Heavy Dark Matter Annihilation from Effective Field Theory

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We formulate an effective field theory description for $\mathrm{SU}(2)_L$ triplet fermionic dark matter by combining nonrelativistic dark matter with gauge bosons in the soft-collinear effective theory. For a given dark matter mass, the annihilation cross section to line photons is obtained with 5% precision by simultaneously including Sommerfeld enhancement and the resummation of electroweak Sudakov logarithms at next-to-leading logarithmic order. Using these results, we present more accurate and precise predictions for the gamma-ray line signal from annihilation, updating both existing constraints and the reach of future experiments.

If weakly interacting massive particles (WIMPs) exist at the TeV scale, their annihilations in the present day—and hence their signatures in indirect dark matter (DM) searches—experience large corrections that are not well described by a simple perturbative expansion in the coupling. On the one hand, exchanges of electroweak gauge bosons and photons between heavy DM particles in the initial state give rise to a long-range attractive potential. This "Sommerfeld enhancement" effect sums large corrections $\sim \sum_k (\alpha_2 m_\chi / m_W)^k$ and has been extensively studied in the literature (e.g., [1–4]). On the other hand, a large hierarchy between the DM mass m_χ and gauge boson mass m_W will generate large logarithmic corrections to exclusive channels, and this has not yet been explored in detail. A study [5] of wino DM annihilation at one-loop found $\mathcal{O}(1)$ corrections, which change the predicted annihilation cross section by a factor of a few. This is a signal of large logarithmic corrections, $\sim \sum_k (\alpha_2 \ln^{2,1} m_\chi^2 / m_W^2)^k$, whose resummation is the focus of this work.

This goal is not an abstract one: existing ground-based gamma-ray telescopes can probe the annihilation of multi-TeV DM [6,7], and future colliders could also have sensitivity [8]. Null results from the LHC already place stringent lower bounds on the SUSY spectrum; so, while direct constraints on DM from the LHC are still not especially strong, the lack of a detection of new physics below the TeV scale motivates consideration of heavier-than-TeV DM and its properties. As one example, models of "split supersymmetry" [9,10] can preserve the unification of gauge couplings with fermionic superpartners at the TeV scale [11]. It is therefore imperative to understand how to translate models of heavy DM into signal predictions with accurate theoretical cross sections.

We focus on pure wino DM and its annihilation to line gamma rays, $\chi^0 \chi^0 \rightarrow \gamma\gamma, \gamma Z$. At the weak scale and above, such spectral lines have zero astrophysical background, so detection would be a smoking gun for new physics.

We show that Sommerfeld enhancement effects can be factorized from large logs to all orders in α_2, and compute the cross section at next-to-leading logarithmic (NLL) order for line photon production, including an estimate of theoretical uncertainties.

Dark matter model.—We do not know yet what the nongravitational interactions of dark matter (DM) are. Here we are interested in DM being an $\mathrm{SU}(2)_L$ triplet of Majorana fermions, a scenario under active investigation [5,6,8,12] both in the context of the SUSY wino and more generally. The DM triplet can be written as

$$\chi = \begin{pmatrix} \chi^0/\sqrt{2} & \chi^+ \\ \chi^- & -\chi^0/\sqrt{2} \end{pmatrix}, \qquad (1)$$

which transforms from left and right under the $\mathrm{SU}(2)_L$ gauge group of the standard model (SM). We extend the SM Lagrangian by including $\mathcal{L}_{\mathrm{DM}} = \frac{1}{2}\mathrm{Tr}\bar{\chi}(i\slashed{D} - M_\chi)\chi$, where the trace sums over the $\mathrm{SU}(2)_L$ indices and the covariant derivative couples the DM to SM gauge bosons $W^{1,2,3}$ or, equivalently, γ, W, Z (χ has zero hypercharge). In principle, the mass mixing and splitting can be described by an arbitrary matrix M_χ; however, in the minimal scenario it is $M_\chi = m_\chi \mathbb{1}$. A small mass splitting between χ^0 and χ^- is generated radiatively, and we take it to be $\delta = 0.17$ GeV for the Sommerfeld calculation [1,13], but ignore it in the Sudakov calculation. The presence of this splitting means the χ^0 constitutes all the stable DM. However, initial-state exchange of W bosons allows excitation from a $\chi^0\chi^0$ two-body state into an (off-shell) $\chi^+\chi^-$ state, and in calculating the Sommerfeld-enhanced cross section the matrix elements for annihilation from $\chi^0\chi^0$ and $\chi^+\chi^-$ initial states must therefore be included.

