A MODEL OF MARKET POWER IN CUSTOMER MARKETS*

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We develop a model for studying dynamic competition in environments with frictions that lead to partial lock-in of customers to products. The dynamic aspects associated with customer retention and acquisition introduce pricing incentives that do not exist in more traditional, static product markets. The proposed model, while highly stylized, maintains certain symmetry properties that allow us to obtain equilibrium existence and uniqueness. We then study the comparative statics of the model and derive a closed-form relationship between average equilibrium markups and the Herfindahl index. We illustrate how the model can be used by analyzing mergers in such a dynamic environment.

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*We are grateful to three anonymous referees, John Asker (the Editor), Kyle Bagwell, Tim Bresnahan, Jakub Kastl, Michael Riordan, and seminar participants at Northwestern (Searle Symposium), San Andres, Stanford, and Universidad de Chile (TOI 3) for helpful comments and suggestions. Financial support from the Martin Lee Johnson Graduate Fellowship (Somaini) and the National Science Foundation (Einav) is greatfully acknowledged. Einav also thanks the hospitality of the Center for Advanced Study in the Behavioral Sciences at Stanford.

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I. INTRODUCTION

Executives and marketing professionals view customer retention and customer acquisition as first-order objectives across a broad range of industries, such as credit card and insurance, grocery stores and other retail outlets, and business-to-business markets of intermediate goods. Underlying this jargon is presumably the sensible idea that customer base is sticky, and is therefore an important determinant of firms' assets and success. This importance can be driven by various non-exclusive mechanisms, such as idiosyncratic preferences (Bronnenberg, Dube and Gentzkow [forthcoming]), costly search (Hall [2008]), or costly switching (Klemperer [1987]).

As pointed out in the seminal contribution of Klemperer [1987], switching costs provide firms with two offsetting incentives relative to frictionless markets. On one hand, firms have incentives to compete more aggressively in order to acquire new customers, who will be subsequently captured. On the other hand, competition would be softer over customers who are already captured and therefore less price elastic. Our goal in this paper is to develop a simple framework that would allow us to investigate the importance of this issue in the context of imperfect competition and antitrust policy.

Specifically, we develop a stylized model of dynamic oligopoly in customer markets. The setting we propose extends the standard Hotelling model to more than two firms, in a similar way to the pyramidal model proposed by von Ungern-Sternberg [1991] and the spokes model proposed by Chen and Riordan [2007]. Unlike other extensions that are common in the literature, the model maintains certain linearity and symmetry assumptions that produce several attractive properties, which are maintained in the dynamic framework. In particular, we obtain an equilibrium existence and uniqueness result, which lead to unambiguous equilibrium predictions and to sharp and intuitive comparative statics. For instance, in the case of single-product firms the equilibrium prices are perfectly positively correlated with firms' (possibly heterogenous) costs, and perfectly negatively correlated with the market shares. As a result, each of our various welfare measures can be written as a linear function of the mean and variance of costs, market shares, or prices. The theoretical framework we propose is rich enough to capture ideas of market power and imperfect competition, asymmetric firms, dynamic consumers, product differentiation, and multi-product firms. Yet, we make many strong assumptions in order to obtain the equilibrium existence and uniqueness results. We illustrate the possible application of the model in the context of merger analysis in a dynamic environment. For example, similar to the famous result by Cowling and Waterson [1976] in the context of static Cournot competition, we show that equilibrium in our model also gives rise to average markups that are proportional to the Hirschman Herfindahl Index (HHI). We also use the model to analyze the price and welfare effects of mergers, and to assess how important dynamic considerations may be relative to static measures of concentration.

The paper proceeds as follows. In the next section we describe the general setting and its relationship to the existing literature in more detail. Section III analyzes equilibrium in a static context, which is a special case of our subsequent dynamic analysis. Section IV describes the way we introduce dynamics, and Section V defines the equilibrium concept we use and establishes (in

Theorem 1) our main uniqueness results. Sections VI describes the comparative statics properties of the model, and Section VII illustrates how one can use the model by using it to analyze the effect of mergers. The last section concludes. We relegate all proofs and many other technical details to the appendix.

II. GENERAL SETTING AND RELATED LITERATURE

Setting. We consider a spatial setting with $J \geq 2$ horizontally differentiated products. The special case of $J = 2$ of our setting reduces to the familiar linear city (of length L), where consumers are spread along the city and both products located at the edges. However, while authors often use circular city models (with equidistant products) to extend spatial models to cases with more than two products, we propose a different extension. Our setting with J products considers consumers that are spread along the $J(J-1)/2$ segments of the $(J-1)$ simplex, with the products located at the vertices. Thus, with $J = 3$ consumers are located along the three edges of an equilateral triangle, with $J = 4$ along the six segments of a regular tetrahedron, and so on. While we allow products to be associated with different costs of production (see later), we assume that products are identical from the consumers' perspective, so – net of transport cost – all consumers value all products the same and take their locations as given.

Because our main focus is on investigating dynamic price competition, this type of setting has certain attractive features. Most importantly, while circular city models describe a symmetric product space, a given product is always closer to some products and further away from others, leading to high-dimensional off-equilibrium-path strategies. In contrast, our setting makes all competing products equally distant from a given product, leading to (off-equilibrium-path) equilibrium strategies that depend on a single sufficient statistic, dramatically simplifying the analysis.

This "global competition" feature of the model (as opposed to the "local competition" feature of a circular city model, a distinction emphasized by Anderson, de Palma and Thisse [1992]) makes our setting similar to symmetric logit and CES (constant elasticity of substitution) demand models. Our particular formulation, however, gives rise to attractive algebraic features of competition that allow the state variables to enter linearly, facilitating closed-form solutions which would not have been possible with alternative (and perhaps more familiar) models. This attractive algebraic feature is driven by the assumption that the market is fully covered, which we employ throughout the analysis. We also assume throughout that consumers face linear transport cost, which are normalized to one per unit distance.

Throughout the analysis we make the strong simplifying assumption that consumers on a given segment only consider the two products located at the end of the segment.[1] We assume that

[1] This assumpion is primarily made to simplify the analysis. It can be motivated by narrow awareness or consideration sets, or by sufficiently convex transport costs. In the symmetric static version of the model, it is easy to verify that the assumption is not binding in equilibrium. Yet, in the dynamic context there are several potential ways by which one could relax the assumption, so investigating the sensitivity of the results to this assumption in the dynamic context is more open-ended.

consumers are uniformly distributed across segments and along each segment, with density f_J. Provided that the density is constant across segments, it will not play an important role in the subsequent analysis; moreover, letting it be a function of J allows it to capture potential market expansion effects resulting from additional (differentiated) products in the market. It is natural to think of f_J as decreasing in J. Depending on the extent of business stealing associated with additional products, the ratio f_J/f_{J+1} will be between one (no business stealing) and $(J+1)/(J-1)$ (no market expansion). Finally, it will be convenient to define M as the market size, or the number of consumers in the market. With the above assumptions, it is easy to see that $M = \frac{J(J-1)}{2} L f_J$.

Relationship to the literature. From a modeling perspective, our static framework described above is very similar to the one proposed by von Ungern-Sternberg [1991], whose initial description of the model begins as a collection of stochastic Salop circles, but then transitions to describe the product space as a "pyramidal" structure, similar to ours. The focus of his paper is on presenting the setting in a static context, suggesting its usefulness in the context of international trade applications. The setting above is also isomorphic to the spokes model proposed by Chen and Riordan [2007]. They also extend the Hotelling model, but have a different motivation; they only focus on a static analysis, as in von Ungern-Sternberg [1991]. In fact, Chen and Riordan's [2007] spokes model nests the symmetric static version of our model as a special case.

Our primary contribution is the extension and application of this type of setting to study "customer markets" and dynamic price competition. As we describe below, the dynamic extension builds heavily on Doganoglu [2010], who extends a standard Hotelling duopoly setting in a similar fashion to ours. One of our key insights is that the combination of the (static) multi-firm setting proposed by von Ungern-Sternberg [1991] and Chen and Riordan [2007] with the dynamic extension proposed by Doganoglu [2010] is quite attractive from a theoretical standpoint, and it allows us to derive sharp equilibrium results for oligopolistic dynamic price competition. The subsequent extension of the analysis to multi-product firms and cost asymmetries present additional contributions of our paper.

From an economic perspective, our work contributes to the theoretical literature that analyze pricing incentives in markets with switching costs (von Weizsacker [1984], Beggs and Klemperer [1992], Cabral, [2008], Doganoglu [2010], and Dube, Hitsch and Rossi [2010]). Our approach and emphasis are different, however. Much of the existing literature is focused on the analysis of duopolistic competition, and on the question of whether switching costs lead to higher or lower prices. In contrast, we focus on developing a stylized framework that can be applied for markets with more than two, potentially asymmetric firms. Our work is also related to the influential work of Farrell and Shapiro [1990], who analyze the price and welfare effects of mergers in Cournot oligopoly. Unlike them, our framework allows for product differentiation (leading the merged entity to obtain some competitive edge after the merger) and asymmetric firms, thus allowing the exploration of a rich and heterogeneous set of possible mergers.

III. EQUILIBRIUM IN THE STATIC CASE

While our focus is on dynamic consumers and dynamic price competition, we start by illustrating the static case. This may be useful to build intuition and also as a benchmark for our subsequent analysis.

Demand. Given the above setting, demand for product i on each segment $i \leftrightarrow j$ is given by

$$D_{i\leftrightarrow j}(p_i, p_j) = \frac{1}{2}\left(L - p_i + p_j\right) f_J, \tag{1}$$

where L is the length of the segment, f_J is the (uniform) density of consumers along the segment, and p_i is the price of product i.[2] Total demand (across all segments) for product i, as a function of its price and the prices of the other products p_{-i}, is then given by

$$D_i(p_i, p_{-i}) = \sum\nolimits_{j\neq i} D_{i\leftrightarrow j}(p_i, p_j) = \sum\nolimits_{j\neq i} \frac{1}{2}\left(L - p_i + p_j\right) f_J = \frac{J-1}{2}\left(L - p_i + \overline{p_{-i}}\right) f_J, \tag{2}$$

already illustrating how the features of the model make the average price of competing products, $\overline{p_{-i}}$, sufficient to summarize residual demand faced by product i.[3]

Equilibrium with single-product firms. Consider first the case of single-product firms; we analyze a case with multi-product firms later in the paper. We assume that the per-unit cost of producing product i (by firm i) is given by c_i, so firm i sets its price to solve

$$\max_{p_i}\,(p_i - c_i)\, D_i(p_i, p_{-i}) = (p_i - c_i)\,\frac{J-1}{2}\left(L - p_i + \overline{p_{-i}}\right) f_J. \tag{3}$$

The first order condition is given by $L - 2p_i + \overline{p_{-i}} + c_i = 0$, resulting in a best response function of

$$p_i = \frac{1}{2}\left(L + c_i + \overline{p_{-i}}\right). \tag{4}$$

As long as costs are not too heterogeneous, in equilibrium all firms have an internal solution, so that equilibrium prices are given by

$$p_i = \bar{c} + \frac{(J-1)}{2J-1}\left(c_i - \bar{c}\right) + L. \tag{5}$$

In a symmetric case ($c_i = \bar{c}$ for all i), this expression simplifies to $p = \bar{c} + L$. That is, the static equilibrium markup only depends on the length of the segment, or on how differentiated the products are. In the more general case of equation (5), we can still decompose the equilibrium price to two components, with the first driven only by production costs and the second only by product differentiation. This property will extend to the dynamic model, allowing for simple comparative statics.

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[2] We note that the expression for demand derived above is only valid when $\frac{1}{2}\left(L - p_i + p_j\right)$ is in $[0, L]$ for all i and j. We later verify that this condition indeed holds in equilibrium.

[3] Carlson and McAfee [1982] present a model that shares this feature; in their model consumers have heterogenous search costs. The model developed by Vogel [2008] also has a similar equilibrium feature; in his case, this is due to endogenous location choice by firms.

Since prices are strategic complements, equilibrium prices are monotone in production costs. That is, if one firm's costs increase it will raise its own price, leading to further price increases by its competitors; hence, all equilibrium prices will increase. Moreover, the derivative of price with respect to cost is less than one, making equilibrium price dispersion lower than the heterogeneity in costs. These two standard properties of price competition will also extend to the more general dynamic model.

It is instructive to link this model to the Hirschman-Herfindahl Index, $HHI = \sum_i x_i^2$ where x_i is firm i's market share. Using equation (5), equilibrium markups are given by $p_i - c_i = \frac{J(\bar{c}-c_i)}{2J-1} + L$. Substituting for the equilibrium prices, one can also derive the market share of firm i to be $x_i = \frac{1}{JL}\left(L + \frac{J(\bar{c}-c_i)}{2J-1}\right)$. Taken together, notice that $p_i - c_i = x_i JL$ and that

$$\sum\nolimits_i (p_i - c_i)\, x_i = JL \sum\nolimits_i x_i^2 = JL \cdot HHI. \tag{6}$$

Thus, conditional on the number of products, the average markup is proportional to the Herfindahl Index, a similar result to the one derived by Cowling and Waterson [1976] in the context of a Cournot model.[4] In equilibrium, more efficient firms will be able to maintain larger markups and market shares. As a result, both the HHI index and the average markup increase with cost dispersion. We will derive a similar expression for the dynamic model we present below.

IV. INTRODUCING DYNAMICS

We now extend the model to allow for dynamics. We introduce consumer dynamics by borrowing heavily from the work of Doganoglu [2010], who makes similar modeling assumptions in order to develop a model of duopolistic competition with consumer switching in equilibrium. Like Doganoglu [2010], we assume that consumers live for two periods with overlapping generations. Each period of the model a new generation of consumers arrive at the market, so overall demand in a given period is driven by a generation of "young" consumers who are buying for the first time, and a generation of "old" consumers who are buying for the second time and are already affiliated with a certain product. We assume a constant population growth rate g, so that if f_J represents the (uniform) density of old consumers, gf_J will represent the density of young consumers. We note that g could be either greater or less than one.

Consumers who purchase one product in the first period and a different product in the second period have to incur switching costs, which are denoted by s. We also assume that consumers' locations remain on the same segment in both periods, but their specific location *within* the segment is redrawn in the second period independently of where the consumers where located earlier. While this assumption is not as natural (although perhaps can be motivated by a taste shock or a learning story), it is not as crucial either. It essentially introduces smoothness into the residual demand function and is somewhat analogous to any other formulation of noise injected to individual

[4] In a Cournot model with homogeneous products and linear demand $P = A - BQ$ the average markup is equal to: $\frac{J(A-\bar{c})}{(J+1)} HHI$.

demands (from the firms' perspective). This smoothness naturally simplifies the analysis, avoiding discontinuities in the marginal profit function and possible related problems of non-existence of pure-strategy equilibrium.

Following these assumptions, old consumers (those in the second period of their lives) are drawn uniformly over the various segments of the market, but because of the switching costs these consumers are split to those who purchased previously one product (and therefore have an increased incentive to purchase it again) and those who purchased the other product. Static incentives of young consumers (those in the first period of their lives) are as in the static version of the model, but their value function includes the discounted utility they obtain in the subsequent period. We make the natural assumption that consumers know the game firms play and can therefore perfectly predict firms' future pricing behavior.

Demand. Given the assumptions above, demand for product i is generated from three sources: existing (old) consumers of product i, existing (old) consumers of competing products, and new (young) consumers. Deriving demands from old consumers is analogous to the static model, except that it also includes the switching cost s. Thus, demand on segment $i \leftrightarrow j$ from existing (old) consumers of product i and existing (old) consumers of product j is given, respectively, by

$$D^{old,i}_{i\leftrightarrow j}(p_i,p_j) = \frac{1}{2}(L-p_i+p_j+s)f_J \tag{7}$$

$$D^{old,j}_{i\leftrightarrow j}(p_i,p_j) = \frac{1}{2}(L-p_i+p_j-s)f_J. \tag{8}$$

Let $x_{i\leftrightarrow j}$ denote product i's market share among old consumers on segment $i \leftrightarrow j$, so aggregate demand for product i from old consumers is given by

$$\begin{aligned} D^{old}_i(p_i,p_{-i},x_i) &= \sum\nolimits_{j\neq i}\left[x_{i\leftrightarrow j}D^{old,i}_{i\leftrightarrow j}(p_i,p_j)+(1-x_{i\leftrightarrow j})D^{old,j}_{i\leftrightarrow j}(p_i,p_j)\right] \\ &= \sum\nolimits_{j\neq i}\left[x_{i\leftrightarrow j}\frac{1}{2}(L-p_i+p_j+s)f_J+(1-x_{i\leftrightarrow j})\frac{1}{2}(L-p_i+p_j-s)f_J\right] \\ &= \frac{J-1}{2}(L-p_i+\overline{p_{-i}})f_J+\frac{J-1}{2}(2\overline{x_i}-1)sf_J, \end{aligned} \tag{9}$$

where $\overline{x_i}$ is the average share of product i across the different segments. Because product i's overall market share is, mechanically, $x_i = \frac{(J-1)\overline{x_i}}{(J-1)J/2} = \frac{2\overline{x_i}}{J}$, we can express demand from old consumers in terms of x_i, so that

$$D^{old}_i(p_i,p_{-i},x_i) = \frac{(J-1)f_J}{2}\left[(L-p_i+\overline{p_{-i}})+(Jx_i-1)s\right]. \tag{10}$$

An important observation is that the demand from old consumers depends only on $\overline{p_{-i}}$, p_i, and x_i. Moreover, it is linear in these three variables.[5]

Demand from new consumers in segment $i \leftrightarrow j$ is more subtle, as consumers are forward looking and account for the equilibrium effect of current prices on future prices. In Appendix A we show

[5] We note that, as in the static case (see footnote 2), the expressions for demand derived above are only valid when the prices are close enough so that there is an interior solution. We later verify that this condition indeed holds in equilibrium.

the derivation, which leads to

$$D_{i\leftrightarrow j}^{young}(p_i,p_j)=\frac{1}{2}\left(L-p_i+p_j-\delta_c\frac{s}{L}\left(p_i'(\cdot)-p_j'(\cdot)\right)\right)gf_J, \tag{11}$$

which aggregates over segments to

$$D_i^{young}(p_i,p_{-i})=\frac{J-1}{2}\left(L-p_i+\overline{p_{-i}}\right)gf_J-\frac{1}{2}\delta_c\frac{s}{L}\left((J-1)p_i'(\cdot)-\sum\nolimits_{j\neq i}p_j'(\cdot)\right)gf_J. \tag{12}$$

Here, δ_c is the rate at which consumers discount second period utility and $p_i'(\cdot)$ and $p_j'(\cdot)$ are the equilibrium prices consumers (correctly) expect to face in the subsequent period. We intentionally do not specify the variables that enter these pricing functions as we later specify these explicitly as part of our equilibrium definition.

