# ELECTROMAGNETIC ANALYSIS AND COLD TEST OF A DISTRIBUTED WINDOW FOR A HIGH POWER GYROTRON


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## ABSTRACT

A novel distributed window for output radiation from a high power 110 GHz gyrotron is made of thin slats of sapphire interleaved and brazed to hollow metal vanes. We report the results of electromagnetic theory and cold test of this distributed window. We calculate the frequency dependence of the reflectivity of a Gaussian beam from the window. The theory indicates a significant frequency shift of the minimum reflectivity with temperature rise of the sapphire slab. This effect is of great importance for high power operation. In cold test, the distributed window reflectivity was measured while the window was heated. The cold test results are in good agreement with the theory.


## 1. INTRODUCTION

Development of high power microwave sources for fusion applications is limited by the availability of suitable vacuum windows in the 100 to 200 GHz frequency range. A novel distributed window for high power gyrotrons operating at frequencies 110 and 170 GHz has been proposed in [1]. Cooling the whole area of the window allows operation at a very high power of the incident wave.

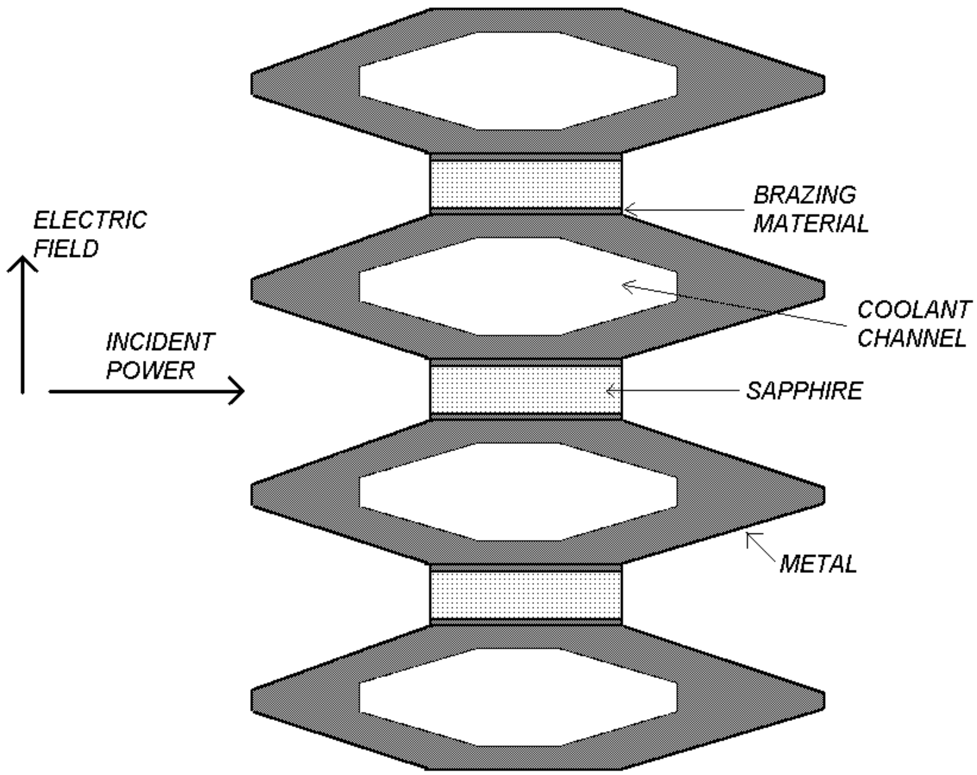


**Fig. 1.** Four sections of the distributed window.

As shown in Fig.1, the distributed window consists of arrays of thin sapphire slats separated by tapered metal (niobium) vanes. The sapphire slats are brazed to the metal. The electric field of the incident linear polarized wave is perpendicular to the vanes, and the sapphire slats are resonant for the operating frequency of 110 GHz, so that the reflectivity from the window is small. Every vane is hollow so that water coolant can pass through it. Efficient cooling of the sapphire will allow high power (1MW), long pulse (10sec) operation of the gyrotron.