We focus on the late-time annihilation of triplet DM and thus assume there are no on-shell $\chi^\pm$ present in the DM halo. We also assume *s*-wave annihilation, since *p*-wave

and higher terms are suppressed by at least the square of the small DM velocity in the local halo ($v \sim 10^{-3}$). This ensures the $\chi^0\chi^0$ initial state is a spin singlet. This also implies that annihilation to three neutral gauge bosons is forbidden by CP conservation [5], so we consider only two-body final states.

Electroweak corrections in NRDM-SCET.—The soft-collinear effective theory (SCET) [14–17] has been used to describe electroweak radiative corrections in high-energy processes via exchanges of weak gauge bosons of the SM gauge group [18,19]. We generalize this formalism to the case with heavy nonrelativistic dark matter (NRDM) in the initial state, and use it to calculate $\chi\chi \to ZZ, Z\gamma, \gamma\gamma$. The calculation can be broken into pieces: constructing operators, matching at a high scale $\mu \simeq 2m_\chi$, running down to $\mu \simeq m_Z$, and calculating matrix elements at this low scale which include the Sommerfeld enhancement.

EFT and high scale matching.—At the high scale $\mu_{m_\chi} \simeq \sqrt{s} = 2m_\chi$ we match the annihilation process in the full theory $\mathcal{L}_{SM} + \mathcal{L}_{DM}$ onto a set of leading order operators O_r in our effective theory NRDM-SCET:

$$\mathcal{L}_{ann}^{(0)} = \sum_{r=1}^{2} C_r(m_\chi, \mu) O_r(m_{W/Z}, v, \mu). \quad (2)$$

There are only two operators in the complete basis for spin-singlet S-wave annihilation of DM:

$$O_r = (\chi_v^{aT} i\sigma_2 \chi_v^b)(S_r^{abcd} \mathcal{B}_{n\perp}^{ic} \mathcal{B}_{\bar{n}\perp}^{jd}) i\epsilon^{ijk}(n-\bar{n})^k,$$
$$S_1^{abcd} = \delta^{ab}(\mathcal{S}_n^{ce} \mathcal{S}_{\bar{n}}^{de}), \qquad S_2^{abcd} = (\mathcal{S}_v^{ae} \mathcal{S}_n^{ce})(\mathcal{S}_v^{bf} \mathcal{S}_{\bar{n}}^{df}). \quad (3)$$

Here, $v = (1,0,0,0)$, $n = (1,\hat{n})$, and $\bar{n} = (1,-\hat{n})$ with $\hat{n}$ the direction of an outgoing gauge boson. χ_v^a is a nonrelativistic two-component fermion DM field in the adjoint representation, $\mathcal{B}_{n,\bar{n}}$ contain the observed (collinear) gauge bosons, and the $\mathcal{S}_\kappa = \mathcal{S}_\kappa[\kappa \cdot A_s]$ are adjoint Wilson lines of soft gauge bosons along the $\kappa = n, \bar{n}, v$ directions. Without soft gauge bosons there are only two possible contractions of gauge indices, $\delta^{ab}\delta^{cd}$ and $\delta^{ac}\delta^{bd}$, since $(\chi_v^{aT} i\sigma_2 \chi_v^b) = \chi_v^{a\alpha}\chi_v^{b\beta}\epsilon_{\alpha\beta}$ is symmetric in (ab). Because of the factorization properties of soft gauge bosons for heavy particles v, or collinear particles $n, \bar{n}$, the addition of the soft $\mathcal{S}_\kappa$ Wilson lines does not change this, see [17]. The final state gauge bosons are also in a spin singlet with orthogonal polarizations so they must be contracted with ϵ^{ijk}. The outgoing energetic gauge bosons appear in the adjoint collinear gauge invariant building block $\mathcal{B}_{n\perp}^{\mu a} = i/(i\bar{n} \cdot \partial_n)\bar{n}_\nu G_n^{\nu\mu b} \mathcal{W}_n^{ba} = A_{n\perp}^{\mu a} - (k_\perp^\mu/\bar{n} \cdot k)\bar{n} \cdot A_n^a + \cdots$, where $A_n^{\mu a}$ is the n-collinear gauge boson field, $G_n^{\nu\mu b}$ is the field strength, and $\mathcal{W}_n^{ba} = \mathcal{W}_n^{ba}[\bar{n} \cdot A_n]$ is a collinear Wilson line in the adjoint representation. For the definition of $\mathcal{B}_{\bar{n}\perp}^{\mu a}$ simply swap $n \leftrightarrow \bar{n}$. In addition to the hard annihilation process encoded in $\mathcal{L}_{ann}^{(0)}$, we will also use the

