Proceedings of the ASME 2014 International Design Engineering Technical Conferences &
Computers and Information in Engineering Conference
IDETC/CIE 2014
August 17-20, 2014, Buffalo, New York, USA

DETC2014-35065

EFFECTS OF PROSTHESIS MASS ON HIP ENERGETICS, PROSTHETIC KNEE TORQUE, AND PROSTHETIC KNEE STIFFNESS AND DAMPING PARAMETERS REQUIRED FOR TRANSFEMORAL AMPUTEES TO WALK WITH NORMATIVE KINEMATICS

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ABSTRACT

We quantify how the hip energetics and knee torque required for an above-knee prosthesis user to walk with the kinematics of able-bodied humans vary with the inertial properties of the prosthesis. We also select and optimize passive mechanical components for a prosthetic knee to accurately reproduce the required knee torque.

Previous theoretical studies have typically investigated the effects of prosthesis inertial properties on energetic parameters by modifying both mass and mass distribution of the prosthesis and computing kinetic and energetic parameters only during swing. Using inverse dynamics, we determined the effects of independently modifying mass and mass distribution of the prosthesis, and we computed parameters during both stance and swing. Results showed that reducing prosthesis mass significantly affected hip energetics, whereas reducing mass distribution did not. Reducing prosthesis mass to 25% of the mass of a physiological leg decreased peak stance hip power by 26%, average swing hip power by 74%, and absolute hip work over the gait cycle by 22%.

Previous studies have also typically optimized prosthetic knee components to reproduce the knee torque generated by able-bodied humans walking with normative kinematics. However, because the prosthetic leg of an above-knee prosthesis user weighs significantly less than a physiological leg, the knee torque required for above-knee prosthesis users to walk with these kinematics may be significantly different. Again using inverse dynamics, it was found that changes in prosthesis mass and mass distribution significantly affected this required torque. Reducing the mass of the prosthesis to 25% of the mass of the physiological leg increased peak stance torque by 43% and decreased peak swing torque by 76%.

The knee power required for an above-knee prosthesis user to walk with the kinematics of able-bodied humans was analyzed to select passive mechanical components for the prosthetic knee. The coefficients of the components were then optimized to replicate the torque required to walk with the kinematics of able-bodied humans. A prosthetic knee containing a single linear spring and two constant-force dampers was found to accurately replicate the targeted torque (R^2=0.90 for a typical prosthesis). Optimal spring coefficients were found to be relatively insensitive to mass alterations of the prosthetic leg, but optimal damping coefficients were sensitive. In particular, as the masses of the segments of the prosthetic leg were altered between 25% and 100% of able-bodied values, the optimal damping coefficient of the second damper varied by 330%, with foot mass alterations having the greatest effect on its value.

INTRODUCTION

A fundamental goal of the design of lower-limb prostheses is to enable lower-limb amputees to walk with the gait of able-bodied humans. However, the gait of amputees using existing prostheses differs from the gait of able-bodied humans in two major ways. First, prosthesis users typically expend significantly more metabolic energy than able-bodied humans during walking [1-4]. Second, prosthesis users typically do not walk with the kinematics of able-bodied humans [5-7]. Designers have taken specific design approaches to resolve each of these issues, but these approaches have their respective limitations.

Designers have primarily attempted to reduce the metabolic energy expenditure of prosthesis users by altering the inertial properties of prostheses (e.g., reducing mass) [8]. Researchers have evaluated this design approach by conducting experimental and theoretical studies to measure or compute the metabolic and/or mechanical energy expended by amputees using prostheses with different inertial properties [9-16]. However, these studies have had two major limitations. First, most studies altered the inertial properties of prostheses by applying mass perturbations (i.e., the application of a physical or simulated mass at a specified location on the prosthesis). Mass perturbations alter both the mass and the mass distribution of the prosthesis, making it difficult to identify which quantity produces the observed results. Second, because of the difficulty of predicting the ground reaction force (GRF), few theoretical studies have investigated how inertial properties affect the energy expenditure of amputees during stance. Thus, the design approach of reducing prosthesis mass in order to reduce the metabolic energy expenditure of prosthesis users has potential for further evaluation.

