

1 **Topographic stress and rock fracture: A two-dimensional numerical model for arbitrary**  
2 **topography and preliminary comparison with borehole observations**

3 Running title: "Topographic stress and rock fracture"

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22

23 **Abstract**

24       Theoretical calculations indicate that elastic stresses induced by surface topography may be  
25 large enough in some landscapes to fracture rocks, which in turn could influence slope stability,  
26 erosion rates, and bedrock hydrologic properties. These calculations typically have involved  
27 idealized topographic profiles, with few direct comparisons of predicted topographic stresses and  
28 observed fractures at specific field sites. We use a numerical model to calculate the stresses  
29 induced by measured topographic profiles and compare the calculated stress field with fractures  
30 observed in shallow boreholes. The model uses a boundary element method to calculate the  
31 stress distribution beneath an arbitrary topographic profile in the presence of ambient tectonic  
32 stress. When applied to a topographic profile across the Susquehanna Shale Hills Critical Zone  
33 Observatory in central Pennsylvania, the model predicts where shear fractures would occur based  
34 on a Mohr-Coulomb criterion, with considerable differences in profiles of stresses with depth  
35 beneath ridgetops and valley floors. We calculate the minimum cohesion required to prevent  
36 shear failure,  $C_{\min}$ , as a proxy for the potential for fracturing or reactivation of existing fractures.  
37 We compare depth profiles of  $C_{\min}$  with structural analyses of image logs from four boreholes  
38 located on the valley floor, and find that fracture abundance declines sharply with depth in the  
39 uppermost 15 m of the bedrock, consistent with the modeled profile of  $C_{\min}$ . In contrast,  $C_{\min}$   
40 increases with depth at comparable depths below ridgetops, suggesting that ridgetop fracture  
41 abundance patterns may differ if topographic stresses are indeed important. Thus, the present  
42 results are consistent with the hypothesis that topography can influence subsurface rock fracture  
43 patterns and provide a basis for further observational tests.

44

45 **Introduction**

46 *Motivation and purpose*

47 Fractures in bedrock influence numerous processes that drive landscape evolution,  
48 including weathering (Anderson et al., 2007), erosion (Moon and Selby, 1983; Augustinus, 1995;  
49 Whipple et al., 2000; Molnar et al., 2007; Moore et al., 2009; Dühnforth et al., 2010), slope  
50 stability (Terzaghi, 1962; Wieczorek and Snyder, 1999; Muller and Martel, 2000; Chigara, 2000;  
51 Clarke and Burbank, 2010, 2011; Stock et al., 2012), and groundwater flow (LeGrand, 1949;  
52 Keller et al., 1986; Trainer, 1988; Borchers, 1996; Berkowitz, 2002; Neuman, 2005). Various  
53 studies have suggested that landforms might in turn influence bedrock fractures by perturbing the  
54 ambient stress field (e.g., McTigue and Mei, 1981; Savage et al., 1985; Savage and Swolfs,  
55 1986) and altering the type, orientation and abundance of fractures in different parts of the  
56 subsurface (e.g., Miller and Dunne, 1996). This mutual influence could lead to feedbacks  
57 between evolving topography and fracture patterns (Miller and Dunne, 1996; Molnar, 2004).  
58 However, few studies have tested whether modeled topographic stresses appear to have  
59 influenced observed bedrock fracture patterns, in part because prior theoretical approaches were  
60 too simplified to apply to specific topographic and tectonic settings. This paper has two  
61 purposes: (1) to provide a modeling framework for calculating topographic stresses beneath  
62 arbitrary topography subject to approximately two-dimensional (plane-strain) conditions; and (2)  
63 to conduct a preliminary test for the effects of topographic stresses by comparing a stress model  
64 for a specific field site with fractures observed in shallow boreholes.

65 *Previous studies of topographic stress and rock fracture*

66 Early studies of topographic stresses focused on the effects of large-scale topography  
67 (hundreds of kilometers or more in horizontal extent) on the lithosphere at depths of kilometers  
68 or more (see review in McNutt, 1980). Holzhausen (1978) and McTigue and Mei (1981) were  
69 among the first to study the stress distribution immediately beneath local topographic features.  
70 Holzhausen (1978) examined an elastic medium with a gently sloping sinusoidal surface using a  
71 perturbation method. McTigue and Mei (1981) also used a perturbation method valid for gently  
72 sloping surfaces but considered more complex topographic forms with the aid of a spectral  
73 approach. McTigue and Mei (1981) used these approximate solutions to show that even if  
74 regional horizontal stresses are absent, topography induces horizontal compression under  
75 ridgetops and horizontal tension under valley floors, and that the effect of topography decreases  
76 with depth. They also showed that the addition of regional horizontal compressive stress can  
77 induce horizontal tension at topographic highs, a result noted earlier by Scheidegger (1963).

78 Savage et al. (1985) and Savage and Swolfs (1986) subsequently found exact analytical  
79 solutions for the elastic stress distribution under certain landforms using the approach of  
80 Muskhelishvili (1953). Their solutions are valid for a set of idealized, symmetric, isolated valleys  
81 and ridges with shapes described by a conformal coordinate mapping. Savage et al. (1985)  
82 calculated the effect of gravity in a laterally constrained medium and showed that topographic  
83 stresses are on the order of  $\rho gb$ , where  $\rho$  is the rock density,  $g$  is the acceleration of gravity, and  $b$   
84 is ridge height or valley depth. Like Holzhausen (1978) and McTigue and Mei (1981), Savage et  
85 al. (1985) predict horizontal compressive stress under ridgetops, horizontal tensile stress under  
86 valley floors, and stresses approaching those beneath a horizontal surface as depth increases.  
87 Savage and Swolfs (1986) additionally evaluated how topography perturbs a regional horizontal  
88 tectonic stress and then superposed their solutions on those of Savage et al. (1985) to obtain the

89 stress distribution due to both tectonic and gravitational stresses. They demonstrated that the  
90 effect of regional tectonic compression is reduced near ridge crests and amplified in valleys. Pan  
91 and Amadei (1994) proposed a similar method that uses a numerical conformal mapping  
92 procedure to accommodate irregular but smooth two-dimensional topographic profiles.

93 Savage et al. (1985) and Savage and Swolfs (1986) calculated subsurface stresses  
94 assuming that the ratio  $k$  of the vertical gradient of horizontal stress to the vertical gradient of  
95 vertical stress was positive but less than one. Miller and Dunne (1996) compiled published  
96 crustal stress values from different geographic locations and concluded that in many places  $k > 1$ .  
97 Noting that many of the reported stresses indicated high regional compressive tectonic stress,  
98 they used the approach of Savage et al. (1985) and Savage and Swolfs (1986) to calculate  
99 stresses for cases with  $0 < k < 1$  and  $k > 1$ . They also noted that the magnitudes of the modeled  
100 stresses exceeded typical mechanical strengths of various rock types, and used brittle fracture  
101 criteria based on laboratory experiments to predict the fracture patterns that would develop for  
102 different combinations of landform shape and tectonic stress state.

103 Miller and Dunne (1996) discussed the implications of their fracture predictions for  
104 landscape evolution. Noting that elastic stresses scale with topographic relief, they proposed that  
105 fracturing might occur only if the topographic relief (ridge height above valley floor) is  
106 sufficiently high that the stresses exceed a brittle failure threshold. They also proposed a positive  
107 feedback between topographic stresses and landscape evolution in which valley incision triggers  
108 fracturing in the valley floor, which makes bedrock more erodible, accelerates valley incision,  
109 and further enhances the fracturing effect. Molnar (2004) revisited the examples discussed by  
110 both Savage et al. (1985) and Miller and Dunne (1996). He developed a framework for

111 quantifying the positive feedback in sustaining a valley that Miller and Dunne (1996) proposed,  
112 and showed the dependence of stress concentration in a valley on its shape, especially the  
113 sharpness of the “V” at the valley axis. Molnar (2004) also suggested that static fatigue (gradual  
114 crack growth at stresses below the macroscopic failure threshold) could be an important  
115 mechanism by which topographic stresses influence rock erodibility and landscape evolution.

116         These studies have advanced our understanding of topographic stresses and the possible  
117 consequences for landscape evolution, but only a few investigations have compared these  
118 predictions with observed stresses, fracture patterns, or proxies for rock damage. Savage and  
119 Morin (2002) and Morin and Savage (2002) used the analytical solutions of Savage et al. (1985)  
120 to approximate the near-surface stresses at a study site in the Davis Mountains of Texas, and  
121 compared their predictions to stress orientations inferred from borehole breakouts. They found  
122 that breakout orientation appeared to change with depth in a manner consistent with their stress  
123 calculations. However, they did not attempt to compare predicted stresses with fracture patterns.  
124 Morin et al. (2006) processed sonic logs and borehole images to study the mechanical properties  
125 of a fractured basalt aquifer and the permeability of subsurface fractures. They used a two-  
126 dimensional finite element model to calculate the stress distribution beneath the Annapolis  
127 Valley, Nova Scotia, and found that topographic stresses appear to influence the permeability of  
128 pre-existing fractures. Martel (2011) predicted the areal distribution of sheeting joints in part of  
129 Yosemite National Park using an exact solution for the gradient in the normal stress  
130 perpendicular to the surface. He accounted for the site-specific topography but assumed the  
131 surface-parallel compressive stresses were constant. Observations of sheeting joints on domes,  
132 ridges, and saddles with wavelengths of a few hundred meters were consistent with the  
133 hypothesis that surface-parallel stresses and topography account for the formation of sheeting

134 joints (Martel, 2011). Leith et al. (2013a) applied a model of coupled stress and elasto-plastic  
135 rock deformation (Leith et al., 2013b) to the Matter Valley in the Swiss Alps, and found that a  
136 zone where the model predicts extensional fracturing corresponds to a break in slope on the  
137 valley walls where enhanced glacial erosion has created an inset U-shaped gorge. However, they  
138 did not attempt to compare their model systematically with rock fracture patterns. An opening  
139 clearly exists for a more complete examination of the coupling between topography and stress  
140 state, the importance of this coupling at depth, and the combined impact on fractures at a specific  
141 site.

142 *New contribution and outline*

143         In this paper, we implement a numerical method for calculating stresses beneath  
144 topographic profiles of arbitrary form and compare predicted topographic stresses with fracture  
145 patterns mapped from shallow boreholes. We first describe our adaptation of the boundary  
146 element model of Martel (2000) and Martel and Muller (2000), which calculates two-  
147 dimensional elastic stresses due to the combined effects of ambient tectonic stress and gravity  
148 acting on topography. We then apply the model to idealized, synthetic topographic profiles to  
149 illustrate the sensitivity of calculated stresses and fracture patterns to tectonic stress, rock  
150 mechanical parameters, and the shapes of ridges and valleys. Next, we calculate stresses beneath  
151 a topographic profile across the Shale Hills, an experimental watershed in central Pennsylvania.  
152 We use the modeled stresses to calculate a proxy for shear fracture susceptibility and compare  
153 that proxy to the fracture abundance observed in optical logs of shallow boreholes in the valley  
154 floor. Finally, we comment on the implications of our results for the hypothesized effects of  
155 topographic stress on rock fracture, and we discuss shortcomings in our current understanding of

156 rock damage accumulation that must be addressed to fully characterize the influence of stress on  
157 landscape evolution.

158

## 159 **Boundary element model for stresses induced by arbitrary topography**

### 160 *Model description*

161         The need to study stresses induced by real topography and their interactions with variable  
162 tectonic stresses motivated Martel and Muller (2000) to develop a flexible numerical method.  
163 They adapted a boundary element method (BEM) based on Crouch and Starfield's (1983) two-  
164 dimensional displacement discontinuity code (TWODD), which is in turn based on an analytical  
165 solution for a constant discontinuity displacement over a finite linear segment in an infinite, two-  
166 dimensional elastic solid. A typical application of Crouch and Starfield's (1983) method is to  
167 calculate the stresses around a crack in a body of rock. The crack is approximated by a set of  
168 linear segments, and the displacement discontinuity – opening, closing and/or sliding – across  
169 the crack is approximated by a set of discrete displacement discontinuities across the linear  
170 segments. Normal and shear tractions on the linear segments are specified as boundary  
171 conditions. The TWODD code then solves for the displacements on opposing sides of the linear  
172 segments that satisfy these boundary conditions. Once these displacements are known, the  
173 resulting stresses at any point in the body of rock can be calculated. This solution method can  
174 account for an arbitrary ambient horizontal stress field.

175         Martel and Muller (2000) adapted the method of Crouch and Starfield (1983) by treating  
176 a topographic surface as a long, traction-free crack in a body of rock that is exhumed by the



177 removal of overburden. We adopt and extend their method here. In this section, we give an  
 178 overview of the computational procedure; for a more detailed explanation, see Martel and Muller  
 179 (2000) and Crouch and Starfield (1983). The topographic surface is approximated by a set of  
 180 connected linear segments passing through a laterally confined body in plane strain under the  
 181 influence of gravity with a traction-free horizontal surface (Figure 1). The ambient stresses in  
 182 this body are

$$\sigma_{xx}^a = k\rho gy + \sigma_{xx}^t \quad (1a)$$

$$\sigma_{yy}^a = \rho gy \quad (1b)$$

$$\sigma_{xy}^a = \sigma_{yx}^a = 0 \quad (1c)$$

183 where  $x$  is the horizontal coordinate (increasing to the right),  $y$  is the vertical coordinate  
 184 (increasing upward),  $\rho$  is rock density,  $g$  is gravitational acceleration, and stress subscripts use  
 185 the “on-in” convention (Pollard and Fletcher, 2005). Compressive stresses are defined as  
 186 negative. The term  $\sigma_{xx}^t$  in Equation 1a is a constant ambient horizontal tectonic stress that may  
 187 exist in addition to the gravitationally induced stresses. The parameter  $k$  scales the vertical  
 188 gradient in the ambient horizontal normal stress relative to the vertical gradient in the ambient  
 189 vertical normal stress. We use  $k$  rather than the term  $\nu/(1-\nu)$ , where  $\nu$  is Poisson’s ratio, to  
 190 emphasize that the in-plane stress solutions are independent of the elastic parameters of the rock,  
 191 provided it is homogeneous and isotropic (Timoshenko and Goodier, 1970).