leading order SCET$_{II}$ Lagrangian [17] $\mathcal{L}_{SCET}^{(0)}$ and leading order nonrelativistic Lagrangian for DM $\mathcal{L}_{NRDM}^{(0)} = \chi_v^\dagger(iv \cdot \partial + \vec{\nabla}^2/2m_\chi)\chi_v + \hat{V}[\chi_v^{(\dagger)}](m_{W,Z})$, where $\hat{V}$ is an operator giving the Yukawa and Coulombic potentials from potential exchange of the W, Z, γ.

To determine the Wilson coefficients C_r at the high scale we match from the full theory onto the effective theory. Since C_r only contain ultraviolet physics this matching can be done in the unbroken SM with $m_W = m_Z = 0$. At tree level we find $C_1(\mu_{m_\chi}) = -C_2(\mu_{m_\chi}) = -\pi\alpha_2(\mu_{m_\chi})/m_\chi$, where $\alpha_2 = g^2/4\pi = \alpha/\sin^2\bar{\theta}_W$.

Sommerfeld-Sudakov factorization.—To the order we are working, $\mathcal{L}_{NRDM}^{(0)}$ does not contain interactions with soft or collinear gauge bosons beyond those that set the scale for the couplings in the potential, and $\mathcal{L}_{SCET}^{(0)}$ contains no interactions with χ_vs, so the matrix element for the $\chi^0\chi^0$ evolution and annihilation factorizes from the matrix element involving the final state X:

$$C_r\langle X|O_r|\chi^0\chi^0\rangle = [C_r i\epsilon^{ijk}(n-\bar{n})^k\langle X|S_r^{abcd}\mathcal{B}_{n\perp}^{ic}\mathcal{B}_{\bar{n}\perp}^{jd})|0\rangle]$$
$$\times \langle 0|\chi_v^{aT} i\sigma_2 \chi_v^b|\chi^0\chi^0\rangle. \quad (4)$$

For the spin-singlet state $|(\chi^a\chi^b)_S\rangle = \epsilon^{\beta\alpha}|\chi_\alpha^a\chi_\beta^b\rangle/\sqrt{2}$, the Sommerfeld enhancement factors are encoded in

$$\langle 0|\chi_v^{3T} i\sigma_2 \chi_v^3|(\chi^0\chi^0)_S\rangle = 4\sqrt{2}m_\chi s_{00},$$
$$\langle 0|\chi_v^{+T} i\sigma_2 \chi_v^-|(\chi^0\chi^0)_S\rangle = 4m_\chi s_{0\pm}, \quad (5)$$

where the matrix elements are evaluated using the potential $\hat{V}$. For these channels the corresponding matrix elements on the first line of Eq. (4) can be denoted F_0^X and $F_\pm^X$, thus giving an all-orders factorized result for the spin-singlet annihilation amplitudes

$$\mathcal{M}_{\chi^0\chi^0 \to X} = 4m_\chi(\sqrt{2}s_{00}F_0^X + s_{0\pm}F_\pm^X),$$
$$\mathcal{M}_{\chi^+\chi^- \to X} = 2\sqrt{2}m_\chi(\sqrt{2}s_{\pm 0}F_0^X + s_{\pm\pm}F_\pm^X). \quad (6)$$

In the one-loop calculation of [5], the coefficients $s_0^{[5]} = s_{00}$ and $s_\pm^{[5]} = s_{0\pm}$ were also included as multiplicative factors, which is consistent with this factorization. We obtain the Sommerfeld coefficients s_{00} and $s_{0\pm}$ by solving the Schrödinger equation numerically (see, e.g., Appendix A of [6] for details). Note that at tree level $s_{00} = s_{\pm\pm} = 1$ and $s_{0\pm} = s_{\pm 0} = 0$.

