DETC2014-35320

DEVELOPMENT OF A DESIGN TOOL FOR FLOW RATE OPTIMIZATION IN THE TATA SWACH WATER FILTER

Sean T. Ricks
Global Engineering and Research Lab
Department of Mechanical Engineering
Massachusetts Institute of Technology
Cambridge, Massachusetts, USA
Email: sricks@mit.edu

John R. Lewandowski
Laboratory for Manufacturing and
Productivity
Department of Mechanical Engineering
Massachusetts Institute of Technology
Cambridge, Massachusetts, USA
Email: dowski@mit.edu

Emmanuel G. Lim
Department of Mechanical Engineering
Massachusetts Institute of Technology
Cambridge, Massachusetts, USA
Email: eglim@mit.edu

Dawn M. Wendell
Department of Mechanical Engineering
Massachusetts Institute of Technology
Cambridge, Massachusetts, USA

Amos G. Winter, V
Global Engineering and Research Lab
Department of Mechanical Engineering
Massachusetts Institute of Technology
Cambridge, Massachusetts, USA

ABSTRACT

When developing a first-generation product, an iterative approach often yields the shortest time-to-market. In order to optimize its performance, however, a fundamental understanding of the theory governing its operation becomes necessary. This paper details the optimization of the Tata Swach, a consumer water purifier produced for India. The primary objective of the work was to increase flow rate while considering other factors such as cost, manufacturability, and efficacy. A mathematical model of the flow characteristics through the filter was developed. Based on this model, a design tool was created to allow designers to predict flow behavior without prototyping, significantly reducing the necessity of iteration. Sensitivity analysis was used to identify simple ways to increase flow rate as well as potential weak points in the design. Finally, it was demonstrated that maximum flow rate can be increased by 50% by increasing the diameter of a flow-restricting feature while simultaneously increasing the length of the active purification zone. This can be accomplished without significantly affecting cost, manufacturability, and efficacy.

INTRODUCTION

Many millions of people around the world are affected by a lack of clean drinking water. According to the WHO/UNICEF JMP 2012 update, there are over 780 million people worldwide who do not have access to an improved drinking-water source [1]. This issue particularly affects the poor in developing countries and leads to many thousands of deaths each year due to water-borne illness [1].

Tata Chemicals Ltd. decided to address this problem by developing a point-of-use filtration system designed specifically for the Indian market. This product, the Tata Swach, provides bacteria- and virus-free water to households at an affordable price.

Out of a desire to get the product to market as quickly as possible, the filter was designed using an iterative approach. Components were prototyped and tested until an acceptable level of performance was reached. Now that there is a desire to optimize the product and increase the rate of filtration, it is necessary to understand the physical phenomena behind the filter's operation.

This paper outlines the development of a mathematical model and design tool that can be used to optimize flow rate in the Tata Swach. A description of the filter's basic operation and key features is provided. The theory behind the mathematical model is also explained. A design tool is presented which allows filter designers to input certain physical parameters and predict the resulting flow rate through the filter. This design tool will be used to show that flow rate can be increased by 50% by making minimal changes to the design. Parameters for achieving the optimized flow rate are specified. Experimental

results, which confirm the conclusions of the design tool, are presented. Finally, the broader implications of this research are provided, and recommendations for future development are given.

NOMENCLATURE

A	Area
atm	Atmospheric (pressure)
D	Diameter
EOL	End-of-life tablet
f	Friction factor
g	Gravitational acceleration
H	Hydraulic (diameter)
h	Height
I	Current
K	Minor loss coefficient
k	Permeability
L	Length
M	Major (losses)
m	Minor (losses)
P	Pressure head
Pe_w	Wetted Perimeter
ρ	Density
Q	Flow rate
R	Resistance
Re	Reynolds number
RHA	Rice husk ash
tot	Total
τ	Residence time
μ	Viscosity
V	Voltage
V	Volume
v	Velocity
w	Width

FILTER OPERATION

The Swach operates without electricity or access to a running water source. As shown in Fig. 1, dirty water is poured into the upper container through a mesh pre-filter. The static water creates a pressure head, which drives the water through the filter element, or bulb, where the purification takes place. Once it has passed through the bulb, the water is stored in the lower container until used by the consumer.

