Robust Power Allocation for Energy-Efficient Location-Aware Networks

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***Abstract*—In wireless location-aware networks, mobile nodes (agents) typically obtain their positions through ranging with respect to nodes with known positions (anchors). Transmit power allocation not only affects network lifetime, throughput, and interference, but also determines localization accuracy. In this paper, we present an optimization framework for robust power allocation in network localization to tackle imperfect knowledge of network parameters. In particular, we formulate power allocation problems to minimize the squared position error bound (SPEB) and the maximum directional position error bound (mDPEB), respectively, for a given power budget. We show that such formulations can be efficiently solved via conic programming. Moreover, we design an efficient power allocation scheme that allows distributed computations among agents. The simulation results show that the proposed schemes significantly outperform uniform power allocation, and the robust schemes outperform their non-robust counterparts when the network parameters are subject to uncertainty.**

***Index Terms*—Localization, wireless networks, resource allocation, semidefinite programming (SDP), second-order conic programming (SOCP), robust optimization.**

I. Introduction

Positional information is of critical importance for future wireless networks, which will support an increasing number of location-based applications and services [1]–[9]. Example applications include cellular positioning, search and rescue work, blue-force tracking, etc., covering civilian life to military operations. In GPS-challenged environments, wireless network localization typically refers to a process that determines the positions of mobile nodes (agents) based on the measurements with respect to mobile/static nodes with known positions (anchors), as illustrated in Fig. 1. With the rapid development of advanced wireless techniques, wireless network localization has attracted numerous research interests in the past decades [10]–[20].

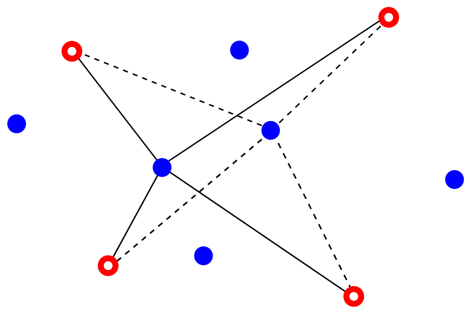

Fig. 1: Location-aware networks: the anchors (red circle) localize the agents (blue dot) based on inter-node range measurements.

Localization accuracy is a critical performance measure of wireless location-aware networks. In recent work [5], [6], the fundamental limits of wideband localization have been derived in terms of the squared position error bound (SPEB) and directional position error bound (DPEB). It shows that localization accuracy is related to several aspects of design, including network topology, signal waveforms, and transmit power. Power allocation for wireless network localization plays a critical role in reducing localization errors or energy consumption, when the nodes are subject to limited power resources or quality-of-service (QoS) requirements [21]–[23]. Optimal or near-optimal trade-off between localization errors and energy consumption can be obtained by optimization methods, which have played an important role in maximizing communication and networking performance under limited resources [24]–[31]. The authors in [32] formulated several optimization problems for anchor power allocation in wideband localization systems, and derived the optimal solution for single-agent networks. In [33], it exploited the geometrical interpretation of localization information to minimize the maximum DPEB (mDPEB).[1] In [34], it investigated the localization using MIMO radar systems, and adopted the constraint relaxation and domain decomposition methods to obtain sub-optimal solutions for power allocation. In general, how to optimally allocate the transmit power in location-aware networks still remains as an open problem.

Power allocation schemes should be adapted to the instan-

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[1] The mDPEB characterizes the maximum position error of an agent over all directions.

taneous network conditions, such as network topology and channel qualities, for optimizing the localization performance. Previous work on power allocation in location-aware networks assumes that the network parameters such as nodes' positions and channel conditions are perfectly known [32]–[34]. However, these parameters are obtained through estimation and hence subject to uncertainty. The power allocation based on imperfect knowledge of network parameters often leads to suboptimal or even infeasible solutions in realistic networks [35]–[37]. Therefore, it is essential to design a robust scheme to combat the uncertainty in network parameters.

In this paper, we present an optimization framework for robust power allocation in network localization to tackle imperfect knowledge of network parameters. Specifically, we treat the fundamental limits of localization accuracy, i.e., SPEB and mDPEB, as the performance metrics. The main contributions are summarized as follows.

- We formulate optimization problems for power allocation to minimize SPEB/mDPEB subject to limited power resources, and prove that these formulations can be transformed into conic programs.[2]
- We propose a robust optimization method for the worst-case SPEB/mDPEB minimization in the presence of parameter uncertainty. The proposed robust formulations retain the same form of conic programs as their non-robust counterparts.
- We develop a distributed algorithm for robust power allocation, which decomposes the original problem into several subproblems enabling parallel computations among all the agents without loss of optimality.

The rest of the paper is organized as follows. In Section II, we describe the system model and introduce the performance metrics. In Section III, we formulate the power allocation problems into conic programs. In Section IV, robust power allocation schemes are proposed to combat the uncertainty in network parameters. In Section V, we further decompose our robust formulation into several subproblems that can be independently solved by each agent. In Section VI, the performance of the proposed schemes is investigated through simulations. Finally, the paper is concluded in Section VII.

Notations: We use lowercase and uppercase bold symbols to denote vectors and matrices, respectively; $\det(\mathbf{A})$ and $\text{tr}(\mathbf{A})$ denote the determinant and trace of matrix $\mathbf{A}$, respectively; the superscript $(\cdot)^{\text{T}}$ and $\|\cdot\|$ denote the transpose and Euclidean norm of its argument, respectively; matrices $\mathbf{A} \succeq \mathbf{B}$ denotes that $\mathbf{A} - \mathbf{B}$ is positive semidefinite. We define the unit vector $\mathbf{u}(\phi) = [\cos\phi \;\; \sin\phi]^{\text{T}}$. We use calligraphic symbols, e.g., $\mathcal{N}$, to denote sets, and $\mathbb{E}\{\cdot\}$ and $\Pr\{\cdot\}$ to denote the expectation and probability operators, respectively.

II. System Model

In this section, we describe the system model, and introduce two performance metrics of location-aware networks.

[2]Conic programs can be efficiently solved by off-the-shelf optimization tools [27], [38]

A. Network Settings

Consider a 2-D location-aware network consisting of N_{a} agents and N_{b} anchors, where the sets of agents and anchor are denoted by $\mathcal{N}_{\text{a}} = \{1, 2, \ldots, N_{\text{a}}\}$ and $\mathcal{N}_{\text{b}} = \{N_{\text{a}}+1, N_{\text{a}}+2, \ldots, N_{\text{a}}+N_{\text{b}}\}$, respectively. The 2-D position of node k is denoted by $\mathbf{p}_k$. The angle and distance between nodes k and j are given by ϕ_{kj} and d_{kj}, respectively. The anchors are mobile/static nodes with known positions, and subject to limited power resources. The agents aim to determine their positions based on the radio signals transmitted from the anchors. For instance, agents can obtain the signal metrics such as time-of-arrival (TOA) from the received signals, and then calculate their positions via triangulation [5].

The multipath received waveform at agent k from anchor j is modeled as [5]

$$r_{kj}(t) = \sum_{l=1}^{L_{kj}} \sqrt{x_{kj}} \cdot \alpha_{kj}^{(l)}\, s\big(t - \tau_{kj}^{(l)}\big) + z_{kj}(t),\; t \in [0, T_{\text{ob}}) \quad (1)$$

where x_{kj} is the power of the transmit waveform from anchor j to agent k, $s(t)$ is a known transmit waveform, $\alpha_{kj}^{(l)}$ and $\tau_{kj}^{(l)}$ are the amplitude and delay, respectively, of the lth path, L_{kj} is the number of multipath components, $z_{kj}(t)$ represents additive white Gaussian noise (AWGN) with two-side power spectral density $N_0/2$, and $[0, T_{\text{ob}})$ is the observation interval.

We consider that the measurements between anchors and agents do not interfere each other by using medium access control, and the network is synchronized such that the inter-node distance is estimated using one-way time-of-flight (TOF).[3] Our work can be extended to asynchronous networks where round-trip TOF is employed for distance estimation, and it will be discussed in Section III.

B. Position Error Bound

The SPEB introduced in [5] is a performance metric that characterizes the localization accuracy, defined as

$$\mathcal{P}(\mathbf{p}_k) \triangleq \text{tr}\big\{\mathbf{J}_{\text{e}}^{-1}(\mathbf{p}_k; \{x_{kj}\})\big\} \quad (2)$$

where $\mathbf{J}_{\text{e}}(\mathbf{p}_k; \{x_{kj}\})$ is the equivalent Fisher information matrix (EFIM) for agent k's position $\mathbf{p}_k$. Using the information inequality [39], we can show that the squared position error is bounded below as

$$\mathbb{E}\left\{\|\hat{\mathbf{p}}_k - \mathbf{p}_k\|^2\right\} \geq \mathcal{P}(\mathbf{p}_k)$$

where $\hat{\mathbf{p}}_k$ is an unbiased estimate of the position $\mathbf{p}_k$. The EFIM in (2) can be derived based on the received waveform in (1) as a 2×2 matrix [5]

$$\mathbf{J}_{\text{e}}(\mathbf{p}_k; \{x_{kj}\}) = \sum_{j \in \mathcal{N}_{\text{b}}} \xi_{kj}\, x_{kj}\, \mathbf{J}_{\text{r}}(\phi_{kj}) \quad (3)$$

where $\mathbf{J}_{\text{r}}(\phi_{kj}) = \mathbf{u}(\phi_{kj})\mathbf{u}(\phi_{kj})^{\text{T}}$ is a 2×2 matrix, and ξ_{kj} is a positive coefficient determined by the channel properties,

[3]There are two common ways for inter-node distance estimation based on TOA: one-way TOF (only anchor transmits) or round-trip TOF (both anchor and agent transmit). The former requires anchors and agents to be synchronized for distance estimation.

given by,[4]

$$\xi_{kj} = \frac{8\pi^2 W^2}{c^2}(1-\chi_{kj})\frac{(\alpha_{kj}^{(1)})^2}{N_0} \tag{4}$$

with W as the effective bandwidth, c as the light speed, χ_{kj} as path-overlap coefficient characterizing the effect of multipath propagation for localization, N_0 as the noise spectrum density.[5]

Since the SPEB characterizes the fundamental limit of localization accuracy and is achievable in high SNR regimes, we will use it as a performance metric for location-aware networks, and allocate the transmit power to optimize the system performance by minimizing the SPEB.

