	193.74	182.52	192.19
1/2 nuke	173.26	175.60	120.38	141.39	178.14	165.21	176.81
	$L^1 + L^2$						
Unmet Load	Mean	Median	Std.Dev.	Min.	Max.	10 %	90 %
Base case	18.34	10.14	433	0	161.69	0	48.96
2% load	263.73	253.67	2,031	56.93	639.06	137.64	399.87
1/2 nuke	294.44	287.60	1,972	82.85	747.19	178.31	418.23
Fuel Costs							
Base case	205,662	199,866	25,236	165,145	383,878	180,878	236,517
2% load	283,940	274,906	35,397	230,908	568,621	250,611	326,202
1/2 nuke	252,288	245,157	29,569	206,441	467,949	223,639	288,325
CO2 Emiss.							
Base case	152.82	155.24	125.67	120.47	158.27	144.36	156.56
2% load	190.22	192.10	104.75	160.91	195.07	183.58	193.36
1/2 nuke	174.34	176.82	125.33	142.14	179.35	165.95	177.92

6 Conclusions

There is currently a pressing need for network expansions in a number of electricity markets. The reasons differ from place to place. They range from paving the way to further penetration of renewable sources to developing integrated markets, to boosting security of supply, or meeting a growing demand.

Uncertainty pervades most of the issues involved in transmission expansions. Just obtaining the necessary planning permissions for possible routes may be challenging. Uncertainties about the final cost of a new line come close. As McGarvey [44] points out, greater certainty over futures costs and revenues alongside settled regulatory rules (Deb [18]) could render transmission a more appealing investment.

These great cost uncertainties are second order compared with those in the estimation of the benefits; Turvey [58]. Consequently, the cost-effectiveness of expansion projects is typically subject to great uncertainty. Alleviating the assessment of benefits could encourage greater cooperation among players to get needed facilities sited and built.

We develop a valuation model to assess investments to expand the transmission network. These investments are related to those in power generation as they change each other's future revenues and thus profits. Our model treats both electricity generation and transmission jointly. It allows for uncertainties stemming from a number of sources. Thus, physical generation and transmission assets are subject to failure. Future loads are not known with great certainty. The prices of fossil fuels like coal and natural gas are hard to forecast. If there are carbon constraints in place, the allowance price is another reason for concern. And with respect to renewable technologies, wind and hydro resources are unpredictable as well as intermittent. Though we set up a transmission network with just two nodes, nothing precludes extending our model to a more general setting with an arbitrary number of nodes.

The model combines optimization techniques with Monte Carlo simulation. We place greater emphasis on the long-term valuation of transmission assets than on short-term optimization. At any time it is necessary to solve for the optimal power flow taking account of the (technical and economic) circumstances. In particular, load must be met at minimum cost. This cost includes obviously the cost to electricity generators. But we also consider explicitly a demand-side cost which arises whenever there is unserved load. Assuming particular stochastic processes for each source of uncertainty allows to perform simulation analysis (with the optimization problem nested in it).

To demonstrate how the model works we heuristically consider the case of the Western subsea HVDC link in Great Britain. We assume that this project has a useful life of twenty years. We adopt the forecast for generation capacity from 2011 to 2017 and keep it constant at 2017 levels until 2030. Numerical estimates of the required parameters are taken from official statistics or estimated from actual prices on futures markets.

According to our results, the construction of the proposed corridor would bring significant savings in system cost. In the base case, which assumes no growth in load, undiscounted average benefits would enable recovery of the project expenses in some six years. These benefits mainly accrue from congestion benefits, with reliability benefits playing a minor role. In the alternative scenario with load growing at 2 % annually, the time required for cost recovery falls to some five years. On the other hand, if there were a major shortfall in nuclear generation (assuming stable loads), the expansion project would take almost half of its useful life to pay for itself.

