Global population growth, technology, and Malthusian constraints: A quantitative growth theoretic perspective

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Bruno Lanz,* Simon Dietz† and Tim Swanson‡

Abstract

We structurally estimate a two-sector Schumpeterian growth model with endogenous population and finite land reserves to study the long run evolution of global population, technological progress and the demand for food. The estimated model closely replicates trajectories for world population, GDP, sectoral productivity growth and crop land area from 1960 to 2010. Projections from 2010 onwards show a slowdown of technological progress, and because it is a key determinant of fertility costs, significant population growth. By 2100 global population reaches 12 billion and agricultural production doubles, but the land constraint does not bind because of capital investment and technological progress.

Contents

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1. INTRODUCTION

World population has doubled over the last fifty years and quadrupled over the past century (United Nations, 1999). During this period and in most parts of the world, productivity gains in agriculture have confounded Malthusian predictions that population growth would outstrip food supply. Population and income have determined the demand for food and thus agricultural production, rather than food availability determining population. However, recent evidence suggests a widespread slowdown of growth in agricultural output per unit of land area (i.e. agricultural yields, see Alston *et al.*, 2009; Alston and Pardey, 2014), and the amount of land that can be brought into the agricultural system is physically finite. For reasons such as these, several prominent contributions from the natural sciences have recently raised the concern that a much larger world population cannot be fed (e.g. Godfray *et al.*, 2010; Tilman *et al.*, 2011). Our aim in this paper is to study how population and the demand for land interacted with technological progress over the past fifty years, and derive some quantitative implications for the years to come.

Despite the importance of understanding global population change and how fertility trends interact with per-capita income, food availability and the pace of technological progress, few economists have contributed to the debate about *future* population growth. This is especially surprising given the success of economic theories in explaining the demographic transition in developed countries, and in particular the role of technological progress (e.g. Galor and Weil, 2000; Jones, 2001; Bar and Leukhina, 2010; Jones and Schoonbroodt, 2010, and other contributions reviewed below). Instead, the *de facto* standard source of demographic projections is the United Nations' series of *World Population Prospects*, updated every two years. The latest edition (United Nations, 2013) projects a global population, on a medium scenario, of 9.6 billion in 2050 and 10.9 billion in 2100, by which time the population growth rate is close to zero. The crucial assumption of the medium scenario, displayed in **Figure 1**, is that all countries around the world converge towards a replacement fertility rate of 2.1 over the next century, irrespective of their starting point.[1]

The UN projections are highly sensitive to the assumed trajectory for fertility and small variations in the fertility trajectories for countries in Asia and Africa in particular account for most of the variance in population projections.[2] These are precisely the regions for which uncertainty about the evolution of fertility is large, and empirical evidence in developing countries suggests no clear pattern of convergence towards a low fertility regime (Strulik and Vollmer, 2015). In explaining long-run fertility development, economic research emphasises the role of technology

[1] The UN uses a so-called 'cohort-component projection method', i.e. it works from the basic demographic identity that the number of people in a country at a particular moment in time is equal to the number of people at the last moment in time, plus the number of births, minus the number of deaths, plus net migration, all of this done for different age groups. This requires assumptions about fertility, mortality and international migration rates.

[2] Using the UN's cohort-component method, imposing the 'high' fertility scenario in these regions alone, so that they converge to a fertility rate of 2.6 rather than 2.1, implies a global population of around 16 billion by 2100. Interestingly, over the past ten years the bi-annual UN projections have been revised systematically upwards, with the 2008 projections of a steady state at around 9 billion still used in many policy discussions.

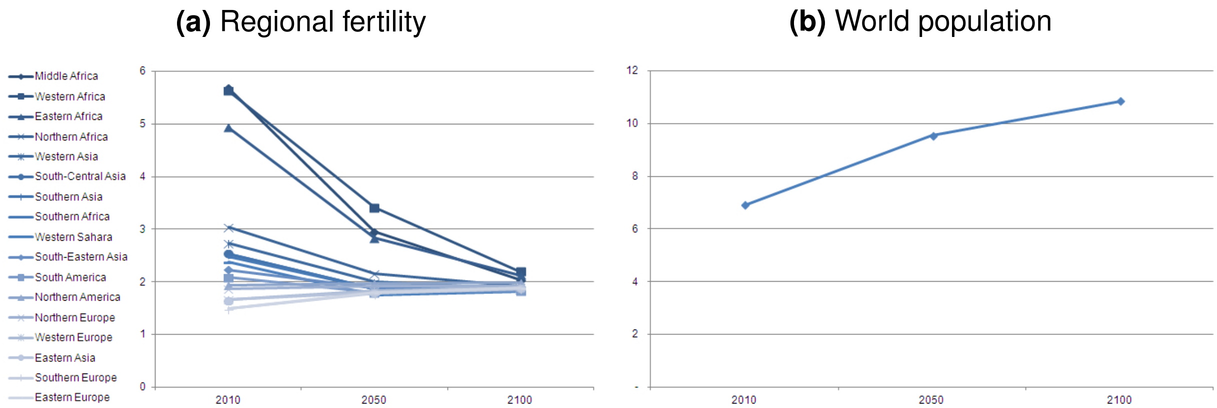

Figure 1. United Nations population projections 2010 – 2100 (United Nations, 2013)

and per-capita income (Herzer *et al.*, 2012) and households' demand for education (Rosenzweig, 1990), inducing a well-documented complementarity between human capital and the level of technology (Goldin and Katz, 1998). Per-capita income is also a determinant of the demand for food (e.g. Subramanian and Deaton, 1996), while technological progress in agriculture is a key contributor to growth in food production and associated demand for land, which may ultimately constrain human development.

In this paper we propose an integrated, quantitative approach to study the interactions between global population, technological progress, per-capita income, demand for food and agricultural land expansion. More specifically, we formulate a model of endogenous growth with an explicit behavioural representation linking child-rearing decisions to technology, per-capita income and availability of food, making the path for fertility an outcome rather than an assumption. In the tradition of Barro and Becker (1989) households in the model have preferences over own consumption, the number of children they have and the utility of their children. Child-rearing is time intensive, and fertility competes with other labour-market activities. In order to capture the complementarity between human capital and the level of technology (Goldin and Katz, 1998), the cost of fertility increases with technological progress. Thus technological progress implies a higher human capital requirement, so that population increments need more education and are thus more costly. As in Galor and Weil (2000), the opportunity cost of fertility increases over time, implying a quality-quantity trade-off and a gradual transition to low fertility regime.

Besides the cost of rearing and educating children, the other key constraint to population growth in our model is food availability. We make agricultural output a necessary condition to sustain population, and assume that food production requirements increase with both the size of the population and per-capita income, the latter capturing changes in diet as affluence rises. An agricultural sector, which meets the demand for food, requires land as an input, and agricultural land has to be converted from a stock of natural land. Therefore, as population and income grow, the demand for food increases, raising the demand for agricultural land. In the model land is an treated as a scarce form of capital, which has to be converted from a finite resource stock of natural land. The cost of land conversion and the fact that it is physically finite generate a potential Malthusian constraint to long run population development.

In our model technology plays a central role for both fertility and land conversion decisions. On the one hand, technological progress raises the opportunity and human capital cost of children. On the other hand, whether land conversion acts as a constraint to population growth mainly depends on technological progress. We model the process of knowledge accumulation in the Schumpeterian framework of Aghion and Howitt (1992), where the growth rate of total factor productivity (TFP) increases with labour hired for R&D activities. A well known drawback of such a representation of technological progress is the population scale effect (see Jones, 1995a).[3] This is important in a setting with endogenous population, as it would imply that accumulating population would increase long-run technology and income growth. Following Chu *et al.* (2013), we 'neutralise' the scale effect by making the growth rate of TFP a function of the *share* of labour allocated to R&D. This implies that long run growth can occur without the need for the population to grow.[4]

To fix ideas, we start with a simple illustration of the theoretical mechanism underlying fertility and land conversion decisions in our model. However, the main contribution of our work is to structurally estimate the model and use it to study the quantitative behaviour of the system. More specifically, most of the parameters of the model are either imposed or calibrated from external sources, but those determining the marginal cost of population, labour productivity in R&D and labour productivity in agricultural land conversion are structurally estimated with simulation methods. We use 1960-2010 data on world population, GDP, sectoral TFP growth and crop land area to define a minimum distance estimator, which compares observed trajectories with those simulated by the model. We show that trajectories simulated with the estimated vector of parameters closely replicate observed data for 1960 to 2010, and that the estimated model also provides a good account of non-targeted moments over the estimation period, notably agricultural output and its share of total output. We then employ the estimated model to jointly project outcomes up to 2100.

The key results are as follows. Trajectories from the estimated model suggest a population of 9.85 billion by 2050, further growing to 12 billion by 2100. These numbers are slightly above the

[3] The population scale effect, or positive relationship between population and productivity growth, can be used to explain the take-off phase that followed stagnation in the pre-industrial era (e.g. Boserup, 1965; Kremer, 1993). However, empirical evidence from growth in recent history is difficult to reconcile with the scale effect (e.g. Jones, 1995b; Laincz and Peretto, 2006). See Strulik *et al.* (2013) on how the transition between the two growth regimes can be explained endogenously through accumulation of human capital.

[4] As we further discuss below, Chu *et al.* (2013) show that the qualitative behaviour of our Schumpeterian representation of R&D is in line with more recent representations of technological progress, put forward by Dinopoulos and Thompson (1998), Peretto (1998), and Young (1998) among others and thus provides a good basis to study growth in contemporary history.

UN's current central projection (United Nations, 2013), but probabilistic projections using the UN's 2012 revision suggest that there is a 95 percent chance that in 2100 the population will lie between 9 and 13 billion (Gerland *et al.*, 2014). Although population *growth* declines over time, population does not reach a steady state over the period we consider. Indeed the pace of technological progress, which, given our assumptions, is the main driver of the demographic transition, declines over time, so that population growth remains positive over the horizon we consider. Despite a doubling of agricultural output associated with growth in population and per capita income, however, agricultural land expansion stops by 2050 at around 1.8 billion hectares, a 10 percent increase on 2010, which is roughly similar to Alexandratos and Bruinsma (2012).[5] A direct implication of our work is that the land constraint does not bind, even though (i) our population projections are higher than conventional wisdom; and (ii) our projections are rather conservative in terms of technological progress (agricultural TFP growth in both sectors is below one percent per year and declining from 2010 onwards).

One important feature of these dynamics is that they derive entirely from the structure of the model, rather than changes in the underlying parameters. We also consider the sensitivity and robustness of our results to a number of assumptions, notably the discount factor and substitution possibilities in agriculture. Overall we find that projections from the model are fairly robust to plausible changes in the structure of the model. Some variations suggest an optimal population path that is higher than our baseline case, although the evolution of agricultural land is only marginally affected. The robustness of our results essentially derives from estimating the model with 50 years of data, tying down trajectories over a long time horizon.

1.1 Related Literature

Our work relates to at least three strands of economic research. First, there is unified growth theory, which studies economic development and population over the long run. Seminal contributions include Galor and Weil (2000) and Jones (2001) (see Galor, 2005, for a survey). Jones (2003) and Strulik (2005) analyse the joint development of population, technological progress and human capital (see also Tournemaine and Luangaram, 2012, for a recent investigation and comprehensive overview of the literature), while Hansen and Prescott (2002) and Strulik and Weisdorf (2008) consider the role of agriculture and manufacturing activities along the development path. The structure of our model, linking technology and economic growth with child rearing and education decisions, and the implied quality-quantity trade-off, is closely related to these papers.

In unified growth theory models, the initial phase of economic development relies on the scale effect to generate a take-off. A key departure from these papers is that we focus on post-1960

[5] This corresponds to the conversion of a further 150 million hectares of natural land into agriculture, roughly the area of Mongolia or three times that of Spain. Because developed countries will likely experience a decline in agricultural land area (Alexandratos and Bruinsma, 2012), land conversion in developing countries will need to be more than that.

growth and rule out the existence of a scale effect. Our work thus also relates to recent growth theories that circumvent the scale effect with 'product line' representations of R&D (see Dinopoulos and Thompson, 1998; Peretto, 1998; Young, 1998, for seminal contributions).[6] These models have been used to develop theories of endogenous population and resource constraints, most notably Peretto and Valente (2011) and Bretschger (2013), and these theoretical contributions are thus close in spirit to our work. Our treatment of land as a scarce form of capital is, however, novel, and by taking our model to the data we are also able to draw quantitative implications about the importance of resource constraints for global development.

A final set of papers has in common with us the use of a quantitative macroeconomic models to study particular aspects of unified growth theory, especially economic development and the demographic transition. These include Mateos-Planas (2002), Doepke (2005), Strulik and Weisdorf (2008; 2014), Bar and Leukhina (2010), Jones and Schoonbroodt (2010), and Ashraf *et al.* (2013). These papers demonstrate that macroeconomic growth models are able to capture essential features of the demographic transition in countries where such a transition has already taken place. Our contribution is to show that models like these can not only closely replicate recent history, they can also be used to model the joint determination of population, technology and land use in the future, and thus to evaluate the potential role of Malthusian constraints.

As mentioned above, our approach also complements existing population and agricultural projections, most notably those by the United Nations (2013). A key source of global agricultural forecasts is the work of the Food and Agriculture Organisation (Alexandratos and Bruinsma, 2012), which is based on detailed modelling of crop yields and land. These projections are based on highly disaggregated, detailed approaches, but require exogenous assumptions about key drivers such as per-capita income and fertility. Moreover, these methodologies are carried out in isolation from each other, yet mutually rely on one another. By contrast, our approach lacks disaggregation and detail, but provides a novel, integrative perspective.

