

Robust Minimum Energy Wireless Routing for Underwater Acoustic Communication Networks

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Abstract—Marine robots are an increasingly attractive means for observing and monitoring the ocean, but underwater acoustic communications remain a major challenge. The channel exhibits long delay spreads with frequency-dependent attenuation; moreover, it is time-varying. We consider the minimum energy wireless transmission problem [MET], augmented by the practical condition that constraints on link power must be satisfied in probability. For this, we formulate the robust counterpart of the multicommodity mixed-integer linear programming (MILP) model from Haugland and Yuan [1], and derive scaled power levels that account for uncertainty. Our main result is that the deterministic formulation with these scaled power levels recovers exactly the optimal robust solution in the absence of correlations, and therefore allows for efficient solution via MILP. This approach achieves significant power improvements over heuristics, and naturally lends itself to vehicle networks.

I. INTRODUCTION

Oil spills, toxic algal blooms and undersea volcanic eruptions are all dynamic ocean processes that need to be measured and monitored in order to enhance our understanding and safe utilization of the ocean [2]. For such tasks, multiple-vehicle fleets will need to work collaboratively [3].

Acoustics is used today for underwater communications over kilometer-plus ranges; compared to RF, acoustic communication (“acomms”) has low bandwidth, high latency and poor reliability. Range and data throughput depend on modem power and carrier frequency [4], and as a result, ocean network deployments are often over-powered or limited in scale to improve robustness. However, excess power causes interference and depletes limited energy sources in untethered vehicles and nodes.

In this work, we consider underwater acomms routing with power control via a centralized robust approach, with emphasis on multicast. While the large size and ad-hoc nature of many RF wireless applications motivate distributed routing methods based on network discovery [5], the high latency and unreliability of acomms suggests that these algorithms could exhibit poor convergence in the underwater domain. In addition, considering large-scale ocean missions, data assimilation and planning are typically centralized today and the marine assets are expensive and tracked carefully [6]. These aspects of acomms and ocean missions motivate

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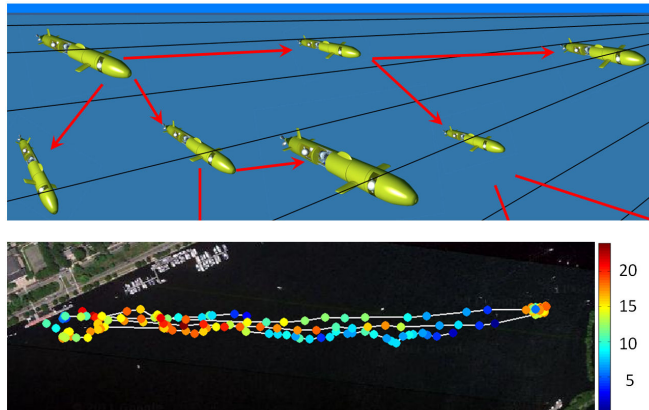


Fig. 1: Top: Multiple marine robots performing a strongly coordinated mission using acoustic communications. Bottom: Acoustic modem performance (receive SNR) in the Charles River (Boston, MA). Two out-and-back trips with a vehicle collecting statistics are shown.

optimization methods which can take into account motion plans, global channel information, and operator input [7].

Wireless network design via centralized approaches is of course a rich and active area of research. Convex optimization for routing in multi-hop RF wireless networks is presented in [8]; see also [9] for an approach specific to acomms. These works do not consider robustness, however. Chang *et al.* consider robustness to uncertain packet success rates in lossy network coding subgraph generation [10]. Regarding power control in routing, several non-robust, acoustics-focused approaches have been proposed, including [11]. Quek *et al.* consider robust power allocation for two-hop RF wireless relay networks [12]; we consider multicast over arbitrary numbers of hops using acoustic channel models. Although acomms possesses the broadcast advantage, multicast has received little attention in underwater acoustic networks [13].

We base our approach on the multicommodity MET-F2 formulation by Haugland and Yuan [1], and the main idea is to use robust convex optimization to account for uncertainty in required power levels for acomms. We give the problem statement in Sec. II. Stochastic acomms models motivated by data are discussed in Sec. III. The supporting formulations are outlined in Sec. IV, and our new approach for Robust MET is presented in Sec. V. In Sec. VI, we show that the deterministic formulation with properly-scaled power data can be used to solve the robust problem. We present

computational results in Sec. VII, and discuss conclusions and some realistic extensions to our formulation in Sec. VIII.

