Diagnosing Chaos Using Four-Point Functions in Two-Dimensional Conformal Field Theory

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(Received 10 March 2015; revised manuscript received 14 May 2015; published 22 September 2015)

We study chaotic dynamics in two-dimensional conformal field theory through out-of-time-order thermal correlators of the form $\langle W(t)VW(t)V\rangle$. We reproduce holographic calculations similar to those of Shenker and Stanford, by studying the large c Virasoro identity conformal block. The contribution of this block to the above correlation function begins to decrease exponentially after a delay of $\sim t_* - (\beta/2\pi)\log\beta^2 E_w E_v$, where t_* is the fast scrambling time $(\beta/2\pi)\log c$ and E_w, E_v are the energy scales of the W, V operators.

DOI: 10.1103/PhysRevLett.115.131603 PACS numbers: 11.25.Hf, 03.67.-a, 05.45.Mt

Introduction.—In classical mechanics, the butterfly effect is a vivid diagnostic of chaos: small perturbations grow rapidly to affect the entire system. In quantum mechanics, studies of chaos have often focused on less direct measures, such as the statistics of level spacings in the energy spectrum [1,2], or the local properties of energy eigenvectors [3–5]. In this Letter, following Refs. [6–9] we will focus on a notion of quantum chaos that is closely related to the classical butterfly effect.

Consider a pair of rather general Hermitian operators V and W in a quantum mechanical system. If the system is strongly chaotic, we expect perturbations by V to affect later measurements of W, for almost any choice of operators V and W [10]. In other words, we expect the commutator $[V, W(t)]$ to become large. A useful diagnostic is the expectation value of the square of the commutator [11]:

$$-\langle [V, W(t)]^2\rangle_\beta = \langle VW(t)W(t)V\rangle_\beta + \langle W(t)VVW(t)\rangle_\beta \quad (1)$$

$$- \langle VW(t)VW(t)\rangle_\beta - \langle W(t)VW(t)V\rangle_\beta. \quad (2)$$

Here, $\langle\cdot\rangle_\beta$ indicates the thermal expectation value at inverse temperature β. Let us assume that the operators commute at $t = 0$. For small times, the terms on the first line cancel the terms on the second line. As we move to large time, the terms on the first line will each approach the order-one value $\langle WW\rangle_\beta\langle VV\rangle_\beta$. This can be understood by viewing the $VWWV$ ordering as an expectation value of WW in a state given by acting with V on the thermal state. If the energy injected by V is small, the state will relax and the expectation value will approach the thermal value, $\langle WW\rangle_\beta$, multiplied by the norm of the state, $\langle VV\rangle_\beta$.

By contrast, in a suitably chaotic system, the correlation functions on the second line will become small for large t. From Eq. (1), it is clear that this will imply a large commutator, and hence a quantum butterfly effect. We believe that this happens for practically any [12] choice of W and V and that, in fact, this behavior is a basic diagnostic of quantum chaos.

This behavior has been confirmed for theories holographically dual to Einstein gravity in recent work [6–9,13]. There, the thermal state is represented by a black hole, and the V, W operators create quanta that collide near the horizon. The key effect leading to a large commutator is the exponential blueshift relative to later slices as the W perturbation falls into the black hole.

The purpose of this Letter is to reproduce part of that analysis without using holography directly. We will work in a 2D conformal field theory (CFT) where thermal expectation values are related to vacuum expectation values by a conformal transformation. We will also restrict attention to a particular contribution to the four-point function given by the large c Virasoro identity conformal block. This resums the terms corresponding to factorization on powers and derivatives of the stress tensor. The close relationship between $2+1$ gravity and the identity Virasoro block has been demonstrated recently in Refs. [14–16]; our work should be understood as an application of the techniques in these papers to the problem studied in Refs. [6–9,13]. We will find exact agreement between the Virasoro block calculation and the corresponding holographic calculation in pure 3D gravity. In particular, we will find that the contribution of the identity block to $\langle VW(t)VW(t)\rangle_\beta$ begins to decay exponentially around the fast scrambling time [17,18] $t_* = (\beta/2\pi)\log c$.

