

Stress tensor correlators in three dimensional gravityArjun Bagchi,^{1,*} Daniel Grumiller,^{2,†} and Wout Merbis^{2,‡}¹*Center for Theoretical Physics, Massachusetts Institute of Technology,
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We calculate holographically arbitrary n -point correlators of the boundary stress tensor in three-dimensional Einstein gravity with negative or vanishing cosmological constant. We provide explicit expressions up to 5-point (connected) correlators and show consistency with the Galilean conformal field theory Ward identities and recursion relations of correlators, which we derive. This provides a novel check of flat space holography in three dimensions.

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I. INTRODUCTION

Correlation functions are the main observational and theoretical entities in many branches of physics, in particular in any system that effectively is described by some quantum field theory (QFT), with numerous applications in condensed matter physics, cosmology, particle physics and many other research fields. Given their ubiquity, it is useful to have several tools available to calculate correlators.

A remarkable tool for strongly coupled QFTs is provided by holography [1,2]. In particular, the anti-de Sitter/conformal field theory (AdS/CFT) correspondence [3,4] allows us to calculate correlators in a specific strongly coupled CFT by mapping this (typically) hard calculation to a simple one in a specific gravity theory [5,6].

As the simplest example, the 1-point function [i.e., the vacuum expectation value (VEV)] of the stress tensor $T_{\mu\nu}$ is calculated by taking the first variation of the corresponding gravity action S_{AdS} with respect to the metric, evaluated on solutions of the equations of motion (EOM):

$$\langle T_{\mu\nu} \rangle_{\text{CFT}} \sim \left. \frac{\delta S_{\text{AdS}}}{\delta g^{\mu\nu}} \right|_{\text{EOM}}. \quad (1)$$

Note that the right-hand side is nonzero and finite due to boundary terms [7–9]. The holographic calculation of arbitrary (connected) n -point correlators of the stress tensor is a straightforward generalization of (1) and requires us to evaluate the n th variation of the gravity action. The n -point correlators of the stress tensor can also be calculated on the CFT side, so ultimately these calculations are a check of the holographic Ward identities.

Of course, calculating, say, the 42nd variation of the Einstein-Hilbert action in order to compare with the 42-point correlator of the stress tensor is not necessarily the most efficient way of performing such checks, so it is useful to

have an alternative tool available. Moreover, for more recent holographic correspondences that go beyond AdS/CFT, such as flat space holography [see [10–34] for some recent advances in three dimensions (3D)], similar checks would contribute to increasing the credibility of the purported relations between certain QFTs and their conjectured gravity duals.

The main goal of the present work is to provide the check outlined above for flat space Einstein-Hilbert (EH) gravity in 3D by calculating arbitrary n -point correlators of the stress tensor components on the QFT side and, independently, on the gravity side. So far only the 0- and 1-point functions were calculated holographically [18,21,22,25]. Our results are the first explicit check of the flat space holographic 2- and higher-point correlators and allow us to present for the first time explicit results for the 3-, 4- and 5-point correlators of the stress tensor components in two-dimensional (2D) Galilean conformal field theories (GCFTs) as well as their Ward identities. In addition we derive recursive relations for the n -point correlation functions of the stress tensor in 2D GCFTs. GCFTs arise in various systems such as nonrelativistic hydrodynamics [35], Galilean electrodynamics [36] and Galilean Yang-Mills theories, and tensionless string theory [37] and are expected to govern fixed points in renormalization group flows for nonrelativistic theories. So our present results are relevant for a vast range of applications over and above the subject of holography in flat space.

II. ADS₃/CFT₂

We start with the AdS₃/CFT₂ derivation of n -point correlators to exhibit the required tools in a familiar context. The key result is that all n -point correlators of the stress tensor in a CFT₂ are determined by the underlying symmetry algebra, which consists of two copies of the Virasoro algebra (n, m are integers),

$$[\mathcal{L}_n, \mathcal{L}_m] = (n - m)\mathcal{L}_{n+m} + \frac{c}{12}n(n^2 - 1)\delta_{n,-m} \quad (2)$$

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and similarly for the barred generators $\bar{\mathcal{L}}_n$, with the central charge c replaced by its barred counterpart \bar{c} . (The same remark applies to all remaining CFT formulas, so to reduce clutter we display only unbarred quantities.)

In their seminal paper, Belavin *et al.* proved a recursion relation between n - and $(n-1)$ -point correlators of the stress tensor in any CFT₂ [38]. For the connected part of the correlators this recursion relation reads

$$\langle T^1 T^2 \dots T^n \rangle = \sum_{i=2}^n \left(\frac{2}{z_{1i}^2} + \frac{1}{z_{1i}} \partial_{z_i} \right) \langle T^2 \dots T^n \rangle \quad (3)$$

where we introduced the abbreviation $z_{ij} = z_i - z_j$. Here $T^i = T_{zz}(z_i)$ is the holomorphic component of the stress tensor and z, \bar{z} are coordinates on the plane with metric $g_{zz} = 0 = g_{\bar{z}\bar{z}}$ and $g_{z\bar{z}} = 1$. The disconnected contribution to the n -point correlators is the sum of all permutations of products of lower-point functions such that all products have n points. Since holographic computations yield connected correlators we focus on them. All correlators in this paper are solely the connected part.