192 The stress field below the topographic surface is found by evaluating the stress  
 193 perturbation caused by the generation of the topography (via the erosion of overburden) and then  
 194 superposing this perturbation on the ambient stress field. The shear tractions,  $\sigma_s$ , and normal  
 195 tractions,  $\sigma_n$ , on the elements defining the topographic surface are set to zero. This traction-free

196 condition does not necessarily imply a *stress*-free surface, because the surface-parallel normal  
 197 stress can be nonzero. The ambient stresses at the element locations due to the overburden are  
 198 calculated from Equations 1a-c, resolved into shear and normal tractions, and subtracted from the  
 199 zero-traction condition to define the boundary conditions for a boundary element problem  
 200 describing the topographic stress perturbation (that is, they describe the effect of the topographic  
 201 “crack” on the ambient stress). The resulting tractions are stored in a vector  $\mathbf{b}$ . The TWODD  
 202 method is then used to calculate a matrix  $\mathbf{A}$  of influence coefficients. Each element of  $\mathbf{A}$  gives  
 203 the change in either shear or normal traction on one boundary element due to a unit shear or  
 204 normal displacement discontinuity at another boundary element. This defines a system of linear  
 205 equations,

$$\mathbf{Ax} = \mathbf{b} , \quad (2)$$

206 which is solved to yield the unknown displacement discontinuities,  $\mathbf{x}$ , at the elements.

207 Once these displacement discontinuities are known, the stress perturbation at any  
 208 observation point below the topographic surface can be calculated. A set of observation points is  
 209 chosen, and TWODD is used to calculate a second matrix of influence coefficients,  $\mathbf{A}^{\text{obs}}$ . Each  
 210 element of  $\mathbf{A}^{\text{obs}}$  gives the effect of a unit shear or normal displacement discontinuity at one of the  
 211 boundary elements on the horizontal normal stress, vertical normal stress, or shear stress at one  
 212 of the observation points. The total stress perturbations at the observation points,  $\boldsymbol{\sigma}^{\text{obs}}$ , are  
 213 calculated as

$$\mathbf{A}^{\text{obs}} \mathbf{x} = \boldsymbol{\sigma}^{\text{obs}} . \quad (3)$$

214 The final step is to add the stress perturbations  $\mathbf{b}$  and  $\sigma^{\text{obs}}$  to the ambient stresses (Equations 1a-  
215 c) at the elements and the observation points, respectively. This yields the total stresses at and  
216 below a traction-free topographic surface due to gravity and any horizontal tectonic stress.

217 The shear and normal tractions on the elements that define the land surface are known  
218 (they are specified as boundary conditions), but the TWODD method does not calculate the  
219 surface-parallel (tangential) normal stresses on the elements. Following Crouch and Starfield  
220 (1983, section 5.10), we approximate the tangential normal stresses using finite differences. The  
221 shear and normal displacements of the elements are differenced to obtain an approximation for  
222 the tangential normal strain at each element midpoint, and the tangential stress is calculated from  
223 Hooke's Law. Along with the shear and normal tractions on the elements, this completely  
224 defines the stress field at and below the land surface.

225 A few additional considerations should be noted when using the TWODD method to  
226 calculate topographic stresses. To ensure an equilibrium stress state, the left and right ends of the  
227 topographic surface should be tapered to the same elevation (Figure 1). Tapering to a horizontal  
228 surface that extends beyond the area of interest (Figure 1) will minimize edge effects. The  
229 additional computational cost introduced by this tapering is not large, because longer elements  
230 can be used near the ends of the profile, and because observation points need only be chosen  
231 within the area of interest. The calculated stresses are very accurate at element midpoints and at  
232 observation points farther than an element length from element endpoints, but can be inaccurate  
233 at observation points within an element length of element endpoints. Two strategies that mitigate  
234 this effect are to center near-surface observation points beneath element midpoints and make

235 elements short enough that the zones of inaccuracy around element endpoints only extend a short  
236 distance below the land surface.

237 The BEM presented here uses a very different approach to calculate topographic stresses than  
238 previous analytical methods, so it is useful to confirm that the different approaches yield  
239 equivalent results for the same scenarios. We compared BEM solutions with analytical solutions  
240 (Savage, 1985; Savage and Swolfs, 1986) for an isolated valley with and without ambient  
241 tectonic stress (Figure 9 in Savage and Swolfs, 1986). In the analytical solutions, stresses were  
242 calculated at points evenly spaced in  $u$  and  $v$  in the coordinate mapping of Savage et al. (1985).  
243 In the BEM solutions, stresses were calculated at points defined with the meshing scheme  
244 described above. Parameter values are listed in Table 1. Figure 2 compares  $\sigma_{xx}$  in the analytical  
245 solutions (Figure 2a,b) with the BEM solutions (Figure 2c,d). The analytical and BEM solutions  
246 agree closely, consistent with the results of Martel and Muller (2000). The largest discrepancies  
247 occur at the land surface due to the reduced accuracy of the finite-difference approximation used  
248 to calculate tangential normal stress on the boundary elements (Crouch and Starfield, 1983,  
249 section 5.10), but these discrepancies are sufficiently small that they are not apparent in Figure 2.  
250 Unlike the analytical methods, the BEM can be used to calculate stresses beneath any  
251 topographic profile, a property we exploit in this study.

### 252 *Predicting fracture mode and orientation*

253         Given a calculated stress field, we seek to estimate the mode, orientation and abundance  
254 of any resulting fractures as a basis for comparison with observed fractures. Following Miller  
255 and Dunne (1996), we use experimentally determined criteria for opening-mode and shear failure  
256 of rock. In plane strain, two of the principal stresses are in the plane of the topographic cross-

257 section, and the third principal stress is the normal stress perpendicular to the cross-section,  $\sigma_{zz}$ .  
 258 In most of the scenarios considered here, the most compressive principal stress  $\sigma_3$  and the least  
 259 compressive principal stress  $\sigma_1$  are in the plane of the cross-section,  $\sigma_{zz}$  is the intermediate  
 260 principal stress, and the three are related by  $\sigma_{zz} = \nu(\sigma_1 + \sigma_3)$ . However, there are two situations in  
 261 which  $\sigma_{zz}$  is not the intermediate principal stress. If  $\sigma_{lc}/\sigma_{mc} < \nu/(1-\nu)$ , where  $\sigma_{lc}$  is the least  
 262 compressive in-plane principal stress and  $\sigma_{mc}$  is the most compressive in-plane principal stress,  
 263 then  $\sigma_{zz}$  is the least compressive principal stress. This can only occur where both  $\sigma_{lc}$  and  $\sigma_{mc}$  are  
 264 compressive. If  $\sigma_{mc}/\sigma_{lc} > \nu/(1-\nu)$ , then  $\sigma_{zz}$  is the most compressive principal stress. This can only  
 265 occur where both  $\sigma_{lc}$  and  $\sigma_{mc}$  are tensile.

266 The well-known graphical representation of a two-dimensional stress state at a point is  
 267 the Mohr circle (Figure 3). The simplest and most commonly applied shear failure criterion is the  
 268 Coulomb criterion,

$$|\tau| = -\sigma \tan \phi + C, \quad (4)$$

269 which states that the shear stress,  $\tau$ , acting on a plane at the point of shear failure equals the sum  
 270 of the cohesive strength of the material,  $C$ , and the frictional resisting stress, given by the term  
 271  $-\sigma \tan \phi$ , where  $\phi$  is the internal friction angle of the material and  $\sigma$  is the normal stress acting  
 272 on the failure plane. The Coulomb criterion defines a pair of linear failure envelopes that are  
 273 symmetric about the  $\sigma$  axis on the Mohr diagram (Figure 3). If the Mohr circle touches this  
 274 failure envelope, shear fractures can form on either of two conjugate planes with normal vectors  
 275 oriented at angles  $\theta = 45^\circ + \phi/2$  with respect to the most compressive principal stress direction,

276 such that the fractures themselves are oriented  $90^\circ - \theta = 45^\circ - \phi/2$  away from  $\sigma_3$ . The Coulomb  
277 criterion can also be written in terms of the principal stresses (Jaeger et al., 2007),

$$-\sigma_3 = 2C \tan \theta - \sigma_1 \tan^2 \theta. \quad (5)$$

278 If pre-existing planes of weakness occur with lower cohesion or a different friction angle than  
279 intact rock, sliding (slip) on these planes may also occur. The Mohr diagram of Figure 3, which  
280 illustrates the case of pre-existing fractures with no cohesion and a friction angle  $\phi_s$ , shows that  
281 sliding would occur on any planes with normal vectors oriented at angles between  $\theta_1$  and  $\theta_2$  with  
282 respect to the most compressive principal stress.

283         If the normal stress on potential shear failure planes is not compressive, shear fractures  
284 typically do not occur, and the Coulomb criterion is not a relevant description of the failure  
285 mode. Laboratory experiments with low confining pressures typically produce opening-mode  
286 fractures oriented perpendicular to the least compressive (most tensile) principal stress,  $\sigma_1$   
287 (Paterson and Wong, 2005). The transition between opening mode and shear failure is complex,  
288 and can include hybrid fractures with characteristics of both end-member types (Ramsey and  
289 Chester, 2004). Following Miller and Dunne (1996), we use a simple approximation for the  
290 transition suggested by Jaeger et al. (2007). The stress state corresponding to the value of  $\sigma_3$  for  
291 which the normal stress on a potential shear failure plane ceases to be compressive can be  
292 visualized by drawing the Mohr circle that is tangent to the shear failure envelope where it  
293 crosses the  $\tau$  axis (Figure 3). From trigonometry, and using Equation (5), this implies that  
294  $-\sigma_3 = C \tan \theta$  and  $\sigma_1 = C / \tan \theta$  (Jaeger et al., 2007). Thus, shear fracture is predicted if

$$-\sigma_3 \geq C \tan \theta \text{ and } C < C_{\min}, \quad (6)$$

295 where  $C_{\min}$  is obtained by solving for  $C$  in Equation (5),

$$C_{\min} = \frac{-\sigma_3 + \sigma_1 \tan^2 \theta}{2 \tan \theta}, \quad (7)$$

296 and opening-mode fracture perpendicular to  $\sigma_1$  is predicted if

$$-\sigma_3 < C \tan \theta \text{ and } C < \sigma_1 \tan \theta. \quad (8)$$

297 Graphically, this is equivalent to truncating the Coulomb failure envelope at  $\sigma = C / \tan \theta$  and  
 298 extending the failure envelope vertically to the  $\sigma$ -axis (Figure 3). Miller and Dunne (1996) also  
 299 assume that opening-mode failure occurs in unconfined compression ( $\sigma_1 = 0$ ,  $\sigma_3 < 0$ ) if  $\sigma_3$   
 300 exceeds the unconfined compressive strength of the rock,  $q_u$ . To facilitate comparisons with their  
 301 results, we also use this criterion. In most of the examples considered here, unconfined  
 302 compression occurs only at the land surface, so this criterion predicts surface-parallel opening-  
 303 mode fractures right at the surface.

304 Recognizing that rocks in natural settings are typically influenced by heterogeneities that  
 305 are not described by these simple failure criteria, and that rock mechanical properties are seldom  
 306 known precisely, we seek an additional proxy for the likelihood of shear failure. From Equation  
 307 6, the minimum cohesion needed to prevent the development of new shear fractures at a given  
 308 location in the rock is  $C_{\min}$ , which is defined in Equation 7.  $C_{\min}$  corresponds to the  $\tau$ -axis  
 309 intercept of the failure envelope tangent to the Mohr circle that describes the state of stress. For a  
 310 given friction angle  $\phi$ , larger values of  $C_{\min}$  indicate that shear fractures are more likely to form

311 and that sliding on existing fractures is more likely to occur, whereas smaller values of  $C_{\min}$   
 312 indicate that shear fractures and sliding are less likely. An alternative measure proposed by  
 313 Iverson and Reid (1992) is the failure potential,  $\Phi$ , defined as

$$\Phi = \left| \frac{\sigma_3 - \sigma_1}{\sigma_3 + \sigma_1} \right| \quad (9)$$

314 For a cohesionless material on the verge of shear failure (described by a Mohr circle tangent to a  
 315 failure envelope that intercepts the origin of a plot such as Figure 3),  $\Phi = \sin \phi$ , and thus  $\Phi$   
 316 describes the internal friction needed to prevent failure of a cohesionless material.  $\Phi$  is a  
 317 dimensionless, scale-independent measure of shear failure potential, whereas  $C_{\min}$  is dimensional  
 318 and scale-dependent. In this paper, we are most interested in dimensional topographic scenarios  
 319 in which rocks have nonzero cohesion.  $C_{\min}$  is therefore a more useful proxy for our purposes,  
 320 and we use it in the examples below.

321 Tests confirm that the fracture patterns predicted by the BEM are consistent with  
 322 previous analytical approaches. We compared fracture patterns for one of the scenarios  
 323 investigated by Miller and Dunne (1996), who used the analytical stress solutions of Savage  
 324 (1985) and Savage and Swolfs (1986). The scenario is a valley with no ambient horizontal  
 325 compression at the far-field land surface and  $k = 1.5$  in Equation 1a, denoting a rapid increase in  
 326 horizontal compression with depth (Figures 4 and 7 of Miller and Dunne, 1996). Figure 4 shows  
 327 that the two approaches predict essentially identical fracture patterns, which is expected given  
 328 the close agreement of the stress fields (Fig. 2) and the use of the same fracture criteria.

329 *Sensitivity of stress and fracture patterns to model parameters and topography*



330 An example using a simple topographic surface illustrates how differences in model  
331 parameters and topography influence the calculated stress field and predicted fractures. We used  
332 the BEM to calculate stresses at and beneath a sinusoidal topographic profile with a wavelength  
333 of 500 m, an amplitude of 50 m, and a compressive horizontal tectonic stress of -6 MPa (Fig. 5).  
334 Values of other parameters are listed in Table 1. In this reference case, the horizontal  
335 compressive stress is concentrated beneath the valley floor and reduced beneath the ridgetops  
336 (Fig. 5a) (Savage and Swolfs, 1986). The differential stress (the difference between  $\sigma_1$  and  $\sigma_3$  in  
337 Fig. 5d) and the potential for shear failure, as measured by  $C_{\min}$  (Fig. 5e), are therefore much  
338 larger beneath the valley than beneath the ridges. For modest rock cohesion of 1 MPa and  
339 unconfined compressive strength  $q_u = 3$  MPa, the zone of fracturing (opening mode at the  
340 surface, shear mode at depth) is restricted to a crescent-shaped zone beneath the valley (Fig. 5f).

