# The Impact of a Target on Newsvendor Decisions

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Goal achieving is a commonly observed phenomenon in practice, and it plays an important role in decision making. In this paper we investigate the impact of a target on newsvendor decisions. We take into account the risk and model the effect of a target by maximizing the satisficing measure of a newsvendor's profit with respect to that target. We study two satisficing measures: i) CVaR (Conditional Value at Risk) satisficing measure that evaluates the highest confidence level of CVaR achieving the target; and ii) entropic satisficing measure that assesses the smallest risk tolerance level under which the certainty equivalent for exponential utility function achieves the target. For both satisficing measures, we find that the optimal ordering quantity increases with the target level. We determine an optimal order quantity for a target based newsvendor and charactize its properties with respect to, for example, product's profit margin.

*Key words*: newsvendor; risk measure; target profit
*History*:

## 1.  Introduction

Meeting a pre-specified profit target has been a key performance metric for firms. A quick search in The Wall Street Journal with the keyword "target" returns numerous results on firms meeting or missing financial goals. This is widely believed to affect firms' stock prices (Rappaport 1999). For example, in the fourth quarter of 2004, Ebay reported earnings of 23 cents per share, which missed the target of 24 cents per share. After the report was issued, Ebay's stock tumbled more than 11% within a few hours (CNNMoney 2005). Despite the important role a target plays in practice, it has not received much attention from the Operations Management research community. Our aim is to bring new perspectives to newsvendor decisions, and to provide an alternative approach in studying newsvendor problems that can appeal to target-attainment oriented decision makers.

The classical newsvendor model assumes that decision makers are risk neutral and maximize the expected profit, which drives a fractile-based solution. This is a prevalent approach for scenarios where the risks can be effectively hedged, such as when the decisions can be repeated for a large number of times or when a firm owns many newsvendors. However, when it comes to capacity

decisions or strategic investment decisions, i.e., those critical and irreversible, the risks cannot be considered as negligible. To take into account the risks in such contexts, researchers have explored alternative objectives such as minimizing a risk measure or maximizing the expected utility. Some representative work includes Eeckhoudt et al. (1995), Ahmed et al. (2007), Özer et al. (2007), Choi and Ruszczynski (2008), and Chen et al. (2009). None of these, however, take into account goal achieving. The only exception, to the best of our knowledge, is Lau (1980), who solves the problem of maximizing the probability of the random profit attaining a target level, which we call *attainment probability*. Although intuitively appealing, the attainment probability measure focuses on the probability of a shortfall. It ignores the magnitude of the loss, which is a critical dimension of concern for decision making (Payne et al. 1980). Therefore, it does not fully capture the role of a target in decision making with risk.

To incorporate the effect of a target as well as alleviate the drawbacks of attainment probability, we undertake an alternative approach to study the impact of a target on the newsvendor's decision by adopting the recently developed satisficing measure (Brown and Sim 2009, Brown et al. 2012). It is a class of risk measures that evaluate the ability of a certain metric – which is associated with the underlying random payoff – achieving a target. In particular, we focus on two satisficing measures that are based on the following two underlying metrics, respectively: CVaR (Conditional Value at Risk) and Certainty Equivalent for an exponential utility function (Mas-Colell et al. 2005). These two metrics represent two different ways decision makers perceive risks. While CVaR is focused on downside risk, Certainty Equivalent takes into account all realizations of the underlying randomness and hence captures the attention on full scale risk. By studying both, we are able to generate insights for newsvendors with different risk focuses.

CVaR was developed by Rockafellar and Uryasev (2000, 2002), the original definition of which evaluates the downside risk of an uncertain cash-flow. It measures the expected value of a random cash flow – profit, in our newsvendor context – that falls below a certain quantile value. We call it worst-case-scenario expected profit, where "worst" is associated with a confidence level. To incorporate the fact that people are not always risk averse, we extend the definition of CVaR such that it also measures the expected value of the profit that is above a certain quantile value. We call it best-case-scenario expected profit, where "best" is also associated with a confidence level. The Certainty Equivalent for a risky alternative is the certain amount that is equally preferred to the alternative, which reveals a decision maker's risk attitude toward the risky alternative. Similarly, we study the certainty equivalent for both the risk-averse and risk-seeking scenarios.

Corresponding to these two metrics, we consider CVaR satisficing measure (CSM) and entropic satisficing measure (ESM), respectively. The former evaluates the confidence level of CVaR achieving the target and the latter assesses the risk tolerance level under which the certainty equivalent

achieves the target. Note that it is desirable to have a CSM value as big as possible, as this suggests that one can be highly confident about the expected profit achieving the target even if the random profit is realized in an undesirable region. Similarly, higher ESM value is preferred because it implies that a highly conservative decision maker can still have the certainty equivalent exceeding the target and accept the underlying decision. As such, the objective of the newsvendor is to find an order quantity that maximizes the CVaR (entropic) satisficing measure.

Before presenting the models and analyses, we summarize our contributions to literature. I) We build a model that captures the effect of a target on newsvendor decisions, and with which incorporate the decision makers' risk attitude. Our model does not require a modeler to calibrate the model parameters, e.g., the parameters for a utility function. Given a target, our model prescribes an ordering strategy. II) We determine an optimal ordering quantity for a target-based newsvendor and characterize its properties with respect to factors such as profit margin and target level. We also compare the optimal order quantity to the well-known critcal-fractile solution that assumes risk neutrality. III) By analyzing the newsvendor decision with both the criteria of CSM and ESM, we find that the optimal ordering quantities under these two criteria exhibit the same properties. This demonstrates the robustness of our model with respect to how risks are recognized. Finally, we take one step further and show that if the target is set in a particular way, our model gives exactly the same solution as an expected utility model does.

## 2.   Newsvendor Decision with CVaR Satisficing Measure

In this section we investigate the quantity decision for a target-oriented newsvendor who focuses on the downside (upside) risk of the random profit. The newsvendor decides how many units of a product to order before the selling season. Each unit is purchased at a cost $c$ and sold at a price $p$. The random demand $D$ is assumed to be bounded by $[\underline{d}, \bar{d}] \subseteq \Re_+$ and without loss of generality, continuously distributed. In this paper, we use capital letters to denote random variables and lower case letters for their realizations. We assume that the unsatisfied demand is lost and the salvage value for unsold items is zero. Given an order quantity $y$, the newsvendor's random profit is:

$$V(y) = -cy + p\min(y, D). \tag{1}$$

The metric that is often used to evaluate decision alternatives with respect to the downside (upside) risk is CVaR. This by itself, however, does not take into account goal achieving. To capture the effect of a target, this target-oriented newsvendor aims to find the order quantity that corresponds to the highest confidence level of the CVaR of the random profit achieving a pre-set target. In particular, let the target profit be $\tau \in \Re$, the newsvendor solves the following problem:

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$$\begin{aligned}
\sup \quad & \eta \\
\text{s.t.} \quad & CVaR_\eta\left(V(y)\right) \geq \tau \\
& \eta \in (-1, 1) \\
& y \geq 0,
\end{aligned} \tag{2}$$

where $CVaR_\eta$ is defined for any random profit $V$ as follows,

$$CVaR_\eta(V) = \begin{cases} \max\limits_{a \in \Re}\left\{a + \frac{1}{1-\eta}\mathbb{E}[\min\{V - a, 0\}]\right\} & \text{if } \eta \in [0, 1), \\ -CVaR_{-\eta}(-V) & \text{if } \eta \in (-1, 0). \end{cases} \tag{3}$$

When $V$ is continuously distributed, an equivalent but more intuitive definition for $CVaR_\eta$ is:

$$CVaR_\eta(V) = \begin{cases} \mathbb{E}\left[V \,|\, V \leq q_{1-\eta}(V)\right], \text{ if } \eta \in [0, 1), \\ \mathbb{E}\left[V \,|\, V \geq q_{-\eta}(V)\right], \text{ if } \eta \in (-1, 0), \end{cases} \tag{4}$$

where $q_\eta(V)$ is the unique $\eta$-quantile of $V$, i.e., $Prob(V \leq q_\eta(V)) = \eta$.