The remainder of the paper proceeds with a simple analytical model capturing the key features of our analysis (Section 2). The structure of our quantitative model and estimation strategy are presented in Section 3. Section 4 reports the results of the quantitative analysis, namely the estimation results, projections, and sensitivity analysis. We discuss some broader implication of our results in Section 5 and provide some concluding comments in Section 6.

2. SIMPLE ANALYTICS OF HOUSEHOLD FERTILITY, TECHNOLOGY AND LAND

In order to provide some intuition for the mechanisms driving the demographic transition and land conversion decisions in our quantitative model, this section studies the problem of a repre-

[6] In a product-line representation of technological progress, the number of products grows over time, thereby diluting R&D inputs, so that long-run growth doesn't necessarily rely on the population growth rate, but rather on the share of labour in the R&D sector. Another strategy to address the scale effect involves postulating a negative relationship between labour productivity in R&D and the existing level of technology, giving rise to "semi-endogenous" growth models (Jones, 1995a). In this setup, however, long-run growth is driven by population growth, which is also at odds with empirical evidence (Ha and Howitt, 2007).

sentative household in a simplified set-up. In particular, we treat technological progress in both sectors as exogenous and also omit capital. Population and land are the remaining state variables. As we will show, this distills the problem into one of allocating labour between several competing uses. Even with all this simplification, we still have a problem that is too complex to yield analytical solutions for the whole development path, but we can nonetheless obtain useful results relating to optimal fertility (and agricultural land expansion) between any two successive time-periods.

We consider a representative agent that lives for only one period and has preferences over its own consumption of a homogeneous, aggregate manufactured good c_t, the number of children it produces n_t, and the utility that each of its children experiences in the future $U_{i,t+1}$. We use the class of preferences suggested by Barro and Becker (1989) defined recursively as:

$$U_t = u(c_t) + \beta b(n_t) \sum_{i=1}^{n_t} U_{i,t+1} \tag{1}$$

where $u(\cdot)$ is the per-period utility function and we assume that $\partial u(\cdot)/\partial c_t > 0$, $\partial^2 u(\cdot)/\partial c_t^2 < 0$, and that $u(\cdot)$ also satisfies the Inada conditions such that $\lim_{c\to 0} \partial u(\cdot)/\partial c_t = \infty$ and $\lim_{c\to\infty} \partial u(\cdot)/\partial c_t = 0$. The function $b(\cdot)$ specifies preferences for fertility and is assumed to be isoelastic, an assumption made in the original Barro and Becker (1989) paper and that we will maintain throughout. $\beta \in (0, 1)$ is the discount factor.

We further assume that children are identical, so that $\sum_{i=1}^{n_t} U_{i,t+1} = n_t U_{t+1}$, and write the motion equation for population as $N_{t+1} = n_t N_t$.[7] Given these assumptions, the recursive nature of Barro-Becker preferences allows us to the define the utility function of the dynastic household head as:

$$U_0 = \sum_{t=0}^{\infty} \beta^t u(c_t) b(N_t) N_t \tag{2}$$

The steps involved are described in APPENDIX A. Consistent with our quantitative analysis in which $U_t > 0$, a preference for fertility that is subject to diminishing returns, and in turn overall concavity of (2), requires that $\partial N b(N)/\partial N > 0$ and $\partial^2 N b(N)/\partial N^2 > 0$ (see Jones and Schoonbroodt, 2010). This also implies that fertility and the utility of children are complements in parents' utility (which is easiest to see in the context of (1), where our combination of assumptions yield $\partial^2 U_t/\partial n_t \partial U_{t+1} > 0$). We further assume that $\lim_{N\to 0} \partial b(\cdot)/\partial N = \infty$ and $\lim_{N\to\infty} \partial b(\cdot)/\partial N = 0$.

Each agent is endowed with one unit of time in each period, which can be spent rearing and educating children, or working on a competitive market for manufacturing labour at wage w_t.

[7] As discussed in APPENDIX A introducing mortality in this context requires the further assumption that parents' welfare in $t + 1$ and that of their children is identical (Jones and Schoonbroodt, 2010).

The household's budget constraint is then $c_t N_t = w_t L_{t,mn}$, where $L_{t,mn}$ is the absolute amount of time spent working in the manufacturing sector. Bringing up children hence competes with labour-market activities as it does in the standard model of household fertility choice (Becker, 1960; Barro and Becker, 1989). In addition, we characterise a complementarity between technology and skills (Goldin and Katz, 1998) by postulating an increasing relationship between the time-cost of rearing and educating children and the level of technology in the economy (specifically in manufacturing), where the latter is denoted $A_{t,mn}$.[8] Technological progress increases the returns to education, which increases the time needed to produce effective labour units. Based on these assumptions, we write the population increments as a function of both the labour time devoted by all agents to child-rearing and education $L_{t,N}$ and the level of technology in the economy:

$$n_t N_t = \chi(L_{t,N}, A_{t,mn}) \,,$$

with $\partial\chi(L_{t,N}, A_{t,mn})/\partial L_{t,N} > 0$, $\partial^2\chi(L_{t,N}, A_{t,mn})/\partial L_{t,N}^2 < 0$, $\partial\chi(L_{t,N}, A_{t,mn})/ \partial A_{t,mn} < 0$, $\partial^2\chi(L_{t,N}, A_{t,mn})\partial A_{t,mn}^2 > 0$ and $\partial^2\chi(L_{t,N}, A_{t,mn})\partial L_{t,N}\partial A_{t,mn} < 0$.

In our model there is an additional constraint bearing upon the household, which is that sufficient food must be available for it to eat at all times. The aggregate food requirement is the product of total population N_t and per-capita food requirements $\overline{f_t}$:[9]

$$\overline{f_t} N_t = A_{t,ag} Y_{ag}(L_{t,ag}, X_t) \tag{3}$$

In this simplified model, food is directly produced by the household by combining 'agricultural' labour $L_{t,ag}$ and land X_t with production function $Y_{ag}(\cdot)$, given agricultural TFP $A_{t,ag}$. We assume strictly positive and diminishing returns to labour and land, and that the Inada conditions also hold on both.

There is a finite supply of land $\overline{X}$ that is in full, private ownership of household at all times. Land can be converted into agricultural land with the use of the household's labour $L_{t,X}$. The state equation for land is then

$$X_{t+1} = \psi(L_{t,X}), \quad X_t \leq \overline{X}$$

where $\partial\psi(\cdot)/\partial L_X > 0$, $\partial^2\psi(\cdot)/\partial L_X^2 < 0$ and the Inada conditions again hold.[10] Land that is

[8] In our quantitative model, the cost of children is proportional to an output-weighted average of TFP in manufacturing and agriculture, although the consequent weight on the former is much larger.

[9] An important simplification that will remain throughout is that food consumption does not enter the utility function of household, but is rather a complement to the consumption of other goods c_t. We return to this assumption below.

[10] In this formulation agricultural land is "recolonised" by nature every period, i.e. the depreciation rate is 100 percent. This is obviously a simplification and we introduce a more realistic depreciation pattern in our quantitative analysis.

prepared for agricultural use thus acts as a productive stock of capital that is physically finite.

Combining the budget constraint with the food constraint (3) and the overall constraint on the household's allocation of labour $N_t = L_{t,mn} + L_{t,N} + L_{t,X} + L_{t,ag}$, the dynastic head's optimisation problem can be written as:

$$\begin{aligned} \max_{\{L_{t,j}\}} \quad & \sum_{t=0}^{\infty} \beta^t u(c_t) b(N_t) N_t \\ \text{s.t.} \quad & N_{t+1} = \chi(L_{t,N}, A_{t,mn}) \, ; \quad X_{t+1} = \psi(L_{t,X}) \, ; \quad X_t \leq \overline{X} \\ & c_t N_t = w_t L_{t,mn} \, ; \quad N_t = L_{t,mn} + L_{t,N} + L_{t,X} + L_{t,ag} \, ; \quad \overline{f_t} N_t = A_{t,ag} Y_{ag}(L_{t,ag}, X_t) \\ & N_0, \, X_0 \quad \text{given} \end{aligned}$$

Necessary and sufficient conditions for a maximum allow us to obtain the following useful result:

Lemma 1. *At the optimum, fertility and hence population growth are chosen to equate the marginal costs and benefits of increasing the population in the next period, specifically*

$$\begin{aligned} & \underbrace{u'(c_t) b(N_t) w_t \Big/ \frac{\partial \chi(L_{t,N}, A_{t,mn})}{\partial L_{t,N}}}_{A} + \underbrace{\beta u'(c_{t+1}) b(N_{t+1}) w_{t+1} \overline{f_{t+1}} \Big/ \left[A_{t+1,ag} \frac{\partial Y_{ag}(L_{t+1,ag}; X_{t+1})}{\partial L_{t+1,ag}} \right]}_{B} \\ & \quad = \underbrace{\beta u(c_{t+1}) \left[b'(N_{t+1}) N_{t+1} + b(N_{t+1}) \right]}_{C} + \underbrace{\beta u'(c_{t+1}) b(N_{t+1}) w_{t+1}}_{D} \end{aligned} \tag{4}$$

Proof. See APPENDIX A. □

As Lemma 1 shows, the marginal costs of increasing the population in the next period are twofold. First, there is the opportunity cost of present consumption foregone (A), as time is spent rearing and educating children rather than working in the manufacturing sector. Second, there is the discounted opportunity cost of consumption foregone in the next period by having to provide additional food to sustain the extra mouths (B). On the other hand, the marginal benefits of increasing the population in the next period are also twofold: the discounted marginal utility of fertility (C), plus the discounted marginal utility of additional consumption, made possible by expanding the pool of labour that can supply work to manufacturing (D).

We can use this result to explore the effect of technological progress on population growth. This requires explicit characterisation of the manufacturing sector. Identical, competitive manufacturing firms employ household labour and combine it with the exogenously given technology $A_{t,mn}$ to produce the composite good that households consume. Production of the representative firm is hence:

$$Y_{t,mn} = A_{t,mn} \cdot Y_{mn}(L_{t,mn})$$

where $Y'_{mn} > 0$, $Y''_{mn} < 0$ and the Inada conditions hold.

Let the evolution of TFP in the manufacturing sector be described by $A_{t+1,mn} = (1+g_{t,mn})A_{t,mn}$. Then the following proposition describes the resulting condition for an increase in the level of TFP in period t to reduce fertility.

Proposition 1. *An increase in the level of manufacturing TFP in period t results in a reduction in fertility and population growth if and only if*

$$\underbrace{u'(c_t)b(N_t)Y'_{mn}\left[\partial\chi(L_{t,N},A_{t,mn})/\partial L_{t,N} - A_{t,mn}\,\partial^2\chi(L_{t,N},A_{t,mn})/\partial L_{t,N}\partial A_{t,mn}\right]}_{A'}+$$
$$\underbrace{\left[\beta u'(c_{t+1})b(N_{t+1})Y'_{mn}(L_{t+1,mn})(1+g_{t,mn})\overline{f_{t+1}}\right]\Big/\left[A_{t+1,ag}\frac{\partial Y_{ag}(L_{t+1,ag};X_{t+1})}{\partial L_{t+1,ag}}\right]}_{B'} >$$
$$\underbrace{\beta u'(c_{t+1})b(N_{t+1})Y'_{mn}(L_{t+1,mn})(1+g_{t,mn})}_{D'}$$

Proof. See APPENDIX A. □

An increase in $A_{t,mn}$ increases the opportunity cost of present consumption foregone (A'), because an effective labour unit is more time-consuming to rear and educate, while it also increases the discounted opportunity cost of providing additional food in the next period (B'). On the other hand, an increase in $A_{t,mn}$ increases consumption in the next period (D'). In general, whether an increase in the level of manufacturing TFP reduces fertility thus depends on the positive effect on the marginal costs of fertility (A' + B') being larger than the positive effect on the marginal benefits of fertility (D').

Proposition 1 gives us a feel for the incentives at work at the household level in driving a decline in population growth linked to technological progress. Equally, however, it can be used to understand what happens if the rate of technological progress itself slows down. In this case, the opportunity costs of fertility at the margin will fall relative to a counterfactual with higher technological progress, as will the marginal benefits. Provided the former effect exceeds the latter, fertility will hold up and population growth will not slow down as much.

From Proposition 1 we can also extract a sufficient condition for population growth to slow in the face of technological progress that is linked to the food requirement.

Corollary 1. *A sufficient condition for an increase in the level of manufacturing TFP in period t to reduce fertility and population growth is that it is not too cheap to meet food requirements, specifically:*

$$\frac{\overline{f_{t+1}}}{A_{t+1,ag}\frac{\partial Y_{t+1,ag}(L_{t+1,ag};X_{t+1})}{\partial L_{t+1,ag}}} > 1$$

Proof. Given our assumptions,

$$u'(c_t)b(N_t)Y'_{mn}\left[\partial\chi(L_{t,N},A_{t,mn})/\partial L_{t,N} - A_{t,mn}\,\partial^2\chi(L_{t,N},A_{t,mn})/\partial L_{t,N}\partial A_{t,mn}\right] > 0$$

The Corollary follows immediately. □

Corollary 1 is more likely to hold the larger is the per-capita food requirement $\overline{f_{t+1}}$ and the less productive is agricultural labour, $A_{t+1,ag}\,\partial Y_{t+1,ag}(L_{t+1,ag};X_{t+1})/\partial L_{t+1,ag}$. This points to the link between population growth and technological progress in agriculture. The following proposition establishes that an increase in the level of agricultural TFP unambiguously increases fertility in this model, by relaxing the food constraint and therefore one of the marginal costs of fertility.

