

PARAMETRIC OPTIMIZATION OF S-TYPE CABLES

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Abstract

A We formulated an S-curve parameterization for re-routing a cable path for a given offset displacement. In our approach we assume the cable path to follow a tangent hyperbolic curve. In our formulation, after taking the geometric constraints into account, two free parameters exist. The objective of our design is to maximize the curvature radius. We performed a full search in the two-dimensional design space to find the curvature radius for all possible design configurations. The two-dimensional distribution of curvature radius versus the design parameters is shown in a contour plot that can be readily used for design purposes.

1 Problem Formulation

The problem can be formulated in the following way:

$$y = \beta \tanh(\alpha x/L)$$

with $x \in [-L, L]$ where three different control parameters, α, β and L appear. A constraint is specified by enforcing the cable to symmetrically pass through two points vertically $2H$ units apart from each other at $x = \pm L$. Enforcing this condition reduces the effective number of control parameters to two by setting:

$$H = \beta \tanh(\alpha)$$

Thus

$$\alpha = \tanh^{-1}(H/\beta)$$

Throughout this report we set $H = 1$:

$$H = 1;$$

Therefore the length is non-dimensionalized with H , and different values of H scales the length everywhere accordingly.

Here four samples of S-curve for different values of $L = 0.5, 1.5$ and $\beta = 1.1, 1.5$ are shown.

```
LW = 'linewidth'; FS = 'fontsize'; MS = 'markersize'; CL = 'color';
L = 1.5;
beta = 1.1; alpha = atanh(1/beta);
x = chebfun('x', [-2,2]); y = beta*tanh(alpha*x/L);
plot(y ,LW, 1.6); hold on;
beta = 1.5; alpha = atanh(1/beta);
```

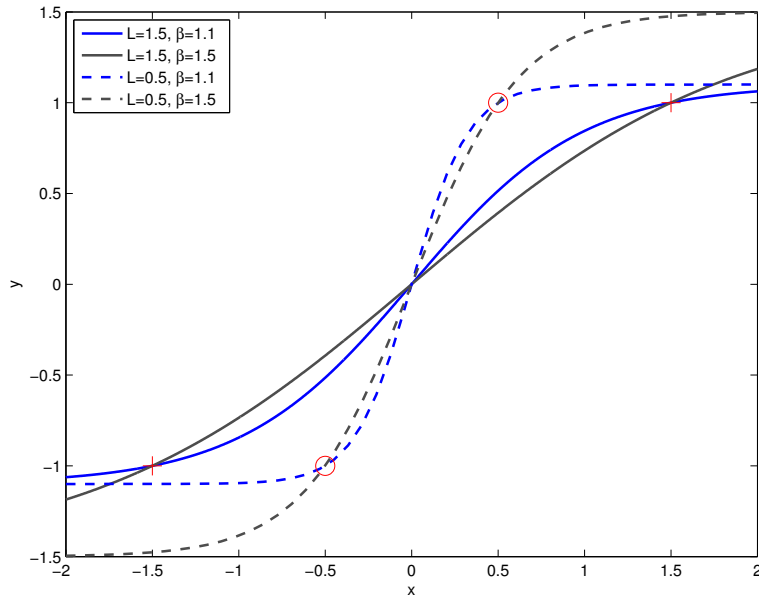


Figure 1: S-type curves for different parameters' choices.

```

x = chebfun('x', [-2,2]); y = beta*tanh(alpha*x/L);
plot(y ,LW, 1.6,CL,[0.3,0.3,0.3]);
%-----
L = 0.5;
beta = 1.1; alpha = atanh(1/beta);
x = chebfun('x', [-2,2]); y = beta*tanh(alpha*x/L);
plot(y ,LW, 1.6,'LineStyle','--','numpts',50); hold on;
beta = 1.5; alpha = atanh(1/beta);
x = chebfun('x', [-2,2]); y = beta*tanh(alpha*x/L);
plot(y ,LW, 1.6,'LineStyle','--',CL,[0.3,0.3,0.3],'numpts',50);

set(gca, FS, 10);
L=1.5; plot([-L,L],[-H,H],'+',CL,'r', MS, 12);
L=0.5; plot([-L,L],[-H,H],'o',CL,'r', MS, 12);
xlabel('x'); ylabel('y')
legend('L=1.5, \beta=1.1','L=1.5, \beta=1.5','L=0.5, \beta=1.1','L=0.5, \beta=1.5',2)

```

Effect of L

It is clear that larger values of L result in larger curvature radius, as increasing L tends to stretch out the curve. However one of the objective is to use a small L due to the space restriction.