Our model can be improved in several ways. One involves adapting it to more complex network topologies. Another would be a better characterization of the strategic behavior of generators and the exercise of market power (for example, using the Lerner and the residual supply indexex); note, though, that market power depends on industry regulation, and assuming the *status quo* as fixed over the project lifetime for valuation purposes may not be appropriate. Our model does not address three related aspects of the investment issue, namely where to invest, how much capacity to add, and when to invest. Our simplified two-node network finesses the 'where' question and considers the project capacity as given. As for the optimal time to invest, properly addressing this issue calls for dynamic planning. This is left for future research.

References

[1] Abadie L.M and Chamorro J.M. (2009): "Income risk of EU coal-fired power plants". *Energy Policy* 37(12): 5304-5316.

[2] Abadie L.M., Chamorro J.M., and González-Eguino M. (2011). "Optimal abandonment of EU coal-fired stations". *Energy Journal* 32(3): 175-207.

[3] Blanco G.A., Olsina F.G., Ojeda O.A., and Garcés F.F. (2009): "Transmission expansion planning under uncertainty". Presented at the 2009 IEEE Bucharest Power Tech Conference.

[4] Blanco G.A., Waniek D., Olsina F.G., Garcés F.F., and Rehtanz C. (2011): "Flexible investment decisions in the European interconnected transmission system". *Electric Power Systems Research* 81(4): 984-994.

[5] Blanco G.A., Olsina F.G., Garcés F. and Rehtanz C. (2011): "Real option valuation of FACTS investments based on the Least Square Monte Carlo method". *IEEE Transactions on Power Systems* 26(3): 1389-1398.

[6] Blumsack S., Lave L.B., and Ilić M. (2007): "A quantitative analysis of the relationship between congestion and reliability in electric power networks". *Energy Journal* 28(4): 73-100.

[7] Blumsack S., Lave L.B. and Ilić M. (2008): "The Real Problem with Merchant Transmission". *The Electricity Journal* 21(2): 9-19.

[8] Bohn R.E., Caramanis M.C., and Schweppe F.C. (1984). "Optimal pricing in electrical networks over space and time". *Rand Journal of Economics* 15(3): 360-376.

[9] Boyle G., Guthrie G., and Meade R. (2006): "Real Options and Transmission Investment: the New Zealand Grid Investment Test". New Zealand ISCR Working Paper.

[10] Bresesti P., Calisti R., Cazzol M.V., Gatti A., Provenzano D., Vaiani A., Vailati R. (2009): "The benefits of transmission in competitive electricity markets". *Energy* 34: 274-280.

[11] Buijs P., Bekaert D., and Belmans R. (2010): "Seams issues in European transmission investments". *The Electricity Journal* 23(10): 18-26.

[12] Buijs P., Bekaert D., Cole S., Van Hertem D., and Belmans R. (2011): "Transmission investment problems in Europe: Going beyond standard solutions". *Energy Policy* 39(3).

[13] Bushnell J. and Stoft S. (1996): "Grid investment: Can a market do the job?". *The Electricity Journal* 9(1): 74-79.
[14] Buygi M.O., Balzer G., Shanechi H.M., and Shahidehpour M. (2004): "Market-based

transmission expansion planning". *IEEE Transactions on Power Systems* 19(4): 2060-2067.

[15] Campaign to Limit Onshore Windfarm Developmnts (2011): UK Metered Wind Energy Data.

[16] Consortium of Electric Reliability Technology Solutions (2004): "Economic evaluation of transmission interconnection in a restructured market". Consultant Report 700-04-007 for the California Energy Commission.

[17] Çetin U. and Verschuere M. (2009): "Pricing and hedging in carbon emission markets". *International Journal of Theoretical and Applied Finance* 12(7): 949-967.

[18] Deb R.K (2004): "Transmission Investment Valuations: Weighing Project Benefits". *The Electricity Journal* March: 55-67.

[19] de la Torre T., Feltes J.W., Gómez T., and Merrill H.M. (1999): "Deregulation, privatization, and competition: Transmission planning under uncertainty". *IEEE Transactions on Power Systems* 14(2): 460-465.

[20] de Neufville R. and Scholtes S. (2011): *Flexibility in Engineering Design*. The MIT Press.

