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Plasma Fusion Center
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139 USA

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TABLE OF CONTENTS

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Abhay K. Ram[1], Abraham Bers[1], Steven D. Schultz[1], Vladimir Fuchs[2]
A. Becoulet[3], B. Saoutic[3]

[1]Plasma Fusion Center, M.I.T.,Cambridge, Massachusetts, USA
[2]Centre Canadien de Fusion Magnetique, Varennes, Québec, Canada
[3]Association Euratom-CEA sur la Fusion, Cadarache, France

1. INTRODUCTION

In the vicinity of the ion-ion hybrid resonance in a plasma with two ion species, ion-Bernstein waves can be excited by mode conversion of fast Alfvén waves. Detailed theoretical analysis has shown that, due to toroidal effects, there is an increase in $|k_\parallel|$ of the IBW as it propagates in a tokamak [1]. This increase leads to an effective damping of IBW's on electrons. The upshift in $|k_\parallel|$ can be so large that the IBW damps before it propagates out to the high-field edge of the plasma, so that the damping of IBW's can be essentially "single pass" [1]. In ICRF heating experiments (in Alcator C-Mod, Tore Supra, and TFTR) where the ion-ion hybrid layer is inside the plasma, direct heating of electrons has been observed. One of the reasons that this heating has been attributed to mode-converted IBW's is that the spatial region of electron heating remains close to the ion-ion hybrid resonance as the position of the resonance is changed over the plasma cross-section.

In an effort to understand the present experiments, and in order to determine the role of IBW's in future advanced tokamak scenarios, two issues need to be addressed. Since the IBW's are generated through mode conversion of FAW's excited from the low magnetic field side, it is important to determine the efficiency of this conversion process. Recent theoretical developments on modelling mode conversion are described in Section 2. After the excitation of IBW's, it is necessary to determine the subsequent propagation and damping of IBW's. This is discussed in Section 3.

2. MODE CONVERSION TO ION-BERNSTEIN WAVES

Previous theories on mode conversion have studied the propagation of fast waves in the proximity of the left-hand cutoff and the ion-ion hybrid resonance. In this case the analysis is similar to the Budden analysis of propagation through a resonance-cutoff pair [2,3]. It was subsequently noted that the high magnetic-field side right-hand cutoff of the fast wave can affect the mode-conversion efficiency [4]. The Budden-type theories have since been extended to include the right-hand cutoff in the studies on mode conversion [5–7].

In a simple, one-dimensional (equatorial plane) description, the propagation of FAW is given by:

$$\frac{d^2y}{dx^2} + Q(x)y = 0 \tag{1}$$

where y is the normalized (poloidal) component of the electric field, x is the normalized spatial coordinate along the equatorial plane, and $Q(x)$ is the potential function which for a cold plasma is:

$$Q(x) = \frac{(L - n_\parallel^2)(R - n_\parallel^2)}{S - n_\parallel^2} \tag{2}$$

where $n_\parallel = ck_\parallel/\omega$, and S, R, L are the usual Stix tensor elements. The WKB solution to (1) gives the FAW dispersion relation $n_\perp^2 \equiv (ck_\perp/\omega)^2 = Q(x)$. $R = n_\parallel^2$ gives the

1

positions of the right-hand cutoffs (RHC). There are usually two such cutoffs: one on the low magnetic field side near the antenna, and another on the high magnetic field side. The positions where $L = n_\parallel^2$ and $S = n_\parallel^2$ correspond to the left-hand cutoff and to the ion-ion hybrid resonance, respectively. For our studies of the mode-conversion process, $Q(x)$ is chosen to be of the form:

$$
Q(x) = \begin{cases} -\alpha x + \gamma - \dfrac{\beta}{x}, & \text{if } x > 0 \\ \tilde{\alpha} x + \tilde{\gamma} - \dfrac{\beta}{x}, & \text{if } x \le 0 \end{cases} \tag{3}
$$

where $x = 0$ is the location of the resonance, $x > 0$ corresponds to the low magnetic field side, and $\alpha, \beta, \gamma, \tilde{\alpha}$, and $\tilde{\gamma}$ are all positive parameters that depend on $k_\parallel$ and are determined from a fit to the FAW dispersion relation (2). The Budden potential corresponds to $\alpha = \tilde{\alpha} = 0$ and $\tilde{\gamma} = \gamma$. Then (1) describes the propagation of the FAW through the left-hand cutoff and resonance, and the power mode conversion coefficient is found to be [2]:

$$
C_B = T_B(1 - T_B) \quad \text{where} \quad T_B = \exp(-\pi\eta), \quad \eta = \beta/\sqrt{\gamma} . \tag{4}
$$

The high-field side right-hand cutoff is included by adding a non-zero $\tilde{\alpha}$. From the approximate analytical solution, obtained by the method of uniform asymptotic matching [8], we find that the power mode-conversion coefficient is:

$$
C \approx 4T_B (1 - T_B) \cos^2\left(\frac{\phi}{2} + \psi\right) \tag{5}
$$

where ψ is the argument of $\Gamma(-i\eta/2)$, Γ being the Gamma function,

$$
\phi \approx \frac{4}{3}\sqrt{\gamma}|x_R| + \eta \ln(8\sqrt{\gamma}|x_R|) \tag{6}
$$

and x_R is the distance between the right-hand cutoff and the resonance (normalized to the FAW wavelength $1/\sqrt{\gamma}$). From (5) it is possible to get a conversion coefficient of 1, whereas from (4) the maximum mode-conversion coefficient is $1/4$. Thus, the right-hand cutoff can significantly modify the conversion coefficient. Physically, the latter case can be thought of as a coupling of the FAW power to an intrinsic plasma resonator formed by the Budden (left-hand) cutoff-resonance pair and the right-hand cutoff. The condition for 100% mode conversion corresponds to "critical coupling" to this resonator. In Fig. 1 we show contours of maximum mode conversion, obtained from (5), as a function of the peak electron density and the antenna $k_\parallel$ spectrum for TPX parameters.