Proposition 2. *An increase in the level of agricultural TFP results in an increase in fertility and population growth.*

Proof. Let the evolution of TFP in the agricultural sector be described by $A_{t+1,ag} = (1+g_{t,ag})A_{t,ag}$. The partial derivative of (4) with respect to $A_{t,ag}$ is

$$-\beta u'(c_{t+1})b(N_{t+1})Y'_{mn}(L_{t+1,mn})(1+g_{t,A,mn})\overline{f_{t+1}}\Bigg/\left[(1+g_{t,ag})\left(A_{t,ag}\right)^2\frac{\partial Y_{ag}(L_{t+1,ag};X_{t+1})}{\partial L_{t+1,ag}}\right] < 0$$

□

Over the period 1960-2005, agricultural productivity as measured by output per unit area – agricultural yield – increased by a factor of 2.4, although the growth rate declined from 2.03% per year from 1960 to 1990 to 1.82% per year from 1990 to 2005 (Alston *et al.*, 2009). Hence we can view Proposition 2 from the opposite angle as supplying intuition for how a sustained slowdown in the pace of technological improvements in agriculture might start to put a brake on population growth.

The constraint on the expansion of agricultural land also has the potential to affect population growth. In the extreme case where the constraint binds ($X_t = \overline{X}$), there are no improvements to agricultural TFP, and labour and land are perfect complements in food production, no further increase in the population can take place. More generally, the extent to which the population can grow despite the constraint binding depends on technological improvements in agriculture and on the substitutability of labour and land in agricultural production.

It is in fact useful to briefly inspect the optimal dynamics of agricultural land:

Remark 1. Optimal expansion of agricultural land in period t, under the assumption that the land constraint does not bind, requires that

$$u'(c_t)b(N_t)w_t/\psi'(L_{t,X}) = \beta u'(c_{t+1})b(N_{t+1})w_{t+1}\overline{f_{t+1}}\,\frac{\partial Y_{ag}(L_{t,+1ag},X_{t+1})}{\partial X_{t+1}}\Bigg/\frac{\partial Y_{ag}(L_{t+1,ag},X_{t+1})}{\partial L_{t+1,ag}} \tag{5}$$

The term on the left-hand side is the marginal cost of land conversion, in terms of present consumption foregone by diverting labour away from manufacturing. In the case where the land constraint binds, the shadow price of the constraint will appear as a cost in the form of a scarcity rent. The term on the right-hand side is the discounted marginal benefit of land conversion. Notice that the marginal benefit of land conversion is higher, the higher is the marginal productivity of land in agriculture relative to the marginal productivity of labour in the same sector.

One important implication of (5) is associated with the fact that labour used to invest in the stock of agricultural land is subject to decreasing returns. Therefore as the agricultural land area expands, the land input becomes relatively more expensive. In our simulation this will be the main driver of a slow-down in land conversion. Investing in the stock of land becomes relatively more costly, and, with substitutability and technological progress, land as a factor of production becomes relatively less important over time.

3. QUANTITATIVE MODEL

In this section, we present the quantitative model and then describe how we estimate key parameters of the model for trajectories to match key economic time series for 1960-2010. The model is an extension of the simple farming-household problem discussed above in which we add capital to the set of factor inputs. The problem is one of allocating labour and capital across sectors, as well as by selecting the savings/investment rate to build up the stock of capital. In addition, sectoral technological progress is endogenously determined by the allocation of labour to R&D activities. This implies that the change in the opportunity cost of children, and associated demographic transition, will occur endogenously.

Our empirical strategy relies on simulation methods, selecting the parameters of interest to minimise the distance between observed and simulated trajectories. The estimation procedure requires computing the model a very large number of times, and for that reason we consider only the optimal solution to the problem. Specifically a social planner maximises households' utility by selecting aggregate quantities subject to the technology that characterises the economy. First, this formulation of the problem makes conditions for the problem to be convex transparent, so that a solution to the problem exists and is unique (see Alvarez, 1999). Second, the social planner formulation affords a number of simplifications, and permits the use of efficient solvers for constrained non-linear optimisation problems, making simulation-based estimation practical. However, by definition the planner internalises all externalities (e.g. those associated with R&D, see Romer, 1994, for example), and thus market imperfections affecting the determination of the targeted variables over the estimation period will be factored into our estimates.[11]

[11] Importantly, even though we solve the model as a social planner problem, we do not rely on the existence of a social planner per se. The quantitative model rationalises observed outcomes 'as if' these resulted from the decisions of a social planner. Thus externalities will be reflected in our estimates. We return to this below.

3.1 The Economy

3.1.1 Production

In agriculture and manufacturing aggregate output is represented by a constant-returns-to-scale production function with endogenous, Hicks-neutral technological change.[12] In manufacturing, aggregate output in period t is given by a standard Cobb-Douglas production function:

$$Y_{t,mn} = A_{t,mn} K_{t,mn}^{\vartheta} L_{t,mn}^{1-\vartheta} , \tag{6}$$

where $K_{t,mn}$ is capital allocated to manufacturing and $\vartheta \in (0,1)$ is a share parameter. Conditional on technical change being Hicks-neutral, the assumption that output is Cobb-Douglas is consistent with long-term empirical evidence (Antràs, 2004).

In agriculture, we posit a two-stage constant-elasticity-of-substitution (CES) functional form (e.g. Kawagoe *et al.*, 1986; Ashraf *et al.*, 2008):

$$Y_{t,ag} = A_{t,ag} \left[(1-\theta_X) \left(K_{t,ag}^{\theta_K} L_{t,ag}^{1-\theta_K} \right)^{\frac{\sigma-1}{\sigma}} + \theta_X X_t^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} , \tag{7}$$

where $\theta_{X,K} \in (0,1)$, and σ is the elasticity of substitution between a capital-labour composite factor and agricultural land. This specification provides flexibility in how capital and labour can be substituted for land, and it nests the Cobb-Douglas specification as a special case ($\sigma = 1$). While a Cobb-Douglas function is often used to characterise aggregate agricultural output (e.g. Mundlak, 2000; Hansen and Prescott, 2002), it is quite optimistic in that, in the limit, land is not required for agricultural production. Long-run empirical evidence reported in Wilde (2013) indeed suggests that $\sigma < 1$.

3.1.2 Innovations and Technological Progress

The evolution of sectoral TFP is based on a discrete-time version of the Schumpeterian model by Aghion and Howitt (1992). In this framework innovations are drastic, so that a firm holding the patent for the most productive technology temporarily dominates the industry until the arrival of the next innovation. The step size of productivity improvements associated with an innovation is denoted $s > 0$, and we assume that it is the same in both sectors.[13] Without loss of generality, we assume that there can be at most $I > 0$ innovations over the length of a time period, so that the maximum growth rate of TFP each period is $S = (1+s)^I$. For each sector $j \in \{mn, ag\}$, the growth rate of TFP is then determined by the number of innovations arriving within each time-

[12] Assuming technological change is Hicks-neutral, so that improvements to production efficiency do not affect the relative marginal productivity of input factors, considerably simplifies the analysis at the cost of abstracting from a number of interesting issues related to the direction or bias of technical change (see Acemoglu, 2002).

[13] In general, the "size" of an innovation in the Aghion and Howitt (1992) framework is taken to be the step size necessary to procure a right over the proposed innovation. For the purposes of patent law, an innovation must represent a substantial improvement over existing technologies (not a marginal change), which is usually represented as a minimum one-time shift.

period, and this rate can be specified in relation to maximum feasible TFP growth S:[14]

$$A_{t+1,j} \;=\; A_{t,j} \cdot (1 + \rho_{t,j} S) \;, \quad j \in \{mn, ag\} \,. \tag{8}$$

where $\rho_{t,j}$ is the arrival *rate* of innovations each period, in other words how many innovations are achieved compared to the maximum number of innovations.

Arrival of innovations in each sector is a function of labour hired for R&D activities:

$$\rho_{t,j} \;=\; \overline{\lambda}_{t,j} \cdot L_{t,A_j} \;, \quad j \in \{mn, ag\} \,,$$

where L_{t,A_j} is labour employed in R&D for sector j and $\overline{\lambda}_{t,j}$ measures labour productivity. As mentioned in the introduction, the standard Aghion and Howitt (1992) framework implies a scale effect by virtue of which a larger population implies a large equilibrium growth rate of the economy, which is at odds with empirical evidence on modern growth. Instead we work with the scale-invariant formulation proposed by Chu *et al.* (2013), where $\overline{\lambda}_{t,j}$ is specified as a decreasing function of the scale of the economy. In particular, we define

$$\overline{\lambda}_{t,j} \;=\; \lambda_j L_{t,A_j}^{\mu_j - 1} / N_t^{\mu_j}$$

where $\lambda_j > 0$ is a productivity parameter and $\mu_j \in (0, 1)$ is an elasticity. Including population N_t in the denominator, so that innovation depends on the share of labour allocated to R&D, neutralises the scale effect and is in line with more recent representations of technological change (see Dinopoulos and Thompson, 1998; Peretto, 1998; Young, 1998, for example). In particular, using the share of employment in R&D can be seen as a proxy for average employment hired to improve the quality of a growing number of product varieties (see Laincz and Peretto, 2006). Furthermore, our representation of R&D implies decreasing returns to labour in R&D through the parameter μ_j, which captures the duplication of ideas among researchers (Jones and Williams, 2000).

3.1.3 Land

As in the simple analytical model above, land used for agriculture has to be converted from a finite stock of reserve land $\overline{X}$. Converting land from the available stock requires labour, therefore there is a cost in bringing new land into the agricultural system. Once converted, agricultural land gradually depreciates back to the stock of natural land in a linear fashion. Thus the allocation of labour to convert land determines agricultural land available each period, and over time the stock

[14] The arrival of innovations is a stochastic process, and we implicitly make use of the law of large numbers to integrate out the random nature of growth over discrete time-intervals. Our representation is qualitatively equivalent, but somewhat simpler, to the continuous time version of the model where the arrival of innovations is described by a Poisson process.

of land used in agriculture develops according to:

$$X_{t+1} \;=\; X_t(1-\delta_X) + \psi \cdot L_{t,X}^{\varepsilon}\,, \quad X_0 \text{ given}\,, \quad X_t \leq \overline{X}\,, \tag{9}$$

where $\psi > 0$ measures labour productivity in land clearing activities, $\varepsilon \in (0,1)$ is an elasticity, and the depreciation rate δ_X measures how fast converted land reverts back to natural land.

One important aspect of equation (9) is the decreasing returns to labour in land-clearing activities, which imply that the marginal cost of land clearing increases with the amount of land already converted. More specifically, as the amount of land used in agriculture increases, labour requirements to avoid it depreciating back to its natural state increase more than proportionally. Intuitively, this captures the fact that the most productive agricultural plots are converted first, whereas marginal land still available at a later stage of land conversion is less productive. Labour can be used to bring these marginal plots into agricultural production, although the cost of such endeavours increases as the total land area under agriculture use increases.

3.1.4 Preferences and Population Dynamics

We now further specialise households' preferences described in Section 2. We again use the dynastic representation that is associated with Barro and Becker (1989) preferences, so that the size of the dynasty coincides with the total population N_t (see APPENDIX A). We use the standard constant elasticity function $u(c_t) = \frac{c_t^{1-\gamma}-1}{1-\gamma}$, where $1/\gamma$ is the intertemporal elasticity of substitution, and specify $b(n_t) = n_t^{-\eta}$, where η is an elasticity determining how the utility of parents changes with n_t. The utility of the dynasty head is then:

$$U_0 = \sum_{t=0}^{\infty} \beta^t\, N_t^{1-\eta}\, \frac{c_t^{1-\gamma}-1}{1-\gamma}\,, \tag{10}$$

Parametric restrictions ensuring overall concavity of the objective and in turn existence and uniqueness of the solution are easy to impose. For $\gamma > 1$, which is consistent with macro-level evidence on the intertemporal elasticity of substitution (Guvenen, 2006), concavity of Equation (10) in (c_t, N_t) requires $\eta \in (0,1)$. This implies that, depending on η, preferences of the dynastic head correspond with both classical and average utilitarian objectives, in terms of social planning, as limiting cases.[15]

Aggregate consumption $C_t = c_t N_t$ is derived from the manufacturing sector. Given a social planner representation, manufacturing output can either be consumed by households or invested into a stock of capital:

$$Y_{t,mn} \;=\; C_t + I_t\,, \tag{11}$$

[15] See Baudin (2011) for a discussion of the relationship between dynastic preferences and different classes of social welfare functions.

The accumulation of capital is then given by:

$$K_{t+1} \;=\; K_t(1-\delta_K) + I_t\,, \quad K_0 \text{ given}\,, \tag{12}$$

where δ_K is a per-period depreciation rate. In this formulation investment decisions mirror those of a one-sector economy (see Ngai and Pissarides, 2007, for a similar treatment of savings in a multi-sector growth model).

In each period, fertility n_t determines the change in population together with mortality d_t:

$$N_{t+1} \;=\; N_t + n_t N_t - d_t\,, \quad N_0 \text{ given}\,. \tag{13}$$

We make the simplifying assumption that population equals the total labour force, so that $n_t N_t$ and d_t represent an increment and decrement to the stock of effective labour units, respectively. The mortality rate is assumed to be constant, so that $d_t \;=\; N_t\delta_N$, where $1/\delta_N$ captures the expected working lifetime.

As described above, the cost of fertility consists of time spent both rearing and educating children. We exploit the social-planner representation, which allows us to treat these as a single activity:

$$n_t N_t \;=\; \overline{\chi}_t \cdot L_{t,N}\,,$$

where $\overline{\chi}_t$ is an inverse measure of the cost of producing effective labour units. Treating child-rearing and education as an activity implies that there is an opportunity cost to population increments, and we assume that it increases with the technological advancement of the economy through the following function:

$$\overline{\chi}_t \;=\; \chi L_{t,N}^{\zeta-1}/A_t^{\omega}$$

where $\chi > 0$ is a productivity parameter, $\zeta \in (0,1)$ is an elasticity representing scarce factors required in child-rearing and education,[16] A_t is an economy-wide index of technology, a weighted average of sectoral TFP where the weights are the relative shares of sectors' output in GDP, and $\omega > 0$ measures how the cost of children increases with the level of technology.