341 We repeated these calculations for different model parameters and topographic profiles to  
342 examine the sensitivity of the results in Fig. 5 to ambient stress, rock properties, and topography.  
343 Figure 6 shows how calculated regions of rock failure respond to variations in ambient stress and  
344 rock properties. Ambient tectonic stress has a strong effect on the occurrence and mode of  
345 predicted fractures. In the presence of gravity but the absence of horizontal tectonic compression,  
346 the topography alone is insufficient to fracture rock with modest  $C$  and  $q_u$  (Fig. 6b). If the  
347 horizontal tectonic stress becomes tensile, shear fractures are predicted at depth, and a shallow  
348 zone of steeply dipping opening-mode fractures is predicted beneath the valley (Fig. 6c). For the  
349 reference case with a compressive horizontal tectonic stress of -6 MPa, increasing  $C_{\min}$  and  $q_u$   
350 restricts the opening-mode fractures to a progressively smaller section of the valley floor and the  
351 shear fractures to a progressively narrower and shallower zone beneath the valley floor (Fig. 6d-  
352 f). These trends can be understood by examining the magnitudes of the surface-parallel principal

353 stress (Fig. 5d) and  $C_{\min}$  (Fig. 5e), respectively. Finally, increasing the internal friction angle  
354 restricts shear failure to a shallower and horizontally narrower zone beneath the valley (Fig. 6g-  
355 i). This trend occurs because a larger friction angle, which corresponds to a steeper failure  
356 envelope (Fig. 3), allows rock to withstand a larger differential stress without fracturing.

357 Figure 7 shows how the predicted region of rock failure depends on the relief, shape and  
358 asymmetry of the topographic profile. Higher relief (for a fixed wavelength) deepens the zone of  
359 shear fractures on either side of the valley axis (Fig. 7a-c). Sharp ridges (Fig. 7f) have little effect  
360 on the valley-centered fracture patterns in the reference case examined here, but a sharp valley  
361 (Fig. 7d) reduces the depth of the shear fracture zone to zero at the valley axis and deepens it  
362 elsewhere, creating two lobate zones on either side of the valley. Asymmetry of hillslope relief  
363 (Fig. 7g) or hillslope length (Fig. 7i) makes the predicted regions of rock failure asymmetric.

364 These calculations for synthetic topography illustrate three general trends. First, tectonic  
365 compression favors shear fracture in the subsurface based on laboratory fracture criteria.  
366 Although Miller and Dunne (1996) emphasized the potential for opening-mode fracture beneath  
367 ridges and valleys, this result was partly a consequence of the absence of a constant horizontal  
368 compression term in their ambient stress field ( $\sigma_{xx}^t = 0$  in Equation 1a). For topographic  
369 scenarios like those in Figures 5-7, we find that modest regional compression ( $\sigma_{xx}^t < 0$ ) inhibits  
370 the formation of tensile fractures in the subsurface. (This result is limited to “modest”  
371 compression because large surface-parallel compressive stresses can cause surface-parallel  
372 tensile fractures to form beneath ridges (Martel, 2006, 2011).) Second, in a compressive tectonic  
373 regime ( $\sigma_{xx}^t < 0$ ), the greatest susceptibility to shear fracture occurs in a zone beneath the valley  
374 floor, which extends deeper for larger tectonic compression, lower rock cohesion, or a smaller

375 rock friction angle (Fig. 6). Third, the shape of this zone of shear fracture is most sensitive to the  
376 sharpness of the “V” formed by the valley side slopes (Fig. 7d), and less sensitive to the ridgeline  
377 shape, valley relief, and valley asymmetry. In the next section, we use these observations to  
378 guide our analysis of a field site with a more irregular topographic profile.

379

### 380 **Application to the Shale Hills, Pennsylvania, USA**

381 The ability to calculate stresses beneath an irregular topographic profile permits direct  
382 comparisons of modeled stresses with observed rock damage at real field sites with irregular  
383 topography. An exhaustive comparison is beyond the scope of this study; instead, we offer a  
384 comparison that demonstrates the potential for future investigations. We sought a site with  
385 uniform rock type in a region where the tectonic stress field is reasonably well constrained; with  
386 a valley extending roughly perpendicular to the maximum horizontal stress, such that a plane  
387 strain approximation can reasonably be applied along a valley cross-section; where a high-  
388 resolution topographic survey has been performed, and where bedrock fractures have been  
389 imaged at depths comparable to the topographic relief. Few sites satisfy all these criteria and  
390 offer extensive opportunities to observe subsurface rock damage. However, an established study  
391 site in Pennsylvania, USA, has all of these characteristics and is suitable for a preliminary  
392 investigation.

#### 393 *Site description*

394 The Susquehanna Shale Hills Critical Zone Observatory (hereafter referred to as the Shale Hills  
395 or SSHO) is a 0.08 km<sup>2</sup> catchment located in the Shaver’s Creek drainage basin in the uplands of

396 the Valley and Ridge physiographic province of central Pennsylvania, USA (Figure 8). The  
397 forested, soil-mantled valley has an average local relief (valley floor to ridgetop) of 20 m, side  
398 slopes with gradients of 25-35%, and a stream with an average channel gradient of 4.5% that  
399 flows west-southwest in its headwaters and northwest near the basin outlet. The catchment is  
400 eroded into the Silurian Rose Hill Formation of the Clinton Group, which consists of shale with  
401 minor interbedded limestones of variable thickness (Jin et al., 2010). Although the area is  
402 currently tectonically inactive, the bedrock has experienced a long history of deformation  
403 associated with the ancient orogenic events that formed the Valley and Ridge. Outcrops of shale  
404 beds on the valley floor have an average strike of S54°W and an average dip of N76°W (Jin et al,  
405 2010), but much shallower bedding dips of approximately 30° are observed in borehole image  
406 logs (Kuntz et al., 2011).

407 The SSHO site meets several key criteria listed above. Aside from local structural  
408 differences, the shale bedrock is relatively uniform. Near the outlet of the valley, where the  
409 stream flows northwest, the valley axis is roughly perpendicular the maximum regional  
410 horizontal stress, as described below. The site's topography has been surveyed with high-  
411 resolution airborne laser altimetry through the State of Pennsylvania's PAMAP program and by  
412 the National Center for Airborne Laser Mapping (NCALM). Finally, optical image logs from a  
413 set of boreholes located at the downstream end of the valley permit detailed observations of  
414 fractures (Kuntz et al., 2011).

#### 415 *Regional tectonic stress*

416 We used measurements compiled in the World Stress Map database (Heidbach et al.,  
417 2008) to estimate the orientation and magnitude of the ambient horizontal tectonic stress,  $\sigma_{xx}^t$ . A

418 compilation of earthquake focal mechanisms, borehole breakouts, overcoring measurements,  
 419 hydrofracture events, and geologic features indicates that the horizontal stresses in the region are  
 420 compressive and that the most compressive horizontal stress is oriented roughly northeast-  
 421 southwest (Fig. 9). Most of the observations in the database only include a direction, but a small  
 422 number include a horizontal stress magnitude and the depth at which it was measured. These  
 423 observations are marked in Figure 9 and listed in Table 2.

424 To convert the horizontal stress estimates at depth,  $\sigma_h$ , to horizontal surface stress,  $\sigma_h^0$ ,  
 425 we assumed that the depth variation in stress magnitude is entirely due to overburden, such that

$$\sigma_h = \sigma_h^0 + \rho g d \frac{\nu}{1 - \nu}, \quad (10)$$

426 where  $d$  is depth beneath the surface and  $\nu$  is Poisson's ratio. We solved Equation 10 for  $\sigma_h^0$  and  
 427 used  $\rho = 2650 \text{ kg/m}^3$ . Table 2 lists the resulting estimates of maximum and minimum horizontal  
 428 stress for  $\nu = 1/4$  and  $\nu = 1/3$ . Maximum and minimum horizontal surface stresses are both  
 429 compressive for all sites in Table 2, with the maximum compressive stress typically about twice  
 430 the magnitude of the minimum stress. The means and standard deviations for  $\nu = 1/3$  are  $\sigma_{h,\max}^0$   
 431  $= -10.8 \pm 1.9 \text{ MPa}$  and  $\sigma_{h,0,\min} = -5.0 \pm 2.1 \text{ MPa}$ , and the estimates for  $\nu = 1/4$  are slightly larger.  
 432 In subsequent calculations for the SSHO, we use a conservative estimate of  $\sigma_{xx}^t = -10 \text{ MPa}$ , with  
 433 the  $x$  direction in the model approximately aligned with the direction of the most compressive  
 434 horizontal stress.

435 *Modeled stress and fracture patterns*

436 We used the PAMAP laser altimetry data to extract a topographic profile along a 1.5 km  
437 transect oriented  $45^\circ$  east of north, which is perpendicular to the valley axis at the location of the  
438 boreholes (Figure 8) and nearly parallel to the orientation of the most compressive regional  
439 horizontal stress (Table 2, Figure 9). The transect passes through one of the boreholes, CZMW 1  
440 (Kuntz et al., 2011), and within 10 meters of the other three boreholes. Because the boreholes are  
441 located near the basin outlet, the extracted profile has somewhat lower relief than the middle part  
442 of the valley. The three-dimensional shape of the valley (curved valley axis, longitudinal slope,  
443 downstream variations in relief) undoubtedly produces some three-dimensional stress variations,  
444 but given the elongated valley shape and relatively gentle longitudinal profile, we assume that  
445 the three-dimensional effects are small relative to the topographic stresses generated by the  
446 valley cross-sectional shape and that a two-dimensional treatment of the state of stress is a  
447 reasonable approximation. We subtracted the mean elevation from the profile and tapered the  
448 ends of the profile to an elevation of zero with a Tukey (tapered cosine) window. This tapering  
449 of the endpoints to a common elevation, which is performed to ensure a stress equilibrium in the  
450 model calculation, only affects the topography far from the Shale Hills catchment (Fig. 8).

451 The topographic profile was resampled by linear interpolation to a horizontal point  
452 spacing of  $\Delta x = 3.5$  m, and the resampled points were used as the endpoints of the boundary  
453 elements. We then defined a mesh of subsurface observation points with  $x$  locations centered  
454 beneath element midpoints and spanning 150 m on either side of the valley axis. A uniform  
455 number of  $y$  locations are chosen beneath each element midpoint, with the  $y$  locations below a  
456 particular element midpoint being spaced at equal intervals from one element length below the  
457 surface to a depth of 100 m below the ends of the tapered profile. The depth intervals of the  
458 observation points therefore vary along the profile, as shown schematically in Figure 1.

459 We used the boundary element model to calculate stresses beneath the SSHO topographic  
460 profile in the presence of a -10 MPa ambient horizontal tectonic stress, using typical parameters  
461 for shales in the region (Table 1; Goodman, 1989). Figure 10 shows horizontal and vertical  
462 normal stresses, vertical shear stresses, principal stress orientations and magnitudes, the shear  
463 fracture proxy  $C_{\min}$ , and predicted fracture modes and orientations. Horizontal normal stresses  
464 ( $\sigma_{xx}$ ) are compressive throughout the landscape, but are least compressive under the ridgetops  
465 and most compressive under the valley floor (Fig. 10a). Vertical normal stresses ( $\sigma_{yy}$ ) follow a  
466 similar pattern (Fig. 10b). The most compressive stress trajectories are horizontal or nearly so,  
467 forming a pattern that resembles a subdued version of the topographic surface (Fig. 10d). The  
468 topographic effect on the shear fracture proxy,  $C_{\min}$ , is pronounced (Fig. 10e). The location most  
469 susceptible to shear fracturing is a shallow zone extending approximately 10 m beneath the  
470 valley floor, where rocks with a cohesion less than about 5 MPa are expected to fail.  $C_{\min}$   
471 declines rapidly with depth beneath this shallow zone. Beneath the ridges,  $C_{\min}$  increases slightly  
472 with depth down to a depth of a few tens of meters, and then declines gradually at greater depths.  
473 Rock with cohesion less than about 2 MPa is expected to fracture in shear at all subsurface  
474 locations shown in the figure (Fig. 10f). The only location where opening mode fractures are  
475 predicted by the criteria of Miller and Dunne (1996) is at the land surface, where the rocks are  
476 unconfined and subject to surface-parallel compression (Fig. 10d,f).

477 We examined the sensitivity of these stress and predicted fracture patterns to the ambient  
478 tectonic stress and rock mechanical parameters by repeating the calculation in Figure 10 for  
479 different values of  $\sigma_{xx}^t$ ,  $k$ , and  $\phi$ . Figure 11 shows the effects on  $C_{\min}$  and fracture patterns when  
480 each of these parameters deviates from the value used in Figure 10. Variations in ambient

481 tectonic stress have a strong effect: halving  $\sigma_{xx}^t$  from -10 MPa (Figure 11c) to -5 MPa (Figure  
482 11b) reduces  $C_{\min}$  and causes predicted shear fractures to be confined to a zone that is shallower  
483 beneath the valley than beneath the ridges. In the absence of tectonic stress, the topography alone  
484 is not enough to cause shear fractures in two areas (white areas in Fig. 11a): (1) a zone that  
485 begins about 10 meters below the valley floor and broadens at greater depths, and (2) a zone  
486 beneath the higher ridge on the northeast side of the valley. Even outside these zones, rock  
487 cohesion exceeding about 1 MPa will prevent fractures from forming (Fig. 11a). Variations in  $k$   
488 have a weak effect, with smaller values generating a somewhat steeper decline in  $C_{\min}$  beneath  
489 the valley floor (Fig. 11d-f). As the internal friction angle increases, predicted shear fractures  
490 take on shallower dips and the magnitude of  $C_{\min}$  declines, indicating a lower potential for shear  
491 fracture, but the relative spatial pattern of  $C_{\min}$  is largely unchanged (Fig. 11g-i). These  
492 sensitivity tests show that the most robust features of the modeled stress and fracture patterns are  
493 the relative differences in the potential for shear fracturing (as expressed by  $C_{\min}$ ) beneath ridges  
494 and valleys and the trends in  $C_{\min}$  with increasing depth. We therefore focus our observational  
495 comparisons on these features.

#### 496 *Borehole fracture mapping*

497 To produce an observational dataset for comparison with the stress and fracture  
498 calculations, we mapped fractures in optical images of the four boreholes in the valley bottom.  
499 Image logs of the borehole walls, such as the example in Figure 12, were acquired with an  
500 Optical Borehole Imaging (OBI) televiewer manufactured by Mount Sopris Instruments. The  
501 OBI produces a vertically continuous, 360° image of the borehole wall using a charge-coupled  
502 device (CCD) camera and uses a 3-axis magnetometer and two accelerometers to measure the



503 compass orientation of the image and deviation of the borehole from vertical. Resolution of the  
504 images was approximately 0.5 mm vertically and 0.33 mm azimuthally, though image noise  
505 typically created a coarser effective resolution. The image logs were processed, oriented to  
506 magnetic north and analyzed with WellCAD, a PC-based software package. No deviation  
507 corrections were made, as the wells are vertical. The wells are cased with polyvinyl chloride  
508 (PVC) pipes from the ground surface to a depth of 3 meters, so fractures could not be mapped in  
509 this shallow zone. No casing exists below that depth, obviating the need for casing-effect  
510 corrections of the images.