In the paper [1], a simple theory of reflection from a dielectric slab was applied to calculate the sapphire thickness. The numerical thermal and stress calculations as well as the electromagnetic calculations using the time-domain code have been done in [2]. In this paper we focus on the electromagnetic analysis of the distributed window. Specifically, we examine the effect of the shift of resonant frequency of the window caused by the tapers contiguous to the sapphire slats. In addition, we calculate the reflectivity of a Gaussian beam from the window.

The experiments with the gyrotron window indicated a 100K temperature rise of the sapphire during long pulse operation. The temperature rise changes the dielectric constant ε of the sapphire by an amount Δε which results in a resonant frequency shift $\Delta f/f=-\Delta\varepsilon/(2\varepsilon)$. Below we analyze this frequency shift using dielectric constant data [3,4]. We assume that the heating affects the sapphire dielectric constant only. The temperature expansion of the sapphire and metal part can be shown to be a much smaller effect.

## 2. REFLECTION OF A PLANE WAVE

The symmetry of the incident wave permits us to pass over from the problem of diffraction at the periodic structure depicted in Fig. 1 to the problem of the TEM-mode reflection and conversion by the irregular planar waveguide with a dielectric slat. This waveguide (Fig.2) corresponds to a unit cell of the structure. The regular waveguide dimension 2*a* and the sapphire height 2*b* (Fig.2) are chosen to provide single-mode operation of the waveguide; that is, $2a<\lambda$ and $2b<\lambda/n$ , where λ is the free space wavelength, $n=\sqrt{\varepsilon}$ is the reflective index.

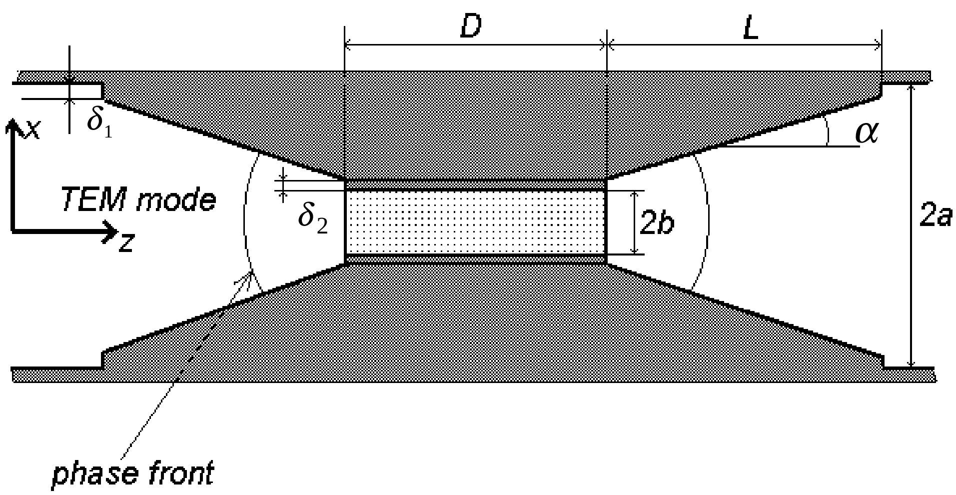


**Fig. 2.** Planar tapered waveguide with a dielectric slab modeling one period of the window structure.

Since the operating frequency is well below the cut-off of the first higher-order mode, the step-wise irregularity results in excitation of an evanescent higher-order mode. The amplitude of the evanescent mode can be estimated using the theory of irregular waveguides [5]. To make this amplitude negligible, the following condition has to be fulfilled $\left(\frac{2\delta_1}{a}\right)^2 \frac{\pi}{\tilde{h}^3 a^2 \lambda} << 1$ ,

where $\delta_1$ is a step-dimension (Fig.2), $\tilde{h} = \sqrt{\left(\frac{\pi}{a}\right)^2 - \left(\frac{2\pi}{\lambda}\right)^2}$ is the propagation constant for the first higher order mode.