With SU(2)$_L$ symmetry the gauge index structure of the first line of Eq. (4) implies that the SCET perturbative corrections at any order are encoded in just two Sudakov form factors, Σ_1 and Σ_2. The gauge boson masses induce symmetry breaking corrections at NLL, which are included by using $\Sigma_{1,2}^W$ for the W^+W^- final state, so

$$F_0^{\gamma\gamma} = P_{\gamma\gamma}(\Sigma_1 - \Sigma_2), \qquad F_{\pm}^{\gamma\gamma} = 2P_{\gamma\gamma}\Sigma_1,$$
$$F_0^{W^+W^-} = P_W\Sigma_1^W, \qquad F_{\pm}^{W^+W^-} = P_W(2\Sigma_1^W - \Sigma_2^W), \quad (7)$$

where the prefactors are $P_{\gamma\gamma} = -e^2\epsilon_{n\perp}^i\epsilon_{\bar{n}\perp}^j\epsilon^{ijk}\hat{n}^k/(2m_\chi)$ and $P_W = (g^2/e^2)P_\gamma$. For $F_0^{\gamma Z}$ and F_0^{ZZ} one simply replaces $P_{\gamma\gamma}$ by $P_{\gamma Z} = \cot\bar{\theta}_W P_{\gamma\gamma}$ or $P_{ZZ} = \cot^2\bar{\theta}_W P_{\gamma\gamma}$. At tree level the form factors are all unity, $\Sigma_1 = \Sigma_2 = 1$.

For the $\gamma\gamma$ and γZ final states there is no tree-level annihilation from $\chi^0\chi^0$, so we normalize by writing

$$\sigma_{\chi^0\chi^0 \to X} = \sigma_{\chi^+\chi^- \to X}^{\text{tree}} |s_{00}(\Sigma_1 - \Sigma_2) + \sqrt{2}s_{0\pm}\Sigma_1|^2. \quad (8)$$

Sudakov resummation.—We now calculate the Sudakov form factors $\Sigma_{1,2}$. For simplicity, in this calculation we take all DM components to have a common mass m_χ. The operators $O_{1,2}$ in (3) mix under renormalization and the resummation of $\alpha_2\ln^{2,1}(m_\chi^2/m_W^2)$ corrections is achieved by finding their SCET anomalous dimension matrix, and running between the high scale $\mu_{m_\chi} \simeq 2m_\chi$ and the low scale $\mu_Z \simeq m_Z$. For NLL order resummation we need the two-loop cusp and one-loop noncusp anomalous dimensions, plus the high scale matching at tree level. The one-loop anomalous dimension matrix for an operator with standard model quantum numbers and any number of single collinear building blocks was derived in Ref. [18], and we will make use of their results, including the Δ-regulator [18,20]. Our case differs from this general result because the incoming nonrelativistic DM fields are in the same direction v, and hence we have two soft S_v Wilson lines that can interact with each other or self-interact.

The anomalous dimension matrix for $(C_1 C_2)^T$ is

$$\hat{\gamma} = 2\gamma_{W_T}\mathbb{1} + \hat{\gamma}_S. \quad (9)$$

Here, γ_{W_T} is the collinear anomalous dimension of $\mathcal{B}_{n\perp}^{ia}$ which only mixes into itself, and hence multiplies a diagonal matrix. Including the two-loop cusp and one-loop noncusp terms it is equal to [18]

$$\gamma_{W_T}^{\text{NLL}} = \frac{\alpha_2}{4\pi}\Gamma_0^g\ln\frac{2m_\chi}{\mu} - \frac{\alpha_2}{4\pi}b_0 + \left(\frac{\alpha_2}{4\pi}\right)^2\Gamma_1^g\ln\frac{2m_\chi}{\mu}, \quad (10)$$

where here and below $\alpha_2(\mu)$ is in the $\overline{\text{MS}}$ scheme, and for SU(2) in the SM, $C_A = 2$, $b_0 = 19/6$ is the one-loop β function, the cusp anomalous dimensions are $\Gamma_0^g = 4C_A = 8$ and $\Gamma_1^g = 8(\frac{70}{9} - \frac{2}{3}\pi^2)$. When integrating, we will also need the two-loop β function $b_1 = -35/6$.