A key point that will emerge in our analysis is the following. Each of the Lorentzian correlators on the right-hand side of Eq. (1) can be obtained by analytic continuation of the same Euclidean four-point function. The sharp difference in behavior between the first line and

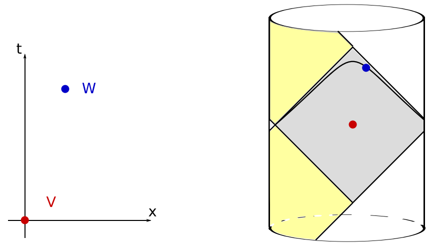

FIG. 1 (color online). Left: The spacetime arrangement of the W and V operators. Right: Their locations after the conformal mapping, viewed in the Rindler patch on the boundary of AdS_3 (gray) covered by x, t. The union of the gray and yellow regions is the Poincaré patch covered by $z, \bar{z}$.

the second line arises because the continuation defines a multivalued function, with different orderings corresponding to different sheets. To see the butterfly effect (via the decay of the terms on the second line), one has to move off the principal sheet.

It is important to emphasize that although the Virasoro identity block does reproduce the gravitational calculation of the four-point function, it is not the full answer. Indeed, a single Virasoro primary with spin greater than two can easily dominate the contribution from the identity in a certain range of times t. In holographic terms, this sensitivity to the spectrum is related to the fact that the high-energy collision depends on stringy corrections. Just as the gravitational calculation is a useful model for a more accurate string-corrected analysis [19], the Virasoro identity block provides a model for a more complete CFT calculation.

Although it is not our purpose to prove that the four-point-function diagnostic described above agrees with other definitions of quantum chaos, we will provide a sanity check by evaluating the above four-point functions in the two-dimensional Ising model. For this system, we will see that certain out-of-time-order four-point functions do not tend to zero for large t.

CFT calculations.—Conventions and review: In this Letter, we will study thermal four-point correlation functions of W and V of the form in Eqs. (1) and (2). Eventually, these operators will be arranged in the timelike configuration shown in Fig. 1, where V is at the origin and W is at position $t > x > 0$. However, we will obtain these correlation functions by starting with the Euclidean correlator and analytically continuing. In 2D CFT, we can map thermal expectation values to vacuum expectation values through the conformal transformation

$$z(x,t) = e^{(2\pi/\beta)(x+t)}, \qquad \bar{z}(x,t) = e^{(2\pi/\beta)(x-t)}. \tag{3}$$

Here, x, t are the original coordinates on the spatially infinite thermal system and $z, \bar{z}$ are coordinates on the vacuum system. Explicitly,

$$\langle \mathcal{O}(x,t)\cdots\rangle_\beta = \left(\frac{2\pi z}{\beta}\right)^h \left(\frac{2\pi \bar{z}}{\beta}\right)^{\bar{h}} \langle \mathcal{O}(z,\bar{z})\cdots\rangle, \tag{4}$$

where $h, \bar{h}$ are the conformal weights of the $\mathcal{O}$ operator, related to the dimension and spin by $\Delta = h + \bar{h}$ and $J = h - \bar{h}$. On the left-hand side, we have a thermal expectation value, at inverse temperature β, and on the right-hand side we have a vacuum expectation value on the $z, \bar{z}$ space. It is common to work with units in which $\beta = 2\pi$, but we prefer to keep the β dependence explicit.