[21] de Nooij M. (2011): "Social cost-benefit analysis of electricity interconnector investment: A critical appraisal" . *Energy Policy* 39(6): 3096-3105.

[22] UK Department of Energy and Climate Change (2009): "Special feature: Generation and hydro changes to electricity tables". *Energy Trends Articles*. September.

[23] UK Department of Energy and Climate Change (2010): *Digest of United Kingdom energy statistics* (DUKES 2010).

[24] UK Department of Energy and Climate Change (2010): *Energy Trends*. December.

[25] UK Department of Energy and Climate Change (2011): *Energy Trends*. March.

[26] UK Department of Energy and Climate Change (2011): *Digest of United Kingdom energy statistics* (DUKES 2011).

[27] Dixit A.K. and Pindyck R.S. (1994): *Investment under Uncertainty*. Princeton Univ. Press.

[28] Electric Power Group (2003): *Transmission bottleneck project report*. March 19.

[29] Electricity Networks Strategy Group (2009): *Our electricity transmission network: A vision for 2020*. March.

[30] European Network of Transmission System Operators for Electricity (2010): *Ten-year*

network development plan 2010-2020.

[31] Federico G. and Vives X. (2008): *Competition and Regulation in the Spanish Gas and Electricity Markets*. Public-Private Sector Research Center 1. IESE Business School and Orkestra.

[32] Foley A.M., Ó Gallachóir B.P., Hur J., Baldick R., and McKeogh E.J. (2010): "A strategic review of electricity systems models". *Energy* 35: 4522-4530.

[33] Garca R.C., Contreras J., Correia P.F., and Muñoz J.I. (2010). "Transmission assets investment timing using net present value curves". *Energy Policy* 38: 598-605.

[34] Hedman K.W., Gao F., and Sheblé G.B. (2005): "Overview of transmission expansion planning using real options analysis". Proceedings 37th Annual North American Power Sympos.

[35] Ilić M.D., Bachovchin K.D. and Lewis A.S. (2011): "Costs and benefits of transmission congestion management". Power Systems Conference and Exposition (PSCE), IEEE/PES, March.

[36] IPCC (2006): *Guidelines for National Greenhouse Gas Inventories*.

[37] IPCC (2007): *4th Assessment Report Climate Change 2007. Mitigation and Climate Change*.

[38] Joskow P. (2005): "Transmission policy in the United States". *Utilities Policy* 13(2): 95-115.

[39] Kazerooni A.K. and Mutale J. (2010): "Transmission Network Planning Under Security and Environmental Constraints". *IEEE Transactions on Power Systems* 25(2): 1169- 1178.

[40] Kindler J. (2011): "Incentivizing investments in transmission". MIT CEEPR Spring Workshop, Cambridge MA, May 6.

[41] Kirschen D. and Strbac G. (2004): "Why investments do not prevent blackouts". *The Electricity Journal* 17(2): 29-36.

[42] Kurlinski R.E. (2008): "Valuing transmission: Reliability, congestion, and uncertainty". *USAEE WP* 08-004.

[43] Macilwain C. (2010): "Supergrid". *Nature* Vol. 468, pp. 624-625, 2 December.

[44] McGarvey J. (2006): "Transmission investment: A primer". *Electricity Journal* 19(8): 71-82.

[45] Newbery D. (2005): "Electricity liberalisation in Britain: The quest for a satisfactory wholesale market design". *Energy Journal* 26, Special issue: 43-70.

[46] National Grid (2010): *National Electricity Transmission System (NETS) Seven Year Statement*.

[47] National Grid (2010): *Winter Outlook Report 2010-11*. October.

[48] National Grid (2011): *National Electricity Transmission System (NETS) Seven Year Statement*.

[49] National Grid (2011): *Transmission Networks Quarterly Connections Update*. April.

[50] Ojeda O.A., Olsina F., and Garcés F. (2009): "Simulation of the long-term dynamic of a market-based transmission interconnection". *Energy Policy* 37(8): 2889-2899.