Figure 2 shows the path that water takes as it passes through the bulb. First it enters the outer body of the bulb and flows to the bottom where it begins to filter up through the inner body. The inner body contains compacted rice husk ash (RHA), which has been infused with silver nanoparticles. As the water passes through the RHA, two purification processes take place. First, the carbon particles (which comprise 10-15% of the RHA) trap organic contaminants, which can affect odor and taste. Second, silver ions are released into the water, neutralizing viruses and bacteria. This process is time sensitive
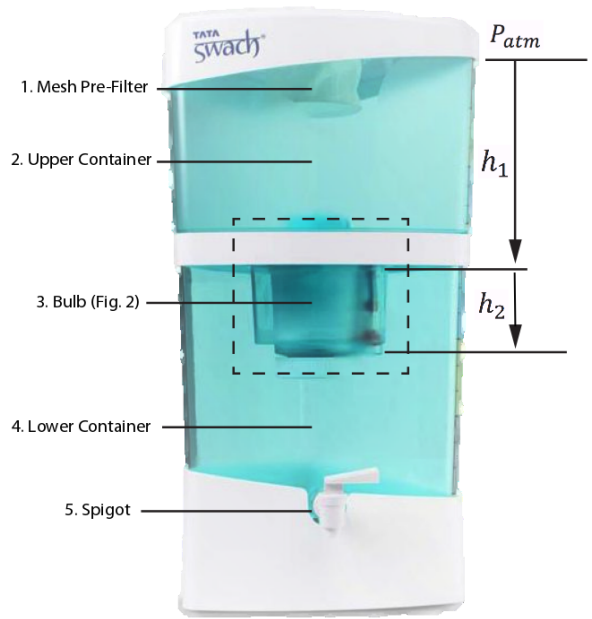

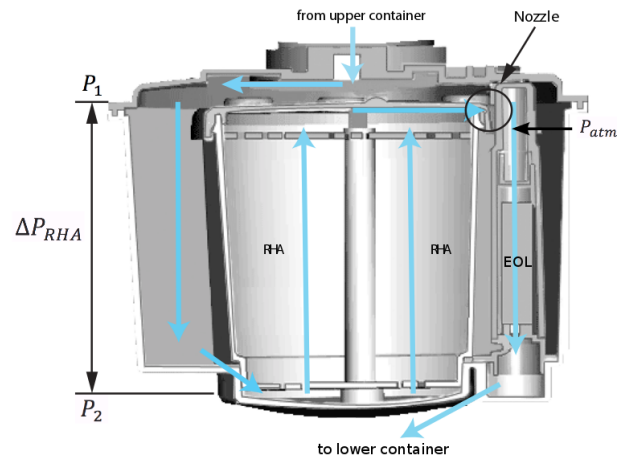Figure 1. BASIC COMPONENTS OF THE TATA SWACH WATER PURIFIER

Figure 2. TATA SWACH BULB FLOW AND CRITICAL COMPONENTS.

(i.e. the longer the water is in contact with the silver nanoparticles, the more ions will be released and the more microbes will be destroyed). For this reason residence time, defined as the amount of time the water spends within the RHA, is an important design parameter. After passing through the RHA, the water passes through a small nozzle (referred to as the flow-restricting nozzle) and enters the end-of-life (EOL) mechanism. The EOL has a water-soluble tablet that is designed to dissolve completely after 3 m^3 (3000 L), allowing a spring to close a valve and stop flow through the bulb. This is

meant to provide a cue to the consumer that the effective life of the bulb is spent and the bulb requires replacement. After exiting the EOL, the water passes through a mesh post filter, which traps any RHA particles that escape from the bulb. Once through the post filter, the water enters the lower container and filtration is complete.