For the single-mode waveguide we can use the elementary formula of the slab theory to calculate the coefficient of reflection at the dielectric slab [6]

$$r_s = \frac{-r_n + r_n \exp(-2iknD)}{1 - r_n{}^2 \exp(-2iknD)} \ , \qquad (1)$$

where $D$ is the sapphire thickness, $r_n = -\frac{n-1}{n+1}$ is the reflection coefficient from a dielectric-vacuum boundary, $k = \frac{2\pi}{\lambda}$ is the wavenumber.

If the angle of tapering α is small, α<<1, and the steps $\delta_{1,2}$ <<λ , the coefficient of reflection *r* from the dielectric slat contiguous to two tapers is slightly different from $r_s$ (Eq. 1). We calculate it using a modification of the theory of slightly irregular waveguides [5].

The coefficient $r_t$ of the TEM-mode reflection from the taper is given by the expression:

$$r_t = \frac{\delta_2}{2b} + 1 - \frac{1}{b+\delta_2} \int_0^{b+\delta_2} \exp\left(-i\frac{k\nu x^2}{2(b+\delta_2)}\right)dx +$$
$$+\left[1 - \frac{1}{a-\delta_1}\int_0^{a-\delta_1} \exp\left(i\frac{k\nu x^2}{2(a-\delta_1)}\right)dx + \frac{\delta_1}{2a}\right]\exp(-2ikL/\cos\alpha)$$

(2)

where $L$ is the taper length, $\nu = \tan\alpha$ . Here we take into account that the TEM-mode phase-front is cylindrical in the taper (Fig. 2). The phase-front radius varies from $(b+\delta_2)/\nu$ to $(a-\delta_1)/\nu$ along the taper.

The coefficient of reflection from the dielectric-vacuum boundary including the taper is given by:

$$r_{nt} = \frac{r_n + r_t}{1 + r_n r_t} \tag{3}$$

Finally, the coefficient of reflection from the waveguide including two tapers and the slat can be expressed as

$$r = \frac{1 + r_{nt}}{1 + r_{nt}^{*}} \frac{\left\{- r_{nt}^{*} + r_{nt} \exp(-2iknD)\right\}}{1 - r_{nt}^{2} \exp(-2iknD)} \quad . \tag{4}$$

where $*$ denotes the complex conjugate value. Using Eq. (4) we can evaluate the sapphire slat resonant frequency shift due to the tapers contiguous to the slat:

$$\frac{\Delta f}{f} = -\frac{\nu}{k_0 b} \frac{n-1}{n+1} \frac{1}{8 k_0 n D} \quad , \tag{5}$$

where $k_0$ is the resonance value of the wavenumber calculated from the slab theory.

The reflection coefficient, Eq. (4), derived for the plane wave, is equal to 0 at the resonant frequency. However, in practice a Gaussian beam of a finite waist is utilized. Diffraction scattering of this beam at the window results in non-zero reflectivity at the resonance.

## 3. GAUSSIAN BEAM SCATTERING THEORY

We will next consider the diffraction of a linearly-polarized Gaussian beam at the grating of the distributed window. The electric field of the incident beam is perpendicular to the waveguide edges and has a Gaussian field distribution:

$$E_x \propto \exp\left( -\frac{x^2 + y^2}{2w^2} \right) \quad ,$$

where *w* is the waist dimension, the *x*-coordinate is transverse to the vanes while the *y*-coordinate is along the vanes.