Notice that s, p_i, and c_i are all expressed in monetary units. The transportation cost, which captures the degree of product differentiation in the model, is also expressed in those terms; the transportation cost from one vertex to another vertex along the connecting segment is L monetary units. To economize on notation, we normalize by setting $L=1$; therefore, switching costs s, prices p_i, and production costs c_i should all be interpreted relative to L.

Market shares and welfare under stationary prices. Suppose that consumers anticipate that firms will set prices equal to $p=\{p_i\}_{i=1}^J$ in every future period, as would be the case in a stationary equilibrium. From equation (12) we know the market share of firm i among young consumers would then be

$$x_i=\frac{1}{J}-(1+\delta_c s)\frac{p_i-\bar{p}}{J-1}. \tag{13}$$

Price dispersion will be translated into shares dispersion according to the long-run price sensitivity of young consumers demand, denoted by $\kappa_X\equiv(1+\delta_c s)$. From equation (10), we know that firm i will sell to a fraction m_i of the market in each period, where

$$m_i=\frac{gx_i+D_i^{old}(p_i,p_{-i},x_i)M^{-1}}{1+g}=\frac{1}{J}-\frac{1+(1+\delta_c s)(g+s)}{1+g}\frac{(p_i-\bar{p})}{(J-1)}. \tag{14}$$

We denote by $\kappa_M\equiv\frac{1+(1+\delta_c s)(g+s)}{1+g}$ the long-run price sensitivity of total demand.[6]

Equations (13) and (14) describe the steady state allocation associated with each stationary price vector. We show that these linear relationships between prices and market shares imply that steady state welfare analysis requires only to know three equilibrium outcomes: the mean and the variance of the vector of steady state prices and the covariance between prices and costs, that is: (i) $\bar{p}=J^{-1}\sum p_i$; (ii) $\sigma_p^2=(J-1)^{-1}\sum(p_i-\bar{p})^2$; and (iii) $\sigma_{cp}=(J-1)^{-1}\sum(p_i-\bar{p})(c_i-\bar{c})$.

Welfare analysis in an overlapping generation model with population growth where agents have different discount factors involves non-trivial inter-personal and intertemporal comparisons. We

[6] Both κ_M and κ_X are increasing in s and δ_c. Patient young consumers are more sensitive to steady state price differences in markets where switching costs are high. They prefer to become attached to the least expensive products. Old consumers are even more responsive to stationary prices than young consumers. Old consumers will be more likely to choose the lowest priced product not only because it is cheaper today but also because it is more likely that they purchased it and got attached to it in the previous period. As a result, when there is a small proportion of old consumers (high g) the total market share is less responsive to stationary prices, i.e., κ_M is decreasing in g.

adopt a rather simple approach. Since the market is fully covered in each period and each consumer always ends up consuming exactly one product, we focus on the per-period costs associated to each steady state allocation. Total costs include transportation, switching, and production costs. Consumer costs include transportation and switching costs, and the purchase price. Firms profits are revenues minus production costs. We normalize our measures of costs and profits by the number of consumers in each period: $M(1+g)$.

The allocation associated with state price vector p implies the following transportation and switching costs:

$$TSC(p) = \frac{1+g+(2-s)s}{4(1+g)} + \frac{1+g(1+\delta_c s)^2}{2(1+g)}\sigma_p^2. \tag{15}$$

These costs are increasing with switching costs and with the share of old consumers because only old consumers pay switching costs. Production costs are

$$PC(p) = \sum m_i c_i = \bar{c} - \frac{1+(1+\delta_c s)(g+s)}{(1+g)}\sigma_{cp}. \tag{16}$$

Production costs are decreasing with the covariance of costs and prices. If high-cost firms set high prices, their demand will be lower and their production share will fall, leading to higher efficiency in production. Consider the normalized firms' revenues,

$$R(p) = \sum m_i p_i = \bar{p} - \frac{1+(1+\delta_c s)(g+s)}{(1+g)}\sigma_p^2. \tag{17}$$

Holding the average price $\bar{p}$ constant, price dispersion reduces total revenue as high-price firms have lower demand. Revenues are just transfers from consumers to firms.

Total costs are easily calculated as

$$TSC(p) + PC(p) = \bar{c} + \frac{1+g+(2-s)s}{4(1+g)} + \frac{1+g(1+\delta_c s)^2}{2(1+g)}\sigma_p^2 - \frac{1+(1+\delta_c s)(g+s)}{(1+g)}\sigma_{cp}. \tag{18}$$

For a given vector of marginal costs c and average price $\bar{p}$, the vector of prices that minimizes total costs is such that $(p_i - \bar{p}) = \gamma^*(c_i - \bar{c})$, where the optimal pass-through γ^* is $\frac{1+g+s+\delta_c gs+\delta_c s^2}{1+g(1+\delta_c s)^2} > \frac{1}{2}$ (see Appendix B).

Consumer costs are:

$$TSC(p) + R(p) = \bar{p} + \frac{1+g+(2-s)s}{4(1+g)} - \frac{(1+(1+\delta_c s)(1-\delta_c s)g + 2(1+\delta_c s)s)}{2(1+g)}\sigma_p^2. \tag{19}$$

Consumers are worse off when switching costs are high and when g is low because only old consumers pay switching costs. Price dispersion reduces consumers' costs as they substitute to the less expensive products. As a result, consumers benefit from a high pass-through from costs to prices.

Firms' profits are revenues minus production costs:

$$R(p) - PC(p) = \bar{p} - \bar{c} + \frac{1+(1+\delta_c s)(g+s)}{(1+g)}\left(\sigma_{cp} - \sigma_p^2\right). \tag{20}$$

Firms profits are increasing in the average markup. Higher profits are associated with a high correlation between production costs and prices and low dispersion of prices. For a given vector of marginal costs c and average price $\bar{p}$, the vector of prices that maximizes firms profits is such that $(p_i - \bar{p}) = \frac{1}{2}(c_i - \bar{c})$. In other words, the pass-through that maximizes firms' profits is $\frac{1}{2}$.

The analysis of consumer costs, firms profits, and total welfare (costs) can be decomposed to the analysis of the average, dispersion, and covariance of prices and costs: $\left(\bar{p}, \bar{c}, \sigma_p^2, \sigma_{cp}\right)$. For example, high average prices $\bar{p}$ increase firms' profits and consumers' costs. Higher price dispersion, on the other hand, reduces total welfare, consumer costs and firms' profits. Finally, equation (14) implies that in a steady state, there is a tight relationship between the HHI and the variance of prices. In particular,

$$HHI = \sum m_i^2 = \frac{1}{J} + \frac{\kappa_M^2}{J-1}\sigma_p^2. \tag{21}$$

V. EQUILIBRIUM OF THE DYNAMIC GAME

We model competition as a discrete-time infinite-horizon game, where firms maximize their discounted profits. As emphasized in the introduction, the key driver of dynamic incentives arises from switching costs on the consumer side. In this section we analyze the case of single-product firms. As before, there are J products, each owned by a different firm, i.e. $N = J$. Other than their locations, consumers treat products symmetrically, but marginal cost c_i associated with each product could vary.

Markov perfect equilibrium. Each period each firm can offer a single price to all its consumers and cannot discriminate between young and old consumers or between its own past customers and its competitors'. We restrict attention to a Markov perfect equilibrium (MPE) in which strategies (prices) can only depend on current market shares among old consumers in each segment. That is, the relevant state variables can be described by a vector $\mathbf{x} \in [0,1]^{J(J-1)/2}$ which summarizes the attachments (market shares) of old consumers to firms, along each segment. That is, $x_{i\leftrightarrow j}$, a generic element of $\mathbf{x}$, is the share of old consumers along the segment $i \leftrightarrow j$ (with $i < j$) who have purchased product i in the previous period. Note that product i's overall market share is given by $x_i = \frac{2}{(J-1)J}\sum_{j\neq i} x_{i\leftrightarrow j}$.

Let $\mathbf{D}^{young}(\mathbf{p}) = \left\{D_{i\leftrightarrow j}^{young}(p_i, p_j)\right\}_{j\neq i}$ denote the vector of demands from young consumers and $\mathbf{D}^{old}(\mathbf{p}, \mathbf{x}) = \left\{D_{i\leftrightarrow j}^{old,i}(p_i, p_j | x_{i\leftrightarrow j}^i)\right\}_{j\neq i}$ denote the vector of demand from old consumers. In each period, firms first set prices as a function of the state $\mathbf{x}$. Consumers then make their purchasing decisions according to the demands derived in the previous section, and these choices determine the state variables for the subsequent period, where one simply needs to normalize by the size of the cohort of young consumers, which is Mg, so that state variables evolve according to $\mathbf{x}_{+1}(\mathbf{x}) = \frac{1}{Mg}\mathbf{D}^{young}(\mathbf{p}(\mathbf{x}))$.

In equilibrium, each firm solves the following Bellman equation:

$$V_i(\mathbf{x}) = \max_{p_i}\left[(p_i - c_i)\left(D_i^{young}(p_i, \overline{p_{-i}}) + D_i^{old}(p_i, \overline{p_{-i}}, x_i)\right) + r_f g V_i\left(\frac{1}{Mg}\mathbf{D}^{young}(\mathbf{p}(\mathbf{x}))\right)\right], \tag{22}$$

where r_f is the firms' common *actual* discount factor (which may or may not be the same as consumers'). Notice that because of the population growth rate, the *effective* discount factor is equal to $\delta_f = r_f g$. For the rest of the paper we will refer to δ_f as the firms discount factor and derive comparative statics with respect to it.

Before characterizing the equilibrium properties, it may be useful to develop an intuition regarding the role of switching costs and dynamics in the model. First, notice that if there are no switching cost ($s = 0$) the model reduces to the static case analyzed earlier since there are no payoff relevant intertemporal linkages. The existence of switching costs creates the incentive to extract rents from existing customers: firms with large market shares will find it optimal to charge higher prices. This is the "milking" incentive. On the other hand, forward looking firms also value greater customer base, which would provide an incentive to acquire new customers via lower prices. The larger the discount factor δ_f, the stronger the incentive to invest in customer acquisition and reduce prices. This is the "investment" incentive. On the demand side, forward looking consumers anticipate firms' incentive to milk their existing customers. Therefore, consumers exhibit a weaker response to (off-equilibrium) price changes. This anticipation effect is larger when the discount factor δ_c is large and when the milking incentive is strong. Finally, for a constant δ_f, the market growth rate g governs the composition effect between young, less responsive consumers and old consumers. These are the main incentives and effects that come into play in the model. Notice that all of these effects depend on the existence of some incentives to milk, i.e. positive switching costs $s > 0$.

Indeed, it will be convenient to partition the parameters of the model into two groups. The first includes parameters that would affect the static equilibrium (number of products and ownership structure). The remaining parameters (discount factors, switching cost s, and growth rate g) only matter when switching costs are positive. These will be referred to as the dynamic parameters of the model.

Equilibrium definition and refinement. Within the class of Markov-strategies, we search for an equilibrium in which strategies have a simple structure by applying a similar equilibrium concept to the one used in Beggs and Klemperer [1992]. Specifically, we restrict attention to firms' pricing strategies which are linear in the firms' own market share. As we will see, the setting we propose implies that such an equilibrium exists (and is, in fact, unique). That is, when competing firms have such linear pricing strategies, the optimal strategy by each firm is also linear in its own market share. The following definitions will allow us to state formally the permissible strategies we consider in our search for an equilibrium.

Definition 1 *A Markov strategy is a function from states to actions,* $p_i(\mathbf{x})$*. A linear Markov strategy is a Markov strategy such that* $p_i(\mathbf{x}) = \boldsymbol{\lambda}_i'\mathbf{x}$ *for some vector* $\boldsymbol{\lambda}_i$*. A simple linear Markov strategy is a linear Markov strategy such that* $p_i(\mathbf{x}) = \alpha_i + \beta_i x_i$*.*

Restricting the set of permissible strategies allows us to refine the set of equilibria, and makes the analysis tractable, as it constrains us to work within a linear-quadratic framework. The following definition is a restatement of the Markov perfect equilibrium definition that introduces the refinements that we will use in the paper.

Definition 2 *An MPE in simple linear Markov strategies is a profile of simple linear Markov strategies that yields a Nash equilibrium in every proper subgame.*

Definition 3 *An MPE in parallel strategies is a J+1-tuple* $(\alpha_1, ..., \alpha_J, \beta)$ *such that the profile of simple linear strategies* $\{p_i(\mathbf{x}) = \alpha_i + \beta x_i\}_{i=1}^{J}$ *is an MPE.*

This concept of MPE in parallel strategies is the one we will use as a solution concept for the dynamic game. To be clear, this equilibrium solution is a refinement – not a modification – of the Markov Perfect Equilibria solution concept.

Once firms believe that opponents have linear strategies and that young consumers' beliefs about future prices conform with these strategies, firms' optimal behavior would be, indeed, linear in their market share. We show that there is an equilibrium in which firms play linear strategies that are parallel. That is, all firms' equilibrium strategies have the same slope coefficient β with respect to their own share, but have different intercepts α_i's. Moreover, we show that this equilibrium is unique within this class and derive the equilibrium strategies explicitly.

Equilibrium existence and uniqueness. Each $\{\alpha_i\}_{i=1}^{J}$ and β are unknown constants for which we will solve. Let x'_j be consumers' beliefs about firm j's market share among young consumers (who would become old in the subsequent period). Substituting $p'_j(\cdot) = \alpha_j + \beta x'_j$ (for all j) into equation (12) implies

$$D_i^{young}(p_i, p_{-i}) = \left(\frac{J-1}{2}\left(1 - p_i + \overline{p_{-i}}\right) - \frac{\delta_c s}{2}\left((J-1)\left(\alpha_i - \overline{\alpha_{-i}} + \beta x'_i\right) - \beta \sum\nolimits_{j \neq i} x'_j \right) \right) g f_J, \tag{23}$$

where $\overline{\alpha_{-i}} = \frac{1}{J-1}\sum_{j\neq i}\alpha_j$. Since $\sum_{j\neq i} x'_j = 1 - x'_i$, we obtain

$$D_i^{young}(p_i, p_{-i}) = \frac{J-1}{2}\left(1 - p_i + \overline{p_{-i}}\right) g f_J - \frac{\delta_c s}{2}\left((J-1)\left(\alpha_i - \overline{\alpha_{-i}}\right) + \beta J x'_i - \beta \right) g f_J. \tag{24}$$

Moreover, correct beliefs also imply that

$$x'_i = \frac{1}{Mg} D_i^{young}(p_i, p_{-i}), \tag{25}$$

which together (see Appendix C) imply

$$D_i^{young}(p_i, p_{-i}) = Mg\left(\frac{1}{J} + \frac{(J-1)\left(\left(\overline{p_{-i}} - p_i\right) - \delta_c s\left(\alpha_i - \overline{\alpha_{-i}}\right)\right)}{J\left((J-1) + \delta_c s \beta\right)} \right). \tag{26}$$

The key observation is that the linearity of demand in firm i's own price and its opponents' (average) price is maintained.[7]

Given this demand from young consumers and demand from old consumers, in equilibrium each firm solves the following Bellman equation:

$$V(x_i, x_{-i}) = \max_{p_i} \left[\begin{array}{c} (p_i - c_i)\left(D_i^{young}(p_i, \overline{p_{-i}}) + D_i^{old}(p_i, \overline{p_{-i}}, x_i)\right) + \\ ... + \delta_f V\left(\frac{1}{Mg} D_i^{young}(p_i, \overline{p_{-i}}), \frac{1}{Mg} D_{-i}^{young}(p_i, p_{-i}) \right) \end{array} \right], \tag{27}$$

[7] Again, as with old consumers (see footnote 2), here too we note that the expression for demand derived above is only valid under certain restrictions that we later verify to hold in equilibrium.