[51] Platss (2011): "Subsea links on a roll". February 15, available at: http://www.platss.com.

[52] Roques F.A., Nuttall W.J., Newbery D.M., de Neufville R., and Connors S. (2006): "Nuclear power: A hedge against uncertain gas and carbon prices?" *Energy Journal* 27(4): 1-23.

[53] Sakhrani V. and Parsons J.E. (2010): "Electricity network tariff architectures: A comparison of four OECD countries". *MIT CEEPR 10-008*, July.

[54] Sauma E.E. and Oren S.S. (2009): "Do generation firms in restructured electricity markets have incentives to support social-welfare-improving transmission investments?". *Energy Economics* 31(5): 676-689.

[55] Scottish Power Distribution (2010): *Distribution Long Term Development Statement for the years 2010/11 to 2014/15*. November.

[56] Skantze P.L. and Ilić M.D. (2001): *Valuation, hedging and speculation in competitive electricity markets: A fundamental approach*. Kluwer Academic Publishers.

[57] Trigeorgis L. (1996): *Real Options*. The MIT Press.

[58] Turvey R. (2006): "Interconnector economics". *Energy Policy* 34(13): 1457-1472.

[59] Valeri L.M. (2009): "Welfare and competition effects of electricity interconnection between Ireland and Great Britain". *Energy Policy* 37(11): 4679-4688.

[60] Wu F.F., Zheng F.L., and Wen F.S. (2006): "Transmission investment and expansion planning in a restructured electricity market". *Energy* 31(6-7): 954--966.

Appendix A. Discrete-time version for estimation and simulation

Loads. The differential equations for load changes can be expressed in discrete time by means of the following difference equations:

$$D^1_{t+\Delta t} - D^1_t = k_1(L^1 - D^1_t)\Delta t + \sigma_1 D^1_t \sqrt{\Delta t} u^1_t,$$
$$D^2_{t+\Delta t} - D^2_t = k_2(L^2 - D^2_t)\Delta t + \sigma_2 D^2_t \sqrt{\Delta t} u^2_t,$$
$$\sigma_1 \sqrt{\Delta t} u^1_t \sigma_2 \sqrt{\Delta t} u^2_t = \rho_{12}\Delta t,$$

where both u^1_t and u^2_t follow a standard normal distribution function $N(0,1)$. These expressions can be rearranged as:

$$\frac{\Delta D^1_t}{D^1_t} = \frac{k_1 L^1 \Delta t}{D^1_t} - k_1\Delta t + \sigma_1 \sqrt{\Delta t} u^1_t,$$
$$\frac{\Delta D^2_t}{D^2_t} = \frac{k_2 L^2 \Delta t}{D^2_t} - k_2\Delta t + \sigma_2 \sqrt{\Delta t} u^2_t.$$

Nodal loads can well be correlated if they are affected by common factors. Fortunately, these expressions can be rewritten in a way which is suitable for seemingly unrelated regression equations (SURE) analysis. The constant term and the coefficient of the independent variable for each load are defined by:

$$\beta^1_0 \equiv -k_1\Delta t; \beta^1_1 \equiv k_1 L^1 \Delta t;$$
$$\beta^2_0 \equiv -k_2\Delta t; \beta^2_1 \equiv k_2 L^2 \Delta t.$$

Estimation of $\{\beta^1_0, \beta^1_1; \beta^2_0, \beta^2_1\}$, following econometric techniques, can be undertaken from individual (deseasonalised) time series for each of the loads D^1 and D^2. From these estimates then it is possible to get an estimate of the underlying parameters $\{k_1, L^1, \sigma_1; k_2, L^2, \sigma_2\}$. From the series of the residuals we can approach the correlation coefficient between both loads, ρ_{12}.

Hence Monte Carlo simulation can proceed. Regarding the time step, Δt, note that it may take on a particular value in the econometric stage (depending on the frequency of the data, e.g. 1/12) and a different one in the simulation stage (as required by the analysis, e.g. 1/365); this poses no further problem since the underlying parameters are meant to be instantaneous (as can be seen in the stochastic differential equations).