The soft anomalous dimension $\hat{\gamma}_S$ encodes the running and mixing of the soft factors $S_{1,2}^{abcd}$ and hence has non-trivial structure. After canceling the regulator dependent part with the zero-bin subtracted [21] collinear graphs, the remaining nonzero one-loop contributions come from wave function renormalization from self-contracting a

S_v, connecting the two S_v Wilson lines, and connecting the S_n and $S_{\bar{n}}$ Wilson lines. The wave function renormalization is the same as in heavy quark effective theory, $\gamma_{h_v} = -C_A\alpha_2/(2\pi)$. The full result needed at NLL is

$$\hat{\gamma}_S^{\text{NLL}} = \frac{\alpha_2}{\pi}(1 - i\pi)\begin{pmatrix} 2 & 1 \\ 0 & -1 \end{pmatrix} - \frac{2\alpha_2}{\pi}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \quad (11)$$

At the low scale $\mu_Z \simeq m_Z$ the operators O_1, O_2 are matched onto operators with W, Z, γs, and effects associated with the gauge boson masses are included from low scale matching (or using the rapidity renormalization group [22,23]). Here we are interested in neutral transverse final state gauge bosons, where the matching at NLL order reads [19] $\mathcal{B}_\perp^3 \to \exp(D)(Z_\perp\cos\theta_W + A_\perp\sin\theta_W)$ with

$$D(\mu_Z) = \frac{\alpha_2(\mu_Z)}{2\pi}\ln\frac{4m_\chi^2}{\mu_Z^2}\ln\frac{m_W^2}{\mu_Z^2}. \quad (12)$$

The Sommerfeld enhancement factors in (5) are low scale matrix elements which are also calculated at $\mu_Z \simeq m_Z$ in the $\overline{\text{MS}}$ scheme.

Analytical resummation formula at NLL order.—The resummed amplitude is

$$\begin{bmatrix} C_1(\mu_Z) \\ C_2(\mu_Z) \end{bmatrix} = e^{D(\mu_Z)} P \exp\left(\int_{\mu_{m_\chi}}^{\mu_Z} \frac{d\mu}{\mu}\hat{\gamma}\right) \begin{bmatrix} C_1(\mu_{m_\chi}) \\ C_2(\mu_{m_\chi}) \end{bmatrix}. \quad (13)$$

This equation can be integrated analytically using $d\mu/\mu = d\alpha_2/\beta_2[\alpha_2]$. For $X = ZZ, \gamma Z, \gamma\gamma$ we find

$$\Sigma_1 = \frac{e^{\Omega+D}}{3}(2z^{-(4\psi/b_0)} + z^{(2\psi/b_0)}),$$

$$\Sigma_1 - \Sigma_2 = \frac{2e^{\Omega+D}}{3}(z^{-(4\psi/b_0)} - z^{(2\psi/b_0)}), \quad (14)$$

where $\psi = 1 - i\pi$, $z = \alpha_2(\mu_Z)/\alpha_2(\mu_{m_\chi})$, D is in Eq. (12), and Ω at NLL order equals

$$\Omega = \frac{-2\pi\Gamma_0^g(z\ln z + 1 - z)}{b_0^2\alpha_2(\mu_Z)} - \frac{\Gamma_0^g b_1(\ln z - z - \frac{\ln^2 z}{2} + 1)}{2b_0^3}$$
$$- \frac{\ln z}{2b_0}\left[8\left(\ln\frac{4m_\chi^2}{\mu_{m_\chi}^2} - 1\right) - 2b_0\right] - \frac{\Gamma_1^g}{2b_0^2}(z - \ln z - 1).$$
$$(15)$$