The n -point correlators are completely determined by the Ward identities and the 2-point correlators, which in turn are uniquely determined by conformal invariance (2):

$$\langle T^1 T^2 \rangle = \frac{c}{2z_{12}^4}. \quad (4)$$

Using this in the recursion relation (3) gives the higher-point correlators [39–42]:

$$\langle T^1 T^2 T^3 \rangle = \frac{c}{z_{12}^2 z_{23}^2 z_{13}^2} \quad (5a)$$

$$\langle T^1 T^2 T^3 T^4 \rangle = \frac{2c g_4(\gamma)}{z_{14}^2 z_{23}^2 z_{12} z_{13} z_{24} z_{34}} \quad (5b)$$

$$\langle T^1 T^2 T^3 T^4 T^5 \rangle = \frac{4c g_5(\gamma, \zeta)}{\prod_{1 \leq i < j \leq 5} z_{ij}}. \quad (5c)$$

The functions

$$g_4(\gamma) = \frac{\gamma^2 - \gamma + 1}{\gamma} \quad (5d)$$

$$g_5(\gamma, \zeta) = \frac{\gamma + \zeta}{2(\gamma - \zeta)} - \frac{(\gamma^2 - \gamma\zeta + \zeta^2)}{\gamma(\gamma - 1)\zeta(\zeta - 1)(\gamma - \zeta)} \\ \times ([\gamma(\gamma - 1) + 1][\zeta(\zeta - 1) + 1] - \gamma\zeta) \quad (5e)$$

depend on the cross-ratios

$$\gamma = \frac{z_{12} z_{34}}{z_{13} z_{24}}, \quad \zeta = \frac{z_{25} z_{34}}{z_{35} z_{24}}. \quad (5f)$$

This procedure can be iterated to arbitrary n -point correlators, with increasingly lengthy expressions [43].

Thus, if we can holographically establish the recursion relation (3) and calculate the 2-point correlators (4) we have essentially succeeded in holographically computing all the n -point correlators of the stress tensor. To this end, we consider a deformation of a free CFT action S_0 by a source term μ for the stress tensor,

$$S_\mu = S_0 - \int d^2 z \mu(z, \bar{z}) T(z). \quad (6)$$

We then localize the source at some point (z_2, \bar{z}_2) :

$$\mu(z, \bar{z}) = \epsilon \delta^{(2)}(z - z_2, \bar{z} - \bar{z}_2) \quad (7)$$

Here ϵ is a small expansion parameter introduced to keep track of the order of μ . The 1-point function with respect to the μ -deformed vacuum yields the 2-point correlator with respect to the free vacuum,

$$\langle T^1 \rangle_\mu = \langle T^1 \rangle + \epsilon \langle T^1 T^2 \rangle + \mathcal{O}(\epsilon^2). \quad (8)$$

We now compute this expansion in the dual AdS₃ gravity theory. According to the AdS/CFT dictionary, the source μ corresponds to the boundary value of the metric component $g_{\bar{z}\bar{z}}$. To compute the n -point function we need the n th variation of the bulk action with respect to the source. Such a computation is lengthy in the metric formulation, but fortunately simplifies significantly in the Chern-Simons (CS) formulation of 3D gravity [44,45]. In this framework, the EH action is the difference of two CS actions: $I_{\text{EH}} = I_{\text{CS}}[A] - I_{\text{CS}}[\bar{A}]$ with

$$I_{\text{CS}}[A] = \frac{k}{4\pi} \int \text{Tr} \left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right) \quad (9)$$

where A and \bar{A} are two $sl(2, \mathbb{R})$ -valued connections and the CS level k is related to the AdS length ℓ and Newton's constant G_N through $k = \frac{\ell}{4G_N}$. We write asymptotically AdS spacetimes in a ‘‘highest-weight gauge’’ [46]:

$$A = b^{-1}(d + a)b \quad b = e^{\rho L_0} \quad (10a)$$

$$a_z = L_+ + \frac{\mathcal{L}}{k} L_- \quad a_{\bar{z}} = -\mu L_+ + \dots \quad (10b)$$

Similar expressions hold for the barred sector. Here L_\pm, L_0 are the $sl(2, \mathbb{R})$ generators and ρ is the radial coordinate of AdS. The dots denote terms of lower weight which are completely fixed by the EOM. In this gauge, the ‘‘chemical potential’’ μ corresponds precisely to the source of the boundary stress tensor and \mathcal{L} is its VEV [47]; see also [48,49]. The EOM are the CFT Ward identities:

$$-\bar{\partial} \mathcal{L} = \frac{k}{2} \partial^3 \mu + 2\mathcal{L} \partial \mu + \mu \partial \mathcal{L}. \quad (11)$$