Wind load factor. Since the underlying process assumed is the same as for load, the discrete-time equation displays the same structure. Ideally, independent wind load factors should be estimated for each node, W^1 and W^2. Clearly this depends on the availability of the required time series. In our case below we do not have separate production series for wind farms in nodes 1 and 2. Therefore, the load factor is assumed equal in both:

$$W_{t+\Delta t} = W_t + k_W(W_m - W_t)\Delta t + \sigma_W W_t \sqrt{\Delta t} u_{W,t}.$$

Based on past (say, monthly) data it is possible to get a numerical estimate of the above parameters $\{k_W, W_m, \sigma_W\}$. Later on they can be used to simulate random paths over a number of years.

Hydro load factor. Again the underlying process assumed is the same as for load. Consequently the discrete-time equation has also the same components. For natural flow and pumped storage, respectively, we have:

$$H_{t+\Delta t} = H_t + k_H(H_m - H_t)\Delta t + \sigma_H H_t \sqrt{\Delta t} u_{H,t};$$
$$P_{t+\Delta t} = P_t + k_P(P_m - P_t)\Delta t + \sigma_P P_t \sqrt{\Delta t} u_{P,t}.$$

In the absence of separate data for each node, common estimates of $\{k_H, H_m, \sigma_H\}$ will be used for natural flow generation in both nodes 1 and 2, and similarly for pumped storage generation $\{k_P, P_m, \sigma_P\}$.

Commodity prices. Stochastic mean-reverting coal and natural gas prices are approached in discrete time by:

$$C_{t+\Delta t} = C_t[k_C(C_m - C_t) - \lambda_C C_t]\Delta t + \sigma_C C_t \sqrt{\Delta t} u_{C,t},$$

$$G_{t+\Delta t} = G_t + (f_G(t+\Delta t) - f_G(t)) + [k_G G_m - (k_G + \lambda_G)(G_t - f_G(t))]\Delta t + \sigma_G (G_t - f_G(t))\sqrt{\Delta t} u_{G,t}.$$

The non-stationary emission allowance price, instead, is governed in discrete time by:

$$A_{t+\Delta t} = A_t e^{(\alpha_A - \lambda_A - \frac{1}{2}\sigma_A^2)\Delta t + \sigma_A \sqrt{\Delta t} u_{A,t}}.$$

As already mentioned, here commodity price processes are estimated from observed futures prices. Regarding coal we are interested in several (composite) parameters $\left\{\frac{k_C C_m}{k_C + \lambda_C}, k_C + \lambda_C, \sigma_C, C_0\right\}$. As for natural gas, we need a numerical estimate of $\left\{\frac{k_G G_m}{k_G + \lambda_G}, k_G + \lambda_G, \sigma_G, G_0, \gamma_G, \varphi_G\right\}$. In the case of the allowance price our interest falls on $\{\alpha - \lambda_A, \sigma_A, A_0\}$. Last, estimates of the cross-correlations $\{\rho_{GC}, \rho_{GA}, \rho_{CA}\}$ are also required. Upon completion of the estimation stage we can move forward to simulate random paths over time and solve for the optimal dispatch at any instant.

Appendix B. Estimation of the long-term fixed margin

The estimation of the long-term fixed margin M_m requires defining the margin in the first place. The margin at time t, M_t (in €$/MWh$), is computed as follows:

$$M_t = S_t - \frac{G_t + 0.20196 A_t}{E_G},$$

where S denotes electricity price (€$/MWh$), G is the price of natural gas (€$/MWh$), E_G is the net thermal efficiency of a gas plant, and A stands for the price of an EU emission allowance (€/t CO_2).