Population dynamics are further constrained by food availability, as measured by agricultural output. As in our analytical model, we have the following constraint:

$$Y_{t,ag} = N_t\overline{f_t}$$

Per-capita demand for food $\overline{f_t}$ determines the quantity of food required for maintaining an individual in a given society, and captures both physiological requirements (e.g. minimum per-capita caloric intake) and the positive relationship between food demand and per-capita income, which

[16] More specifically, ζ captures the fact that the costs of child-rearing over a period of time may increase more than linearly with the number of children (see Barro and Sala-i Martin, 2004, p.412, Moav, 2005, and Bretschger, 2013).

captures changing diet as affluence rises. The relationship between food expenditures and per-capita income is not linear, however, so we specify food demand as a concave function of per-capita income: $\overline{f}_t = \xi \cdot \left(\frac{Y_{t,mn}}{N_t}\right)^{\kappa}$, where ξ is a scale parameter and $\kappa > 0$ is the income elasticity of food consumption. Therefore, while food consumption does not directly enter the utility function of households, food availability will affect social welfare through its impact on population dynamics.[17]

3.2 Optimal Control Problem and Empirical Strategy

We consider a social planner choosing paths for C_t, $K_{t,j}$ and $L_{t,j}$ by maximising the utility of the dynastic head (10) subject to technological constraints (6), (7), (8), (9), (11), (12), (13) and feasibility conditions for capital and labour:

$$K_t \;=\; K_{t,mn} + K_{t,ag}\,, \quad N_t \;=\; L_{t,mn} + L_{t,ag} + L_{t,A_{mn}} + L_{t,A_{ag}} + L_{t,N} + L_{t,X}\,.$$

Aggregate consumption is provided by allocating capital and labour to the manufacturing sector, as well as labour to manufacturing R&D. Increases in the population require time to be spent rearing and educating children. In addition, sufficient food must be provided at all times to feed the population, by allocating capital, labour and land to agriculture, as well as labour to agricultural R&D. Insofar as increasing agricultural production requires greater inputs of land, labour must also be allocated to converting reserves of natural land into agricultural land.

Since consumption grows over time and since fertility and the welfare of children are complements in parents' utility, the main driver of any slowdown in fertility will be the cost of fertility itself and how it evolves over time. Building on Section 2, we can identify several components to this evolution. First, technological progress increases human-capital requirements and in turn lowers the marginal productivity of labour in the production of children, because more time is required for their education. Second, as the economy develops the marginal productivity of labour in rearing and educating children changes relative to the marginal productivity of labour in other activities. This implies among other things that technological progress, which will raise labour productivity in the two production sectors, will tend to increase the opportunity cost of labour in child-rearing and education. Third, there are diminishing returns to labour in the production of children, implying that the marginal cost of fertility with respect to labour is an increasing and convex function. This is the usual assumption for the cost of education (Moav, 2005), and can also represent a form of congestion (see Bretschger, 2013). Fourth, a cost of fertility is meeting food requirements, and the demand for food increases with per-capita income (at a decreasing

[17] Making per-capita food demand proportional to income drastically limits substitution possibilities between food and manufacturing products, which magnifies the role of Malthusian constraints in the analysis. In other words, if land is a limiting factor to development, the relative cost of food would increase, and allowing households to substitute more of the manufactured product for food would essentially make land constraints irrelevant. Note that from a decentralised perspective we are effectively creating an externality, although this simplification bears no consequence for the central-planner formulation.

rate). Thus growth in population and per-capita income are associated with an increasing demand for agricultural output. This can be achieved either through technological progress, or by allocating primary factors, i.e. labour, capital and land, to agriculture. However, agricultural land is ultimately fixed, either because it is constrained by physical availability or because its conversion cost increases with the area already converted. Thus over time the cost of agricultural output will increase, adding a further break to population growth.

3.2.1 Numerical Solution Concept

The optimisation problem is an infinite horizon optimal control problem, and we use mathematical programming techniques to solve for optimal trajectories. In particular, we employ a solver for constrained non-linear optimisation problems, which directly mimics the welfare maximisation program: the algorithm searches for a local maximum of the concave objective function (the discounted sum of utility), starting from a candidate solution and improving the objective subject to maintaining feasibility as defined by the technological constraints.[18]

A potential shortcoming of direct optimisation methods, as compared to dynamic programming for example, is that they cannot explicitly accommodate an infinite horizon.[19] As long as $\beta < 1$, however, only a finite number of terms matter for the solution, and instead we solve the associated finite-horizon problem truncated to the first T periods. The truncation may induce differences between the solution to the infinite-horizon problem and its finite-horizon counterpart because the shadow values of the stock variables are optimally zero in the terminal period T, whereas they will be so only asymptotically if the planning horizon is infinite. Since we are interested in trajectories over the period from 2010 to 2100 (1960 to 2010 for the estimation of the model), we check that the solution over the first $T' = 90$ periods are not affected by the choice of T, finding that $T = 300$ is sufficient to make computed trajectories over the first T' periods independent of T.[20] Similarly, re-initialising the model in $T' = 90$ and solving the problem onwards, we remain on the same optimal path with a precision of 0.1 percent for all the variables in the model. Given the truncation over 300 periods and appropriate scaling of variables, the model solves in a matter of seconds.

[18] The program is implemented in GAMS and solved with KNITRO (Byrd *et al.*, 1999, 2006), which alternates between interior-point and active-set methods.

[19] By definition, the objective function is a sum with an infinite number of terms, and the set of constraints includes an infinite number of elements, which is incompatible with finite computer memory. The main alternative class of numerical solution methods is dynamic programming (see Judd, 1998), and exploiting a recursive formulation could accommodate an infinite horizon. Because dynamic programming is subject to the curse of dimensionality with respect to the number of continuous state variables, the computational burden associated with recursive methods would make simulation-based estimation impractical.

[20] For the estimation the model is initialised in 1960 and solved up to 2260. For projection the model is initialised in 2010 and solved up to 2310.

3.2.2 Empirical strategy

Having defined the numerical optimisation problem, our empirical strategy proceeds in three steps. First, a number of parameters are determined exogenously. Second, we calibrate some of the parameters to match observed quantities, mainly to initialise the model based on 1960 data. Third, we estimate the remaining parameters with simulation methods. These are the crucial parameters determining the cost of fertility (χ, ζ, ω), technological progress (μ_{mn}, μ_{ag}) and land conversion (ψ, ε). We now discuss each step in turn. The full set of parameters of the model is listed in **Table 1**.

Exogenous parameters

Starting with production technology, we need to select values for the share parameters ϑ, θ_K and θ_X, and for the elasticity of substitution σ. In manufacturing, the Cobb-Douglas functional form implies that the output factor shares (or cost components of GDP) are constant over time, and we use a standard value of 0.3 for the share of capital (see for example Gollin, 2002). In agriculture, the CES functional form implies that the factor shares are not constant, and we choose θ_X to approximate a value for the share of land in global agricultural output of 0.25 in 1960. For the capital-labour composite, we follow Ashraf *et al.* (2008) and also use a standard value of 0.3 for the share of capital. Taken together, these estimates of the output value shares in agriculture are broadly in agreement with factor shares for developing countries reported in Hertel *et al.* (2012).[21]

As mentioned previously, the long-run elasticity of substitution between land and the capital-labour composite input is a crucial parameter for long-run growth. If land is an essential input into agriculture it is expected to be less than one (Cobb-Douglas being the limiting case where it is essential only asymptotically), which is confirmed by empirical evidence reported in Wilde (2013). Using long-run data on land and other inputs in pre-industrial England, he finds robust evidence that $\sigma \simeq 0.6$. While external validity of these estimates may be an issue (in particular for the currently developing countries with rapidly growing population), it reflects long-run substitution possibilities that are consistent with our CES functional form (7). We consider $\sigma = 0.6$ to be the best estimate available, and derive trajectories assuming $\sigma = 0.2$ and $\sigma = 1$ in the sensitivity analysis.

The yearly rate of capital depreciation δ_K is set to 0.1 (see Schündeln, 2013, for a survey and evidence for developing countries), and maximum TFP growth per year S is set to 5 percent. The latter number is consistent with evidence on yearly country-level TFP growth rates from Fuglie (2012), which do not exceed 3.5 percent. The labour productivity parameter in R&D λ_j is not separately identified from S, and we set it to 1 without affecting our results.

[21] For 2007, the factor shares for the global agricultural sector reported in Hertel *et al.* (2012) are 0.15 for land, 0.47 for labour and 0.37 for capital. While there are no data on the global land factor share in 1960, it has been shown to be negatively correlated with income (Caselli and Feyrer, 2007), so that factor shares for developing countries are probably a better estimates of the value shares prevailing at the global level in 1960. That said, our results are not significantly affected by variations in the estimated value shares within a plausible range.

Table 1. List of parameters of the model and associated numerical values

Imposed parameters			
ϑ	Share of capital in manufacturing	0.3	
θ_K	Share of capital in capital-labour composite for agriculture	0.3	
θ_X	Share of land in agriculture	0.25	
σ	Elasticity of substitution between land and the capital-labour composite	0.6	
δ_K	Yearly rate of capital depreciation	0.1	
S	Maximum increase in TFP each year	0.05	
$\lambda_{mn,ag}$	Labour productivity parameter in R&D	1	
γ	Inverse of the intertemporal elasticity of substitution	2	
η	Elasticity of altruism towards future members of the dynasty	0.001	
κ	Income elasticity of food demand	0.25	
β	Discount factor	0.99	
Initial values for the stock variables and calibrated parameters			
N_0	Initial value for population	3.03	
X_0	Initial the stock of converted land	1.35	
$A_{0,mn}$	Initial value for TFP in manufacturing	4.7	
$A_{0,ag}$	Initial value for TFP in agriculture	1.3	
K_0	Initial value for capital stock	20.5	
ξ	Food consumption for unitary income	0.4	
δ_N	Exogenous mortality rate	0.022	
δ_X	Rate of natural land reconversion	0.02	
Estimated parameters (range of estimates for relaxed goodness-of-fit objective in parenthesis)			
χ	Labour productivity parameter in child-rearing	0.153	(0.146 – 0.154)
ζ	Elasticity of labour in child-rearing	0.427	(0.416 – 0.448)
ω	Elasticity of labour productivity in child-rearing w.r.t. technology	0.089	(0.082 – 0.106)
μ_{mn}	Elasticity of labour in manufacturing R&D	0.581	(0.509 – 0.585)
μ_{ag}	Elasticity of labour in agricultural R&D	0.537	(0.468 – 0.545)
ψ	Labour productivity in land conversion	0.079	(0.078 – 0.083)
ε	Elasticity of labour in land conversion	0.251	(0.238 – 0.262)

The next set of imposed parameters determines preferences over consumption and fertility. First, the income elasticity of food demand is 0.25, which is consistent with evidence across countries and over time reported in Subramanian and Deaton (1996), Beatty and LaFrance (2005), and Logan (2009). Second, the elasticity of intertemporal substitution is set to 0.5 in line with estimates from Guvenen (2006), which corresponds with $\gamma = 2$. Given the constraint on η to maintain concavity of the objective function, we initially set it to 0.01 so that the planner effectively has a classical utilitarian objective. Intuitively, this implies that parents' marginal utility of fertility is almost constant, or that altruism towards the welfare of children remains constant as the number of children increases. Correspondingly, we also assume a high degree of altruism by setting the discount factor to 0.99, which implies a pure rate of time preference of 1 percent per year. We report sensitivity analysis for the case where altruism declines with n_t, in particular $\eta = 0.5$, and for a discount factor of 0.97.[22]

Initial values and external calibration
Starting values for the state variables are calibrated to observed quantities in 1960. Initial population N_0 is set to an estimate of the world population in 1960 of 3.03 billion (United Nations, 1999). Initial crop land area X_0 is set to 1.348 billion hectares (Goldewijk, 2001) and the total stock of natural land reserves that can be converted for agriculture is 3 billion hectares (see Alexandratos and Bruinsma, 2012). For the remaining state variables, sectoral TFP $A_{0,ag}$, $A_{0,mn}$ and the stock of capital K_0, there are no available estimates, and we target three moments. First, we use an estimate of world GDP in 1960 of 8.79 trillion 1990 international dollars (Maddison, 1995; Bolt and van Zanden, 2013). Second, we obtain an estimate of world agricultural production by assuming that the share of agriculture in total GDP in 1960 is 15% (see Echevarria, 1997). Third, we assume that the marginal product of capital in 1960 is 15 percent. While this may appear relatively high, it is not implausible for developing economies (see Caselli and Feyrer, 2007). Solving for the targeted moments as a system of three equations with three unknowns gives initial values of 4.7 and 1.3 for TFP in manufacturing and agriculture respectively, and a stock of capital of 20.5.

Three other parameters of the model are calibrated to observed quantities. First, the parameter measuring food consumption for unitary income (ξ) is calibrated such that the demand for food in 1960 represents about 15% of world GDP, which is consistent with the calibration targets for initial TFP and capital stock. This implies $\xi = 0.4$. Second, the mortality rate δ_N is calibrated by assuming an average adult working life of 45 years (United Nations, 2013), which implies $\delta_N = 0.022$. We vary that assumption in the sensitivity analysis, using $\delta_N = 0.015$ instead, in other words a 65 year working life. Finally we set the period of regeneration of natural land to 50 years so that $\delta_X = 0.02$.

