TITLE:

Polar vortex formation in giant planet atmospheres under moist convection

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ABSTRACT:

A strong cyclonic vortex has been observed on each of Saturn's poles, coincident with a local maximum in observed tropospheric temperature[1,2,3]. Neptune also exhibits a hot, though much more transient[4], region on the South Pole. The creation and maintenance of Saturn's polar vortices, and their presence or absence on the other giant planets, are not understood. Additionally, highly energetic, small-scale storm-like features have been observed on each of the giant planets, originating from the water cloud level or perhaps lower. Previous studies suggest that these small storms are moist convective and play a significant role in global heat transfer from the hot interior to space[5,6]. Here we show that simple 'storm' forcing, motivated by moist convection, can create a strong polar cyclone through the depth of the troposphere. Using a shallow water model, we find that shallow polar flows on giant planets may be qualitatively expressed by two parameters: a scaled planetary size and a scaled energy density of the atmosphere. We also suggest that the observed difference in a typical eddy length scale between Saturn and Jupiter may preclude a Jovian polar cyclone, a question that will be resolved by the *Juno* mission in 2016.

BODY:

Saturn's polar cyclones have been compared to terrestrial hurricanes[2]. Saturn cannot harbor classical hurricanes because there is no thermal discontinuity such as a sea surface from which they can gain energy. Additionally, on Earth, wind stress across the frictional sea surface induces convergence of cyclonic flows, and gas giants lack a source of such stress in the weather layer. There must be another persistent energy source for these longlived, highly stable[7] cyclones, and it may be moist convection driven by Saturn's hot interior, which is also considered a leading candidate for the maintenance of the jets[8].

Moist convection, which commonly manifests as cumulus clouds, should induce localized divergence at its top, consistent with observations of storms on Jupiter[5]. In the neighborhood of the south polar vortex on Saturn, the majority of small bright cloud features with measurable relative vorticity were found to be anticyclonic[2]. We suggest that a fraction of the anticyclonic anomalies, co-located with small cloudy features, are the tops of moist convective storms with vertical vorticity dipoles, implying cyclonic counterparts at depth (Figure 1).

These vorticity anomalies react to the planetary vorticity gradient differently[9], due to nonlinear advective interaction with surrounding fluid. While anticyclonic anomalies should migrate equatorward[10], cyclonic anomalies should migrate poleward; in each case, until their magnitude equals the magnitude of the background vorticity[8]. This latitudinal advection, together with a significant zonal component, is called beta drift, and is responsible for much of the motion of hurricanes on Earth. Previous work examined the effect of beta drift in polar vortex formation in single-layer models[8], finding that anticyclonic 'patches' do move equatorward and cyclonic patches condense into a larger circumpolar vortex.

The previous models use single-layer forcing and are too simple to say anything about the vertical structure of the forcing or resulting vortex. GCM simulations also frequently exhibit a circumpolar vortex[11,12]. However, the energy injection in these and many similar models is commonly concentrated at a particular wavelength, and does not include any moist convective analogue, in which a scale separation exists between fast rising motions and broad, slow subsidence. We hypothesize that the vertical dipoles of vorticity anomalies, representing moist convection, may separate due to opposing meridional migration. The resulting flow may then be governed primarily by single-layer dynamics.

Since the atmospheres of the giant planets merge smoothly into their interiors, we use a 2.5 layer 'reduced gravity' shallow water model with an abyssal lower layer, which is more realistic for a gas giant than a rigid bottom boundary. We employ the shallow water equations without the quasi-geostrophic approximation because Saturn's polar cyclones appear at least as deep as extant observations (~1 bar[7]) and likely indicate significant pressure perturbations in the polar region.

At the large scale, energy is injected and removed solely through adjustments in the layer thickness perturbations. Energy injection simulates 'storms'[13] by thinning the bottom layer in small Gaussian perturbations randomly distributed around the domain, and by increasing the thickness of the top layer immediately above. Layer mass is conserved at each time step via subsidence. The horizontal velocity field responds by trying to reach geostrophic balance, creating vertically stacked counter-rotating vortices. Energy is removed through a simple Rayleigh damping scheme on each layer's thickness perturbations, to simulate radiative cooling[11].