511         The main structures visible in the borehole image logs are natural fractures and bedding  
512 planes. Neither drilling-induced fractures nor borehole breakouts are apparent in the images.  
513 Planar features that intersect a cylindrical borehole wall have sinusoidal traces on the flattened  
514 image logs (Fig. 12). The phase angle of a sinusoidal trace relative to a reference mark yields the  
515 strike of the plane, and the trace amplitude yields the dip (Serra, 1989; Luthi, 2001). In the  
516 SSHO boreholes, bedding planes have consistent attitudes at a given depth and parallel planar  
517 changes in rock color, whereas fractures commonly crosscut bedding planes (Fig. 12). We used  
518 rock color and cross-cutting relationships to classify each planar feature as a bedding plane or a  
519 fracture (Fig. 13). We generally could not identify offsets along fractures. Each feature was  
520 traced eight times (by mapping all features, recording the sinusoidal traces, deleting the traces,  
521 and starting again), and the strike and dip of a feature were recorded as the arithmetic averages of  
522 the eight strike directions and dip angles. Strike and dip orientations were corrected for magnetic  
523 declination to yield orientations relative to geographic north.

524 Stereonet plots of the poles to fracture and bedding planes (Fig. 14) show generally  
525 consistent orientations in all four boreholes, as expected for boreholes within about 12 meters of  
526 one another. Fractures generally dip steeply to the NNW or SSE, except for a few fractures that  
527 dip steeply E or W (Fig. 14a), whereas bedding is either approximately horizontal or dips gently  
528 to the NNW. The gentle bedding dips contrast with the report of Jin et al. (2010) but agree with  
529 our observations of outcrops as well as those of Kuntz et al. (2011).

### 530 *Comparison of fractures with modeled topographic stresses*

#### 531 Fracture orientation

532 The observation that most fractures in the boreholes dip steeply to the NNW or SSE (Fig.  
533 14a) suggests that they may be conjugate sets of shear fractures. However, these fractures are  
534 approximately orthogonal to the expected NE or SW dip directions of shear fractures triggered  
535 by stresses along the cross-valley SSHO transect (Fig. 8). One possible explanation for this  
536 difference in orientation is that the observed fractures were caused by topographic stresses, but  
537 the three-dimensional effects of the topography near the outlet of the SSHO valley caused the  
538 stress field to deviate from the plane strain conditions assumed in our model calculation. Another  
539 possible explanation is that the fractures are older features that may (or may not) have  
540 experienced renewed sliding under the influence of recent topographic and tectonic stress. The  
541 fact that both fractures (Fig. 14a) and dipping bedding planes (Fig. 14b) strike in approximately  
542 the same direction as regional Valley and Ridge structural features (Fig. 8) supports the latter  
543 explanation.

544 Either scenario makes it difficult to test the topographic fracture hypothesis by comparing  
545 observed fracture orientations with a two-dimensional stress model. But in both scenarios, the

546 presence of the SSHO valley should still perturb the stress field and alter the potential for rock  
547 damage beneath ridges and valleys. We therefore turn to a more general measure of spatial  
548 variations in rock damage that may have been influenced by topography: trends in fracture  
549 abundance with depth beneath the surface.

#### 550 Fracture abundance

551 We seek a measure of the intensity of fracturing that is also relevant to near-surface  
552 hydrologic and geomorphic processes. One simple proxy is the number of fractures per unit  
553 distance along a linear path through the rock, which we refer to as the linear fracture abundance.  
554 Fractures that intersect a linear path and are highly inclined with respect to that path appear to be  
555 spaced farther apart than fractures that are more nearly perpendicular to the path (Terzaghi,  
556 1965; Martel, 1999). Steeply dipping fractures are therefore likely to be under-represented in our  
557 vertical boreholes. We accounted for this bias by calculating a weighted linear fracture  
558 abundance. We divided each borehole into five equal depth intervals and binned the fractures  
559 according to the depth at which the fracture plane intersects the borehole axis. Each fracture was  
560 assigned a weight equal to  $1/\cos \psi$ , where  $\psi$  is the fracture's dip angle (Terzaghi, 1965; Martel,  
561 1999). The weighted linear fracture abundance is the sum of the weights in each bin divided by  
562 the vertical length of the bin. Linear fracture abundance declines steeply with depth in all four  
563 SSHO boreholes, ranging from approximately 30 fractures per meter at a depth of 3 meters (the  
564 bottom of the borehole casing) to fewer than 2 fractures per meter at a depth of 15 meters (Figure  
565 15c).

566 These observed trends in fracture abundance can be compared with  $C_{\min}$ , the shear  
567 fracture proxy (Fig. 10e). The specific quantity to be compared with fracture abundance depends

568 on how fractures accumulate as rock is exhumed toward the land surface. If fractures only form  
 569 close to the surface, or if fractures heal rapidly (through recrystallization or secondary mineral  
 570 deposition, for example) relative to the rate of exhumation, the abundance of active fractures at a  
 571 given location in the subsurface should be most influenced by the present-day stress state, and  
 572 the most appropriate quantity to compare would be the local value of  $C_{\min}$ . If fractures form over  
 573 a range of depths and times and never heal, then the abundance of active fractures at a given  
 574 location in the subsurface will reflect the cumulative effects of the different stress states the rock  
 575 experienced on its way to its present location. In this case, a more appropriate proxy to compare  
 576 would be the integrated  $C_{\min}$  between the depth at which the rock began to accumulate fractures  
 577 and its present depth. (This assumes that the topography and tectonic stress have not changed as  
 578 the rock was exhumed, an idea we return to in the Discussion.) We calculate the vertically  
 579 integrated value of  $C_{\min}$  at a depth  $z$  as

$$\int_{z_0}^z C_{\min}(z') dz' \quad (11)$$

580 where  $z_0$  is the depth at which the rock begins to accumulate fractures, here taken to be the depth  
 581 at which  $C_{\min} = 1$  MPa (approximately 200 m for the scenario in Fig. 10e). The actual history of  
 582 fracture accumulation may lie between these two end-member cases.

583 Figure 15 compares the observed depth profiles of fracture abundance beneath the valley  
 584 bottom (Figure 15c) with the present-day depth profile of  $C_{\min}$  (Figure 15a) and the vertically  
 585 integrated  $C_{\min}$  (Figure 15b). For comparison, we also plot in Figures 15a and 15b the predicted  
 586 trends beneath the highest ridgetop, where the vertical trends in  $C_{\min}$  differ most from those  
 587 beneath the valley floor (Figure 11e).  $C_{\min}$  declines rapidly with depth in the uppermost 15 m

588 beneath the valley floor, then declines more gradually at greater depths (Figure 15a). This differs  
589 from the vertically integrated  $C_{\min}$  values, which decay more gradually relative to their surface  
590 values over the entire depth range shown in Figure 15b. Under the ridgetop,  $C_{\min}$  increases with  
591 depth for the shallow depths shown in Figure 15a (the opposite of the modeled trend beneath the  
592 valley), whereas the vertically integrated  $C_{\min}$  declines gradually with depth, similar to the trend  
593 under the valley floor but with a larger magnitude (Figure 15b). The parameters  $\phi$ ,  $k$  and  $\sigma'_{xx}$   
594 scale the magnitude of  $C_{\min}$ , but do not have a strong effect on the shapes of these trends (Figure  
595 11). The observed decline in fracture abundance with depth in Figure 15c resembles the rapid  
596 decline in  $C_{\min}$  more than the gradual decline in vertically integrated  $C_{\min}$ . This suggests that, if  
597 the fractures were indeed influenced by topography, the present-day stress field may exert a  
598 stronger control on fracture patterns than the stresses experienced by the rocks earlier in their  
599 exhumation history. More generally, the similar trends in fracture abundance and  $C_{\min}$  are  
600 consistent with the hypothesis that topographic stresses have influenced bedrock fracture patterns  
601 at the SSHO site.

602         Given the distinct trends in  $C_{\min}$  beneath ridges and valleys (Fig. 15a), a comparison of  
603 fracture abundance in boreholes located on ridgetops with those in the valley wells would  
604 provide an additional test of the topographic fracture hypothesis. A few ridgetop boreholes exist  
605 at the SSHO, but small borehole diameters and fully cased holes prevented us from collecting  
606 image logs.

## 607 **Discussion**

608 *Testing the topographic fracture hypothesis*

609           The decline in fracture abundance beneath the valley floor at the SSHO site is consistent  
610 with the hypothesis that topographic stresses have shaped the distribution of bedrock fractures.  
611 However, this trend could have a different origin. With our current dataset, for example, we  
612 cannot rule out the possibility that pre-existing fractures are reactivated more frequently near the  
613 surface due to the reduced overburden, and that the similar depth trends of fracture abundance  
614 and  $C_{\min}$  are only coincidental.

615           Additional observations would provide a more definitive test of the topographic fracture  
616 hypothesis. At the SSHO site, fracture mapping in boreholes located on ridgetops (or other  
617 locations along the valley cross-section) would reveal whether the trend of fracture abundance  
618 with depth differs beneath ridges and valleys, as predicted by the topographic stress model, or  
619 whether the trend is similar throughout the landscape. More broadly, comparisons of modeled  
620 stresses and observed fractures at sites with different characteristics than SSHO could provide a  
621 test under simpler conditions. Sites with crystalline bedrock containing fewer pre-existing  
622 fractures should make topographically influenced fractures stand out more prominently. Higher  
623 topographic relief would strengthen the topographic perturbation to the stress field, which should  
624 also make any topographic effects on fracture patterns more apparent. Sites with similar  
625 lithologic and topographic characteristics but different ambient tectonic stress fields should have  
626 different topographically driven fracture patterns, especially if the horizontal tectonic stresses  
627 have opposite signs.

#### 628 *Rock damage and exhumation*

629           If the decline in fracture abundance with depth is a consequence of topographic effects,  
630 then its apparent association with the present-day stress field (Figure 15a) rather than integrated

631 effects of stresses over depth (Figure 15b) merits some discussion. This observation would seem  
632 to suggest that fracture patterns beneath the SSHO valley are surprisingly insensitive to the stress  
633 history experienced by a parcel of rock during its exhumation. We see at least two possible  
634 explanations. First, rock cohesion may be sufficiently large that the rock only becomes  
635 susceptible to topographically influenced fracture when it is very near the surface, within the  
636 shallow zone where  $C_{\min}$  is largest (Figure 10e). In this scenario, the decline in fracture  
637 abundance with depth beneath the valley floor would be a signature of the transition from a  
638 shallow zone in which fractures have formed or reactivated, to a deeper zone in which they have  
639 not. The second possibility is that fractures formed at depth heal as the rock is exhumed, such  
640 that the distribution of active fractures near the surface mainly reflects the local stress field. At  
641 sites with relatively slow erosion, such as the Appalachians, fractures might have had enough  
642 time to heal as they neared the surface. At typical regional erosion rates of a few tens of meters  
643 per million years (Portenga and Bierman, 2011), for example, exhumation through a depth  
644 comparable to the SSHO topographic relief would take nearly one million years.

645 *Stress modeling framework: advantages, limitations, and potential improvements*

646 The analysis presented here demonstrates the potential for comparing stress models with  
647 mapped fractures to test for topographic effects on rock damage. The boundary element model  
648 calculations of stresses beneath synthetic and natural topography illustrate the advantages of a  
649 flexible numerical approach, especially the ability to model the effects of irregular topography  
650 and multiple adjacent landforms, and the ability to calculate subsurface stresses only in the area  
651 of interest. In addition to modeling the effects of gravity and topography on stresses, the

652 boundary element method can account for the effects of slip on faults (Gomberg and Ellis, 1994;  
653 Muller and Martel, 2000; Martel and Langley, 2006; Mitchell, 2000).

654         Applying the boundary element model to a field site also reveals some limitations that  
655 could be removed through improvements to the modeling approach. A two-dimensional stress  
656 model may provide a good approximation of the stress field in certain parts of some landscapes,  
657 but a three-dimensional model would offer a more flexible and accurate tool for calculating  
658 stresses beneath arbitrary topography. All landscapes are really three-dimensional, and even  
659 landforms that are nearly two-dimensional may be oriented oblique to regional tectonic stress  
660 directions. The displacement discontinuity method of Crouch and Starfield (1983) has been  
661 extended to three dimensions (Thomas, 1993; Gomberg and Ellis, 1994), and could be used to  
662 calculate topographic stresses with an approach similar to the one presented here. The model  
663 employed here considers only elastic stresses. This is reasonable for landforms with relatively  
664 low relief, but in other scenarios, such as mountain-scale topography, the effects of plastic  
665 deformation on stress relaxation in rock may also be important (Leith et al., 2013a,b).

666 *Topographic stress and landscape evolution*

667         Fractures are well known to enhance rock erodibility (Molnar et al., 2007), and studies  
668 have documented the influence of fractures on the effectiveness of specific erosional processes.  
669 Whipple et al. (2000) show with a series of field examples that fractures accelerate bedrock river  
670 incision by promoting plucking of large blocks of material, whereas relatively unfractured rock  
671 appears to erode more gradually through abrasion. Dühnforth et al. (2010) showed that glaciers  
672 in the Sierra Nevada eroded through more granite during the most recent glacial advance at  
673 locations with highly fractured rock than at locations with less fractured rock. Moore et al.



674 (2009) found that the retreat rate of bedrock walls above talus slopes increases exponentially as a  
675 composite measure of rock mass strength declines, with joint orientation relative to the rock face  
676 exerting the strongest control on retreat rate. These are only a few examples among many. The  
677 influence of fractures on rock erodibility prompted Miller and Dunne (1996) and Molnar (2004)  
678 to propose hypothetical feedbacks between bedrock fracturing and topography, in which  
679 topographic stresses influence the development of bedrock fractures, which in turn alter the  
680 evolution of topography by creating spatially variable erodibility. Noting the tendency of valleys  
681 to concentrate stresses, they emphasized the possibility of a positive feedback between  
682 topographic stresses and valley incision. Leith et al. (2013a) present evidence of such a feedback  
683 in the form of a deep alpine valley that appears to have developed a pronounced inner gorge as a  
684 result of glacial erosion accelerated by topographic stresses. The correlation of topographic  
685 stresses and bedrock fracture patterns at the SSHO supports the idea that such feedbacks could  
686 occur even in landscapes with relatively low relief.