C. Directional Decoupling of SPEB

We then introduce the notations of DPEB and mDPEB [6]. The EFIM (3) can be written, by eigen decomposition, as

$$\mathbf{J}_\text{e}(\mathbf{p}_k;\{x_{kj}\}) = \mathbf{U}_{\theta_k}\begin{bmatrix}\mu_{1,k} & 0\\ 0 & \mu_{2,k}\end{bmatrix}\mathbf{U}_{\theta_k}^\text{T}$$

where $\mu_{1,k}$ and $\mu_{2,k}$ are the ordered eigenvalues of EFIM ($\mu_{1,k} \geq \mu_{2,k}$), given by

$$\mu_{1,k}, \mu_{2,k} = \frac{1}{2}\bigg(\sum_{j\in\mathcal{N}_\text{b}} \xi_{kj}\, x_{kj} \pm \Big\|\sum_{j\in\mathcal{N}_\text{b}} \xi_{kj}\, x_{kj}\, \mathbf{u}(2\phi_{kj})\Big\|\bigg)$$

and $\mathbf{U}_{\theta_k}$ is a rotation matrix with angle θ_k, given by

$$\mathbf{U}_{\theta_k} = \begin{bmatrix}\cos\theta_k & -\sin\theta_k\\ \sin\theta_k & \cos\theta_k\end{bmatrix}.$$

Geometrically, the EFIM for agent k can be viewed as an information ellipse given by $\{\mathbf{z}\in\mathbb{R}^2 : \mathbf{z}^\text{T}\mathbf{J}_\text{e}^{-1}(\mathbf{p}_k;\{x_{kj}\})\mathbf{z} = 1\}$ (see Fig. 2), where $2\sqrt{\mu_{1,k}}$ and $2\sqrt{\mu_{2,k}}$ give the major axis and minor axis, respectively.

Definition 1: The directional position error bound (DPEB) of agent k along the direction φ is defined as

$$\mathcal{P}(\mathbf{p}_k;\varphi) \triangleq \mathbf{u}(\varphi)^\text{T}[\mathbf{J}_\text{e}^{-1}(\mathbf{p}_k,\{x_{kj}\})]\mathbf{u}(\varphi).$$

Proposition 1: The mDPEB of agent k is

$$\max_{\varphi\in[0,2\pi)}\{\mathcal{P}(\mathbf{p}_k;\varphi)\} = \frac{1}{\mu_{2,k}}. \tag{5}$$

Proof: See Appendix A. □

Proposition 1 can also be understood via the information ellipse of EFIM. The information for localization achieves the maximum along the major axis and the minimum along the minor axis. Due to the reciprocal, the SPEB is dominated by the mDPEB, which is the inverse of the smaller eigenvalue of the EFIM. Therefore, in order to improve the localization performance, it is more helpful to maximize the smaller eigenvalue of EFIM, equivalently to minimize the mDPEB that characterizes the maximum position error of an agent over all directions. We will use mDPEB as another performance metric of localization accuracy.

[4]The derivation of ξ_{kj} is given in [5], and this parameter can be obtained through channel estimation.

[5]Although the structure of SPEB is derived based on the received waveforms for wideband systems in [5], it is also observed in other TOA- or RSS-based localization systems, e.g., [16], [40]–[42].

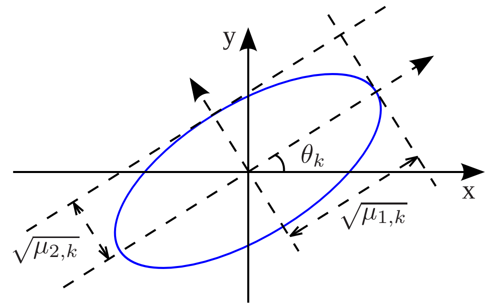

Fig. 2: Geometrical interpretation of the EFIM for agent k.

III. Optimal Power Allocation via Conic Programming

In this section, we formulate the power allocation problem using SPEB and mDPEB as the objective functions, respectively. We show that the SPEB minimization is a semidefinite program (SDP) and the mDPEB minimization is a second-order conic program (SOCP).

A. Problem Formulation Based on SPEB

We first consider the problem of optimal power allocation that minimizes the total SPEB while the network is subject to a budget of power consumption. The problem can be formulated as[6]

$$\mathscr{P}_1: \quad \min_{\{x_{kj}\}} \quad \sum_{k\in\mathcal{N}_\text{a}} \text{tr}\big\{\mathbf{J}_\text{e}^{-1}(\mathbf{p}_k;\{x_{kj}\})\big\} \tag{6}$$

$$\text{s.t.} \quad \sum_{k\in\mathcal{N}_\text{a}}\sum_{j\in\mathcal{N}_\text{b}} x_{kj} \leq P^\text{tot} \tag{7}$$

$$x_{kj}\geq 0, \quad \forall k\in\mathcal{N}_\text{a},\ \forall j\in\mathcal{N}_\text{b} \tag{8}$$

where (7) gives the total transmit power budget P^tot for all the anchors. We first show the convexity of the above problem in the following proposition.

Proposition 2: The problem $\mathscr{P}_1$ is convex in x_{kj}.

Proof: See Appendix B. □

Since $\mathscr{P}_1$ is a convex problem, the optimal solution can be achieved by the standard convex optimization algorithms, e.g., interior point method. We next show that such problem can be converted to a SDP problem, which is a more favorable formulation since many fast real-time optimization solvers are available for SDP [43], [44].

To obtain an equivalent formulation to $\mathscr{P}_1$, we replace the EFIMs in (6) with auxiliary matrices $\mathbf{M}_k$, and add another constraint

$$\mathbf{M}_k \succeq \mathbf{J}_\text{e}^{-1}(\mathbf{p}_k;\{x_{kj}\}).$$

Since $\mathbf{J}_\text{e}(\mathbf{p}_k)$ is a positive semidefinite matrix, due to the property of Schur complement, the above inequality is equivalent to

$$\begin{bmatrix}\mathbf{M}_k & \mathbf{I}\\ \mathbf{I} & \mathbf{J}_\text{e}(\mathbf{p}_k;\{x_{kj}\})\end{bmatrix} \succeq 0\,.$$

[6]The structure of the problem retains with additional linear constraints, such as the maximum transmit power from anchor j to agent k, and the maximum total transmit power from anchor j. See Remark 2 for more discussion.

Then, we can obtain a SDP formulation $\mathscr{P}_1^{\text{SDP}}$ equivalent to $\mathscr{P}_1$,

$$\begin{aligned}\mathscr{P}_1^{\text{SDP}}\text{:} \min_{\{x_{kj}\},\,\mathbf{M}_k} & \sum_{k\in\mathcal{N}_\text{a}} \text{tr}\{\mathbf{M}_k\} \\ \text{s.t.} \quad & \begin{bmatrix}\mathbf{M}_k & \mathbf{I} \\ \mathbf{I} & \mathbf{J}_\text{e}(\mathbf{p}_k;\{x_{kj}\})\end{bmatrix} \succeq 0\,,\ \forall k\in\mathcal{N}_\text{a} \\ & (7) - (8).\end{aligned}$$

Hence, the optimal solution of $\mathscr{P}_1$ can be efficiently obtained by solving the SDP formulation $\mathscr{P}_1^{\text{SDP}}$.

B. Problem Formulation Based on mDPEB

We now consider the minimization of total mDPEB as our objective. The problem can be formulated as

$$\begin{aligned}\mathscr{P}_2: \quad \min_{\{x_{kj}\}} \quad & \sum_{k\in\mathcal{N}_\text{a}} \frac{1}{\mu_{2,k}} \\ \text{s.t.} \quad & (7) - (8)\end{aligned}$$

which can be equivalently converted to

$$\begin{aligned}\mathscr{P}_2^{\text{SOCP}}\text{:} \min_{\{x_{kj},r_k\}} & \sum_{k\in\mathcal{N}_\text{a}} \frac{1}{\sum_{j\in\mathcal{N}_\text{b}}\xi_{kj}\,x_{kj} - r_k} \\ \text{s.t.} \quad & r_k \geq \Big\| \sum_{j\in\mathcal{N}_\text{b}} \xi_{kj}\,x_{kj}\,\mathbf{u}(2\phi_{kj})\Big\|,\ \forall k\in\mathcal{N}_\text{a} \quad (9) \\ & (7) - (8).\end{aligned}$$

The constraints (9) define N_a second-order cones given by

$$\mathcal{Q}_k = \{(r_k,\mathbf{z}_k)\in\mathbb{R}\times\mathbb{R}^2 : r_k \geq \|\mathbf{z}_k\|\}, \quad \forall k\in\mathcal{N}_\text{a}$$

where $\mathbf{z}_k = \sum_{j\in\mathcal{N}_\text{b}}\xi_{kj}\,x_{kj}\,\mathbf{u}(2\phi_{kj})$. Moreover, the objective is convex in $\{x_{kj},r_k\}$, since the denominator is a positive linear combination of $\{x_{kj},r_k\}$, and the reciprocal is a convex and decreasing function which preserves convexity [45]. Thus, we obtain a nonlinear SOCP problem which is convex in x_{kj}.

Remark 1: We consider a general model where each anchor can use different transmit power, and our work can be applied to the anchor broadcasting scenario by simply adding constraint $x_{kj} = x_j,\ \forall k\in\mathcal{N}_\text{a}$.

Remark 2: Additional linear constraints on transmit power can be imposed depending on the realistic requirements of location-aware networks. For example, we can consider $P_{kj}^{\min} \leq x_{kj} \leq P_{kj}^{\max}$ where $P_{kj}^{\min}$ and $P_{kj}^{\max}$ are the lower and upper limit of the transmit power from anchor j to agent k, respectively; or $\sum_{k\in\mathcal{N}_\text{a}} x_{kj} \leq P_j^{\text{tot}}$ where P_j^{tot} is the upper limit of the total transmit power from anchor j. Due to the linearity of these constraints, the convexity of the problem is retained, and the optimal solution can be obtained via conic programming.