PAST WORK

In order to ensure that the filter achieves a 6-log reduction of bacteria and a 4-log reduction of viruses (meeting or exceeding US-EPA standards), Tata Chemicals conducted extensive tests on their prototypes [2]. Given a certain amount of RHA in a prototype, an acceptable flow rate was determined experimentally without respect to residence time. The flow rate was controlled by the size of the orifice in the flow-restricting nozzle and successively lower flow rates were tested until the desired purification was achieved. The current design of the bulb yields a maximum flow rate of $1.67E^{-6}$ m^3/s (6 L/hr) when the upper container is full and the pressure head is at its greatest. By developing a physical understanding of how each element within the filter affects the flow rate and residence time, an optimized flow rate can be achieved while ensuring the same level of purification.

MATHEMATICAL MODEL

Flow through the bulb is analogous to current in an electrical circuit as shown in Eqs. (1) and (2). In an electrical circuit, current is equivalent to a potential difference divided by an electrical resistance; in a fluid circuit, flow rate is equivalent to a pressure difference divided by a flow resistance.

$$I = \frac{\Delta V}{R} \qquad (1)$$

$$Q = \frac{\Delta P}{R} \qquad (2)$$

The total pressure difference of the system is created by the hydraulic head above the bulb and is given by the hydrostatic equation:

$$\Delta P = P_1 - P_{atm} = \rho g h_1 \qquad (3)$$

where P_1 is the pressure at location 1 (refer to Fig. 2). The pressure just after the flow-restricting nozzle (inside the EOL) is atmospheric because vents in the side of the mesh post filter and the fact that the EOL never fills with water (flow is slow enough that water simply drips through the EOL) allow atmospheric air to reach the top of the EOL. In order to simplify calculations, h_1 is measured to the top of the bulb

rather than the bottom (i.e. h_2 is neglected). The hydrostatic contribution of h_2 is compensated for by the fact that elevation gain through the RHA is also neglected.

There are several resistances that contribute to the total resistance of the system: the resistance of the RHA, major losses, and minor losses.

RHA Resistance

The resistance of the RHA can be determined from Darcy's Law for flow through a porous medium:

$$Q = \frac{kA\Delta P_{RHA}}{\mu L} \qquad (4)$$

where k is the permeability of the RHA, A is the cross-sectional area of the RHA, ΔP_{RHA} represents the change in pressure across the porous medium, μ is the viscosity of the water, and L is the length of RHA. Note that this form of Darcy's Law assumes flow driven by pressure only (i.e. the elevation gain through the RHA is neglected). Comparing this equation with Eq. (2), it can be seen that the effective resistance of the RHA is given by

$$R_{RHA} = \frac{\mu L}{kA} \qquad (5)$$

Because k is dependent on factors such as grain size and particle arrangement, which are difficult to determine, its value was chosen based on experimental results [3]. Several experiments were conducted (detailed later in this paper) during which flow rate and pressure were measured. With all other variables known, k was back-calculated to fit the data.

Note also that there are two mesh screens on either side of the RHA that prevent the medium from dispersing with the water. Since it was impossible to isolate these screens from the RHA without disrupting the RHA and therefore changing its permeability, it was determined to lump the resistance of these screens into the value of k. In other words, the RHA and the mesh screens were treated as one resistor with an experimentally determined permeability.

Major Losses

Major and minor losses are pressure drops across elements due to viscous effects [4]. These viscous effects create flow resistances that, for the sake of simplicity, we will refer to as major and minor loss resistances.