taking as given opponents' strategies. However, if $p_j = \alpha_j + \beta x_j$ for each firm j then $\overline{p_{-i}} = \overline{\alpha_{-i}} + \beta \overline{x_{-i}} = \overline{\alpha_{-i}} + \beta \frac{1-x_i}{J-1}$, which only depends (linearly!) on x_i. We can then simplify firm i's problem to only depend on a single state variable x_i:

$$V_i(x_i) = \max_{p_i} \left(\begin{array}{c} (p_i - c_i) \left(D_i^{young} \left(p_i, \overline{\alpha_{-i}} + \beta \frac{1-x_i}{J-1} \right) + D_i^{old} \left(p_i, \overline{\alpha_{-i}} + \beta \frac{1-x_i}{J-1}, x_i \right) \right) + \\ ... + \delta_f V_i \left(\frac{1}{Mg} D_i^{young} \left(p_i, \overline{\alpha_{-i}} + \beta \frac{1-x_i}{J-1} \right) \right) \end{array} \right). \quad (28)$$

Since demand is linear in p_i and x_i (through the linearity in $\overline{p_{-i}}$), the problem has an attractive linear-quadratic structure. As is familiar with such problems, we can then continue by guessing that the value function is quadratic in the state variable,

$$V_i(x_i) = A_i + B_i x_i + C_i x_i^2. \quad (29)$$

We can then solve for the optimal pricing strategy and obtain new expressions for α_i and β, substitute these expression back in equation (28) and find the new expressions for A_i, B_i, and C_i. We first note that the coefficient C_i does not depend on any marginal cost c or price intercept α. Similarly, the resulting slope β does not depend on any of these firm-specific variables either. Both coefficients depend only on the guessed C and β and on parameters of the model $(g, s, \delta_c, \delta_f, J)$. Therefore, in an MPE in parallel strategies the pair (β, C) has to be the same for each and every firm. We find a fixed point (β, C) by the method of undetermined coefficients. In Appendix D we show that finding a fixed point is equivalent to finding the roots of a quartic equation in β with coefficients that depend on the parameters of the model. We show that there is only one root that gives rise to an interior and stationary equilibrium path, i.e. that lies in the interval $\left[-\frac{J-1}{1+\delta_c s}, \frac{J-1}{1-\delta_c s}\right]$.[8] We also show that provided that other firms play non-divergent strategies the profit maximization problem is always concave. Since the roots of a quartic polynomial have closed form solutions, both β and C have closed form solutions. Once we found (β, C) we can calculate $\{\alpha_i, B_i\}_{i=1}^J$ by solving a $2J$-by-$2J$ system of linear equations. Subsequently, we obtain the value function intercept, A_i, for each firm. The following theorem formalizes this result:

Theorem 1 *In the model with $s < L = 1$, $\delta_c < 1$, $\delta_f < 1$, $g > 0$, and $J \geq 2$ there exists an MPE in parallel strategies. Moreover, this equilibrium is unique within the class of MPEs in parallel strategies, and it has a closed form solution. In this equilibrium, the common slope β is positive such that $\beta \in \left[0, \frac{J-1}{1-\delta_c s}\right)$, the value function is convex ($C > 0$) and $\alpha_i = \bar{c} + \mu_0 + \mu_1 (c_i - \bar{c})$, where $\mu_0 > 0$ and $\mu_1 \in (0, 1)$.*

The proof is in Appendix D. Notice that if all firms' costs were to increase by the same magnitude, equilibrium prices will also rise by the same magnitude. However, asymmetric cost variation will not translate completely into price variation. In equilibrium, an increase in firm i's unit cost is only partially compensated by a price increase.

[8] The other three roots are associated with strategies that give rise to divergent dynamics of market shares. If firms expect the vector of market shares to lie on the boundary of the $J-1$th dimensional simplex they will not play linear strategies. Therefore, any MPE in linear strategies should have non-divergent dynamics.

Let us try to provide some intuition. Equilibrium outcomes (β, μ_0, μ_1) depend only on the dynamic parameters and not on the distribution of costs.[9] If switching costs s are positive, the location of the residual demand curve of firm i by old consumers will depend positively on its market share. If g is low, the old consumers represent a large fraction of total population, thus market shares are a more important determinant of the position of the residual demand curve. These two parameters are the key determinants of β, the slope of the equilibrium pricing strategy with respect to own market share. High s and low g lead to high β.

Consumers will realize that a firm that undercuts its prices today will increase its market share in the next period, when it will price higher. If they are patient (high δ_c), they will be less responsive to price changes. As a result, the slope of firms' residual demand curve by young consumers is steeper when δ_c is high.

Except for the traditional tradeoff between price and quantity, firms face a trade-off between pricing low to invest in more locked-in customers that are valuable in the future and pricing high to extract rents from their current attached customers. If firms are patient the investment motive is stronger and they price low (high δ_f leads to low μ_0). If consumers are patient, on the other hand, the slope of firms' residual demand curve will be steeper, and prices will be higher (high δ_c leads to high μ_0). A large g implies that the share of young consumers with steeper residual demand curves is larger and prices are higher (high g leads to high μ_0). A firm with no locked consumers ($x_i = 0$) would face a lower residual demand curve when switching costs are high, thus high switching costs are associated with lower policy intercept μ_0.

The pass-through from costs to prices depends on the slope of the residual demand and marginal revenue curves. A flatter (linear) residual demand curve results in higher pass-through. Therefore, the policy function will be more sensitive to cost differentials when δ_c and g are low (low δ_c and g leads to high μ_1).

Equilibrium prices and welfare. We now discuss the implications of the equilibrium – and in particular the steady state that it gives rise to – for the two primary objects of interest, prices and welfare. As we emphasize throughout, a common theme is that the framework we propose has a simple linear-quadratic structure that makes it easy to describe many equilibrium objects of interest as functions of a small number of summary statistics of the environment: the average, variance and covariance of prices and costs.

By Theorem 1, equilibrium prices are given by

$$p_i(x_i) = \mu_0 + \mu_1 (c_i - \bar{c}) + \bar{c} + \beta x_i. \tag{30}$$

That is, the equilibrium price (and markup) of each firm is a linear combination of its own cost c_i, its own state variable (market share among young consumers) x_i, and the average cost in the industry $\bar{c}$. The average price across firms is:

$$\bar{p} = \bar{c} + \mu_0 + \frac{\beta}{J}. \tag{31}$$

[9] For example, equation (5) shows that if $s = 0$ then $\beta = 0$, $\mu_0 = L = 1$ and $\mu_1 = \frac{(J-1)}{2J-1}$

To derive the steady state, we can substitute $p_i(x_i)$ into the demand from young consumers (equation (26)) and solve for a fixed point, such that $(Mg)^{-1} D_i^{young}(p_i(x_i), p_{-i}(x_{-i})) = x_i$ for every i.

Corollary 2 *Each MPE in parallel strategies has a steady state where*

$$x_i = \frac{1}{J} - \frac{\beta(1+\delta_c s)}{J-1+\beta(1+\delta_c s)}\mu_1(c_i - \bar{c}), \text{ and} \tag{32}$$

$$p_i = \bar{p} + \frac{J-1}{J-1+\beta(1+\delta_c s)}\mu_1(c_i - \bar{c}). \tag{33}$$

The steady state pass-through is $\gamma^e = \frac{J-1}{J-1+\beta(1+\delta_c s)}\mu_1$. Notice that in the steady state equilibrium:

$$\sigma_p^2 = (\gamma^e)^2 \sigma_c^2 \text{ and } \sigma_{cp} = \gamma^e \sigma_c^2. \tag{34}$$

The welfare measures in equations (18), (19), and (20) all depend on $\bar{p}$, pass-through γ^e, and the distribution of costs. The next section analyzes how $\bar{p}$ and γ^e depend on the dynamic parameters and their effect on welfare.

Finally, firms' equilibrium profits can be written as a function of the HHI. Using the results above, it follows that

$$\sum\nolimits_i (p_i - c_i) m_i = \frac{J-1}{\kappa_M} \frac{(1-\gamma^e)}{\gamma^e} \cdot HHI. \tag{35}$$

This expression is similar to the one obtained for the static case in equation (6); the average equilibrium markup is proportional to the Herfindahl Index.

VI. COMPARATIVE STATICS

We illustrate the comparative statics of the model, and in particular how prices and consumer welfare change with the dynamic parameters. We will first focus on the baseline case of symmetric firms with marginal costs normalized to zero. The steady state average price becomes $\mu_0 + \frac{\beta}{J}$ and consumer costs are $\mu_0 + \frac{\beta}{J} + \frac{1+g+(2-s)s}{4(1+g)}$. Next, we analyze separately the effect of cost asymmetries.

Figure 1 illustrates the model predictions as to the way that the price and consumer welfare respond to these primitives. We arbitrarily fix the values of the parameters and let them vary one by one. We plot the equilibrium price (solid line, values on the left y-axis) and consumer costs (dashed line, values on the right y-axis). We do so assuming parameter values of $g = 1$, $J = N = 5$, $\delta_c = 0.7$, $\delta_f = 0.5$, and $s = 0.4$ and solving for the (stationary) equilibrium levels of prices and welfare. Recall that throughout the analysis we normalize the extent of product differentiation, so that $L = 1$. The results below are qualitatively similar for different parameter values.

Panel (a) presents prices and consumer costs as a function of the number of firms, N. As long as the degree of differentiation remains constant, the number of firms has a modest impact on prices and welfare, with the overall patterns replicating standard comparative static results from static models. As mentioned earlier, this is a model in which the product space expands with additional firms, implying that firms have market power even as N goes to infinity, so equilibrium prices converge to a constant which is greater than marginal costs.

Panel (b) graphs prices and consumer costs as a function of market growth rate, showing that faster growing markets are predicted to generate higher prices (and lower consumer costs).[10] A large g means that there are more young consumers than old ones. The marginal young consumer accounts for the fact that lower prices today would provide incentives for higher prices tomorrow and is therefore less price responsive compared to the marginal old consumer (for whom there is no future). Thus, the overall demand is less elastic which leads to less competition in equilibrium. However, as g goes to infinity, the composition effect disappears since young consumers now anticipate that firms that are going to have larger market share in the future are not going to charge higher prices. Therefore, they are as responsive as old consumers. Young consumers never incur switching costs and, as a consequence, consumer costs decline with g despite the fact that average prices increase with g.

In panel (c) we observe that higher discount factor of consumers is associated with higher prices and consumer costs. Patient young consumers will be less price elastic since they anticipate that lower prices today will lead to larger market share and higher prices tomorrow. As a result, competition is softer. The reverse occurs when firms are patient. In such a case, firms will compete fiercely to gain a larger market share in the future which results in lower prices in steady state. The intuition is confirmed by panel (d).

Finally, in panel (e) we show that at $s = 0$ the static results hold and $p = 1$. For small values of s the effect described by Doganoglu [2010] holds: prices are lower than the static benchmark. However, prices are increasing with s for moderate or large values of s. There are two counteracting effects that arise due to changes in s: the investment incentive and the anticipation effect. By the investment incentive, higher s provides incentives to reduce prices and capture a larger market share for the next period. This incentive is stronger if δ_f is high. By the anticipation effect, an increase in s makes young consumers less responsive to current prices and the equilibrium price is higher. The anticipation effect is stronger when both s and δ_c are high. For low values of s, the investment motive prevails (if $\delta_f > 0$) and prices are decreasing in s. As we consider higher values of s, the anticipation effect becomes stronger and prices will eventually increase with s. This will happen for lower values of s if δ_f is low and δ_c is high.

A different way to describe the predictions of the model regarding the two key outcomes of interest (prices and welfare, measured by consumer costs) is to report partial correlations between the primitives of the model and the outcomes of interest. To do so, we generate a grid of the primitives, calculate the equilibrium price for each point on the grid, and then report regression results in which the dependent variables are prices and consumer welfare, and the regressors are various transformations of the model primitives. The results are reported in Table I. One could view these regression results as a possibly useful index. It seems natural to assume that we can observe (or know) the number of firms, the growth rate g, and the discount factors. Moreover, if we

[10] Notice that we are increasing g, while holding constant $\delta_f = r_f g$.

observe churn rate h, we could recover the switching costs. To see this, notice that in equilibrium the churn rate is $h = \frac{1-s/L}{2}$.[11] Therefore, under the normalization ($L = 1$) we obtain $s = 1 - 2h$, and we could simply redefine switching costs in terms of churn rates. Thus, if these are known, one could in principal calculate equilibrium prices (and welfare) using our model or the polynomial approximation.

To sum up, according to our comparative analysis, competitive environments in our baseline model are characterized by a large number of firms, impatient consumers, patient firms, low share of young consumers, and large switching costs. In contrast, less competitive environments are characterized by a small number of firms, patient consumers, impatient firms, a moderately large share of young consumers, and large switching costs. Notice that large switching costs are compatible with both extreme cases of competition, but may interact in important ways with other model primitives. The comparative statics of market growth rate holding constant the r_f are ambiguous because the composition effect is typically anti-competitive, while the investment effect is pro-competitive.

Finally, Corollary 2 and expressions (17) and (19) imply that prices and consumer costs are linearly increasing in the average cost, $\bar{c}$ and linearly decreasing in the product of the squared pass-through rate times the variance of cost, $(\gamma^e)^2 \sigma_c^2$. In appendix B we show that when $s > 0$ the equilibrium pass-through γ^e is less than $\frac{1}{2}$ and that both firms and consumers would be better off if it were larger. For high switching costs environments, a higher pass-through occurs when firms are patient, consumers are impatient and the share of young consumers in the market is large. In sum, the dynamic model exhibits a lower pass-through than the static model, which hurts both consumers and firms. The next section analyzes the effects of mergers on consumer and overall welfare and shows that a merger between two firms may increase the pass-through.

VII. ILLUSTRATIVE APPLICATION: THE EFFECT OF A MERGER

Although algebraically cumbersome, the linear-quadratic structure is tractable to allow us to extend the dynamic model in various directions, and in Appendix E we provide more details. Many such extensions are possible, and we verify for some of them (in Appendix E) that they lead to a similar analysis, although extending Theorem 1 to prove existence and, in particular, uniqueness in these richer settings becomes much more difficult. As one of our primary motivations to developing this model is related to antitrust policy, we highlight in this section an extension of the model that will allow us to address mergers.

Our framework also allows us to introduce mergers or multi-product firms in a relatively elegant way. As before, we fix the product space to include J products with identical values (to the consumers; the products are still associated with different production costs). A multi-product firm owns more than a single product. We further assume that a multi-product firm sets a single

[11] In any given segment, from all the consumers that bought from firm j in the previous period, only a proportion $\left(\frac{L+p_j-p_i+s}{2L}\right)$ will buy again from j. In equilibrium $p_j = p_i$ so the proportion becomes $(1 + s/L)/2$. Therefore, the churn rate is $h = (1 - s/L)/2$. In the general case with asymmetric costs the churn rate also depends on the consumer's discount factor and on the variance of prices: $h = \frac{1-s/L}{2} - (1 + \delta_c s/L) \frac{\sigma_P^2}{L^2} m$.

price that applies to all its products. While the primary reason for this assumption is analytical convenience, we should also note that the assumption can also be motivated by various realistic pricing restrictions. For example, clothing stores of the same chain often do not set different prices (for the same item) at different stores in order to preserve the chain reputation and to avoid logistical complications that would arise with returned merchandise. Grocery chains face a similar constraint, mostly because they distribute identical flyers that advertise in-store prices without knowing which specific store the customer would visit. A firm that owns more than a single product can extract monopoly rents from those consumers who are located along the segment whose edges are owned by the firm. However, as the firm sets a single price and cannot price discriminate across consumers, it will trade off the incentive to increase price and extract monopolistic rents from segments it fully controls and the incentive to decrease prices and be more competitive on other, competitive segments. The more products it owns, the higher the former incentive would be, as in any other pricing model with market power.[12]

It may be informative to contrast the conceptual analysis of mergers in the static version of the model with that of Farrell and Shapiro [1990], who analyze the price and welfare effects of mergers in Cournot oligopoly. A key feature that they emphasize is the output response of competing firms to the merger. The limitation of a homogeneous product framework as in Cournot is that (absent cost synergies) equilibrium forces make the merged entity look like any of the other non-merged firms after the merger, reducing the unilateral incentives to merge (and presumably driving much of the strong and influential results derived in Farrell and Shapiro [1990]). While our framework also delivers this endogenous output response by competing firms, the differentiated products framework implies that the merged entity remains larger and with greater market power due to its increased product variety relative to non-merged firms. This greater product coverage seems an important consideration for many mergers, and will increase the unilateral merger incentives.