The model is on a Cartesian grid with a pole at the center, which avoids the polar singularity seen in spectral and latitude-longitude grids. The spherical curvature near the pole is approximated by the 'polar β-plane', where the total Coriolis frequency is represented by a Taylor expansion about the pole, $2\Omega - \beta r^2$. Here Ω is the angular speed of the planet and r is the distance from the pole. The parameter $\beta = 2\Omega/(2a^2)$ for planetary radius a.

This is the simplest model that permits a realistic, vertically variable (baroclinic) forcing to create a broad, vertically homogeneous (equivalent barotropic) vortex in the weather layer, while forcing and dissipating only potential energy, which is relevant for upper atmospheres of gas giants with no surface. Here we examine which aspects of baroclinic, moist-convective forcing are conducive to polar vortex genesis. A key length scale is the internal Rossby radius, which is closely associated with moist convection and eddies that possess available potential energy (APE).

The nondimensionalized model has 11 control parameters including the Burger number Br_2 (internal Rossby radius L_{D2} squared over storm size squared), a convective Rossby number[14] (a convective vertical velocity scaled by the layer height and Coriolis frequency), and dimensionless storm lifetime and frequency. Simulations in statistical equilibrium exhibit behavior that falls into several broad regimes that can approximately be expressed[15] in a 2-dimensional parameter space: a nondimensional $\tilde{\beta}$ = ($L_{D2}^2/2a^2$) and a nondimensional 'energy parameter[13]' E_p (energy density scaled by Br_2). The only energy source considered is latent heating from moist convection driven by the hot interior of the planet[16]; seasonal insolation is neglected. The regimes (Figure 2) span all of the polar behavior observed so far on the giant planets in our solar system, as well as the circumpolar cyclone precession commonly seen in simulations[8,11,17] and weak, eddy-driven jet behavior without a polar cyclone. In all simulations with sufficiently high E_p, an energy cascade from the internal Rossby radius toward the larger external Rossby radius allows coherent equivalent barotropic vortices to form and merge (Figure 3). This proves essential for polar cyclone genesis.

For simulated planets with an internal Rossby radius 30 times smaller than the planetary radius or less (we will call these planets `small'), $\tilde{\beta}$ is relatively large and storms experience more poleward drift before they are dissipated or sheared. If a polar cyclone forms, it is relatively more stable on the pole than for 'larger' planets, because of the strong restoring force of $\tilde{\beta}$. On a small planet, the major determining factor of whether there will be a polar cyclone is the energy parameter E_p. If it is too low, storms radiate energy away as Rossby waves[18] before they can be meaningfully advected by the beta drift mechanism. The turbulence forms weak, eddy driven jets that fill the domain. Medium values of E_p cause a very asymmetric and time-varying polar concentration of cyclonic vorticity. Larger E_p causes a symmetric, largely barotropic cyclone, which wobbles within one or two Rossby radii from the pole. As E_p continues to increase, this vortex begins to precess around the pole. This precession is reminiscent of the polar vortex simulations by other authors mentioned above, and has yet to be observed on a real planet.

'Large' planets (internal Rossby radius 40 or more times smaller than the planetary radius: small $\tilde{\beta}$) experience a different set of regimes with increasing E_p. Low E_p simulations are similar to those for a small planet, though with a higher number of weak jets. Rossby wave radiation prevents cyclone growth and merger, but as E_p increases there is no polar concentration of vorticity. This is because $\tilde{\beta}$ is so low that its effect on storm motion is smaller than the influence of neighboring storms. Instead, with increasing E_p, multiple coherent vortices form, grow and move about the domain, virtually unaffected by the location of the pole.