To calculate the 2-point function, we localize the sources as in (7) and expand

$$\mathcal{L}(z) = \mathcal{L}^{(0)}(z) + \epsilon \mathcal{L}^{(1)}(z) + \mathcal{O}(\epsilon^2) \quad (12)$$

where $\mathcal{L}^{(0)}$ is the background value, which we take to be AdS₃ in the Poincaré patch ($\mathcal{L}^{(0)} = \bar{\mathcal{L}}^{(0)} = 0$), while $\mathcal{L}^{(1)}$ corresponds to the linear term in ϵ in (8). This is precisely the 2-point function. From (11) we find

$$\bar{\partial} \mathcal{L}^{(1)} = -\frac{k}{2} \partial^3 \delta^{(2)}(z - z_2, \bar{z} - \bar{z}_2). \quad (13)$$

We solve this differential equation using the Green function of the Laplacian $\partial \bar{\partial}$ in flat space,

$$G(z_{12}, \bar{z}_{12}) = \ln(z_{12} \bar{z}_{12}). \quad (14)$$

Then

$$\mathcal{L}^{(1)} = -\frac{k}{2} \partial_{z_1}^4 G(z_{12}) = \frac{3k}{z_{12}^4} \quad (15)$$

gives exactly the 2-point function (4) with central charge

$$c = 6k = \frac{3\ell}{2G_N}, \quad (16)$$

compatible with [50].

The n -point functions can be obtained similarly. We localize the source at $n - 1$ points,

$$\mu(z, \bar{z}) = \sum_{i=2}^n \epsilon_i \delta^{(2)}(z - z_i, \bar{z} - \bar{z}_i) := \sum_{i=2}^n \epsilon_i \delta_i. \quad (17)$$

On the CFT side, this gives the expansion

$$\begin{aligned} \langle T^1 \rangle_\mu &= \langle T^1 \rangle + \sum_{i=2}^n \epsilon_i \langle T^1 T^i \rangle + \dots \\ &+ \left(\prod_{i=2}^n \epsilon_i \right) \langle T^1 T^2 \dots T^n \rangle + \mathcal{O}(\epsilon^n). \end{aligned} \quad (18)$$

The EOM at order ϵ^{n-1} give

$$-\bar{\partial} \mathcal{L}^{(n-1)} = \sum_{i=2}^n (2\mathcal{L}^{(n-2)} \partial \delta_i + \delta_i \partial \mathcal{L}^{(n-2)}). \quad (19)$$

Using the Green function G to solve for $\mathcal{L}^{(n-1)}$ yields

$$\mathcal{L}^{(n-1)}(z_1) = \sum_{i=2}^n \left(\frac{2}{z_{1i}^2} + \frac{1}{z_{1i}} \partial_{z_i} \right) \mathcal{L}^{(n-2)}(z_i). \quad (20)$$

This expression yields the recursion relation (3).

We have now derived holographically the 2-point function and the CFT recursion relation for the correlators (3) in the CS formulation and therefore know that all n -point correlators of the stress tensor match between gravity and the CFT side.

The results generalize from the plane to the cylinder after taking $z = e^{i\omega}$, with $\omega = it + \varphi$ and $\varphi \sim \varphi + 2\pi$, so that $(\omega, \bar{\omega}) \sim (\omega + 2\pi, \bar{\omega} + 2\pi)$. In all correlators

effectively the only change is that one needs to replace the quantities $z_{ij} \rightarrow 2 \sin[(\omega_i - \omega_j)/2] := s_{ij}$ and (3) becomes $[c_{ij} := \cot[(\omega_i - \omega_j)/2]]$

$$\langle T^1 T^2 \dots T^n \rangle_C = \sum_{i=2}^n \left(\frac{2}{s_{1i}^2} + \frac{c_{1i}}{2} \partial_{\omega_i} \right) \langle T^2 \dots T^n \rangle_C. \quad (21)$$

On the gravity side the relevant vacuum state is now global AdS₃ with background values $\mathcal{L}^{(0)} = \bar{\mathcal{L}}^{(0)} = \frac{k}{4}$.

This concludes the AdS₃ proof that the holographic calculation of (the connected part of) n -point correlators of the stress tensor matches precisely the CFT calculation. In the remainder of the paper we focus on 3D flat space, starting with the field theory side.