II. APPROACH AND PROBLEM DEFINITION

We consider a single source transmitting to multiple destinations, and design minimum-power broadcast trees and node power levels which meet individual connectivity requirements with a specified probability. Node locations are considered static and known; the primary sources of uncertainty are in transmission loss and noise at the receiver and transmitter. While we recognize the importance of protocol effects, we do not consider link throughput rates, impacts of interference on medium access, nor correlated uncertainty across links in this work in order to focus on the key aspects of robust minimum-power routing. However, the formulation of Robust MET via convex optimization is a key underlying construction onto which protocol aspects may be added and analyzed.

Since we are designing power levels at the nodes, we choose to model uncertainty in the transmit power necessary to achieve a minimum SNR at the receiver: $p_{ij} = \bar{p}_{ij} + \tilde{p}_{ij}$. The mean power for link (i, j) to have successful transmission is \bar{p}_{ij} (the no-uncertainty power), and the normal random variable describing the uncertainty in the power is \tilde{p}_{ij} .¹ The mean and variance for each link, along with the desired probability of link connectivity, are inputs to the optimization.

Robust optimization considers the worst-case realization of the random variable p_{ij} ; under the assumption of a Gaussian distribution we use the mean power plus a properly-scaled addition to account for uncertainty. Our solution is thus feasible for the worst-case realization within a certain probabilistic bound. We call the mean power plus the scaled power \hat{p}_{ij} and will show in Sec. VI that it can be set deterministically.

The wireless network is described by a graph $G(V, E)$, where E is set of possible (undirected) edges and V is the set of nodes. The set of directional arcs derived from E is A . The multicast source node is s and the set of destinations is D . The transmit power of node i is P_i . Additionally,

$$x_{ij}^t = \text{Flow on arc } (i, j) \in A \text{ for commodity } t \in D$$

$$y_{ij} = \begin{cases} 1 & \text{if the power of node } i \geq \hat{p}_{ij} \\ 0 & \text{otherwise} \end{cases}$$

The x variables are binary and an arc is included in the routing if it has flow for any commodity.

The minimum energy transmission [MET] problem was first introduced in [14], and concerns the optimal node transmission powers and associated routing tree for a wireless single-source broadcast or multicast network. To be consistent with our notation we use \bar{p}_{ij} to denote the

¹As will be discussed in Sec. IV-B, the assumption of a Gaussian distribution is simply used to formally size the uncertainty sets used in the optimization. Other distributions can be better-suited for acoustic channel variability, and the size of the uncertainty sets could be approximated under different distributions or based directly on data.

deterministic power model. The formal problem statement is:

[MET] Find a power vector $(P_1, P_2, \dots, P_N) \in \mathbb{R}_+^N$ of minimum sum, such that the induced graph (V, E^P) , where $E^P = \{(i, j) \in A : P_i \geq \bar{p}_{ij}\}$, has a path from s to each $t \in D$.

Broadcast has $D=V \setminus \{s\}$ while multicast has $D \subset V \setminus \{s\}$. The MET problem can be transformed into an equivalent Steiner tree problem and is thus NP-complete [1].

The robust formulation of MET requires the power constraints, which relate the power P_i at a node to the inter-node minimum power levels p_{ij} , to be satisfied in probability:

$$E^P = \{(i, j) \in A : \mathbf{prob}(P_i \geq p_{ij}) \geq \eta\} \quad (1)$$

Successful transmission occurs when the power at the receiver exceeds a minimum SNR threshold.

III. ACOUSTIC COMMUNICATIONS MODEL

The unique characteristics of the acoustic communications channel leave many transmission parameters to be optimized, such as center-frequency, bandwidth, frequency allocation, power level, and modulation schemes [4]. We develop our simple models with an eye towards practical implementation using currently available hardware. The WHOI MicroModem [15] is commonly used in acoustic communication research, and operates at one of three hardware-defined frequency bands; we thus assume center frequencies, bandwidth, and frequency allocation to be fixed in our propagation models. New versions of the MicroModem allow for transmit power to be set in the range of 140-150 [dB], whereas the standard source level is 185 [dB] [16].