Major losses are given by the Darcy-Weisbach equation:

$$\Delta P = \frac{fL\rho v^2}{2D_H} \qquad (6)$$

where f is the Darcy friction factor and is given by

$$f = \frac{64}{Re_D} \tag{7}$$

L is the length of the element in the flow direction, v is the velocity of water through the element and is given by

$$v = \frac{Q}{A} \tag{8}$$

D_H is the hydraulic diameter and is given by

$$D_H = \frac{4A}{Pe_w} \tag{9}$$

Re_D is the Reynolds number and is given by

$$Re_D = \frac{\rho v D_H}{\mu} \tag{10}$$

A is the area perpendicular to the direction of flow, and Pe_w is the wetted perimeter of the element. Substituting Eqs. (7), (8), and (10) into Eq. (6) yields

$$\Delta P = \frac{32 L \mu Q}{D_H^2 A} \tag{11}$$

From Eqs. (2) and (11), it can be shown that the effective resistance associated with major losses, R_M, is given by

$$R_M = \frac{32 L \mu}{D_H^2 A} \tag{12}$$

Since $A \propto D_H^2$, $R_M \propto D_H^{-4}$. Therefore, as D_H becomes large, the major loss resistance quickly becomes negligible. Table 1 shows the hydraulic diameter (or similar characteristic length) of several cross-sections within the system. Because the hydraulic diameter of the flow-restricting nozzle is much smaller than the hydraulic diameter anywhere else in the system (by at least an order of magnitude), major loss resistances can be neglected everywhere except in the flow-restricting nozzle.

Table 1. HYDRAULIC DIAMETER OF VARIOUS COMPONENTS IN SYSTEM.

Component	Hydraulic Diameter, D_H (m)
Flow-Restricting Nozzle	0.001
EOL Tube	0.025
Bulb Inlet	0.034
Pre-RHA Tube	0.050
Bulb Outlet	0.190

Also, since Eq. (7) is only valid for laminar flow, care was taken to ensure that the Reynolds number did not exceed 2300 in the nozzle. For a flow rate of 1.67E^{-6} m^3/s (6 L/hr), the Reynolds number in the nozzle was found to be 1870 (note that the nozzle has a *square* cross-section of edge length .001 m).

An assumption was made that flow in the nozzle is fully developed. This is unlikely to be true since the nozzle length is only .005 m. However, it was determined that since the major loss over the nozzle is the smallest of the losses considered to be significant, any error associated with this assumption would be small when applied to the entire system.

Minor Losses

Minor losses are given by

$$\Delta P = \frac{K_L \rho v^2}{2} \tag{13}$$

where K_L is the minor loss coefficient and can be obtained from tabulated data. Substituting Eq. (8) yields

$$\Delta P = \frac{K_L \rho Q^2}{2 A^2} \tag{14}$$

Comparing with Eq. (2) gives the minor loss resistance as

$$R_m = \frac{K_L \rho Q}{2 A^2} \tag{15}$$

For the situation of contraction of the fluid into the nozzle, $K_L = 0.48$ and A is the area inside the nozzle [4]. Again, because the resistance is dependent on A^2, minor losses in other parts of the bulb can be neglected by the order of magnitude argument. Note that there is no corresponding resistance

associated with expansion of the fluid after the nozzle. This is because flow stops after the nozzle and water simply drips through the EOL.

The only significant resistances in the system, then, are the resistance of the RHA, major loss resistance through the flow-restricting nozzle, and minor loss resistance due to contraction of the fluid into the nozzle. Because the fluid must pass through each of these resistances, they can be treated as resistors in series and summed to give a total resistance of the system. Thus,

$$R_{tot} = R_{RHA} + R_M + R_m \qquad (16)$$

Finally, for any chosen nozzle geometry and RHA configuration (assuming permeability is held constant), Eq. (1) can be used to predict flow rate:

$$Q = \frac{\Delta P_{tot}}{R_{tot}} \qquad (17)$$

Note that since R_m is dependent on Q, Eq. (17) is implicit and must be solved using numerical methods.