Static equilibrium with multi-product firms. Before we move on to analyze the fully dynamic model, we discuss the effects of a merger in the static case. Consider the case of $N = J - 1$ firms, where firms 1 through $N-1$ each own product 1 through $J-2$, respectively, and firm N owns both product $J-1$ and product J. The single-product firms solve the same pricing problem as in the previous (single-product) case, so their best response functions are given by equation (5) derived earlier. Firm N would solve

$$\max_{p_N} \left[(p_N - c_{J-1})\, D_{J-1}(p_{J-1} = p_N, p_J = p_N; p_1, ..., p_{J-2}) + (p_N - c_J)\, D_J(p_{J-1} = p_N, p_J = p_N; p_1, ..., p_{J-2})\right]. \tag{36}$$

Solving for the resulting best response and then for the equilibrium prices (see Appendix F), we

[12] Recall our assumption that consumers value the good enough so that the market is fully covered. With monopoly power over some segments, we now need to also assume that consumers do not value the good enough to provide incentives to the merged entity to forgo all consumers in the competitive segments and sell only to the segment it controls.

obtain that the steady-state equilibrium prices are given by

$$p_i^s = \left(1+\frac{1}{J(J-2)}\right)+\bar{c}+\frac{J-1}{2J-1}(c_i-\bar{c}_s)+\frac{1}{J}(\bar{c}_s-\bar{c}_m) \quad (37)$$

$$p_N^m = \left(1+\frac{(J+1)}{2J(J-2)}\right)+\bar{c}-\frac{(J-3)}{2J}(\bar{c}_s-\bar{c}_m) \quad (38)$$

where p_i^s is the equilibrium price of a single-product firm $i=1,2,...,N-1$, and p_N^m is the equilibrium price of the one multi-product firm N. In these equations, $\bar{c}_s$ is the average cost of all the single-product firms and $\bar{c}_m$ is the average cost of the multi-product firm.

The merger will change the average prices $\bar{p}$, price dispersion σ_p^2, and the covariance between costs and prices σ_{cp}–the three sufficient statistics to describe the effects on our measures of welfare. The unweighted average price of all products sold in the steady state equilibrium after the merger is

$$\bar{p}^m = \left(1+\frac{2J-1}{J^2(J-2)}\right)+\bar{c}+\frac{1}{J^2}(\bar{c}_s-\bar{c}_m). \quad (39)$$

The pre-merger average price is just: $\bar{p}=1+\bar{c}$. The merger has two effects on this average. First, the merger has a direct anti-competitive effect equal to $\frac{2J-1}{J^2(J-2)}$. Intuitively, the merged firm can extract rents from the segment that it fully controls and will have incentives to price higher. Because prices are strategic complements, it will induce an overall increase in prices. Second, if the merger is between two low-cost firms prices will be higher. The intuition behind this result can be illustrated in a simple case with four firms, two high-cost and two low-cost. If the two low-cost firms merge they will have plenty of room to exercise their market power because they will only compete against the two high-cost firms. If the two high-cost firm merge they will still face a very competitive market. Both the direct effect and the effect of the cost differences decrease at a quadratic rate with J.

The variance of prices after the merger is

$$\sigma_p^{2,m} = \frac{(J-1)^2}{(2J-1)^2}\sigma_c^2+\frac{(J-1)}{2(J-2)J^3}-\frac{(J-1)}{J^3}(\bar{c}_s-\bar{c}_m)-\frac{(J-2)(4J-1)(J-1)}{2J^3(2J-1)^2}(\bar{c}_s-\bar{c}_m)^2. \quad (40)$$

The pre-merger variance is just: $\sigma_p^2=\frac{(J-1)^2}{(2J-1)^2}\sigma_c^2$. The merger has two effects on this variance. First, a direct positive effect decreases with J at a cubic rate. Even if all firms have the same cost, the merger increases the variance of costs because the merged entity will have incentives to price higher. Second, the variance of prices will fall if the merger is between two low-cost firms. Before the merger the low-cost firms were charging low prices. After the merger they set higher prices. As a result, the overall price dispersion falls. This effect decreases at a quadratic rate with J.

The covariance between prices and costs is

$$\sigma_{pc}^m = \frac{J-1}{(2J-1)}\sigma_c^2-\frac{1}{J^2}(\bar{c}_s-\bar{c}_m)-\frac{(J-2)}{J^2(2J-1)}(\bar{c}_s-\bar{c}_m)^2. \quad (41)$$

The pre-merger covariance is $\sigma_{pc}=\frac{(J-1)}{(2J-1)}\sigma_c^2$. Notice that if two low-cost firms merge the covariance falls. This effect decreases at a quadratic rate with J.

Mergers harm consumers and benefit firms. These effects are stronger when two low-cost firms merge. A merger between two low-cost firms generates a larger price increase, reduces price dispersion and shifts production towards less efficient firms reducing total welfare. A merger between two high-cost firms increases the covariance between prices and costs, shifts production towards more efficient firms and may increase total welfare.

Figures 2 and 3 illustrate the effect of mergers on average prices, consumer costs and total costs for different values of J and $(\overline{c}_s - \overline{c}_m)$. The results of the static model discussed here are represented by the "x" symbols in the figures. Figure 2 shows the effect on average prices for different values of J holding $\overline{c}_s - \overline{c}_m = 0$, e.g., a merger in a market with $J = 6$ goods increases average prices by 0.074. The effect of the merger on prices is decreasing in J. Figure 3 shows the effect on consumers' and total costs. A merger in a market with $J = 5$ goods where $\overline{c}_s - \overline{c}_m = 0.5$, i.e., when two low-cost firms merge, increases consumers costs by 0.147 and total costs by 0.017. If two high-cost firms merge, $\overline{c}_s - \overline{c}_m = -0.5$, consumers' costs increase only by 0.091 and total costs drop by 0.007. These effects are decreasing in J.

The dynamic analysis. We now analyze the effects of a merger in a dynamic market with switching costs and compare them with those in the static case.

The dynamic model with single-product firms benefitted from linear residual demand functions that were completely determined by the average price charged by the competitors. To solve the dynamic model after a merger, all one needs to observe is that the same linearity properties still apply, but now a sufficient statistic for competition is the type-by-type average of price and market share. That is, a single firm can partition competing firms to types based on the number of products they own, and then condition pricing policies on its own market share among firms of its own type, and on the overall market shares of each type. With K types, this implies K state variables, but the linear-quadratic structure remains. Appendix E provides more details about this particular extension and its solution. To illustrate the analysis and the results, we focus again on the specific case where the initial market structure has $N = J$ single-product firms, while the post-merger structure has $N = J - 2$ such firms and a single merged entity that owns two products. In such a case, the pre-merger equilibrium is our baseline model, while the post-merger equilibrium has firms of two types (the merged firm and everyone else).[13]

The unweighted average price after the merger is

$$\bar{p}^m = p^m_{ss0} + \bar{c} + \zeta^m_0 \left(\bar{c}_s - \bar{c}_m\right), \tag{42}$$

where p^m_{ss0} and ζ^m_0 depend on the dynamic parameters (s, δ_f, δ_c and g) and J. The direct effect of the merger on average prices is the difference between p^m_{ss0} and $\mu_0 + \frac{\beta}{J}$. This difference is positive for all parameter configurations. Average prices are higher when two low-cost firms merge, i.e., $\zeta^m_0 > 0$.

[13] We focus the discussion on comparing the steady state of the market prior to the merger with the post-merger steady state. Our framework also allows us to investigate and analyze the transition from one steady state to the other. As one may expect given the discrete-nature of consumer cohorts in our model, this transitions follows an oscillating pricing (and market share) patterns that (relatively quickly) converge to the new steady state.

The price variance after the merger is

$$\sigma_p^{2,m} = (\gamma^m)^2 \sigma_c^2 - \chi_0^m (\bar{c}_s - \bar{c}_m) + \chi_1^m - \chi_2^m (\bar{c}_s - \bar{c}_m)^2 , \tag{43}$$

where $\gamma^m, \chi_0^m, \chi_1^m, \chi_2^m > 0$ depend on the dynamic parameters. γ^m, the post-merger pass-through, is lower than the pre-merger pass-through γ^e and than the static pass-through $\frac{J-1}{2J-1}$. Price dispersion will be lower if two low-cost firms merge. The covariance between prices and costs after the merger is

$$\sigma_{pc}^m = \gamma^m \sigma_c^2 - \delta_0^m (\bar{c}_s - \bar{c}_m) - \delta_2^m (\bar{c}_s - \bar{c}_m)^2 , \tag{44}$$

where $\delta_0^m, \delta_2^m > 0$ are functions of the dynamic parameters. This covariance will be higher if the merger is between two high-cost firms. The effect of the merger on price average, variance, and covariance with costs works in the same direction as in the static model. The intuition behind the welfare effects of the merger is also very similar to the static case. The magnitude of these effects, however, will be reduced or amplified by dynamic considerations.

To quantify the effect of the merger in dynamic environments we first consider a symmetric cost environment and calculate the effects on the average price and welfare, and then consider the effects of cost asymmetries.

We start with the case of symmetric marginal costs, by computing the merger effect for a range of parameter values that span the entire parameter space. Figure 2 reports the results for the price effect of such mergers, starting from different numbers of initial number of (symmetric) firms. As a way of comparison, recall that when $s = 0$, in which case the model reduces to the static model, the price is equal to 1. The results in Figure 2 are presented so that each set of parameter value is a point, and together these points cover the entire range of possible price effects, for a given $N = J$. Clearly, because the equilibrium outcomes are continuous in the primitives, the implied effects generate compact sets. The key point to notice in Figure 2 is that for relatively concentrated markets ($N < 8$), the number of firms (or products) is the most important variable to determine the merger effect of prices and consumer welfare: for concentrated markets, the ranking of concentration matches the predicted effect of the merger, with no overlap in the range of the predicted effect. In other words, the effect of a merger in an industry with $N < 8$ symmetric firms is greater than the effect in an industry with $N + 1$ symmetric firms regardless of the values of the other market primitives. This in a way can justify the use of a simple Hirschman-Herfindahl Index (HHI) as a quick way to evaluate mergers to the extent that the index reflects $N = J$ (the number of differentiated products in the market).

Once markets become less concentrated, the effect of N becomes smaller and more comparable to the effect of other economic primitives of the market. This can be seen in Figure 2 by the fair amount of overlap in the predicted effects of the merger as we move from $N = 10$ to $N = 11$ and to $N = 12$. This patterns suggests that for markets with intermediate levels of concentration, it may be insufficient to focus on concentration index, as other market primitives may play an equally important role.

It may be also instructive to understand which parameters lead to the greatest and smallest effects of a merger on prices and consumer welfare. The greatest effects (on both prices and consumer costs) arise in less competitive environments, i.e., when $s \to 1$, g moderately large, $\delta_c \to 1$, and $\delta_f \to 0$. The smallest effects arise in competitive environments, when $s \to 1$, $g \to 0$, $\delta_c \to 0$ and $\delta_f \to 1$. When consumers are myopic and firms care more about the future, merger will lead to smaller price and welfare effects. When consumers are forward looking and firms care more about static incentives, the merger effects are greater. Indeed, applying regression analysis to the simulated effects, we find that one of the best way to predict the post-merger outcome is to condition the analysis on pre-merger outcome, which capture in a reduced form way the competitiveness of the industry.

Panel A of Table II shows the result of regressing the post-merger price $\bar{p}^m$ on the pre-merger price $\bar{p}$ for a grid of dynamic parameters. Conditional on N, the per-merger price operates almost as a sufficient statistic for the post-merger price, with all regression having values of R^2 that are essentially one, despite having only two coefficients. The constant coefficient captures the effect of the merger that is independent of the dynamic parameters. The effect of the merger is larger when the number of products/firms is low. The coefficient on $\bar{p}$ shows that higher pre-merger prices are associated to higher post-merger prices. Moreover, these coefficients are greater than one and decreasing in N which implies that the effect of the merger, $\bar{p}^m - \bar{p}$, is larger when the dynamic configuration is such that $\bar{p}$ is high. Panel B of Table II shows a set of similar results for consumer costs. Intuitively, the anti-competitive effect of the merger on prices (consumer costs) is larger in markets where the dynamic parameters are such that the pre-merger equilibrium prices (consumer costs) are higher.

The main effect of the merger is to generate incentives for the merged firm to increase prices. Since prices are strategic complements, all the small firms will also increase their prices. As a result both prices and consumer costs will increase. Because relative prices do change, there will also be an additional effect on consumer costs due to changes in switching and transportation costs. This allocation effect on consumer costs is small relative to the effect of prices and the effect of the merger on consumer costs is essentially the same as the effect on prices. The effect on total costs is positive but small.

Consider now a case with asymmetric marginal costs. Both the price effect and welfare effect depend on whether the production cost of the merged products are low or high relative to the rest of the products in the market. In the static model we found that the price effect of a merger between less efficient firms is smaller. We also found that a such a merger may be efficiency enhancing because it provides additional incentives to the merged entity to increase prices. In the dynamic model, these results also hold but their magnitudes can be different.

Figure 3 presents results for three different cases of mergers, which vary in the production cost of the merged products relative to the production costs of the single-owned products. Specifically, we use three different values (positive, zero, and negative) for $\bar{c}_s - \bar{c}_m$, the difference between the average cost of the merged products and the average costs of the other products. For simplicity, we let $c_i = \bar{c}_s$ for all single-product firm i. We plot the range of possible values that consumer costs and total costs can take under different set of dynamic parameter configurations. The dynamic parameters seem to exacerbate the effects of the cost differences. In markets with high switching costs, the welfare consequences of a merger depend crucially on the cost differences between the merged entity and the rest of the firms.

VIII. CONCLUSIONS

In this paper we proposed a stylized model of oligopolistic competition that allows for dynamic demand, product differentiation, and firm asymmetries. A key aspect of the model is that unlike much of the related literature that focuses on pricing incentives in duopoly, our proposed model can be applied to any general number of firms. Many of our modeling assumptions are made in order to lead to a tractable analysis and sharp equilibrium predictions. Indeed, in our central result (Theorem 1) we show that equilibrium in our model always exists, and that it is unique once we apply certain equilibrium refinements.

We illustrate some of the potential benefits of our model by using it to assess the effect of a merger on pricing and welfare. In particular, we find that despite the dynamic demand and the existence of customer relationship, the single most important primitive that affects prices and welfare is concentration. Yet, we show that once markets are less concentrated, the evaluation of mergers would benefit from further analysis of dynamic aspects of the market, such as the extent of market growth, as well as firms' and consumers' discount factors. Just as concentration measures are broadly used as a "quick" guide in initial evaluations of proposed merger, we think our framework may allow other aspects of market dynamics that are relatively easy to observe (churn rate, market growth rate, etc.) to get incorporated into such preliminary analysis. Of course, this stylized model cannot substitute for a deeper look into specific markets, and further investigation should look at much greater details at the specific institutions and practices of particular markets.

We find that switching costs could make markets either more or less competitive, depending on other dynamic parameters such as discount rates and market growth rate. Customers' markets with patient consumers and inpatient firms are particularly anti competitive. We find that mergers increase the average price, consumer costs, and total costs, and that these effects are amplified in less competitive dynamic environments or when the merger is between two low-cost firms. We also find that mergers between high-cost firms may improve total welfare without any cost synergies. Finally, we observe that if switching costs are high, the effects of the merger become more sensitive to the cost differences between the merged firm and other firms.

Finally, from a pure applied theory perspective, we view our specific extension of the Hotelling framework as attractive. In this paper we illustrate its tractability to address issues associated with dynamic demand. For instance, in our single-product firm model the steady state vector of prices is perfectly positively correlated with the vector of costs and perfectly negatively correlated with the vector of market shares. As a result, each of our various welfare measures can be written as a linear function of the mean and variance of costs, market shares, or prices.

The framework can be extended in various ways, and we think that many of these extensions would maintain many of the tractable equilibrium properties. For example, on the demand side, one can imagine extending the framework to allow for various forms of heterogeneity across consumers, e.g. in their switching costs, transport costs, discount factors, or the (possibly stochastic) number of periods they remain in the market. On the supply side, one could allow firms to price discriminate or to endogenously choose switching costs imposed on consumers. Thus, we hope that further research would find ways to utilize our theoretical framework to investigate other important applied questions.

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Table I

The effect of dynamic parameters (symmetric firms)

Dependent Variable:	Baseline equilibrium price			Baseline equilibrium consumer costs		
	(1)	(2)	(3)	(4)	(5)	(6)
constant	1.059	0.999	1.000	1.381	1.249	1.250
s	-0.108	-0.074	-0.044	0.020	0.280	0.408
g	0.038			-0.004		
n_hat	-0.073			-0.073		
delta_c	0.041			0.041		
delta_f	-0.166			-0.166		
s * g		0.071	0.137		-0.006	-0.086
s * (n_hat)		-0.145	-0.067		-0.145	-0.067
s * delta_c		0.092	0.030		0.092	0.030
s * delta_f		-0.319	-0.479		-0.319	-0.479
s^2		0.114	0.063		-0.015	-0.145
s * g^2			-0.037			0.001
s * g * (n_hat)			-0.023			-0.023
s * g * (delta_c)			0.005			0.005
s * g * (delta_f)			0.096			0.096
s^2 * g			-0.043			0.020
s * (n_hat)^2			0.039			0.039
s * (n_hat) * (delta_c)			-0.043			-0.043
s * (n_hat) * (delta_f)			-0.220			-0.220
s^2 * (n_hat)			0.002			0.002
s * (delta_c)^2			-0.002			-0.002
s * (delta_c) * delta_f			0.006			0.006
s^2 * (delta_c)			0.118			0.118
s * delta_f^2			0.107			0.107
s^2 * delta_f			0.150			0.150
s^3			-0.021			-0.021
R-squared	0.742	0.924	0.991	0.706	0.919	0.989
Number of grid points		100,000			100,000	

We constructed a 100,000 cell-grid where we let $n \in \{2, 3, ..., 9, 10, 20, 100\}$; δ_c, δ_f, s take 10 equi-spaced values between 0.01 and 0.99; and g takes 10 equi-spaced values between 0.025 and 2.5. We construct n_hat $= 1 - 1/n$. In columns (1) and (4) we report a simple linear projection. The sign of the coefficients are consistent with most of our comparative statics except for the effect of switching costs on prices; this restricted specification does not allow for a U-shaped effect.