To account for a finite width *w* of the beam in the *y*-direction, we modify Eq. (4) for the reflection coefficient:

$$r = \frac{1 + r_{nt}}{1 + r_{nt}^{\ *}} \frac{\left\{- r_{nt}^{\ *} + r_{nt} \exp(-2i\Phi_b)\right\}}{1 - r_{nt}^{\ 2} \exp(-2i\Phi_b)} \ , \qquad (6)$$

where

$$\Phi_b = knD - \frac{1}{2}\tan^{-1}\frac{D}{knw^2}$$

is the Gaussian beam phase difference at the dielectric thickness. The expression for $\Phi_b$ indicates that the beam phase velocity is different to that of the plane wave. Comparing Eqs. (4) and (6), we obtain the resonance frequency shift due to a finite width of the beam: $\Delta f/f = 1/(2k^2n^2w^2)$ .

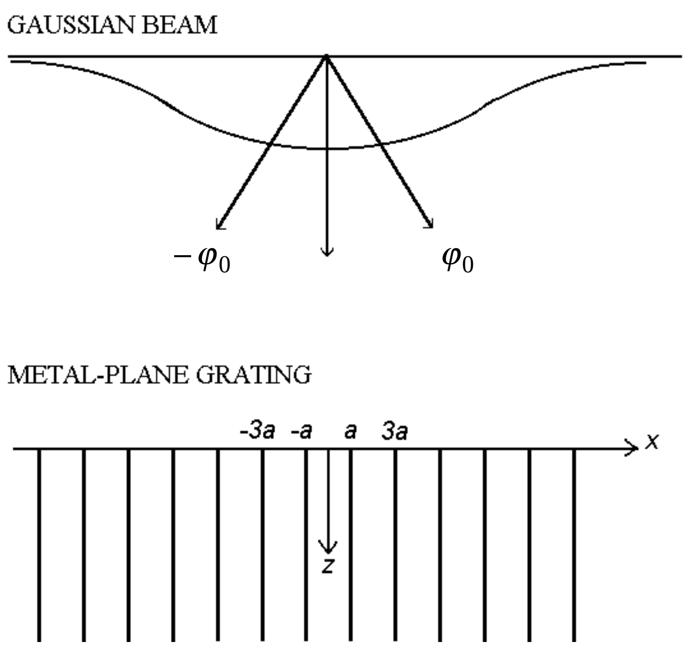


**Fig. 3.** Gaussian beam scattering at the metal grating.

The other, more important issue about the Gaussian beam is that a non-uniform field distribution in the *x*-direction leads to excitation of higher-order modes in the tapered waveguides between the vanes. A higher-order mode, which has been excited in the down-tapered waveguide, reflects from the taper.

This increases the reflectivity as compared to that for a plane wave (Eq. 4), so that the reflectivity is finite at the resonance. To examine the scattering of the Gaussian beam at the window we consider only the first TM-mode with the electric field distribution $E_x \propto \sin(\pi x / 2a)\exp(-ih_1 z)$ in the waveguide section of the window grating; $h_1 = \sqrt{k^2 - (\pi / 2a)^2}$ is the longitudinal wave number.

To model the excitation of the TEM-mode and TM-mode in the waveguide sections of the distributed window, we examine scattering of the Gaussian beam at the grating consisting of the metal planes $x=(2m+1)a$ , $m=0,\pm 1, \pm 2,\ldots$, $z>0$ (Fig. 3).

We consider three plane waves at $z<0$ and with the propagation angles 0, $\varphi_0$ and ($-\varphi_0$):

$$E_x = \exp(-ikz) + \frac{1}{2}\exp(ikx\sin\varphi_0 - ikz\cos\varphi_0) + \frac{1}{2}\exp(-ikx\sin\varphi_0 - ikz\cos\varphi_0)$$

This superposition represents the Gaussian beam $E_x \propto \exp\left(-\frac{x^2}{2w^2}\right)$ at $z=0$ and within the interval $-\frac{\pi}{k\sin\varphi_0} < x < \frac{\pi}{k\sin\varphi_0}$, with $\varphi_0$ given by $k\sin\varphi_0 \cong \pi / (2.5w)$.