Residence Time

The residence time is the amount of time the water spends inside the RHA and in contact with the purifying agent (silver nanoparticles).

$$\tau = \frac{v}{L} \qquad (18)$$

where v is the velocity of water through the RHA and L is the length of the RHA in the flow direction. Substituting Eq. (8) yields

$$\tau = \frac{Q}{V_{RHA}} \qquad (19)$$

where V_{RHA} is the total volume of the rice husk ash. It was assumed that as long as two prototypes had equivalent residence times, the purification level achieved by each would also be equivalent. Therefore, any increase in flow rate must correspond with an appropriate increase in V_{RHA}.

DESIGN TOOL AND OPTIMIZATION

The equations described in the previous section were used to create a design tool in Microsoft Excel. This tool allows designers to input the physical parameters of the bulb (nozzle dimensions, RHA properties, and fluid properties) and predict the resulting flow rate through the bulb. Resistances are calculated using Eqs. (5), (12), and (15), and Eq. (17) is used to give the flow rate for a given pressure head. Because R_m is dependent on the flow rate, an initial guess for Q is required, and the solver is used to ensure that the initial guess and the predicted value of Q converge to the same value. The residence time is also calculated so that designers can ensure that minimum purification requirements are met.

This provides a snapshot of the flow at a single moment in time. In order to see how the flow behaves with respect to time, the dynamic pressure head must be taken into account (i.e. as water is filtered and passes to the other side of the bulb, the pressure driving the flow is reduced). To accomplish this, the calculated values are used as initial conditions for a step-wise solution where the resistances and flow rate are recalculated every five seconds. Automatically-generated figures based on these data allow designers to see how the filter will operate for an entire filtration cycle. The design tool thus provides a convenient method for designers to virtually prototype various bulb configurations and observe performance without developing expensive physical models.

Once the design tool was complete, it was used to optimize the configuration of the Tata Swach bulb. The goal of the optimization problem was to reach a maximum flow rate with the constraints that the residence time could be no shorter than in the original bulb (to preserve efficacy) and the plastic body of the bulb could not be significantly altered (to preserve manufacturability and cost). Measurements of bulb dimensions showed that the current length of the RHA is 32 mm, and there is approximately 16 mm of empty space between the top of the RHA and the bottom of the nozzle. The RHA length, therefore, could be increased by 50% without changing the plastic body of the bulb. This means that the flow rate could also be increased by 50% while maintaining the current minimum residence time. With this objective in mind, the design tool was used to determine the nozzle parameters that would achieve the desired flow rate. It was determined that a nozzle orifice size of 4 mm would result in a new flow rate of $2.51E^{-6}$ m^3/s (9 L/hr), a 50% increase from the current flow rate of $1.67E^{-6}$ m^3/s (6 L/hr). Note that these flow rates are the maximum flow rate achieved when the upper container of the system is completely full. This was considered to be an optimized flow rate as it is the maximum allowable flow rate that can be achieved in the current embodiment of the bulb without altering residence time (efficacy) or manufacturability and cost associated with the plastic container.

EXPERIMENTAL VERIFICATION

Equation (19) shows that in order to increase volumetric flow rate through the bulb by a certain factor without changing residence time, the length of the RHA filtration element must be increased by the same factor (area held constant). This was the basis upon which a prototype was created.

The stock Swach bulb was first studied by measuring its total volume output at uniform time intervals, given 15 cm of initial pressure head, h_0, as measured in the upper container (refer to Fig. 1). The same method was used to generate similar data for other Swach bulbs that were modified from the original. A prototype bulb was made by removing the flow-restricting nozzle (making it a 4.0 mm wide square hole), and adding RHA material to the inner bulb to increase its length by 50% (corresponding to a 50% increase in volumetric flow rate as per the logic above). Bulbs with different nozzle dimensions (1.3, 2.0, 8.0 mm side lengths) were fabricated by modifying nozzles of existing bulbs, and were tested in the same fashion as before. These experiments were performed to compare the original bulb with the prototype that was designed using the flow model described above, to demonstrate the sensitivity of volumetric flow rate to nozzle size when the nozzle is small (< 2.0 mm), and to demonstrate the significance of the nozzle resistance for large nozzle dimensions.