Estimation of the remaining parameters
The seven remaining parameters $\{\mu^{mn,ag}, \chi, \zeta, \omega, \psi, \varepsilon\}$ are conceptually more difficult to tie down using external sources, and we therefore estimate them using simulation-based structural methods. The moments we target are taken from observed trajectories over the period 1960 to 2010 for world GDP (Maddison, 1995; Bolt and van Zanden, 2013), world population (United Nations, 1999, 2013), crop land area (Goldewijk, 2001; Alexandratos and Bruinsma, 2012) and

[22] In fact, as we show below, the estimation error is significantly higher if we assume $\eta = 0.5$, and only slightly lower for $\beta = 0.97$.

sectoral TFP (Martin and Mitra, 2001; Fuglie, 2012; Alston and Pardey, 2014).[23] For each time series, we target one data point for each five-year interval, denoted τ, yielding 11 data points for each targeted quantity (55 points in total).[24] The data are reported in APPENDIX B.

The targeted quantities in the model are respectively $Y_{t,mn}+Y_{t,ag}$,[25] N_t, X_t, $A_{t,mn}$ and $A_{t,ag}$, and we formulate a minimum distance estimator as follows. For a given vector of parameters v, we solve the model and obtain the values for each targeted quantity, which we denote $Z^*_{v,k,\tau}$, where k indexes targeted quantities. We then compute the squared deviations between the solution of the model and observed data points $Z_{k,\tau}$, and sum these both over k and τ to obtain a measure of the estimation error over time and across targeted variables. Formally the error for a vector of parameters v is given by:

$$error_v = \sum_k \left[\sum_\tau \left(Z^*_{k,\tau} - Z_{k,\tau}\right)^2 \Big/ \sum_\tau Z_{k,\tau} \right], \tag{14}$$

where the error for each variable is scaled to make these comparable. Therefore, our estimation procedure is essentially non-linear least squares defined over several jointly determined model outcomes. Importantly the error for each vector of parameters is computed for all targeted variables in one run of the model, so that all the parameters are jointly rather than sequentially estimated.

In order to select the vector of parameters that minimises the goodness-of-fit objective (14), we simulate the model over the domain of plausible parameter values, starting with bounds of a uniform distribution, which is our initial 'prior' for the parameters. For elasticity parameters, these bounds are 0.1 and 0.9 and for the labour productivity parameters we use 0.03 and 0.3. We then solve the model for 10,000 randomly drawn vectors of parameters and evaluate the error between the simulated trajectories and those observed. Having identified a narrower range of parameters for which the model approximates observed data relatively well, we reduce the range of values considered for each parameter and draw another 10,000 vectors to solve the model. This algorithm gradually converges to the estimates reported in Table 1.[26]

[23] Data on TFP is derived from TFP growth estimates and are thus more uncertain than other trajectories. Nevertheless, a robust finding of the literature is that the growth rate of economy-wide TFP and agricultural TFP is on average around 1.5-2% per year. To remain conservative about the pace of future technological progress, we assume TFP growth was at 1.5 percent between 1960 and 1980, declined to 1.2 percent from 1980 to 2000, and was equal to 1 percent over the last decade of the estimation period.

[24] Considering five-year intervals smooths year-on-year variations and allows us to focus on the long-run trends in the data. Using yearly data would not change our results. Similarly, we use the level of TFP rather than its growth rate to mitigate the impact of discontinuities implied by the TFP growth rates.

[25] In the model investments in sectoral TFP $I_{t,A_j} = \lambda_j \left(L_{t,A_j}/N_t\right)^{\mu_j}$ and in land conversion $I_{t,X} = \psi L^{\varepsilon}_{t,X}$ are not intermediate goods (they are not used in period t production) and hence could be included in our simulated measure of GDP. In practice, however, these activities represent a very small share of total production, and their exclusion does not affect our quantitative results.

[26] As for other simulation-based estimation procedures involving highly non-linear models, the uniqueness of the solution to the estimation of the parameters cannot be formally proved (see Gourieroux and Monfort, 1996). Our

4. QUANTITATIVE RESULTS

This section provides the quantitative results of the analysis. We start by reporting targeted and non-targeted trajectories over the estimation period, and discuss the fit of the model and associated parameter estimates. We then present implications of the model up to 2100. Finally we present sensitivity of our results to a number of assumptions underpinning our approach.

4.1 Estimation Results: 1960 – 2010

Trajectories for the targeted quantities over the period 1960 to 2010 are reported in **Figure 2**. More specifically, we compare the observed trajectories for world GDP, world population, crop land area and sectoral TFP against simulated trajectories obtained from the estimated model. By definition the estimated parameters are selected to minimise the distance between observed and simulated trajectories through equation (14), and they are reported in Table 1.

The model is able to closely replicate observed trajectories, with a relative squared error of 3.52 percent across all variables. The difference between the model and observed trajectories is mainly driven by the error on output (3.3 percent), followed by land (0.1 percent) and population (0.03 percent). In Figure 2 we also report runs for which the goodness-of-fit objective is relaxed by 10% relative to the best fit achieved, as represented by the shaded area. In other words, the shaded area reports the set of simulated trajectories with an error of 3.9 percent at most. The associated range of parameters is reported in Table 1.

Having considered the fit of the model to targeted trajectories we now consider non-targeted trajectories. First, because of our focus on fertility and population, the model should also closely match changes in the population growth rate even though it is not directly targeted by the estimation. Indeed, because observed population growth rates are more volatile than the level of population, providing a good fit in terms of the population level does not necessarily imply that the model provides a good representation of the decline in population growth. As shown in the top right panel of Figure 2 the model closely replicates the decline of population growth observed in the past fifty years.

A second measure not directly targeted in the estimation that is important for the analysis is the evolution of agricultural output over time. According to Alexandratos and Bruinsma (2012), global agricultural output has grown by 2 percent per year on average from 1960 to 2010, or an equivalent of 269 percent over that period. As shown in **Figure 3** agricultural output in our model increased by 279 percent over the same period. An implication is that the model provides a good account of the industrialisation process as measured by the size of the agricultural sector relative to total GDP. Similarly, the model provides a good account of growth in agricultural yields,

experience with the model suggests however that the solution is unique, with no significantly different vector of parameters providing a comparable goodness-of-fit objective. In other words, estimates reported in Table 1 provide a global solution to the estimation problem. The fact that we simultaneously estimate the whole vector of parameters makes the criteria highly demanding, as changing one parameter will impact trajectories across all variables in the model.

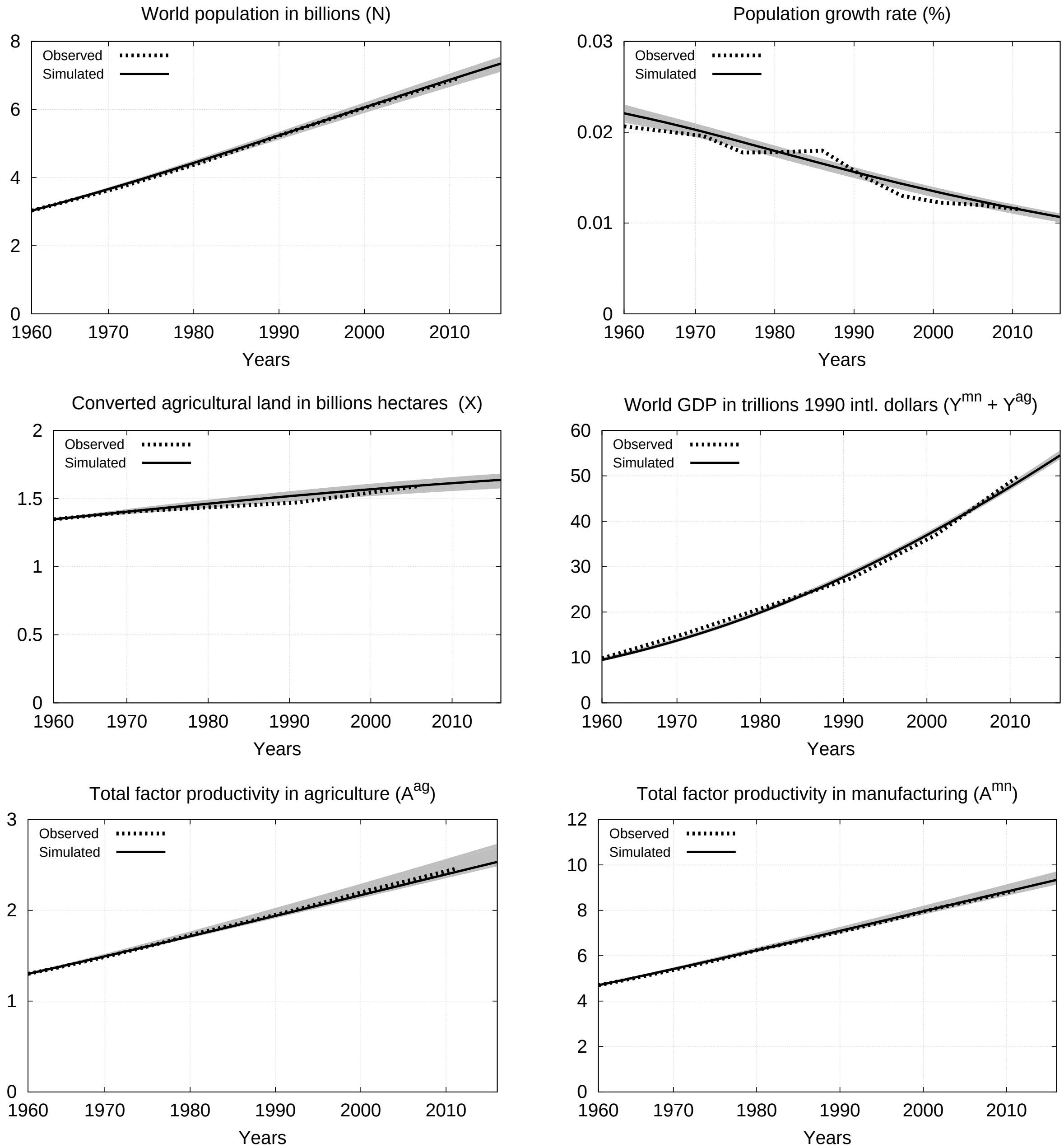

Figure 2. Estimation of the model 1960-2010

shown in Figure 3, as compared to figures reported in Alston and Pardey (2014), 2% per year from 1961 to 1990 and 1.8% from 1990 to 2005, and Alexandratos and Bruinsma (2012), 1.9% per year from 1960 to 1985 and 1.4% from 1985 to 2007.

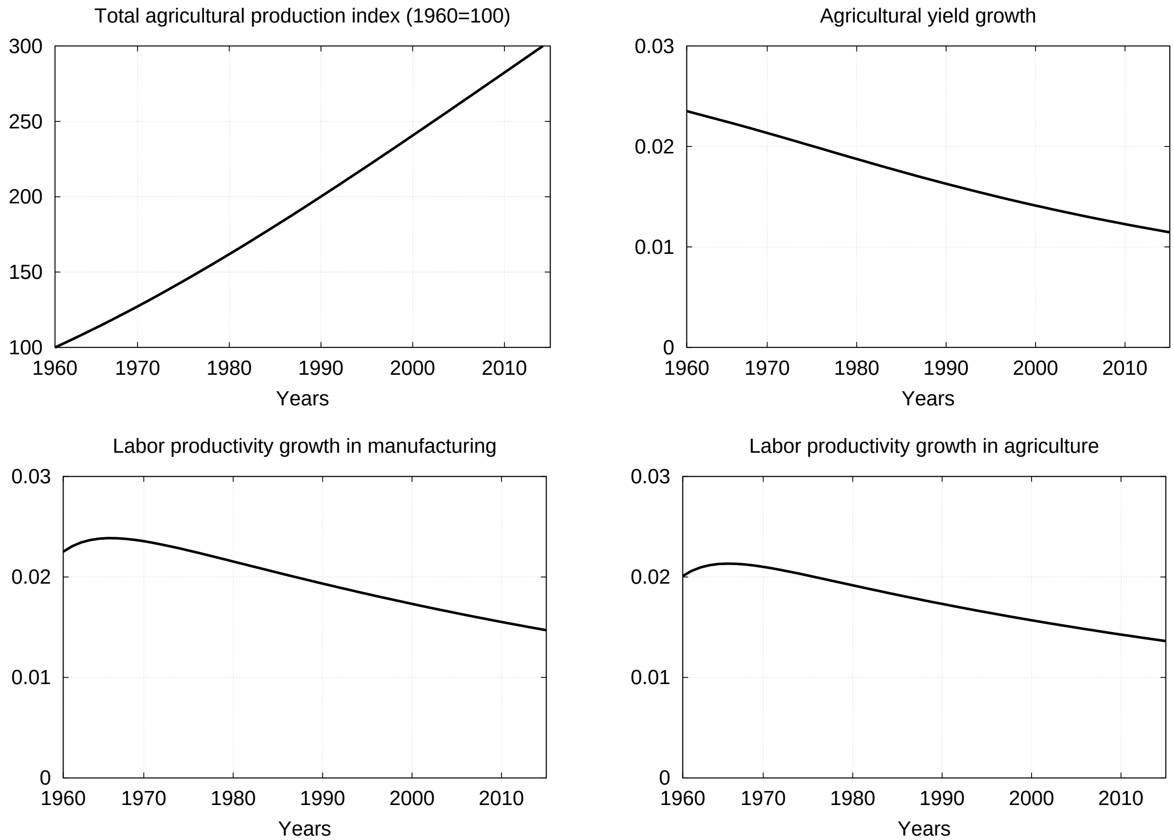

Figure 3. Non-targeted trajectories 1960-2010

The model does less well regarding the control variables, namely the allocation of capital and labour (aside from fertility which provides plausible figures for the cost of children, discussed below). In particular, the share of labour allocated to agriculture relative to the manufacturing sector declines from around 40% in 1960 to 27% in 2010, which is lower than observations (in 2010 around 40% of the world labour force was employed in agriculture).[27] Nevertheless, Figure 3 shows that labour productivity growth (in terms of output per worker) in both manufacturing and in agriculture are in line with expectations.