2 Effect of β

For a fixed L , increasing β results in *straightening out* the curve and thus larger curvature radius. However, increasing β also increases the slope of the cable at two ends. This effect is also undesirable.

3 Qualitative Definition of the Optimization Problem

The objective of the optimization problem is to find L and β such that

1. The minimum curvature radius is above an admissible value.
2. The start and end angles are as small as possible (the amount of the start and end angles are penalized).
3. The value of L is as small as possible (the value of L is penalized).

4 Solution Strategy

To this end, we perform an exhaustive search in the control space, i.e. two dimensional space of L and β . We search for

$$L \in [0.2, 10]$$

and

$$\beta \in [1.01, 1.5].$$

We discretize this space with a uniform mesh

```
imax = 51;  
L = linspace(0.2,10,imax);  
beta = linspace(1.01,1.5,imax);
```

We then evaluate the curvature and end angles at all discrete points in a full search. Note that the curvature is obtained from

$$\kappa = \frac{y''}{(1 + (y')^2)^{3/2}},$$

and curvature radius is:

$$R = \frac{1}{\kappa}.$$

The minimum radius curvature is defined by:

$$R_{min} = \min(R(x)) \quad \forall x \in [-L, L].$$

The end angles are calculated by:

$$\theta = \tan^{-1}(y')|_{x=\pm L}.$$

This follows:

```

for j=1:imax
    for i=1:imax
        alpha      = atanh(1/beta(j));
        x          = chebfun('x',[-L(i),L(i)]);
        y          = beta(j)*tanh(alpha*x/L(i));
        yp         = diff(y);
        ypp        = diff(y,2);
        k          = ypp./(1+yp.^2).^(3/2);          % curvature as a function of x
        K(i,j)     = max(k);                        % maximum curvature
        Theta(i,j) = atand(yp(L(i)));               % start and end angles
    end
end
end

```

5 Curvature Radius and End Angles

The values of minimum radius curvature and the end angles are shown in the following figure. Depending on the minimum admissible curvature radius and the maximum admissible end angles, the desirable design region can be determined from the following figure.

```

[X,Y] = meshgrid(L,beta);
figure
contour(X',Y',1./K,[5 10 20 30 40 50 60 70 80],'ShowText','on')
xlabel('L')
ylabel('\beta')
title('Minimum Radius')
hold on
contour(X',Y',Theta,[0.5 1 2 3 4 5 10], 'LineStyle','--','ShowText','on')
legend('Minimum Radius','End Angles',2)