Designers have also attempted to enable above-knee prosthesis users to walk with the kinematics of able-bodied humans by carefully selecting components for prosthetic knees. In particular, they have optimized the components to reproduce the knee torques generated by able-bodied humans during walking [17,18]. However, this design approach has a major limitation. Because the masses of prosthetic legs are typically less than the masses of physiological legs, the knee torques generated by able-bodied humans during walking may be significantly different from the torques required by prosthesis users to walk with the kinematics of able-bodied humans. This design approach may not significantly affect the kinematics of prosthesis users wearing active microprocessor-controlled knees, as these devices can typically sense and compensate for undesired kinematics. However, the approach may significantly affect the kinematics of prosthesis users wearing passive mechanical knees.

In this paper, we present a theoretical analysis that aims to quantify and extend the previously described design approaches to better enable above-knee prosthesis users to walk with the gait of able-bodied humans. To further evaluate the design approach of altering inertial properties to reduce the energy expenditure of prosthesis users, we use inverse dynamics to determine the effects of independently varying prosthesis mass and mass distribution on the hip energetics required for prosthesis users to walk with the kinematics of able-bodied humans. We perform this analysis during both swing and stance. To extend previous optimizations of prosthetic knee components, we use inverse dynamics to quantify how varying prosthesis mass and mass distribution affects the knee torque required for prosthesis users to walk with the kinematics of able-bodied humans. We analyze knee power to select components (i.e., springs and dampers) for a passive knee that can accurately reproduce this knee torque, instead of the torque generated by able-bodied humans. Finally, we optimize the coefficients of the components (i.e., spring and damping coefficients) to maximize the accuracy of the reproduction.

The motivation for our research is to design a high-performance, low-cost, passive prosthetic knee for use in the developing world. By performing these biomechanical analyses, we aim to formulate detailed design requirements for a low-cost, passive prosthetic knee that can enable transfemoral amputees to walk with the gait of able-bodied humans.

METHODS

Modeling of Prosthetic Leg

A two-dimensional, three-segment link-segment model of the prosthetic leg of a unilateral transfemoral amputee wearing an above-knee prosthesis was designed (Figure 1). The model consisted of an upper leg segment (socket and stump), a lower leg segment (shank), and a foot segment. The foot segment was modeled as having a distal arc equivalent to the roll-over shape of the physiological foot [19]. The lengths of the segments of the model were prescribed according to standard anthropometric ratios of able-bodied humans [20,21] scaled to the average American body height [22].

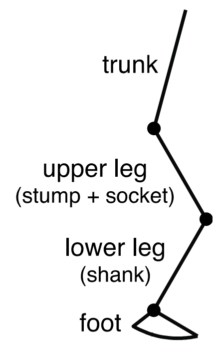

FIGURE 1: THREE-SEGMENT LINK-SEGMENT MODEL OF THE PROSTHETIC LEG.

Alteration of Inertial Properties

The masses and moments of inertia of each of the segments of the model were then collectively and independently varied. Specifically, the following types of inertial alterations were applied to the segments:

1) Varying mass and moment of inertia by the same factor (equivalent to altering the mass of the segment, but holding mass distribution constant)
2) Varying moment of inertia and holding mass constant (equivalent to altering the mass distribution of the segment, but holding mass constant)
3) Varying mass and holding moment of inertia constant (equivalent to altering the mass of the segment and inversely altering the mass distribution)

All inertial alterations will be reported with respect to able-bodied values (e.g., a lower leg mass of 25% indicates that the mass of the lower leg was altered to 25% of the corresponding able-bodied value). Able-bodied masses and moments of inertia for each segment were found by scaling standard anthropometric ratios of able-bodied humans [21,23,24] to the average American body mass [22].

Application of Kinematics

Since we aimed to determine the hip energetics and prosthetic knee torque required for prosthesis users to walk with the kinematics of able-bodied humans, the kinematics of able-bodied humans walking at a natural cadence [25] were applied to the model.

Calculation of Ground Reaction Forces

The GRF acting on the model was then computed. It was first observed that the GRF acting on a generic multi-rigid-body model of the body is equal to sum of the inertial forces minus the gravitational forces over all the segments in the model [21]. Mathematically, this equivalence can be written as follows:

$$\overrightarrow{GRF} = \sum_{i=1}^{N} m_i(\ddot{\vec{r}}_{COM} + g\hat{y}) \quad (1)$$

where N is the number of segments in the model of the body, m_i is the mass of the i^{th} segment, $\vec{r}_{COM}$ is the position vector of the COM of the i^{th} segment relative to the origin of an inertial reference frame, g is the gravitational constant, and y is the unit vector in the positive vertical direction.