For our mean power model we use classical descriptions of underwater acoustic propagation, as well as the conversion from sound pressure level (traditionally denoted in acoustics in [dB rel μPa]) to absolute power in [W]. To reach a threshold SNR of SNR_0 [dB], with ambient noise N_{RX} [dB rel μPa], the transmit power in [W] as a function of distance r [m] is approximated as

$$\bar{p}(r) = Ar^\kappa \left(10^{(\alpha r)/10}\right) \left(10^{(SNR_0 + N_{RX} + 60 - 185)/10}\right) + B \quad (2)$$

The first term (r^κ) is due to spreading ($\kappa = 2$ for spherical), while the second term is a linear approximation of absorption loss in seawater [9]. The constant factor that is a function of SNR_0 and N_{RX} represents the desired power at the receiver, and $(60 - 185)$ represents the conversion from [dB rel μPa] to [W].

Uncertainty derives from different types of nodes (static sensor nodes, AUVs, surface ships), different operating locations (harbor, open-ocean, shipping lane) and different ocean conditions (mixing water masses, varying wind/wave conditions, varying bathymetry). These can all affect both the ambient noise at the receiver and the transmission loss. Consequently, we define multiplicative and additive uncertainty on each link: $A_{ij} = 1 + \tilde{A}_{ij}$, and $B_{ij} = 0 + \tilde{B}_{ij}$, with \tilde{A}_{ij} and \tilde{B}_{ij} as zero-mean Gaussian random variables. To first

order, multiplicative uncertainty can approximate physical uncertainty in path loss (large-scale fading), uncertainty in distance (navigation), as well as ambient noise at the receiver. Additive uncertainty corresponds to uncertain power levels or conditions at the transmitter, specifically including local noise sources.

References [17], [7] discuss two specific MicroModem datasets which are supportive of the mean power model in Eqn. 2, and have a path loss variance in decibels which is constant with distance. Constant variance in decibels roughly equates with our multiplicative uncertainty model in Watts. These data were taken in moderately deep water and in relatively good channel conditions. Conversely, Fig. 1 shows data with higher variability obtained in experiments with MicroModems in the Charles River (Boston, MA), a very shallow acoustic environment. Statistical analysis of modem performance in this environment is ongoing work.

IV. SUPPORTING FORMULATIONS

A. MET-F2 MILP formulation

Here we summarize a compact integer programming model for MET introduced by Haugland and Yuan [1]; our notation matches theirs. The strength of “MET-F2” over previous formulations comes from multi-commodity flows: each commodity corresponds to a unique destination. Continuity is defined in a standard way by relating the flows of each commodity, \mathbf{x}^t , the graph G , and the supply/demand vector \mathbf{b}_{st} : $\mathbf{x}^t \in \mathcal{F}(G, \mathbf{b}_{st}), t \in D \setminus \{s\}$, where \mathcal{F} is the set of admissible flows. For each commodity, the source has a supply of one, and the destination has a demand of one.

The multicommodity flow formulation allows for the broadcast advantage to be represented compactly, using constraints which relate the y_{ij} variables to the flows x_{ij}^t using a specific ordering of power levels. For any node $i \in V$, let $\pi_i : \{1, \dots, N-1\} \mapsto V \setminus \{i\}$ be a bijection such that $p_{i, \pi_i(1)}, \dots, p_{i, \pi_i(N-1)}$ is monotonically non-decreasing. As shorthand, the subscript (i, k) defines the variables in non-decreasing order of power required, where k refers to the k th-closest node to node i . The formal problem is [1]:

[MET-F2]

$$\text{minimize} \quad \sum_{\{i,j\} \in A} p_{ij} y_{ij} \quad (3)$$

$$\text{subject to} \quad \mathbf{x}^t \in \mathcal{F}(G, \mathbf{b}_{st}), t \in D \setminus \{s\}, \quad (4)$$

$$\sum_{l=k}^{N-1} x_{(il)}^t \leq \sum_{l=k}^{N-1} y_{(il)}, \quad (5)$$

$$i \in V, k \in 1, \dots, N-1, t \in D \setminus \{s\},$$

$$\mathbf{y} \in \{0, 1\}^{|A|}, \quad (6)$$

$$\mathbf{x} \in \{0, 1\}^{|A||D|}, \quad (7)$$

where the minimum mean link powers p_{ij} , the sets A and D , the source s , and the ordering π_{ij} are given. The node powers are then set as $P_i = \sum_{j \in V} p_{ij} y_{ij}$.