III. GCFT STRESS TENSOR CORRELATORS

The asymptotic symmetries of 3D flat space at null infinity are governed by the BMS₃ algebra [10,51] whose nonvanishing commutators are (n, m are integers)

$$[L_n, L_m] = (n - m)L_{n+m} + \frac{c_L}{12} n(n^2 - 1) \delta_{n,-m} \quad (22a)$$

$$[L_n, M_m] = (n - m)M_{n+m} + \frac{c_M}{12} n(n^2 - 1) \delta_{n,-m}. \quad (22b)$$

This algebra also arises out of non- or ultrarelativistic contractions of two copies of the Virasoro algebra (2) and is known as 2D Galilean conformal algebra (GCA). QFTs with GCA symmetries are called GCFTs. If there exists a dual formulation of 3D flat space gravity in terms of a 2D QFT it has to be a GCFT₂ [10,12,13].

The stress tensor components $\mathcal{M}_P, \mathcal{N}_P$ of a GCFT₂ on the plane can be mode-expanded in terms of L_n, M_n [37,52],

$$\begin{aligned} \mathcal{M}_P &= \sum_n M_n \xi^{-n-2}, \\ \mathcal{N}_P &= \sum_n \left(L_n - (n-2) \frac{\eta}{\xi} M_n \right) \xi^{-n-2}. \end{aligned} \quad (23)$$

The mapping from the plane to the cylinder is given by $\xi = e^{i\varphi}, \eta = iue^{i\varphi}$ with $\varphi \sim \varphi + 2\pi$. On the cylinder one has the following mode expansions [37]:

$$-M + \frac{c_M}{24} = \xi^2 \mathcal{M}_P = \sum_n M_n e^{-in\varphi} \quad (24a)$$

$$-N + \frac{c_L}{24} = \xi^2 \mathcal{N}_P - 2\xi \eta \mathcal{M}_P = \sum_n (L_n - inu M_n) e^{-in\varphi}. \quad (24b)$$

For our explorations of flat holography we work on the cylinder and consider highest-weight representations of the GCFT₂ [53,54]. We define the vacuum of the theory $|0\rangle$ as $L_n|0\rangle = 0 = M_n|0\rangle, \forall n \geq -1$. Using the above relation

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and its conjugate ($\langle 0|L_n = 0 = \langle 0|M_n, n \leq 1$), together with (25) and (22), we can calculate any correlation function of the GCFT₂ stress tensor components. In particular, the nonvanishing 2-point functions in the cylinder representation are given by

$$\langle M^1 N^2 \rangle = \frac{c_M}{2s_{12}^4} \quad \langle N^1 N^2 \rangle = \frac{c_L - 2c_M \tau_{12}}{2s_{12}^4} \quad (25)$$

with the definitions $M^i = M(\varphi_i)$, $N^i = N(u_i, \varphi_i)$, $s_{ij} = 2\sin[(\varphi_i - \varphi_j)/2]$, $c_{ij} = \cot[(\varphi_i - \varphi_j)/2]$ and $\tau_{ij} = (u_i - u_j)c_{ij}$.

Using the mode expansion and the algebra, the higher-point correlators can be derived similarly. Note that (the connected part of) any correlator with two or more M -insertions vanishes. The first few nonvanishing higher-point functions read

$$\langle M^1 N^2 N^3 \rangle = \frac{c_M}{s_{12}^2 s_{13}^2 s_{23}^2} \quad (26a)$$

$$\langle N^1 N^2 N^3 \rangle = \frac{c_L - c_M \tau_{123}}{s_{12}^2 s_{13}^2 s_{23}^2} \quad (26b)$$

$$\langle M^1 N^2 N^3 N^4 \rangle = \frac{2c_M g_4(\gamma)}{s_{14}^2 s_{23}^2 s_{12} s_{13} s_{24} s_{34}} \quad (26c)$$

$$\langle N^1 N^2 N^3 N^4 \rangle = \frac{2c_L g_4(\gamma) + c_M \Delta_4}{s_{14}^2 s_{23}^2 s_{12} s_{13} s_{24} s_{34}} \quad (26d)$$

$$\langle M^1 N^2 N^3 N^4 N^5 \rangle = \frac{4c_M g_5(\gamma, \zeta)}{\prod_{1 \leq i < j \leq 5} s_{ij}} \quad (26e)$$

$$\langle N^1 N^2 N^3 N^4 N^5 \rangle = \frac{4c_L g_5(\gamma, \zeta) + c_M \Delta_5}{\prod_{1 \leq i < j \leq 5} s_{ij}} \quad (26f)$$

where $\Delta_4 = 4g_4'(\gamma)\eta_{1234} - (\tau_{1234} + \tau_{14} + \tau_{23})g_4(\gamma)$ and $\Delta_5 = 4\partial_\gamma g_5(\gamma, \zeta)\eta_{1234} + 4\partial_\zeta g_5(\gamma, \zeta)\eta_{2345} - 2g_5(\gamma, \zeta)\tau_{12345}$. In the above, we have defined $\tau_{1\dots n} = \sum_{1 \leq i < j \leq n} \tau_{ij}$, e.g. $\tau_{123} = \tau_{12} + \tau_{13} + \tau_{23}$. The function $g_4(\gamma)$ [$g_5(\gamma, \zeta)$] is again given by (5d) [(5e)], where z_{ij} has to be replaced by s_{ij} in the cross-ratios (5f). The quantity η_{pqrs} is defined as $\eta_{pqrs} = \sum' (-1)^{1+i-j} \frac{(u_i - u_j) \sin(\varphi_k - \varphi_l)}{s_{pr}^2 s_{qs}^2}$ and the sum \sum' goes over all six permutations of $\{i, j, k, l\} = \pi(p, q, r, s)$ where $i < j$ and $k < l$.