It will be essential in this Letter to study correlation functions with operators at complexified times t_i. In our convention, real t corresponds to Minkowski time and imaginary t corresponds to Euclidean time. Notice from Eq. (3) that $\bar{z}$ is the complex conjugate of z only if the time t is purely Euclidean. In order to make contact with standard CFT formulas for the four-point function, we will begin with a purely Euclidean arrangement of the operators. This means a choice of $z_1, \bar{z}_1, \ldots, z_4, \bar{z}_4$ with $\bar{z}_i = z_i^*$. With such a configuration, the ordering of the operators is unimportant, and global conformal invariance on the $z, \bar{z}$ plane implies that the four-point function can be written,

$$\begin{aligned}&\langle W(z_1,\bar{z}_1)W(z_2,\bar{z}_2)V(z_3,\bar{z}_3)V(z_4,\bar{z}_4)\rangle \\ &\quad = \frac{1}{z_{12}^{2h_w} z_{34}^{2h_v}} \frac{1}{\bar{z}_{12}^{2\bar{h}_w} \bar{z}_{34}^{2\bar{h}_v}} f(z,\bar{z}),\end{aligned} \tag{5}$$

in terms of a function f of the conformally invariant cross ratios

$$z = \frac{z_{12}z_{34}}{z_{13}z_{24}}, \qquad \bar{z} = \frac{\bar{z}_{12}\bar{z}_{34}}{\bar{z}_{13}\bar{z}_{24}}, \qquad z_{ij} \equiv z_i - z_j. \tag{6}$$

According to the general principles of CFT, we can expand f as a sum of global conformal blocks, explicitly [20,21],

$$f(z,\bar{z}) = \sum_{h,\bar{h}} p(h,\bar{h}) z^h \bar{z}^{\bar{h}} F(h,h,2h,z) F(\bar{h},\bar{h},2\bar{h},\bar{z}), \tag{7}$$

where F is the Gauss hypergeometric function, the sum is over the dimensions of global $SL(2)$ primary operators, and the constants p are related to operator product expansion (OPE) coefficients $p(h,\bar{h}) = \lambda_{WW\mathcal{O}_{h,\bar{h}}} \lambda_{VV\mathcal{O}_{h,\bar{h}}}$.

Continuation to the second sheet: In order to apply the above formulas to the correlators (1) and (2), we need to understand how to obtain them as analytic continuations of the Euclidean four-point function. That this is possible follows from the fact that all Wightman functions are analytic continuations of each other [22]. The procedure involves three steps. First, one starts with the Euclidean function, assigning small and different imaginary times $t_j = i\epsilon_j$ to each of the operators. Second, with the imaginary times held fixed, one increases the real times of the operators to the desired Lorentzian values. Finally, one smears the operators in real time and then takes the imaginary times $\{\epsilon_i\}$ to zero [24]. The result will be a Lorentzian correlator ordered such that the leftmost operator corresponds to the smallest value of ϵ, the second operator corresponds to the second smallest, and so on.

This elaborate procedure is necessary because Eq. (5) is a multivalued function of the independent complex variables

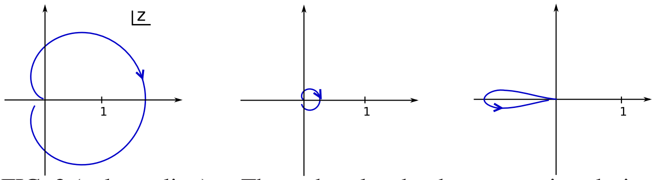

FIG. 2 (color online). The paths taken by the cross ratio z during the continuations corresponding to (from left to right) $\langle WVWV\rangle$, $\langle WWVV\rangle$, and $\langle WVVW\rangle$. Only in the first case does the path pass around the branch point at $z=1$.

$\{z_i,\bar{z}_i\}$. The interesting multivaluedness comes from $f(z,\bar{z})$. By crossing symmetry, this function is single valued on the Euclidean section $\bar{z}=z^*$, but it is multivalued as a function of independent z and $\bar{z}$, with branch cuts extending from one to infinity. Different orderings of the W,V operators correspond to different sheets of this function. To determine the correct sheet, we must assign $i\epsilon$'s as above, and follow the path of the cross ratios, watching to see if they pass around the branch loci at $z=1$ and $\bar{z}=1$.