The problem of scattering of a plane wave at a grating of metal planes has been solved in [7]. According to this theory, the amplitudes $T_0$ of the TEM-mode and $T_1$ of the first TM-mode excited in the waveguide of the grating by the plane wave with the propagation angle $\varphi_0$ are the following:

$$T_0(\varphi_0) = \frac{\sin\pi\eta}{\pi\eta}\frac{2\cos\varphi_0}{1+\cos\varphi_0}\frac{h_1 + k}{h_1 + k\cos\varphi_0} \quad , \qquad (7)$$

$$T_1(\varphi_0) = -\frac{2i}{\pi}\frac{\eta\cos\pi\eta}{\eta^2 - 1/4}\frac{2\cos\varphi_0}{1+\cos\varphi_0}\frac{h_1}{k}\frac{h_1 + k}{h_1 + k\cos\varphi_0} \quad ,$$

here $\eta = ka\sin\varphi_0 / \pi$ .

Using Eq. (7) for $T_0(\varphi_0)$ of the TEM-mode excitation we find the coefficient of Gaussian-beam reflection from the window grating

$$|r_b|^2 = |r|^2 + \frac{1}{3}\left(1 - |r|^2\right)\left(1 - |T_0|^2 \frac{1}{2M+1}\sum_{m=-M}^{M}\cos^2 2\pi m\eta\right) \quad , \qquad (8)$$

where $r$ is expressed by Eq. (6), and $2M+1$ is the number of the waveguides covered by the Gaussian beam.

## 4. RESULTS OF CALCULATIONS AND MEASUREMENTS

In the cold test, the transmittance of the Gaussian beam at frequencies around 110 GHz was measured. The Gaussian beam was launched from a corrugated waveguide and had a flat phase front at the waist $w$=7.2 mm at the window. The distributed window was heated during the measurements.

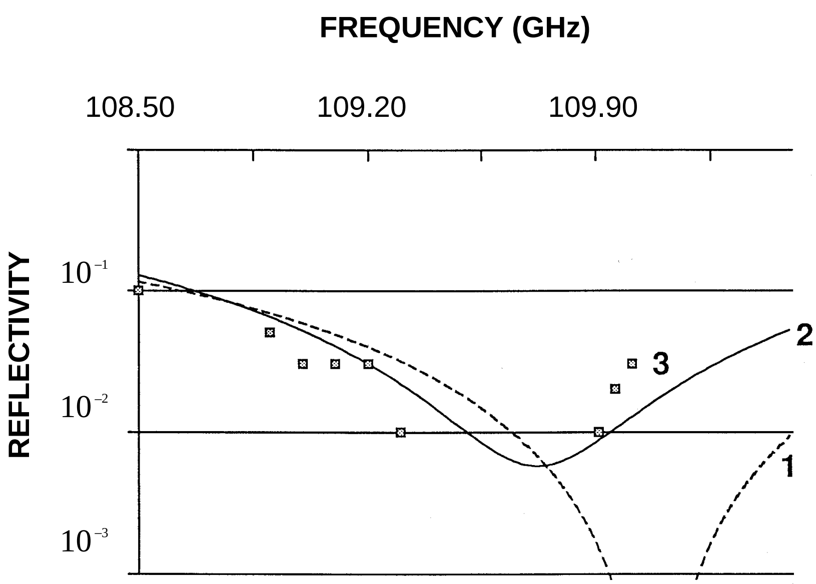


**Fig. 4.** Reflectivity of distributed window vs. frequency at $T$=300K: sapphire dielectric constant ε=9.40, 1 – slab theory, 2 - Gaussian beam scattering theory, 3 (squares) – measurement.

We use Eq. (8) to calculate the distributed window reflectivity as a function of frequency for the Gaussian beam with $w$=7.2 mm, and compare the calculations to the measurements. Based on the experimental results [3,4], we suppose that the sapphire dielectric constant varies from 9.40 to 9.47 while the temperature rises from 300K to 370K.