The total GRF acting on a prosthesis user walking with the kinematics of able-bodied humans is simply equal to the total GRF acting on an able-bodied human [25], minus the portion of the GRF of the able-bodied human that acts on the additional mass of an able-bodied human when compared to a prosthesis user (i.e., the additional mass due to the higher weight of a physiological leg relative to a prosthetic leg). Mathematically, this equivalence can be written as follows:

$$\overrightarrow{GRF}_{prosth} = \overrightarrow{GRF}_{able} - \begin{pmatrix} (m_{ul_{able}} - m_{ul_{prosth}})(\ddot{\vec{r}}_{ul_{COM}} + g\hat{y}) & + \\ (m_{ll_{able}} - m_{ll_{prosth}})(\ddot{\vec{r}}_{ll_{COM}} + g\hat{y}) & + \\ (m_{f_{able}} - m_{f_{prosth}})(\ddot{\vec{r}}_{f_{COM}} + g\hat{y}) & \end{pmatrix} \quad (2)$$

where $\overrightarrow{GRF}_{prosth}$ is the total GRF acting on the prosthesis user; $\overrightarrow{GRF}_{able}$ is the total GRF acting on an able-bodied human; $m_{ul_{able}}$, $m_{ll_{able}}$, and $m_{f_{able}}$ are the masses of the upper leg, lower leg, and foot in an able-bodied human, respectively; and $m_{ul_{prosth}}$, $m_{ll_{prosth}}$, and $m_{f_{prosth}}$ are the masses of the same segments in an above-knee prosthesis. During single support, the GRF acting on the prosthetic leg is exactly equal to $\overrightarrow{GRF}_{prosth}$. During double-support phases, the GRF acting on the prosthetic leg is indeterminate. During these phases, the GRF acting on the prosthetic leg was approximated by a linear transition between initial and final values (i.e., during early stance, a linear transition from zero to the initial single-support value, and during late stance, a linear transition from the final single-support value to zero).

Calculation of Knee Torque and Hip Energetic Parameters

A standard 2-dimensional inverse dynamics procedure [26] was used to compute knee torque, hip power, and absolute hip work[1]. In order to calculate absolute hip power, hip torque was calculated, and hip power was calculated as the product of hip torque with hip angular velocity. Absolute hip work was then computed as the integral of the absolute value of hip power over the gait cycle. These kinetic and energetic parameters were calculated for all prosthesis mass configurations examined. Thus, the effects of independently varying mass and moment of inertia on the hip energetics and knee torque required for prosthesis users to walk with the kinematics of able-bodied humans were determined.

Component Selection

Passive mechanical components for the prosthetic knee were then selected to reproduce the knee torque required for prosthesis users to walk with the kinematics of able-bodied humans. To select the type of component (i.e., spring or damper), it was insightful to first examine the knee power required to walk with the kinematics of able-bodied humans (Figure 2). Knee power was calculated as the product of knee torque with knee angular velocity. The integral of knee power with respect to time (i.e., the positive or negative area under the graph) is knee work, which is equivalent to the kinetic energy generated at the knee. From the graph, it was seen that the gait cycle could be divided into three energy-based phases: one in which kinetic energy was stored and released in nearly equal proportion by the knee, and two in which kinetic energy was dissipated by the knee. Thus, it was concluded that the prosthetic knee should engage a spring during phase 1 and dampers during phases 2 and 3.

Component Optimization

The coefficients of the components selected for the prosthetic knee were then optimized to reproduce the knee torque calculated earlier (i.e., the knee torque required for a prosthesis user to walk with the kinematics of able-bodied humans). Since nonlinear springs and dampers can be designed, the torque-angle relationship of the spring and torque-angular velocity relationship of the damper were expressed as generic second-order polynomials. The coefficients of the polynomials were then optimized using a genetic algorithm to reproduce the targeted torque.

RESULTS

Knee Torque and Hip Energetic Parameters

Figure 3 shows the effects of alterations in the masses and moments of inertia (about COM) of the prosthetic leg segments on knee torque, and Figure 4 shows the effects of the same alterations on hip power.

For knee torque, decreasing the masses and moments of inertia (about COM) of all segments of the leg increased the peak magnitude of late stance torque by up to 43% and

[1] Absolute hip work is a measure of mechanical energy expenditure at the hip that, like the metabolic energy expenditure of muscles, increases in value during both concentric and eccentric contractions [27].