To carry this out directly, we write

$$z_1=e^{(2\pi/\beta)(t'+i\epsilon_1)},\qquad \bar{z}_1=e^{-(2\pi/\beta)(t'+i\epsilon_1)}, \tag{8}$$

$$z_2=e^{(2\pi/\beta)(t'+i\epsilon_2)},\qquad \bar{z}_2=e^{-(2\pi/\beta)(t'+i\epsilon_2)}, \tag{9}$$

$$z_3=e^{(2\pi/\beta)(x+i\epsilon_3)},\qquad \bar{z}_3=e^{(2\pi/\beta)(x-i\epsilon_3)}, \tag{10}$$

$$z_4=e^{(2\pi/\beta)(x+i\epsilon_4)},\qquad \bar{z}_4=e^{(2\pi/\beta)(x-i\epsilon_4)} \tag{11}$$

as a function of the continuation parameter t'. When $t'=0$, we have a purely Euclidean correlator, on the principal sheet of the function $f(z,\bar{z})$. When $t'=t>x$, we have an arrangement of operators as shown in Fig. 1.

The cross ratios $z,\bar{z}$ are determined by these coordinates as in Eq. (6). Their paths, as a function of t', depend on the ordering of operators through the associated $i\epsilon$ prescription. Representative paths for the three cases of interest are shown in Fig. 2. The variable $\bar{z}$ never passes around the branch point at one, and the z variable does so only in the case corresponding to $WVWV$ [25].

In the final configuration with $t'=t$, the cross ratios are small. For $t\gg x$, we have

$$z\approx -e^{(2\pi/\beta)(x-t)}\epsilon_{12}^*\epsilon_{34},\qquad \bar{z}\approx -e^{-(2\pi/\beta)(x+t)}\epsilon_{12}^*\epsilon_{34}, \tag{12}$$

where we introduced the abbreviation

$$\epsilon_{ij}=i(e^{(2\pi/\beta)i\epsilon_i}-e^{(2\pi/\beta)i\epsilon_j}). \tag{13}$$

For the orderings $WWVV$ and $WVVW$, no branch cuts are crossed, so the limit of small cross ratios can be taken on the principal sheet of Eq. (7). The contribution from the identity operator dominates, verifying our statement in the Introduction that both $\langle W(t)VVW(t)\rangle_\beta$ and $\langle W(t)W(t)VV\rangle_\beta$ approach $\langle WW\rangle\langle VV\rangle_\beta$ for large t.

For $WVWV$, z passes around the branch point at one. The hypergeometric function $F(a,b,c,z)$ has known monodromy around $z=1$, returning to a multiple of itself, plus a multiple of the other linearly independent solution to the hypergeometric equation, $z^{1-c}F(1+a-c,1+b-c,2-c,z)$. For small $z,\bar{z}$, we then have

$$f(z,\bar{z})\approx\sum_{h,\bar{h}}\tilde{p}(h,\bar{h})z^{1-h}\bar{z}^{\bar{h}}, \tag{14}$$

where $\tilde{p}$ has been defined to absorb the transformation coefficient. On this sheet, as $z,\bar{z}$ become small, global primaries with large spin become important. As a function of x,t, individual terms in this sum grow like $e^{(h-\bar{h}-1)t}e^{-(h+\bar{h}-1)x}$. For sufficiently large t, this sum diverges, and it must be defined by analytic continuation. In other words, we must do the sum over $h,\bar{h}$ before we continue the cross ratios. In a CFT dual to string theory in AdS_3, we expect this divergence even at a fixed order in the large c expansion, because of the sum over higher spin bulk exchanges [19].