B. Robust LP

A deterministic LP uses constraints of the form $\mathbf{a}_i^T \mathbf{z} \leq b_i$, where \mathbf{a}_i^T and b_i are known. The robust optimization framework of Ben-Tal and Nemirovski [18] requires the solution to hold for all constraint parameters in an uncertainty set. We use the second-order cone program (SOCP) formulation from [19], which models \mathbf{a}_i as Gaussian random variables and sizes the uncertainty sets such that the constraints are met in probability. We desire:

$$\text{prob}(\mathbf{a}_i^T \mathbf{z} \leq b_i) \geq \eta. \quad (8)$$

The corresponding SOC constraint is:

$$\bar{\mathbf{a}}_i^T \mathbf{z} + \Phi^{-1}(\eta) \left\| Q_i^{1/2} \mathbf{z} \right\|_2 \leq b_i \quad (9)$$

where Φ^{-1} is the inverse cdf of the standard normal distribution. The probability η must be ≥ 0.5 , which results in $\Phi^{-1}(\eta) \geq 0$, making (9) a valid SOC constraint. Q_i is the covariance matrix of the independent Gaussian random vectors \mathbf{a}_i ; there are no correlations between \mathbf{a}_i and \mathbf{a}_j represented. Notice that this formulation uses continuous decision variables, while there are binary variables in MET-F2. We will address this in the next section.

V. ROBUST LP FOR MET-F2

In the deterministic MET-F2 formulation, the P_i variables are used, since they are redundant with p_{ij} and y_{ij} . In order to pose the problem as a robust LP, we re-introduce them. Substituting the stochastic definition of p_{ij} from Sec. III, and enforcing the power constraint probabilistically, we require

$$P_i \geq \sum_{j \in V} (\bar{p}_{ij} + \tilde{p}_{ij}) y_{ij}, \text{ with probability } \eta. \quad (10)$$

We define the vector of decision variables, with N P_i variables, $|A||D|$ x_{ij}^t variables, and $|A|$ y_{ij} variables:

$$\mathbf{z} = [P_1, \dots, P_N, x_{12}, \dots, x_{N-1,N}, y_{12}, \dots, y_{N-1,N}] \quad (11)$$

Following the procedure of Sec. IV-B, we can manipulate the constraints of [MET-F2] into the form $\mathbf{a}_i^T \mathbf{z} \leq b_i$, and arrive at a new set of SOC constraints:

$$-P_i + \sum_{j=1}^N (\bar{p}_{ij} y_{ij}) + \Phi^{-1}(\eta) \left\| Q_i^{1/2} \mathbf{z} \right\|_2 \leq 0, \quad i = 1, \dots, N \quad (12)$$

For the Robust MET-F2 problem, Q_i is a large matrix with blocks corresponding to the constituents of \mathbf{z} (P_i , x_{ij}^t , and y_{ij}). For a given node i , y_{ij} is a singleton vector which we denote \mathbf{y}_i . Since uncertainty is modeled in the parameter p_{ij} , multiplying the variables y_{ij} , the only nonzero block of Q_i is the one corresponding to \mathbf{y}_i . We denote this block $Q_{i,yy}$, and restrict it to be diagonal.

With inter-node variances of p_{ij} denoted as σ_{ij}^2 , we define the vector of variances from node i to each other node $\boldsymbol{\sigma}_i^2 = [\sigma_{i1}^2, \dots, \sigma_{iN}^2]$. Thus, $Q_{i,yy} = \text{diag}(\boldsymbol{\sigma}_i^2)$. The full robust MET-F2 optimization problem is:

[Robust MET-F2]

$$\begin{aligned}
& \text{minimize} && \sum_{i \in V} P_i && (13) \\
& \text{subject to} && (4), (5), (6), (7), \\
& && -P_i + \sum_{j=1}^N (\bar{p}_{ij} y_{ij}) + \Phi^{-1}(\eta) \sigma_i^T \mathbf{y}_i \leq 0, \\
& && i = 1, \dots, N, && (14) \\
& && P_i^{\min} \leq P_i \leq P_i^{\max}, \quad i = 1, \dots, N && (15)
\end{aligned}$$

This model has two major features. First, the diagonal $Q_{i,yy}$ restriction reduces the second-order cone constraint of the robust counterpart to a linear constraint. Second, the ordering based on power used in constraint (5) must be modified to use \hat{p}_{ij} instead of the deterministic (or mean) powers in order to account for the effects of uncertainty. In the next section we show exactly how to set \hat{p}_{ij} .

VI. ANALYSIS AND DETERMINATION OF SCALED POWERS

A. Determination of \hat{p}_{ij}

We show that the scaled powers \hat{p}_{ij} are a function of the mean and variance of p_{ij} , and further, that if \hat{p}_{ij} is used as input to the deterministic MET-F2 MILP formulation, the results are the optimal solution to Robust MET.