The manufacturing specifications and variation of the Swach bulb were measured for 5 samples and are shown in Table 2. Variation of measurement between these samples were used in the mathematical model to account for manufacturing error.

Table 2. MANUFACTURING SPECIFICATIONS OF RHA AND NOZZLE.

Component	Specifications
RHA mass	100 ± 5 g
RHA length	35 ± 1 mm
Nozzle diameter	1.00 ± 0.05 mm

The permeability of the RHA was found by fitting the mathematical model to 4 experimental data sets with different nozzle and RHA dimensions (Tab. 3), but with the same RHA medium. This was found to be approximately $5.5E^{-11}$ m^2. The strong consistency between experiment and theory for all four data sets suggests the suitability of this model to the flow characteristics in the Swach bulb.

Table 3. NOZZLE AND RHA DIMENSIONS FOR EXPERIMENTAL TESTS.

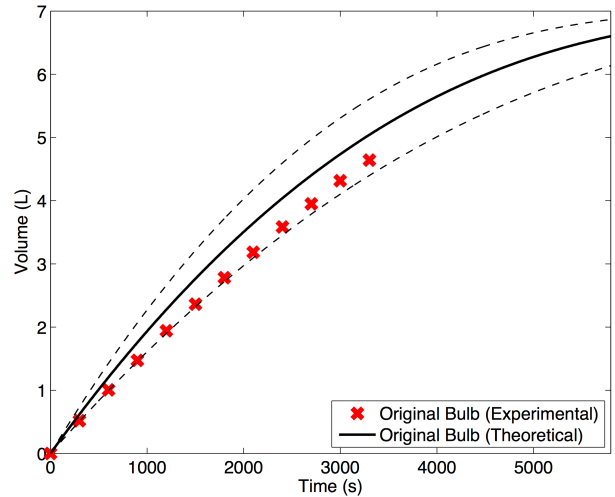| Test	RHA Length, L_{RHA} (mm)	Nozzle Hydraulic Diameter, D_H (mm)
1	32	1.15
2	50	4
3	32	4
4	32	8

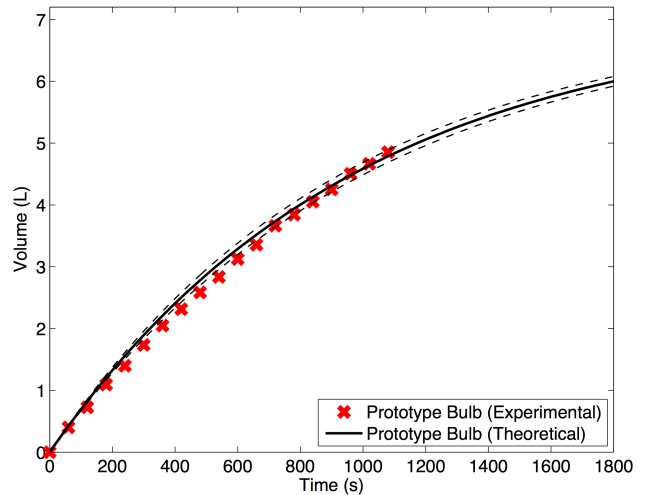Figure 3. THEORETICAL FLOW MODEL OF ORIGINAL BULB WITH ERROR BARS AND EXPERIMENTAL VALIDATION.

Figure 4. THEORETICAL FLOW MODEL OF PROTOTYPE BULB WITH ERROR BARS AND EXPERIMENTAL VALIDATION.