Virasoro identity block: The primary focus of this Letter is reproducing the Einstein gravity calculation of the correlation function. This calculation was done by studying free propagation on a shock wave background, which implicitly sums an infinite tower of ladder exchange diagrams. In the CFT, these diagrams are related to terms involving powers and derivatives of the stress tensor in the OPE representation of the four-point function. In a two-dimensional CFT, all such terms can be treated simultaneously using the Virasoro conformal block of the identity operator, which itself is an infinite sum of $SL(2)$ conformal blocks. Including only these terms in the OPE amounts to replacing

$$f(z,\bar{z})\to\mathcal{F}(z)\bar{\mathcal{F}}(\bar{z}), \tag{15}$$

where $\mathcal{F}$ is the Virasoro conformal block with dimension zero in the intermediate channel. This substitution is appropriate for a large N CFT with a sparse spectrum of single-trace higher spin operators [29].

The function $\mathcal{F}$ is not known explicitly, but there are several methods for approximating it [14–16]. We will use a formula from Ref. [15], which is valid at large c, with h_w/c fixed and small and h_v fixed and large. Here, the formula reads

$$\mathcal{F}(z)\approx\left(\frac{z(1-z)^{-6h_w/c}}{1-(1-z)^{1-12h_w/c}}\right)^{2h_v}. \tag{16}$$

This function has a branch point at $z=1$, as expected. Following the contour around $z=1$ and taking z small, we find

$$\mathcal{F}(z)\approx\left(\frac{1}{1-\frac{24\pi i h_w}{cz}}\right)^{2h_v}. \tag{17}$$

The trajectory of $\bar{z}$ does not circle the branch point at $\bar{z}=1$, so for small $\bar{z}$, we simply have $\bar{\mathcal{F}}(\bar{z})\approx 1$, the contribution of the identity operator itself. Substituting Eq. (17) in Eq. (15) and then in Eq. (5), we find

$$\frac{\langle W(t+i\epsilon_1)V(i\epsilon_3)W(t+i\epsilon_2)V(i\epsilon_4)\rangle_\beta}{\langle W(i\epsilon_1)W(i\epsilon_2)\rangle_\beta\langle V(i\epsilon_3)V(i\epsilon_4)\rangle_\beta}\approx\left(\frac{1}{1+\frac{24\pi i h_w}{\epsilon_{12}^*\epsilon_{34}}e^{(2\pi/\beta)(t-t_*-x)}}\right)^{2h_v}, \tag{18}$$

where we define the fast scrambling time t_* [17,18] with the convention

$$t_* = \frac{\beta}{2\pi}\log c. \tag{19}$$

Equation (18) is the main result of our Letter. The correlation function (18) begins to decrease at a time $t \sim (\beta/2\pi)\log c/h_w$.

This formula agrees precisely with the bulk analysis (reviewed in Appendix A) in the above scaling. It is also interesting to consider the scaling h_v, h_w fixed, $h_w \gg h_v \gg 1$ with $c \to \infty$. The bulk analysis suggests that Eq. (18) is also correct in this scaling, but even without the Virasoro block analysis, the $SL(2)$ block of the stress tensor [which gives a contribution $\propto h_w h_v/(cz)$ on the second sheet] is enough to show that, for general dimensions, the time until the identity Virasoro block is affected is of order $t_* - (\beta/2\pi)\log h_w h_v$, where the second term is order one in this scaling. This gives a field-theoretic explanation for the origin of the fast scrambling time.

We are grateful to Ethan Dyer, Tom Hartman, Alexei Kitaev, and Steve Shenker for helpful discussions. D. A. R. is supported by the Fannie and John Hertz Foundation and is very thankful for the hospitality of the Stanford Institute for Theoretical Physics during the completion of this work. D. A. R. also acknowledges the U.S. Department of Energy under cooperative research agreement Contract No. DE-SC00012567. D. S. is supported by NSF Grant No. PHY_1314311/Dirac.