The above definition of CVaR is an extended version based on the one proposed by Rockafellar and Uryasev (2000, 2002). It suggests that for $\eta \in [0, 1)$, $CVaR_\eta(V)$ measures the expectation of $V$ in the worst $(1 - \eta)$ case realizations, whereas for $\eta \in (-1, 0)$, it measures the expectation in the best $(1 + \eta)$ case realizations. Figure 1 provides an example to illustrate these definitions. Graph 1 in Figure 1 illustrates the PDF of the random profit $V$, and Graphs 2 to 6 illustrate the values of $CVaR_\eta(V)$ for $\eta \in \{3/4, 1/3, 0, -1/4, -2/3\}$, which are the expectations taken over the grey areas in the graphs, e.g., $CVaR_{3/4}(V) = 1/6$. We can see that as the value of $\eta$ decreases, the expectation is taken over a more optimistic area of the profit realization, and $CVaR_\eta$ increases (from $1/6$ to $2/3$). In other words, a smaller $\eta$ in the $CVaR_\eta$ evaluation suggests that the decision maker is less risk averse and more optimistic about the random profit realization.
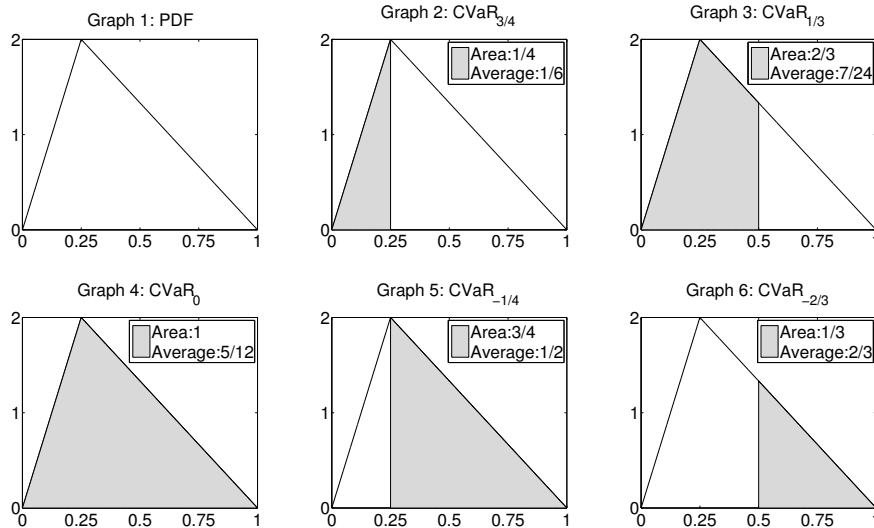


**Figure 1      Illustration for CVaR**

Note that Problem (2) can be equivalently formulated based on the following satisficing measure that evaluates a random profit's risk with respect to the target profit $\tau$.

**Definition 1** *Given a target profit $\tau \in \Re$, the CVaR satisfying measure (CSM) of the random profit $V$ is defined as:*

$$\rho_\tau(V) = \begin{cases} \sup\{\eta \in (-1,1) : CVaR_\eta(V) \geq \tau\}, & if\ feasible, \\ -1 & otherwise. \end{cases} \tag{5}$$

By Definition 1, Problem (2) is exactly the same as the following one that maximizes the CSM of the random profit,

$$\rho_\tau^* = \max_{y \geq 0} \rho_\tau(V(y)). \tag{6}$$

According to Definition 1, CSM measures the highest $\eta$ that guarantees $CVaR_\eta$ achieving a target. Take Figure 1 for illustration. Suppose the target $\tau = 1/2$. We can see that for any $\eta > -1/4$ (e.g., Graphs 2 to 4 in Figure 1), $CVaR_\eta$, the expectation over the corresponding grey area, is less than the target. When $\eta \leq -1/4$, however, $CVaR_\eta \geq 1/2$. As such, $\rho_\tau(V) = -1/4$, is the largest $\eta$ that ensures $CVaR_\eta$ to achieve the target. From a risk management perspective, it is desirable for a random payoff to have a high CSM value, as it implies that even if the uncertainties realize in a not-so-optimistic region, the expectation can still achieve the target. In other words, the random payoff $V$ is more secure with respect to the target $\tau$.

Problems (2) and (6) are two alternative formulations of the same problem. Both aim to find the optimal order quantity so that the CVaR of the random profit can achieve the target with the highest confidence level. In other words, the satisficing measure of the random profit resulting from the order decision is maximized. In what follows, we first describe the solution procedure, and then investigate the properties of the optimal ordering decisions. In discussing the properties, we follow the framework of the problem in (6) and often talk about the optimal satisficing measures.

To solve for the optimal order quantity, we first observe that following the definition in (3), $CVaR_\eta$ is non-increasing in $\eta$. Therefore, we can find the optimal solution for the problem in (2) by performing a binary search on $\eta$. For each $\eta \in (-1,1)$, we need to solve the following subproblem:

$$\max_{y \geq 0} CVaR_\eta(V(y)). \tag{7}$$

Proposition 1 below provides the solution to (7). We defer all proofs to the appendix.

**Proposition 1** *For any $\eta \in (-1,1)$, we have*

$$\arg\max_{y \geq 0} CVaR_\eta(V(y)) = \begin{cases} F^{-1}(\xi - \eta\xi) & if\ \eta \in [0,1), \\ F^{-1}(\xi - \eta(1-\xi)) & if\ \eta \in (-1,0), \end{cases}$$

*where $\xi = \frac{p-c}{p}$ is called critical fractile, and $F$ is the cumulative distribution of $D$.*

We now proceed to examine how the target profit affects the ordering decision.

**Theorem 1** *Assume that $\tau_1 \geq \tau_2$. Then we have: 1) $\rho_{\tau_1}^* \leq \rho_{\tau_2}^*$; and 2) there exists $y_1 \geq y_2 \geq 0$ such that $y_i \in \arg\max_{y \geq 0} \rho_{\tau_i}(V(y))$, $i \in \{1,2\}$.*

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Theorem 1 part 1 shows that the newsvendor's maximum CSM is decreasing with the target profit. Intuitively, the same random profit will be more secure if we have a lower target, and be riskier if we have a higher target. Consequently, the best quantity decision under a low target must make the profit at least as secure as that under a high target.

Theorem 1 part 2 suggests that the newsvendor will order more if the target is higher. We note that a high target is an indication of the newsvendor's soaring ambition, which cannot be realized unless the newsvendor places a large order. To further illustrate, we consider an extreme case. Suppose the target is $\tau = (p-c)\underline{d}$, where $\underline{d}$ is the lower bound of the random demand. Then the newsvendor would order exactly at $\underline{d}$ since it yields a deterministic profit of $(p-c)\underline{d}$ and ensures the target attainment. However, if the target $\tau > (p-c)\underline{d}$, then the ordering quantity of $\underline{d}$ is no longer optimal since the profit is consistently lower than the target. Therefore, the newsvendor has to increase the ordering quantity.

In the newsvendor problem, an important benchmark is the risk neutral solution, $y_N$, which maximizes the expected profit and is known to be $y_N = F^{-1}(\xi)$. Theorems 2 and 3 and their corresponding corollaries below investigate the relationship between $y_N$ and the target-based solution.

**Theorem 2** *If $\tau = \mathbb{E}\left[V\left(y_N\right)\right]$, then $y_N \in \arg\max_{y \geq 0} \rho_\tau\left(V(y)\right)$.*

Theorem 2 says that if the newsvendor's target is set to be the maximum expected profit, then the risk-neutral newsvendor solution gives the highest CSM. This is intuitive because for any other order quantity, the risk neutral expectation of the profit is less than the target, $\tau = \mathbb{E}[V(y_N)]$. As such, to enable its CVaR to reach the target, it is only possible by looking at the best-case profit realization when $\eta < 0$, whereas the risk-neutral solution can do so for $\eta = 0$. Based on Theorem 2, we have the following Corollary.