The mathematical model was found to agree with stock Swach bulbs (Fig. 3) as well as a prototype bulb (Fig. 4). Both the variation in nozzle size and RHA length due to manufacturing error have been accounted for in the theoretical model (error bars).

A 50% increase in the maximum volumetric flow rate from the original bulb is achieved in the prototype bulb (Fig. 4). Here, the flow restricting nozzle piece was removed to create a 4.0 mm square hole, which effectively removed this resistance. In order to maintain residence time, the RHA length was increased by 50% by adding 18 mm of RHA length to the original 35 mm.

Error bars are more pronounced for the stock bulbs (Fig. 3) because changes in nozzle size when the nozzle is small lead to
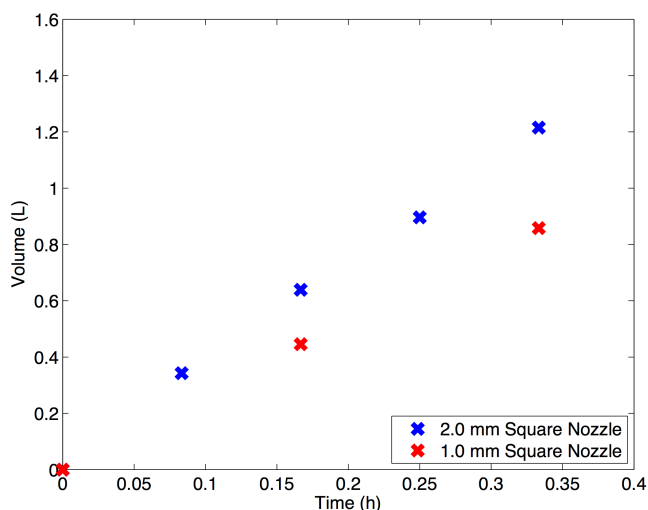
Figure 5. HIGH SENSITIVITY OF THE FLOW
RATE TO NOZZLE SIZE WHEN THE NOZZLE IS
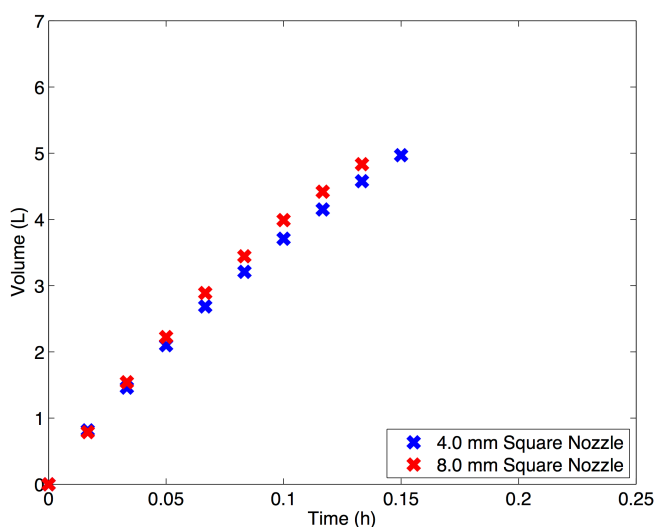SMALL (< 2.0 mm).

Figure 6. NOZZLE RESISTANCE IS NEGLIGIBLE
WHEN THE NOZZLE IS LARGE (> 4.0 mm).

magnified changes in flow rate. The sensitivity in flow rate to nozzle size was demonstrated experimentally in Fig. 5, which displays tests performed for bulbs with nozzles of 1.3 mm and 2.0 mm side lengths. Flow rate in the mathematical model was shown to be very sensitive to nozzle size below a 2.0 mm dimension due to the dominance of the nozzle resistance in the electrical circuit analogy, which was consistent with experimental results.