687         The stress modeling approach presented here could also provide a flexible framework for  
688 modeling co-evolution of topography and stresses. The boundary element model could be  
689 combined with an erosional model of landscape evolution to iteratively describe how time-  
690 varying stresses and erodibility alter the trajectory of topographic change. A challenging aspect  
691 of this problem is that if topography or tectonic stresses change as rock is exhumed, it is  
692 necessary to account for the changing stress field by tracking the position and state of damage of  
693 a parcel of rock through time as it is advected toward the surface. If fractures heal slowly relative  
694 to the rate of exhumation, the population of fractures that reaches the surface will partly reflect  
695 the stress effects of prior topographic surfaces. In this scenario, topography, stresses, and fracture  
696 patterns could still co-evolve, but the feedbacks could be somewhat damped.

697           Evaluating possible feedbacks between topographic stresses and landscape evolution will  
698 require solutions to other fundamental problems related to bedrock erosion. One challenge is  
699 quantifying how rock damage affects erodibility. As rock damage increases, the rock probably  
700 erodes more easily (Molnar et al., 2007), but the functional relationship between fracture  
701 characteristics and the rate of bedrock erosion is not as clear. A few studies have begun to  
702 measure these functional relationships for specific erosion processes in the field (Moore et al.,  
703 2009), and valuable clues come from engineering applications such as drilling and dredging  
704 (Molnar et al., 2007, and references therein). Molnar et al. (2007) also note a distinction between  
705 fractures reducing rock strength and fractures transforming rocks into discrete particles small  
706 enough to be transported away by, both of which influence erodibility.

707           A second challenge is understanding how the rock beneath an eroding landscape  
708 accumulates damage, and whether that damage can heal. We have focused on macroscopic brittle  
709 fracture, which is undoubtedly an important type of damage, but others could be significant as  
710 well. Citing laboratory experiments that document static fatigue accumulation – time-dependent  
711 crack growth in rock samples subject to differential stresses below the macroscopic fracture  
712 threshold – Molnar (2004) suggests that modulation of static fatigue by topographic stresses  
713 could dominate bedrock damage in eroding landscapes. The lack of a straightforward measure of  
714 rock damage or a rate law for damage accumulation (and possibly annealing) underscores the  
715 need for a better understanding of the mechanisms that generate damage as well as field  
716 measurements that characterize the spatial distribution of damage.

717           The effects of topographic stress and rock fractures on landscape evolution may extend  
718 beyond bedrock erodibility. Spatial trends in fracture abundance and orientation may create

719 characteristic patterns of permeability, and therefore infiltration and shallow groundwater flow  
720 may vary systematically across drainage basins. Reduction of rock strength and enhancement of  
721 rock surface area and groundwater flow by fractures should also influence rates of chemical  
722 weathering and soil production. As more observations of the subsurface become available,  
723 comparisons with modeled stresses will reveal the extent to which topographic stresses shape the  
724 deep critical zone.

725

## 726 **Summary and Conclusions**

727 We used a two-dimensional boundary element method to calculate elastic stresses  
728 beneath an arbitrary topographic profile due to the combined effects of gravity and tectonics.  
729 Calculated stresses and macroscopic fracture patterns for a range of hypothetical profiles across  
730 ridges and valleys reveal how the modes and spatial extents of predicted fractures depend on  
731 both the ambient tectonic stress and the shape of the topography. In the presence of large  
732 regional horizontal compression, the expected fracture mode in the subsurface is typically shear  
733 based on a Mohr-Coulomb criterion, with the greatest potential for shear fracture in a shallow  
734 zone beneath the valley floor and adjacent slopes. The minimum cohesion needed to prevent  
735 shear failure,  $C_{\min}$ , serves as a proxy for the susceptibility of the rock to the formation or  
736 reactivation of shear fractures. We used the boundary element method and estimates of regional  
737 tectonic stresses to calculate stresses beneath a topographic cross section through the  
738 Susquehanna Shale Hills Critical Zone Observatory, an experimental watershed in Pennsylvania,  
739 USA. The model predicts a steep decline in  $C_{\min}$  with increasing depth beneath the valley floor,  
740 which compares well with a measured decline in the abundance of fractures mapped from optical

741 image logs of four boreholes in the valley. The similarity of these trends is consistent with the  
742 hypothesis that topographic stresses influence the formation or reactivation of fractures, and it  
743 suggests that feedbacks between topographic stress, rock fracture, and landscape evolution may  
744 occur. Future observations of fractures in different topographic settings or in sites with different  
745 topographic, lithologic or tectonic characteristics would provide a more complete test of the  
746 topographic fracture hypothesis, and could rule out or support alternative explanations for the  
747 measured trend in fracture abundance. In sites where topographic stresses influence rock  
748 fracture, the model presented here provides a framework for studying the effects of topography  
749 on subsurface hydrology and rock weathering, as well as possible feedbacks between rock  
750 fracture and landform evolution.

751

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767

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Zoback ML, Zoback MD. 1989. Tectonic stress fields of the continental United States. In Geophysical framework of the continental United States, Pakiser LC, Mooney W D (eds). Geological Society of America Memoir 172: Boulder, Colorado.

994 Table 1. Parameters used in model calculations\*.  
995

Figure	$k$	$E$ (GPa)	$\sigma'_{xx}$ (MPa)	$C$ (MPa)	$q_u$ (MPa)	$\phi$	$\Delta x^\dagger$ (m)
2‡	0.5	1	- $b$ to 0				0.05
4‡	1.5	1	0	0	0	30°	0.05
5	0.5	50	-6	1	3	30°	5
6	0.5	50	-6 to +2	0 to 2	0 to 6	15° to 45°	10
7	0.5	50	-6	1	3	30°	10
10	0.5	30	-10	1	3	20°	3.5/1.15
11	0.25 to 0.67	30	-10 to 0	1	3	10° to 30°	3.5
15	0.5	30	-10	1	3	20°	1.75

996 \*Dimensionless calculations (Figures 2 and 4) used  $\rho g = 1$ . Dimensional calculations (Figures 5 through 15) used  $\rho$   
997  $= 2650 \text{ kg/m}^3$ ,  $g = 9.81 \text{ m/s}^2$ . All calculations used  $\sigma'_{yy} = \sigma'_{xy} = 0$ .

998 †Average horizontal spacing of element endpoints, which is shorter than but approximately equal to the average  
999 element length.

1000 ‡Units do not apply to dimensionless calculations in Figures 2 and 4.

1001

1002

1003 Table 2. Regional stress estimates. All stresses in MPa.  
1004

WSM ISO*	Lat, Lon	Azimuth	Depth (m)	$\sigma_{h,max}$	$\sigma_{h,min}$	$\nu = 1/4$		$\nu = 1/3$	
						$\sigma_{h,max}^0$	$\sigma_{h,min}^0$	$\sigma_{h,max}^0$	$\sigma_{h,min}^0$
CAN336 <sup>oc,1</sup>	43.10°, -79.20°	66°	18	-11.6	-8.3	-11.4	-8.1	-11.4	-8.0
CAN337 <sup>oc,2</sup>	43.90°, -78.80°	68°	21	-11.3	-6.6	-11.1	-6.4	-11.0	-6.4
USA926 <sup>hf,3,4</sup>	42.08°, -78.00°	77°	510	-16	-10.1	-11.6	-5.7	-9.4	-3.5
USA58 <sup>hf,3,4,5</sup>	39.50°, -82.50°	64°	808	-24	-14	-17.0	-7.0	-13.5	-3.5
USA939 <sup>hf,4,6</sup>	40.64°, -83.92°	70°	110	-10.1	-5.1	-9.1	-4.1	-8.7	-3.7
Mean						-12.1	-6.3	-10.8	-5.0
s.d.						2.9	1.5	1.9	2.1

1005 \*World Stress Map identification code (Heidback et al., 2008). Techniques: <sup>oc</sup> = overcoring, <sup>hf</sup> = hydrofracture.  
1006 References: <sup>1</sup>Palmer and Lo (1976), <sup>2</sup>Lo (1981a,b), <sup>3</sup>Haimson (1974), <sup>4</sup>Haimson and Doe (1983), <sup>5</sup>Overbey and  
1007 Rough (1968), <sup>6</sup>Haimson (1983).  
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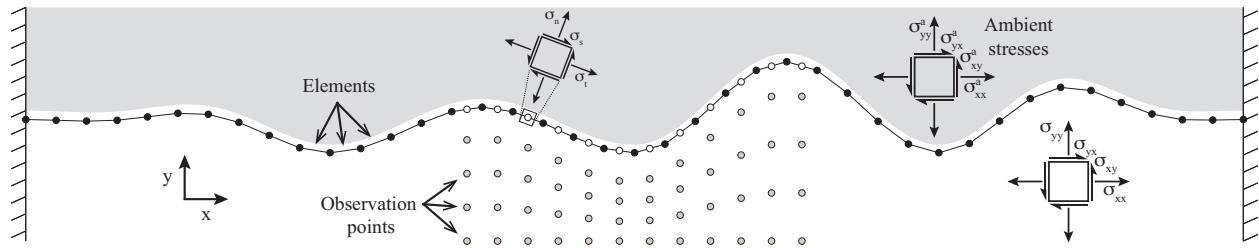
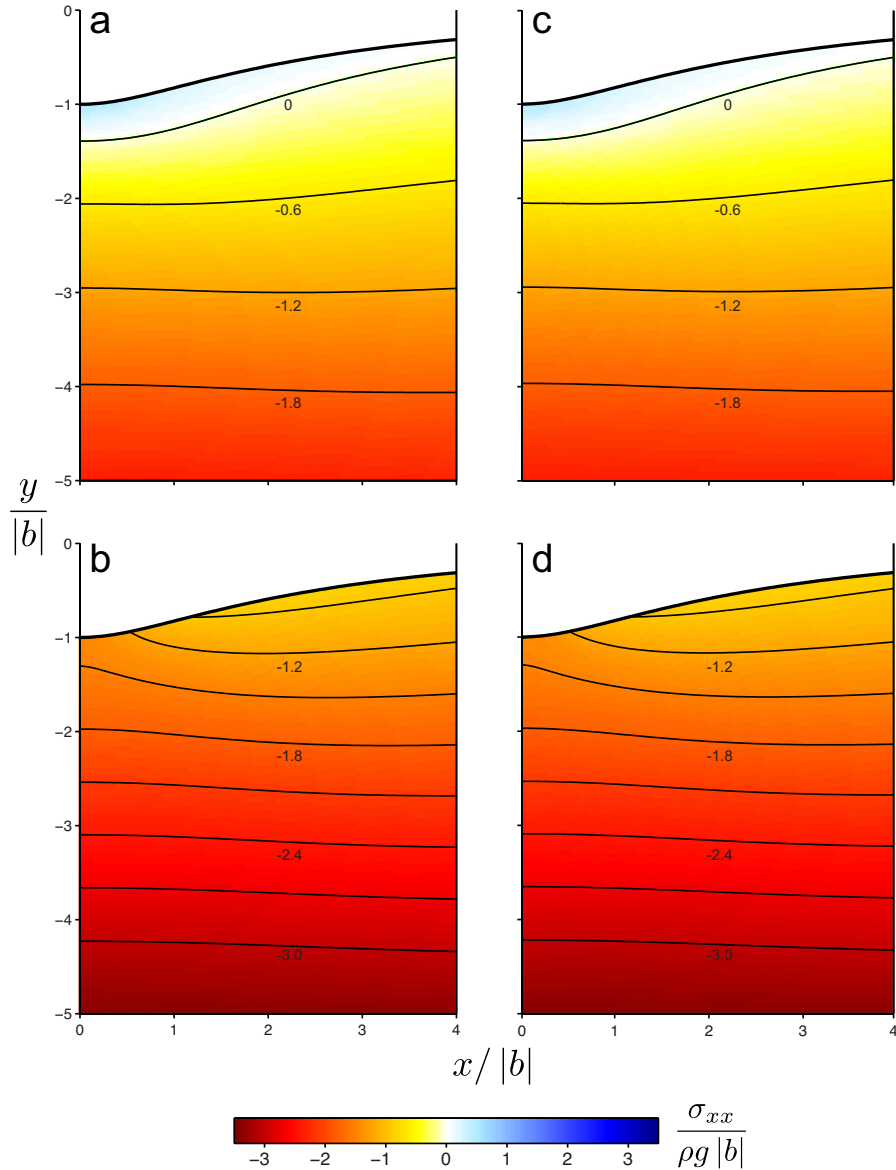


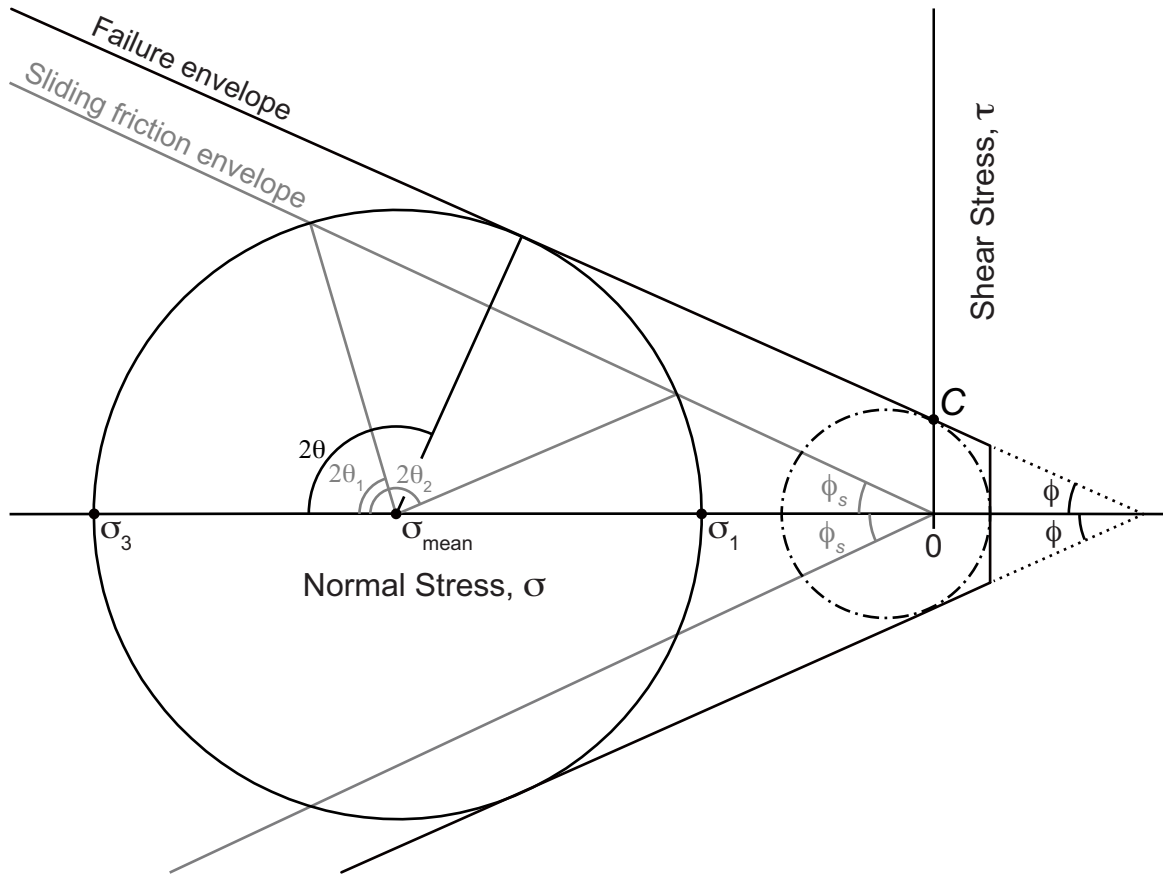
Fig. 1. Schematic diagram illustrating the coordinate system and setup of the boundary element model. The gray region above the land surface represents the overburden that is mathematically removed from the underlying rock to generate the topographic surface. The small gap between the gray region and the land surface represents the traction-free “crack” described by the displacement discontinuity method. Black circles represent element endpoints, white circles represent element midpoints, and gray circles represent observation points where stresses are calculated at depth. Squares with arrows show the orientations of stresses defined in the text. Hatched left and right boundaries represent lateral confinement.