At $s = 0$, the equilibrium price is equal to $L = 1$ (regardless of the value of other parameters). We use this information in columns (2) and (5) to obtain projections in which the primitives all interact with s. We include a quadratic term for s that allows for a U-shaped effect. Indeed, this improves the fit substantially. For a large set of parameters the approximated price is decreasing for small values of s and increasing for large values of s. The effect on the consumer costs is more likely to be positive, i.e., the competitive effect of higher switching costs in unlikely to compensate for the increase in the real cost of switching. The number of firms has a modest effect on both prices and consumer welfare. The coefficient on δ_c reflects that patient young consumers anticipate future prices and are less responsive to current prices, which relaxes competition. The coefficients on δ_f confirm the intuition that more patient firms have a greater incentive to invest in building a larger customer base which reduces equilibrium prices and consumer costs. The coefficients on g are consistent with the fact that a larger share of young consumers in the market relaxes competition because young consumers are less responsive to price changes.

Columns (3) and (6) report regression results from a richer polynomial approximation. While the comparative statics are not as straightforward, they are still consistent with our previous results. The regression of columns (3) and (6) show that relatively simple parameterizations can capture the model prediction extremely well, with the R^2 getting extremely close to 1.

Table II

The predicted effect of a merger (Symmetric firms)

Panel A

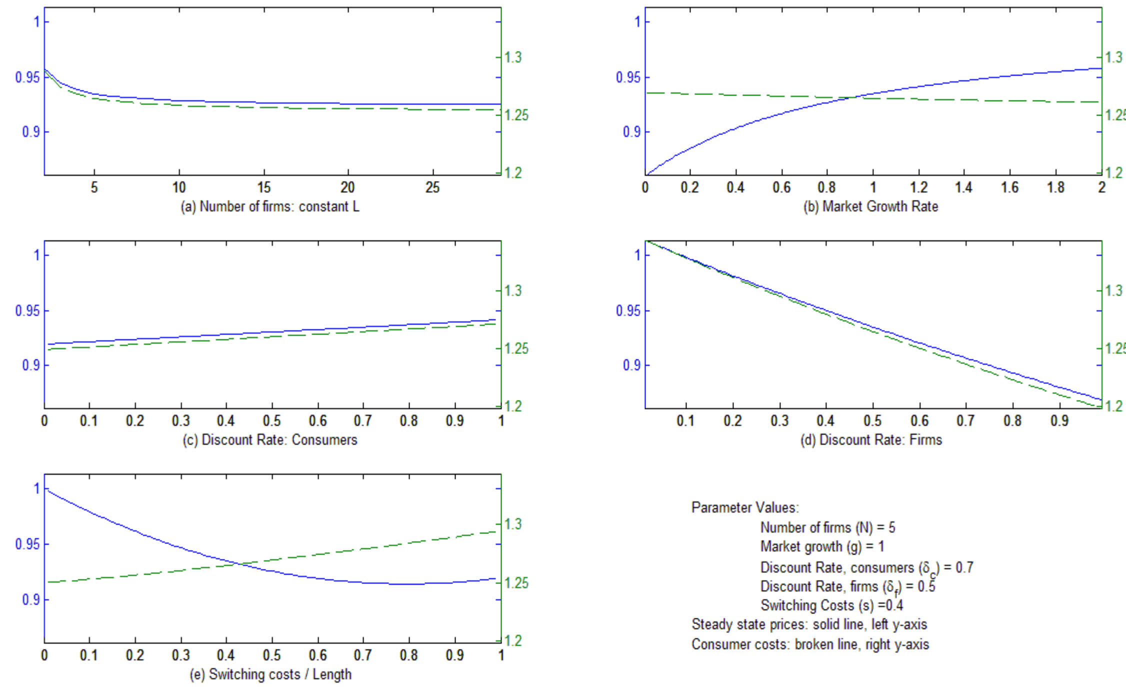
Dependent Variable: Average price post-Merger				
N	constant	Pre-merger price	R-squared	Pre-merger price range
(1)	(2)	(3)	(4)	(5)
3	0.253	1.269	0.989	0.65 - 1.14
4	0.097	1.112	0.998	0.61 - 1.13
5	0.054	1.061	0.999	0.59 - 1.13
6	0.036	1.038	1.000	0.58 - 1.13
7	0.025	1.026	1.000	0.57 - 1.13
8	0.019	1.019	1.000	0.56 - 1.13
9	0.015	1.015	1.000	0.56 - 1.12
10	0.012	1.011	1.000	0.55 - 1.12
11	0.010	1.009	1.000	0.55 - 1.12
12	0.008	1.008	1.000	0.55 - 1.12

Panel B

Dependent Variable: Consumer cost post-merger				
N	constant	pre-merger consumer cost	R-squared	Pre-merger cost range
(1)	(2)	(3)	(4)	(5)
3	0.241	1.220	0.934	1.11 - 1.52
4	0.080	1.099	0.987	1.09 - 1.52
5	0.041	1.057	0.996	1.07 - 1.52
6	0.025	1.037	0.999	1.06 - 1.51
7	0.017	1.026	0.999	1.05 - 1.51
8	0.013	1.020	1.000	1.05 - 1.51
9	0.010	1.015	1.000	1.04 - 1.51
10	0.008	1.012	1.000	1.04 - 1.51
11	0.006	1.010	1.000	1.04 - 1.51
12	0.005	1.008	1.000	1.04 - 1.51

For Panel A, we use the same 100,000 cell-grid as in the regressions reported in Table I to perform a series of simple regression of $\bar{p}^m$, the post-merger price on $\bar{p}$. We run this regression for each value of $N = 3, ...12$. Panel B presents the same regressions for the consumer costs (price paid, transportation and switching costs). The results show that the pre-merger outcome explains almost all the variation in the post-merger outcome due to different dynamic parameters. Notice that the merger effect that is independent of the pre-merger outcome – the constant term is column (2) – is decreasing in N. Moreover, the coefficients in column (3) are greater than one and decreasing in N implying that the effect of mergers on prices (consumers' costs) is larger for dynamic parameter configurations for which the pre-merger price (consumers' costs) is high.

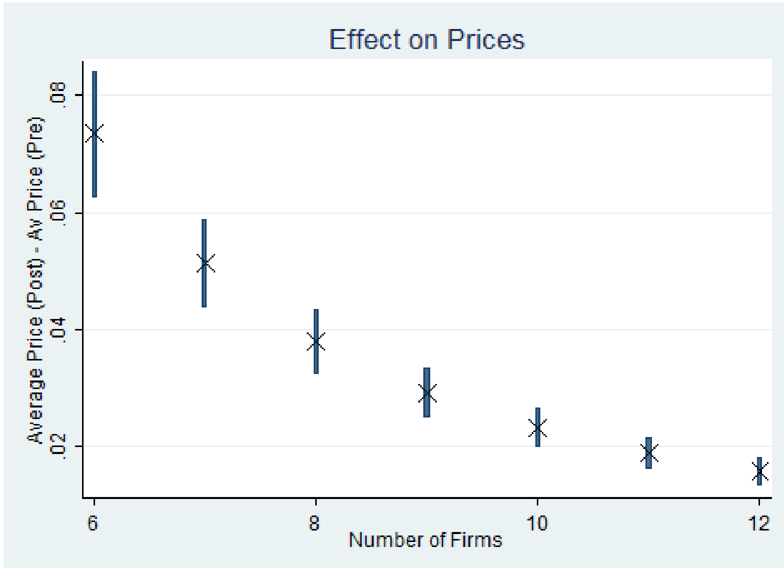
Figure 1
Comparative statics in the baseline model



The figure reports comparative statics of the baseline (symmetric) model. We set the parameter values as indicated at the bottom right corner of the figure, and then let each parameter vary, holding the rest of the parameters fixed. Each graph presents equilibrium price (solid line, values on the left y-axis) and consumer costs (dashed line, values on the right y-axis).

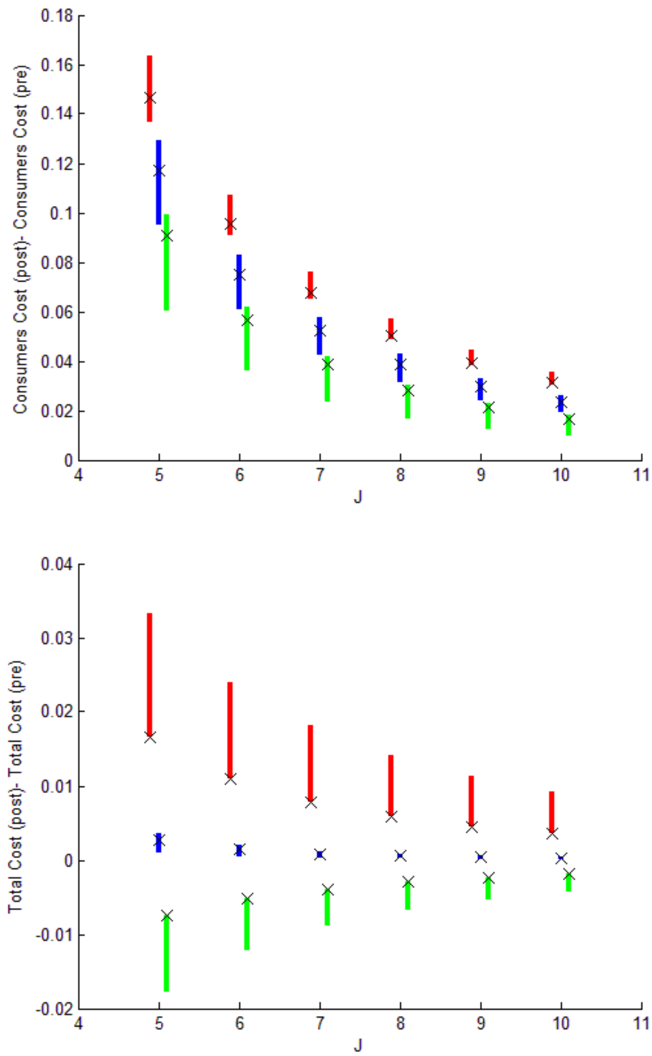
Figure 2

The predicted effect of a merger (symmetric firms)



The figure reports the model's predictions regarding a merger effect on prices (the effect on consumer costs is very similar). To generate it, we use a large grid for the values of the model primitives to calculate the effect of the merger on equilibrium outcomes; the figure reports each vector of parameter values as a point. For a given value of J, the scattered points give rise to the vertical lines in the figure. Our key interest is the extent of overlap in the effect as we change J. That is, the extent to which other economic forces may be greater than the market concentration in predicting the price (and welfare) effect of mergers. The 'x' in each line represents the static case (with $s = 0$).

Figure 3
The predicted effect of a merger (asymmetric firms)



The figure plots the range of merger effects on consumer costs (top panel) and total cost (bottom panel) by number of firms. The top vertical lines in each panel present results for a merger between two low cost firms: $\overline{c}_s - \overline{c}_m = 0.5$. The middle vertical lines in each panel present results for a merger between average cost firms $\overline{c}_s - \overline{c}_m = 0$. The bottom vertical lines in each panel present results for a merger between high cost firms $\overline{c}_s - \overline{c}_m = -0.5$. The 'x' in each line represents the static case (with $s = 0$) in each case. The welfare effects of mergers depend on the cost difference between the merged entity and the rest of the firms, especially when switching costs are high.

APPENDIX

A. Deriving demand from young consumers

In this appendix we derive equation (11) of the paper. Consider the problem of a consumer located on the segment $i \leftrightarrow j$ at a distance θ from i. In the first period she must decide whether to buy product i or product j. Let ψ_i and ψ_j be the expected cost of choosing i and j, respectively:

$$\psi_i = \theta + p_i + \delta_c \mathbb{E}_\kappa \left(\min \left[\kappa + p_i', L - \kappa + s + p_j' \right] \right) \tag{45}$$

$$\psi_j = L - \theta + p_j + \delta_c \mathbb{E}_\kappa \left(\min \left[\kappa + s + p_i', L - \kappa + p_j' \right] \right), \tag{46}$$

where δ_c is the consumers' discount factor, κ is the location of the consumer in the next period, and p_i' (p_j') is the price that firm i (j) will charge in the future. Because there is no uncertainty at the firm level, consumers perfectly anticipate future prices.

The indifferent consumer will be located at θ^*, which is defined by

$$\theta^* = \frac{L - p_i + p_j + \delta_c \mathbb{E}_\kappa \left(\xi(\kappa) \right)}{2}, \tag{47}$$

where $\xi(\kappa) = \min \left[\kappa + s + p_i', L - \kappa + p_j' \right] - \min \left[\kappa + p_i', L - \kappa + s + p_j' \right]$. To compute $\mathbb{E}_\kappa (\xi(\kappa))$ we condition on three different events

$$\mathbb{E}_\kappa (\xi(\kappa)) = \mathbb{E}_\kappa (\xi(\kappa) | A_1) \mathbb{P}(A_1) + \mathbb{E}_\kappa (\xi(\kappa) | A_2) \mathbb{P}(A_2) + \mathbb{E}_\kappa (\xi(\kappa) | A_3) \mathbb{P}(A_3), \tag{48}$$

where:

$$A_1 = \left\{ \kappa \in [0, L] : \kappa \geq \frac{1}{2} \left(L + p_j' - p_i' + s \right) \right\} \tag{49}$$

$$A_2 = \left\{ \kappa \in [0, L] : \kappa \leq \frac{1}{2} \left(L + p_j' - p_i' - s \right) \right\} \tag{50}$$

$$A_3 = \left\{ \kappa \in [0, L] : \kappa \notin A_1 \cup A_2 \right\}. \tag{51}$$

Using the assumption that κ is uniformly distributed on $[0, L]$ we obtain that

$$\begin{array}{ll} \mathbb{E}_\kappa (\xi(\kappa) | A_1) = -s & \mathbb{P}(A_1) = (2L)^{-1} \left(L + p_i' - p_j' - s \right) \\ \mathbb{E}_\kappa (\xi(\kappa) | A_2) = s & \mathbb{P}(A_2) = (2L)^{-1} \left(L + p_j' - p_i' - s \right) \\ \mathbb{E}_\kappa (\xi(\kappa) | A_3) = 0 & \mathbb{P}(A_3) = \frac{s}{L} \end{array}, \tag{52}$$

and it then follows that

$$\mathbb{E}_\kappa (\xi(\kappa)) = -\frac{s}{L} \left(p_i' - p_j' \right) \tag{53}$$

and

$$\theta^* = \frac{L - p_i + p_j - \delta_c \frac{s}{L} \left(p_i' - p_j' \right)}{2}. \tag{54}$$

We note that $D_{i \leftrightarrow j}^{young,i}(p_i, p_j) = \theta^* g f_J$, and we obtain equation (11).

B. The optimal pass-through

In the main text we discuss that transportation and switching costs increase with the variance of prices and that production costs are decreasing with the covariance of prices and costs. The vector of prices that minimizes the sum of total costs in steady state is such that the covariance of costs and prices is maximized while at the same time the variance of prices is minimized. The optimal price vector is aligned with the vector of costs, i.e. for all $i : (p_i - \bar{p}) = \gamma (c_i - \bar{c})$. Finding the optimal price vector is equivalent to finding γ such that:

$$\min_\gamma \frac{1}{2} \left(1 + g (1 + \delta_c s)^2 \right) \gamma^2 - \left(1 + g + s + \delta_c g s + \delta_c s^2 \right) \gamma \tag{55}$$

It is straightforward to show that:

$$\gamma^* = \frac{1 + g + s + \delta_c g s + \delta_c s^2}{1 + g (1 + \delta_c s)^2} \tag{56}$$

The optimal pass-trough coefficient γ^* is decreasing in g. It ranges from one half (when $g \to \infty$, $\delta_c = 1$, $s = 1$) to three (when $g \to 0$, $\delta_c = 1$, $s = 1$). This result might be perplexing at first. It seems that the maximal allocative efficiency should be achieved by setting $\gamma = 1$ and making each and every consumer internalize all the economic costs of her decision. However, that is not correct under our welfare measure. When a young consumer is deciding between two products, she takes into account the expected costs that that decision will cause her in the future and weights those costs according to her discount factor δ_c, so future switching costs are weighted by δ_c. The social planner weights switching costs according to the share of old consumers in the population. This share is $\frac{1}{1+g}$, therefore switching costs are weighted by a decreasing function of g. Because the social planner and consumers weight expected switching costs differently, it should not be surprising that the social optimum does not involve $\gamma = 1$. The intuition that the optimum is achieved by making the consumers internalize the total economic costs of their actions holds only if switching costs are 0 or if $\delta_c g = 1$.