**Corollary 1**     1. *If $\tau \leq \mathbb{E}\left[V\left(y_N\right)\right]$, then $\exists y^* \leq y_N$ such that $y^* \in \arg\max_{y \geq 0} \rho_\tau\left(V(y)\right)$.*
    2. *If $\tau \geq \mathbb{E}\left[V\left(y_N\right)\right]$, then $\exists y^* \geq y_N$ such that $y^* \in \arg\max_{y \geq 0} \rho_\tau\left(V(y)\right)$.*

A newsvendor is said to *under-order* if she orders less than $y_N$, and *over-order* if she orders more than $y_N$. Corollary 1 shows that the newsvendor under-orders when the target is lower than the maximum expected profit, and over-orders when the target is higher than that. This is probably because if the target is very high, then the decision maker may just take the chance and "pray for odds", and hence over-orders. If the target is low, however, then it makes more sense to be conservative, and hence under-orders.

So far we have treated the target profit as exogenously given, without considering how it is set and what form it takes. In comparison to the substantial body of empirical research on the effect of target (e.g., Brown and Tang 2006), the research on how people form their targets is rather

scant. To the best of our knowledge, the only descriptive research is a field study by Merchant and Manzoni (1989), who show that in practice the targets are usually set in a way such that they can be achieved in eighty to ninety percent of the time. The other stream of research, which can be considered as a guide on how to set targets normatively, mainly focus on how the challenging level of the goal impacts employee performance (e.g., Tubbs 1986, Locke & Latham 2002, and Fried & Slowik, 2004). While how the targets should be set is an interesting issue to investigate, it is beyond the scope of this paper. It, however, can certainly be a future research direction. In what follows, we first follow the path of Köszegi and Rabin (2006) and make the assumption that the newsvendor's target profit is determined by following simple heuristics, and study the property of optimal order quantities. We then show that if the target is set in a particular way, our model provides the same solution as a model maximizing CVaR does.

**Theorem 3** *Assume $\tau = (p-c) \times \alpha(D)$, where $\alpha(D)$ is a positive value that depends on the knowledge of D alone. Then there exists a threshold value $\zeta$, which increases with $\alpha(D)$, such that: (i) if $\frac{p-c}{p} \geq \zeta$, we can find $y^* \leq y_N$, where $y^* \in \arg\max_{y \geq 0} \rho_\tau(V(y))$; and (ii) if $\frac{p-c}{p} \leq \zeta$, we can find $y^* \geq y_N$, where $y^* \in \arg\max_{y \geq 0} \rho_\tau(V(y))$.*

By Theorem 3, if the target is proportional to the unit marginal profit as well as a demand-related factor, the newsvendor will under-order high-profit products and over-order low-profit products. Here $\tau = (p-c) \times \alpha(D)$ can be considered as a simple and intuitive heuristic for the newsvendors to set their targets. For example, a newsvendor can simply treat the random demand as a deterministic one with the value equal to its expectation. After taking 20% off as the cost of uncertainty, her target profit is set to be $\tau = 80\% \times (p-c)\mathbb{E}[D]$. Hence, for this newsvendor we have $\alpha(D) = 0.8\mathbb{E}[D]$. Likewise, the target can be $\tau = 0.6(p-c) \times m(D)$, where $m(D)$ is the mode of the demand distribution.

It is worth noting that high-profit and low-profit are benchmarked against the threshold value $\zeta$. From Theorem 3, a high value of $\alpha(D)$ leads to a larger $\zeta$, implying a wider range of products to be considered as low-value. This is probably because the newsvendor with higher $\alpha(D)$ has a higher target profit and is more ambitious. As such, she is more likely to consider a product as low-profit and over-order it. We highlight this conclusion in Corollary 2.

**Corollary 2** *A newsvendor with a higher target to achieve is more likely to overorder.*

Corollary 3 below further illustrates Theorem 3 by showing the specific form of $\zeta$ when the demand is uniformly distributed.

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**Corollary 3** *Assume the random demand $D$ is uniformly distributed in $[\underline{d}, \bar{d}] \subset \Re^+$, and $\tau = (p - c) \times \alpha(D)$, where $\alpha(D)$ is a positive value that depends on the knowledge of $D$ alone. Then there exists a threshold value*

$$\zeta_U = 2 \times \frac{\alpha(D) - \underline{d}}{\bar{d} - \underline{d}} \tag{8}$$

*such that if $\frac{p-c}{p} \geq \zeta_U$, we can find $y^* \leq y_N$ with $y^* \in \arg\max\limits_{y \geq 0} \rho_\tau(V(y))$; and if $\frac{p-c}{p} \leq \zeta_U$, we can find $y^* \geq y_N$ with $y^* \in \arg\max\limits_{y \geq 0} \rho_\tau(V(y))$.*

By (8) we can see that the threshold value $\zeta_U$ increases with $\alpha(D)$. To have a more concrete example, let $\underline{d} = 100$, $\bar{d} = 200$, $\alpha(D) = k\mathbb{E}[D] = 150k$ with $k$ being a constant that falls in the range of $(0, 1)$. Hence, the decision maker forms the target profit as $\tau = 150k(p - c)$. A start-up company may set a conservative target such that $k$ has a low value of 80%, i.e., $\tau = 80\%(p - c)\mathbb{E}[D]$. From (8) we know that the threshold value is $\zeta_U = 0.4$. That is, the company would consider a product to be a high-profit one and under-order it if and only if the product has $\frac{p-c}{p} > 0.4$. In contrast, if the company is well-established and has higher tolerance for risk, it may set a more ambitious target such that $k = 90\%$, i.e., $\tau = 90\%(p - c)\mathbb{E}[D]$. Similarly we can get the threshold value $\zeta_U = 0.7$, which means that a product is considered high-profit if and only if $\frac{p-c}{p} > 0.7$. Figure 2 provides a clear illustration on how the under-order and over-order regions change with the threshold values.
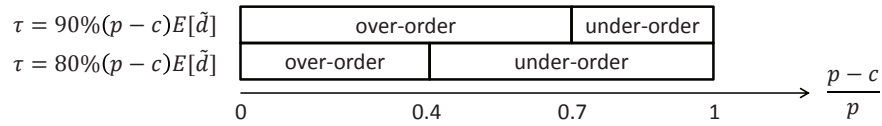


**Figure 2    Ordering behavior for high and low profit products.**

While the focus of our paper is not on how to set the target, our results in Proposition 2 below show that if the target is set in a particular way, the decision prescribed by our target-based framework is the same as the one that maximizes $CVaR_\eta$ for a given $\eta$. While abundant literature (e.g., Locke & Latham 1990, Rasch and Tosi 1992, and Barrick et al. 1993) promotes using goal achieving to incentivize employees' performance, an important issue is whether the employees' decisions, which are driven by the target, would also optimize the firm's objective (e.g., Shi et al. 2010). For example, if the firm aims to maximize the $CVaR_\eta$ of the random profit, then in the target-based framework, is there a way for the firm to use a target to drive the managers to a decision that maximizes $CVaR_\eta$? Proposition 2 sheds some light on this issue.

**Proposition 2** *With the target value $\tau = \max\limits_{y \geq 0} CVaR_\eta(V(y))$, we have $\arg\max\limits_{y \geq 0} \rho_\tau(V(y)) = \arg\max\limits_{y \geq 0} CVaR_\eta(V(y))$.*

According to Proposition 2, if the firm sets the target level to be the optimal $CVaR_\eta$ value that can be achieved, the manager's decision based on the CSM decision criterion will be exactly the same as the solution maximizing the $CVaR_\eta$ criterion.

## 3. Newsvendor with ESM

In this section we study the ordering decision of a target-oriented newsvendor who is concerned about the full scale risk of the random profit. For conciseness, we only present the satisficing measure modelling framework as in (6). The satisficing measure we consider here is the entropic satisficing measure (ESM), which is focused on certainty equivalent achieving the target. Different from CSM, which considers only the worst/best case expectation, ESM captures all possible realizations of the random profit and hence represents decision makers' preference over the full scale. By assuming an exponential utility function, we define the ESM as follows:

**Definition 2** *Given a target profit $\tau$, the entropic satisficing measure (ESM) of the random profit V is defined as:*

$$\rho_\tau^E(V) = \begin{cases} \sup\{\eta : C_\eta(V) \geq \tau\}, & if\ feasible, \\ -\infty & otherwise, \end{cases} \tag{9}$$

*where $C_\eta(V)$ is defined as:*

$$C_\eta(V) = \begin{cases} -\frac{1}{\eta}\ln\mathbb{E}\left[\exp(-\eta V)\right] & if\ \eta \neq 0, \\ \mathbb{E}\left[V\right] & if\ \eta = 0. \end{cases} \tag{10}$$

In Definition 2, $C_\eta$ is the certainty equivalent for the well-accepted exponential utility function, $u_\eta(x) = 1 - \exp(-\eta x)$, where $\eta$ represents the risk aversion level of a decision maker. As $\eta$ increases, the utility function becomes more concave (see the left panel of Figure 3). This implies that the decision maker is more risk averse. By Definition 2, ESM represents the highest degree of risk aversion such that the certainty equivalent is above the target. To illustrate, let us take the random payoff $V$ with CDF as in Figure 1 for example. We plot its certainty equivalent, $C_\eta(V)$, in the right panel of Figure 3 with varying $\eta$. Suppose we have a target $\tau = 0.2$, then $\rho_\tau^E(V) = 20$, which implies that for any decision maker who has an exponential utility with $\eta \leq 20$, $C_\eta(V) \geq \tau$, i.e., $V$ is more appealing than the certain value that is equal to the target profit.