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1021

1022 Fig. 2. Comparison of horizontal stress in boundary element model solutions (c,d) with analytical  
 1023 solutions of Savage and Swolfs (1986) (a,b) for the scenarios shown in their Fig. 9. The  
 1024 symmetric valley topography (only half of which is shown) is defined by the conformal  
 1025 coordinate mapping of Savage et al. (1985) with  $a = 3$  and  $b = -1$ . The scenario in (a) and (c) has  
 1026 an ambient horizontal surface stress of  $\sigma_{xx}^{a,0} = 0$ , and the scenario in (b) and (d) has a  
 1027 compressive ambient horizontal surface stress of  $\sigma_{xx}^{a,0} = -\rho g b$ . Coordinates and stresses are  
 1028 normalized as indicated. Contours are stresses in the same units as the color scale. Other  
 1029 parameter values are listed in Table 1.

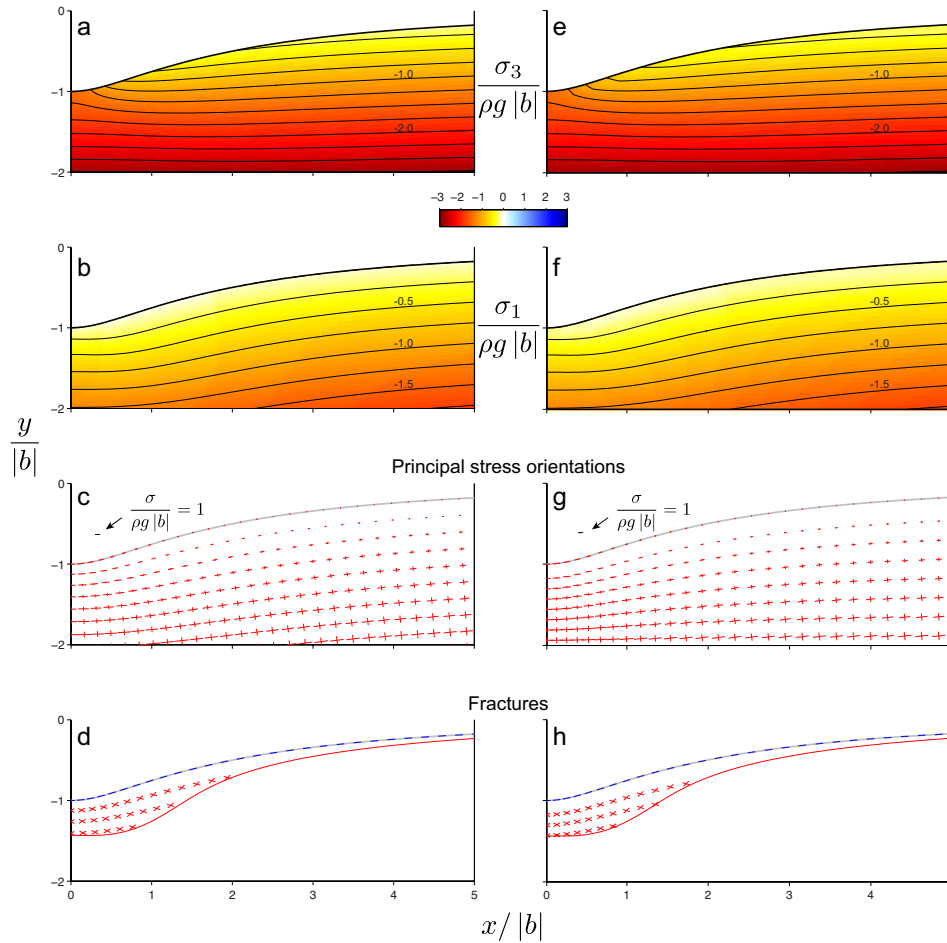


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1032 Fig. 3. Mohr diagram for a specific stress state defined by the most compressive ( $\sigma_3$ ) and least  
 1033 compressive ( $\sigma_1$ ) principal stresses. Compression is negative. The value of the mean stress,  
 1034  $\sigma_{\text{mean}}$ , is indicated. Quantities related to the formation of new fractures are black, and quantities  
 1035 related to sliding on existing fractures are gray. Dotted lines represent the extension of the shear  
 1036 failure envelope to the  $\sigma$  axis. Dash-dot Mohr circle represents the state of stress at which the  
 1037 failure criterion prescribes a transition from shear fracture to tensile fracture.

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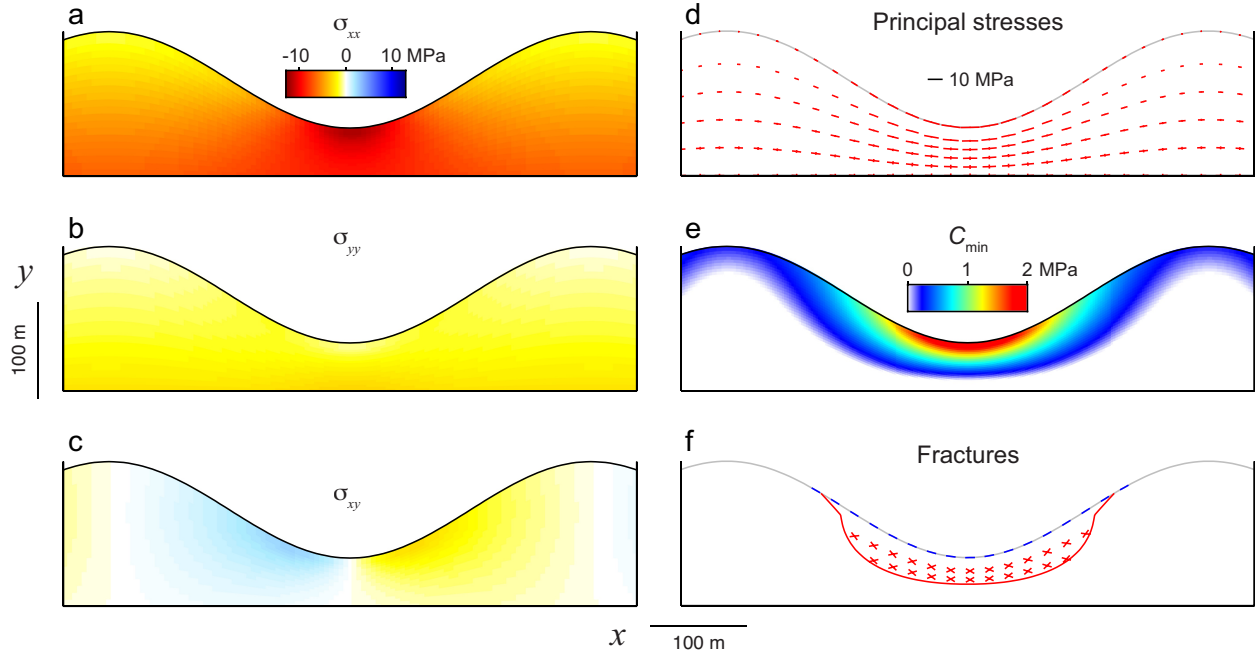


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1041 Fig. 4. Comparison of principal stresses and fracture patterns in boundary element model  
 1042 solutions (e-h) with analytical solutions of Miller and Dunne (1996) (a-d) for the “high regional  
 1043 compression” scenarios shown in their Figs. 4 and 7. Despite the description, this scenario has an  
 1044 ambient horizontal surface stress of  $\sigma_{xx}^{a,0} = 0$ . The symmetric valley topography (only half of  
 1045 which is shown) is defined by the conformal coordinate mapping of Savage et al. (1985) with  $a =$   
 1046  $2.5$  and  $b = -1$ . Other parameter values are listed in Table 1. Coordinates and stresses are  
 1047 normalized as indicated. Contours in (a), (b), (e) and (f) are stresses in the same units as the color  
 1048 scale. Orientations and lengths of line segments in (c) and (g) show orientations and magnitudes  
 1049 of principal stresses, with red segments indicating compression. In (d) and (h), blue line  
 1050 segments indicate potential opening-mode fractures and red line segments indicate potential  
 1051 shear fractures. Locations of stress and fracture symbols differ between the analytical and  
 1052 numerical solutions because symbols in (c,d) follow the conformal coordinate mapping whereas  
 1053 symbols in (g,h) are located at gridded BEM observation points.



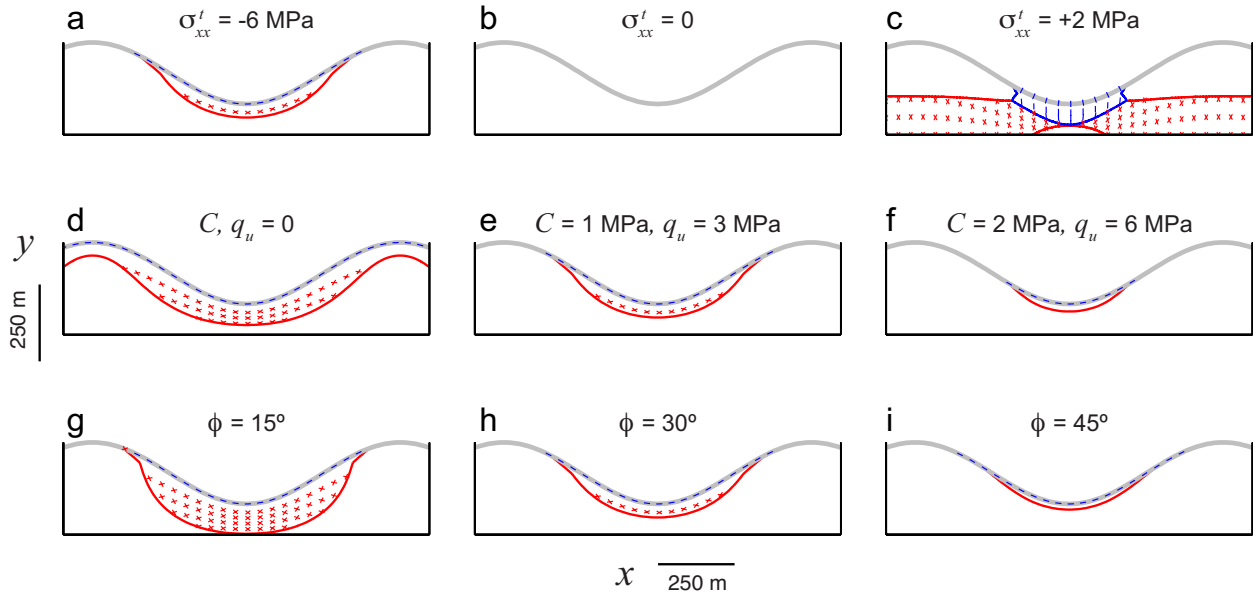


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1056 Fig. 5. Boundary element model solution for stresses (a-d), the minimum cohesion to prevent  
 1057 shear fracture,  $C_{\min}$  (e), and fracture modes and orientations (f) beneath a sinusoidal topographic  
 1058 profile subjected to a compressive ambient horizontal stress of -6 MPa. See Table 1 for other  
 1059 parameters. The entire topographic profile used in the calculation extends several wavelengths  
 1060 beyond the portion shown, and tapers to a level surface at the ends. Color scale in (a) applies to  
 1061 (a), (b), and (c). Orientations and lengths of line segments in (d) show orientations and  
 1062 magnitudes of principal stresses, with red segments indicating compression. In (f), blue line  
 1063 segments indicate potential opening-mode fractures and red line segments indicate potential  
 1064 shear fractures.

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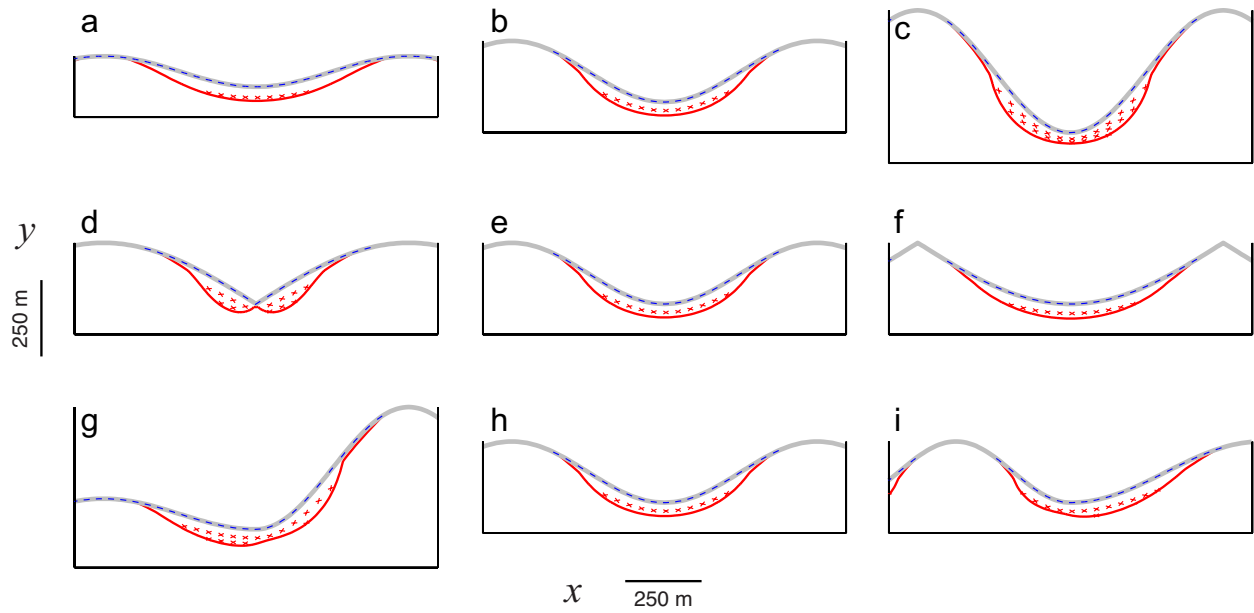


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1068 Fig. 6. Boundary element model solutions showing the sensitivity of the predicted rock fracture  
 1069 regions to variations in ambient horizontal tectonic stress (a-c), rock strength (d-f), and rock  
 1070 friction angle (g-i). Panels (a), (e), and (h) use the same parameters as Fig. 5. Parameters for  
 1071 other panels are the same as in Fig. 5 except where indicated, and except  $\Delta x = 10$  m (see Table  
 1072 1). Gray line is the land surface. Blue line segments indicate opening-mode fractures and red line  
 1073 segments indicate shear fractures. Red lines mark the boundaries of the zones where shear  
 1074 fractures are predicted; where no upper boundary is indicated, shear fractures are predicted up to,  
 1075 but not including, the land surface. Blue line in (c) marks the boundary of the zone where  
 1076 opening mode fractures in tension are predicted; in panels (a) and (d-i), opening mode fractures  
 1077 are predicted only at the land surface, where the rock is in unconfined compression.

