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HEATING AND CURRENT DRIVE BY MODE-CONVERTED ION-BERNSTEIN WAVES

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ABSTRACT

Previous theoretical analysis has shown that mode-converted ion-Bernstein waves generated at the ion-ion hybrid resonance in an ICRF heated plasma damp effectively on electrons. Recent experiments in tokamaks observe substantial electron heating which is attributed to these ion-Bernstein waves. In this paper, some of the latest theoretical develepements on mode conversion, propagation, and damping of ion-Bernstein in tokamaks are presented. The eventual purpose of these studies is to determine the role of ion-Bernstein waves in advanced tokamak scenarios.

I. INTRODUCTION

A most successful way of delivering radio frequency power into a tokamak plasma has been through fast Alfvén waves (FAW), excited from the low magnetic field side (LFS) of a tokamak, in the ion-cyclotron range of frequencies (ICRF). The coupling, propagation, and damping of FAW's have been extensively studied theoretically and demonstrated experimentally in a variety of tokamaks. In a plasma consisting of at least two ion species, the ion-ion hybrid resonance can be present in the plasma if the frequency of the RF wave is between the ion-cyclotron frequencies of the two ion species. In the vicinity of this resonance, the FAW can couple to the ion-Bernstein wave (IBW), which then propagates away from the resonance towards the high-field side of a tokamak. Theoretical analysis has shown that, due to toroidal effects, there is an increase in $|k_{\|}|$ of the IBW as it propagates away from the resonance region. This increase leads to an effective damping of IBW's on electrons [1]. The upshift in $|k_{\|}|$ can be so large that the IBW damps before it propagates out to the high-field edge of the plasma. Thus, the spatial location in a plasma where the IBW's transfer their energy to the electrons is essentially determined by the location of the region of mode-conversion (MC) from FAW to IBW. This control of the region of interaction of IBW's with electrons is a useful advantage offered by IBW's. Theoretical analysis shows that the damping rate of IBW's on electrons can be large so that IBW's damp before reaching the high-field edge of the plasma. Furthermore, IBW's can interact with energetic electrons. These properties make IBW's good candidates in tokamak plasmas for depositing incoming ICRF power onto electrons. Recently, there have been a series of ICRF heating experiments in which

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the ion-ion hybrid resonance has been present inside the plasma (TFTR, Tore Supra). The observed electron heating in these experiments has been attributed to the interaction of IBW's with electrons. In an earlier set of experiments on JET, it was observed that the lower-hybrid current drive (LHCD) efficiency increased in the presence of ICRF heating [2]. The corresponding experimental conditions suggested that IBW's could be excited in the plasma. Theoretical analysis of the interaction of ICRF with LHCD showed that the interaction of IBW's with the LH-generated suprathermal electrons leads to an enhancement in the LHCD efficiency [3]. In light of these experiments, it is useful to examine the MC with LFS excitation, propagation, and damping of IBW's and the role that these IBW's can play in future advanced tokamak scenarios, e.g., in TPX.

In this paper, I present some of the recent theoretical results on the excitation and damping of IBW's. Since these IBW's are indirectly excited in a plasma — through the MC of FAW's at the ion-ion hybrid resonance — it is important to understand the mechanism of mode conversion and its dependence on plasma parameters. A simple model of the MC process, which ignores the LFS coupling to the antenna at the plasma edge, is discussed in section II. In section III, this analysis is extended to the low-field plasma edge where the effect of the antenna spectrum on the MC process is examined. The propagation and damping of IBW's are summarized in section IV.

II. MODE CONVERSION WITH REFLECTION ON THE HIGH-FIELD SIDE

In a simple, one-dimensional (equatorial plane) description, the approximate cold-plasma dispersion relation for the FAW is:

$$n_\perp^2 = \frac{(L - n_\parallel^2)(R - n_\parallel^2)}{S - n_\parallel^2} \tag{1}$$

where $n_\perp = ck_\perp/\omega$, $n_\parallel = ck_\parallel/\omega$, and S, R, L are the usual Stix tensor elements. $R = n_\parallel^2$ gives the positions of the right-hand cutoffs (RHC). There are usually two such cutoffs: one on the LFS near the antenna, and another on the high magnetic field side (HFS). The positions where $L = n_\parallel^2$ and $S = n_\parallel^2$ correspond to the left-hand cutoff (LHC) and to the ion-ion hybrid resonance (IHR), respectively. The propagation of the FAW through the resonance and the cutoffs is described by a differential equation:

$$\frac{d^2y}{dx^2} + Q(x)y = 0 \tag{2}$$

where y is the normalized (poloidal) component of the electric field, x is the normalized spatial coordinate along the equatorial plane, and $Q(x)$ is the potential function [4], which for a cold plasma is equal to the right-hand side of (1).