We assume that the optimal routing y_{ij} has been determined, and define $j^*(i) = j$ s.t. $y_{ij} = 1$; $j^*(i)$ is the node in the routing which requires the largest power for connectivity with node i . The robust constraint (14) reduces to:

$$P_i \geq \bar{p}_{ij^*(i)} + \Phi^{-1}(\eta) \sigma_{ij^*(i)}, \quad (16)$$

where $\sigma_{ij^*(i)}$ is the standard deviation of the uncertainty for the transmit power of link $ij^*(i)$. Since the objective is to minimize the sum of the node powers P_i , and P_i appear only in this constraint, the inequality (16) is tight. The resulting equality relation for P_i allows for substitution of the RHS of (16) in the objective,² which becomes:

$$\text{minimize} \quad \sum_{i=1}^N P_i = \sum_{i=1}^N (\bar{p}_{ij^*(i)} + \Phi^{-1}(\eta) \sigma_{ij^*(i)}) \quad (17)$$

The only remaining difference between the constraint sets of the deterministic MET-F2 formulation and the robust version is that the ordering used in constraint (5) is different.³ Robust MET requires ordering based on the scaled powers \hat{p}_{ij} , while ordering in deterministic MET-F2 is set based on the deterministic (or mean) powers. However, by the same equality argument as for Eqn. (16), it is clear that:

$$\hat{p}_{ij} = \bar{p}_{ij} + \Phi^{-1}(\eta) \sigma_{ij}. \quad (18)$$

Substituting \hat{p}_{ij} for p_{ij} in deterministic MET-F2 results in an equivalent formulation to Robust MET. This is important

²Substituting the robust definition of P_i into the objective can also be viewed as a special case of the robust optimization approach for cost coefficients with ellipsoidal uncertainty sets by Bertsimas and Sim [20].

³Additionally, if maximum and minimum node power levels are desired, the P_i variables must be retained; the effect on overall problem size and tractability is negligible.

computationally because MET-F2 (a MILP) solves much faster than the general robust counterpart of a MILP (a MISOCP). We refer to [1] for solution times; networks up to fifty nodes are tractable to solve to optimality today.

The case of a nondiagonal $Q_{i,yy}$ represents correlations, which is outside our current scope. However, correlations could be treated approximately by solving the MISOCP with constraint (12), using the ordering based on \hat{p}_{ij} as given above. If it is desired, a fully linear approximation could also be made through the relation:

$$\left\| Q_{i,yy}^{1/2} \mathbf{y}_i \right\|_2 \leq \left\| Q_{i,yy}^{1/2} \right\|_2. \quad (19)$$

B. Special case: constant multiplicative uncertainty

Multiplicative uncertainty (described by \tilde{A} in Sec. III) which is constant across all links is amenable to further analysis. This model would be valid if all nodes have similar characteristics and the ocean conditions are approximately uniform across the operating region. The uncertainty for link ij in absolute power [W] at the sender becomes a simple fraction of the mean power for the link in [W]:

$$\sigma \left(\tilde{A}_{ij} \right) = \sigma_{ij} = \frac{\bar{p}_{ij}}{\mathbb{C}} \quad (20)$$

We insert this model for σ_{ij} into the objective as defined in (17) and collect terms:

$$\sum_{i=1}^N P_i = \left(1 + \frac{\Phi^{-1}(\eta)}{\mathbb{C}} \right) \left(\sum_{i=1}^N \bar{p}_{ij^*(i)} \right) \quad (21)$$

Since $\Phi^{-1}(\eta)$ and \mathbb{C} are both constants, the ordering based on \hat{p}_{ij} is the same as the ordering based on \bar{p}_{ij} . Thus, this formulation has the same feasible set as deterministic MET-F2 and the optimal solution to Robust MET is:

- The optimal routing x_{ij}^t and y_{ij} from deterministic MET-F2
- Node powers set according to:

$$P_i = \left(1 + \frac{\Phi^{-1}(\eta)}{\mathbb{C}} \right) \left(\sum_{j=1}^N \bar{p}_{ij} y_{ij} \right) \quad (22)$$

The optimal topology and routing are invariant, but the power levels change with the uncertainty level.