On the other hand, the model shows that above certain nozzle dimensions (> 4.0 mm side lengths), the flow rate through the bulb is no longer sensitive. This was experimentally confirmed by testing bulbs with the same RHA length but with 4.0 mm and 8.0 mm sized nozzles (Fig. 6). In
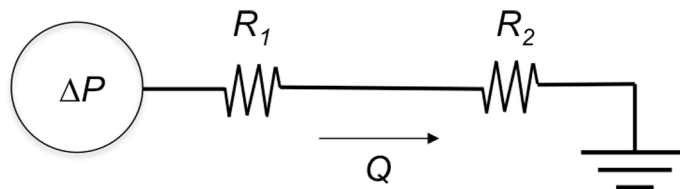

Figure 7. A DEPICTION OF RESISTANCES IN THE
SWACH BULB, WHERE R_2 REPRESENTS A FLOW-
RESTRICTING NOZZLE.

this size range the nozzle resistance is negligible, so any changes in size and therefore resistance lead to minimal changes in flow rate.

Let R_1 and R_2 represent two flow resistances, where R_2 is a flow-restricting nozzle (Fig. 7). If $R_2 \gg R_1$ (e.g. where R_2 is a very small nozzle), then the system is very sensitive to R_2. Fig. 5 demonstrates this sensitivity when the flow-restricting nozzle is < 2.0 mm. On the other hand, if R_2 is reduced and $R_2 \ll R_1$, then changes in R_2 will not affect the flow. Fig. 6 demonstrates the low sensitivity of flow rate to nozzle size when the flow restricting nozzle is > 4.0 mm.

The flow model was shown to closely track flow data from experiments performed using the Tata Swach. Its use as a design tool additionally describes sensitivities that Tata can now account for. It demonstrates that Tata may increase flow rate by increasing nozzle size while increasing RHA length. The existing dimensions of the Swach bulb allow this to be done without any drastic changes to geometry. At the same time, if nozzle sizes were increased beyond 4.0 mm, Tata may not need to improve manufacturing tolerances due to the reduced sensitivity of flow rate to nozzle size in this regime.

CONCLUSIONS

The impact of this work is manifold. First, by developing a mathematical model of the flow, the effects of certain features on filter performance are elucidated. For example, it is now apparent that flow rate is highly sensitive to the size of the orifice in the flow-restricting nozzle. The model shows that R_M and R_m are proportional to D_H^{-4}. This means that very small changes in orifice size can result in drastically different flow rates. The importance of tolerances is quantified and failure points are highlighted, allowing Tata Chemicals Ltd. to develop a more robust system.

Also, the design tool allows Tata Chemicals to create prototypes virtually before creating them physically. Concepts with poor performance can be ruled out before expensive building and testing occurs. This will reduce new product development times and costs.

The optimization methodology demonstrated here can be applied to other similar systems. Individuals or organizations interested in water purification or hydraulic systems can use a similar approach to characterize flow and optimize system performance.

Future work should focus on determining an accurate value for the permeability of the RHA. This should be left to Tata Chemicals as the value of k will be highly dependent on their standard packing and assembly procedure. At that point, it may also be useful to separate the effects of the two mesh screens from the permeability of the RHA. Finally, consideration of resistances that were neglected in this work may lead to greater accuracy, though researchers should bear in mind the relative magnitude of errors associated with each assumption as well as the ultimate purpose of the model in order to determine if such work is worthwhile.

ACKNOWLEDGMENTS

REFERENCES

[1] World Health Organization and UNICEF, 2012. "Progress on Drinking Water and Sanitation". On the WWW at http://www.unicef.org/media/files/JMPreport2012.pdf

[2] Shah, J., 2013. Tata Swach: Making safe drinking water an affordable reality. *Tata Chemicals Ltd.*, Mumbai, India.

[3] Zimmerman, R. W., 2003. "Flow in Porous Media". MS Thesis, Department of Earth Science and Engineering, Imperial College, London, UK, May.

[4] Shaughnessy Jr., E., Katz, I., Schaffer, J., 2005. *Introduction to Fluid Mechanics*, Oxford University Press, Inc., New York, NY