APPENDIX A: BULK CALCULATIONS

Here, we will compute the correlation function (18) using the gravitational shock wave methods of Refs. [6,8]. If $h_w \gg h_v \gg 1$, we can calculate the correlation function by treating the W operator as creating a shock wave and calculating the two-point function of the V operator on that background. The analysis breaks into two parts: (i) finding the geometry of the shock sourced by W and (ii) computing the correlation function of the V operators in that background.

The metric of a localized shock wave [30] in $(2+1)$-dimensional AdS-Rindler space is [6,8,26,31,32]

$$ds^2 = -\frac{4}{(1+uv)^2}dudv + \frac{(1-uv)^2}{(1+uv)^2}dx^2 + 4\delta(u)h(x)du^2, \tag{A1}$$

where h will be defined below. We will consider a shock sourced by a stress tensor,

$$T_{uu}(u', v', x') = P\delta(u')\delta(x'-x), \tag{A2}$$

appropriate for a particle sourced by the W operator, traveling along the $u=0$ horizon at transverse position x. The metric (A1) can be understood as two halves of AdS-Rindler, glued together at $u=0$ with a shift

$$\delta v(x) = h(x) \tag{A3}$$

in the v direction. Plugging into Einstein's equations, we determine h as

$$h(x') = 2\pi G_N P e^{-|x'-x|}. \tag{A4}$$

To relate this geometry to the state $|\psi\rangle$, we need to fix P. In other words, we need to evaluate

$$\frac{\langle\psi|\int dx'du'T_{uu}|\psi\rangle}{\langle\psi|\psi\rangle}, \tag{A5}$$

where we take the integral to run over the slice $v=0$. We will assume that W is dual to a single-particle operator in the bulk, so that the state $|\psi\rangle$ can be described by a Klein-Gordon wave function K. This wave function is a bulk-to boundary propagator from the location (x,t) of the W operator. It is given in terms of the regularized geodesic distance d from the boundary point, as $K \propto (\cosh d)^{-2h_w}$. At $v=0$, we find

$$K(t,x;u',x') = \frac{\mathcal{N}}{[e^t u' + \cosh(x-x')]^{2h_w}}. \tag{A6}$$

The norm $\langle\psi|\psi\rangle$ is a Klein-Gordon inner product

$$\langle\psi|\psi\rangle = 2i\int dx'du'K(t+i\tau, x; u', x')^* \times \partial_{u'}K(t+i\tau, x; u', x'). \tag{A7}$$

The u' integral can be done using contour integration, and the x' integral can be done in terms of Γ functions:

$$\langle\psi|\psi\rangle = \mathcal{N}^2\frac{4\pi^{3/2}}{(2\sin\tau)^{4h_w}}\frac{\Gamma[4h_w]}{\Gamma[2h_w]\Gamma[2h_w+\frac{1}{2}]}. \tag{A8}$$

For the numerator, the stress tensor for the Klein-Gordon field is given by the expression $T_{uu} = \partial_u\varphi\partial_u\varphi$. Contracting bulk operators with boundary operators using K, we have

$$\langle\psi|\int dx'du'T_{uu}|\psi\rangle = 2\int dx'du'\partial_{u'}K(t_w+i\tau, x; u', x')^* \times \partial_{u'}K(t_w+i\tau, x; u', x'), \tag{A9}$$

where the factor of 2 comes from the two different ways of doing the contractions. The integrals can be done the same way as before:

$$\langle\psi|\int dx'du'T_{uu}|\psi\rangle = \mathcal{N}^2\frac{8\pi^{3/2}e^{t_w}}{(2\sin\tau)^{4h_w+1}}\frac{\Gamma[4h_w]\Gamma[2h_w+\frac{1}{2}]}{\Gamma[2h_w]^3}. \tag{A10}$$