The newsvendor problem under the ESM framework is to find an ordering quantity such that $\rho_\tau^E$ is maximized. From Definition 2 we can see that, a random profit with higher $\rho_\tau^E$ attracts a greater subset of individuals who are willing to prefer the random profit over the target profit with certainty. In other words, an optimal newsvendor decision under the ESM framework is one such that this decision is favorable even to decision makers with very low risk tolerance level. This decision criterion can be especially useful for group decision making where each group member may have a different level of risk tolerance.
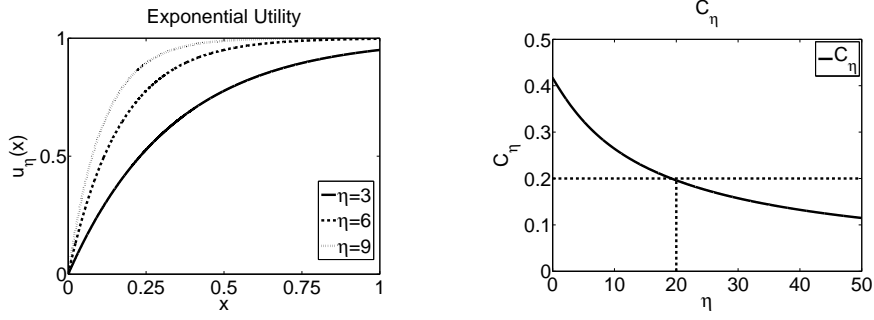
**Figure 3**     **Illustration for ESM**

Interestingly, we show that for newsvendors under ESM, all the results in Section 2 still hold. Theorem 4 summarizes the results. We especially highlight that if the target is set to be the maximum certainty equivalent (equivalently, expected utility) that can be achieved, the ordering decision prescribed by our ESM framework is the same as the one maximizing the expected utility.

**Theorem 4** *For newsvendors maximizing the entropic satisficing measure, the following holds:*

1. *Assume $\tau_1 \geq \tau_2$. Then there must exist $y_1 \geq y_2 \geq 0$ such that $y_i \in \arg\max\limits_{y \geq 0} \rho^E_{\tau_i}(V(y))$, $i \in \{1, 2\}$.*

2. *If $\tau$ is greater than (equal to, or less than) $\mathbb{E}[V(y_N)]$, then we can find $y^*$ greater than (equal to, or less than, respectively) $y_N$ such that $y^* \in \arg\max\limits_{y \geq 0} \rho^E_{\tau_i}(V(y))$.*

3. *If $\tau = \alpha \times (p - c)$, where $\alpha$ is a positive value depends on the knowledge of $D$ alone. Then $\exists \zeta \in [0, 1]$ such that if $\frac{p-c}{p} \geq \zeta$, there exists $y^* \leq y_N$ and $y^* \in \arg\max\limits_{y \geq 0} \rho^E_{\tau_i}(V(y))$; and if $\frac{p-c}{p} \leq \zeta$, there exists $y^* \geq y_N$ and $y^* \in \arg\max\limits_{y \geq 0} \rho^E_{\tau_i}(V(y))$.*

4. *With the target value $\tau = \max\limits_{y \geq 0} C_\eta(V(y))$, we have $\arg\max\limits_{y \geq 0} \rho^E_\tau(V(y)) = \arg\max\limits_{y \geq 0} C_\eta(V(y))$.*

Comparing the results in Theorem 4 and those in Section 2, we see that under CSM and ESM (which are two different ways to capture risks), the findings on the effect of target are the same. This suggests that our target-based newsvendor model is robust on how decision makers recognize risks.

## 4. Computational Analysis

In this section we conduct a numerical study to compare the ordering decisions using our target based approaches (maximizing CSM and ESM) with those from maximizing expected profit, maximizing attainment probability, and the model of mean-variance analysis which is formulated by Choi et al. (2008) as follows:

$$\begin{aligned}
\min \quad & \mathbb{E}\left[\left(V(y) - \mathbb{E}[V(y)]\right)^2\right] \\
\text{s.t.} \quad & \mathbb{E}[V(y)] \geq \tau \\
& y \geq 0.
\end{aligned} \tag{11}$$

As such, altogether we compare five models with different decision criteria. We let the demand follow a discrete uniform distribution over $\{1, 2, \ldots, 100\}$. Note that for a given demand, a newsvendor instance can be characterized by the selling price and the critical-fractile. To capture a wide range of scenarios, we generate 50 newsvendor instances. For each instance, the price is randomly sampled from the distribution $U[10, 20]$, and the critical fractile from $U[0.2, 0.8]$. For the models involving a target profit, we set the target $\tau = \varphi \max_{y \geq 0} \mathbb{E}[V(y)]$, where $\varphi$ measures how far the target is away from the maximum expected profit. We have $\varphi \in \{0.7, 0.8, 0.9, 1.1, 1.2, 1.3\}$. For each instance, we first find the optimal ordering quantity for each of the five models, and then compare the performances of the optimal decisions with respect to different performance measures. Note that computational complexity is not an issue for the newsvendor problem because it involves only a one dimensional search if a closed form solution is unavailable. We do not take the expected utility approach for comparison as it is unclear what utility function to use, and hence a fair comparison is hard to achieve. Even with the commonly used exponential utility, $u(x) = 1 - \exp(-\eta x)$, the solution is sensitive to the risk averse parameter $\eta$. For example, with the current demand distribution, if $p = 12$ and $c = 6$, the optimal ordering quantity will be 44 if $\eta = 0.001$, 20 if $\eta = 0.01$, 5 if $\eta = 0.1$, and 1 if $\eta = 1$.

Table 1 shows the average performance of all the 50 instances for each target level specified by $\varphi$. Note that when $\varphi > 1$, the mean-variance model in (11) is infeasible so we provide the solutions for this model only for $\varphi \leq 1$. Here Expected Loss (EL) is the expected value of the loss with respect to the target; Conditional Expected Loss (CEL) is the expected value of the loss conditioning on that there is a *strictly* positive loss. Value at Risk (VaR) is the threshold value that the newsvendor's loss does not exceed with a specified probability level. Note that for EL, CEL, and VaR, low values are desirable because they all measure losses.

We first observe that the optimal ordering decision for the attainment probability model is the most conservative one. Compared to the optimal solutions for other models, it gives the lowest expected profit, standard deviation, expected loss, conditional expected loss, and VaR. This is because the optimal solution of the attainment probability model, $\tau/(p - c)$ (Lau 1980), is one such that the profit can never exceed $\tau$. This is very conservative in nature. But for other models involving targets, we require the expected profit (mean variance model), conditional expected profit (CSM model), or certainty equivalent (ESM model) to be no less than the target. Therefore, the optimal decisions of these models must not be as conservative as that in the attainment probability model.