Another approach to evaluate the goodness-of-fit of the quantitative model is to assess whether the estimated parameters are in a plausible range of values.[28] The magnitude of estimated pa-

[27] The share of capital allocated also declines from around 40% in 1960 to 30% in 2010, although the stock of capital used in agriculture increases over time.

[28] Note that the estimated parameters are conditional on the model, which complicates comparison with external sources. It is also important to bear in mind that the estimates we report cannot be interpreted as the technology parameters of a representative firm operating in a decentralised setting because of the presence of externalities.

rameters is probably of greatest importance for our specification of child rearing and education. For example, Jones and Schoonbroodt (2010) report calibrated value for the cost of children in terms of years of output for the U.S. around 1970, which ranges from 4.5 to 15.4. Jones and Schoonbroodt (2014) further estimate the cost of children in terms of both time and goods. The time cost amounts to 15 percent of work time, while the goods cost amounts to around 20 percent of household income. In our model the *implied* time cost of children increases from 7.5 years ($\overline{\chi}_t$ = 0.133) in 1960 to 17.9 years in 2010 ($\overline{\chi}_t$ = 0.056). While our 2010 estimate then appears to be high, remember that it combines the time and goods costs of children.

A key component of the cost of fertility is the advancement of technology, and the elasticity of fertility with respect to technology (ω) can also be compared to the empirical evidence derived from Herzer *et al.* (2012). In particular, they estimate that the long-run elasticity of fertility with respect GDP growth of is around -0.0018.[29] In our model, a one percent increase in TFP (and hence GDP) reduces fertility by -0.00089 *in the same period*, or about half of the long-term impact. Our elasticity estimate is hence in the same ballpark.

The elasticity of labour in R&D activities (μ_j) is also discussed in the literature. However, there is disagreement on what this parameter should be. In particular, Jones and Williams (2000) argue that it is around 0.75, while Chu *et al.* (2013) use a value of 0.2. These two papers however rely on thought experiments to justify their choices. According to our results, a doubling of the share of labour allocated to R&D would increase TFP growth by around 50%. We are also not aware of comparable evidence for our estimates related to land clearing. Note however that that these estimates rationalise the relatively slow development of agricultural land area as compared to agricultural output and thus reflect forces determining the allocation of land, such as the demand for pastures and urban areas.

Despite the difficulties in assessing the magnitude of estimates, estimation results suggest that the implications of the quantitative model are broadly in agreement with global development trends observed over the past 50 years. In fact, given that the model is based on several components whose empirical relevance have been demonstrated in the literature, the finding that it can rationalise several key features of global development dynamics is not a surprise. Nevertheless, it provides confidence that the model can be used to study implications for the future evolution of the system.

Importantly though, our objective is not to obtain estimates for the structural parameters of a representative firm. Rather, we want the model to rationalise observed trajectories in order to study the joint determination of outcomes, and the estimated parameters provide the flexibility for doing this.

[29] More specifically they estimate a long-run cointegrating relationship between the crude birth rate and the log of GDP, with their central estimate being -5.83. For a one percent increase in GDP, this implies a reduction of the crude birth rate of -0.058, or -0.0018 percent at their mean fertility level of 33. In a model with country-specific time trends, they report an elasticity of -3.036, which is associated with an elasticity of -0.0009 and almost identical to our own estimate.

4.2 Global Projections: 2010 – 2100

We next describe projections implied by the estimated model. **Figure 4** displays the growth rate of key variables from 2010 to 2100. The main feature of these paths is that they all decline towards a balanced growth trajectory where population, land and capital reach a steady state. Agricultural land area is the first state variable to reach a steady state as its growth rate becomes negligible by 2050. Thus the total amount of land that can be used for agriculture is never exhausted. Population growth on the other hand remains significantly above zero over the whole century, being around 0.3 percent by 2100. Thus the model is far from predicting a complete collapse of population growth over the coming fifty years. Nevertheless population growth continues to decline after that, being around 0.1 percent in 2150.

The pace of technological progress also declines over time, starting at around one percent per year and reaching about half of one percent by the end of the century. This has the consequence that, over time, labour productivity and the educational costs of children grow less quickly than in the period 1960-2010. This is the main explanation for why population growth does not fall more quickly, which in turn implies a relatively high population *level* reported in **Figure 5** (see also APPENDIX B). In particular, world population is around 9.85 billion by 2050, which is broadly consistent with the latest revision of UN's projections (United Nations, 2013), but not with older projections in which global population peaked at around 9 billion. Our model further suggests that population growth continues over the entire century, so that the global population reaches more than 12 billion by 2100. This estimate lies towards the upper bound of the probabilistic forecasts of United Nations (2013) recently reported in Gerland *et al.* (2014).

Interestingly the shaded band for the population growth rate, which represents a range of alternative pathways for vectors of parameters with a slightly lower fit, shrinks over time. This demonstrates that the estimation of the parameters does not affect the long-run growth rate of population, whereas different transition paths imply a range of possible population levels between 11 and 13 billion by 2100.

Our model indicates that a significant increase of population over the century is compatible with food production possibilities. Between 1960 and 2010, agricultural output in the model increased by 279 percent, and projections from the fit indicate an increase by a further 67 percent between 2010 and 2050. These figures are close to the 58 percent increase in global agricultural output projected by Alexandratos and Bruinsma (2012) for the period 2010 to 2050, although these are based on lower population numbers from the 2010 revision of United Nation's projections. After 2050, our model suggests a further increase in agricultural output of 31 percent by 2100, so that by the end of the century agricultural output roughly doubles relative to the current level. This can be compared to 80 percent growth in population and a 95 percent increase in per-capita income.

In light of these results, the fact that agricultural land area stabilises at around 1.77 billion hectares is an important finding. First, this number is slightly higher than land conversion pro-

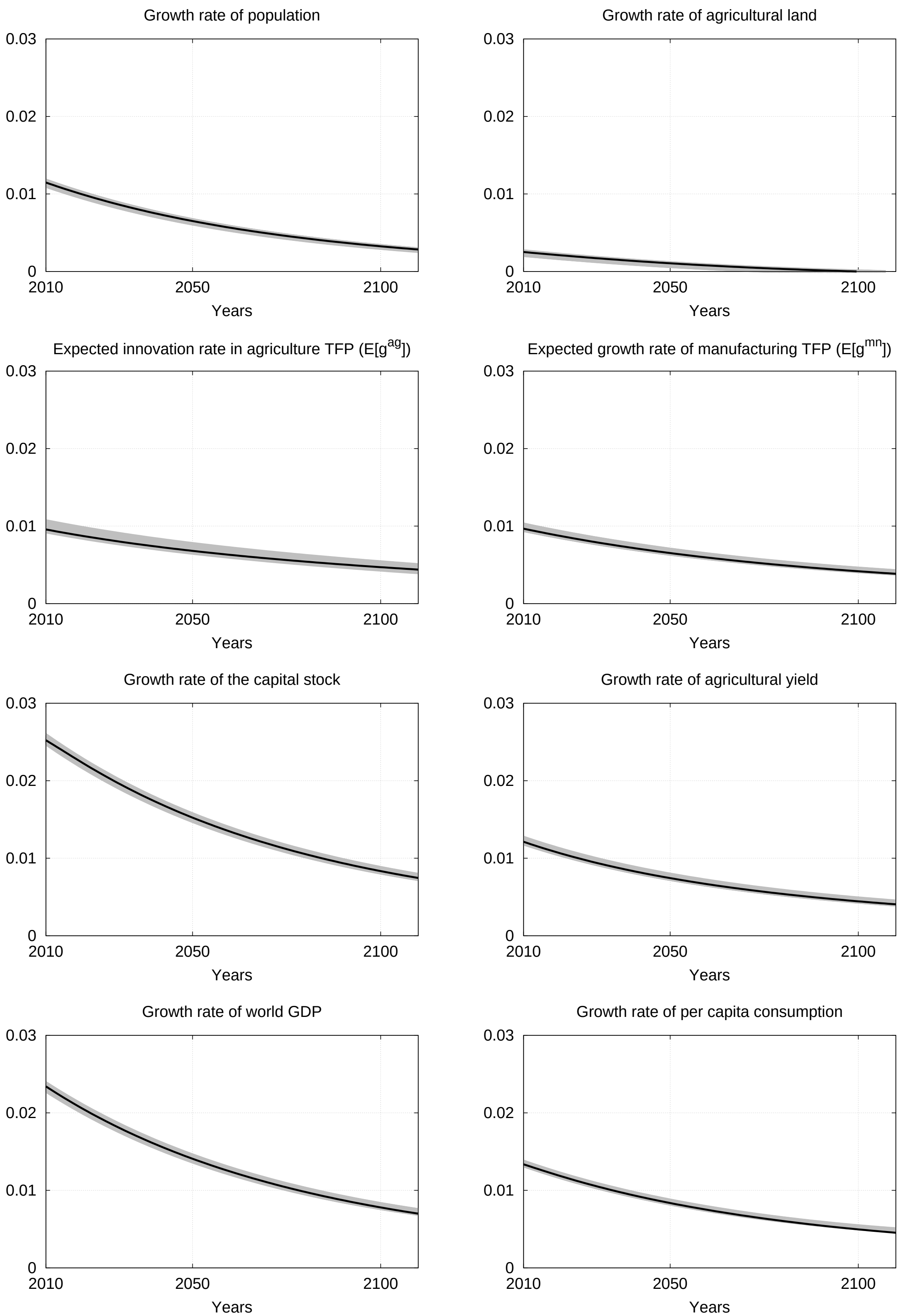

Figure 4. Growth rate of selected variables 2010 – 2100

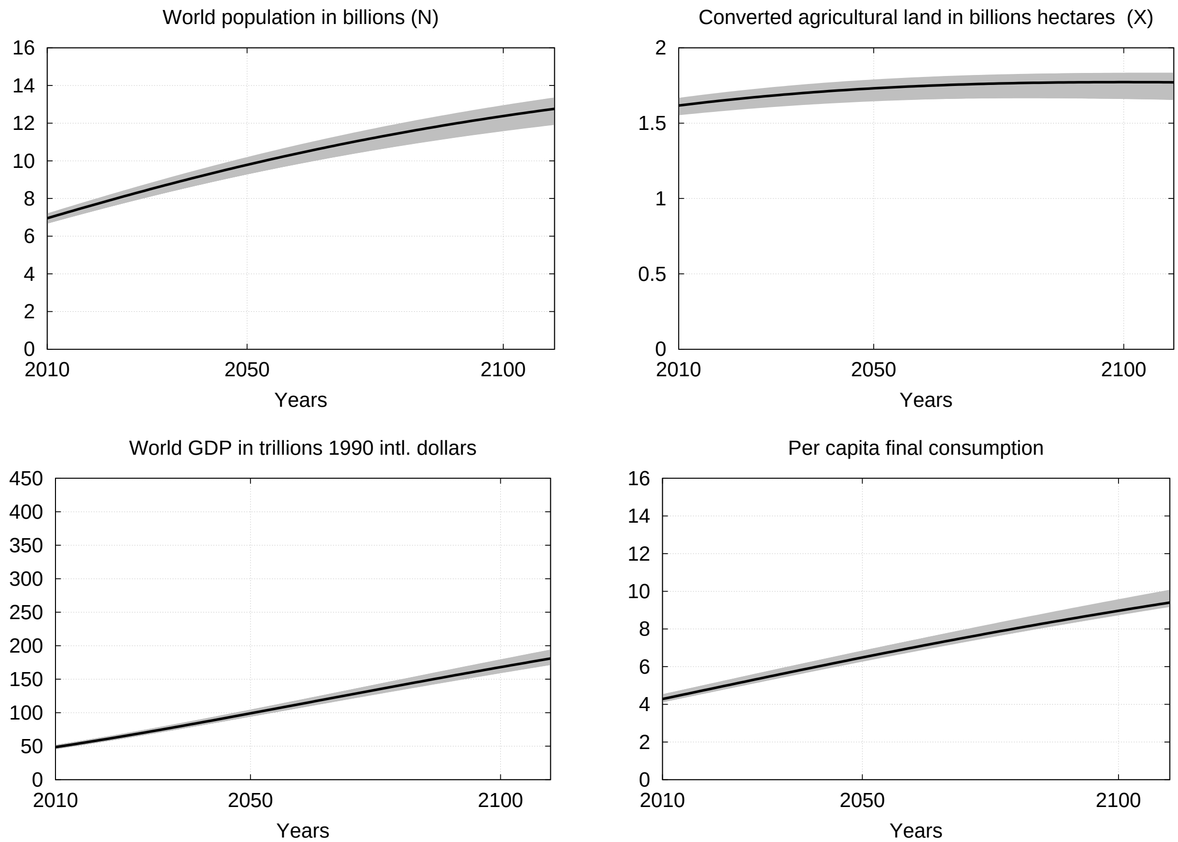

Figure 5. Projections for selected variables 2010 – 2100

jections by Alexandratos and Bruinsma (2012), in which cropland expansion is expected to stop at around 1.66 billion hectares. As with population growth, land conversion will mostly occur in developing countries, while agricultural area in developed countries has declined and presumably will continue to do so on economic grounds. Second, TFP growth in agriculture remains below 1 percent, which is a fairly conservative assumption. In other words the pace of technological progress does not need to be very high to allow for sustained growth in agricultural output. Third, the halt of agricultural land expansion suggests that the elasticity of substitution (σ) is high enough to allow agricultural output to grow from the accumulation of capital (we return to the role of σ in the sensitivity analysis). Indeed, although the share of capital allocated to agriculture declines over time, the stock of capital in agriculture almost doubles between 2010 and 2050.[30] This would mainly represent improvements to irrigation facilities. Both technology improvement and capital accumulation are reflected in the growth rate of agricultural yield (Figure 4), measuring growth in agricultural output per hectare used in agricultural production.