Remark 3: For the asynchronous networks where round-trip TOF is employed for distance estimation, we need to allocate the transmit power for both anchors and agents. Let x'_{kj} denote the power of the transmit waveform from agent k to anchor j. In addition to the total anchor power constraint in (7), we also impose a total power constraint on agents, i.e.,

$$\sum_{k\in\mathcal{N}_\text{a}}\sum_{j\in\mathcal{N}_\text{b}} x'_{kj} \leq P'^{\text{tot}} \quad (10)$$

where

$$x'_{kj} \geq 0, \quad \forall k\in\mathcal{N}_\text{a},\ \forall j\in\mathcal{N}_\text{b}. \quad (11)$$

It can be shown that the EFIM of agent k is given by

$$\mathbf{J}_\text{e}(\mathbf{p}_k;\{x_{kj}\}) = \sum_{j\in\mathcal{N}_\text{b}} \xi_{kj}\, g(x_{kj},x'_{kj})\,\mathbf{J}_\text{r}(\phi_{kj})$$

where the equivalent power $g(x_{kj},x'_{kj}) = 4\big(x_{kj}^{-1} + x'^{-1}_{kj}\big)^{-1}$. To derive the maximum total equivalent power, we consider the following problem

$$\begin{aligned}\max_{\{x_{kj},x'_{kj}\}} \quad & \sum_{k\in\mathcal{N}_\text{a}}\sum_{j\in\mathcal{N}_\text{b}} g(x_{kj},x'_{kj}) \\ \text{s.t.} \quad & (7) - (8) \\ & (10) - (11).\end{aligned}$$

Using the Karush-Kuhn-Tucker conditions [46], it can be proved that the optimal value is reached as a constant $g(P^{\text{tot}},P'^{\text{tot}})$ if and only if

$$x'_{kj} = \frac{P'^{\text{tot}}}{P^{\text{tot}}}\,x_{kj}. \quad (12)$$

Hence, in order to achieve the maximum total equivalent power, the power allocated on anchors and agents should be proportional and consequently, the EFIM for asynchronous network is

$$\mathbf{J}_\text{e}(\mathbf{p}_k;\{x_{kj}\}) = \sum_{j\in\mathcal{N}_\text{b}} \xi_{kj}\,\frac{4P'^{\text{tot}}}{P'^{\text{tot}}+P^{\text{tot}}}\,x_{kj}\,\mathbf{J}_\text{r}(\phi_{kj})$$

which is with the same structure as the EFIM of synchronous network in (3). Therefore, the power allocation on both anchors and agents in asynchronous networks can be equivalently converted into anchor power allocation in synchronous networks.

C. Formulations with QoS Guarantee

We next briefly show that the proposed framework also applies to another two types of problem formulations based on different QoS requirements.

1) Energy-efficient Formulation: The objective is to minimize the total transmit power subject to the requirements for agents' SPEBs, i.e.,

$$\begin{aligned}\min_{\{x_{kj}\}} \quad & \sum_{k\in\mathcal{N}_\text{a}}\sum_{j\in\mathcal{N}_\text{b}} x_{kj} \\ \text{s.t.} \quad & \text{tr}\left\{\mathbf{J}_\text{e}^{-1}(\mathbf{p}_k;\{x_{kj}\})\right\} \leq \gamma_k, \quad \forall k\in\mathcal{N}_\text{a} \quad (13) \\ & (8).\end{aligned}$$

Similarly, a formulation for the mDPEB case can be obtained by replacing (13) with

$$\frac{1}{\mu_{2,k}} \leq \gamma_k\,, \quad \forall k\in\mathcal{N}_\text{a}. \quad (14)$$

2) Min-max SPEB Formulation: The objective is to minimize the maximum SPEB among all the agents, i.e.,

$$\begin{aligned} \min_{\{x_{kj}\}} \quad & \max_{k} \left\{ \text{tr}\left\{ \mathbf{J}_\text{e}^{-1}(\mathbf{p}_k;\{x_{kj}\}) \right\} \right\} \\ \text{s.t.} \quad & (7) - (8). \end{aligned}$$

It can be equivalently transformed into

$$\begin{aligned} \min_{\{x_{kj}\},\,\gamma} \quad & \gamma \\ \text{s.t.} \quad & \text{tr}\left\{ \mathbf{J}_\text{e}^{-1}(\mathbf{p}_k;\{x_{kj}\}) \right\} \le \gamma\,, \quad \forall k \in \mathcal{N}_\text{a} \\ & (7) - (8) \end{aligned}$$

which turns out to be with the same structure as the energy-efficient formulation. Similarly, a min-max formulation for the mDPEB case can be obtained by replacing the SPEB with the mDPEB in the constraint.

Note that since the above formulations with QoS guarantee have the same structure as $\mathscr{P}_1$ or $\mathscr{P}_2$, which can be solved efficiently by conic programing, we will focus on $\mathscr{P}_1$ and $\mathscr{P}_2$ in the following.

To obtain the optimal solutions of $\mathscr{P}_1$ and $\mathscr{P}_2$, it requires the network parameters, i.e., the channel parameter ξ_{kj} and the angle ϕ_{kj}. However, ξ_{kj}'s and ϕ_{kj}'s are usually not perfectly known in realistic networks, and only estimated values are available. When ξ_{kj}'s and ϕ_{kj}'s are subject to uncertainty, the formulation $\mathscr{P}_1$ or $\mathscr{P}_2$ may fail to provide reliable solutions, since the actual SPEB/mDEPB is not necessarily minimized. Therefore, it is essential to design a power allocation scheme which is robust to the uncertainty in network parameters.

IV. Robust Power Allocation under Imperfect Knowledge of Network Parameters

In this section, we consider the location-aware networks with imperfect knowledge of network parameters, and propose robust optimization methods to minimize the worst-case SPEB/mDPEB under parameter uncertainty.

A. Robust Counterpart of SPEB Minimization

In realistic location-aware networks, the network parameters, i.e., ξ_{kj} and ϕ_{kj}, can be obtained through channel estimation or inferred based on the prior information of agents' positions,[7] and hence are both subject to uncertainty. We adopt robust optimization methodology, which is developed in recent years to handle the optimization problems with data uncertainty [36]. Typically, the data defining the optimization problem is assumed to lie in a certain bounded set, referred to as *uncertainty set*. Here we consider the actual channel parameters and angles lie in linear uncertainty sets, i.e.,[8]

$$\begin{aligned} \xi_{kj} &\in \mathcal{S}_{kj}^{\xi} \triangleq [\,\hat{\xi}_{kj} - \varepsilon_{kj}^{\xi}\,,\, \hat{\xi}_{kj} + \varepsilon_{kj}^{\xi}\,] \\ \phi_{kj} &\in \mathcal{S}_{kj}^{\phi} \triangleq [\,\hat{\phi}_{kj} - \varepsilon_{kj}^{\phi}\,,\, \hat{\phi}_{kj} + \varepsilon_{kj}^{\phi}\,] \end{aligned}$$

where $\hat{\xi}_{kj}$ and $\hat{\phi}_{kj}$ denote channel parameter and angle with uncertainty, respectively, and ε_{kj}^{ξ} and ε_{kj}^{ϕ} are both small positive numbers denoting the maximum uncertainty in the channel parameter and angle, respectively.[9]

To deal with the network parameter uncertainty, we adopt robust optimization techniques to guarantee the worst-case performance. Instead of using the estimated values, we consider minimizing the largest SPEB over the possible set of actual network parameters, i.e.,

$$\begin{aligned} \mathscr{P}_{\text{R-0}}: \quad \min_{\{x_{kj}\}} \quad & \max_{\{\xi_{kj}\in\mathcal{S}_{kj}^{\xi},\,\phi_{kj}\in\mathcal{S}_{kj}^{\phi}\}} \sum_{k\in\mathcal{N}_\text{a}} \text{tr}\left\{ \mathbf{J}_\text{e}^{-1}(\mathbf{p}_k;\{x_{kj}\}) \right\} \\ \text{s.t.} \quad & (7) - (8). \end{aligned}$$

Since $\text{tr}\left\{ \mathbf{J}_\text{e}^{-1}(\mathbf{p}_k;\{x_{kj}\}) \right\}$ is a monotonically non-increasing function of ξ_{kj}, the maximum SPEB over ξ_{kj} is independent of ϕ_{kj}. Hence, the maximization over ξ_{kj} simply follows that

$$\widetilde{\xi}_{kj} \triangleq \arg \max_{\{\xi_{kj}\in\mathcal{S}_{kj}^{\xi}\}} \text{tr}\left\{ \mathbf{J}_\text{e}^{-1}(\mathbf{p}_k;\{x_{kj}\}) \right\} = \hat{\xi}_{kj} - \varepsilon_{kj}^{\xi}.$$

On the other hand, however, the maximization over ϕ_{kj} is not trivial, because

$$\begin{aligned} \{\widetilde{\phi}_{kj}\} &\triangleq \arg \max_{\{\phi_{kj}\in\mathcal{S}_{kj}^{\phi}\}} \text{tr}\left\{ \mathbf{J}_\text{e}^{-1}(\mathbf{p}_k;\{x_{kj}\}) \right\} \\ &= \arg \max_{\{\phi_{kj}\in\mathcal{S}_{kj}^{\phi}\}} \Big\| \sum_{j\in\mathcal{N}_\text{b}} \xi_{kj}\, x_{kj}\, \mathbf{u}(2\phi_{kj}) \Big\|^2 \end{aligned} \tag{15}$$

and the right-hand side of (15) is not a convex problem. Hence, it is difficult to obtain a close-form solution of $\{\widetilde{\phi}_{kj}\}$ since it depends on $\{x_{kj}\}$.

We next consider a relaxation for the robust optimization with respect to $\{\phi_{kj}\}$ and introduce a new matrix

$$\mathbf{Q}_\text{r}(\hat{\phi}_{kj}, \delta_{kj}) = \mathbf{J}_\text{r}(\hat{\phi}_{kj}) - \delta_{kj} \cdot \mathbf{I} \tag{16}$$

to replace $\mathbf{J}_\text{r}(\phi_{kj})$ in the SPEB in (2). We will show that the worst-case SPEB over ϕ_{kj} can be bounded above by the new function for sufficiently large δ_{kj}. The details are given in the following proposition.

Proposition 3: If $\sum_{j\in\mathcal{N}_\text{b}} \xi_{kj}\, x_{kj}\, \mathbf{Q}_\text{r}(\hat{\phi}_{kj}, \delta_{kj}) \succeq 0$ and $\delta_{kj} \ge \sin \varepsilon_{kj}^{\phi}$, the maximum SPEB over the actual angle ϕ_{kj} is always upper bounded as

$$\begin{aligned} \max_{\{\phi_{kj}\in\mathcal{S}_{kj}^{\phi}\}} & \text{tr}\big\{ \mathbf{J}_\text{e}^{-1}(\mathbf{p}_k;\{x_{kj}\}) \big\} \\ & \le \ \text{tr}\bigg\{ \Big(\sum_{j\in\mathcal{N}_\text{b}} \xi_{kj}\, x_{kj}\, \mathbf{Q}_\text{r}(\hat{\phi}_{kj}, \delta_{kj}) \Big)^{-1} \bigg\}. \end{aligned} \tag{17}$$

Moreover, the tightest upper bound in (17) is attained by

$$\sin \varepsilon_{kj}^{\phi} = \arg \min_{\delta_{kj}} \text{tr}\bigg\{ \Big(\sum_{j\in\mathcal{N}_\text{b}} \xi_{kj}\, x_{kj}\, \mathbf{Q}_\text{r}(\hat{\phi}_{kj}, \delta_{kj}) \Big)^{-1} \bigg\}.$$

Proof: See Appendix C. □

In the rest of the paper, we take the minimizer $\delta_{kj} = \sin \varepsilon_{kj}^{\phi}$ and denote the matrix

$$\mathbf{Q}_\text{r}(\hat{\phi}_{kj}) = \mathbf{J}_\text{r}(\hat{\phi}_{kj}) - \sin \varepsilon_{kj}^{\phi} \cdot \mathbf{I}$$

[7]The prior position information is available in applications such as navigation.