At LL order we would only have $\Sigma_1 = \Sigma_2 = \exp(\Omega_0)$, where Ω_0 is the first term in (15). Treating Sommerfeld effects at tree level the ratio of cross sections is given by the Sudakov form factors
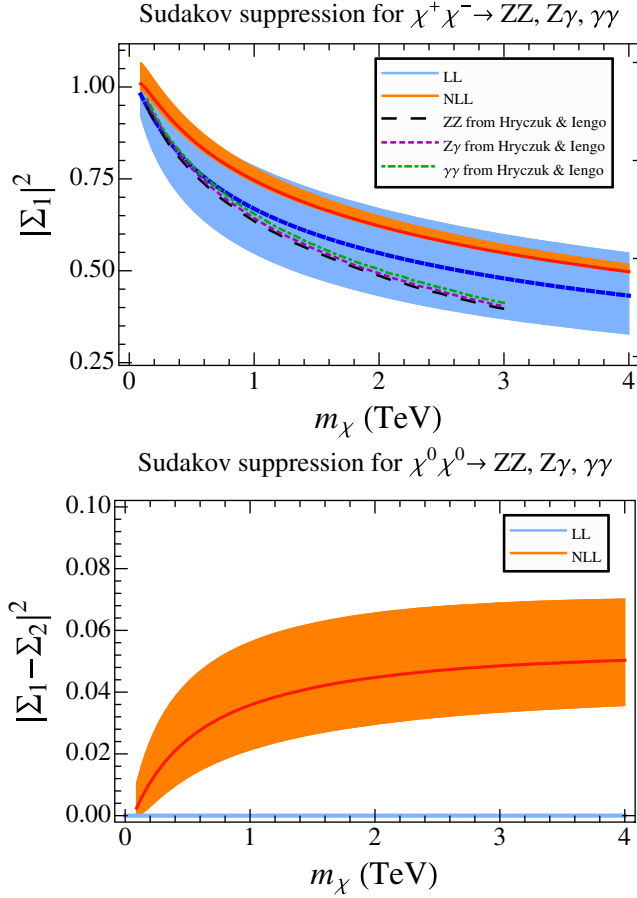
Sudakov suppression for $\chi^+\chi^- \to ZZ, Z\gamma, \gamma\gamma$

Sudakov suppression for $\chi^0\chi^0 \to ZZ, Z\gamma, \gamma\gamma$

FIG. 1 (color online).   Resummed leading and next-to-leading logarithmic electroweak corrections for $\chi^+\chi^-, \chi^0\chi^0 \to ZZ, Z\gamma, \gamma\gamma$. Only high scale variation by a factor of 2 from $\mu_{m_\chi} = 2m_\chi$ is shown. Low scale variation has a 20% smaller error band for the top plot and a 5% bigger error band for the bottom plot.

$$\frac{\sigma_{\chi^+\chi^-\to X}^{NLL+SE}}{\sigma_{\chi^+\chi^-\to X}^{tree}} = |\Sigma_1|^2, \qquad \frac{\sigma_{\chi^0\chi^0\to X}^{NLL+SE}}{\sigma_{\chi^+\chi^-\to X}^{tree}} = |\Sigma_1 - \Sigma_2|^2. \quad (16)$$

This nonzero result for $\chi^0\chi^0 \to ZZ, Z\gamma, \gamma\gamma$ at short distances starts at NLL in $|\Sigma_1 - \Sigma_2|^2$, and occurs because there

is a Sudakov mixing between the W^+W^- and W^3W^3 from soft gauge boson exchange. This is similar in spirit to the Sommerfeld mixing of the initial states.

In Fig. 1 we plot $|\Sigma_1|^2$ and $|\Sigma_1 - \Sigma_2|^2$ as a function of m_χ. To obtain theoretical uncertainty bands we use the residual scale dependence at LL and NLL obtained by varying $\mu_{m_\chi} = [m_\chi, 4m_\chi]$ and $\mu_Z = [m_Z/2, 2m_Z]$. The one-loop fixed order results of [5] are within our LL uncertainty band. Our NLL result yields precise theoretical results for these electroweak corrections. To test our uncertainties we added nonlogarithmic $\mathcal{O}(\alpha_2)$ corrections to $C_{1,2}(\mu_{m_\chi})$, of the size found in [5], and noted that the shift is within our NLL uncertainty bands.

Indirect detection phenomenology.—Combining Eqs. (8) and (14) with the standard Sommerfeld enhancement (SE) factors s_{00} and $s_{0\pm}$, we can now compute the total cross section for annihilation to line photons at NLL + SE and compare to existing limits from indirect detection. We sum the rates of photon production from $\chi^0\chi^0 \to \gamma\gamma, \gamma Z$, as the energy resolution of current instruments is typically comparable to or larger than the spacing between the lines (see, e.g., [6] for a discussion).