The above analyses ignore the FAW power reflected from the left-hand cutoff back towards the antenna. A proper accounting of the mode-converted power in this case requires an analysis that includes the coupling to an antenna. Towards this end, the model potential of (3) has to be used in its entirety so that both of the right-hand cutoffs are included in the description of the FAW [9]. The radiation impedance of the antenna is then related to the inverse of the plasma admittance, Y_p, at the low-field boundary ($x = x_0$) of the plasma [10]:

$$
Y_p(x = x_0) = i\sqrt{\frac{\epsilon_0}{\mu_0}} \left[\frac{1}{y}\frac{dy}{dx}\right]_{x_0} . \tag{7}
$$

2

The coupling of the resonator with mode conversion to the ICRF antenna allows us to determine a quantity that is accessible to physical measurements, namely, the characteristics of the antenna radiation resistance. An approximate expression for $Y_p(x)$ can be obtained by solving (1) with (3) using the method of uniform asymptotic matching [8,9]. We find that $|1/Y_p(x_0)|$ exhibits resonant peaks as a function of the $k_\parallel$ spectrum of the antenna [7]. The underlying physics of this behavior is equivalent to the coupling of the FAW to a global plasma resonator. The global plasma resonator contains the intrinsic (internal) resonator discussed above and the low-field side right-hand cutoff. The peaks correspond to critical coupling of the antenna to the resonator and determine the $k_\parallel$ for which the conversion coefficient is a maximum. In Fig. 2, we plot $|F(k_\parallel)/Y_p(x_0)|$, where $F(k_\parallel)$ is the launched power spectrum of the antenna, as a function of $k_\parallel$ for Tore Supra parameters [11]. This result compares very favorably with the results obtained from Alcyon – a full-wave code with a hot, toroidal plasma [7,9,11]. Since in our model we do not have finite temperature effects, the only "damping" observed is due to conversion of FAW's to IBW's. The peaks in Fig. 2 correspond to $k_\parallel$ for which there is substantial mode conversion.

3. PROPAGATION AND DAMPING OF IBW'S

In the mode-conversion region and its immediate vicinity, the IBW, like the FAW, is electromagnetic. Thus, as is case with FAW, any damping of the IBW on electrons in this region will be essentially due to transit time magnetic pumping (TTMP). In present day experiments, the electron-β is not high enough for a large fraction of the IBW and/or FAW power to damp via TTMP. As the IBW's propagate away from the mode-conversion region, their $|k_\perp|$'s increase, they become electrostatic, and they damp via electron Landau damping [1]. Associated with the propagation of IBW's is a change in the poloidal mode numbers due to toroidicity. In the presence of the poloidal magnetic field, this leads to a change in $|k_\parallel|$ over short distances of propagation so that the IBW phase velocity becomes comparable to the electron thermal velocity. This occurs even when the initial $k_\parallel$ at mode conversion is very small. Consequently, the IBW's damp on electrons, usually in the vicinity of the mode-conversion region [1]. Thus, by an appropriate choice of the mode conversion layer in a tokamak, one can choose the spatial location where the IBW's will impart their energy and momentum to electrons. This is an important benefit of mode-converted IBW's. One can, in particular, choose the mode-conversion layer to be on the high-field side where IBW's do not interact with magnetically trapped electrons.

4. CONCLUSIONS

Our analysis shows that IBW's are promising candidates for electron heating in tokamaks. Appropriate advanced tokamak scenarios, e.g., in TPX, can be found where a large fraction of the FAW power can be mode converted to IBW's. The advantage of our simple models, which account for the essential physics of the conversion process, is that they allow for a quick analysis of a large parameter space. The results from these models compare very favorably with more elaborate full-wave codes with hot plasma and toroidal effects. We find that IBW's can be made to damp on electrons in the vicinity of the mode conversion layer. This allows for a dynamic control of the region of electron heating. Studies are underway to determine the use of these IBW's to drive plasma current.

ACKNOWLEDGEMENTS

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FIGURE CAPTIONS

Figure 1: Contours of maximum mode conversion for TPX parameters: $f_{ICRF} = 45$ MHz, D-^{3}He plasma with $n_D/n_e = 0.7$ and $n_{^3He}/n_e = 0.15$, $B_0 = 4$ T, $R_0 = 2.25$ m, $a = 0.5$ m, density profile (normalized to the peak density) of the form: $0.2 + 0.8(1 - x^2/a^2)^{0.4}$.

Figure 2: The plasma impedance $|F(k_\parallel)/Y_p(x_0)|$ as a function of $k_\parallel$ for Tore Supra parameters [11]: $n_{e0} = 6.2 \times 10^{19}$ m^{-3}, $B_0 = 3.69$ T, H-^{3}He plasma with $n_H/n_e = 0.4$ and $n_{^3He}/n_e = 0.3$, $f_{ICRF} = 48.3$ MHz, $R_0 = 2.365$ m, and $a = 0.715$ m. F is the imposed antenna spectrum and $Y_p(x_0)$ is the plasma admittance calculated at the LFS edge of the plasma.

FIGURE 1

FIGURE 2