[8]We consider the parameter ξ_{kj} related to the channel properties to be always positive, i.e., $\hat{\xi}_{kj} - \varepsilon_{kj}^{\xi} > 0$.

[9]If uncertainty exists in anchor positions, it can be equivalently converted into the uncertainty in channel qualities [6].

by omitting the variable δ_{kj} in (16) for simplicity. Then, we replace the matrix $\mathbf{J}_{\text{r}}(\phi_{kj})$ with $\mathbf{Q}_{\text{r}}(\hat{\phi}_{kj})$ in the previous formulation, and propose a robust counterpart of $\mathscr{P}_1$ given by

$$\begin{aligned}\mathscr{P}_{\text{R-1}}:\quad &\min_{\{x_{kj}\}} \sum_{k\in\mathcal{N}_{\text{a}}} \text{tr}\Big\{\Big(\sum_{j\in\mathcal{N}_{\text{b}}} \widetilde{\xi}_{kj}\, x_{kj}\, \mathbf{Q}_{\text{r}}(\hat{\phi}_{kj})\Big)^{-1}\Big\}\\ &\text{s.t.} \sum_{j\in\mathcal{N}_{\text{b}}} \widetilde{\xi}_{kj}\, x_{kj}\, \mathbf{Q}_{\text{r}}(\hat{\phi}_{kj}) \succeq 0,\ \forall\, k\in\mathcal{N}_{\text{a}} \qquad (18)\\ &\quad (7) – (8).\end{aligned}$$

Again by the property of Schur complement as in $\mathscr{P}_1^{\text{SDP}}$, the problem $\mathscr{P}_{\text{R-1}}$ is equivalent to a SDP formulation, given by

$$\begin{aligned}\mathscr{P}_{\text{R-1}}^{\text{SDP}}:\ &\min_{\{x_{kj}\},\,\mathbf{M}_k} \sum_{k\in\mathcal{N}_{\text{a}}} \text{tr}\{\mathbf{M}_k\}\\ &\text{s.t.} \begin{bmatrix}\mathbf{M}_k & \mathbf{I}\\ \mathbf{I} & \sum_{j\in\mathcal{N}_{\text{b}}} \widetilde{\xi}_{kj}\, x_{kj}\, \mathbf{Q}_{\text{r}}(\hat{\phi}_{kj})\end{bmatrix} \succeq 0,\ \forall k\in\mathcal{N}_{\text{a}} \qquad (19)\\ &\quad (7) – (8).\end{aligned}$$

Remark 4: The formulation with QoS guarantee proposed in Section III-C can also be extended to its robust formulation using the above method. By such, the SPEB of each agent is always guaranteed to satisfy its position error requirement. However, if using the non-robust formulation, the requirements for agents' SPEBs, e.g., (13) or (14), can easily be violated due to imperfect knowledge of network parameters.

Note that from Proposition 3, the new formulation $\mathscr{P}_{\text{R-1}}$ is a valid relaxation for $\mathscr{P}_{\text{R-0}}$ when the condition (18) holds. Since $\mathbf{Q}_{\text{r}}(\hat{\phi}_{kj})$ is not positive definite due to $\det\big(\mathbf{Q}_{\text{r}}(\hat{\phi}_{kj})\big) = \sin\varepsilon_{kj}^{\phi}(\sin\varepsilon_{kj}^{\phi}-1) \leq 0$, such a condition does not necessarily hold for all power allocation $\{x_{kj}\}$. However, we will show that it holds for the optimal power allocation of $\mathscr{P}_{\text{R-0}}$ with high probability (w.h.p.) when the number of anchors is large or the uncertainty in angle is small.

Before giving the proposition, we introduce an equivalent expression for the channel parameter ξ_{kj} in (4) as $\xi_{kj} = \zeta_{kj}/d_{kj}^{2\beta}$, where ζ_{kj} is a positive coefficient characterizing shadowing effect and small-scale fading process, and β is the amplitude loss exponent.[10]

Proposition 4: Consider a network where all the nodes are uniformly located in a $R \times R$ square region, the minimum distance between two nodes is r_0, and the coefficient ζ_{kj} has a support on $[\zeta_{\min}\ \zeta_{\max}]$ where $0 < \zeta_{\min} \leq \zeta_{\max}$. Let $\{x_{kj}^*\}$ be the optimal solution of $\mathscr{P}_{\text{R-0}}$, and $\delta = \sin\varepsilon^{\phi}$ where $\varepsilon^{\phi} = \max\{\varepsilon_{kj}^{\phi}\}$, then

(a) when $N_{\text{b}} \to \infty$ and $\delta \leq \delta_{\max}$, where $\delta_{\max}$ is the smallest positive root of equation $4\delta^4 - 4\delta^2 - 2\zeta_{\max}/\zeta_{\min}\delta + 1 = 0$, we have

$$\Pr\Big\{\sum_{j\in\mathcal{N}_{\text{b}}} \widetilde{\xi}_{kj}\, x_{kj}^*\, \mathbf{Q}_{\text{r}}(\hat{\phi}_{kj}) \succeq 0\Big\} = 1 - \mathcal{O}\big(\exp(-\eta\cdot N_{\text{b}})\big),\quad \forall k\in\mathcal{N}_{\text{a}}$$

where η is a fixed positive number;

(b) when $\varepsilon^{\phi} \to 0$, we have

$$\Pr\Big\{\sum_{j\in\mathcal{N}_{\text{b}}} \widetilde{\xi}_{kj}\, x_{kj}^*\, \mathbf{Q}_{\text{r}}(\hat{\phi}_{kj}) \succeq 0\Big\} = 1 - \mathcal{O}\big((\varepsilon^{\phi})^{N_{\text{b}}/2}\big),\quad \forall k\in\mathcal{N}_{\text{a}}.$$

Proof: See Appendix D. □

Remark 5: Proposition 4 implies that the condition (18) holds w.h.p. at the rate indicated by the $\mathcal{O}$ notation, where $\mathcal{O}(f(n))$ means that the function value is on the order of $f(n)$ [47].

Remark 6: Note that Proposition 4 holds for $\{x_{kj}^*\}$, which implies that the optimal solution of the original robust formulation $\mathscr{P}_{\text{R-0}}$ is included in the feasible set of the proposed formulation $\mathscr{P}_{\text{R-1}}$ (or $\mathscr{P}_{\text{R-1}}^{\text{SDP}}$) w.h.p.

B. Robust Counterpart of mDPEB Minimization

We investigate the robust power allocation based on mDPEB formulation $\mathscr{P}_2$. To circumvent the intractable maximization in (15), we consider the robust SPEB formulation $\mathscr{P}_{\text{R-1}}$. Specifically, the objective of $\mathscr{P}_{\text{R-1}}$ can be written as

$$\text{tr}\Big\{\Big(\sum_{j\in\mathcal{N}_{\text{b}}} \widetilde{\xi}_{kj}\, x_{kj}\, \mathbf{Q}_{\text{r}}(\hat{\phi}_{kj})\Big)^{-1}\Big\} = \frac{1}{\widetilde{\mu}_{1,k}} + \frac{1}{\widetilde{\mu}_{2,k}} \qquad (20)$$

where $\widetilde{\mu}_{1,k}$ and $\widetilde{\mu}_{2,k}$ are the two eigenvalues of the matrix $\sum_{j\in\mathcal{N}_{\text{b}}} \widetilde{\xi}_{kj}\, x_{kj}\, \mathbf{Q}_{\text{r}}(\hat{\phi}_{kj})$, given by

$$\begin{aligned}\widetilde{\mu}_{1,k},\ \widetilde{\mu}_{2,k} = \frac{1}{2}\Big(&\sum_{j\in\mathcal{N}_{\text{b}}} \widetilde{\xi}_{kj}\, x_{kj}(1 - 2\sin\varepsilon_{kj}^{\phi})\\ &\pm \Big\|\sum_{j\in\mathcal{N}_{\text{b}}} \widetilde{\xi}_{kj}\, x_{kj}\, \mathbf{u}(2\hat{\phi}_{kj})\Big\|\Big). \qquad (21)\end{aligned}$$

Geometrically, $\widetilde{\mu}_{1,k}$ and $\widetilde{\mu}_{2,k}$ are similar to the DPEB's in two orthogonal directions. Using Proposition 4, we can show that $\widetilde{\mu}_{2,k} \geq 0$ w.h.p. when N_{b} is large or ε^{ϕ} is small. Since $\widetilde{\mu}_{1,k} \geq \widetilde{\mu}_{2,k}$, the smaller eigenvalue $\widetilde{\mu}_{2,k}$ dominates the function in (20). Hence, we formulate a robust counterpart of $\mathscr{P}_2$ based on $\widetilde{\mu}_{2,k}$, given by

$$\begin{aligned}\mathscr{P}_{\text{R-2}}:\quad &\min_{\{x_{kj}\}} \sum_{k\in\mathcal{N}_{\text{a}}} \frac{1}{\widetilde{\mu}_{2,k}}\\ &\text{s.t.}\ \widetilde{\mu}_{2,k} \geq 0,\quad \forall k\in\mathcal{N}_{\text{a}} \qquad (22)\\ &\quad (7) – (8).\end{aligned}$$

Given that $\widetilde{\mu}_{2,k} \geq 0$, the problem $\mathscr{P}_{\text{R-2}}$ is equivalent to the following SOCP problem:

$$\begin{aligned}\mathscr{P}_{\text{R-2}}^{\text{SOCP}}:\ &\min_{\{x_{kj},r_k\}} \sum_{k\in\mathcal{N}_{\text{a}}} \frac{1}{\sum_{j\in\mathcal{N}_{\text{b}}} \widetilde{\xi}_{kj}\, x_{kj}\big(1 - 2\sin\varepsilon_{kj}^{\phi}\big) - r_k} \qquad (23)\\ &\text{s.t.}\ r_k \geq \Big\|\sum_{j\in\mathcal{N}_{\text{b}}} \widetilde{\xi}_{kj}\, x_{kj}\, \mathbf{u}(2\hat{\phi}_{kj})\Big\|,\quad \forall k\in\mathcal{N}_{\text{a}} \qquad (24)\\ &\quad r_k \leq \sum_{j\in\mathcal{N}_{\text{b}}} \widetilde{\xi}_{kj}\, x_{kj}\big(1-2\sin\varepsilon_{kj}^{\phi}\big),\ \forall k\in\mathcal{N}_{\text{a}}\\ &\quad (7) – (8).\end{aligned}$$

[10]We introduce the path loss model here to facilitate the proof of the Proposition 4. However, the robust power allocation schemes do not require β, since the channel parameter ξ_{kj} can be obtained directly through channel estimation.