Taking the ratio at large h_w, we find

$$P = \frac{2h_w e^{t_w}}{\sin\tau}. \tag{A11}$$

The second step, following Ref. [6], is to compute the two-sided correlation function of the V operators in this shock background. We will do this using the geodesic approximation

$$\langle\psi|V_L V_R|\psi\rangle \propto e^{-md}, \tag{A12}$$

where d is the regularized geodesic distance and the mass m is approximately $2h_v$, the conformal weight of V. Following Ref. [6], we find

$$d = 2\log 2r_\infty + \log[1 + h(0)]. \tag{A13}$$

After subtracting the divergent distance in the unperturbed thermofield double state, $d_{\mathrm{TFD}} = 2\log 2r_\infty$, we plug the distance into Eq. (A12), finding

$$\frac{\langle\psi|V_L V_R|\psi\rangle}{\langle\psi|\psi\rangle\langle V_L V_R\rangle} = \left(\frac{1}{1 + \frac{4\pi G_N h_w}{\sin\tau} e^{t-x}}\right)^{2h_v}. \tag{A14}$$

This agrees with Eq. (18) after (i) using $G_N = 3/2c$ to express the gravitational constant in terms of the central charge, (ii) plugging in $\epsilon_1 = -\tau$, $\epsilon_2 = \tau$, $\epsilon_3 = 0$, $\epsilon_4 = \beta/2$, and (iii) using $\beta = 2\pi$.

APPENDIX B: ISING MODEL

It is interesting to contrast the behavior of out-of-time-order correlators in a chaotic theory to those in an integrable theory. As an example, we consider the two-dimensional Ising CFT. This theory has $c = 1/2$ and three Virasoro primary operators: I, σ, and ϵ, corresponding to the identity, "spin," and "energy" operators. The different combinations of four-point correlators of these primaries are well known [33–35]. We will present these by giving the functions $f(z,\bar{z})$ in Eq. (5):

$$f_{\sigma\sigma}(z,\bar{z}) = \frac{1}{2}(|1+\sqrt{1-z}| + |1-\sqrt{1-z}|), \tag{B1}$$

$$f_{\sigma\epsilon}(z,\bar{z}) = \left|\frac{2-z}{2\sqrt{1-z}}\right|^2, \tag{B2}$$

$$f_{\epsilon\epsilon}(z,\bar{z}) = \left|\frac{1-z+z^2}{1-z}\right|^2, \tag{B3}$$

where the operators are ordered $WVWV$ with the configuration specified by Eqs. (8)–(11). Following the contour across the branch cut for the two correlators that do have a second sheet and taking z small, we find

$$\frac{\langle\sigma\sigma\sigma\sigma\rangle_\beta}{\langle\sigma\sigma\rangle_\beta^2} = 0, \quad \frac{\langle\sigma\epsilon\sigma\epsilon\rangle_\beta}{\langle\sigma\sigma\rangle_\beta\langle\epsilon\epsilon\rangle_\beta} = -1, \quad \frac{\langle\epsilon\epsilon\epsilon\epsilon\rangle_\beta}{\langle\epsilon\epsilon\rangle_\beta^2} = 1. \tag{B4}$$

Only $\langle\sigma\sigma\sigma\sigma\rangle$ vanishes at large t.

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[29] It is interesting to note that, because of the rapid $z^{1-h}\bar{z}^{\bar{h}}$ behavior on the second sheet, even one single-trace operator

with spin greater than two and dimension parametrically independent of c will dominate over the Virasoro identity block near time t_*. This means that the universal contribution to the correlator identified in this Letter applies only to CFTs with a very sparse spectrum of single-trace operators. This matches the sensitivity of high-energy scattering in gravity to massive higher spin fields, which grow more rapidly with energy. In a consistent bulk theory of higher spin fields, such as string theory, a Regge-type resummation of an infinite number of stringy operators is necessary. For further discussion of this point in the present context, see Ref. [19].

[30] Although we refer to these geometries as shock waves, the terminology is somewhat misleading in $2+1$ dimensions, since the geometry is locally pure AdS_3 away from the source.

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