In one extreme, suppose that g is close to zero. The share of old consumers in the economy will be close to 1 and the social planner will only care about minimizing the costs born by old consumers. To minimize switching costs, he would allocate young consumers to the firm that has higher share in steady state, he can do so by setting prices that exaggerate the cost difference for young consumers. In the other extreme, suppose that $g \to \infty$, then in each period the population is composed mainly of young consumers. The social planner will pay little attention to switching costs, he would allocate young consumers to the firm with lower current total cost. Forward looking young consumers react to price differences by a magnitude of $1 + \delta_c s$, the social planner wants them to react to cost differences by a magnitude of 1. Therefore, he would set $\gamma = (1 + \delta_c s)^{-1}$ which is exactly $\lim_{g\to\infty} \gamma^*$.

The allocation that results from γ^* does not achieve the maximum allocative efficiency. The optimal pass-through γ^* was obtained by choosing the set of prices that minimize costs. However, we restricted prices to be the same across consumers. A better allocation could be achieved by a price system in which young and old consumers are charged different prices.

C. Deriving equation (26)

Substituting equation (25) into equation (24) and replacing M by $(J\,(J-1)\,f_J/2)$, we obtain

$$x_i' = \frac{(1 - p_i + \overline{p_{-i}})}{J} - \delta_c \frac{s}{J\,(J-1)} \left((J-1)\,(\alpha_i - \bar{\alpha}_{-i}) + \beta J x_i' - \beta\right) \tag{57}$$

or

$$\begin{aligned} x_i' \left[\frac{J - 1 + \delta_c s \beta}{(J-1)}\right] &= \frac{(1 - p_i + \overline{p_{-i}})}{J} + \frac{\delta_c s\,(\beta - (J-1)\,(\alpha_i - \bar{\alpha}_{-i}))}{J\,(J-1)} = \\ &= \frac{(-p_i + \overline{p_{-i}})}{J} + \frac{(J-1) + \delta_c s\,(\beta - (J-1)\,(\alpha_i - \bar{\alpha}_{-i}))}{J\,(J-1)}. \end{aligned} \tag{58}$$

This leads to

$$x_i' = \frac{(J-1)\,(\overline{p_{-i}} - p_i - \delta_c s\,(\alpha_i - \bar{\alpha}_{-i}))}{J\,(J - 1 + \delta_c s \beta)} + \frac{1}{J}. \tag{59}$$

Thus,

$$D_i^{young}(p_i, p_{-i}) = g\left(\frac{J(J-1)}{2} f_J\right)\left(\frac{(J-1)\,(\overline{p_{-i}} - p_i - \delta_c s\,(\alpha_i - \overline{\alpha_{-i}}))}{J\,(J - 1 + \delta_c s\beta)} + \frac{1}{J}\right). \tag{60}$$

D. Proof of Theorem 1

In the main text we derived the demand from young and old consumers. To avoid carrying unnecessary notation throughout this appendix we make a few normalizations that are without loss of generality. First, we divide both demands by the market size $M = J(J-1)Lf_J/2$. Since the instantaneous payoff is re-scaled, the resulting value function will also be re-scaled by the same factor. Second, we normalize $L = 1$ and interpret switching costs s, unit costs c and prices p in terms of transportation costs, e.g. $s = 0.5$ means that the switching costs are equivalent to the transportation costs incurred from travelling from the middle of a segment to one of the firms in the vertices.

Now we investigate the best response of firm i given that competitors and its future incarnations play parallel strategies: $p_j = \alpha_j + \beta x_j$. The demands given other players' actions are:

$$\tilde{D}_i^{young}(p_i, \boldsymbol{\alpha}, \beta, x) = \left(\frac{1}{J} + \frac{(J-1)\left(\bar{\alpha}_{-i} + \beta \frac{1-x_i}{J-1} - p_i - \delta_c s\left(\alpha_i - \bar{\alpha}_{-i}\right)\right)}{J\left((J-1) + \delta_c s \beta\right)} \right) \quad (61)$$

$$\tilde{D}_i^{old}(p_i, \boldsymbol{\alpha}, \beta, x) = \frac{1 - p_{-i} + \bar{\alpha}_{-i} + \beta \frac{1-x_i}{J-1} - s.}{J} + x_i s. \quad (62)$$

The value function is:

$$V_i\left(x_i\right) = \max_{p_i} \left(\begin{array}{c} \left(p_i - c_i\right)\left(g\tilde{D}_i^{young}(p_i, \alpha_i, \bar{\alpha}_{-i}, \beta, x) + \tilde{D}_i^{old}(p_i, \bar{\alpha}_{-i}, \beta, x)\right) + \\ ... + r_f g V\left(\tilde{D}_i^{young}(p_i, \alpha_i, \bar{\alpha}_{-i}, \beta, x)\right) \end{array} \right) = A_i + B_i x_i + C_i x_i^2. \quad (63)$$

The Theorem assumes that $0 < \delta_c, s < 1$, $J > 2$, $g > 0$ and $0 < \delta_f = r_f g < 1$. The last assumption ensures discounting. A symmetric MPE will be J tuples (α_i, A_i, B_i) and a pair (β, C) such that: $V_i\left(x_i\right) = A_i + B_i x_i + C x_i^2$ satisfies the functional equation for every firm when each firm j follows the linear policy function $\alpha_j + \beta x_j$; and that such policy is a best response when all other firms also play that policy.

The method of undetermined coefficients will leave us with a system of $3J+2$ non-linear equations and $3J+2$ unknowns that might have several solutions. The purpose of this appendix is to show that there is one and only one of those solutions that can constitute an equilibrium. There is an additional qualification that our solution has to satisfy to be an equilibrium. We have to show that the second order conditions hold and that there are no profitable non-local deviations. We are able to prove that the second order condition holds and that in equilibrium each firm faces a concave maximization problem if firms are not allowed to exit a particular segment. A firm exits a segment if it sets a price so high that no young consumers will buy from it in that period. We can rationalize this behavior as in Beggs and Klemperer (1992) by assuming that firms have to pay a sufficiently high exit cost if they decide to exit a segment. It is only for cases with large switching costs that this additional assumption is necessary.

Let us first sketch the next steps, which are detailed below. First, we note that the dynamics implied by any equilibrium policy function (α_i, β) has to satisfy a non-divergence condition that will bound the possible values of β. Second, we show that for each value of β there is only one value of C that satisfies the functional equation, we denote such function as $C_1(\beta)$, more importantly, we show that the solution for C does not depend on any α or c. Third, we show that plugging the resulting $C_1(\beta)$ in the functional equation yields a concave problem for the firm for every β. Fourth, we define a function $\beta' = f(\beta)$ that returns a best response β' for every β employed by other firms and future incarnations of the same firm, we also show that $f(\beta)$ does not depend on any α or c. Fifth, we show that this function is continuous and that it has at least one fixed point in the relevant interval of β. Sixth, we show that there is a unique fixed point in the interval. Seventh, we show how the other coefficients (α_i, A_i, B_i) are uniquely determined for each firm once β and C are known.

1. Lower and upper bounds for β. Since our space state is bounded, a set of linear policies that generates divergent dynamics cannot be an MPE. If all firms employ the policy function $\alpha_i + \beta x_i$, then equation (61) implies that:

$$x_i' = \frac{J - 1 + \beta\left(1 + \delta_c s\right) + (J-1)\left(\left(1 + \delta_c s\right)\left(\bar{\alpha}_{-i} - \alpha_i\right)\right)}{J\left(J - 1 + \beta\delta_c s\right)} - \frac{\beta}{\left(J - 1 + \beta\delta_c s\right)} x_i. \quad (64)$$

Therefore, the dynamics will be non-divergent if and only if $\beta \in \left[\underline{\beta}, \overline{\beta}\right]$ where $\underline{\beta} = -(J-1)/(1-\delta_c s)$ and $\overline{\beta} = (J-1)/(1+\delta_c s)$.

2. Define the function $C_1(\beta)$.Substituting $p_i = \alpha_i + \beta x_i$ in equation (61) and evaluating equation (63) results in a quadratic function of the state x_i. The quadratic coefficient will be linear in C. Equating the quadratic coefficient to C and solving for C yields the following function:

$$C_1(\beta) = \frac{\beta\left(J - 1 + \delta_c s\beta\right)\left((J-1)^2 s + (J-1)\left(1 + g\delta_c s^2\right)\beta - \delta_c s\beta^2\right)}{(J-1)\left(\left(J - 1 + \delta_c s\beta\right)^2 - \delta_f \beta^2\right)}. \quad (65)$$

Notice that this function depends on β and parameters of the model but it does not depend on any α, A, B or c. This function is continuous for all $\beta \in \left[\underline{\beta}, \overline{\beta}\right]$ since the denominator is always positive in that interval. The numerator has two roots lesser than $\underline{\beta}$, one root at 0 and one root in the interval $\left[0, \overline{\beta}\right]$. Therefore, $C_1(\beta)$ is negative for $\beta \in \left[\underline{\beta}, 0\right)$; $C_1(0) = 0$; $C_1(\beta) > 0$ for $\beta \in (0, k)$ and $C_1(\beta) < 0$ for $\beta \in \left(k, \overline{\beta}\right]$.

3. Third Step: Show that the maximand is concave for all (C,β) *such that* $C = C_1(\beta)$ *and* $\beta \in [\underline{\beta},\overline{\beta}]$. For any bounded value function, the maximand in equation (63) is concave if:

$$2\left(g\frac{\partial \tilde{D}_i'^{young}(p_i,\boldsymbol{\alpha},\beta,x)}{\partial p_i}\left(1+\delta_f C\frac{\partial \tilde{D}_i'^{young}(p_i,\boldsymbol{\alpha},\beta,x)}{\partial p_i}\right)+\frac{\partial \tilde{D}_i^{old}(p_i,\boldsymbol{\alpha},\beta,x)}{\partial p_i}\right)<0. \tag{66}$$

After replacing C by $C_1(\beta)$ this expression depends only on β and on the parameters J, δ_f, δ_c, g and s. We show that for all permissible parameter values and for all $\beta \in [\underline{\beta},\overline{\beta}]$, the second derivative of the maximand is concave. The denominator of the expression above is positive and the numerator is a cubic function of β. It can be shown that the intercept of the cubic function is negative and its real roots are all outside the interval $\beta \in [\underline{\beta},\overline{\beta}]$. Therefore, the maximand is concave in the whole interval (the algebraic details are available upon request).

4. Define a function $\beta' = f(\beta)$ *that returns a best response* β' *for every* β *employed by other firms and future incarnations of the same firm.* Given that the maximand is concave we can use the first order conditions to derive the best response from a given firm. Since demands are linear in p_i and x_i, the maximand will be a quadratic form in (p_i, x_i). A_i, B_i and α_i do not appear in the terms where p_i and x_i interact. We obtain the best response $p_i^*(\cdot) = \alpha^*(\cdot) + \beta^*(\cdot)x_i$. While $\alpha^*(\cdot)$ depends on the original $A_i, B_i, C, \alpha_i, \beta$, $\beta^*(\cdot)$ depends only on β and C. Replacing C by $C_1(\beta)$, we obtain $\beta' = f(\beta) = \beta^*(\beta, C_1(\beta))$.

5. $f(\beta)$ *is continuous and has at least one fixed point in the relevant interval of* β. The function $f(\beta) = \frac{Q(\beta)}{P(\beta)}$ is a ratio of a quartic polynomial $Q(\beta)$ over a cubic polynomial $P(\beta)$. $f(\beta)$ is discontinuous at the roots of its denominator. We show that the denominator, $P(\beta)$, is positive for all $\beta \in [\underline{\beta},\overline{\beta}]$ and that its roots are outside the interval. Let $F(\beta) = f(\beta) - \beta = \frac{Q(\beta)-\beta P(\beta)}{P(\beta)}$. A root of the quartic polynomial $R(\beta) = Q(\beta) - \beta P(\beta)$ will be an equilibrium if it is located in the relevant range $[\underline{\beta},\overline{\beta}]$. We show that there is a root of this polynomial in the interval $[0,\overline{\beta}]$ by showing that $R(0) > 0$ and $R(\overline{\beta}) < 0$.

6. Uniqueness. Uniqueness is established by showing that $R(\beta)$ has only one root in the interval $[\underline{\beta},\overline{\beta}]$. Let $R(\beta) = \gamma_0 + \gamma_1\beta + \gamma_2\beta^2 + \gamma_3\beta^3 + \gamma_4\beta^4$, where the γ_k's are a function of the parameters of the model. First, we notice that $R(a) < 0$, $R(\underline{\beta}) > 0$, $R(0) > 0$ and $R(\overline{\beta}) < 0$ where $a = -(J-1)/(\delta_c s) < \underline{\beta} < 0 < \overline{\beta}$. One root is in the interval $(a,\underline{\beta})$ and another one in the interval $(0,\overline{\beta})$. The proof consists in showing that the other two roots are outside the interval $(\underline{\beta},\overline{\beta})$.

The term γ_4 governs the behavior of R as β goes to minus or plus infinity:

$$\gamma_4 = J\delta_c s\left(\delta_f - \delta_c s(2J-1)\right). \tag{67}$$

If $\gamma_4 > 0$ then there can be only one root in $[\underline{\beta},\overline{\beta}]$. This is because the four roots r_k, $k = 1,2,3,4$ are such that $r_1 < a$, $a < r_2 < \underline{\beta}$, $0 < r_3 < \overline{\beta}$ and $\overline{\beta} < r_4$.

If $\gamma_4 = 0$, then $R(\beta)$ is a cubic function with roots in $r_1 < a$, $a < r_2 < \underline{\beta}$ and $0 < r_3 < \overline{\beta}$ (if $\gamma_3 < 0$), or in $a < r_1 < \underline{\beta}$, $0 < r_2 < \overline{\beta}$ and $\overline{\beta} < r_3$ (if $\gamma_3 < 0$). If the function is quadratic, $\gamma_4 = \gamma_3 = 0$, it has two roots: $a < r_1 < \underline{\beta}$, $0 < r_2 < \overline{\beta}$.

If $\gamma_4 < 0$, we know the location of only two real roots of the polynomial. If the other two roots are complex, then our results follows automatically; therefore, we focus in cases where the four roots are real. First, we show that there are no roots in the interval $[\underline{\beta},0]$. For all $\beta \in [\underline{\beta},0]$, $C_1(\beta) < 0$ and for all $\beta \in [\underline{\beta},0]$ and $C < 0$, $\beta^*(\beta,C) > 0$, therefore $f(\beta)$ does not have a fixed point in that interval.

Now, we have to show that for $\gamma_4 < 0$, $R(\beta)$ has only one root in $[0,\overline{\beta}]$. Using Vieta's formulas, we can deduce the location of the other roots once we know the sign of γ_1, γ_2 and γ_3. Since $\gamma_4 < 0$ and $\gamma_0 > 0$, the polynomial has either one or three positive roots. For the cases in the left column there is only one positive root and our result holds. For the four cases in the right column, we need to show that the additional two positive roots lie outside the range $[0,\overline{\beta}]$.

$(\gamma_1,\gamma_2,\gamma_3)$	$(\gamma_1,\gamma_2,\gamma_3)$
$(-,-,-)$	$(-,-,+)$
$(+,-,-)$	$(-,+,+)$
$(+,+,-)$	$(-,+,-)$
$(+,+,+)$	$(+,-,+)$

We show that if $\gamma_1 > 0$ then $\gamma_3 < 0$, which rules out the case $(+,-,+)$. Then, we focus on the cases $(-,-,+)$ and $(-,+,+)$. Notice that at $\beta = 0$, the first derivative is negative (i.e. $R'(\beta) < 0$). We show that this derivative is negative for all $\beta \in \left[0, \overline{\beta}\right]$. To show that, we define the function:

$$T_1(\beta) = \gamma_1 + 2\gamma_2\beta + 3\gamma_3\beta^2. \tag{68}$$

Since $\gamma_4 < 0$, $T_1(\beta) > R'(\beta)$ for all $\beta > 0$. Besides, $\gamma_3 > 0$ and $\gamma_1 < 0$ imply that $T_1(\beta)$ has one positive root and one negative root. We show that $T_1\left(\overline{\beta}\right) < 0$ (i.e. the positive root of $T_1(\beta)$ is greater than $\overline{\beta}$); therefore: $0 > T_1(\beta) > R'(\beta)$ for all $\beta \in \left[0, \overline{\beta}\right]$ and our result follows.

Finally, for the case $(-,+,-)$ we also prove that $R'(\beta) < 0$ for all $\beta \in \left[0, \overline{\beta}\right]$. We define

$$T_2(\beta) = \gamma_1 + 2\gamma_2\beta. \tag{69}$$

Since $\gamma_4, \gamma_3 < 0$, $T_2(\beta) > R'(\beta)$ for all $\beta > 0$. $T_2(\beta)$ is a linear function with negative intercept and positive slope. We show that $T_2\left(\overline{\beta}\right) < 0$; therefore, $0 > T_2(\beta) > R'(\beta)$ for all $\beta \in \left[0, \overline{\beta}\right]$ and our result follows. We have shown that there is a unique $\beta^* \in \left[\underline{\beta}, \overline{\beta}\right]$ that satisfies the functional equation. Since any $\beta \in \left[\underline{\beta}, \overline{\beta}\right]$ implies convergent dynamics, any symmetric MPE in parallel strategies has to satisfy $\beta = \beta^*$.