Note that the uncertainty in angle ε_{kj}^{ϕ} only exists in the objective, and does not affect the second-order conic constraint (24). Hence, the problem $\mathscr{P}_{\text{R-2}}^{\text{SOCP}}$ retains the same structure of $\mathscr{P}_{2}^{\text{SOCP}}$, and its optimal solution can be efficiently obtained.

V. Efficient Robust Algorithm Using Distributed Computations

In this section, we designed a distributed robust algorithm for both SPEB and mDPEB minimization, which decomposes the original formulation into two-stage optimization problems and enables parallel computations among all the agents. The proposed algorithms achieve the global optimal solution with improved computational efficiency.

A. Algorithm for SPEB Minimization

Despite the convexity of the robust SDP formulation $\mathscr{P}_{\text{R-1}}^{\text{SDP}}$, there are multiple positive semidefinite constraints imposed for multiple agents, and the computational complexity depends on the number of SDP constraints. To efficiently obtain the power allocation decision for multi-agent networks, we design a distributed implementation for $\mathscr{P}_{\text{R-1}}^{\text{SDP}}$, which can be solved using parallel computations among the agents.

Specifically, we let $x_{kj} = \rho_{kj} x_k$ where x_k is the total power assigned for locating agent k, and $\rho_{kj} \in [0,1]$ is a fractional number denoting the percentage of x_k allocated to anchor j. By introducing the two variables ρ_{kj} and x_k, the robust formulation for power allocation can be written as

$$\min_{\{\rho_{kj}, x_k\}} \quad \sum_{k\in\mathcal{N}_\text{a}} \frac{1}{x_k}\text{tr}\Big\{\Big(\sum_{j\in\mathcal{N}_\text{b}} \widetilde{\xi}_{kj}\, \rho_{kj}\, \mathbf{Q}_\text{r}(\hat{\phi}_{kj})\Big)^{-1}\Big\}$$

$$\text{s.t.} \quad \sum_{j\in\mathcal{N}_\text{b}} \rho_{kj} \le 1 \tag{25}$$

$$\rho_{kj} \ge 0, \quad \forall k \in \mathcal{N}_\text{a},\ \forall j \in \mathcal{N}_\text{b} \tag{26}$$

$$\sum_{k\in\mathcal{N}_\text{a}} x_k \le P^{\text{tot}} \tag{27}$$

$$x_k \ge 0, \quad \forall k \in \mathcal{N}_\text{a}. \tag{28}$$

Since the constraints on ρ_{kj} and x_k are separable, and x_k and ρ_{kj} are only related to the SPEB of agent k, we can decompose the above problem into two stages. In Stage I, given the total power budget x_k for agent k, we consider the optimal allocation of x_k among all the anchors, i.e.,

$$\mathscr{P}_{\text{R-1},k}^{(\text{I})}: \quad \min_{\{\rho_{kj}\},\, \mathbf{M}_k} \ \text{tr}\{\mathbf{M}_k\}/x_k$$
$$\text{s.t.} \quad \begin{bmatrix} \mathbf{M}_k & \mathbf{I} \\ \mathbf{I} & \sum_{j\in\mathcal{N}_\text{b}} \widetilde{\xi}_{kj}\, \rho_{kj}\, \mathbf{Q}_\text{r}(\hat{\phi}_{kj}) \end{bmatrix} \succeq 0$$
$$(25) - (26).$$

The optimal solution of $\mathscr{P}_{\text{R-1},k}^{(\text{I})}$ is denoted by ρ_{kj}^*, and it is independent of the total power for agent k since x_k only appears as a scaler in the objective and can be removed. Since the problem $\mathscr{P}_{\text{R-1},k}^{(\text{I})}$ is formulated for agent k, there are totally N_a problems to be solved in Stage I.

In Stage II, we allocate the total x_k for localizing agent k. The objective is the total SPEB of the agents, where the parameter ρ_{kj}^*'s are from Stage I $\mathscr{P}_{\text{R-1},k}^{(\text{I})}$. In particular, we let $T_k = \text{tr}\big\{\big(\sum_{j\in\mathcal{N}_\text{b}} \widetilde{\xi}_{kj}\, \rho_{kj}^*\, \mathbf{Q}_\text{r}(\hat{\phi}_{kj})\big)^{-1}\big\}$ and formulate the problem as:

$$\mathscr{P}_{\text{R-1}}^{(\text{II})}: \quad \min_{\{x_k\}} \ \sum_{k\in\mathcal{N}_\text{a}} \frac{T_k}{x_k}$$
$$\text{s.t.} \quad (27) - (28).$$

The problem $\mathscr{P}_{\text{R-1}}^{(\text{II})}$ is convex in x_k, and the optimal solution is given in a closed form as follows.

Proposition 5: Given that ρ_{kj}^* is the optimal solution of $\mathscr{P}_{\text{R-1},k}^{(\text{I})}$, the optimal solution of $\mathscr{P}_{\text{R-1}}^{(\text{II})}$ is given by

$$x_k^* = \frac{P^{\text{tot}}\sqrt{T_k}}{\sum_{k\in\mathcal{N}_\text{a}}\sqrt{T_k}}\,. \tag{29}$$

Proof: See Appendix E. □

The optimal power allocation for the location-aware network is

$$x_{kj}^* = \rho_{kj}^* x_k^* \tag{30}$$

where x_k^* is given in (29). The detailed algorithm is described in the Algorithm 1.

Algorithm 1 Robust power allocation algorithm for multiple-agent networks

Require: the angle $\hat{\phi}_{kj}$ and the distance $\hat{d}_{kj}$ between anchor j ($j \in \mathcal{N}_\text{b}$) and agent k ($k \in \mathcal{N}_\text{a}$)

1: Set $x_k \leftarrow 1$, $\forall k \in \mathcal{N}_\text{a}$
2: Solve the Stage I problems $\mathscr{P}_{\text{R-1},k}^{(\text{I})}$ which gives the optimal solution ρ_{kj}^*
3: Set $\rho_{kj} \leftarrow \rho_{kj}^*$, $\forall k \in \mathcal{N}_\text{a}$, $\forall j \in \mathcal{N}_\text{b}$
4: Solve the Stage II problem $\mathscr{P}_{\text{R-1}}^{(\text{II})}$ by using (29) to compute the optimal solution x_k^*
5: Set $x_{kj}^* \leftarrow \rho_{kj}^* x_k^*$, $\forall k \in \mathcal{N}_\text{a}$, $\forall j \in \mathcal{N}_\text{b}$

Remark 7: Since each Stage I problem $\mathscr{P}_{\text{R-1},k}^{(\text{I})}$ in Algorithm 1 is with a single SDP constraint, its complexity is much lower than the original problem $\mathscr{P}_{\text{R-1}}^{\text{SDP}}$ which contains N_a SDP constraints. Moreover, the N_a Stage I problems $\mathscr{P}_{\text{R-1},k}^{(\text{I})}$ can be separately solved by the N_a agents, since each agent itself does not require any information from other agents. Thus, the computation efficiency can be improved by N_a times using the parallel computations among the agents.

Remark 8: The proposed distributed algorithm can also be applied to the robust power allocation with individual power constraint, e.g., $\sum_{k\in\mathcal{N}_\text{a}} x_{kj} \le P_j^{\text{tot}}$. In particular, we replace such constraint with $\sum_{k\in\mathcal{N}_\text{a}} \rho_{kj} x_k \le P_j^{\text{tot}}$ in the Stage II formulation $\mathscr{P}_{\text{R-1}}^{(\text{II})}$, while the Stage I formulation $\mathscr{P}_{\text{R-1},k}^{(\text{I})}$ remains the same. In such case, the close-form solution in (30) is not available, however, the optimal solution of the Stage II problem can still be efficiently obtained since the problem is convex. Consequently, we can obtain a sub-optimal solution for the overall problem.

B. Algorithm for mDPEB Minimization

A similar decomposition method can be applied to the mDPEB minimization $\mathscr{P}_{\text{R-2}}$, i.e., by introducing two variables

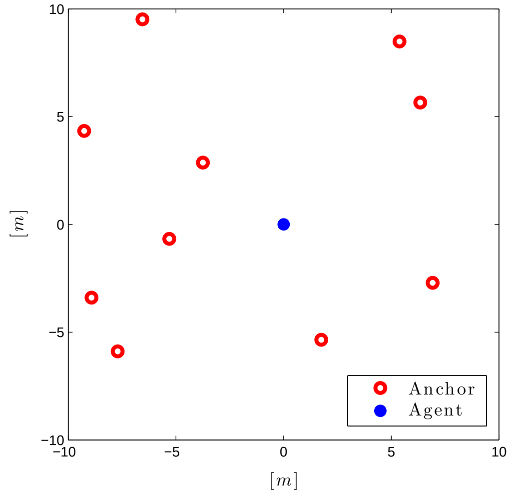

Fig. 3: The location-aware network consisting ten anchors (red circle) and one agents (blue dot), where the anchors are uniformly distributed in the square region.

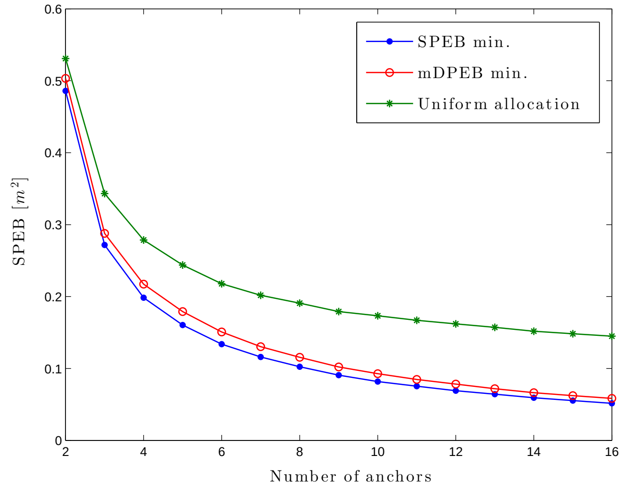

Fig. 4: The SPEB in single-agent networks with respect to the number of anchors, obtained by different power allocation schemes.