In the low target scenarios ($\varphi < 1$), the mean-variance model gives the second most conservative solution. The reason is that the optimal solution for this model is such that the expected profit is equal to the target (Choi et al. 2008). However, the two satisficing models have optimal solutions

| $\varphi$ | Approach | Criterion | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | Expected profit | Standard deviation | Attainment probability | EL | CEL | VaR @ 95% | VaR @ 99% |
| 0.7 | Expected profit | 223 | 246 | 67.42% | 81 | 241 | 258 | 318 |
| | Attainment probability | 132 | 65 | 81.78% | 27 | 137 | 47 | 106 |
| | Mean variance | 158 | 91 | 79.30% | 33 | 151 | 80 | 139 |
| | CSM | 203 | 159 | 74.10% | 51 | 190 | 158 | 217 |
| | ESM | 196 | 146 | 75.38% | 47 | 182 | 140 | 199 |
| 0.8 | Expected profit | 223 | 246 | 65.88% | 89 | 252 | 258 | 318 |
| | Attainment probability | 146 | 78 | 79.16% | 35 | 156 | 64 | 123 |
| | Mean variance | 180 | 120 | 75.54% | 47 | 179 | 113 | 173 |
| | CSM | 214 | 189 | 70.28% | 68 | 218 | 191 | 250 |
| | ESM | 211 | 180 | 71.08% | 64 | 211 | 179 | 238 |
| 0.9 | Expected profit | 223 | 246 | 64.28% | 98 | 263 | 258 | 318 |
| | Attainment probability | 159 | 92 | 76.40% | 45 | 176 | 81 | 141 |
| | Mean variance | 202 | 158 | 71.10% | 65 | 212 | 157 | 216 |
| | CSM | 221 | 218 | 66.58% | 86 | 247 | 225 | 284 |
| | ESM | 220 | 214 | 67.00% | 84 | 244 | 219 | 279 |
| 1.1 | Expected profit | 223 | 246 | 61.20% | 116 | 285 | 258 | 318 |
| | Attainment probability | 181 | 120 | 71.10% | 67 | 215 | 114 | 174 |
| | CSM | 223 | 254 | 60.40% | 120 | 291 | 270 | 329 |
| | ESM | 221 | 275 | 58.58% | 131 | 304 | 297 | 356 |
| 1.2 | Expected profit | 223 | 246 | 59.70% | 126 | 297 | 258 | 318 |
| | Attainment probability | 190 | 135 | 68.50% | 80 | 235 | 131 | 190 |
| | CSM | 222 | 262 | 58.22% | 134 | 308 | 280 | 340 |
| | ESM | 215 | 298 | 54.88% | 155 | 333 | 331 | 390 |
| 1.3 | Expected profit | 223 | 246 | 58.02% | 136 | 307 | 258 | 318 |
| | Attainment probability | 198 | 150 | 65.80% | 94 | 254 | 148 | 207 |
| | CSM | 221 | 270 | 55.98% | 149 | 324 | 290 | 350 |
| | ESM | 207 | 319 | 51.14% | 181 | 359 | 362 | 421 |

EL=Expected Loss; CEL=Conditional Expected Loss; VaR=Value at Risk.

**Table 1    Performance of Various Newsvendor Models**

with $\eta$ greater than zero. A positive $\eta$ under CSM suggests that $CVaR_\eta$ reaches the target, whereas under ESM this implies the certainty equivalent achieving the target. As such, the expected profit under both satisficing measures should be larger than the target.

Compared to the risk-neutral model that maximizes the expected profit, for CSM and ESM, when the target is smaller than $E[V(y_N)]$, the decrease in the target results in a reduction in the expected profit as well as the standard deviation and the loss-related performance measures (EL, CEL, and VaR). However, the decrease in the profit (maximum at 12%, 196 vs. 223) is much more mild than that in the standard deviation and other loss related measures (maximum at 46%, 228 vs. 118). On the other hand, when the target is larger than $E[V(y_N)]$, an increase in the target is associated with a decrease in the expected profit but an increase in the standard deviation and the loss-related measures. Similarly, the profit reduction is relatively mild (maximum at 7%, 207 vs. 223) while the loss-related measures increase quickly (maximum at 40%, 362 vs. 258). This suggests that compared to the risk-neutral model, CSM and ESM perform relatively well when the target is low.

Finally, as the value of $\varphi$ approaches to one (from both sides), we observe that the performance gap between different models is reduced. The reason is that as the target moves close to the maximum expected profit, all target based models (except the attainment probability model) provide solutions close to the risk neutral one. As such, the difference becomes smaller.

## 5. Conclusions

In this paper, we incorporate the concept of achieving a target profit in newsvendor decision making with risk. By adopting the concept of satisficing measure (Brown et al. 2012), we study two measures: 1) CVaR satisficing measure (CSM) which measures the highest confidence level $\eta$ which guarantees $CVaR_\eta$ achieving the target; and 2) entropic satisficing measure (ESM) which measures the smallest risk tolerance level under which the certainty equivalent for an exponential utility function achieves the target.

We provide an easy solution method to find the optimal quantity for the CSM framework. On the other hand, the optimal decision for ESM can be found numerically with very mild effort. For both the two satisficing measures we find that i) the optimal order quantity increases with the target; ii) the newsvendor orders more than the risk neutral solution sometimes and order less other times, depending on the target level; and iii) if the target is proportional to the unit marginal profit and is also affected by only one other demand related factor, then the newsvendor over-orders the low-profit products and under-orders the high-profit products. While these results are obtained based on continuously distributed demand, they can be easily extended to the non-continuous case by redefining the inverse function of the demand probability distribution. Our results are consistent with the behavioral observation made by multiple studies (Schweitzer and Cachon 2000, Benzion et al. 2008, Bolton and Katok 2008, and Bostian et al. 2008). Schweitzer and Cachon (2000) argue that "new techniques may be required to correctly optimize these systems". The consistency of our theoretical results and the existing behavioral observations suggest that our modelling framework may provide a potential direction in looking for new techniques. Nevertheless, we need to be careful about interpreting the connection between our results and the behavioral observations. After all, there is no evidence suggesting that the subjects in these experiments are in fact subconsciously setting a target to reach. What we do establish, however, is that if reaching a target is the decision maker's goal, then our paper prescribes the best decisions they can make as well as describes how the decisions should change with the targets.

Most inventory models up to date focus on the absolute performance such as the expected profit. With ample evidence (e.g., Brown and Tang 2006) suggesting that managers are more concerned about achieving a target, our target-based framework can appeal to a large group of target-attainment oriented decision makers and open a new direction for future research. We hope

that our work would motivate increasing research interest along this avenue. We believe that it is worthwhile extending this framework to other operations management models to investigate how a target affects decision making. It would also be interesting to study how managers form their targets in practice. Are there any heuristics that decision makers can follow or they simply use their intuition? Though past research has highlighted the role of target, little work has touched upon target formation. Although in this paper we proposed an intuitive heuristic for target formation, we expect that more research remains to be done in this direction. Finally, while our theoretical results are consistent with the existing behavioral observations in laboratory experiments, it is unclear whether the subjects in those experiments are subconsciously setting a target to achieve. Further investigation is needed along these lines.

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# Online Supplement to "The Impact of a Target on Newsvendor Decisions"

## Proof of Proposition 1

For the case of $\eta \in [0, -1)$, the result can be referred to Chen et al. (2009). Here we just discuss on the case of $\eta \in (-1, 0)$, where

$$
\begin{aligned}
CVaR_\eta\left(V(y)\right) &= -CVaR_{-\eta}\left(-V(y)\right) \\
&= -\max_{a\in\Re}\left\{a + \frac{1}{1+\eta}\mathbb{E}\left[\min\{-V(y) - a, 0\}\right]\right\} \\
&= \min_{a\in\Re}\left\{a + \frac{1}{1+\eta}\mathbb{E}\left[\left(p\min(y, D) - cy - a\right)^+\right]\right\} \\
&= \min_{a\in\Re} g(y, a),
\end{aligned}
$$

where $g(y, a)$ is defined as

$$
\begin{aligned}
g(y, a) &= a + \frac{1}{1+\eta}\mathbb{E}\left[\left(p\min(y, D) - cy - a\right)^+\right] \\
&= a + \frac{1}{1+\eta}\left(\int_0^y \left(pz - cy - a\right)^+ dF(z) + \left(py - cy - a\right)^+ (1 - F(y))\right).
\end{aligned}
$$

For any given $y \geq 0$, we discuss on the three different cases.

1. $a < -cy$. In this case, $g(y, a) = a + \frac{1}{1+\eta}\mathbb{E}\left[(V((y) - a)\right]$, $\frac{\partial g}{\partial a} = \frac{\eta}{1+\eta} < 0$.