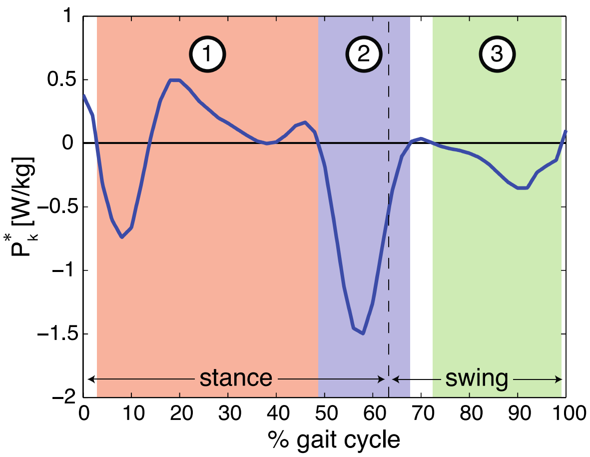

FIGURE 2: THREE ENERGY-BASED PHASES OF GAIT. Knee power versus time graph is shown for a prosthesis user wearing a prosthetic leg with typical mass properties (upper leg mass = 50% of able-bodied value, and lower leg and foot mass = 33% of able-bodied values) walking with the kinematics of able-bodied humans. Symbol $\boldsymbol{P_k^*}$ designates knee power normalized to body mass. Kinetic energy is stored and released in nearly equal proportion in phase 1, and kinetic energy is purely dissipated in phases 2 and 3.

decreased the peak magnitude of swing torque by up to 76%. Reducing masses and holding moments of inertia constant increased the peak magnitude of late stance torque by up to 43% and decreased the peak magnitude of swing torque by up to 60%. Finally, decreasing moments of inertia and holding masses constant decreased the peak magnitude of late stance torque by no more than 2% and decreased the peak magnitude of swing torque by up to 16%. Thus, changes in the masses and mass distributions of the segments of the prosthetic leg had a significant effect on the knee torque required for walking with normative kinematics. However, changes in mass distributions alone had little effect on stance torque.

For hip power, decreasing both masses and moments of inertia (about COM) of all segments of the leg reduced peak stance hip power by up to 26% and average swing hip power by up to 74%. Decreasing masses and holding moments of inertia constant reduced peak stance hip power by up to 20% and average swing hip power by up to 66%. Finally, decreasing moments of inertia and holding masses constant reduced peak stance hip power by no more than 6% and average swing hip power by no more than 8%. Thus, changes in the masses of the segments of the prosthetic leg had a significant effect on hip power, but changes in the moments of inertia of the segments did not have such an effect.

The results from computing absolute hip work mirrored the trends for hip power. Decreasing both masses and moments of inertia (about COM) reduced absolute hip work over the gait cycle by up to 22%, and decreasing masses and holding moments of inertia constant reduced absolute hip work by up to 19%. However, decreasing moments of inertia but holding masses constant reduced absolute hip work by no more than 4%. As with hip power, changes in masses of the segments of the prosthetic leg had a significant effect on absolute hip work, but changes in the moments of inertia did not.

Optimal Component Values

When the coefficients of the springs and dampers were optimized, it was found that a simple combination of a linear spring (i.e., a spring described by a first-order polynomial) during phase 1 and constant-force dampers (i.e., dampers described by zero-order polynomials) during phase 2 could accurately reproduce the torque required for prosthesis users to walk with the kinematics of able-bodied humans. The accuracy was not significantly improved by using springs or dampers with a higher polynomial order. Figure 5 illustrates the typical accuracy with which these components reproduced the knee torque required for prosthesis users to walk with the kinematics of able-bodied humans.

The stiffness coefficient (k_1) of the spring was relatively insensitive to changes in prosthesis mass, varying by no more than 5.6% as the masses of the segments of the prosthetic leg varied between 25% and 100% of able-bodied values. On the other hand, the damping coefficient of the first damper (b_1) and the damping coefficient of the second damper (b_2) varied by up to 36% and 330%, respectively. Thus, changes in the masses of the segments significantly affected the optimal damping coefficients of the dampers within the prosthetic knee, but not the optimal stiffness coefficient of the spring.