The dimensions of the 110 GHz distributed window are given in Table 1.

**Table 1.** Distributed window dimensions.

| | |
|---|---|
| Sapphire thickness *D* (mm) | 2.67 |
| Sapphire height *2b* (mm) | 0.73 |
| Distance between vanes *2a* (mm) | 2.39 |
| Vane transition length *L* (mm) | 4.10 |
| Transition angle $\alpha$ (deg) | 9.58 |
| Small thickness of vane $2\delta_1$ (mm) | 0.18 |
| Brazing material thickness $\delta_2$ (mm) | 0.05 |

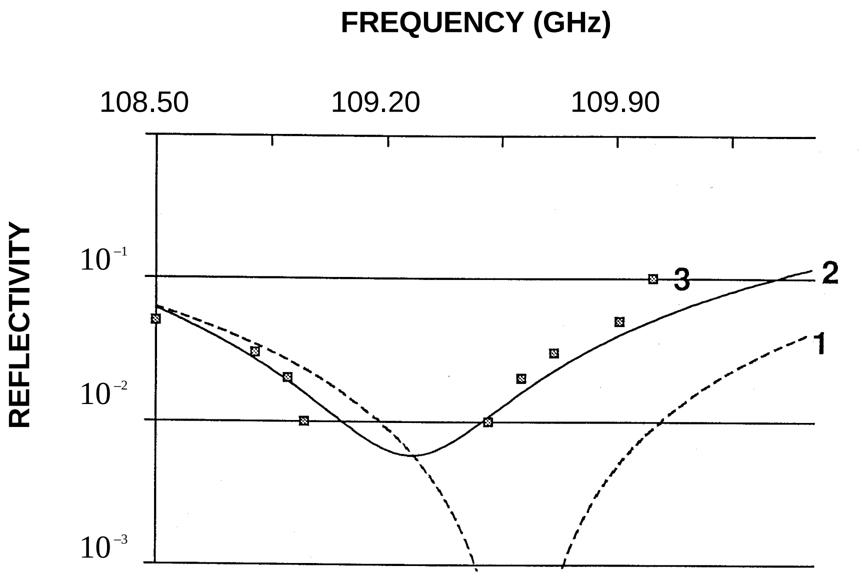


**Fig. 5.** Reflectivity of distributed window vs. frequency at *T*=370K: sapphire dielectric constant $\varepsilon$=9.47, 1 – slab theory, 2 - Gaussian beam scattering theory, 3 (squares) – measurement.

Figure 4 shows the frequency dependence of the reflectivity calculated using Eq. (1) in the simple slab model (curve 1) and using the Gaussian beam scattering theory (curve 2), and the measured reflectivity (curve 3) for the sapphire temperature 300K (dielectric constant $\varepsilon$=9.40). Figures 5 plots the same for the temperature 370K ($\varepsilon$=9.47). Equation (8) is used to calculate the reflectivity of the Gaussian beam, where $2\pi\eta=24^{o}$ and $M=3$. The scattering theory gives a resonance frequency shifted by 0.3% with respect to the slab theory. Both theory and measurements indicate that the resonance frequency shift caused by the temperature rise is significant, it is 0.4% for the temperature rise of 70K. The calculated reflectivity at 110 GHz is 2% at 300K, and increases to 6% as the temperature increases to 370K, and higher with further temperature rise.

## CONCLUSIONS

We developed the theory of transmission and reflection of a Gaussian beam at the grating of the distributed window. The theory adequately models the cold test. The distributed window reflectivity is calculated, and a good agreement between the theory and the measurements is demonstrated. We conclude that the tapered metal sections of the distributed window lead to the resonance frequency shift that we have to take into account for designing the window. A temperature rise as well affects the window performance in a long pulse operation of the gyrotron.

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