2. $-cy \leq a \leq py - cy$. In this case, we have

$$
g(y, a) = a + \frac{1}{1+\eta}\left(\int_{\frac{cy+a}{p}}^y \left(pz - cy - a\right) dF(z) + \left(py - cy - a\right)\left(1 - F(y)\right)\right),
$$

$$
\frac{\partial g}{\partial a} = \frac{F\left(\frac{cy+a}{p}\right) + \eta}{1+\eta}.
$$

3. $a > py - cy$. In this case, $g(y, a) = a$, $\frac{\partial g}{\partial a} = 1 > 0$.

Therefore, let $a^*(y) = \arg\min_{a\in\Re} g(y, a)$, it should satisfy $-cy \leq a^*(y) \leq py - cy$. Hence, in the following discussion it suffices to only consider $a \in [-cy, py - cy]$, which implies $(cy + a)/p \in [0, y]$.

If $y < F^{-1}(-\eta)$, $\frac{\partial g}{\partial a} \leq \frac{F(y)+\eta}{1+\eta} < 0$, $a^*(y) = py - cy$, $CVaR_\eta(V(y)) = py - cy$, and

$$
\frac{\partial CVaR_\eta(V(y))}{\partial y} = p - c > 0.
$$

If $y \geq F^{-1}(-\eta)$, by FOD, $a^*(y) = pF^{-1}(-\eta) - cy$,

$$
CVaR_\eta(V(y)) = -cy + \frac{1}{1+\eta}\left(\int_{F^{-1}(-\eta)}^y pz dF(z) + py(1 - F(y))\right),
$$

$$
\frac{\partial CVaR_\eta(V((y))}{\partial y} = -c + \frac{p(1 - F(y))}{1+\eta}.
$$

By FOD, the maximizer of $CVaR_\eta(V((y))$ is $y^* = F^{-1}\left(1 - \frac{c}{p}(1+\eta)\right) = F^{-1}\left(\xi - \eta(1 - \xi)\right)$. Q.E.D.

2

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**Preliminary Lemmas 1 to 3**

We now introduce Lemmas 1 to 3, which are used for subsequent proof of theorems.

**Lemma 1** *Given a random profit $V$ and a target profit $\tau \in \Re$, $\rho_\tau(V) = 1$ if and only $\mathbb{P}(V \geq \tau) = 1$.*

**Proof.** If $\mathbb{P}(V \geq \tau) = 1$, $\forall \eta \in (0,1)$,

$$
\begin{aligned}
CVaR_\eta(V) &= \max_{a \in \Re} \left\{ a + \frac{1}{1-\eta} \mathbb{E}[\min\{V - a, 0\}] \right\} \\
&\geq \tau + \frac{1}{1-\eta} \mathbb{E}[\min\{V - \tau, 0\}] \\
&= \tau,
\end{aligned}
$$

where the second equality follows from $\mathbb{P}(V \geq \tau) = 1$. Therefore, $\rho_\tau(V) = 1$.

If $\mathbb{P}(V \geq \tau) \in [0, 1)$, denote $\delta = \mathbb{P}(V \leq \tau)$, then we have $\delta \in (0, 1]$, $1 - \frac{\delta}{2} \in [1/2, 1)$, and

$$
CVaR_{1-\frac{\delta}{2}}(V) = \max_{a \in \Re} \left( a + \frac{2}{\delta} \mathbb{E}\left[\min\{V - a, 0\}\right] \right) = \max_{a \in \Re} \{a + h(a)\},
$$

with $h(a) = \frac{2}{\delta} \mathbb{E}\left[\min\{V - a, 0\}\right] \leq 0$. For $a < \tau$, $a + h(a) \leq a < \tau$. For $a = \tau$, $a + h(a) = \tau + h(\tau) < \tau$, where the strict inequality follows from $\mathbb{P}(V \geq \tau) < 1$. For $a > \tau$,

$$
\begin{aligned}
a + h(a) &= a + \frac{2}{\delta} \left( \int_{-\infty}^{\tau} (z - a) d\mathbb{P}(V \leq z) + \int_{\tau}^{a} (z - a) d\mathbb{P}(V \leq z) \right) \\
&\leq a + \frac{2}{\delta} \left( (\tau - a) \cdot \delta + 0 \right) \\
&= 2\tau - a \\
&< \tau.
\end{aligned}
$$

Therefore, we must have $CVaR_{1-\frac{\delta}{2}}(V) = \max_{a \in \Re}\{a + h(a)\} < \tau$. It is obvious that $CVaR_\eta(V)$ is nonincreasing in $\eta$, hence, $\rho_\tau(V) = \sup\{\eta \in (-1, 1) : CVaR_\eta(V) \geq \tau\} < 1$.         Q.E.D.

**Lemma 2** *Given a random profit $V$ and a target profit $\tau \in \Re$, $\rho_\tau^E(V) = \infty$ if and only $\mathbb{P}(V \geq \tau) = 1$.*

**Proof.** If $\mathbb{P}(V \geq \tau) = 1$, we can easily check that $C_\eta(V) \geq \tau$ for all $\eta$. Therefore, $\rho_\tau^E(V) = \infty$.

If $\mathbb{P}(V \geq \tau) < 1$, there exists $\Delta < \tau$ such that $\mathbb{P}(V \leq \Delta) = \delta \in (0, 1]$. Denote the upper bound of $V$ as $\bar{v}$. Observe that

$$
\begin{aligned}
\lim_{\eta \to \infty} C_\eta(V) &= \lim_{\eta \to \infty} -\frac{1}{\eta} \ln \mathbb{E}\left[\exp(-\eta V)\right] \\
&\leq \lim_{\eta \to \infty} -\frac{1}{\eta} \ln \left( \delta \exp(-\eta \Delta) + (1 - \delta) \exp(-\eta \bar{v}) \right) \\
&= \lim_{\eta \to \infty} -\frac{1}{\eta} \ln \left( \exp(-\eta \Delta) \left( \delta + (1 - \delta) \exp(-\eta(\bar{v} - \Delta)) \right) \right)
\end{aligned}
$$

$$= \Delta + \lim_{\eta \to \infty} -\frac{1}{\eta} \ln \left( \delta + (1 - \delta) \exp(-\eta(\bar{v} - \Delta)) \right)$$

$$= \Delta + \lim_{\eta \to \infty} -\frac{1}{\eta} \ln(\delta)$$

$$= \Delta$$

$$< \tau,$$

where the second last equality follows from $\bar{v} > \tau > \Delta$. Therefore, $\rho_\tau(V) < \infty$.                        Q.E.D.

**Lemma 3** *Assume $u_1$, $u_2$, and $f$ are nondecreasing convex functions from $\Re$ to $\Re$, such that $u_1(w) = f(u_2(w))$, $\forall \, w \in \Re$. Then there exists $y_1 \ge y_2 \ge y_N$, such that $y_i \in \arg\max_{y \ge 0} \mathbb{E}\left[u_i(V(y))\right]$, $i \in \{1, 2\}$.*

**Proof**. We first show that $y_1 \ge y_2$. For $i \in \{1, 2\}$, denote $t_i(y) = \mathbb{E}\left[u_i\left(V(y)\right)\right]$. Then $y_i$ is a maximizer of $t_i(\cdot)$ on $\Re_+$, i.e., $t_i(y_i) \ge t_i(y)$ for all $y \ge 0$, $i \in \{1, 2\}$. To prove the existence of $y_1 \ge y_2$, it suffices to show $\forall y \le y_2$, $t_1(y) \le t_1(y_2)$. To this end, we observe

$$
\begin{aligned}
t_1'(y) &= \frac{\partial}{\partial y} \left( \int_0^y u_1(pd - cy) dF(d) + u_1(py - cy)(1 - F(y)) \right) \\
&= -c \int_0^y u_1'(pd - cy) dF(d) + (p - c)u_1'(py - cy)\left(1 - F(y)\right) \\
&= -c \int_0^y f'(u_2(pd - cy))u_2'(pd - cy) dF(d) + (p - c)f'(u_2(py - cy))u_2'(py - cy)\left(1 - F(y)\right) \\
&\ge s(y)t_2'(y),
\end{aligned}
$$

where $s(y) = f'(u_2(py - cy)) \ge 0$ is a nondecreasing function. Therefore, $\forall y \le y_2$,