The usual Budden-type [5] MC analysis studies the asymptotic behavior of the solutions of (2) in the vicinity of the LHC-IHR pair. In this case, $Q(x)$ can be modelled by: $Q_B(x) = \gamma - \beta/x$ where $\sqrt{\gamma}$ is the normalized wavenumber of the FAW, and β/γ defines the normalized location of the LHC with respect to the IHR (at $x = 0$). For a FAW incident from the LFS ($x > 0$), the Budden-type analysis gives the transmission through the LHC-IHR and the reflection from the LHC. From power conservation, the difference between the incident FAW power and the sum of the transmitted and reflected FAW power gives the power that is mode converted. In this case the solution to (2) is in terms of the Whittaker function [6]: $y(x) = W_{-\kappa,\mu}(2ix\sqrt{\gamma})$ where $\mu = 1/2$, $\kappa = -i\eta/2$, and $\eta = \beta/\sqrt{\gamma}$. The asymptotic properties of this function give the power transmission coefficient $T_B = \exp(-\pi\eta)$. The power reflection and power MC coefficients are $R_B = (1 - T_B)^2$ and $C_B = T_B(1 - T_B)$, respectively [5]. Notice that the scattering coefficients depend on η which is the distance between the LHC and the IHR normalized to the FAW wavelength. The maximum power MC coefficient for the Budden potential Q_B is 25% (corresponding to $T_B = 1/2$).

It has been noted that the HFS-RHC can have a significant effect on the MC process [7]. An easy way to see this is as follows [8]. For the usual Budden potential, let us impose a boundary condition that at some point on the HFS, $x = x_R$ ($x_R < 0$), the transmitted part of the FAW is reflected back towards the IHR-LHC. In this case, the overall transmission coefficient is zero since the FAW does not propagate for $x < x_R$. Hence, there is only a LFS power reflection coefficient and a power MC coefficient. The general solution to (2) with this added cutoff is given by:

$$y(x) \;=\; c_1 W_{\kappa,\mu}(z) \;+\; c_2 W_{-\kappa,\mu}(-z) \tag{3}$$

where $z = -2i\sqrt{\gamma}x$, and c_1 and c_2 are arbitrary constants that depend on the boundary conditions. For $x \to -\infty$, $W_{\kappa,\mu}(z)$ represents an incoming (towards $x = 0$) wave while $W_{-\kappa,\mu}(-z)$ represents an outgoing wave. If we assume that there is no damping of the wave between the resonance and the high-field cutoff, then c_1 and c_2 can differ by at most an arbitrary phase. Let us assume that $c_2 = c_1 \exp\{-i(\pi + \phi)\}$. Then, using the asymptotic properties of the Whittaker functions for $x \to \infty$ [5], the power MC coefficient is found to be:

$$C(\eta, \phi) = 4T_B(1 - T_B)\cos^2\left(\frac{\phi}{2} + \psi\right) \tag{4}$$

where $\psi = \arg\{\Gamma(-i\eta/2)\}$, and Γ is the Gamma function [6]. Now the maximum power MC coefficient can be 100%, provided $T_B = 1/2$ and the $\phi/2 + \psi$ is an integer multiple of π. Thus, the effect of a high-field side reflection in the Budden problem can significantly alter the power MC coefficient. Physically, the Budden potential with a HFS cutoff can be thought of as a coupling of the FAW power to an intrinsic plasma resonator. The condition for 100% MC corresponds to a "critical coupling" to this resonator.