The two dimensions $\tilde{\beta}$ and E_p can only be roughly estimated given current observations. It is perhaps coincidence that actual planet size and nondimensional size ($a/L_{D2}=(2\tilde{\beta})^{-1/2}$) among the internally heated giant planets are ordered similarly: Jupiter[19] > Saturn[20] > Neptune[21,22], given estimates of Rossby radii. On the other hand, the Rossby radius may be a function of water abundance[23], which may in turn be a function of planetary formation and mass[24]. The internal heat flux is also directly proportional to total planetary mass[25], and if one assumes consistent energy partitioning to moist convection across Jupiter, Saturn and Neptune, then E_p is also directly proportional to mass. However E_p is highly unconstrained, and even in this simple model is a function of 11 parameters. Jupiter's poles have not been directly imaged, but their near environment lacks significant jets (between 70 and 80 degrees poleward)[26], unlike Saturn. If Jupiter and

Saturn have similar E_p, the difference in $\tilde{\beta}$ may be sufficient to yield polar cyclones only on Saturn.

An interesting difference exists between polar cyclone genesis and polar cyclone maintenance. We find that early on in the simulations, the storm strength is a very important predictor of whether or not a polar cyclone will form, as it controls the magnitude of nonlinear advection. These simulations are initiated with no horizontal wind and so only the beta drift can separate the vorticity anomalies as they develop. However, in mature simulations with a strong polar vortex, horizontal winds can be quite large and storms get sheared into the mean flow as fast as they are injected, which greatly reduces their anomalous vorticity amplitude. The reason that the vortex strength doesn't oscillate in time with this apparent weaker forcing is due to a symmetric region of low but positive vorticity gradient around the polar cyclone (known in hurricane meteorology as a "β-skirt"[27]). In mature simulations of 'small' planets, the actual vorticity gradient that small storms feel is highest in the neighborhood of the polar cyclone (Figure 4), even though the planetary contribution to this gradient goes to zero. This allows mature storms to maintain their strength and stability on the pole. This finding is consistent with Saturn's polar relative vorticity gradients[28], and the observation that few convective features are found within the β-skirt around the south polar vortex[2].

This study offers a weather layer theory for polar vortex genesis and maintenance. By limiting ourselves to mechanisms that are plausible in giant planet atmospheres, we can explore the importance of different parameters for polar flow. We show that the ratio of the internal Rossby radius to the planetary radius is enough to determine the presence or absence of a polar vortex, and that the threshold is modulated by E_p. However, other notable differences between the planets' tropospheres may instead be the culprits, and our model is too simple to account for thermodynamical parameters such as water abundance and latent heating[29], as well as the varying depths of cloud formation. Strong observed horizontal shears violate the barotropic stability criterion in Saturn's subpolar jets, and the present model domain is not large enough to simulate and study them. Additionally, the poles are the only place where the buoyancy and rotational vectors are parallel. This unique alignment may implicate the deep interior in ways that we can't address in a shallow, layered model. Cassini's imminent high eccentricity polar orbit around Saturn will complement Juno's polar orbit from 2016-2018, which will provide detailed observations of the Jovian poles for the first time. These observations will help inform and constrain theories of polar vortex formation.

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METHODS:

Model formulation:

The 2 ½ layer model assumes an infinitely deep and quiescent bottom layer, which precludes a barotropic mode. There is a first baroclinic mode, also known as the 'equivalent barotropic mode' in a reduced gravity model; and a second baroclinic mode. Because the system is nonlinear and divergent, these modes are coupled and cannot fully describe its behavior; yet they provide more physically relevant gravity wave speeds than those for each layer. The second baroclinic mode is associated with the smallest deformation radius ("Rossby radius") of the system, which is the dominant mode of vertical moist convection. We normalize our model by this radius in order to ensure consistent resolution of small scale enstrophy and vortical filaments.