In Fig. 2 we display our results for the line cross sections calculated at LL + SE and NLL + SE. Our theoretical uncertainties are from μ_{m_χ} variation. (The μ_Z variations are very similar. Since both cases are dominated by the variation of the ratio of the high and low scales we do not add them together.) In the left panel we compare to earlier cross section calculations, including "Tree-level + SE" where Sudakov corrections are neglected, the "One-loop fixed-order" cross section where neither Sommerfeld or Sudakov effects are resummed (taken from [7]), and the calculation in [5] where Sommerfeld effects are resummed but other corrections are at one-loop. At low masses, our results converge to the known ones (except [5] which focused on high masses, omitting the term $|F_0^{\gamma\gamma}|^2$, which is important at low masses). At high masses, our NLL + SE result provides a sharp prediction for the annihilation cross section with $\simeq 5\%$ theoretical uncertainty.
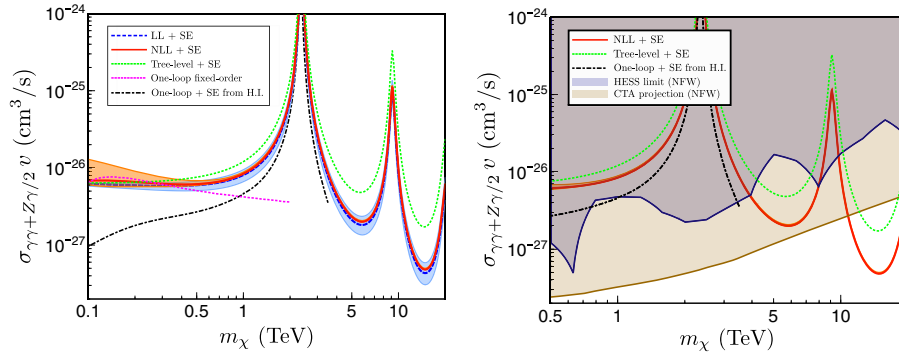

FIG. 2 (color online).   Left panel: Our NLL + SE cross section for $\chi^0\chi^0$ annihilation to line photons from $\gamma\gamma$ and γZ, compared to earlier results. Right panel: current bounds from H. E. S. S and projected reach of 5 hours of CTA observation time, overlaid with our (and previous) cross section predictions, for an NFW profile, with local velocity $v/c \sim 10^{-3}$.

In the right panel of Fig. 2 we compare the NLL cross section to existing limits from H. E. S. S [24] and projected ones from CTA. In the latter case we follow the prescription of [6], based on [25], and in both cases we assume an Navarro-Frenk-White (NFW) profile [26] with local DM density 0.4 GeV/cm^3 (consistent with [27–29]). We assume here that the χ^0 constitutes all the DM due to a nonthermal history (the limits can be straightforwardly rescaled if it constitutes a subdominant fraction of the total DM). For this profile, we see that H. E. S. S already constrains models of this type for masses below ~ 4 TeV, consistent with the results of [6] (which employed the tree-level+SE approximation), and that five hours of observation with CTA could extend this bound to ~ 10 TeV. Any constraint on the line cross section should be viewed as a joint constraint on the fundamental physics of DM and the distribution of DM in the Milky Way [30].

The method we developed here allows systematically improvable effective field theory techniques to be applied to DM, and enabled us to obtain NLL + SE predictions for the DM annihilation cross section to photon lines. This enables precision constraints to be placed on DM.

Note added.—As our Letter was being finalized two papers appeared [34,35] which also investigate DM with SCET. They are complementary to ours: Ref. [34] computes the semi-inclusive cross section for fermionic DM annihilation at LL, and Ref. [35] investigates the exclusive line annihilation cross section for scalar DM up to NLL. The $\mathcal{O}(\alpha)$ corrections obtained from matching at $\mu = 2m_\chi$ in Ref. [35] are consistent with our 5% uncertainty estimate.

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[30] Assuming an Einasto profile [31] with standard parameters ($\alpha = 0.17$, $r_s = 20$ kpc) would strengthen the limits by roughly a factor of 2; allowing a Burkert profile [32] with a core radius greater than ~ 2 kpc would weaken the limits, e.g., by a factor of ~ 20 for a 5 kpc core and ~ 100 for a 10 kpc core (the applicability of such profiles to the Milky Way has been studied in, e.g., Refs. [29,33]).

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