ρ_{kj} and x_k. Instead of solving SDP in SPEB minimization, each agent will separately solve a SOCP problem with linear objective for the mDPEB minimization. Specifically, we rewrite (21) as

$$\widetilde{\mu}_{2,k} = \frac{x_k}{2}\Bigg(\sum_{j\in\mathcal{N}_\text{b}} \widetilde{\xi}_{kj}\,\rho_{kj}\left(1-2\sin\varepsilon_{kj}^{\phi}\right) - \Bigg\|\sum_{j\in\mathcal{N}_\text{b}} \widetilde{\xi}_{kj}\,\rho_{kj}\,\mathbf{u}(2\hat{\phi}_{kj})\Bigg\|\Bigg).$$

Then, the two-stage formulations are given by

$$\mathscr{P}_{\text{R-2}}^{(\text{I})}: \quad \max_{\{\rho_{kj}\}} \quad \widetilde{\mu}_{2,k}/x_k \qquad \text{s.t.} \quad \widetilde{\mu}_{2,k}\ge 0, \quad (25)-(26)$$

and

$$\mathscr{P}_{\text{R-2}}^{(\text{II})}: \quad \min_{\{x_k\}} \quad \sum_{k\in\mathcal{N}_\text{a}} \frac{1}{\widetilde{\mu}_{2,k}} \qquad \text{s.t.} \quad (27)-(28)$$

respectively. The optimal power allocation is the product of the optimal solutions of the two-stage problems, given by (30). The algorithm for mDPEB minimization is similar to that of Algorithm 1, and hence, we omit the details here.

VI. Simulation Results

In this section, we investigate the localization performance by the proposed power allocation schemes. The total power for localization is normalized to $P^{\text{tot}} = 1$, and the channel parameter is given by $\xi_{kj} = 10^3/d_{kj}^2$.[11] The proposed optimization of power allocation, i.e., SDP and SOCP, are solved by the standard optimization solver CVX [49].

[11] We choose the free-space propagation model where the path loss exponent is 2 [48].

A. Power Allocation with Perfect Network Parameters

First, we investigate the SPEB with power allocation as the number of anchors or agents changes. Three schemes of power allocation are compared: the allocation via SPEB minimization formulated in $\mathscr{P}_1^{\text{SDP}}$, the allocation via mDPEB minimization formulated in $\mathscr{P}_2^{\text{SOCP}}$, and the uniform allocation which assigns P^{tot} equally over all the anchors. Given the number of anchors and agents, we run Monte Carlo simulation to generate 10^3 deployments of agents or anchors that are uniformly distributed in a squared region, i.e., $U([-10,10]\times[-10,10])$, and then compute the average SPEB obtained by each scheme.

In Figs. 3 and 4, we consider the network with a single agent at the center and anchors uniformly distributed. An example of the network topology is illustrated in Fig. 3. We plot the SPEBs obtained by the above-mentioned three schemes in Fig. 4. A decreasing tendency in SPEB is observed as the number of anchors increases. This is reasonable since the agent has more freedom to choose "good" anchors when there are more anchors. Moreover, the results show that the mDPEB minimization outperforms the uniform allocation by about 46%, and achieves a SPEB close to the one obtained by SPEB minimization.

Next, we consider a network with multiple agents. Ten anchors are placed with fixed locations, and the agents are uniformly distributed in the region (see Fig. 5). Similarly, we compare the SPEB obtained by the three schemes with respect to the number of agents in Fig. 6. It shows that, even in multiple-agent case, the mDPEB minimization still achieves a similar performance as the SPEB minimization, and remarkably outperforms the uniform allocation. It implies that mDPEB is a meaningful performance metric for the optimization of power allocation. In addition, we observe that the average SPEB increases linearly with the number of agents. This is because each agent tends to obtain less power when the total power budget is fixed. As indicated by the slope, the speed of SPEB increase of optimized allocation is about 60%

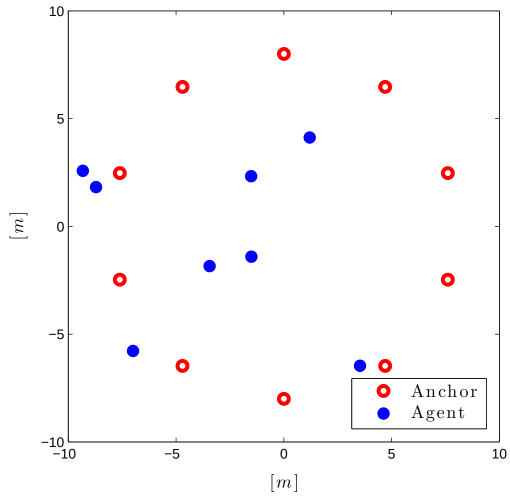

Fig. 5: The location-aware network consisting ten anchors (red circle) and eight agents (blue dot), where the agents are uniformly distributed in the square region.

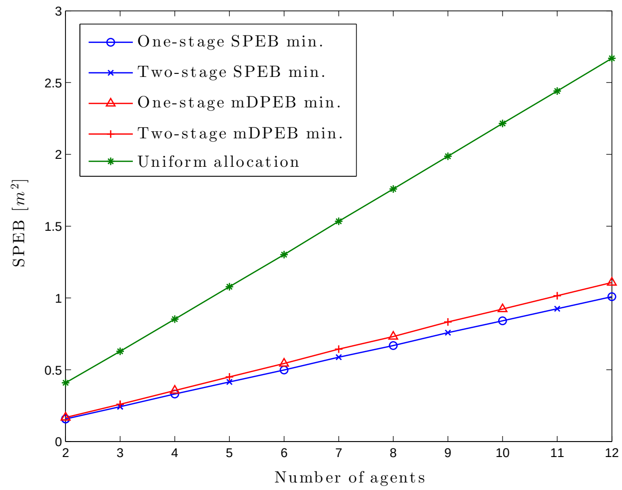

Fig. 6: The average SPEB in multiple-agent networks ($N_b = 10$) by different power allocation schemes. Both one-stage and two-stage optimization are considered.

slower than that of uniform allocation.

Furthermore, we investigate the performance of the two-stage optimization proposed in Section V which exploits the distributed computations among multiple agents. In Fig. 6, we plot the SPEB obtained by the two-stage optimization for both SPEB and mDPEB minimization. The results show that the SPEB solved by two-stage optimization perfectly matches that of one-stage optimization, which validates that the two-stage scheme can obtain the optimal solution while requiring much less computational time.

B. Robust Power Allocation with Imperfect Knowledge of Network Parameters

We then investigate the performance of the power allocation with imperfect knowledge of network parameters. We compared the following schemes: allocation by the robust formulation $\mathscr{P}_{\text{R-1}}^{\text{SDP}}$ and $\mathscr{P}_{\text{R-2}}^{\text{SOCP}}$, allocation by the non-robust formulation $\mathscr{P}_{1}^{\text{SDP}}$ and $\mathscr{P}_{2}^{\text{SOCP}}$, and uniform allocation. We consider the agent's actual position lies within a circle of radius ε^d centering at its estimated position. Then the maximum angular uncertainty is determined by $\varepsilon_{kj}^{\phi} = \arcsin(\varepsilon^d/\hat{d}_{kj})$.[12] The *normalized uncertainty set size* on network parameters is defined to be $\varepsilon = 2\varepsilon^d/20$ which is normalized by the length of the squared region.

In Fig. 7, we investigate the actual SPEB with respect to the number of anchors. We consider a single-agent network, and set the normalized uncertainty set size ε to be 0.2, i.e., $\varepsilon^d = 2$ m. The results show that the robust SPEB minimization ($\mathscr{P}_{\text{R-1}}^{\text{SDP}}$) outperforms the non-robust SPEB minimization ($\mathscr{P}_{1}^{\text{SDP}}$) by 20%, and outperforms uniform allocation by 35%; the robust mDPEB minimization ($\mathscr{P}_{\text{R-2}}^{\text{SOCP}}$) outperforms the non-robust mDPEB minimization ($\mathscr{P}_{2}^{\text{SOCP}}$) by 30%, and outperforms uniform allocation by 70%. Moreover, we observe that the actual SPEB of robust mDPEB minimization is smaller than that of robust SPEB minimization, and the same observation is on the non-robust schemes. It implies that the mDPEB minimization is more robust to the network parameter uncertainty, compared with the SPEB minimization. This can be explained as follows: the robust mDPEB minimization can be viewed as a doubly robust optimization, since it first minimizes the maximum positional error over all the directions. Therefore, $\mathscr{P}_{\text{R-2}}^{\text{SOCP}}$ outperforms $\mathscr{P}_{\text{R-1}}^{\text{SDP}}$ when the uncertainty in network parameters is not negligible (e.g., $\varepsilon = 0.2$).

[12]Without loss of generality, we set $\varepsilon_{kj}^{d} = \varepsilon^d$ for all k, j.

In Fig. 8, we investigate the actual SPEB with respect to the normalized uncertainty set size ε. We consider a single-agent network with ten anchors deployed on a circle (similar to Fig. 5). As we observe, the actual SPEB of non-robust schemes quickly increases as the normalized uncertainty set size goes large. When the normalized uncertainty set size is larger than 0.22 and 0.27, respectively, the non-robust SPEB minimization and non-robust mDPEB minimization even perform worse than the uniform allocation, while the robust schemes always achieves better SPEB than all the other schemes. Moreover, the robust mDPEB minimization outperforms the non-robust mDPEB minimization and robust SPEB minimization by 30% and 23%, respectively, when $\varepsilon = 0.15$. Both Figs. 7 and 8 have demonstrated the advantage of the proposed robust power allocation schemes, especially the mDPEB minimization, in the practical location-aware networks with imperfect knowledge of network parameters.

VII. Conclusion

In this paper, we presented an optimization framework for robust power allocation in network localization based on the performance metrics SPEB and mDPEB. We first showed that the optimal power allocation with perfect network parameters can be efficiently obtained via conic programming, and then proposed robust power allocation schemes to combat uncertainty in network parameters for practical systems. Moreover, we designed an efficient algorithm for robust power allocation

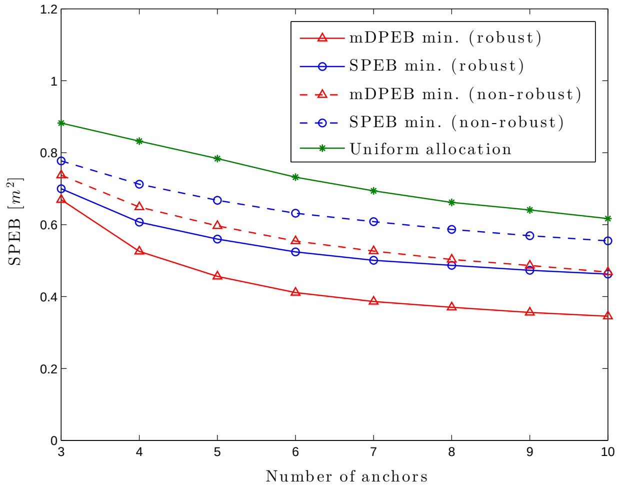

Fig. 7: The actual SPEB with respect to number of anchors, obtained by different power allocation schemes with imperfect knowledge of network parameters ($\varepsilon = 0.2$).