IV. RECURSION RELATIONS

We derive now GCFT recursion relations involving the stress tensor components that encode the corresponding Ward identities [54]. The GCFT stress tensor components on the cylinder, (25), obey the conservation equations

$$\partial_u \langle M(\varphi) \mathcal{O} \rangle = 0 \quad \partial_u \langle N(u, \varphi) \mathcal{O} \rangle = \partial_\varphi \langle M(\varphi) \mathcal{O} \rangle \quad (27)$$

for any operator (or product of operators) \mathcal{O} . The left equation (27) looks very similar to the CFT conservation

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equation [38] $\partial_{\bar{z}} \langle T(z) \mathcal{O} \rangle = 0$ which together with the similarity of the GCA (22) to the Virasoro algebra (2) eventually leads to a recursion relation similar to (21):

$$\langle M^1 N^2 \dots N^n \rangle = \sum_{i=2}^n \left(\frac{2}{s_{1i}^2} + \frac{c_{1i}}{2} \partial_{\varphi_i} \right) \langle M^2 N^3 \dots N^n \rangle \quad (28)$$

The right equation (27) allows us to determine the correlation functions with only N -insertions in terms of correlation functions with one M -insertion (28):

$$\langle N^1 N^2 \dots N^n \rangle = \frac{c_L}{c_M} \langle M^1 N^2 \dots N^n \rangle + \sum_{i=1}^n u_i \partial_{\varphi_i} \langle M^1 N^2 \dots N^n \rangle. \quad (29)$$

The second recursion relation (29) can be derived as follows. Integrating the right equation (27) and using the cyclic symmetry of the $\langle MN \dots N \rangle$ correlators we obtain

$$\langle N^1 \dots N^n \rangle = A + \sum_{i=1}^n u_i \partial_{\varphi_i} \langle M^1 N^2 \dots N^n \rangle. \quad (30)$$

The quantity A is a u -independent integration constant that must be identical to the result in a chiral CFT with central charge c_L , since the u -independent part of N contains only the Virasoro generators L_n . As the $\langle MN \dots N \rangle$ correlators also give chiral CFT results, with central charge c_M , we can write

$$A = \frac{c_L}{c_M} \langle M^1 N^2 \dots N^n \rangle. \quad (31)$$

We have checked that the recursion relations (28), (29) also follow from a suitable contraction of CFT results (21) and that they are compatible with all our explicit results for correlators, (25)–(26f). Thus we have succeeded in deriving the GCFT₂ analog of the CFT recursion relation on the cylinder (21). In the last part of the paper we derive the same results holographically.

V. FLAT SPACE CORRELATORS

The 3D flat space EH action can be rewritten as the Chern-Simons action (9) for the $iso(2,1)$ connection $A = A_M^n M_n + A_L^n L_n$ where L_n, M_n ($n = 0, \pm 1$) generate the global part of the GCA (22) and the trace is defined as $\text{Tr}(L_m M_n) = -2\gamma_{mn}$, where $\gamma = \text{antidiag}(1, -\frac{1}{2}, 1)$; see [31] for our conventions, details and references. Just like in the AdS case, we parametrize the radial dependence of A by going to a gauge where [27,31] $A = b^{-1}(d + a)b$ with the group element $b = \exp(\frac{\xi}{2} M_-)$ and $a = a_u du + a_\varphi d\varphi$ with u, r, φ the (outgoing) Eddington-Finkelstein coordinates. The metric is constructed as

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -4A_M^+ A_M^- + (A_M^0)^2. \quad (32)$$

In the absence of chemical potentials the connection components take the form

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$$a_u = M_+ + \frac{\mathcal{M}}{2k} M_- \quad (33)$$

$$a_\varphi = L_+ + \frac{\mathcal{M}}{2k} L_- + \frac{\mathcal{N}}{2k} M_- \quad (34)$$

Here the CS-level k is related to Newton's constant as $k = 1/(4G_N)$. The EOM restrict the state-dependent functions \mathcal{M}, \mathcal{N} as follows:

$$\partial_u \mathcal{M} = 0 \quad \partial_u \mathcal{N} = \partial_\varphi \mathcal{M}. \quad (35)$$

These equations are equivalent to the GCFT conservation equations (27).