7. The other coefficients are uniquely determined once β^* is known. Once β^* and $C^* = C_1(\beta^*)$ are determined, the equations for $\{B_i, \alpha_i\}$ form a two-by-two linear system with non-zero determinant. We solve for the unique $\{B^*, \alpha^*\}$ that solves the system. The equation for A_i is a linear equation with one unknown that can be easily solved for once $\{B_i^*, C^*, \alpha_i^*, \beta^*\}$ are determined. After finding β and C, we calculate α_i and B_i for each firm i. The best response intercept α_i is a linear function of the average $\overline{\alpha_{-i}}$ and the own marginal cost c_i. More specifically, α_i is given by

$$\alpha_i = \xi_0 + \xi_1\overline{\alpha_{-i}} + (1 - \xi_1)\, c_i, \tag{70}$$

where $\xi_0 > 0$ and $0 < \xi_1 < 1$ depend only on the dynamic parameters of the model and on the equilibrium values of the pair (β, C). Therefore, in equilibrium these coefficients are the same across firms. Taking the J equations described by equation (70) and solving for α_i yields the equilibrium intercept α_i:

$$\alpha_i = \frac{\xi_0}{1-\xi_1} + \left(\frac{J-1+(2-J)\,\xi_1}{J-1+\xi_1} c_i + \frac{(J-1)\,\xi_1}{J-1+\xi_1}\overline{c_{-i}}\right). \tag{71}$$

$$= \bar{c} + \frac{\xi_0}{1-\xi_1} + \frac{(J-1)(1-\xi_1)}{J-1+\xi_1}(c_i - \bar{c}) \tag{72}$$

Let $\mu_0 = \frac{\xi_0}{1-\xi_1} > 0$, $\mu_1 = \frac{(J-1)(1-\xi_1)}{J-1+\xi_1} \in (0,1)$. Notice that in the static case (when $s = 0$), $\xi_0 = \xi_1 = 0.5$, $\mu_0 = 1$ and $\mu_1 = \frac{(J-1)}{2J-1}$.

E. Discussion and derivations of various extensions to the model

In this appendix we provide details about the general model that allows for asymmetric marginal costs and product ownership. We show that the model allows for a linear policy function and a quadratic value function.

We assume that there are J products owned by K firms. If firm k owns product i we say that $k = K(i)$. As in the symmetric case, consumers are spread (uniformly) along the $J(J-1)/2$ segments of the $(J-1)$ simplex, with the products located at the vertices. We allow each of the $J(J-1)/2$ segments to have a different density, denoted by $f_{i\leftrightarrow j}$, but we still require their length to be equal to a constant $L = 1$. If firm k owns products i and j we say that segment $i \leftrightarrow j$ is trapped by k. The segments that are not trapped by any firm are called competitive. We allow for a firm-specific marginal cost, c_k.

Demand. Demand for firm k can come from four sources. The first three sources are the same as in the baseline model, i.e. three types of consumers located in the competitive segments: existing (old) consumers of own products, existing (old) consumers of competing products and new (young) consumers. The fourth source is demand from young and old consumers located in segments trapped by the firm k.

Deriving demands from old consumers in competitive segments is straightforward. Demand on segment $i \leftrightarrow j$ from existing (old) consumers of product i and existing (old) consumers of product j is given, respectively, by

$$D^{old,i}_{i\leftrightarrow j}(p_{K(i)}, p_{K(j)}) = \frac{1}{2}\left(1 - p_{K(i)} + p_{K(j)} + s\right) f_{i\leftrightarrow j} \quad (73)$$

$$D^{old,j}_{i\leftrightarrow j}(p_{K(i)}, p_{K(j)}) = \frac{1}{2}\left(1 - p_{K(i)} + p_{K(j)} - s\right) f_{i\leftrightarrow j}. \quad (74)$$

Let $x_{i\leftrightarrow j}$ denote product i's market share among old consumers on segment $i \leftrightarrow j$, so aggregate demand for firm $k = K(i)$ from old consumers in competitive segments is given by:

$$\begin{aligned} D^{old}_k(p_k, p_{-k}, x_k) &= \sum\nolimits_{i:K(i)=k} \sum\nolimits_{j:K(j)\neq k} x_{i\leftrightarrow j} D^{old,i}_{i\leftrightarrow j}(p_i, p_j) + (1 - x_{i\leftrightarrow j}) D^{old,j}_{i\leftrightarrow j}(p_i, p_j) \\ &= \sum\nolimits_{i:K(i)=k} \sum\nolimits_{j:K(j)\neq k} \frac{f_{i\leftrightarrow j}}{2}\left(1 - p_k + p_{K(j)} - s + 2x_{i\leftrightarrow j} s\right) \\ &= f_k\left(\frac{1 - p_k - s}{2}\right) + \sum_{m\neq k} \frac{f_{m\leftrightarrow k} p_m}{2} + sfx_k \end{aligned} \quad (75)$$

where $f_k = \sum_{i:K(i)=k} \sum_{j:K(j)\neq k} f_{i\leftrightarrow j}$, $f_{m\leftrightarrow k} = \sum_{i:K(i)=k} \sum_{j:K(j)=k} f_{i\leftrightarrow j}$ and $f = \sum_{k=1}^{K} \frac{f_k}{2}$. Notice that f_k, $f_{m\leftrightarrow k}$ and f denote the mass of consumers located, respectively, in competitive segments in which k is one of the competing firm, in segments disputed by m and k, and in all competitive segments. Finally, $x_k = f^{-1} \sum_{i:K(i)=k} \sum_{j:K(j)\neq k} f_{i\leftrightarrow j} x_{i\leftrightarrow j}$ is firm k's market share among consumers in competitive segments. An important observation is that the demand from old consumers depends only on p_k, x_k and some weighted average of competitors' prices. Moreover, it is linear in these three variables.

We showed before that the demand from young customers in a competitive segment $i \leftrightarrow j$ is given by:

$$D^{young,i}_{i\leftrightarrow j}(p_{K(i)}, p_{K(j)}) = \frac{1}{2}\left(1 - p_{K(i)} + p_{K(j)} - \delta_c s\left(p'_{K(i)}(\cdot) - p'_{K(j)}(\cdot)\right)\right) g f_{i\leftrightarrow j} \quad (76)$$

which aggregates to

$$\begin{aligned} D^{young}_k(p_k, p_{-k}) &= g \sum\nolimits_{i:K(i)=k} \sum\nolimits_{j:K(j)\neq k} \frac{1}{2}\left(1 - p_k + p_{K(j)} - \delta_c s\left(p'_k(\cdot) - p'_{K(j)}(\cdot)\right)\right) f_{i\leftrightarrow j} \\ &= g\left(f_k \frac{1}{2}(1 - p_k) + \sum_{m\neq k} \frac{f_{m\leftrightarrow k} p_m}{2} - \frac{\delta_c s}{2}\left(f_k p'_k(\cdot) - \sum_{m\neq k} f_{m\leftrightarrow k} p'_m(\cdot)\right)\right). \end{aligned} \quad (77)$$

As before, δ_c is the rate at which consumers discount second period utility and $p'_k(\cdot)$ and $p'_m(\cdot)$ are the equilibrium prices consumers believe to face in the subsequent period.

Equilibrium. We solve for an MPE where firms' strategies are a linear function of the vector of market shares among old consumers. Once firms believe that opponents have such linear strategies and that young consumers' beliefs about future prices conform with these strategies, firms' optimal behavior would be, indeed, linear in the vector of market shares. Thus, we assume that firms' equilibrium strategies follow this linear form $p_k = \mathbf{p}'_k \mathbf{x}$ where $\mathbf{p}_k, \mathbf{x} \in \mathbb{R}^K$ and $\mathbf{1}'\mathbf{x} = 1$ ($\mathbf{1}$ is an m-dimensional vector of ones). Since the state $\mathbf{x}$ is in the unit simplex we have that for any scalar a and $\mathbf{b} \in \mathbb{R}^K$: $a + \mathbf{b}'\mathbf{x} = (a\mathbf{1} + \mathbf{b})'\mathbf{x}$. Therefore, our definition of a linear policy function can accommodate an intercept.

We are going to derive firm k best response given that of all other firms and future incarnations of all firms play a linear strategy. Substituting $p_m = \mathbf{p}'_m \mathbf{x}$ in equation (75) yields:

$$\begin{aligned} D^{old}_k(p_k, \mathbf{p}_{-k}, \mathbf{x}) &= f_k\left(\frac{1 - p_k - s}{2}\right) + \sum_{m\neq k} \frac{f_{m\leftrightarrow k} \mathbf{p}'_m \mathbf{x}}{2} + sfx_k \\ &= \left(f_k\left(\frac{1 - s}{2}\right)\mathbf{1} + \sum_{m\neq k} \frac{f_{m\leftrightarrow k} \mathbf{p}_m}{2}\right)' \mathbf{x} - \left(\frac{f_k}{2}\right) p_k + (sf)\, x_k \\ &= \mathbf{d}^{old}_{k,x} \mathbf{x} - d^{old}_{k,p} p_k \end{aligned} \quad (78)$$

where $\mathbf{d}_{k,x}^{old} \in \mathbb{R}^K$ is a row vector and $d_{k,p}^{old}$ is a scalar. The demand from young consumers in equation (77) becomes:

$$D_k^{young}(p_k, \mathbf{p}_{-k}, \mathbf{x}, \mathbf{x}_{+1}) = g\left(\frac{f_k}{2}(1-p_k) + \sum_{m\neq k}\frac{f_{m\leftrightarrow k}p_m}{2} - \frac{\delta_c s}{2}\left(f_k p_k'(\cdot) - \sum_{m\neq k} f_{m\leftrightarrow k} p_m'(\cdot)\right)\right) \tag{79}$$

$$= g\left(\frac{f_k}{2}(1-p_k) + \sum_{m\neq k}\frac{f_{m\leftrightarrow k}\mathbf{p}_m'\mathbf{x}}{2} - \frac{\delta_c s}{2}\left(f_k \mathbf{p}_k - \sum_{m\neq k} f_{m\leftrightarrow k}\mathbf{p}_m\right)'\mathbf{x}_{+1}\right) \tag{80}$$

$$= g\left(\frac{f_k}{2}\mathbf{1} + \sum_{m\neq k}\frac{f_{m\leftrightarrow k}\mathbf{p}_m}{2}\right)'\mathbf{x} - g\frac{f_k}{2}p_k - g\frac{\delta_c s}{2}\left(f_k\mathbf{p}_k - \sum_{m\neq k} f_{m\leftrightarrow k}\mathbf{p}_m\right)'\mathbf{x}_{+1}$$

where $\mathbf{x}_{+1}$ is next period's state. Because the entire vector $\mathbf{x}_{+1}$ enters the value function, we also need to derive the effect of p_k on the whole vector of market shares. In particular for $m \neq k$:

$$D_m^{young}(p_k, \mathbf{p}_{-k}, \mathbf{x}, \mathbf{x}_{+1}) = g\left(\frac{f_m}{2}\mathbf{1} - \frac{f_m}{2}\mathbf{p}_m + \sum_{h\neq m,k}\frac{f_{m\leftrightarrow h}\mathbf{p}_h}{2}\right)'\mathbf{x} + g\frac{f_{k\leftrightarrow m}}{2}p_k - g\frac{\delta_c s}{2}\left(f_m\mathbf{p}_m - \sum_{h\neq m} f_{m\leftrightarrow h}\mathbf{p}_h\right)'\mathbf{x}_{+1}. \tag{81}$$

Let $\mathbf{y} = \left\{D_m^{young}(p_m, \mathbf{p}_{-m}, \mathbf{x}, \mathbf{x}_{+1})\right\}_{m=1}^K$ be the vector of demand from young consumers. Notice that $\mathbf{x}_{+1} = \mathbf{y}(gfL)^{-1}$ and that we can write:

$$\begin{aligned}\mathbf{y} &= \mathbf{D}_{k,x}^{young}\mathbf{x} + \mathbf{D}_{k,x_{+1}}^{young}\mathbf{x}_{+1} + \mathbf{d}_{k,p}^{young}p_k \\ &= \mathbf{D}_{k,x}^{young}\mathbf{x} + \mathbf{D}_{k,x_{+1}}^{young}\mathbf{y}(gfL)^{-1} + \mathbf{d}_{k,p}^{young}p_k \\ &= \left(\mathbf{I} - \mathbf{D}_{k,x_{+1}}^{young}(gfL)^{-1}\right)^{-1}\left(\mathbf{D}_{k,x}^{young}\mathbf{x} + \mathbf{d}_{k,p}^{young}p_k\right)\end{aligned} \tag{82}$$

where $\mathbf{D}_{k,x_{+1}}^{young}, \mathbf{D}_{k,x}^{young}$ are K-by-K matrices and $\mathbf{d}_{k,p}^{young} \in \mathbb{R}^K$ is a row vector. The kth element of $\mathbf{y}$ is $y_k = \mathbf{d}_{k,x}^{young}\mathbf{x} - d_{k,p}^{young}p_k$.

We have shown that demand from young and old consumers can be expressed as a linear functional of the current state, $\mathbf{x}$, and firm k price, p_k. Given this demand from young consumers and demand from old consumers, each firm solves the following problem:

$$V_k(\mathbf{x}) = \max_{p_k}\left(\begin{array}{c}(p_k - c_k)\left((1+g)f_{k\leftrightarrow k} + \left(\mathbf{d}_{k,x}^{young} + \mathbf{d}_{k,x}^{old}\right)\mathbf{x} - \left(d_{k,p}^{young} + d_{k,p}^{old}\right)p_k\right) \\ +\delta_f V_k\left(\left((gfL)\mathbf{I} - \mathbf{D}_{k,x_{+1}}^{young}\right)^{-1}\left(\mathbf{D}_{k,x}^{young}\mathbf{x} + \mathbf{d}_{k,p}^{young}p_k\right)\right)\end{array}\right), \tag{83}$$

where c_k is the firm specific unit cost and $f_{k\leftrightarrow k} = \sum_{i:K(i)=k}\sum_{j:K(j)=k}\frac{f_{i\leftrightarrow j}}{2}$ is such that $f_{k\leftrightarrow k}$ is the mass of consumers in segments trapped by k.

We show that the value function that satisfies this functional equation is a quadratic form: $V_k(\mathbf{x}) = \mathbf{x}'\mathbf{V}_k\mathbf{x}$. Since the state $\mathbf{x}$ is in the unit simplex we have that for any scalar a, $\mathbf{b} \in \mathbb{R}^K$ and any K-by-K matrix $\mathbf{C}$: $a + \mathbf{b}'\mathbf{x} + \mathbf{x}'\mathbf{C}\mathbf{x} = \mathbf{x}'\left(a\mathbf{1}\mathbf{1}' + 0.5\mathbf{b}\mathbf{1}' + 0.5\mathbf{1}\mathbf{b}' + \mathbf{C}\right)\mathbf{x}$.

Let:

$$\mathbf{d}_{k,x} = \mathbf{d}_{k,x}^{young} + \mathbf{d}_{k,x}^{old} + (1+g)f_{k\leftrightarrow k} \tag{84}$$

$$d_{k,p} = d_{k,p}^{young} + d_{k,p}^{old} \tag{85}$$

$$\mathbf{Z}_{k,x} = \left((gfL)\mathbf{I} - \mathbf{D}_{k,x_{+1}}^{young}\right)^{-1}\mathbf{D}_{k,x}^{young}\mathbf{x} \tag{86}$$

$$\mathbf{z}_{k,p} = \left((gfL)\mathbf{I} - \mathbf{D}_{k,x_{+1}}^{young}\right)^{-1}\mathbf{d}_{k,p}^{young}p_k \tag{87}$$

then

$$\begin{aligned}V_k(\mathbf{x}) &= \max_p\left((p_k - c_k)(\mathbf{d}_{k,x}\mathbf{x} - d_{k,p}p_k) + \delta_f(\mathbf{Z}\mathbf{x} + \mathbf{z}p_k)'\mathbf{V}_k(\mathbf{Z}\mathbf{x} + \mathbf{z}p_k)\right) \\ &= \max_p\left(\left(\delta_f\mathbf{z}'\mathbf{V}_k\mathbf{z} - d_{k,p}\right)p_k^2 + \left(\left(2\delta_f\mathbf{z}'\mathbf{V}_k\mathbf{Z} + \mathbf{d}_{k,x}\right)\mathbf{x} - c_k d_{k,p}\right)p_k - c_k\mathbf{d}_{k,x}\mathbf{x} + \delta_f\mathbf{x}'\mathbf{Z}'\mathbf{V}_k\mathbf{Z}\mathbf{x}\right)\end{aligned} \tag{88}$$

If this function is concave, the optimal price is linear and given by:

$$p_k = -\frac{\left(\left(2\delta_f \mathbf{z}'\mathbf{V}_k\mathbf{Z} + \mathbf{d}_{k,x}\right)\mathbf{x} - c_k d_{k,p}\right)}{2\left(\delta_f \mathbf{z}'\mathbf{V}_k\mathbf{z} - d_{k,p}\right)} = \tilde{\mathbf{p}}_k'\mathbf{x} \tag{89}$$

which yields a quadratic value function:

$$\begin{aligned} V_k(\mathbf{x}) &= \mathbf{x}'\tilde{\mathbf{p}}_k\left(\delta_f \mathbf{z}'\mathbf{V}_k\mathbf{z} - d_{k,p}\right)\tilde{\mathbf{p}}_k'\mathbf{x} + \mathbf{x}'\tilde{\mathbf{p}}_k\left(2\delta_f \mathbf{z}'\mathbf{V}_k\mathbf{Z} + \mathbf{d}_{k,x}\right)\mathbf{x} + \\ &\quad + \delta_f \mathbf{x}'\mathbf{Z}'\mathbf{V}_k\mathbf{Z}\mathbf{x} - c_k\left(\left(d_{k,p}\tilde{\mathbf{p}}_k' + \mathbf{d}_{k,x}\right)\mathbf{x}\right) \\ &= \mathbf{x}'\tilde{\mathbf{V}}_k\mathbf{x} \end{aligned} \tag{90}$$

Then, if there are no non-local deviations: (i) the relevant state space is the $K-1$ dimensional unit simplex: $\mathbf{X} = \left\{\mathbf{x} \in \mathbb{R}^K : 0 \leq x_k \leq 1, \sum x_k = 1\right\}$; (ii) the best response to linear policy functions,$\mathbf{p}_k'\mathbf{x}$, is a linear policy function: $\tilde{\mathbf{p}}_k'\mathbf{x}$; and (iii) the resulting value function is a quadratic form: $\mathbf{x}'\mathbf{V}\mathbf{x}$.