VII. COMPUTATIONAL RESULTS

We ignore absorption losses and present results for the spherical spreading model $\bar{p}_{ij} = Gr_{ij}^2$ in order to be consistent with literature on MET. Results were computed using AMPL/CPLEX. The results we show are all for a single multicast instance with $N = 30$ nodes, and $|D| = 15$ destinations randomly located in the unit square. We present example results for multiplicative and additive uncertainty separately, all with $\eta = 0.99$. We normalize the powers such that the deterministic objective ($\sigma = 0$) has total power of one. We did not set maximum or minimum power levels for any of these cases, in order to focus on the effects of the robust constraints.

A. Multiplicative uncertainty

The left side of Fig. 2 shows the deterministic routing, and the right side shows a scenario where all links going into destinations have a multiplicative uncertainty of $\sigma_{ij} = \bar{p}_{ij}/2$ and all links going into optional router nodes have a multiplicative uncertainty of $\sigma_{ij} = \bar{p}_{ij}/20$. The routing is notably different between the two cases. The deterministic case would be infeasible with uncertainty.

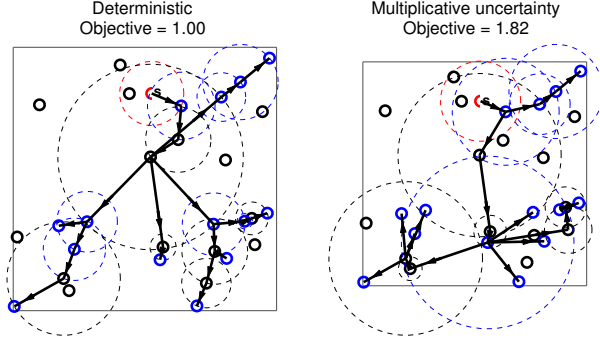


Fig. 2: The left plot is the deterministic solution (shown for reference). The red node labeled s is the source. The right plot is the solution when destination nodes (blue) have multiplicative uncertainty of $\sigma_{ij} = \bar{p}_{ij}/2$ and optional routers (black) have multiplicative uncertainty of $\sigma_{ij} = \bar{p}_{ij}/20$. Note that the deterministic solution would be infeasible for the scenario with uncertainty.

B. Constant additive uncertainty

We consider next uncertainties in transmit power for all links as a single constant: $\sigma(\bar{B}_{ij}) = \sigma_{ij} = \sigma_C$. Fig. 3 shows three cases. The uncertainty is normalized such that a standard deviation of one is equal to the power required to transmit the edge length of the domain. The optimal solutions are compared to the prior heuristic, which takes the deterministic design and increases node power levels in order to meet the robust constraints. The heuristic applied in this case is very poor. As uncertainty increases, the true solution moves from the optimal deterministic solution towards a star network. Fig. 4 shows a summary comparison. Even at low uncertainty, for $\sigma_C = 1/50$ shown in Fig. 3b, Robust MET achieves an objective which is 41% better than that of the heuristic. We note that the optimal solution is piecewise-linear in between changes in routing and topology, although Fig. 4 does not directly show each discrete change.

VIII. DISCUSSION AND FUTURE WORK

Robust MET provides a tractable means for designing efficient geographic routing subject to power uncertainty, a capability which is especially useful in power-constrained marine robotic networks that rely on unreliable acomsms. We have shown that with proper scaling of input power levels, a deterministic MILP formulation may be used to find the optimal robust solution; MILP solvers are faster than mixed-integer SOCP solvers. Additionally, in the case of constant