The baroclinic gravity wave speeds can be expressed as a linear combination of layer gravity wave speeds. Assume modal solutions to the linearized, non-rotating system such that $u_2' = \mu u_1'$ and $h_2' = (H_2/H_1)\mu h_1'$ and let c_1 and c_2 be the upper and lower gravity wave speeds respectively; then:

$$\mu^2 + \left(\frac{c_1^2}{c_2^2} - 1\right)\mu - \frac{\rho_1}{\rho_2}\frac{H_1}{H_2} = 0$$

Our first and second baroclinic (squared) gravity wave speeds are, respectively:

$$c_{e1}^2 = c_1^2 + m^{+} c_2^2$$
$$c_{e2}^2 = c_1^2 + m^{-} c_2^2$$

We scale our dimensional parameters (Table 1 in Supplementary Information) in the following way:

$$\left(x, y\right) = L_{D2}(x^*, y^*)$$
$$h_i = H_i h_i^*$$
$$\vec{u}_i = c_{e2}\vec{u}_i^*$$
$$t = f_0^{-1} t^*$$

where asterisks indicate dimensionless parameters. The nondimensional control parameters are listed in Table 2 of the Supplementary Information.

The model equations are (i=1 is the upper layer; primes are dropped):

$$\frac{\partial \vec{u}_i}{\partial t} = -\left(1 - \tilde{\beta}|\vec{x}_i|^2 + \xi_i\right)\hat{k} \times \vec{u}_i - \nabla\left(\gamma^{i-1}\tilde{c}_1^2 h_1 + \tilde{c}_2^2 h_2 + \frac{1}{2}|\vec{u}_i|^2\right) - \mathrm{Re}^{-1}\nabla^4\vec{u}_i;$$

$$\frac{\partial h_i}{\partial t} = -\nabla \cdot (\vec{u}_i h_i) + \left(-\frac{H_1}{H_2}\right)^{i-1} S_{st} - \frac{h_i - 1}{\tilde{\tau}_{rad}} + \mathrm{Pe}^{-1}\nabla^2 h_i.$$

The forcing function induces storms that are Gaussians in space and boxcars in time:

$$S_{st} = \begin{cases} \displaystyle\sum_{j=1}^{\#} \mathrm{Ro}_{conv} \exp\left[-\mathrm{Br}_2 \frac{\left(\vec{x} - \vec{x}_j\right)^2}{0.36}\right] + \text{subsidence}, & \text{for } \tilde{t}_{clock} \le \tilde{\tau}_{st} \\ 0, \ \text{for } \tilde{\tau}_{st} < \tilde{t}_{clock} \le \tilde{\tau}_{stper} & \end{cases}$$

for a $\tilde{t}_{clock}$ that resets to 0 every time it reaches $\tilde{\tau}_{stper}$.

The parameter $\gamma = \frac{\rho_1}{\rho_2}\frac{c_2^2}{c_1^2}\frac{H_1}{H_2}$ and is equivalent to γ in the 2 ½ layer model of Ref. 30.

The simulated areal fraction of storm coverage $Ar = (\#\pi)/(Br_2 L_{dom}^2)$ is on average 0.075. This is likely an overestimate of planetary storm coverage, because abundant observed anticyclones often have a long lifetime[31], and mass continuity implies that only a small fraction of them are convecting through the weather layer at any time. However, our model is also overdamped by at least one order of magnitude[32], so the overforcing may not strongly affect the steady state behavior.

Energy parameter:

Following Ref. 12, we derive a scaling for the energy density and modify it by the Burger number.

$$E_p = \left(\frac{1}{2}\frac{\rho_1}{\rho_2}\tilde{c}_1^2 + \frac{1}{2}\frac{H_1}{H_2}\tilde{c}_2^2 - \gamma\tilde{c}_1^2\right)\frac{H_1}{H_2}(\mathrm{Ro}_{conv}\tilde{\tau}_{st})^2\,\frac{Ar}{1-Ar}\frac{\tilde{\tau}_{rad}}{\tilde{\tau}_{stper}}\frac{1}{\mathrm{Br}_2}$$

where L_{dom}^2 is the domain area.