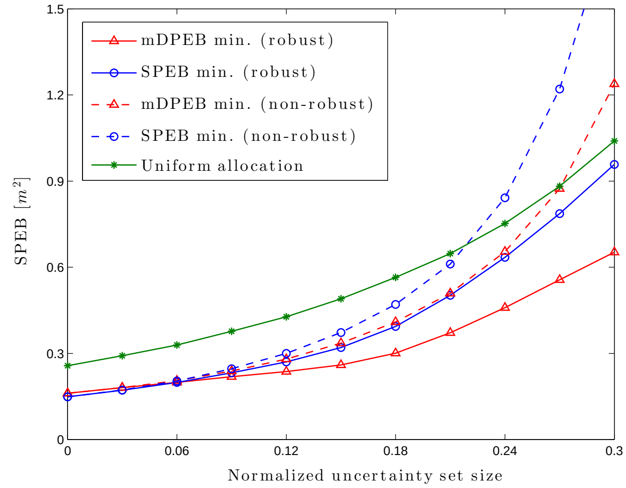

Fig. 8: The actual SPEB with respect to the normalized uncertainty set size on network parameters, obtained by different power allocation schemes.

that allows distributed computations among agents. The simulation results demonstrated that the robust power allocation remarkably outperforms the non-robust power allocation and uniform allocation. Furthermore, we showed that, compared with the SPEB minimization, the mDPEB minimization is more robust to network parameter uncertainty for power allocation.

Appendix A
Proof of Proposition 1

The maximization on DPEB in (5) follows that:

$$\begin{aligned}
&\max_{\varphi\in[0,2\pi)} \{\mathcal{P}(\mathbf{p}_k;\varphi)\}\\
&= \max_{\varphi\in[0,2\pi)} \mathbf{u}(\varphi)^{\mathrm{T}}[\mathbf{J}_{\mathrm{e}}^{-1}(\mathbf{p}_k;\{x_{kj}\})]\mathbf{u}(\varphi)\\
&= \max_{\varphi\in[0,2\pi)} \mathbf{u}(\varphi)^{\mathrm{T}}(\mathbf{U}_{\theta_k}^{-1})^{\mathrm{T}} \begin{bmatrix}\mu_{1,k}^{-1} & 0\\ 0 & \mu_{2,k}^{-1}\end{bmatrix} \mathbf{U}_{\theta_k}^{-1}\mathbf{u}(\varphi)\\
&= \max_{\varphi'\in[0,2\pi)} \mathbf{u}(\varphi')^{\mathrm{T}}[\mathbf{J}_{\mathrm{e}}^{-1}(\mathbf{p}_k;\{x_{kj}\})]\mathbf{u}(\varphi') \qquad (31)
\end{aligned}$$

where the last equality is due to the fact that the product of a unit vector and a rotation matrix $\mathbf{U}_{\theta_k}$ is still a unit vector. Now, let $\varphi' = \theta_k$ in (31), then we have

$$\begin{aligned}
\max_{\varphi\in[0,2\pi)} \{\mathcal{P}(\mathbf{p}_k;\varphi)\} &= \max_{\theta_k} \left\{\mu_{1,k}^{-1}\cos^2\theta_k + \mu_{2,k}^{-1}\sin^2\theta_k\right\}\\
&= \mu_{2,k}^{-1}
\end{aligned}$$

where the last equation is due to $\mu_{1,k} \geq \mu_{2,k}$.

Appendix B
Proof of Proposition 2

Since (7)–(8) are all linear constraints, we only need to show the objective in (6), i.e., the SPEB, is a convex function in x_{kj}. We write the transmit power of agent k as a vector $\mathbf{x}_k = [x_{k1}\ x_{k2}\ \cdots\ x_{kN_{\mathrm{b}}}]^{\mathrm{T}}$, and the SPEB is a function of $\mathbf{x}_k$, given by

$$f(\mathbf{x}_k) \triangleq \mathrm{tr}\left\{\Big(\sum_{j\in\mathcal{N}_{\mathrm{b}}} \xi_{kj}\, x_{kj}\, \mathbf{J}_{\mathrm{r}}(\phi_{kj})\Big)^{-1}\right\}.$$

We choose two arbitrary $\mathbf{x}_k,\ \mathbf{x}'_k \in \mathbb{R}_+^{N_{\mathrm{b}}}$. Given any $\alpha\in[0,1]$, we have

$$\begin{aligned}
&f(\alpha\mathbf{x}_k + (1-\alpha)\mathbf{x}'_k)\\
&= \mathrm{tr}\left\{\Big(\sum_{j\in\mathcal{N}_{\mathrm{b}}} \xi_{kj}\,\big(\alpha x_{kj} + (1-\alpha)x'_{kj}\big)\,\mathbf{J}_{\mathrm{r}}(\phi_{kj})\Big)^{-1}\right\}\\
&= \mathrm{tr}\left\{\Big(\alpha\sum_{j\in\mathcal{N}_{\mathrm{b}}} \xi_{kj}\, x_{kj}\, \mathbf{J}_{\mathrm{r}}(\phi_{kj}) + (1-\alpha)\sum_{j\in\mathcal{N}_{\mathrm{b}}} \xi_{kj}\, x'_{kj}\, \mathbf{J}_{\mathrm{r}}(\phi_{kj})\Big)^{-1}\right\}\\
&\leq \alpha f(\mathbf{x}_k) + (1-\alpha) f(\mathbf{x}'_k). \qquad (32)
\end{aligned}$$

The inequality (32) holds since the function $\mathrm{tr}\{\mathbf{X}^{-1}\}$ is convex in $\mathbf{X} \succ 0$ [45]. If the matrix $\mathbf{X}$ is singular, the inequality (32) still holds. Since ξ_{kj} is a positive scaler, $f(\mathbf{x}_k)$ is convex in $\mathbf{x}_k$.

Appendix C
Proof of Proposition 3

Let $\phi_{kj}^{+} = \phi_{kj} + \hat{\phi}_{kj}$ and $\phi_{kj}^{-} = \phi_{kj} - \hat{\phi}_{kj}$, we have

$$\begin{aligned}
&\mathbf{J}_{\mathrm{r}}(\phi_{kj}) - \mathbf{Q}_{\mathrm{r}}(\hat{\phi}_{kj},\delta_{kj})\\
&= \begin{bmatrix}\delta_{kj} - \sin\phi_{kj}^{+}\,\sin\phi_{kj}^{-} & \cos\phi_{kj}^{+}\,\sin\phi_{kj}^{-}\\ \cos\phi_{kj}^{+}\,\sin\phi_{kj}^{-} & \delta_{kj} + \sin\phi_{kj}^{+}\,\sin\phi_{kj}^{-}\end{bmatrix}.
\end{aligned}$$

We can show that $\mathbf{J}_{\mathrm{r}}(\phi_{kj}) - \mathbf{Q}_{\mathrm{r}}(\hat{\phi}_{kj},\delta_{kj})$ is positive semidefinite if

$$\begin{cases}\delta_{kj} \geq \sin\phi_{kj}^{+}\,\sin\phi_{kj}^{-},\\ \delta_{kj} \geq |\sin\phi_{kj}^{-}|.\end{cases}$$

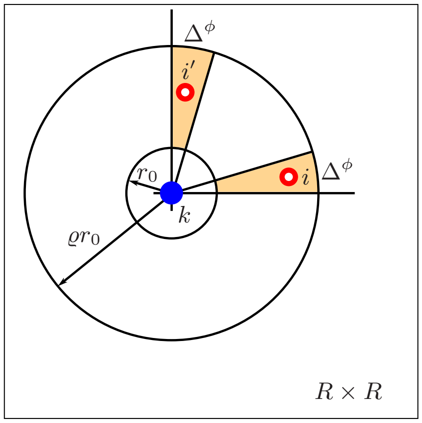

Fig. 9: Geometrical illustration of the proof of Proposition 4(a) where agent is inside the square region. We choose two anchors i and i' in the shaded region.

Since $|\phi^-_{kj}| \le \varepsilon^\phi_{kj}$, the above two inequality conditions are guaranteed by

$$\delta_{kj} \ge \sin \varepsilon^\phi_{kj} \,.$$

Given that $\sum_{j\in\mathcal{N}_\text{b}} \xi_{kj}\, x_{kj}\, \mathbf{Q}_\text{r}(\hat{\phi}_{kj}, \delta_{kj}) \succeq 0$, we have

$$\begin{aligned}&\text{tr}\Big\{\Big(\sum_{j\in\mathcal{N}_\text{b}} \xi_{kj}\, x_{kj}\, \mathbf{J}_\text{r}(\phi_{kj})\Big)^{-1}\Big\}\\ &\le \text{tr}\Big\{\Big(\sum_{j\in\mathcal{N}_\text{b}} \xi_{kj}\, x_{kj}\, \mathbf{Q}_\text{r}(\hat{\phi}_{kj}, \delta_{kj})\Big)^{-1}\Big\}\end{aligned}$$

for all $\phi_{kj} \in \mathcal{S}^\phi_{kj}$. Furthermore, we can show that $\mathbf{Q}_\text{r}(\hat{\phi}_{kj}, \delta_1) \preceq \mathbf{Q}_\text{r}(\hat{\phi}_{kj}, \delta_2)$ for $0 \le \delta_2 \le \delta_1$, which implies that the function $\text{tr}\big\{\big(\sum_{j\in\mathcal{N}_\text{b}} \xi_{kj}\, x_{kj}\, \mathbf{Q}_\text{r}(\hat{\phi}_{kj}, \delta_{kj})\big)^{-1}\big\}$ is a non-decreasing function of δ_{kj}. Hence, the minimum value of the right-hand side of (17) is obtained when $\delta_{kj} = \sin \varepsilon^\phi_{kj}$.