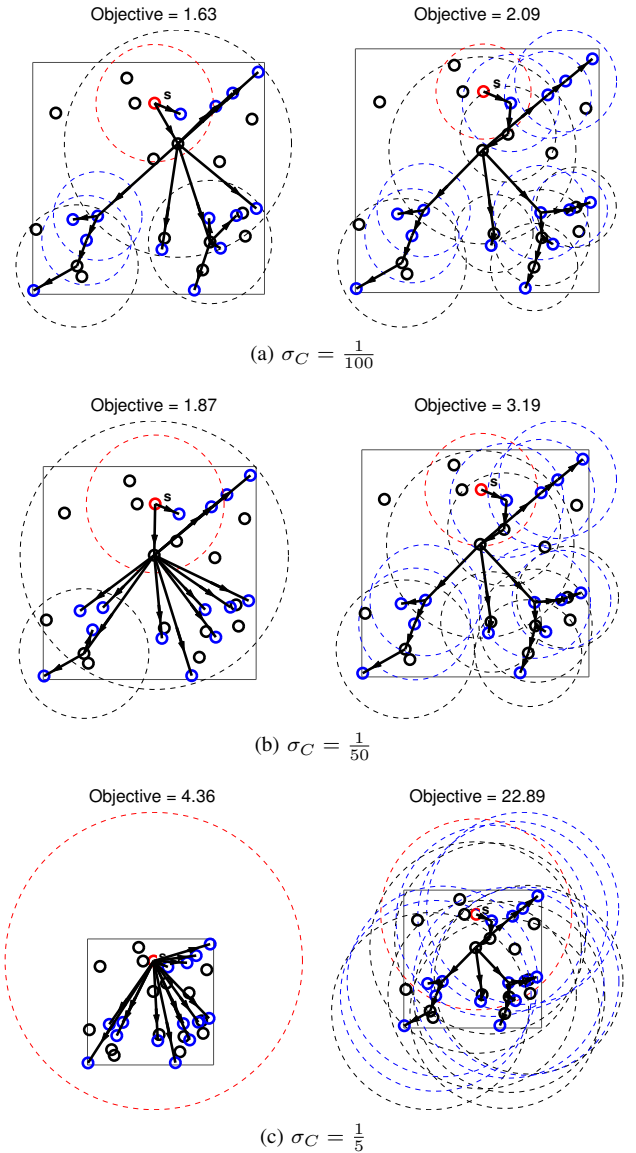


Fig. 3: Robust MET solution (left) compared to baseline heuristic (right) for three different values of constant additive uncertainty. $\sigma_C = 1$ corresponds to uncertainty equal to the power to transmit the distance of an edge of the box. The objective is normalized such that the optimal deterministic objective ($\sigma = 0$) is equal to one.

multiplicative uncertainty the deterministic routing solution plus a linear scaling of node powers is optimal. This suggests that the routing table does not always need to be updated as conditions change. In this case or between shifts in topology for arbitrary uncertainty scenarios, adaptive power-control schemes using feedback, such as in [21], could be used for additional performance benefits as the routing is locally optimal.

Robust MET can be extended in a number of directions, most directly to multi-source solutions via shared broadcast trees [22]. Interference between competing transmissions should be explicitly considered for optimal medium access

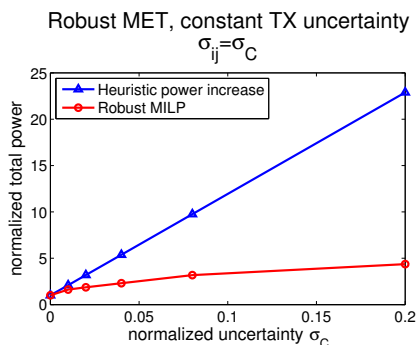


Fig. 4: Normalized sum of transmit powers as a function of constant additive uncertainty for $N = 30$ and $D = 15$. The total power with no uncertainty is 1. Uncertainty with a standard deviation equal to the mean power required to transmit the edge length of the domain is one.

control. We would like to connect our work with the design of reliable protocols above the physical layer, such as network coding. To incorporate mobility, we can re-solve the optimization as vehicles move, and thus integrate our formulation with motion planning. More broadly, we expect that convex optimization can continue to provide a unifying framework for design and analysis in the context of robust multi-agent control in marine applications.

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REFERENCES

- [1] D. Haugland and D. Yuan, "Compact integer programming models for power-optimal trees in ad hoc wireless networks," *Wireless Network Design*, pp. 219–246, 2011.
- [2] R. Camilli, C. Reddy, D. Yoerger, B. Van Mooy, M. Jakuba, J. Kinsey, C. McIntyre, S. Sylva, and J. Maloney, "Tracking hydrocarbon plume transport and biodegradation at deepwater horizon," *Science*, vol. 330, no. 6001, p. 201, 2010.
- [3] J. Bellingham and K. Rajan, "Robotics in remote and hostile environments," *Science*, vol. 318, no. 5853, pp. 1098–1102, 2007.
- [4] M. Stojanovic, "On the relationship between capacity and distance in an underwater acoustic communication channel," *ACM SIGMOBILE Mobile Computing and Communications Review*, vol. 11, no. 4, pp. 34–43, 2007.
- [5] E. Royer and C.-K. Toh, "A review of current routing protocols for ad hoc mobile wireless networks," *Personal Communications, IEEE*, vol. 6, no. 2, pp. 46–55, apr 1999.
- [6] S. Ramp, R. Davis, N. Leonard, I. Shulman, Y. Chao, A. Robinson, J. Marsden, P. Lermusiaux, D. Fratantoni, J. Paduan, *et al.*, "Preparing to predict: the second autonomous ocean sampling network (aosn-ii) experiment in the monterey bay," *Deep Sea Research Part II: Topical Studies in Oceanography*, vol. 56, no. 3-5, pp. 68–86, 2009.
- [7] G. Hollinger, S. Choudhary, P. Qarabaqi, C. Murphy, U. Mitra, G. Sukhatme, M. Stojanovic, H. Singh, and F. Hover, "Underwater data collection using robotic sensor networks," *Selected Areas in Communications, IEEE Journal on*, vol. 30, no. 5, pp. 899–911, 2012.
- [8] R. Cruz and A. Santhanam, "Optimal routing, link scheduling and power control in multihop wireless networks," in *INFOCOM 2003. Twenty-Second Annual Joint Conference of the IEEE Computer and Communications Societies*, vol. 1, march-3 april 2003, pp. 702–711 vol.1.