Numerical considerations:

The Cartesian grid is a staggered Arakawa C-grid. The time-stepping scheme is a 2nd order Adams-Bashforth algorithm. Early tests showed that this provided dynamics nearly identical to the 3rd order Adams-Bashforth scheme. Horizontal hyperviscosity ∇^4 is used instead of viscosity to reduce its impact on the dynamics, which at upper levels on giant planets is virtually inviscid.

For most simulations we impose a resolution constraint on the second baroclinic Rossby radius of $L_{D2} = 5dx$. The equilibrium behavior is found to be relatively insensitive to scaled energy density. We were unable to simulate a planet with $\tilde{\beta}$ relevant (large enough) for Neptune, because its $\tilde{\beta}$ is significantly higher than likely values for Saturn and Jupiter, and the 5dx resolution of the Rossby radius would severely under-resolve Neptune within limits of valid polar β-plane approximation. However, Neptune observations are consistent with low E_p and high $\tilde{\beta}$ qualitatively.

The model is highly dissipative, which is unfortunate but a necessary tradeoff for computational speed, given the enormous parameter space to explore. The Reynolds and Peclet numbers are fixed at the highest value that empirically permits consistent numerical stability (5e4 and 1e5 respectively). This may not strongly impact the dynamics however, because the radiative timescale is very short. Ref. 32 use a simple model of the giant planet atmospheres and consider the frictional time constant as the independent parameter. They find that a frictional time constant is on the same order as the radiative time constant for the giant planets. Here it is one or two orders higher, which suggests that dissipation will not affect the outcome at equilibrium - provided the storm timescale remains much shorter than the radiative timescale, which in all cases presented is true.

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CODE AVAILABILITY: The MATLAB model and output are available upon request from M.E.O.

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FIGURE LEGENDS:

Figure 1: **A schematic of a giant planet troposphere with moist convection**. The shallow troposphere on internally heated giant planets lies below the stratosphere, which

is highly stably stratified, and above an abyssal convective interior. In the troposphere condensable materials like water and ammonium hydrosulfide are able to release latent heat in convecting clouds. Vorticity anomalies may react differently to the planetary vorticity gradient, depending on their sign, leading to a vertical shearing of the convective storm. If the planetary vorticity gradient is high enough, positive anomalies will self-advect poleward and negative anomalies will self-advect equatorward.

Figure 2: **A set of regimes that spans likely planetary polar behavior.** In these simulations only $\tilde{\beta}$ and Ro_{conv} (a proxy for E_p) are varied. Both colors and contours show depth-integrated, time-averaged potential vorticity. Regimes similar to observations of Neptune and Saturn are identified. Jupiter's regime is also speculated. Neptune's very high $\tilde{\beta}$ value was not simulated but simulations of high $\tilde{\beta}$, low E_p consistently demonstrate a transient concentration of polar cyclonic vorticity, concurrent with a transient warm anomaly. Time averaging causes polar regions with randomly-moving vortices to appear smeared; instantaneous fields would exhibit the strongest cyclones for the highest E_p simulations.

Figure 3: **The evolution of a polar cyclone via vortex merger**. The three panels show instantaneous snapshots from the evolution of a simulation with high $\tilde{\beta}$ and high E_p, from left to right. The nondimensional perturbation potential vorticity of the lower model layer has been plotted. The left panel shows a field filled with small storms. The middle panel

shows a snapshot just before vortex merger of the domain's two strongest cyclones. At the end of the simulation, the main polar cyclone is statistically steady and dominates the domain.

Figure 4: **'Small' planets with high energy have a significant β-skirt**. The layer-, azimuthal- and time-averaged radial PV gradient is shown for a range of $\tilde{\beta}$ and E_p values. The black line is the Coriolis gradient, $df/dr = -2\tilde{\beta}r$, for comparison. The largest vortex gradient, or β-skirt, conducive to beta drift is exhibited by high $\tilde{\beta}$, high E_p simulations. The vorticity gradient due to a mature polar vortex can be significantly stronger than the background Coriolis gradient.