[30] As expected, both the share and the quantity of labour allocated to agriculture decline over time.

Finally, the growth rate of GDP falls from more than two percent in 2010 to less than one percent in 2100, which implies that world GDP doubles by 2050 and more than triples by 2100. Similarly, per-capita consumption more than doubles by 2100 relative to 2010.

4.3 Sensitivity Analysis

We now report the results of sensitivity analysis with respect to a number of assumptions we have made: substitution possibilities in agriculture (σ), the elasticity of utility with respect to fertility (η), the discount factor (β) and the expected working lifetime ($1/\delta_N$). For each change in the value of a parameter, it is necessary to re-estimate the vector of parameters to match observed data over the period 1960-2010. Here we focus on trajectories for two of the main variables of interest, population and agricultural land, against our baseline results discussed above. We report the vector of estimates associated with each sensitivity run in **Table 2**.

The parameter σ determines the elasticity of substitution between land and the capital-labour composite input in the agricultural production function. Our baseline case is obtained under the assumption that $\sigma = 0.6$, which follows empirical evidence by Wilde (2013). However, evidence with regard to this parameter remains scarce, and it is the main determinant of the demand for agricultural land, and in turn the ability to produce food and sustain the population.

We therefore re-estimate the parameters of the model assuming that $\sigma = 1$, so that agricultural production is Cobb-Douglas, and $\sigma = 0.2$, which we interpret as a lower bound on substitution possibilities in agriculture. The results reported in **Figure 6** demonstrate that the choice of σ has a small impact on land conversion and virtually no impact on population. As expected, a high value of σ implies less land conversion, since other factors can be more easily substituted when the marginal cost of land conversion increases. Conversely, a lower σ makes land more important in agriculture, so that the overall area of agricultural land is higher. However, estimating the model over 50 years of data largely ties down the trajectory for land use in a robust manner, irrespective of the choice of σ. Estimates of labour productivity in land conversion imply a higher (lower) conversion cost under $\sigma = 0.2$ ($\sigma = 1$). Estimates of the marginal productivity of labour in agricultural R&D also adjust, implying lower productivity for $\sigma = 0.2$, exemplifying interdependencies in our estimation procedure. The fit of the model remains very similar.

The second sensitivity test we conduct targets η, the elasticity of utility with respect to fertility. We consider the case of $\eta = 0.5$, so that the marginal utility of fertility (and population) declines more rapidly than under our baseline assumption of $\eta = 0.01$.[31] We re-estimate the parameters of the model so that the model fits observed trajectories given alternative assumptions about η, and report the resulting trajectories in **Figure 7**. In addition, we also report trajectories obtained with $\eta = 0.5$ but where the baseline parameter estimates are retained. This can be thought of as a comparative-static experiment (we label these trajectories "comparative"). As Figure 7 shows, when the model is not re-estimated trajectories over 1960 to 2010 differ significantly.

[31] Note that in our setting an average utilitarian objective corresponds to $\eta = 0$, but it implies that the objective function is not globally concave.

Table 2. Estimates supporting the sensitivity analysis

Parameter	Baseline	$\sigma = 0.2$	$\sigma = 1$	$\eta = 0.5$	$\beta = 0.97$	$\delta_N = 0.015$
χ	0.153	0.155	0.151	0.205	0.155	0.104
ζ	0.427	0.417	0.426	0.399	0.460	0.516
ω	0.089	0.085	0.088	0.161	0.087	0.091
μ_{mn}	0.581	0.575	0.580	0.751	0.523	0.525
μ_{ag}	0.537	0.549	0.509	0.482	0.383	0.512
ψ	0.079	0.063	0.083	0.078	0.083	0.077
ε	0.251	0.174	0.256	0.239	0.243	0.186
Estimation error	0.035	0.033	0.035	0.189	0.029	0.045

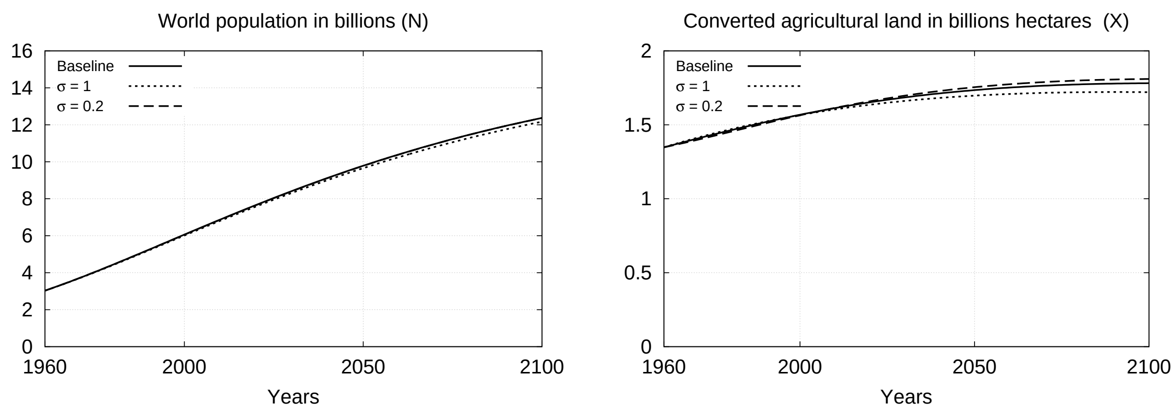

Figure 6. Sensitivity analysis on substitution possibilities in agriculture

As expected, reducing η while keeping the estimated parameters to their baseline values implies lower population growth. This results from putting less weight on the welfare of future members of the dynasty, so that the dynastic head reallocates resources to increase its own consumption at the expense of population growth. However, once we re-estimate the model to observed trajectories over 1960 to 2010, the population path is virtually identical to the baseline trajectory. Note that the estimated parameters under $\eta = 0.5$ are very different from those in the baseline case, and the estimation error is significantly higher (see Table 2).

The third parameter we vary is the discount factor. The baseline value of $\beta = 0.99$ implies a relatively low discount rate, and we instead use $\beta = 0.97$. We report a trajectory where we re-estimate the model to 1960-2010 data under the assumption that $\beta = 0.97$, as well as a comparative-static exercise in which we set $\beta = 0.97$ while keeping other parameters to their baseline values.

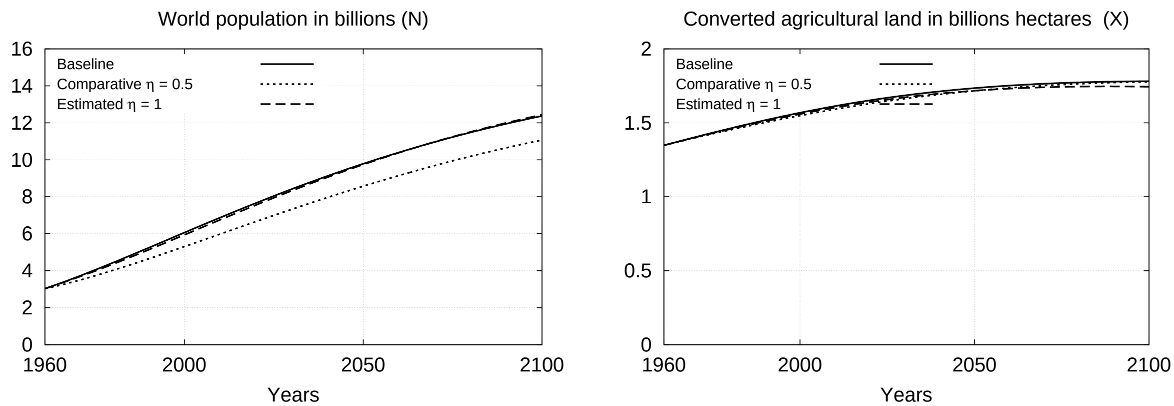

Figure 7. Sensitivity analysis on altruism towards children

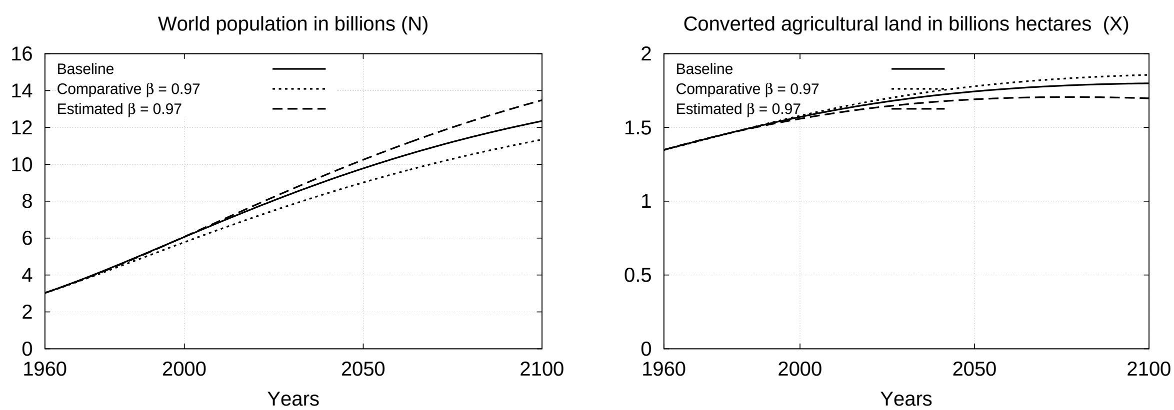

Figure 8. Sensitivity analysis for the discount factor

Results are reported in **Figure 8**.

Reducing β gives less weight to the welfare of future members of the dynasty, thus reducing the demand for children and lowering population growth. This implies that the comparative-static trajectory for population is lower than the baseline trajectory. Moreover reducing the discount factor implies a lower saving rate, so that there is less capital available for agricultural production, and more land is needed to compensate. However, by re-estimating the model to 1960-2010 data under the assumption $\beta = 0.97$, we find that the opposite is true. As compared to the baseline, a lower discount factor implies a higher long-run population, while the agricultural land area is smaller. As Table 2 shows, estimates of the cost of fertility imply higher labour productivity and more weakly decreasing returns to labour, and hence a lower marginal cost of fertility both within and across periods. In turn, the accumulation of labour becomes cheap relative to capital and land, incentivising the accumulation of population as a substitute for the accumulation of

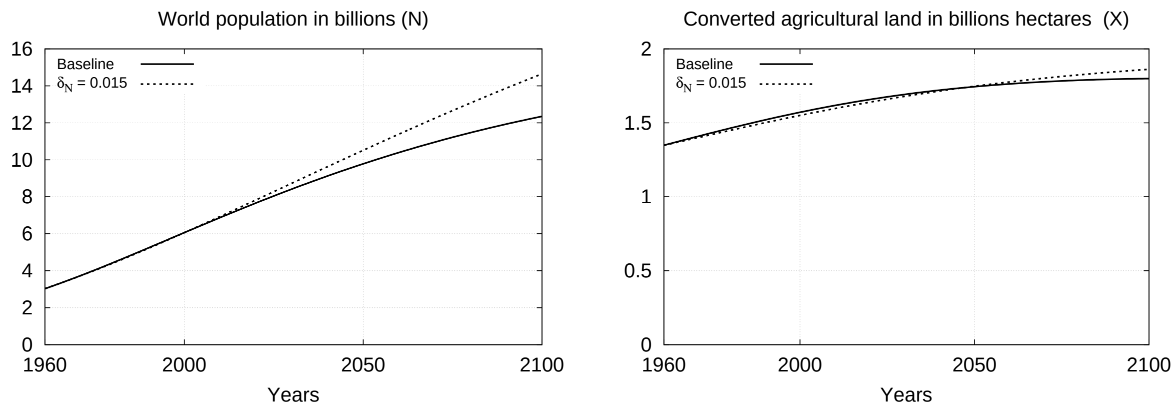

Figure 9. Sensitivity analysis on the expected working lifetime

capital and land. This result contrasts with changes in η, which did not directly affect incentives to accumulate capital and land.

The final sensitivity test is on the death rate δ_N, or equivalently the expected working lifetime $1/\delta_N$. We illustrate the effect of this parameter by using a somewhat extreme value of 65 years, corresponding to $\delta_N = 0.015$. Trajectories are reported in **Figure 9**. As expected this implies a larger long-run population, reaching more than 10 billon in 2050 and around 15 billion by 2100. The impact of this parameter is mostly felt in the long run, as it implies that the growth rate of population declines less rapidly over time, on account of the larger expected benefits associated with effective labour units. This result confirms the importance of δ_N as a driver of population dynamics, as demonstrated by Jones and Schoonbroodt (2010) and Strulik and Weisdorf (2014). In practice however a change of this magnitude is unlikely, as future increases in life expectancy will be at least partly compensated by an increase of mortality associated with an ageing population.

