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Near-lunar proton velocity distribution explained by electrostatic acceleration

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Abstract

The observation of parallel ion velocity in the near-lunar wake approximately equal to external solar wind velocity *can* be explained within uncertainties by an analytic electrostatic expansion model. The one-dimensional model frequently used is inadequate because it does not account for the moon's spherical shape. However, application of a more recent generalization to three-dimensions of the solution along characteristics predicts higher velocities, and is probably sufficient to account for the SARA observations on the Chandrayaan-1 space-craft.

1 Introduction

Recent very detailed observations by the SARA experiment on board the Chandrayaan-1 satellite have provided excellent documentation of proton velocity distribution functions in the deep wake of the moon when it lies in the solar wind, as reported by [1]. For the conditions studied there, the interplanetary magnetic field is oriented almost perpendicular to the solar wind and in the ecliptic. At an altitude of 100km, above the moon's surface ($R_L \approx 1730$km), and at an angle of 50 degrees to the terminator, on the equatorial plane, the proton velocity distribution is observed to be concentrated in a beam whose parallel velocity is approximately equal to its perpendicular velocity. The perpendicular velocity is the solar wind cross-field drift velocity approximately 325km/s in the downstream direction. And the parallel velocity is inward along the magnetic field.

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The authors observe that this parallel velocity cannot be explained by the one-dimensional self-similar expansion-into-vacuum solution reviewed by [2] (but developed substantially earlier, see for example [3]). That theory gives a velocity too small by a factor of approximately 2, and density too large by a factor of 2-20. Alternative electron (kappa) distributions, which have been similarly applied by [4], do not resolve the problem. This inconsistency has motivated further, more detailed, modelling based on electromagnetic hybrid computation [5], although without a clear resolution of the discrepancies. The purpose of this brief communication is to point out that the one-dimensional analytic model effectively treats the moon as a planar disk rather than as a sphere. This approximation, while reasonable for the distant wake, is not appropriate for these near-wake situations. However, there exists an extension of the analytic approach to 3-dimensions, [6, 7] which provides results of equivalent accuracy for objects of essentially arbitrary shape, and in particular for a sphere. This theory, whose formulas are equally compact, is outlined briefly and applied to the situation at hand, predicting substantially higher parallel velocity. Probably, within the approximation of neglecting the ion Larmor radius and other uncertainties, it is sufficient to explain the observed results by self-consistent electrostatic acceleration.

2   Theoretical drift treatments

The self-consistent one-dimensional quasi-neutral expansion of a plasma into vacuum gives rise to self-similar solutions that are a function of distance s and time t. These can be applied to the wake of a semi-infinite planar object past which plasma is flowing at a constant drift velocity $v_\perp$, by identifying s with distance parallel to the magnetic field, and $tv_\perp$ with distance perpendicular to magnetic field in the downstream drift direction. The self-similar solution is a function only of the ratio s/t, and can be written:

$$v_\parallel = c_s + s/t. \tag{1}$$

The general three-dimensional expression for arbitrary object shape (of which the above is a special case) is more conveniently expressed by normalizing all velocities to the sound speed $c_s \equiv \sqrt{ZT_e/m_i}$ and writing the Mach numbers $M = v/c_s$. Then

$$M_\parallel = 1 + M_\perp \cot \theta. \tag{2}$$

Perpendicular ($\perp$) and parallel ($\parallel$) refer to components in directions relative
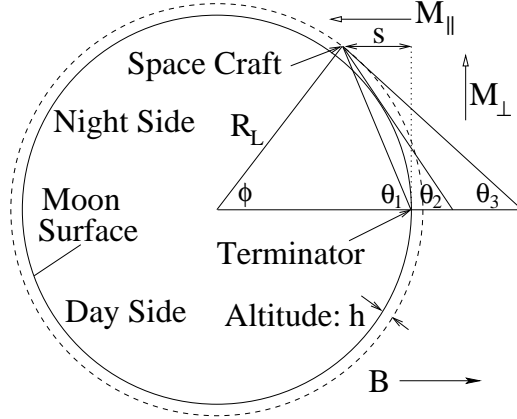

Figure 1: Geometry of the observations in the equatorial plane, which contains both the field (B) and the solar wind ($M_\perp$) directions.

to the magnetic field. The ion motion perpendicular to the field is presumed to be accounted for by drifts arising from the self-consistent electric field, but is effectively the external drift of the solar wind, which is what defines $M_\perp$, a constant. The angle θ, which is the controlling variable in the solution, is shown rigorously by integration of the ion momentum and continuity equations to be the angle to the magnetic field of one of the "characteristics" of the differential equations. The characteristic is that straight line passing through the location of interest which is tangent to the surface of the object.