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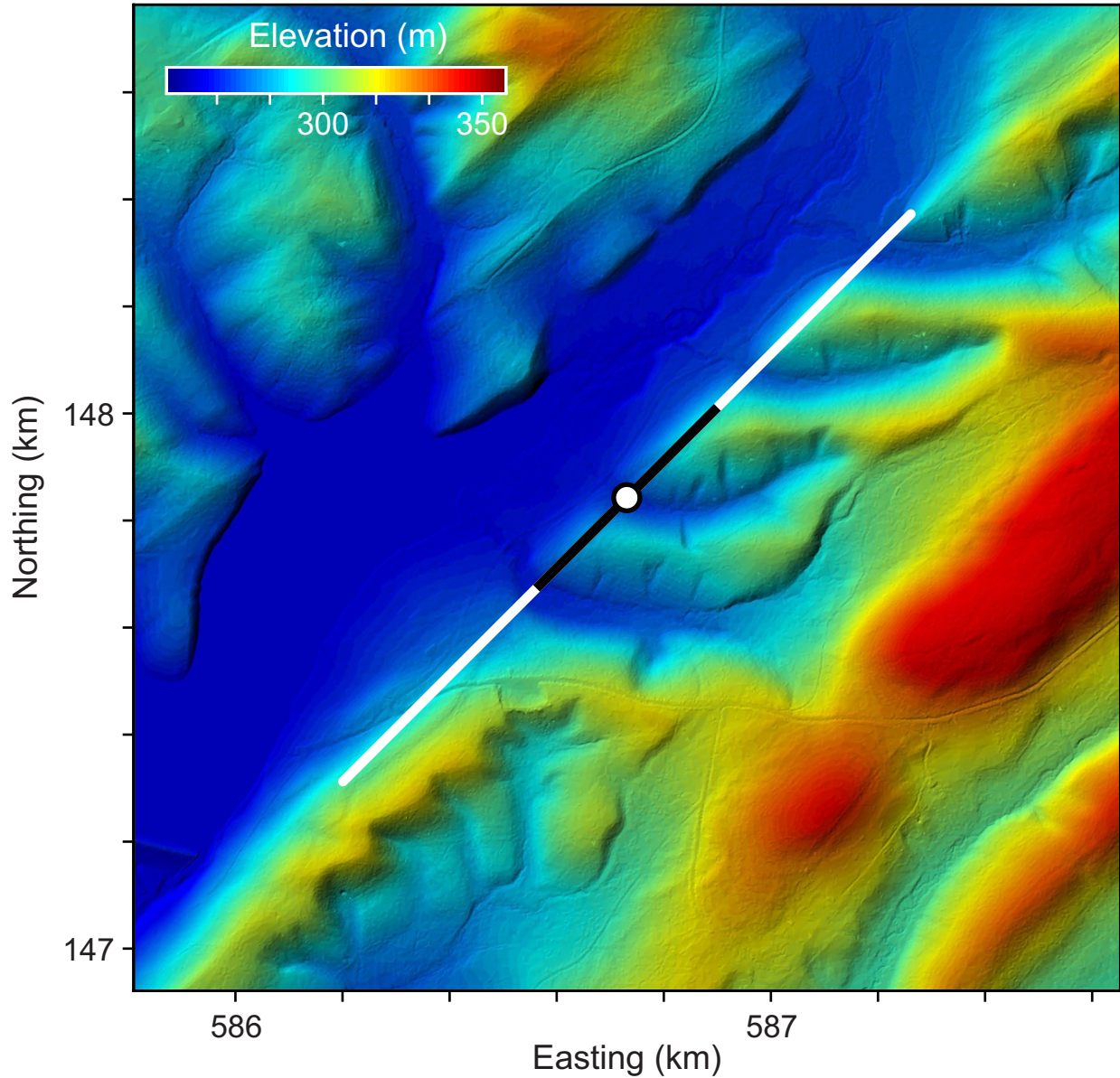


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1080

1081 Fig. 7. Boundary element model solutions showing the sensitivity of predicted fracture patterns  
1082 to variations in relief (a-c), valley and ridge shape (d-f), and asymmetry in valley relief (g) and  
1083 width (i). Model parameters are the same as in Fig. 5, except  $\Delta x = 10$  m (see Table 1). Panels (b),  
1084 (e), and (h) have the same profile shape as Fig. 5. Far-field topography (not shown) and symbols  
1085 are the same as in Fig. 6.

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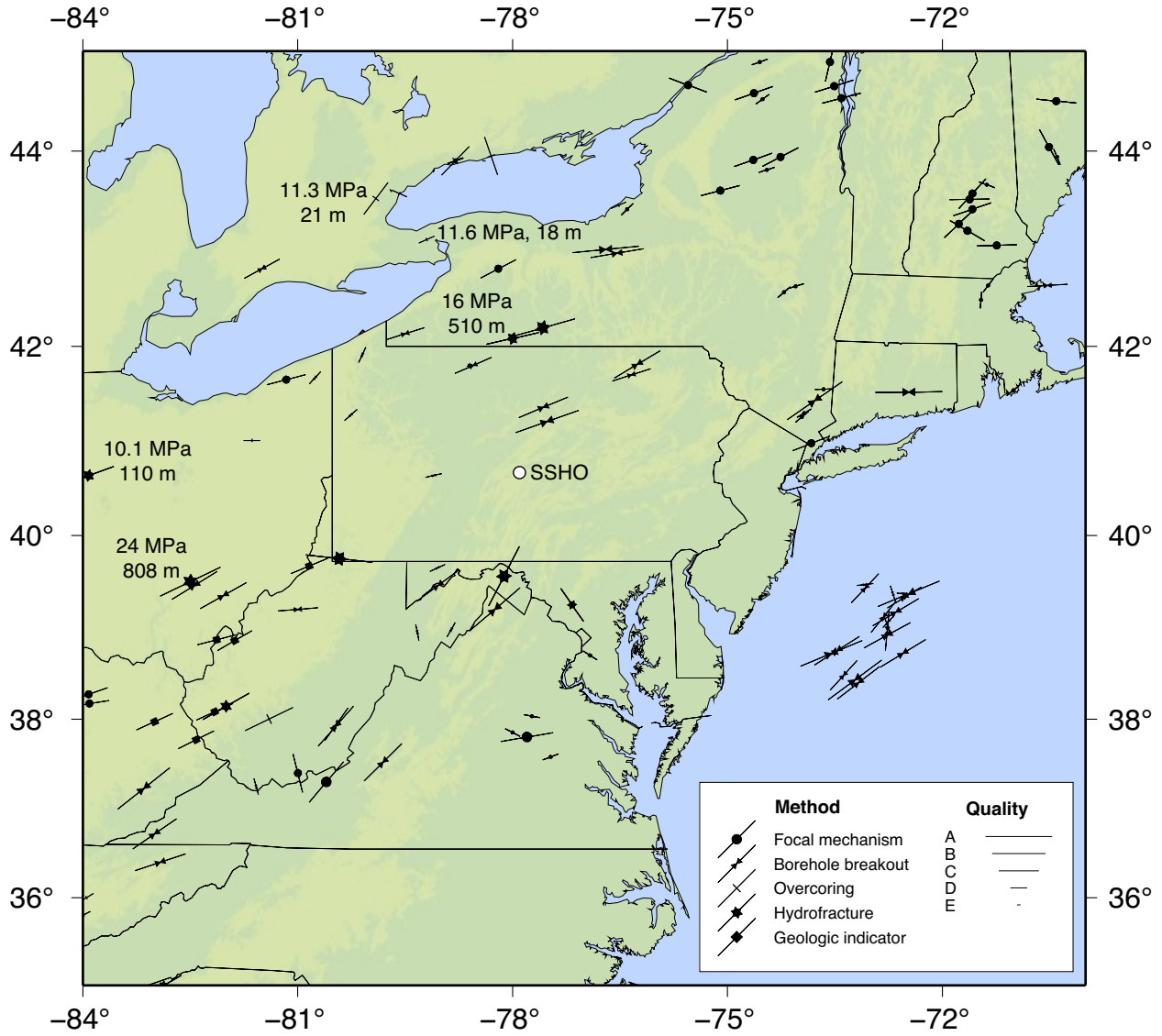


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1089 Fig. 8. Shaded relief map of the Shale Hills study site and surrounding area showing location of  
1090 boreholes (white circle), the transect used in model calculations (white line), and the portion of  
1091 the transect shown in Figures 10 and 11. Pennsylvania South State Plane projection (zone 3702),  
1092 NAD83 datum. Topographic data from the 1/9 arcsecond US National Elevation Dataset. See  
1093 Fig. 9 for the location of the site within the state of Pennsylvania.

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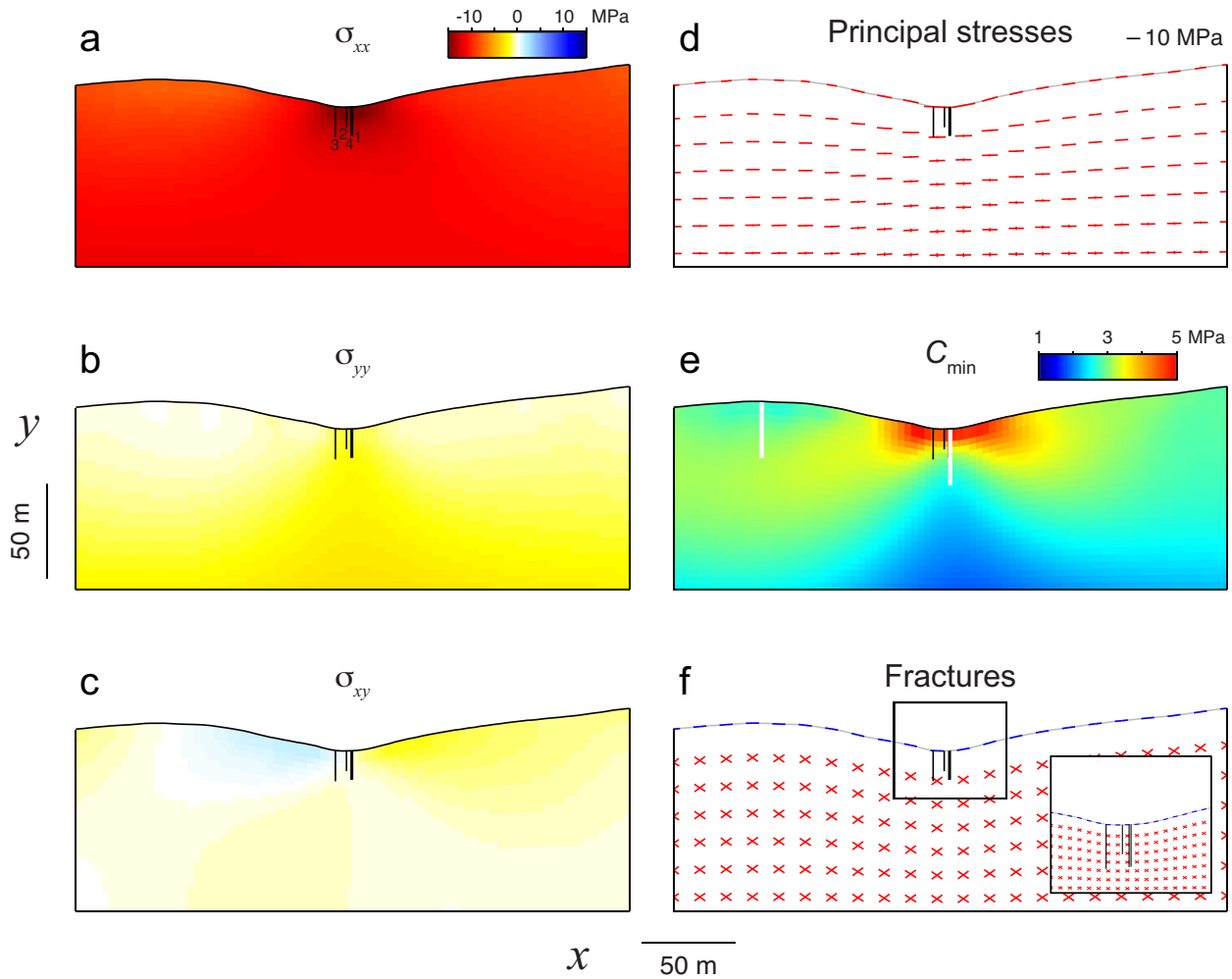
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1097 Fig. 9. Map of orientations of maximum horizontal crustal stress measurements, modified from  
 1098 the World Stress Map (Heidback et al., 2008). Symbols indicate the method used to estimate the  
 1099 stress orientation, and size and length of the symbols correspond to a qualitative measure of the  
 1100 quality of the measurement (see key). Magnitudes and depths of measurements are labeled where  
 1101 magnitude estimates were reported in the literature. Table 2 lists estimates of the maximum and  
 1102 minimum horizontal stresses at the surface based on these reported values, as well as the original  
 1103 references. The location of the SSHO field site is marked with a white circle.