We proceed as in the AdS case and introduce chemical potentials as sources. The component a_φ remains unaffected, while a_u changes to $a_u + \Delta a_u$ with

$$\Delta a_u = -\mu_M M_+ - \mu_L L_+ + a_0 M_0 + a_1 L_0 + a_2 M_- + a_3 L_- \quad (36)$$

The EOM determine completely the functions a_i in terms of the chemical potentials μ_M and μ_L and restrict the state-dependent functions as follows [31]:

$$-\partial_u \mathcal{M} = k \partial_\varphi^3 \mu_L + \mu_L \partial_\varphi \mathcal{M} + 2\mathcal{M} \partial_\varphi \mu_L \quad (37a)$$

$$-\partial_u \mathcal{N} = k \partial_\varphi^3 \mu_M - (1 - \mu_M) \partial_\varphi \mathcal{M} + 2\mathcal{M} \partial_\varphi \mu_M + \mu_L \partial_\varphi \mathcal{N} + 2\mathcal{N} \partial_\varphi \mu_L \quad (37b)$$

Analogous to the AdS case, these equations can be interpreted as GCFT Ward identities.

For our background we choose global Minkowski space, $\mathcal{M}^{(0)} = k/2$, $\mathcal{N}^{(0)} = 0$. To calculate 2-point correlators in this Minkowski background we switch on chemical potentials, localized at (u_2, φ_2) , like in the AdS case (7),

$$\mu_{M/L} = \epsilon_{M/L} \delta^{(2)}(u - u_2, \varphi - \varphi_2) := \epsilon_{M/L} \delta. \quad (38)$$

Here $\epsilon_{M/L}$ are constants. Plugging these localized chemical potentials into the EOM (37) with $\mathcal{M} = k/2 + \mathcal{M}^{(1)}$, $\mathcal{N} = \mathcal{N}^{(1)}$ yields the linearized Ward identities:

$$\partial_u \mathcal{M}^{(1)} = -k \epsilon_L (\partial_\varphi^3 \delta + \partial_\varphi \delta) \quad (39a)$$

$$\partial_u \mathcal{N}^{(1)} = -k \epsilon_M (\partial_\varphi^3 \delta + \partial_\varphi \delta) + \partial_\varphi \mathcal{M}^{(1)}. \quad (39b)$$

To solve them we use the Green function defined as

$$\partial_u \partial_\varphi G(u - u_2, \varphi - \varphi_2) = \delta. \quad (40)$$

Using the method of images yields

$$G(u - u_2, \varphi - \varphi_2) = \ln((u - u_2) \sin[(\varphi - \varphi_2)/2]) \quad (41)$$

and thus we can solve the linearized Ward identities (39a), (39b),

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$$\mathcal{M}^{(1)} = \frac{6k\epsilon_L}{s_{12}^4} \quad \mathcal{N}^{(1)} = \frac{6k(\epsilon_M - 2\epsilon_L \tau_{12})}{s_{12}^4} \quad (42)$$

where s_{ij} and τ_{ij} are defined below (25). The ϵ_L component of $\mathcal{M}^{(1)}$ [and the ϵ_M component of $\mathcal{N}^{(1)}$] is the 2-point function $\langle M^1 N^2 \rangle$ and the ϵ_L part of $\mathcal{N}^{(1)}$ is the 2-point function $\langle N^1 N^2 \rangle$. They agree precisely with the GCFT 2-point functions (25) for $c_L = 0$ and $c_M = 12k = 3/G_N$, compatible with [10,15].

Like in the AdS case we compute the higher-point functions by localizing the sources at multiple points u_i , φ_i and expanding the VEVs of the deformed theory to order ϵ^{n-1} , which eventually establishes recursive formulas:

$$\mathcal{M}^{(n-1)} = \sum_{i=2}^n \epsilon_L^i \left(\frac{2}{s_{1i}^2} + \frac{c_{1i}}{2} \partial_{\varphi_i} \right) \mathcal{M}^{(n-2)} \quad (43a)$$

$$\begin{aligned} \mathcal{N}^{(n-1)} = & \sum_{i=2}^n \left\{ \left(\frac{2}{s_{1i}^2} + \frac{c_{1i}}{2} \partial_{\varphi_i} \right) (\epsilon_M^i \mathcal{M}^{(n-2)} + \epsilon_L^i \mathcal{N}^{(n-2)}) \right. \\ & \left. + u_i \epsilon_L^i \left(\frac{2c_{1i}}{s_{ij}^2} + \frac{\partial_{\varphi_i}}{s_{1i}^2} \right) \mathcal{M}^{(n-2)} \right\} + u_1 \partial_{\varphi_1} \mathcal{M}^{(n-1)}. \end{aligned} \quad (43b)$$

The ϵ_L component of $\mathcal{N}^{(n-1)}$ then corresponds to the n -point function $\langle N^1 N^2 \dots N^n \rangle$. The ϵ_M component of $\mathcal{N}^{(n-1)}$ corresponds to the n -point correlator with one M^i insertion [just as the ϵ_L component of $\mathcal{M}^{(n-1)}$]. The recursion relation (43a) then simply gives (28) with $c_M = 12k$, while the ϵ_L part of (43b) can be simplified to give

$$\langle N^1 N^2 \dots N^n \rangle = \sum_{i=1}^n u_i \partial_{\varphi_i} \langle M^1 N^2 \dots N^n \rangle. \quad (44)$$

These results show precise agreement with the GCFT recursion relations (28), (29), provided that $c_L = 0$ and $c_M = 12k = 3/G_N$, again compatible with [10,15].