Figure 1 illustrates the definition of θ. The tangential characteristic's angle for the space-craft position is θ_2. Ignoring finite gyro-radius, θ_2 is the angle that ought to be used in eq (2). The angle that corresponds to the one-dimensional approximation treating the moon as a disk, is θ_1, whose cotangent is $s/tc_sM_\perp$. [The tangent line to a disk simply passes through its edge.] The angle $\theta_3 = \pi/2 - \phi$ is what would be required if the effective altitude of the observation (h) were taken to be zero. In that case the tangent point is at the space-craft. The longitudinal angle of the space craft from the terminator in the reported observations was $\phi = 50$ degrees.

In all cases the corresponding theoretical density is

$$n = n_\infty \exp(M_{\parallel\infty} - M_\parallel) \tag{3}$$

where $M_{\parallel\infty}$ is the component of external wind velocity parallel to the field, equal to zero in this case.

The most notable new feature of the three-dimensional solution is that the parallel velocity can become very large for positions near the surface on the downstream side where θ may approach zero.

3 Theory-Observation Comparison

Simply evaluating the geometry gives the following values of predicted $M_\parallel$ for the three angles shown in Fig. 1. The final row is obtained by adding

Case:	θ_1	θ_2	θ_3
θ^o	68	59	40
$\cot\theta$	0.395	0.600	1.19
$M_\parallel/M_\perp$	0.495	0.700	1.29

Table 1: Theoretical values for parallel velocity via eq (2), for the three angle cases.

$1/M_\perp \approx 0.1$ to the value of $\cot\theta$. This is based on the (cold-ion) thermal acoustic speed in a proton plasma at the estimated electron temperature (141,000K), approximately 35km/s, and the wind velocity 310-340km/s. The uncertainty in the $1/M_\perp$ value is relatively unimportant. Parallel velocity $M_\parallel/M_\perp \approx 1$ was observed by the space craft, which is what the theory ought to match.

The one-dimensional approximation, θ_1 gives velocity too low by a factor of 2, as noted by [1]. However, using the more appropriate three-dimensional formulation θ_2, a substantial increase of the predicted $M_\parallel/M_\perp$ to 0.7 is obtained. This still does not fully reach the observed value. However, the theory represented in these equations assumes that the gyro-radius is negligible. This is not the case with these observations in which the thermal ion gyro radius ρ_i is approximately equal to the altitude ($h = 100$km). An over-correction of the finite gyro-radius effect would be to consider the moon's effective collecting radius to be not R_L but $R_L + \rho_i$. That would give the θ_3 case, which over-predicts $M_\parallel/M_\perp = 1.29$. Without detailed three-dimensional kinetic calculation far beyond the scope of the drift theory, it is not possible to estimate precisely the effect of finite gyro-radius. It will increase the predicted velocity from the θ_2 case towards, but certainly no more than, the θ_3 case. Incidentally, the drift analysis can be performed accounting for the full parallel ion distribution function in finite ion temperature cases (see [8]),

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thus generalizing the iso-thermal ion approximation used in the fluid theory. The predicted flow does not change substantially.

The enhancement of $M_\parallel$ by a few, is more than sufficient also to reduce the theoretical ion density, eq. (3), to a level consistent with observations.

4 Summary

Analytic theory that accounts properly for the shape of the moon, predicts substantially greater parallel ion velocity arising from self-consistent electrostatic acceleration in the near-moon wake. Within the uncertainty arising from finite ion thermal gyro-radius (not included in the theory), and the other experimental uncertainties, such a theory seems probably to be capable of explaining the observed velocity. Undoubtedly there are many other significant physical effects, including the dynamics of the magnetic field that are represented in electromagnetic models. However, it appears that the electrostatic structure of the wake is probably sufficient to explain the parallel velocity observed by the Chandrayaan-1 space craft.

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