Overall, the sensitivity analysis shows how our estimates of technology parameters are affected by structural assumptions, but at the same time it shows that the resulting projections remain very similar. This can also be interpreted as evidence that the choice of a particular solution concept is unlikely to alter our main conclusions. If we solved for a competitive equilibrium instead of a social planner's allocation, while retaining the *baseline* vector of parameter estimates, externalities would imply that fewer resources are allocated to R&D, among other things. In turn, economic growth would be lower. However, if the model were re-estimated using this decentralized solution concept, technology estimates consistent with growth observed over the last fifty years would imply higher labour productivity in R&D activities, and in turn very similar growth trajectories.[32]

[32] Note that an important assumption here is the absence of a scale effect. If the model featured a scale effect, so that technological progress were a function of population, the planner could exploit it by generating higher popula-

5. DISCUSSION

Our integrated representation endogenises the evolution of quantities that are jointly determined, integrating plausible components from growth theory into an empirical framework. The dynamic relationship between these variables is informed by structurally estimating the model to minimise the distance between observed and simulated trajectories. Our model thus treats the representation of preferences and technology as fixed, with the dynamics being driven exclusively by structural assumptions. This contrasts with existing projections such as those of the United Nations (2013) and Alexandratos and Bruinsma (2012), which do not employ explicit behavioural assumptions and rather rely on exogenously determined drivers as the main source of variations.[33]

Overall, our results confirm the widespread expectation that the long-standing processes of growth in population and land conversion are in decline, and imply a "smooth landing". This stems from a quality-quantity trade-off: shifting from a quantity-based economy with rapid population growth and associated land conversion, towards a quality-based economy with investments in technology and education, and lower levels of fertility. Land is the first quantity to *endogenously* reach a steady state, doing so in the coming half-century. We find, however, that a halt in land conversion is consistent with sustained growth in food demand and agricultural output as well as mildly optimistic assumptions about technological progress in the future.

Structural estimation of the model across several interlinked outcomes and over a relatively long period of time implies that our quantitative results are quite robust to different assumptions. This is notably the case for the land constraint, which is unlikely to bind in most configurations. This result is consistent with the past fifty years, during which agricultural production almost tripled, while growth in agricultural land was below twenty percent. However, this does not imply that food will not remain a problem for many areas of the world. We take a highly aggregated view of the problem, and food security is very likely to remain of concern at the regional level. That is to say, our results should perhaps be interpreted in terms of *potential* food security.

6. CONCLUDING COMMENTS

One of the key challenges associated with global population growth is the ability of the economy to produce food. In this paper we have proposed a model in which population, technology and land use are jointly determined. Being based on plausible ingredients from the economic growth literature, we have shown that the model can match quite well the evolution of key economic time series over recent history. Our results suggest that sustained population growth over the coming century is compatible with an evolution of agricultural output close to what has been

tion growth and in turn higher economic growth. Because the long-run properties of the model would differ, an equilibrium with higher population would presumably prevail.

[33]For example the rapid decline of population growth towards zero implied by existing population projections is an outcome of the assumed convergence of fertility to its replacement level. The basis of this assumption is the observed convergence of *developed* countries to a low fertility regime, although implicitly it has strong implications in terms of technological progress and economic convergence.

observed in the past, mainly on account of technological change and capital accumulation. Furthermore, estimating the model over fifty years implies that our conclusions are fairly robust in their account of future long-run development.

One implication of our work is a novel perspective on population dynamics. Specifically, in our projections population growth declines over time but remains positive (and significantly so) in 2100. While uncertainty over such a time horizon cannot be overstated, a key finding of our analysis is therefore that population does not reach a steady state in the foreseeable future. Population growth falls more slowly than in the existing population projections of the United Nations (2013). We think this is plausible, because of the amount of inertia in the system, and because better economic prospects will sustain the demand for children despite an increasing cost associated with child-rearing and education. In our framework the slowdown of technology accumulation implies a slowdown in the decline in fertility, so to speak, so that the decline in population growth itself slows down.

While this work provides a first attempt to see future population development, technology and potential Malthusian constraints from the perspective of economic growth theory, our approach necessitated a number of simplifications and opens a number of avenues for future research. First, declining fertility implies population ageing, which may affect both the mortality rate and labour productivity, and in turn economic growth. For example, Mierau and Turnovsky (2014) include an age-structured population in a general equilibrium growth model, although they treat the demographic structure as exogenous for the model to remain tractable. Integrating a richer representation of population heterogeneity into a model with endogenous fertility remains an important research topic. Second, we have abstracted from uneven economic development across regions, whereas fundamental drivers of fertility and growth will differ across the globe. Regional heterogeneity also raises interesting questions related to international trade, migration, and technology diffusion. Third, we have focused on baseline trajectories consistent with recent history, and our framework also provides a rich empirical framework to study policies affecting key drivers of long-run growth. Finally, there may be factors (such as water) affecting the ability to produce food, which are not included in the model and whose scarcity may increase in the future. Incorporating such constraints would constitute another interesting area for future work.

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APPENDIX A: Derivations and Proofs

Derivation of the Objective Function

This section details the derivations necessary to obtain the dynastic (social) planner's utility, equations (2) and (10). Most of the steps involve standard assumptions and we closely follow Jones and Schoonbroodt (2010) in their treatment of a positive survival probability.

Starting from the recursively-defined utility function in equation (1):

$$U_t = u(c_t) + \beta b(n_t) \sum_{i=1}^{n_t} U_{i,t+1} \,,$$

we assume that (i) parents survive with probability $1 - \delta_N$, (ii) children are identical, and (iii) parents care about their (surviving) selves as much as they care about their children. This implies:

$$U_t = u(c_t) + \beta b((1 - \delta_N) + n_t)[(1 - \delta_N) + n_t]U_{t+1} \,.$$

Note that assuming $\delta_N = 1$ (agents live only one period) brings us back to the original Barro-Becker preferences considered in Section 2. Denoting $\tilde{n}_t = (1 - \delta_N) + n_t$, the utility of the dynastic head is obtained by sequential substitution starting from $t = 0$:

$$\begin{aligned}
U_0 &= u(c_0) + \beta b(\tilde{n}_0)\tilde{n}_0 U_1 \\
&= u(c_0) + \beta b(\tilde{n}_0)\tilde{n}_0[u(c_1) + \beta b(\tilde{n}_1)\tilde{n}_1 U_2] \\
&= u(c_0) + \beta b(\tilde{n}_0)\tilde{n}_0 u(c_1) + \beta^2 b(\tilde{n}_0) b(\tilde{n}_1)\tilde{n}_0\tilde{n}_1[u(c_2) + \beta b(\tilde{n}_2)\tilde{n}_2 U_3] \\
= \cdots &= \sum_{t=0}^{\infty} \beta^t u(c_t) \left(\prod_{\tau=0}^{t} b(\tilde{n}_\tau)\tilde{n}_\tau \right) + \lim_{t\to\infty} \beta^{t+1} \left(\prod_{\tau=0}^{t} b(\tilde{n}_\tau)\tilde{n}_\tau \right) U_{t+1}
\end{aligned}$$

where the limit term is assumed to be zero. We will further assume that the function $b(\cdot)$ has a standard constant elasticity form, $b(\tilde{n}) = \tilde{n}^{-\eta}$, and write population dynamics (13) as:

$$N_{t+1} = N_t + n_t N_t - \delta_N N_t = N_t[(1 - \delta_N) + n_t] = N_t \tilde{n}_t$$

and hence we have that

$$\begin{aligned}
\prod_{\tau=0}^{t} b(\tilde{n}_\tau) n_\tau &= \tilde{n}_0^{1-\eta} \cdot \tilde{n}_1^{1-\eta} \cdot \tilde{n}_2^{1-\eta} \cdot \ \ldots \ \cdot \tilde{n}_t^{1-\eta} \\
&= \left(\frac{N_1}{N_0}\right)^{1-\eta} \cdot \left(\frac{N_2}{N_1}\right)^{1-\eta} \cdot \left(\frac{N_3}{N_2}\right)^{1-\eta} \cdot \ \ldots \ \cdot \left(\frac{N_t}{N_{t-1}}\right)^{1-\eta} = \left(\frac{N_t}{N_0}\right)^{1-\eta} .
\end{aligned}$$

This gives the following expression for the time zero utility function:

$$U_0 = \left(\frac{1}{N_0}\right)^{1-\eta} \sum_{t=0}^{\infty} \beta^t u(c_t) N_t^{1-\eta}$$

where N_0 is a constant and does not affect choices. This is equation (10), while equation (2) can be obtained by recalling that $N_t^{1-\eta} = b(N_t)N$.

Proof of Lemma 1

Write the dynastic household's optimisation problem as

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} u(c_t)b(N_t)N_t + \mu_{t,N}\left[N_{t+1} - \chi(L_{t,N}, A_{t,mn})\right] + \mu_{t,X}\left[X_{t+1} - \psi(L_{t,X})\right] \\ +\theta_{t,X}[\overline{X} - \psi(L_{t,X})] + \theta_{t,N}\left[N_t - L_{t,mn} - L_{t,N} - L_{t,X} - L_{t,ag}\right] \\ +\theta_{t,ag}\left[A_{t,ag}Y_{ag}(L_{t,ag}, X_t) - \overline{f_t}N_t\right] \end{array} \right\}$$

Substituting in the budget constraint, $c_t = 1/N_t\ w_t L_{t,mn}$, the necessary first-order conditions for a maximum include that

$$\frac{\partial \mathcal{L}}{\partial L_{t,mn}} = u'(c_t)b(N_t)w_t - \theta_{t,N} = 0$$

$$\frac{\partial \mathcal{L}}{\partial L_{t,N}} = -\mu_{t,N}\frac{\partial \chi(L_{t,N}, A_{t.mn})}{\partial L_{t,N}} - \theta_{t,N} = 0$$

$$\frac{\partial \mathcal{L}}{\partial L_{t,ag}} = \theta_{t,ag}A_{t,ag}\frac{\partial Y_{ag}(L_{t,ag}, X_t)}{\partial L_{t,ag}} - \theta_{t,N} = 0$$

$$\frac{\partial \mathcal{L}}{\partial L_{t,X}} = (-\mu_{t,X} - \theta_{t,X})\psi'(L_{t,X}) - \theta_{t,N} \leq 0$$

The marginal effect on household welfare of fertility in period t, at the optimum, can be characterised as

$$\frac{\partial \mathcal{L}}{\partial N_{t+1}} = \beta u(c_{t+1})\left[b'(N_{t+1})N_{t+1} + b(N_{t+1})\right] + \mu_{t,N} + \beta\theta_{t+1,N} - \beta\theta_{t+1,ag}\overline{f_{t+1}} = 0$$

We now proceed by using the first-order conditions on the controls to eliminate the shadow prices. It is straightforward to verify that –

$$\mu_{t,N} = \left[-u'(c_t)b(N_t)w_t\right] \Big/ \frac{\partial \chi(L_{t,N}, A_{t,mn})}{\partial L_{t,N}}\ ,$$

$\theta_{t+1,N} = u'(c_{t+1})b(N_{t+1})w_{t+1}$ and

$$\theta_{t+1,ag} = \left[u'(c_{t+1})b(N_{t+1})w_{t+1}\right] \Big/ \left[A_{t+1,ag}\frac{\partial Y_{ag}(L_{t+1,ag}, X_{t+1})}{\partial L_{t+1,ag}}\right]$$

The Lemma follows immediately. □

Proof of Proposition 1

Partially differentiate (4) with respect to $A_{t,mn}$, i.e. compute $\dfrac{\partial^2 \mathcal{L}}{\partial N_{t+1} \partial A_{t,mn}}$, where $A_{t+1,mn} = (1 + g_{t,mn})A_{t,mn}$.

Using the condition that maximises firm profits, $A_{t,mn} Y'_{mn}(L_{t,mn}) = w_t$, the partial derivative of part (A) with respect to $A_{t,mn}$ is

$$u'(c_t) b(N_t) Y'_{mn} \left[\partial \chi(L_{t,N}, A_{t,mn}) / \partial L_{t,N} - A_{t,mn} \, \partial^2 \chi(L_{t,N}, A_{t,mn}) / \partial L_{t,N} \partial A_{t,mn} \right]$$

Since we assume that $\partial^2 \chi(L_{t,N}, A_t) / \partial L_{t,N} \partial A_t < 0$, this term is positive.

The partial derivative of part (B) with respect to $A_{t,mn}$ is

$$\left[\beta u'(c_{t+1}) b(N_{t+1}) Y'_{mn}(L_{t+1,mn})(1 + g_{t,mn}) \overline{f_{t+1}} \right] \Big/ \left[A_{t+1,ag} \frac{\partial Y_{ag}(L_{t+1,ag}, X_{t+1})}{\partial L_{t+1,ag}} \right]$$

This term is also positive. Part (C) is not a function of $A_{t,mn}$.

The partial derivative of part (D) with respect to $A_{t,mn}$ is

$$\beta u'(c_{t+1}) b(N_{t+1}) Y'_{mn}(L_{t+1,mn})(1 + g_{t,mn})$$

This term is again positive. Combining these three terms yields the Proposition. $\square$

APPENDIX B: Observed and simulated data

The table below reports both observed and simulated data from 1960 to 2100, by 10-year intervals. Note that agricultural area is only available for 2005.

Year	Population (billion)		Population growth (%)		Crop land area (billion ha)		GDP (trillions 1990 intl. $)	
	Observed	Simulated	Observed	Simulated	Observed	Simulated	Observed	Simulated
1960	3.03	3.03	0.021	0.022	1.37	1.35	8.79	9.5
1970	3.69	3.74	0.020	0.020	1.41	1.41	14.46	14.3
1980	4.45	4.51	0.018	0.018	1.43	1.47	19.98	20.6
1990	5.32	5.32	0.015	0.015	1.47	1.52	26.88	28.5
2000	6.13	6.14	0.012	0.013			36.93	38.0
2005					1.59	1.60		
2010	6.92	6.95	0.011	0.011		1.62	49.97	48.6
2020		7.74		0.010		1.65		60.5
2030		8.49		0.009		1.69		73.2
2040		9.19		0.007		1.71		86.6
2050		9.85		0.006		1.73		100.5
2060		10.46		0.006		1.75		114.5
2070		11.02		0.005		1.76		128.5
2080		11.53		0.004		1.77		142.4
2090		12.00		0.004		1.77		156.1
2100		12.42		0.003		1.77		169.3

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