When the masses of individual segments of the prosthetic leg were varied with respect to each other, k_1 was most sensitive to changes in upper leg mass, varying by up to 4.1% as upper leg mass was altered between 25% and 100% of its able-bodied value. Coefficient b_1 was most sensitive to changes lower leg mass, varying by up to 27%, and also moderately sensitive to upper leg mass, varying by up 8.0%. Finally, b_2 was most sensitive to changes in foot mass, varying by up to 180%, and highly sensitive to changes in lower leg mass and upper leg mass, varying by up to 134% and 45%, respectively. Hence, k_1 was most sensitive to changes in upper leg mass, b_1 was most sensitive to changes in lower leg mass, and b_2 was most sensitive to changes foot mass. Coefficients b_1 and b_2 were also notably sensitive to alterations in the masses of other segments of the prosthetic leg.

DISCUSSION

The results regarding the effects of inertial alterations on kinetics and energetics indicate that changes in the masses of the segments of the prosthetic leg have a much greater effect on knee torque, hip power, and absolute hip work than changes in moments of inertia (about COM) of the segments. These findings are supported by physical intuition. The prosthetic leg was modeled by a two-dimensional link-segment model, which approximates the leg as a system of rigid bodies connected by pin joints. For such a system, the torque, power, and work required at a given joint to achieve prescribed kinematics should be more greatly influenced by

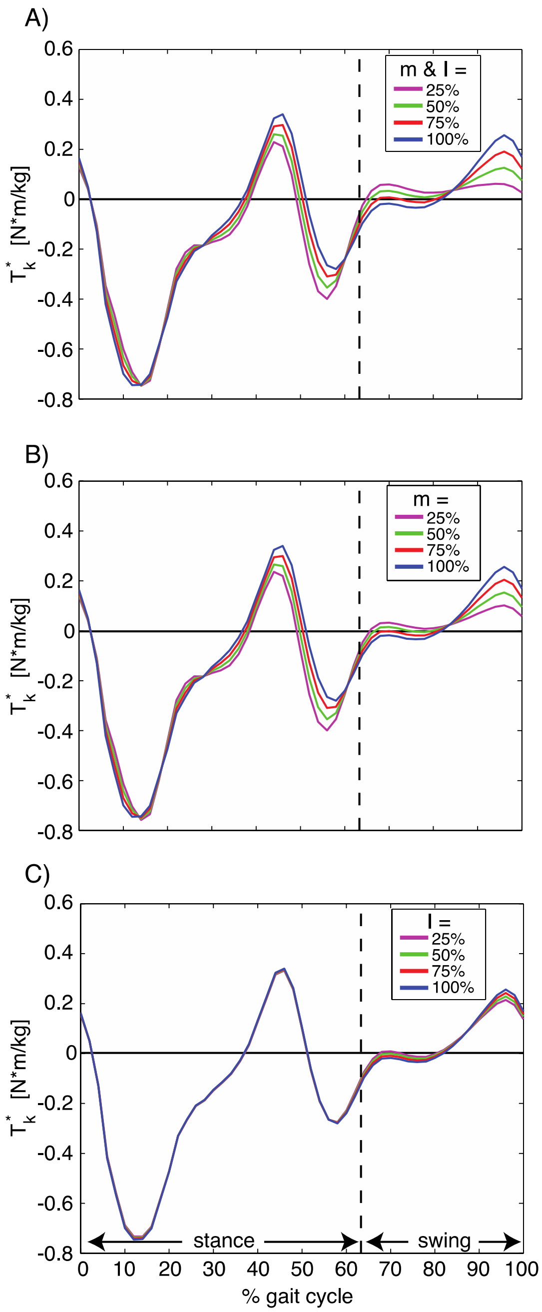

FIGURE 3: EFFECTS OF GROSS ALTERATIONS IN PROSTHESIS MASS AND MOMENT OF INERTIA ON KNEE TORQUE. Symbol $\boldsymbol{T}_{\boldsymbol{k}}^{*}$ designates knee torque normalized to body mass. Inertial alterations are performed uniformly on all segments of the prosthetic leg, and percentages are given relative to corresponding able-bodied values. Figure 3A: Masses and moments of inertia are altered. Figure 3B: Masses are altered while holding moments of inertia constant at able-bodied values. Figure 3C: Moments of inertia are altered while holding masses constant at able-bodied values.