- [9] L. Badia, M. Mastrogiovanni, C. Petrioli, S. Stefanakos, and M. Zorzi, "An optimization framework for joint sensor deployment, link scheduling and routing in underwater sensor networks," in *Proceedings of the 1st ACM international workshop on Underwater networks*, ser. WUWNet '06. New York, NY, USA: ACM, 2006, pp. 56–63.
- [10] C. Chang, T. Ho, and M. Effros, "On robust network coding subgraph construction under uncertainty," in *Signals, Systems and Computers, 2008 42nd Asilomar Conference on*, oct. 2008, pp. 2211–2215.
- [11] J. Jorner, M. Stojanovic, and M. Zorzi, "On joint frequency and power allocation in a cross-layer protocol for underwater acoustic networks," *Oceanic Engineering, IEEE Journal of*, vol. 35, no. 4, pp. 936–947, oct. 2010.
- [12] T. Quek, H. Shin, and M. Win, "Robust wireless relay networks: Slow power allocation with guaranteed qos," *Selected Topics in Signal Processing, IEEE Journal of*, vol. 1, no. 4, pp. 700–713, dec. 2007.
- [13] P. Nicopolitidis, G. Papadimitriou, and A. Pomportsis, "Adaptive data broadcasting in underwater wireless networks," *Oceanic Engineering, IEEE Journal of*, vol. 35, no. 3, pp. 623–634, july 2010.
- [14] J. Wieselthier, G. Nguyen, and A. Ephremides, "On the construction of energy-efficient broadcast and multicast trees in wireless networks," in *INFOCOM 2000. Nineteenth Annual Joint Conference of the IEEE Computer and Communications Societies. Proceedings. IEEE*, vol. 2, 2000, pp. 585–594 vol.2.
- [15] L. Freitag, M. Grund, S. Singh, J. Partan, P. Koski, and K. Ball, "The WHOI micro-modem: an acoustic communications and navigation system for multiple platforms," in *OCEANS, 2005. Proceedings of MTS/IEEE*. IEEE, 2005, pp. 1086–1092.
- [16] E. Gallimore, J. Partan, I. Vaughn, S. Singh, J. Shusta, and L. Freitag, "The WHOI micromodem-2: A scalable system for acoustic communications and networking," in *OCEANS 2010*. IEEE, 2010, pp. 1–7.
- [17] P. Qarabaqi and M. Stojanovic, "Modeling the large scale transmission loss in underwater acoustic channels," in *Communication, Control, and Computing (Allerton), 2011 49th Annual Allerton Conference on*, sept. 2011, pp. 445–452.
- [18] A. Ben-Tal and A. Nemirovski, "Robust convex optimization," *Mathematics of Operations Research*, pp. 769–805, 1998.
- [19] M. S. Lobo, L. Vandenbergh, S. Boyd, and H. Lebret, "Applications of second-order cone programming," *Linear Algebra and its Applications*, vol. 284, pp. 193–228, 1998.
- [20] D. Bertsimas and M. Sim, "Robust discrete optimization under ellipsoid uncertainty," 2004, manuscript, 2004.
- [21] P. Qarabaqi and M. Stojanovic, "Adaptive power control for underwater acoustic communications," in *OCEANS, 2011 IEEE - Spain*, june 2011, pp. 1–7.
- [22] D. Yuan and D. Haugland, "Dual decomposition for computational optimization of minimum-power shared broadcast tree in wireless networks," *IEEE Transactions on Mobile Computing*, vol. 99, no. PrePrints, 2011.