The best response of a linear policy function will be linear if there are no profitable non-local deviations. In particular, firms might decide to give up in one market and exploit the locked in consumers, specially consumers in trapped segments. One way to avoid this type of deviation is to bound consumer values. Our assumption of covered markets is based on a more primitive assumptions of sufficiently high consumer values. Now, we have to assume that values are high enough for markets to be fully covered but low enough such that the firm does not find it profitable to forgo all other markets in order to milk trapped consumers. The maximum consumer valuation that prevents this deviation can be easily derived once we obtained the equilibrium policies. A second way to avoid these deviations is to assume that firms have to pay a sufficiently high exit cost if they decide to exit a segment.

Computation of the equilibrium. So far we do not have existence or uniqueness results for the asymmetric case. However, we expect that the results in the symmetric case translate into similar results for the asymmetric case. We are able to solve numerically for an equilibrium in the asymmetric case. To do that we proceed as follows:

1. Classify the K firms into J types such that firms within each type are symmetric.
2. Set initially $\mathbf{p}_k' = \mathbf{0}$.
3. Obtain the best response for each type of firm $\tilde{\mathbf{p}}_j'$.
4. Set $\mathbf{p}_k' = \mathbf{p}_{J(k)}'$ and repeat steps 3-4 until policies converge.
5. Check that the solution implies convergent dynamics and satisfies second order conditions.
6. Obtain the maximum consumer values such that no firm is willing to milk captured consumers.

Specialization to the case of a symmetric model with a merger. We focus on the specific case of a merger occurring in a symmetric market. The initial market structure has $N = J$ symmetric single-product firms, while the post-merger structure has $J-2$ such firms and a single merged entity that owns two products. If $k = N$ is the merged entity, $f_{N\leftrightarrow N} = f_{k\leftrightarrow l} = f_J$ and $f_{N\leftrightarrow k} = 2f_J$ for any $k, l \neq N$. Moreover, $f_N = (J-2)\,2f_J$, $f_k = (J-1)\,f_J$ and $f = (J-2)(J+1)\,f_J/2$. By symmetry of the $J-2$ firm we can assume that the merged firm reacts symmetrically to the other $J-2$ firms. Therefore, its policy function will depend only on its own share. However, the policy function of each of the $J-2$ small firms will depend not only on their own share but also on the merged firm share.

Once we solved numerically for the equilibrium policies and prices we can compute steady state welfare and profits.

Steady State Equilibrium: we derive the mean and variance of prices in the steady state equilibrium to inform the welfare analysis. To calculate the effect of total costs we also need the steady state covariance between prices and costs.

We have shown that the general multi-product firm model has a linear-quadratic structure which results in linear policy functions. A parallel equilibrium may not exist because the demand structure is asymmetric. The merged entity is going to be a larger firm; therefore, its price is going to have a larger effect on the residual demand curve of its competitors. In equilibrium, the merged entity price according to its share; therefore, competitors are going to condition their prices on their own shares and on the merged entity share. If we introduce more asymmetries on the demand side, we should expect more complex equilibrium pricing behavior.

In the general multi-product firms case, the equilibrium is characterized by two J-by-J matrices B and E such that the equilibrium prices are given by

$$\mathbf{p} = B\mathbf{x} + E\mathbf{c} \tag{91}$$

where $\mathbf{p}$ denotes the price vector, $\mathbf{x}$ is the vector of market shares and $\mathbf{c}$ is the vector of costs. Notice that in the case of a merger between firms j and k, if we additionally impose that the merged entity has to charge the same price for the two products it owns, then the jth and kth rows of B are going to be the identical. Similarly, the jth and kth rows of E are going to be the identical too.

As seen in equation (13), for a steady state vector of prices $\boldsymbol{\rho}$, the market share among young consumers is

$$\mathbf{x} = \eta - \frac{\kappa_X}{J-1} M\mathbf{p} \tag{92}$$

where M is such that $y - \bar{y} = My$ and η is a vector of $1/J$. Plugging (92) in (91) and solving for $\mathbf{p}$ we obtain the unique equilibrium price vector $\mathbf{p}$ that is consistent with a steady state equilibrium:

$$\mathbf{p} = QB\eta + QE\mathbf{c}, \tag{93}$$

where $Q = \left(I + \frac{\kappa_X}{J-1} BM\right)^{-1}$.

The mean price is:

$$\bar{p} = \eta'\mathbf{p} = \eta' QB\eta + \eta' QE\mathbf{c} \tag{94}$$

The first term will be analogous to p_{ss0}, i.e.$p^m_{ss0} = \eta' QB\eta$ is the average price when costs are zero. The second term is the inner product of a set of weights $\omega = \eta' QE$ with the vector of costs. Therefore, in the merger case, the mean price can be written as:[14]

$$\bar{p} = p^m_{ss0} + \bar{c} + 2\left(\frac{1}{J} - \omega_m\right)(\bar{c}_s - \bar{c}_m) \tag{95}$$

ω_m is a scalar: the weight of one of the merged firms in ω. $\bar{c}$,$\bar{c}_m$,$\bar{c}_s$ are mean costs, mean cost among the merged firms and mean cost among non-merging firms, respectively.

The covariance between costs and prices is $(J-1)^{-1}\mathbf{c}'M\mathbf{p}$, where:

$$\mathbf{c}'M\mathbf{p} = \mathbf{c}'MQB\eta + \mathbf{c}'MQE\mathbf{c}. \tag{96}$$

The first term is another set of weights, and the second term is a quadratic form in costs. $H = MQE$ is a symmetric matrix that can be decomposed in its eigenvalues. In the merger case we analyze in the paper, λ_1 is an eigenvalue associated to a one-dimensional eigenspace, λ_2 is an eigenvalue of H associated to a $J-3$ dimensional eigenspace, and the remaining two eigenvalues of H are zero.[15]

$$\sigma_{pc} = (J-1)^{-1}\mathbf{c}'M\mathbf{p} = -\frac{2\phi_m}{(J-1)}(\bar{c}_s - \bar{c}_m) - \frac{2(J-2)}{J(J-1)}(\lambda_2 - \lambda_1)(\bar{c}_s - \bar{c}_m)^2 + \lambda_2\sigma_c^2, \tag{97}$$

where The coefficient ϕ_m is the weight of one of the merged firms in $\phi = MQB\eta$.

Finally, the variance of prices given by

$$p'Mp = \eta' B'Q'MQB\eta + 2\eta' B'Q'MQE\mathbf{c} + \mathbf{c}'E'Q'MQE\mathbf{c} = \phi'\phi + 2\phi' H\mathbf{c} + \mathbf{c}'HH\mathbf{c}. \tag{98}$$

The first term captures the variance in prices due to the fact that one firm owns more than one product. The second and third terms are linear and quadratic forms of $\mathbf{c}$, respectively. In the merger case under consideration,

$$\sigma_p^2 = (J-1)^{-1}p'Mp = \frac{2J\phi_m^2}{(J-2)(J-1)} - \frac{4\phi_m\lambda_1}{(J-1)}(\bar{c}_s - \bar{c}_m) - \frac{2(J-2)}{J(J-1)}\left(\lambda_2^2 - \lambda_1^2\right)(\bar{c}_s - \bar{c}_m)^2 + \lambda_2^2\sigma_c^2, \tag{99}$$

where ϕ_m, λ_1 and λ_2 were defined above.

In the single product firm market the formula above reduces to $(\gamma^e)^2(c_i - \bar{c})'(c_i - \bar{c})$. The effect of a merger can be seen comparing the formula above with the variance decomposition formula for $(c_i - \bar{c})'(c_i - \bar{c})$, i.e. the variance will remain unaffected if $\lambda_1^2 = \lambda_2^2 = (\gamma^e)^2$ and if the merging firms have the same costs. After the merger, λ_1^2

[14] In the single product case: $\eta' QB\eta = p_{ss0}$ and $\eta' QE = \eta'$.

[15] In the single product case, the first term is zero since $MQB\eta = 0$. The second term is $\mathbf{c}'H\mathbf{c}$, where H has $J-1$ non-zero identical eigenvalues equal to γ^e (as defined above) so that: $\mathbf{c}'H\mathbf{c} = \gamma^e \sum_{j=1}^{J}(c_i - \bar{c})^2$

captures the contribution of mean cost differences between the merged entity and the small firms, while λ_2 captures the contribution of cost asymmetries among the small firms to the overall price dispersion.

The equilibrium outcomes that determine the mean and variance of the steady state prices and the correlation with costs are: p_{ss0}^m, the average price when costs are zero; ω_m the weight associated with the one of the merged firms in the weighted average of costs that determine the mean price[16]; ϕ_m, the strength of the covariance between cost and price due to the merger; bias effect in the covariance with costs; λ_1 the effect of intergroup cost variance between the merged firms and the other firms; and λ_2 the effect of cost variance among small firms.

Summary of outcomes:

Outcome	Pre-Merger, $s=0$	Pre-Merger, $s \neq 0$	Merger, $s=0$	Merger, $s \neq 0$
p_{ss0}^m	1	p_{ss0}	$1+\frac{2J-1}{(J-2)J^2}$	$[p_{ss0}^m] > p_{ss0}$
ω_m	$1/J$	$1/J$	$\frac{2J-1}{2J^2}$	$\frac{1}{J+1} < [\omega_m] < \frac{2J-1}{2J^2}$
ϕ_m	0	0	$\frac{J-1}{2J^2}$	$0 < [\phi_m] < \frac{J-1}{2J^2}$
λ_1	$\frac{J-1}{2J-1}$	$\lambda_1 = \lambda_2 < \frac{J-1}{2J-1}$	$\frac{J-1}{2J}$	$[\lambda_1] < \lambda_2, [\lambda_1] < \frac{J-1}{2J}$
λ_2	$\frac{J-1}{2J-1}$	$\lambda_1 = \lambda_2 < \frac{J-1}{2J-1}$	$\frac{J-1}{2J-1}$	$[\lambda_2] < \gamma^e < \frac{J-1}{2J-1}$

The effect of a merger: Recall the effect on the mean price:

$$\triangle\bar{p} = \bar{p}_m - \bar{p} = (p_{ss0}^m - p_{ss0}) + 2\left(\frac{1}{J} - \omega_m\right)(\bar{c}_s - \bar{c}_m) \tag{100}$$

and the effect on the covariance between prices and costs:

$$\begin{aligned} \triangle\sigma_{cp}(J-1) &= c'Mp_m - c'Mp \\ &= -2\phi_m(\bar{c}_s - \bar{c}_m) - \frac{2(J-2)}{J}(\lambda_2 - \lambda_1)(\bar{c}_s - \bar{c}_m)^2 + (\lambda_2 - \gamma^e)(J-1)\sigma_c^2 \end{aligned} \tag{101}$$

The effect on the price variance:

$$\begin{aligned} \triangle\sigma_\rho^2(J-1) &= p'Mp_m - p'Mp \\ &= \frac{2J\phi_m^2}{(J-2)} - 4\phi_m\lambda_1(\bar{c}_s - \bar{c}_m) - \frac{2(J-2)}{J}\left(\lambda_2^2 - \lambda_1^2\right)(\bar{c}_s - \bar{c}_m)^2 + \left(\lambda_2^2 - (\gamma^e)^2\right)(J-1)\sigma_c^2 \end{aligned} \tag{102}$$

Effect on consumer average costs:

$$\triangle\bar{p} - \frac{1 + g\left(1 - \delta_c^2 s^2\right) + 2s(1 + \delta_c s)}{2(1+g)}\triangle\sigma_\rho^2 \tag{103}$$

Effect on total average costs:

$$\frac{1 + g(1 + \delta_c s)^2}{2(1+g)}\triangle\sigma_p^2 - \frac{\left(1 + g + s + \delta_c gs + \delta_c s^2\right)}{(1+g)}\triangle\sigma_{cp} \tag{104}$$

Not all the values of σ^2 and $(\bar{c}_m - \bar{c}_s)$ are consistent with the proposed equilibrium in linear strategies. We know that if $\sigma^2 = 0$ the proposed equilibrium in linear strategies will satisfy the requirement that the marginal consumer in each segment is located strictly in the interior of the segment and that firms make non-negative profits. These requirements may be violated for large σ^2 and $|\bar{c}_m - \bar{c}_s|$. To see why, notice that if there is too much variation in costs, it will generate too much variation in prescribed equilibrium prices, but that may imply that for some edge one firm captures the whole market (among one of the three set of consumers: young, and two set of old consumers according to their affiliation).

Other possible extensions:

1. Consumers live for a fixed number $T > 2$ of periods: The state space grows since now firms will condition their pricing strategy on their market share in each cohort. Moreover, it is necessary to derive the demand of consumers in each of the $t < T$ periods. The demand of consumers in their last period remains unchanged.

[16] If $\omega_1 = \frac{1}{J}$, each product is weighed equally. If $\omega_1 = \frac{1}{2(J-1)}$ each entity receives an equal weight irrespective of how many products it owns.

2. Consumers exit the market stochastically at rate λ: It is necessary to adjust the demand from the young and old consumers to accordingly. The only difference between the two types of consumers is that young consumers are not attached to any good. However, both young and old consumers will take into account the future behavior of firms since with some probability they will be affected by it.

F. Deriving equilibrium prices in the static model with multi-product firms.

We analyze the case of $N = J - 1$ firms where firms 1 to $N - 1$ each own product 1 to $J - 2$, respectively, and firm N owns both products $J - 1$ and product J. For $i \leq N - 1$, the single product firm maximizes

$$\frac{1}{2}(J-1)f_J\left(1+\overline{p_{-i}}-p_i\right)(p_i-c_i) \tag{105}$$

where $\overline{p_{-i}}$ is the average competitors' price in each of the segments where firm i competes (i.e., the multi-product firm appears twice). Notice that if the firm sets $p_i = \overline{p_{-i}}$, it will sell to half of the consumers that are located in segments where firm i competes. The best response to $\overline{p_{-i}}$ is:

$$p_i = \frac{1+\overline{p_{-i}}+c_i}{2}. \tag{106}$$

Notice that

$$\overline{p_{-i}} = \frac{J\overline{p}-p_i}{J-1}, \tag{107}$$

where $\overline{p}$ is the average price across products (the price of the multi-product firm enters twice). Therefore, the best response can be written as

$$p_i = \frac{(J-1)}{2J-1} + \frac{J\overline{p}}{2J-1} - \frac{(J-1)c_i}{2J-1}. \tag{108}$$

Let $\overline{p_s}$ be the average price among the single product firms and let $\overline{c_s}$ be their average cost. Then

$$\overline{p_s} = \frac{(J-1)}{2J-1} + \frac{J\overline{p}}{2J-1} + \frac{(J-1)\overline{c_s}}{2J-1}. \tag{109}$$

Firm N will then maximize

$$f_J\left(L+(J-2)\left(1+\overline{p_{-N}}-p_N\right)\right)\left(p_N-\overline{c_m}\right), \tag{110}$$

where $\overline{c_m}$ is the average cost of the multi-product firm and $\overline{p_{-N}}$ is the average competitors' price. The best response is

$$p_N = \frac{J-1}{2(J-2)} + \frac{\overline{c_m}}{2} + \frac{\overline{p_{-N}}}{2} \tag{111}$$

Notice that

$$\overline{p_{-N}} = \frac{J\overline{p}-2p_N}{J-2}, \tag{112}$$

and the best response can be written as

$$p_N = \frac{1}{2} + \frac{J\overline{p}}{2(J-1)} + \frac{(J-2)\overline{c_m}}{2(J-1)}. \tag{113}$$

The average price equals

$$\bar{p} = \frac{2p_N + (J-2)\bar{p}_s}{J}. \tag{114}$$

We use equations (108), (109), and (113) to solve for the equilibrium prices:

$$p_N^m = \frac{(J-1)(2J-1)}{2(J-2)J} + \frac{(J+1)\overline{c_m}}{2J} + \frac{(J-1)\overline{c_s}}{2J} \tag{115}$$

and

$$p_i^s = \frac{(J-1)^2}{(J-2)J} + \frac{\overline{c_s}(J-1)^2}{J(2J-1)} + \frac{c_i(J-1)}{2J-1} + \frac{\overline{c_m}}{J}. \tag{116}$$

Equations (37) and (38) in the paper follow directly from these expressions.