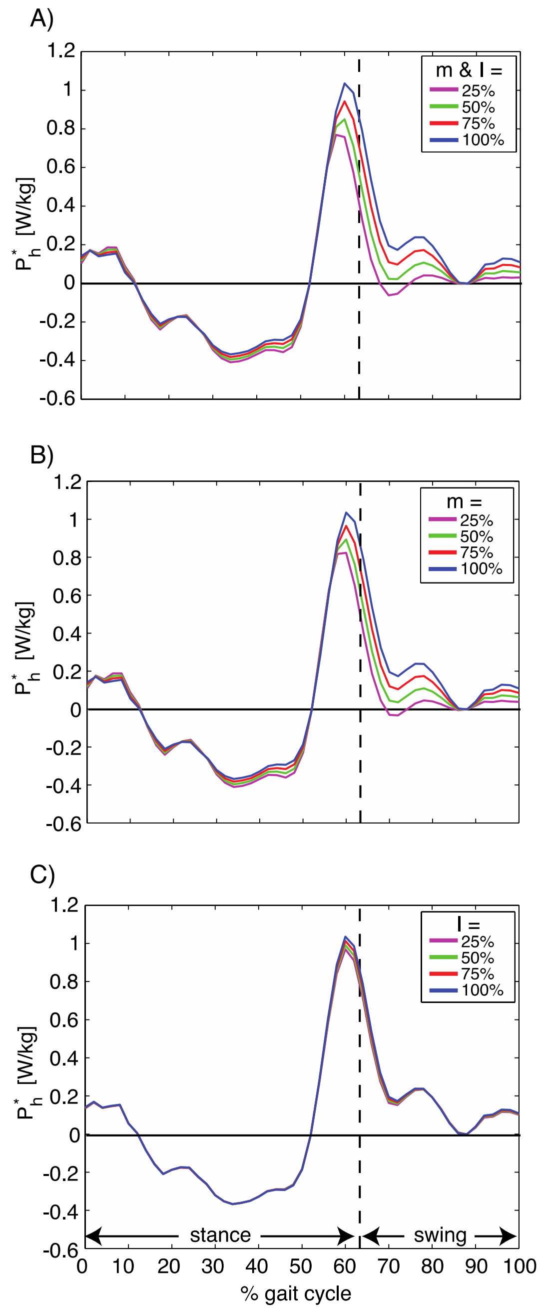

FIGURE 4. EFFECTS OF GROSS ALTERATIONS IN PROSTHESIS MASS AND MOMENT OF INERTIA ON HIP POWER. Symbol $\boldsymbol{P}_{\boldsymbol{h}}^{*}$ designates hip power normalized to body mass. Inertial alterations are performed uniformly on all segments of the prosthetic leg, and percentages are given relative to corresponding able-bodied values. Figure 4A: Masses and moments of inertia are altered. Figure 4B: Masses are altered while holding moments of inertia constant at able-bodied values. Figure 4C: Moments of inertia are altered while holding masses constant at able-bodied values.

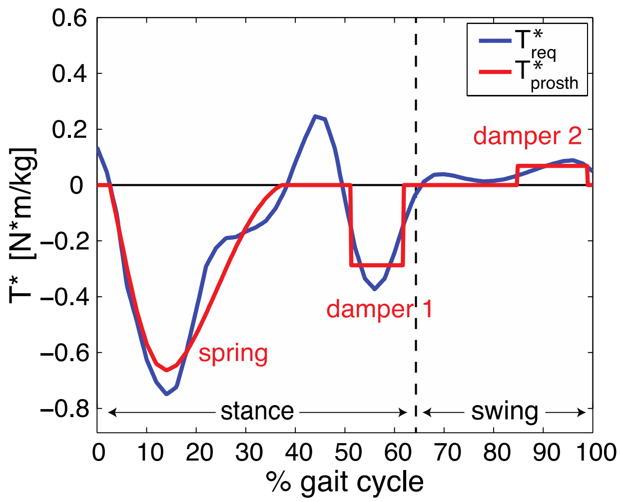

FIGURE 5: COMPARISON OF REQUIRED TORQUE TO PROSTHETIC TORQUE. Symbol T^* designates knee torque normalized to body mass. Symbol T^*_{req} indicates knee torque required for above-knee prosthesis user to walk with kinematics of able-bodied humans. Symbol T^*_{prosth} indicates knee torque produced by optimized prosthetic knee components. Parameter T^*_{prosth} is calculated for prosthesis with typical mass properties (upper leg mass = 50% of able-bodied value, lower leg mass and foot mass = 33% of able-bodied values). For this simulation, R^2 = 0.90, and optimized coefficient values were as follows: k_1 = 2.9 [N*m/(kg*rad)], b_1 = 0.29 [N*m*s/(kg*rad)], and b_2 = 0.069 [N*m*s/(kg*rad)].

altering mass distal to the joint, rather than redistributing the existing mass of any of the segments about its original COM.