$$t_1(y_2) - t_1(y) = \int_y^{y_2} t_1'(x) dx \ge \int_y^{y_2} s(x)t_2'(x) dx \ge \int_{m_1}^{y_2} s(x)t_2'(x) dx,$$

where

$$m_1 = \begin{cases} y & \text{if } t_2'(y) < 0 \\ \max\{r \le y_2 : \forall x \in [y, r], t_2'(x) \ge 0\} & \text{otherwise} \end{cases}$$

and the last inequality holds since either $m_1 = y$, or $\forall x \in [y, m_1]$ we have $t_2'(x) \ge 0$. As $y_2$ is a maximizer of $t_2(\cdot)$, we can let

$$n_i = \max\{r : \forall x \in [m_i, r], t_2'(x) \le 0\}, \quad i = 1, 2, \ldots, N - 1,$$

$$m_{i+1} = \max\{r \le y_2 : \forall x \in [n_i, r], t_2'(x) \ge 0\}, \quad i = 1, 2, \ldots, N - 1,$$

such that $y_2 = m_N$. Therefore,

$$t_1(y_2) - t_1(y) \ge \int_{m_1}^{y_2} s(x)t_2'(x) dx$$

$$= \sum_{i=1}^{N-1} \left( \int_{m_i}^{n_i} s(x)t_2'(x)dx + \int_{n_i}^{m_{i+1}} s(x)t_2'(x)dx \right)$$

$$\geq \sum_{i=1}^{N-1} \left( s(n_i) \int_{m_i}^{m_{i+1}} t_2'(x)dx \right)$$

$$= \sum_{i=1}^{N-3} \left( s(n_i) \int_{m_i}^{m_{i+1}} t_2'(x)dx \right) + s(n_{N-2}) \int_{m_{N-2}}^{m_{N-1}} t_2'(x)dx + s(n_{N-1}) \int_{m_{N-1}}^{m_N} t_2'(x)dx$$

$$\geq \sum_{i=1}^{N-3} \left( s(n_i) \int_{m_i}^{m_{i+1}} t_2'(x)dx \right) + s(n_{N-2}) \int_{m_{N-2}}^{m_N} t_2'(x)dx$$

$$\geq \sum_{i=1}^{N-4} \left( s(n_i) \int_{m_i}^{m_{i+1}} t_2'(x)dx \right) + s(n_{N-3}) \int_{m_{N-3}}^{m_N} t_2'(x)dx$$

$$\vdots$$

$$\geq s(n_1) \int_{m_1}^{m_N} t_2'(y)dy,$$

where the second inequality holds since $s(x)$ is positive nondecreasing and $t_2'(x)$ is non-positive when $x \in [m_i, n_i]$ and non-negative when $x \in [n_i, m_{i+1}]$; the following inequalities hold since $y_2$ is maximizer of $t_2(\cdot)$ and hence $\int_{m_i}^{m_N} t_2'(x)dx \geq 0$ for all $i = 1, \ldots, N-1$.

Therefore, we get $t_1(y_2) - t_1(y) \geq s(n_1)(t_2(y_2) - t_2(m_1)) \geq 0$ and $t_1(y_2) \geq t_1(y)$, $\forall y \in [0, y_2]$. Therefore, we know that there exists $y_1 \geq y_2$ such that $y_1$ is the maximizer of $t_1(\cdot)$ on $\Re_+$.

To show $y_2 \geq y_N$, let $u_3(w) = w$. Then $u_3$ is an increasing convex function and $u_2(w) = u_2(u_3(w))$ for any $w$. From the previous result we can know $y_2 \geq y_N$, since $y_3$ is the maximizer of $E[u_3(V(y))]$ on $\Re_+$.                                                                                  Q.E.D.

**Proof of Theorem 1**

For $i \in \{1, 2\}$, denote $\rho_i = \rho_{\tau_i}^* = \max_{y \geq 0} \rho_{\tau_i}(V(y))$. By the definition of CSM we can get $\rho_1 \leq \rho_2$ since $\tau_1 \geq \tau_2$.

Note that $\forall y \in [0, \underline{d}]$, $\mathbb{P}(V(y) \leq V(\underline{d})) = 1$; and $\forall y \in [\bar{d}, \infty)$, $\mathbb{P}(V(y) \leq V(\bar{d})) = 1$. Hence, there must exist $y \in [\underline{d}, \bar{d}]$ maximizing CSM. Here we just look at the existence of $y_i \in [\underline{d}, \bar{d}]$ to prove the result.

First, we consider the case that $-1 < \rho_1 \leq \rho_2 < 1$. Let

$$y_i = \arg\max CVaR_{\rho_i}(V(y)) = \begin{cases} F^{-1}(\xi - \rho_i \xi) & \text{if } \rho_i \in [0, 1), \\ F^{-1}(\xi - \rho_i(1 - \xi)) & \text{if } \rho_i \in (-1, 0). \end{cases} \tag{12}$$

By definition, we can easily check that $\rho_{\tau_i}(V(y_i)) = \rho_i$, and $y_i \in \arg\max_{y \geq 0} \rho_{\tau_i}(V(y))$. By (12), we have $y_1 \geq y_2$ since $\rho_1 \leq \rho_2$.

Secondly, consider the case that $\rho_1 = -1$, i.e., $\rho_{\tau_1}(V(y)) = -1$ for all $y$. We choose $y_1 = \bar{d}$. For any $y_2 \in [\underline{d}, \bar{d}]$ such that $\rho_{\tau_2}(V(y_2)) = \rho_2$, we have $y_1 \geq y_2$.

Finally, consider the case that $-1 < \rho_1 \le \rho_2 = 1$. Let $y^* \in [\underline{d}, \bar{d}]$ be an order quantity such that $\rho_{\tau_2}(V(y^*)) = 1$. By Lemma 1, $\mathbb{P}(V(y^*) \ge \tau_2) = 1$, which implies $-cy^* + p\underline{d} \ge \tau_2$. Hence, we have $-c\underline{d} + p\underline{d} \ge \tau_2$, and $\rho_{\tau_2}(V(\underline{d})) = 1$. Choose $y_2 = \underline{d}$. For any $y_1 \in [\underline{d}, \bar{d}]$ such that $\rho_{\tau_1}(V(y_1)) = \rho_1$, we have $y_1 \ge y_2$.                                                                 Q.E.D.

## Proof of Theorem 2

By the definition in equation (3), we can get $CVaR_0(V(y_N)) = \mathbb{E}[V(y_N)] = \tau$. Therefore, $\rho_\tau(V(y_N)) \ge 0$. Next we will prove $\rho_\tau(V(y)) \le 0 \; \forall y \ne y_N$. Since $D$ is continuously distributed, $\mathbb{E}[V(y)]$ is uniquely maximized at $y_N$. Hence, $\forall y \ne y_N$, we have $CVaR_0(V(y)) = \mathbb{E}[V(y)] < \mathbb{E}[V(y_N)] = \tau$. That implies $\rho_\tau(V(y)) \le 0 \le \rho_\tau(V(y_N))$.                                   Q.E.D.

## Proof of Corollary 1

It follows immediately from Theorems 1 and 2.                                                             Q.E.D.

## Proof of Theorem 3

Let $r(\xi) = \mathbb{E}[V(y_N)] - \tau = \mathbb{E}[V(F^{-1}(\xi))] - (p-c)\alpha(D)$. According to Corollary 1, $r(\xi) \le 0$ implies over-ordering, and $r(\xi) \ge 0$ implies under-ordering. If $\alpha(D) \le \underline{d}$, we get for all $\xi$,

$$r(\xi) = \mathbb{E}[V(y_N)] - (p-c) \times \alpha(D) \ge \mathbb{E}[V(\underline{d})] - (p-c)\underline{d} = 0.$$

So we just need to choose $\zeta = 0$.

Similarly, if $\alpha(D) \ge \bar{d}$, we get $r(\xi) \le 0$ for all $\xi$. So we just choose $\zeta = 1$.