Now we assume that this spread evolves stochastically over time. In order to get an estimate for M_m, we propose a (theoretical) model for the behavior of M_t; we then estimate this model by following standard econometric techniques. Specifically, the time path of the margin is assumed to follow an Ornstein-Uhlenbeck process. This stochastic process accounts for mean reversion in CSS values along with continuous unpredictable swings. It also allows the margin to take on negative and positive values:

$$dM_t = k_M(M_m - M_t)dt + \sigma_M dW_t^M.$$

M_m is the value which the margin tends to in the long term, and k_M is the speed of reversion toward this value. σ_M denotes the instantaneous volatility of the margin. dW_t^M stands for the increment to a standard Wiener process. It can be shown that:

$$E(M_t) = M_0 e^{-k_M t} + M_m(1 - e^{-k_M t}),$$

where M_0 stands for the value of the margin at time $t =$ 0. For high speeds of reversion ($k_M >>$ 0) the model provides margins that are close to the long-term value. The same holds for times that are far away into the future (since $k_M t >$ 0). In both cases we get $E(M_t) \approx M_m$.

The sample period goes from December 1 2009 to November 30 2010, i.e. a whole year. Natural gas prices were obtained from ICE UK (£/therm). Electricity prices correspond to ICE UK base electricity (£/MWh). Gas and electricity prices have been turned into €$/MWh$ using term exchange rates; the closest to maturity contracts have been used. CO2 allowance prices are taken from ICE EUA futures contracts. In this case, the price for the nearest maturity has been computed by means of a cubic spline starting from the spot price; see Abadie and Chamorro [1].

Full details on the sample data and estimation procedure can be found in Abadie et al. [2] Appendix A2. Here we merely show the numerical estimate that will be used in the (long-term) valuations below: $\hat{M}_m =$ 6.560679. It will be taken as constant henceforth.

Appendix C. Estimation of underlying parameters

Drift and volatility of nodal demands. The data sample consists of monthly Sales of electricity to consumers (Public distribution system) in England/Wales (D^E) and Scotland (D^S) since January 2002 to March 2011 (TWh), i.e. 111 observations (DECC [26]: TABLE 5.5).First we take the seasonal component out of each series; see Table C1. Demand volatility in each case $\hat{\sigma}$ has been computed by multiplying the standard deviation of (monthly) residuals times $\sqrt{12}$. Second, the difference between the original series (for each country) and its homologous, deseasonalised counterpart allows to get the seasonal component of demand in each country. These values are used when simulating the future behavior of both demands over time.

Table C1. Estimates of underlying load parameters.

$\hat{k}_E$	$\hat{L}_E$	$\hat{\sigma}_E$	$\hat{k}_S$	$\hat{L}_S$	$\hat{\sigma}_S$	$\rho_{E,S}$
11.1829	23.8876	0.1546	8.5751	2.5261	0.1464	0.2616

Note that in the official UK statistics Final Consumption is the main component of Total Demad but not the only one. Energy industry use and Losses drive a sizeable wedge between both figures. The size of this wedge has been fairly stable in 2007, 2008 and 2009. The average value over these years has been 17.17 % for the whole UK . Therefore, the data of consumption in both England/Wales and Scotland are multiplied by 1.1717 to approach Total Demand in each country. Note also that since the original series (D^E, D^S) are 'net' demands, pumped storage must be added to both series in order to derive 'gross' demands (d^E, d^S).

Wind electricity: load factor, seasonality, and drift rate. Unfortunately we do not have country-specific time series for wind and hydro generation. The available data refer to the whole UK. Therefore, we are forced to undertake the estimation from these aggregate data and, in a second step, allocate each result to nodes E and S by means of some proportions.

Regarding wind, the sample comprises the monthly percentage ratios between output electricity and installed capacity for the whole UK from April 2006 to December 2010, i.e. 57 observations.[23] See Table C2.

Table C2. Estimates of underlying wind parameters. Monthly load factors 04:2006 to 12:2010		
$\hat{k}_W$	$\hat{W}_m$	$\hat{\sigma}_W$
11.2369	25.7256	0.9088

According to the installed wind capacities in 2009 (DECC [23]) we adopt the decomposition between countries:

$$W^E = 454 \times \frac{W}{100}; W^S = 642 \times \frac{W}{100}.$$

Hydro electricity: seasonality, and drift rates. The sample for natural flow comprises the monthly electricity supplied (net) for the whole UK (in TWh) from January 1997 to March 2011, i.e. 171 observations (DECC [26]: TABLE 5.4). The results appear on the left of Table C3. The sample for pumped storage comprises the monthly electricity supplied (net) for the whole UK from January 1998 to March 2011, i.e. 159 observations (DECC [26]: TABLE 5.4). The results appear on the right of Table C3.