Appendix D
Proof of Proposition 4

We first consider the network with a single agent, and then extend the proof to the multiple-agent case. For a given $k \in \mathcal{N}_\text{a}$, we need to show that the condition (18) holds for $\{x^*_{kj}\}$ w.h.p. for both cases (a) and (b). Note that since

$$\sum_{j\in\mathcal{N}_\text{b}} \tilde{\xi}_{kj}\, x^*_{kj}\, \mathbf{Q}_\text{r}(\hat{\phi}_{kj}) \succeq \sum_{j\in\mathcal{N}_\text{b}} \tilde{\xi}_{kj}\, x^*_{kj}\, \mathbf{J}_\text{r}(\hat{\phi}_{kj}) - \frac{\zeta_\text{max}}{r_0^{2\beta}} P^\text{tot}\, \delta_{kj}\, \mathbf{I}$$

it is sufficient to show that w.h.p.

$$\text{tr}\Big\{\Big(\sum_{j\in\mathcal{N}_\text{b}} \tilde{\xi}_{kj}\, x^*_{kj}\, \mathbf{J}_\text{r}(\hat{\phi}_{kj})\Big)^{-1}\Big\} \le \frac{r_0^{2\beta}}{\zeta_\text{max}} \frac{2}{P^\text{tot}\,\delta} \tag{33}$$

where $\delta = \sin \varepsilon^\phi$ with $\varepsilon^\phi = \max\{\varepsilon^\phi_{kj}\}$.

For (a): we pick two anchors i and i' in the region (see Fig. 9) such that

1) $r_0 \le \tilde{d}_{ki}, \tilde{d}_{ki'} \le \varrho r_0$ with $\varrho > 1$;
2) $0 \le \phi_{ki} \le \Delta^\phi$ and $\pi/2 - \Delta^\phi \le \phi_{ki'} \le \pi/2$ for a small positive Δ^ϕ.

Note that if the agent is at the corner or on the boundary of the square area, we can rotate the angles accordingly to find such a region.

It can be shown that there exists at least one such pair of anchors with probability $1 + (1-2p_0)^{N_\text{b}} - 2(1-p_0)^{N_\text{b}}$, where $p_0 = (\varrho^2 - 1) r_0^2 \Delta^\phi / 2R^2$. Since the probability goes to 1 exponentially with N_b, such a pair of anchors can be found w.h.p.

Consider a power allocation scheme $\{\breve{P}_{ki} = \breve{P}_{ki'} = P^\text{tot}/2\}$, and we show this scheme satisfies the condition (33) for a sufficiently small δ. Based on the definition of the optimal power allocation, we have

$$\begin{aligned}&\text{tr}\Big\{\Big(\sum_{j\in\mathcal{N}_\text{b}} \tilde{\xi}_{kj}\, x^*_{kj}\, \mathbf{J}_\text{r}(\hat{\phi}_{kj})\Big)^{-1}\Big\}\\ &\le \max_{\{\phi_{kj}\}} \text{tr}\Big\{\Big(\sum_{j\in\mathcal{N}_\text{b}} \tilde{\xi}_{kj}\, \breve{P}_{kj}\, \mathbf{J}_\text{r}(\phi_{kj})\Big)^{-1}\Big\}\\ &\le \max_{\{\phi_{kj}\}} \text{tr}\Big\{\Big(\frac{\zeta_\text{min}}{\varrho^{2\beta} r_0^{2\beta}} \frac{P^\text{tot}}{2} \left(\mathbf{J}_\text{r}(\phi_{ki}) + \mathbf{J}_\text{r}(\phi_{ki'})\right)\Big)^{-1}\Big\}\\ &= \frac{\varrho^{2\beta} r_0^{2\beta}}{\zeta_\text{min}} \frac{2}{P^\text{tot}} \frac{2}{\sin^2(\pi/2 - 2\Delta^\phi - 2\varepsilon^\phi)} \,.\end{aligned}$$

Therefore, a sufficient condition for (33) is

$$\frac{\varrho^{2\beta} r_0^{2\beta}}{\zeta_\text{min}} \frac{2}{P^\text{tot}} \frac{2}{\sin^2(\pi/2 - 2\Delta^\phi - 2\varepsilon^\phi)} \le \frac{r_0^{2\beta}}{\zeta_\text{max}} \frac{2}{P^\text{tot}\,\delta}$$

which is equivalent to

$$\frac{2\varrho^{2\beta} \sin \varepsilon^\phi}{\cos^2(2\Delta^\phi + 2\varepsilon^\phi)} \le \frac{\zeta_\text{min}}{\zeta_\text{max}} \tag{34}$$

where $\delta = \sin \varepsilon^\phi$. Note that the left-hand side of (34) is an increasing function in ϱ, Δ^ϕ and ε^ϕ, when Δ^ϕ and ε^ϕ are both small positive numbers. Thus, the maximum ε^ϕ (or equivalently, maximum δ) to satisfy (34) can be obtained by taking the limit $\varrho \to 1$ and $\Delta^\phi \to 0$. It follows that

$$\frac{2 \sin \varepsilon^\phi}{\cos^2(2\varepsilon^\phi)} \le \frac{\zeta_\text{min}}{\zeta_\text{max}}$$

and the inequality holds when $0 < \delta = \sin \varepsilon^\phi \le \delta_\text{max}$, where δ_max is the smallest positive root of the equation

$$4\delta^4 - 4\delta^2 - 2\frac{\zeta_\text{max}}{\zeta_\text{min}}\delta + 1 = 0 \,.$$

We give some numerical examples: $\delta_\text{max} = 0.318$ when $\zeta_\text{max}/\zeta_\text{min} = 1$; $\delta_\text{max} = 0.096$ when $\zeta_\text{max}/\zeta_\text{min} = 5$.

For (b): Consider a small angle $\sqrt{2a\varepsilon^\phi}$ as $\varepsilon^\phi \to 0$, where $a = (2^{\beta+1} R^{2\beta} \zeta_\text{max})/(r_0^{2\beta} \zeta_\text{min})$. The probability that all N_b anchors locate in such a small angle of the $R \times R$ region is at most $(\sqrt{2a\varepsilon^\phi})^{N_\text{b}}$, which goes to 0 at the rate of polynomial power $N_\text{b}/2$ as $\varepsilon^\phi \to 0$. Hence, we can find two anchors, i and i', whose angle separation is larger than $\sqrt{2a\varepsilon^\phi}$ and smaller than $\pi - \sqrt{2a\varepsilon^\phi}$ w.h.p.

We allocate the power equally on these two anchors, and it follows

$$\begin{aligned}
&\mathrm{tr}\bigg\{\Big(\sum_{j\in\mathcal{N}_\mathrm{b}}\widetilde{\xi}_{kj}\,x^*_{kj}\,\mathbf{J}_\mathrm{r}(\hat{\phi}_{kj})\Big)^{-1}\bigg\}\\
&\le \max_{\{\phi_{kj}\}}\ \mathrm{tr}\bigg\{\Big(\sum_{j\in\mathcal{N}_\mathrm{b}}\widetilde{\xi}_{kj}\,\breve{P}_{kj}\,\mathbf{J}_\mathrm{r}(\phi_{kj})\Big)^{-1}\bigg\}\\
&\le \max_{\{\phi_{kj}\}}\ \mathrm{tr}\bigg\{\Big(\frac{\zeta_{\min}}{(\sqrt{2}R)^{2\beta}}\frac{P^{\mathrm{tot}}}{2}\left(\mathbf{J}_\mathrm{r}(\phi_{ki})+\mathbf{J}_\mathrm{r}(\phi_{ki'})\right)\Big)^{-1}\bigg\}\\
&= \frac{2^\beta R^{2\beta}}{\zeta_{\min}}\frac{2}{P^{\mathrm{tot}}}\frac{2}{\sin^2(\sqrt{2a\varepsilon^\phi}-2\varepsilon^\phi)}\,.
\end{aligned}$$

Finally, we need to show that

$$\frac{2^\beta R^{2\beta}}{\zeta_{\min}}\frac{2}{P^{\mathrm{tot}}}\frac{2}{\sin^2(\sqrt{2a\varepsilon^\phi}-2\varepsilon^\phi)} \le \frac{r_0^{2\beta}}{\zeta_{\max}}\frac{2}{P^{\mathrm{tot}}\sin\varepsilon^\phi}$$

or equivalently,

$$a \le \frac{\sin^2(\sqrt{2a\varepsilon^\phi}-2\varepsilon^\phi)}{\sin\varepsilon^\phi}\,.$$

The above inequality holds as $\varepsilon^\phi \to 0$, since the limit of its right-hand side is $2a$.

Now, we extend the above proof to the multiple-agent case. In Section V, we decomposed the one-stage problem $\mathscr{P}_{\text{R-1}}^{\text{SDP}}$ into two-stage optimizations. Let ρ^*_{kj} and x^*_k denote the optimal solution of $\mathscr{P}_{\text{R-1},k}^{(\text{I})}$ and $\mathscr{P}_{\text{R-1}}^{(\text{II})}$, respectively. Since the Stage I problem $\mathscr{P}_{\text{R-1},k}^{(\text{I})}$ is formulated for each single agent, we can show by the above proof that

$$\sum_{j\in\mathcal{N}_\mathrm{b}}\widetilde{\xi}_{kj}\,\rho^*_{kj}\,\mathbf{Q}_\mathrm{r}(\hat{\phi}_{kj}) \succeq 0$$

holds w.h.p. for agent k. Moreover, the optimal power allocation is given in (30) as $x^*_{kj} = \rho^*_{kj}x^*_k$, where x^*_k obtained in Stage II does not affect ρ^*_{kj}. Hence, we can show that the condition (18) holds w.h.p. for multiple-agent networks.

Appendix E
Proof of Proposition 5

The Lagrangian function is given by

$$\mathcal{L}(x_k,u_k,v) = \sum_{k\in\mathcal{N}_\mathrm{a}}\frac{T_k}{x_k} - \sum_k u_k x_k + v\bigg(\sum_{k\in\mathcal{N}_\mathrm{a}}x_k - P^{\mathrm{tot}}\bigg)$$

where $u_k,\ v\ge 0$. The KKT conditions [46] can be derived as

$$\frac{\partial\mathcal{L}}{\partial x_k} = -\frac{T_k}{x_k^2} - u_k + v = 0 \tag{35}$$

$$u_k x_k = 0$$

$$v\bigg(\sum_{k\in\mathcal{N}_\mathrm{a}}x_k - P^{\mathrm{tot}}\bigg) = 0.$$

Since x_k is always positive, we have $u_k = 0$, which leads to $x_k = \sqrt{T_k/v}$ in (35). Moreover, the objective is monotonically decreasing in x_k, which implies the optimal allocation must use all the power resource, i.e., $\sum_{k\in\mathcal{N}_\mathrm{a}}x_k = P^{\mathrm{tot}}$. Hence, the optimal solution is given by (29).

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