The HFS-RHC can be included more generally in the propagation of the FAW by modifying $Q(x)$ such that:

$$Q(x) = \begin{cases} \gamma - \dfrac{\beta}{x}, & \text{if } x > 0 \\ \tilde{\alpha} x + \tilde{\gamma} - \dfrac{\beta}{x}, & \text{if } x \leq 0 \end{cases} \tag{5}$$

where $\tilde{\alpha}$, β, γ, and $\tilde{\gamma}$ are positive parameters that are determined from a fit to the FAW dispersion relation. Then for $x > 0$, $Q(x)$ is the usual Budden potential, while for $x < 0$ the potential has a right-hand cutoff located at $x_R = (-\tilde{\gamma} - \sqrt{\tilde{\gamma}^2 + 4\tilde{\alpha}\beta})/(2\tilde{\alpha})$.

The solution to (2) is obtained by determining the solutions for $x < 0$ and for $x > 0$ and matching them across $x = 0$. For $x < 0$, a closed form solution does not exist. However, an approximate solution can be obtained by the phase-integral method [9] or by uniform asymptotic matching [8]. Both techniques validate the result of (4) and determine an approximate expression for the phase ϕ. For $\tilde{\gamma} = \gamma$, the method of uniform asymptotic expansion gives a simple form for the phase [8]:

$$\phi \approx \frac{4}{3}\sqrt{\gamma}\,|x_R| + \eta \ln(8\sqrt{\gamma}|x_R|) . \tag{6}$$

Notice that ϕ depends on the distance between the HFS-RHC and the IHR normalized to the FAW wavelength. The condition for the MC coefficient to be 100% is that $|y|$ be a local maximum at $x = 0$, while for 0% power MC $|y| = 0$ at $x = 0$.

In Fig. 1 we have plotted contours of maximum possible mode conversion as a function of peak electron density and FAW $k_{\parallel}$ spectrum for a D–^{3}He plasma in TPX. The usual Budden results would be 1/4 of the values indicated in this figure.

III. COUPLING OF MODE-CONVERSION TO AN ANTENNA

While MC analyses of the type discussed above are useful in identifying ideal mode-conversion scenarios in tokamaks, they ignore the FAW power reflected from the MC region towards the antenna. A proper accounting of the power deposition in this case requires an analysis that includes the coupling to an antenna. Towards this end, the procedure of the previous section is extended to the low-field edge of the plasma. The radiation impedance of the antenna is related to the inverse of the plasma admittance at the low-field boundary of the plasma [10]. For insight into the plasma admittance, the potential function is chosen to be of the form:

$$Q(x) = \begin{cases} -\alpha x + \gamma - \dfrac{\beta}{x}, & \text{if } x > 0 \\ \tilde{\alpha} x + \tilde{\gamma} - \dfrac{\beta}{x}, & \text{if } x \leq 0 \end{cases} \tag{7}$$

where $\tilde{\alpha}$ is a positive parameter. The RHC on the LFS is located at $x_{RL} = (\gamma + \sqrt{\gamma^2 - 4\beta\alpha})/(2\alpha)$. The plasma admittance at the low-field boundary ($x = x_0$) is $Y_p(x = x_0) = i\sqrt{(\epsilon_0/\mu_0)}\ [(1/y)(dy/dx)]_{x_0}$. An approximate expression for $Y_p(x)$ can be obtained by solving (2) with (7), using the method of uniform asymptotic matching [8]. The details of the derivation and the lengthy form of $Y_p(x)$ are beyond the limitations of this paper. We find that $|1/Y_p(x_0)|$ exhibits resonant peaks as a function of the $k_\|$ spectrum of the antenna. The underlying physics of this behavior is similar to the coupling of the FAW to a global plasma resonator. However, the global plasma resonator now has the LFS-RHC, and it contains the intrinsic (internal) resonator discussed above. The peaks correspond to critical coupling of the antenna to the resonator and determine the $k_\|$ for which the power MC coefficient is a maximum. In Figure 2, we plot $|F(k_\|)/Y_p(x_0)|$, where $F(k_\|)$ is the launched power spectrum of the antenna, as a function of $k_\|$ for Tore Supra parameters [11]. This result compares very favorably with the results obtained from Alcyon – a full-wave code with a hot, toroidal plasma [11]. Since in our model we do not have finite temperature effects, the only "damping" observed in our model is due to the MC of FAW's to IBW's. The peaks in Fig. 2 correspond to $k_\|$ for which there is substantial mode conversion. For $k_\| = 10.5$ m^{-1}, corresponding to the location of the largest peak in Fig. 2, we plot $|y|$ across the plasma cross-section in Fig. 3. The MC resonance is near $x \approx 0$. It is interesting to note that the field amplitude towards the HFS of the MC region is small. The FAW fields basically exist between the antenna and the MC layer. Again, this agrees qualitatively with the results from Alcyon [11].