```

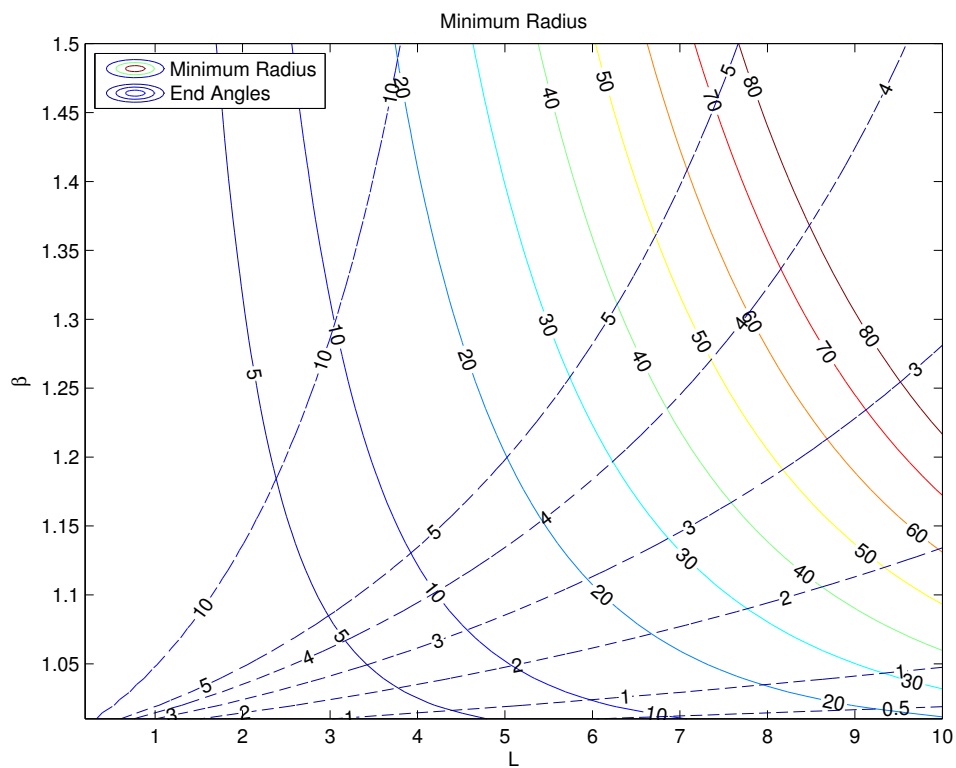


Figure 2: Contour lines of minimum radius R_{min} and end angles θ in two-dimensional design space: solid lines: R_{min} and dashed lines: θ .

6 Results

Problem definition: Consider two parallel lines b inches apart parallel to the y -axis. One line is at $x=a$ and the other is at $x=a+b$. Consider that these two lines are the centerline of a cable of diameter d . The design goal is to have a cable to transition from $x=a$ to $x=a+b$ with the cable being vertical at $y=0$ and $y=l$ without exceeding the radius of curvature. The specific values of interest are $b=9$ inches $d=2$ inches and $l=36$ inches.

Solution: First we have to transform the problem to a non-dimensional one, to be able to use the above chart. In this case the transition distance between two cables is $b=9$ inches. Therefore $b = 2H$. Thus $H = 4.5$ inches. This is the *non-dimensionalizing length*, and every length scales with H .

Similarly $l = 36$ inches becomes $l^* = l/H = 8$ units. We also know that $l^* = 2L$. Therefore $L = 4$ units. The minimum curvature radius is $R_{min} = 16$ inches which becomes $R_{min}^* = R_{min}/H = 3.55$ units.

The non-dimensional definition of the problem then becomes:

Find α and β such that $R > R_{min}^* = 3.55$, $L = 4$ and the end angles are less than one degree.

Now looking at the chart (Figure 3.), there is only a small region along the line $L = 4$ in which $R > 3.55$ and $\theta < 1$ deg. One point in that region is at:

$$\beta = 1.01$$

$$L = 4.$$

$\alpha = \tanh^{-1}(H/\beta) = 2.65$ For this point the minimum curvature radius is $R_{min}^* = 3.69$ units which has the dimensional value of $R_{min} = 3.69 * H = 16.65$ inches. The end angles are $\theta = 0.75$ deg. All these quantities meet the design requirements. At the end the *dimensional* centerline curve of the cable can be given by:

$$y = \beta H \tanh(\alpha x / (H \times L))$$

or

$$y = 1.01 \times 4.5 \tanh(2.65x / (4.5 \times 4)) = 4.545 \tanh(0.1473x) \quad (\text{inches})$$