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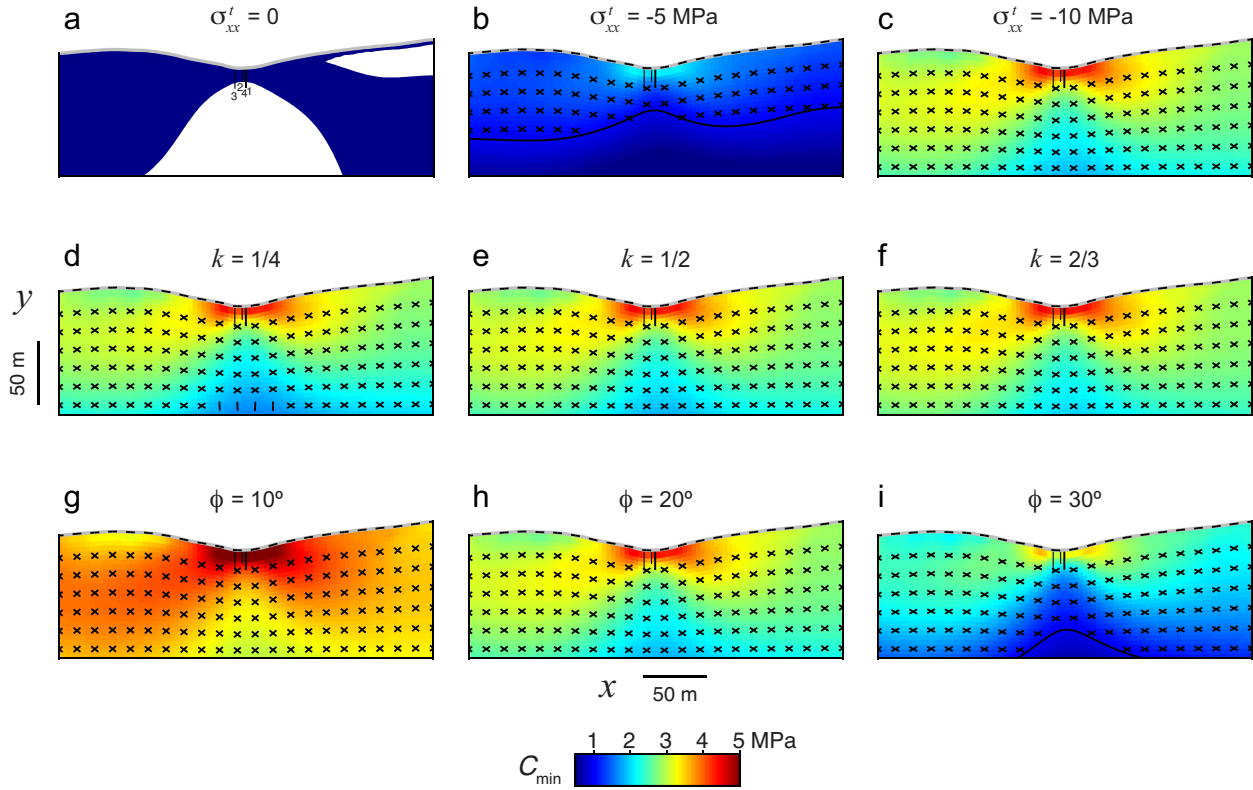
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1108 Fig. 10. Boundary element model solution for stresses (a-d),  $C_{\min}$  (e), and fracture modes and  
 1109 orientations (f) beneath the Shale Hills transect in Fig. 8 for a compressive ambient horizontal  
 1110 surface stress of -10 MPa (Fig. 9, Table 2). See Table 1 for other parameters. Locations of wells  
 1111 are indicated, and well numbers are labeled in (a). Red line segments in (d) indicate compression.  
 1112 Vertical white lines in (e) indicate the locations of vertical profiles plotted in Fig. 15a,b. In (f),  
 1113 blue line segments indicate opening mode fractures at the land surface, and red line segments  
 1114 indicate shear fractures. Inset shows a solution in the vicinity of the wells with higher spatial  
 1115 resolution. Color scale in (a) applies to (a), (b), and (c).

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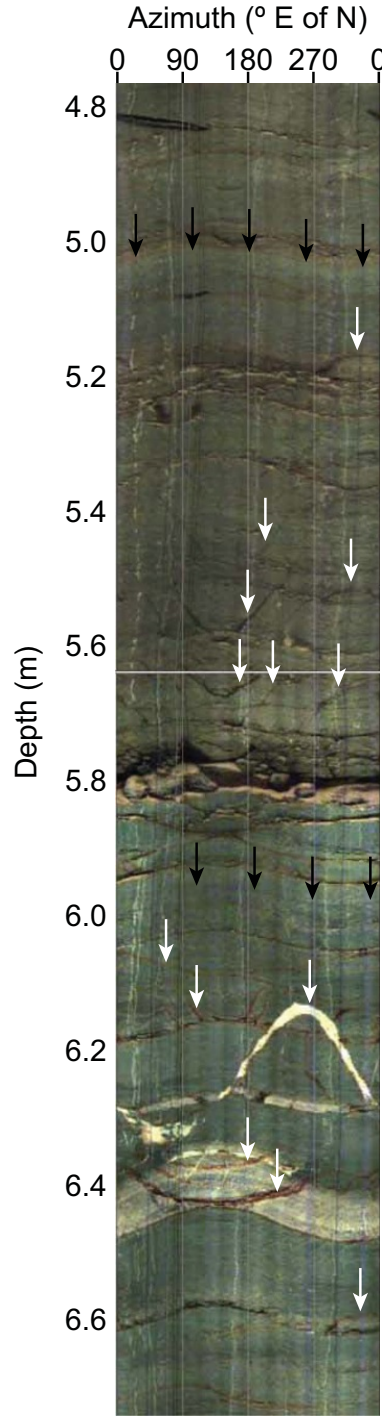


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1119 Fig. 11. Boundary element model solutions showing the sensitivity of the calculated  $C_{min}$  values  
 1120 and fracture patterns in Fig. 10 to variations in ambient horizontal tectonic stress (a-c), the  
 1121 ratio of the depth gradient of horizontal stress to the depth gradient of vertical stress (d-f), and rock  
 1122 friction angle (g-i). Panels (c), (e), and (h) are the same scenario as Fig. 10. Parameters for other  
 1123 panels are the same as in Fig. 10 except where indicated (see Table 1). Gray line is the land  
 1124 surface. Crossing black line segments below the surface represent shear fractures, and single  
 1125 black line segments at the surface represent opening mode fractures. Single near-vertical black  
 1126 segments at the lower edge of (d) represent shear fractures associated with an out-of-plane least  
 1127 compressive principal stress, in which case both shear fracture planes intersect the plane of the  
 1128 cross section along the same line. Solid black curves in (b) and (i) mark the lower boundaries of  
 1129 the zones where shear fractures are predicted for the specified cohesion of 1 MPa; in all panels  
 1130 except (a), shear fractures are predicted up to, but not including, the land surface. Locations of  
 1131 wells are indicated, and well numbers are labeled in (a). Color plots in the background show  
 1132  $C_{min}$ , the minimum cohesion required to prevent shear fracturing.

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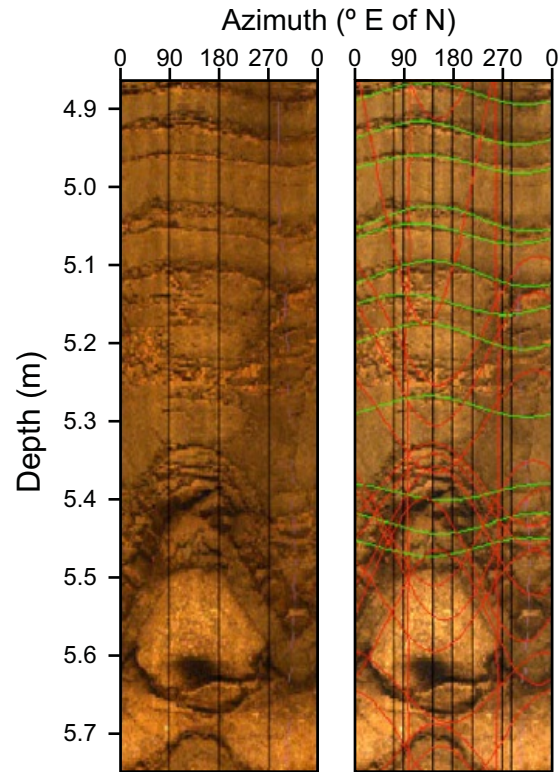
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1136 Fig. 12. Example section of borehole image log from well 1. Black arrows mark examples of  
1137 color differences used to identify bedding planes. White arrows mark examples of fractures.

1138 Planar features that intersect the borehole have sinusoidal traces in this unwrapped view of the  
1139 borehole walls. Image orientations are relative to magnetic north.



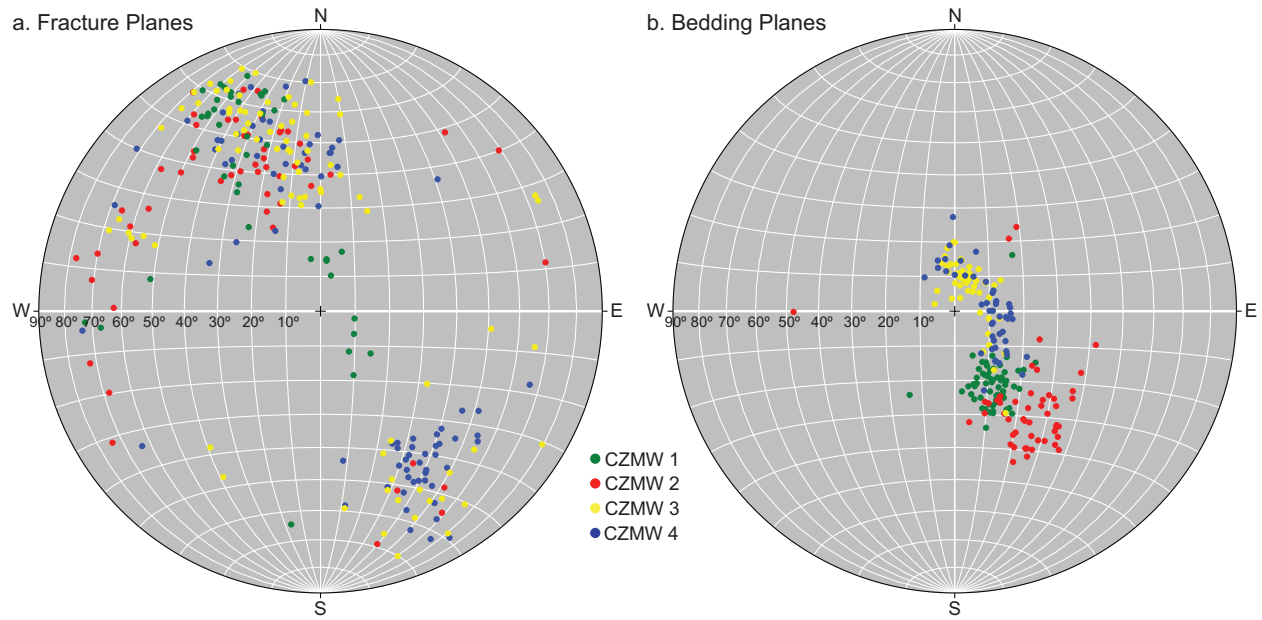


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1142 Fig. 13. Section of a borehole image log from well 4. Left image shows untraced bedding and  
1143 fracture planes. Right image shows the same section of the borehole wall with structural features  
1144 traced. Green low-amplitude traces indicate gently dipping bedding planes, and red high-  
1145 amplitude traces indicate steeply dipping fracture planes. Image orientations are relative to  
1146 magnetic north.

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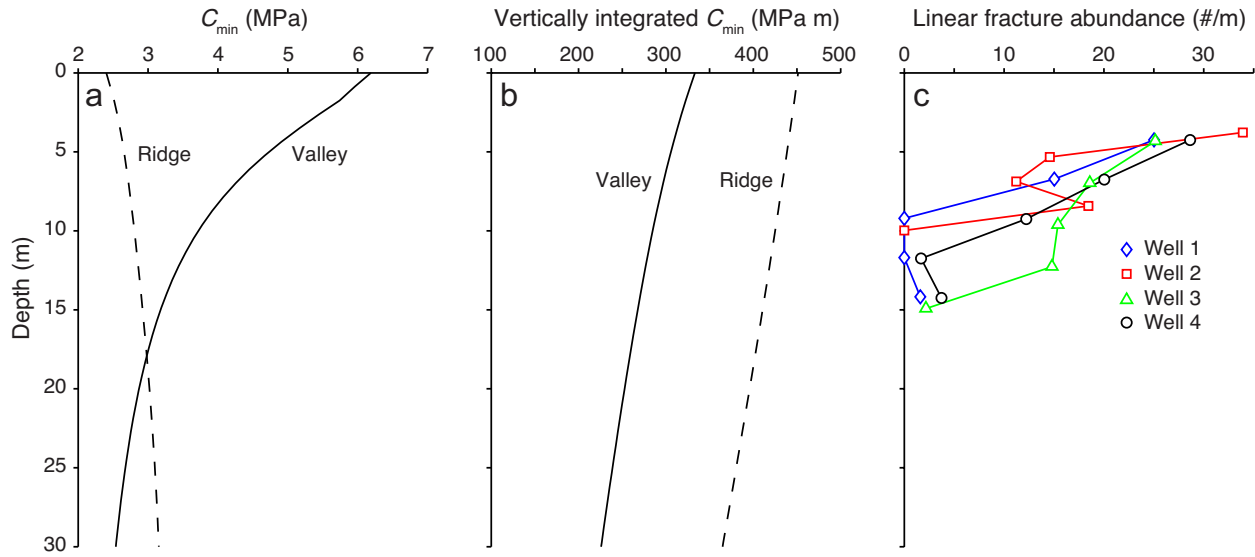


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1150 Fig. 14. Stereonets showing measured orientations of poles to (a) fractures and (b) bedding  
1151 planes in the wells. Points are the intersections of the poles to the planes with the lower  
1152 hemisphere, such that steeper-dipping planes plot closer to the outer circle. Dip angles for planar  
1153 features corresponding to the poles are indicated on the grid. Orientations are relative to  
1154 geographic north.

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1158 Fig. 15. Comparison of measured fracture abundance with model-based proxies for shear  
 1159 fracture. (a) Depth profiles of  $C_{min}$  beneath the valley floor and ridgeline at the locations  
 1160 indicated in Fig. 10e. (b) Depth profiles of vertically integrated  $C_{min}$  at the same locations. (c)  
 1161 Depth profiles of weighted linear fracture abundance beneath the valley floor, based on fracture  
 1162 counts in borehole image logs. The gap in fracture abundance near the surface occurs because the  
 1163 wells are cased from the surface to 3 m depth, so no fractures could be measured in that depth  
 1164 range.