From an absolute perspective, the results also indicate that the effects of mass alterations on kinetics and energetics are large. Knee torque peaks changed by approximately 40-80% throughout the gait cycle, and hip power parameters were observed to change by approximately 30-70%. In addition, when the masses of all the segments were reduced to their minimum tested values, absolute hip work decreased approximately 20% over the gait cycle. Once again, these findings are supported by physical intuition. Altering the masses of the segments of the prosthetic limb by up to 75% of their able-bodied values should significantly affect the joint torques and powers required to achieve prescribed kinematics. Furthermore, reducing the mass properties of a prosthesis should decrease the mechanical power and work required to achieve these kinematics.

Collectively, the preceding results suggest two major conclusions: 1) Prosthesis designers may able to significantly reduce the metabolic energy expenditure of transfemoral amputees by reducing prosthesis mass, and 2) prosthesis designers should calculate the torque required for prosthesis users to walk with the kinematics of able-bodied humans and design prostheses to reproduce this torque, as opposed to reproducing the torque generated by able-bodied humans.

The first conclusion supports the strong trend in the prosthetics industry towards the design of lightweight prostheses. Furthermore, it is supported by literature studies that show that increased prosthesis mass can lead to increased mechanical energy expenditure and muscular effort at the hip in below-knee and above-knee prosthesis users [28,29]. The precise relationship between hip mechanical energy expenditure and the metabolic energy expenditure at the hip muscles is still undetermined in the literature, but it is reasonable to assume that a positive correlation exists.

The second conclusion contrasts the approach that a number of previous studies have taken in designing prosthetic knees [17,18]. However, the conclusion may help explain biomechanical and user-based limitations of knees designed in these studies. For example, one study found significant differences between optimized component coefficients (e.g., spring and damping coefficients) for a prosthetic knee and user-preferred coefficients. The authors hypothesized that inertial differences between able-bodied humans and amputees, which were not considered in the optimization process, may have been the cause [18].

The results of our component optimization indicate that simple, passive mechanical components can accurately reproduce the knee torque required for above-knee prosthesis users to walk with the kinematics of able-bodied humans. The components consist of a linear spring and two constant-force dampers, which could be physically implemented as a torsion spring and friction pads, respectively. The accuracy of the components offers a promising framework for the design of a high-performance, passive prosthetic knee, and the low cost and high availability of the components suggests that such a knee may be affordable for our targeted users (i.e., transfemoral amputees in the developing world).

The optimal coefficients of the spring agree with previous determinations of the torque-angular displacement relationship of the knee during the weight-bearing phase of stance [18,30,31]. Studies with which to accurately compare our optimal damping coefficients could not be found, as similar studies optimized damping coefficients during different phases of gait. The high sensitivity of the optimal damping values to prosthesis mass suggests that designers of prosthetic knees should carefully specify dampers based on the prosthesis mass of each user. For dampers in late stance, lower leg mass has the greatest influence on the optimal damping coefficient, whereas for dampers in swing, foot mass has the greatest influence. Optimizing components based on prosthesis mass is particularly important for passive prostheses, which do not have active control systems to compensate for undesired kinematics. However, even for actively controlled knees, optimally selected components may significantly reduce control effort, algorithm complexity, and power requirements, which could in turn reduce cost and improve performance.

ACKNOWLEDGMENTS

We would like to thank V.N. Murthy Arelekatti for his help revising this manuscript. We would also like to acknowledge Bhagwan Mahaveer Viklang Sahayata Samiti (BMVSS, a.k.a., the Jaipur Foot organization) for their partnership in our project to design a high-performance, low-cost prosthetic knee for the developing world. Funding for this research was provided by the Pappalardo Mechanical Engineering Research Fellowship at the Massachusetts Institute of Technology (MIT), the MIT Public Service Center Fellowship, the MIT Research Support Committee, the National Science Foundation Graduate Research Fellowship No. 1122374, and the Tata Center for Technology and Design.

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