Table C3. Estimates of hydro parameters: natural flow (left) and pumped storage (right).					
Monthly load factors 01:1997 to 03:2011			Monthly load factors 01:1998 to 03:2011		
$\hat{k}_H$	$\hat{H}_m$	$\hat{\sigma}_H$	$\hat{k}_P$	$\hat{P}_m$	$\hat{\sigma}_P$
6.0440	0.3093	1.2314	3.9459	0.0859	0.4472

According to the hydro electricity generated in 2009 (DECC [24]) we adopt the decomposition between countries:

$$H^E = \frac{240}{(240+4{,}055)} \times H; H^S = \frac{4{,}055}{(240+4{,}055)} \times H.$$

According to the electricity generated from pumped storage in 2009 (DECC [24]) we adopt the decomposition:

$$P^E = \frac{2{,}598}{(2{,}598+1{,}087)} \times P; P^S = \frac{1{,}087}{(2{,}598+1{,}087)} \times P.$$

Gross loads d^E and d^S are therefore:

$$d^E = D^E + \frac{2{,}598}{(2{,}598+1{,}087)} \times P; d^S = D^S + \frac{1{,}087}{(2{,}598+1{,}087)} \times P.$$

Estimation of the price processes. Our sample includes daily prices of all futures contracts on natural gas and ARA coal available on the European Energy Exchange (EEX, Germany), irrespective of their maturity, along with all futures contracts on EU emission

[23] The maximum possible output for each month is calculated from the installed capacity of the wind farm: Maximum output (MWh) = Installed capacity (MW) * number of days * 24. The actual output is then expressed as a percentage of the maximum possible output over the same time interval. Source: CLOWD [15].

allowances maturing in December that are traded on the Inter Continental Exchange (ICE, United Kingdom). Details on the estimation procedure can be found in Abadie et al. [2]. The numerical estimates of the relevant (composite) parameters appear in Table C4.

Table C4. Parameter estimates of commodity prices.							
Parameter	Value	Parameter	Value	Parameter	Value	Parameter	Value
$C_m^* = \frac{k_C C_m}{k_C + \lambda_C}$	105.27	σ_C	0.4144	φ_G (days)	-21.7	$\alpha^* = \alpha - \lambda_A$	0.054
$k_C + \lambda_C$	0.69	C_0	74.7898	γ_G	3.29	σ_A	0.20
$G_m^* = \frac{k_G G_m}{k_G + \lambda_G}$	25.04	σ_G	0.6356	ρ_{GC}	0.2652	A_0	13.18
$k_G + \lambda_G$	0.85	G_0	7.2419	ρ_{GA}	0.2572	ρ_{CA}	0.2797

All commodity prices are anticipated to increase from their current levels (G_0, C_0, A_0) in the future. This can be seen in the (risk-neutral) long-term values G_m^* and C_m^* for natural gas and coal, respectively. Regarding emission allowances, it is shown by an expected 5.4 % annual growth (in the risk-neutral world). On the other hand, the risk-free interest rate adopted is r = 3.22%. As the ARA coal is traded on the EEX in US dollars per tonne, it is also necessary to transform the price units; note that natural gas is quoted in €/MWh.[24]

[24] The interest rate corresponds to the rate of German government bonds in November 2009. On the other hand, we transform ARA coal prices from $/tonne to €/tonne using an exchange rate of 1.4934 $/ (the rate on 11/27/2009, when the 15-year interest rates of the euro and the dollar were at similar levels). In a further step, we transform €/tonne into €/MWh considering 29.31 GJ/tonne and using the equivalence 1 GJ = 0.27777 MWh.