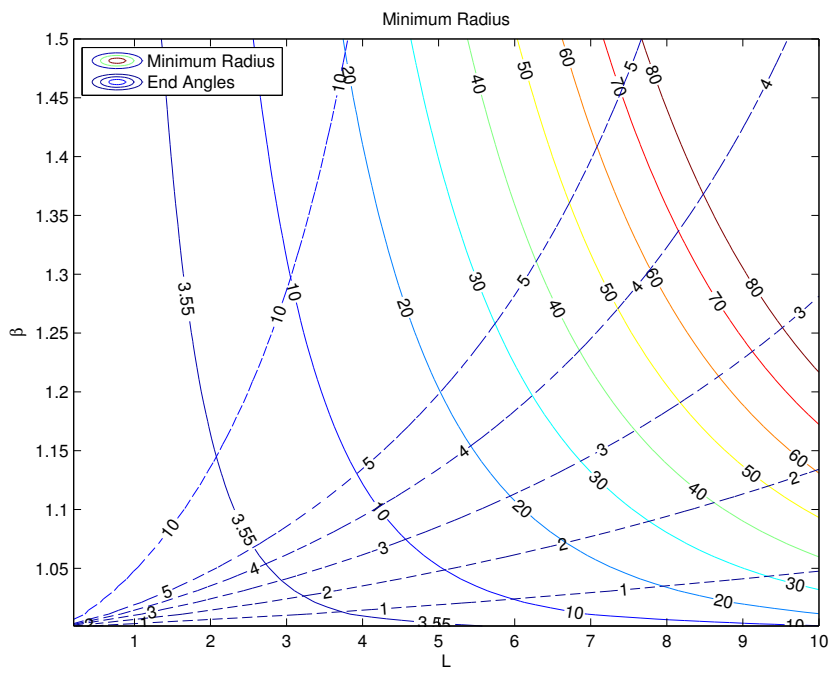


Figure 3: Contour lines of minimum radius R_{min} and end angles θ in two-dimensional design space: solid lines: R_{min} and dashed lines: θ .

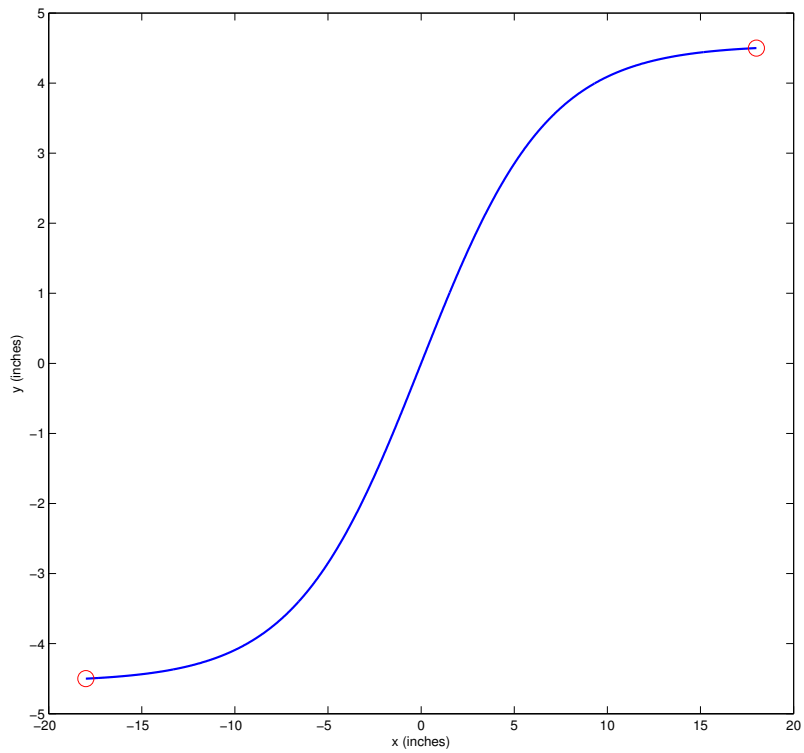


Figure 4: The dimensional centerline curve for cable bending that meets the design requirements. The specific design parameters are: $\beta = 1.01$, $L = 4$ and $\alpha = 2.65$. The dimensional curve relation is $y = 4.545 \tanh(0.1473x)$.