VI. CONCLUSIONS

Since the holographic recursion relations (43a), (44) are exactly the ones that we found in a GCFT (28), (29) with the EH values of the central charges, $c_L = 0$, $c_M = 3/G_N$, and the 2-point correlators coincide as well, we conclude that flat space holography is perfectly consistent with the GCFT Ward identities. In other words, all holographic n -point correlators of the stress tensor components match precisely with the corresponding GCFT (connected) correlators.

It would be interesting to generalize our analysis to other 3D gravity (or gravitylike) theories; see e.g. Refs. [19,20,44,55–63] for various intriguing models.

In this work we have focused mainly on the Ward identities for correlation functions on one half of lightlike infinity. A similar construction (exchanging retarded time u with advanced time v) works for the other half. It would be interesting to investigate the relation between these two

symmetries by matching the appropriate charges through spatial infinity, along the lines of [64].

Finally, it would be interesting to generalize our discussions to other correlation functions, for instance of scalar operators, like in AdS/CFT [6]. This will require the introduction of matter degrees of freedom, both on the field theory side and on the gravity side.

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- [1] G. 't Hooft, in *Salamfestschrift* (World Scientific, Singapore, 1993).
- [2] L. Susskind, *J. Math. Phys. (N.Y.)* **36**, 6377 (1995).
- [3] J. M. Maldacena, *Adv. Theor. Math. Phys.* **2**, 231 (1998).
- [4] O. Aharony, S. S. Gubser, J. M. Maldacena, H. Ooguri, and Y. Oz, *Phys. Rep.* **323**, 183 (2000).
- [5] S. S. Gubser, I. R. Klebanov, and A. M. Polyakov, *Phys. Lett. B* **428**, 105 (1998).
- [6] E. Witten, *Adv. Theor. Math. Phys.* **2**, 253 (1998).
- [7] M. Henningson and K. Skenderis, *J. High Energy Phys.* 07 (1998) 023.
- [8] V. Balasubramanian and P. Kraus, *Commun. Math. Phys.* **208**, 413 (1999).
- [9] R. Emparan, C. V. Johnson, and R. C. Myers, *Phys. Rev. D* **60**, 104001 (1999).
- [10] G. Barnich and G. Compère, *Classical Quantum Gravity* **24**, F15 (2007).
- [11] G. Barnich and C. Troessaert, *J. High Energy Phys.* 05 (2010) 062.
- [12] A. Bagchi, *Phys. Rev. Lett.* **105**, 171601 (2010).
- [13] A. Bagchi and R. Fareghbal, *J. High Energy Phys.* 10 (2012) 092.
- [14] G. Barnich, A. Gomberoff, and H. A. Gonzalez, *Phys. Rev. D* **86**, 024020 (2012).
- [15] A. Bagchi, S. Detournay, and D. Grumiller, *Phys. Rev. Lett.* **109**, 151301 (2012).
- [16] G. Barnich, *J. High Energy Phys.* 10 (2012) 095.
- [17] A. Bagchi, S. Detournay, R. Fareghbal, and J. Simon, *Phys. Rev. Lett.* **110**, 141302 (2013).
- [18] A. Bagchi, S. Detournay, D. Grumiller, and J. Simon, *Phys. Rev. Lett.* **111**, 181301 (2013).
- [19] H. Afshar, A. Bagchi, R. Fareghbal, D. Grumiller, and J. Rosseel, *Phys. Rev. Lett.* **111**, 121603 (2013).
- [20] H. A. Gonzalez, J. Matulich, M. Pino, and R. Troncoso, *J. High Energy Phys.* 09 (2013) 016.
- [21] R. Caldeira Costa, *Phys. Rev. D* **90**, 104018 (2014).
- [22] R. Fareghbal and A. Naseh, *J. High Energy Phys.* 03 (2014) 005.
- [23] A. Bagchi and R. Basu, *J. High Energy Phys.* 03 (2014) 020.
- [24] C. Krishnan, A. Raju, and S. Roy, *J. High Energy Phys.* 03 (2014) 036.
- [25] S. Detournay, D. Grumiller, F. Schöller, and J. Simon, *Phys. Rev. D* **89**, 084061 (2014).
- [26] G. Barnich and B. Oblak, *J. High Energy Phys.* 06 (2014) 129.
- [27] G. Barnich, L. Donnay, J. Matulich, and R. Troncoso, *J. High Energy Phys.* 08 (2014) 071.
- [28] M. Riegler, *Phys. Rev. D* **91**, 024044 (2015).
- [29] R. Fareghbal and A. Naseh, *Classical Quantum Gravity* **32**, 135013 (2015).
- [30] A. Bagchi, R. Basu, D. Grumiller, and M. Riegler, *Phys. Rev. Lett.* **114**, 111602 (2015).
- [31] M. Gary, D. Grumiller, M. Riegler, and J. Rosseel, *J. High Energy Phys.* 01 (2015) 152.
- [32] B. Oblak, *Commun. Math. Phys.* **340**, 413 (2015).
- [33] G. Barnich and B. Oblak, *J. High Energy Phys.* 03 (2015) 033.
- [34] G. Barnich, H. A. Gonzalez, A. Maloney, and B. Oblak, *J. High Energy Phys.* 04 (2015) 178.
- [35] A. Bagchi and R. Gopakumar, *J. High Energy Phys.* 07 (2009) 037.
- [36] A. Bagchi, R. Basu, and A. Mehra, *J. High Energy Phys.* 11 (2014) 061.
- [37] A. Bagchi, *J. High Energy Phys.* 05 (2013) 141.
- [38] A. Belavin, A. M. Polyakov, and A. Zamolodchikov, *Nucl. Phys.* **B241**, 333 (1984).
- [39] P. Di Francesco, P. Mathieu, and D. Senechal, *Conformal Field Theory* (Springer, New York, 1997).
- [40] R. Blumenhagen and E. Plauschinn, *Introduction to Conformal Field Theory with Applications to String Theory*, Lecture Notes in Physics, Vol. 779 (Springer, New York, 2009).
- [41] H. Osborn, *Phys. Lett. B* **718**, 169 (2012).
- [42] M. Headrick, A. Maloney, E. Perlmutter, and I. G. Zadeh, *J. High Energy Phys.* 07 (2015) 059.
- [43] Even though the results for the 5-point function are straightforward to obtain we were not able to find it in the literature and therefore display it here for future use. Note that—like for the 4-point correlator [41]—the disconnected part that we dropped is quadratic in the central charge c . For higher n -point correlators there will be higher powers of c up to $c^{\lfloor n/2 \rfloor}$ appearing in the disconnected parts.
- [44] A. Achúcarro and P. K. Townsend, *Phys. Lett. B* **180**, 89 (1986).
- [45] E. Witten, *Nucl. Phys.* **B311**, 46 (1988).
- [46] M. Bañados, *AIP Conf. Proc.* **484**, 147 (1999).
- [47] M. Banados and R. Caro, *J. High Energy Phys.* 12 (2004) 036.
- [48] J. de Boer and J. I. Jottar, [arXiv:1407.3844](https://arxiv.org/abs/1407.3844).
- [49] W. Li and S. Theisen, *J. High Energy Phys.* 08 (2015) 035.
- [50] J. D. Brown and M. Henneaux, *Commun. Math. Phys.* **104**, 207 (1986).
- [51] A. Ashtekar, J. Bicak, and B. G. Schmidt, *Phys. Rev. D* **55**, 669 (1997).