Now we just consider $\alpha(D) \in (\underline{d}, \bar{d})$. Recall that $y_N = F^{-1}(\xi)$, so we have

$$
\begin{aligned}
r(\xi) &= p\left(\int_{\underline{d}}^{F^{-1}(\xi)} x \cdot dF(x) + \int_{F^{-1}(\xi)}^{\bar{d}} F^{-1}(\xi) dF(x)\right) - cF^{-1}(\xi) - (p-c)\alpha(D) \\
&= p\left(\int_{\underline{d}}^{F^{-1}(\xi)} x \cdot dF(x) - \xi\alpha(D)\right), \\
r'(\xi) &= p\left(F^{-1}(\xi) - \alpha(D)\right).
\end{aligned}
\tag{13}
$$

Therefore $r(0) = 0$, $r'(0) < 0$, and $r(\xi)$ is convex since $r'(\xi)$ is increasing.

If $\alpha \ge \mathbb{E}[D]$, $r(1) \le 0$, and $r(\xi) \le 0$ for all possible $\xi \in [0,1]$, so we can choose $\zeta = 1$.

If $\alpha(D) < \mathbb{E}[D]$, we have $r(1) > 0$, and there exists $\zeta \in (0,1)$ such that $r(\xi) \le 0$ for $\xi \le \zeta$, and $r(\xi) \ge 0$ for $\xi \ge \zeta$.

To prove that $\zeta$ increases with $\alpha(D)$, it suffices to show the monotonicity for $\alpha(D) \in (\underline{d}, \mathbb{E}[D])$, since we have already shown that $\zeta = 0$ for $\alpha(D) \le \underline{d}$, $\zeta \in (0,1)$ for $\alpha(D) \in (\underline{d}, \mathbb{E}[D])$, and $\zeta = 1$ for $\alpha(D) \ge \mathbb{E}[D]$. For any given $\alpha(D) \in (\underline{d}, \mathbb{E}[D])$, by the above analysis we know that $r(\xi) < (=,>) 0$ when $\xi < (=,>) \zeta$. Therefore, if we increase $\alpha(D)$, from (13) we know that $r(\xi) < 0$ for all $\xi < \zeta$, and hence we need to increase $\zeta$ to have $r(\zeta) = 0$.                                   Q.E.D.

**Proof of Corollary 3**

Following the assumption of uniform demand and the proof of Theorem 3, we have

$$
\begin{aligned}
\frac{r(\xi)}{p\xi} &= \frac{1}{\xi}\int_{\underline{d}}^{\underline{d}+\xi(\bar{d}-\underline{d})} \frac{x}{\bar{d}-\underline{d}}dx - \alpha(D) \\
&= \frac{1}{2\xi(\bar{d}-\underline{d})}\left((\underline{d}+\xi(\bar{d}-\underline{d}))^2 - \underline{d}^2\right) - \alpha(D) \\
&= \frac{2\underline{d}+\xi(\bar{d}-\underline{d})}{2} - \alpha(D).
\end{aligned}
$$

Since $p$, $\xi > 0$, we have $r(\xi) \geq 0$, or over-ordering, if and only if $\xi \geq 2 \times \frac{\alpha(D)-\underline{d}}{\bar{d}-\underline{d}} = \zeta_U$.          Q.E.D.

**Proof of Proposition 2**

It is straightforward from the definition of $\rho_\tau$.

**Proof of Theorem 4**

1) With the same argument as we made in the proof of Theorem 1, we only look at the existence of $y_i \in [\underline{d}, \bar{d}]$ to prove the result.

By definition, we can easily check that $\max_{y\geq 0}\rho_{\tau_1}^E(V(y)) \leq \max_{y\geq 0}\rho_{\tau_2}^E(V(y))$ since $\tau_1 \geq \tau_2$.

If $\max_{y\geq 0}\rho_{\tau_1}^E(V(y)) = -\infty$, we have $\rho_{\tau_1}^E(V(y)) = -\infty$ for all $y$. Choose $y_1 = \bar{d}$. For any $y_2 \in [\underline{d}, \bar{d}]$ such that $\rho_{\tau_2}(V(y_2)) = \max_{y\geq 0}\rho_{\tau_2}^E(V(y))$, we have $y_1 \geq y_2$.

If $\max_{y\geq 0}\rho_{\tau_2}^E(V(y)) = \infty$. Let $y^* \in [\underline{d}, \bar{d}]$ be an order quantity such that $\rho_{\tau_2}(V(y^*)) = \infty$. By Lemma 2, $\mathbb{P}(V(y^*) \geq \tau_2) = 1$, which implies $-cy^* + p\underline{d} \geq \tau_2$. Hence, we have $-c\underline{d} + p\underline{d} \geq \tau_2$, and $\rho_{\tau_2}(V(\underline{d})) = \infty$. Choose $y_2 = \underline{d}$. For any $y_1 \in [\underline{d}, \bar{d}]$ such that $\rho_{\tau_1}(V(y_1)) = \max_{y\geq 0}\rho_{\tau_1}^E(V(y))$, we have $y_1 \geq y_2$.

Now we consider the case of $-\infty < \max_{y\geq 0}\rho_{\tau_1}^E(V(y)) \leq \max_{y\geq 0}\rho_{\tau_2}^E(V(y)) < \infty$. Given $\tau$, let $\eta_\tau$ be the maximum value of the ESM, i.e., $\eta_\tau = \max_{y\geq 0}\rho_\tau^E(V(y))$. Further, for the case of $\eta_\tau \in (-\infty, \infty)$, let $y_\tau$ be the maximizer of $C_{\eta_\tau}(V(y))$, i.e., $y_\tau = \arg\max_{y\geq 0}C_{\eta_\tau}(V(y))$.

By definition of ESM, we know $y_\tau$ is also a maximizer of ESM. Observe that

$$
y_\tau = \arg\max_{y\geq 0}C_{\eta_\tau}(V(y)) = \begin{cases} \arg\max_{y\geq 0}\mathbb{E}\left[-\exp\left(-\eta_\tau V(y)\right)\right], & \text{if } \eta_\tau \in (0, \infty); \\ \arg\max_{y\geq 0}\mathbb{E}\left[V(y)\right], & \text{if } \eta_\tau = 0; \\ \arg\max_{y\geq 0}\mathbb{E}\left[\exp\left(-\eta_\tau V(y)\right)\right], & \text{if } \eta_\tau \in (-\infty, 0). \end{cases}
$$

Hence, we know $y_\tau$ is also a maximizer of the expectation of a utility function $u_\tau(\cdot)$, where $u_\tau(w) = -\text{sign}(\eta_\tau)\exp(-\eta_\tau w)$ if $\eta_\tau \neq 0$, and $u_\tau(w) = w$ if $\eta_\tau = 0$.

Since $\tau_1 \geq \tau_2$, we know $-\infty < \eta_{\tau_1} \leq \eta_{\tau_2} < \infty$. If $0 < \eta_{\tau_1} \leq \eta_{\tau_2}$, $u_{\tau_1}(w) = -\exp(-\eta_{\tau_1}w)$ and $u_{\tau_2}(w) = -\exp(-\eta_{\tau_2}w)$. Both are nondecreasing concave functions, and there exists nondecreasing concave function $f(\cdot)$ such that $u_2(w) = f(u_1(w))$ for all $w$. Therefore, $y_{\tau_2} \leq y_{\tau_1} \leq y_N$ (Eeckhoudt et al. 1995).

If $\eta_{\tau_1} \leq \eta_{\tau_2} < 0$, $u_{\tau_1}(w) = \exp(-\eta_{\tau_1} w)$ and $u_{\tau_2}(w) = \exp(-\eta_{\tau_2} w)$. Both are nondecreasing convex functions, and there exists nondecreasing convex function $f(\cdot)$ such that $u_1(w) = f(u_2(w))$ for all $w$. Therefore, by Lemma 3, $y_{\tau_1} \geq y_{\tau_2} \geq y_N$.

If $\eta_{\tau_1} \leq 0 \leq \eta_{\tau_2}$, then $u_1(\cdot)$ is nondecreasing convex function while $u_2(\cdot)$ is nondecreasing concave function. So we get $y_{\tau_1} \geq y_N \geq y_{\tau_2}$.

2) While $\tau = \max_{y \geq 0} \mathbb{E}[V(y)]$, we know $\eta_\tau = 0$ and $y_\tau = y_N$. Others can be derived from part 1).

3) The proof is similar to that for Theorem 3.

4) It is straightforward from the definition of $\rho_\tau^E$.                                   Q.E.D.

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