- [52] A. Bagchi, *J. High Energy Phys.* 02 (2011) 091.
- [53] A. Bagchi and I. Mandal, *Phys. Lett. B* **675**, 393 (2009).
- [54] A. Bagchi, R. Gopakumar, I. Mandal, and A. Miwa, *J. High Energy Phys.* 08 (2010) 004.
- [55] S. Deser, R. Jackiw, and S. Templeton, *Phys. Rev. Lett.* **48**, 975 (1982).
- [56] S. Deser and J. H. Kay, *Phys. Lett.* **120B**, 97 (1983).
- [57] E. A. Bergshoeff, O. Hohm, and P. K. Townsend, *Phys. Rev. Lett.* **102**, 201301 (2009).
- [58] R. Andringa, E. A. Bergshoeff, M. de Roo, O. Hohm, E. Sezgin, and P. K. Townsend, *Classical Quantum Gravity* **27**, 025010 (2010).
- [59] E. A. Bergshoeff, S. de Haan, O. Hohm, W. Merbis, and P. K. Townsend, *Phys. Rev. Lett.* **111**, 111102 (2013).
- [60] M. Henneaux and S.-J. Rey, *J. High Energy Phys.* 12 (2010) 007.
- [61] A. Campoleoni, S. Fredenhagen, S. Pfenninger, and S. Theisen, *J. High Energy Phys.* 11 (2010) 007.
- [62] M. Gary, D. Grumiller, and R. Rashkov, *J. High Energy Phys.* 03 (2012) 022.
- [63] M. Henneaux, G. Lucena Gómez, J. Park, and S.-J. Rey, *J. High Energy Phys.* 06 (2012) 037.
- [64] A. Strominger, *J. High Energy Phys.* 07 (2014) 152.