IV. PROPAGATION AND DAMPING OF ION-BERNSTEIN WAVES

In the MC region and its immediate vicinity, the IBW, like the FAW, is electromagnetic. Thus, as is the case with FAW, any damping of the IBW on electrons in this region will be essentially due to transit time magnetic pumping (TTMP). In present day experiments, the electron-β is not high enough to damp via TTMP a large fraction of the IBW and/or FAW power. As the IBW's propagate away from the MC region, their $|k_\perp|$'s increase, they become electrostatic, and they damp via electron Landau damping [1]. Associated with the propagation of IBW's is a change in the poloidal mode numbers, leading to a change in $|k_\||$ due to toroidicity and the poloidal magnetic field. Even when the initial (at mode conversion) $|k_\||$ is such that the phase velocity of the IBW is much greater than the electron thermal velocity, the toroidal enhancement of $|k_\||$ is usually very rapid over short distances of propagation so that the phase velocity becomes comparable to the electron thermal velocity and the IBW damps on electrons [1]. In effect, the IBW's interact with electrons in the vicinity of the MC region. Thus, by an appropriate choice of the mode conversion layer in a tokamak, one can choose the spatial location where the IBW's will impart their energy and momentum to electrons. This is an important benefit of mode-converted IBW's. One can, in particular, choose the MC layer to be on the HFS of the axis where IBW's do not interact with magnetically trapped electrons.

An analysis of the IBW ray trajectories shows that above the mid-plane of a tokamak, the change in $k_{\parallel}$ is negative, while below the mid-plane the change is positive. This has serious implications when we consider IBW's for driving currents in a plasma. An initial, unidirectional FAW spectrum can lose some of its unidirectionality when IBW's propagate away from the MC region. In Fig. 4, we have plotted $\mathrm{Im}(k_{\perp})$, the spatial damping rate, as a function of the phase velocity of the wave normalized to the electron thermal velocity for the IBW's and the FAW's in TPX. The damping rate is evaluated at a location slightly to the HFS of the MC layer. The IBW damping is significantly higher than the FAW damping as the phase velocities of the waves approach the electron thermal velocity. If the launched unidirectional spectrum of the FAW is at sufficiently high $k_{\parallel}$'s, the mode-converted IBW's could damp before the toroidal changes in $k_{\parallel}$ cause any major alterations in the spectrum. Moreover, Fig. 1 shows that for large $k_{\parallel}$'s the MC coefficient is high.

Another important aspect of the IBW's is that the distance of propagation from the MC layer to the location of electron damping depends on the ratio of the ion temperature to the electron temperature [1]. So, if this ratio is nearly the same over the plasma cross-section, the electron damping of the IBW will always occur near the MC region regardless of whether the MC layer is near the center of the plasma or on the HFS where the plasma is relatively colder.

V. CONCLUSIONS

Our theoretical analysis shows that mode-converted IBW's are promising candidates for electron heating in tokamaks. By using simple theoretical models, appropriate advanced tokamak scenarios, e.g., in TPX, can be found where a large fraction of the FAW power can be mode converted to IBW's. The advantage of these simple models, which account for the essential physics of the problem, is that they allow for a quick analysis of a large parameter space. The results from these models compare very favorably with more elaborate codes. In order to evaluate the potential of IBW's for current drive, work is currently under progress to determine the conditions for optimizing the current drive efficiency. Here, with the imposed constraint of maximizing current drive efficiency, we need to find an appropriate spectrum of the launched waves such that, after mode conversion, the spectrum remains unidirectional as the IBW's damp on electrons.

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FIGURE CAPTIONS

Figure 1: Contours of maximum mode conversion for TPX parameters: $f_{ICRF} = 45$ MHz, D-^{3}He plasma with $n_D/n_e = 0.7$ and $n_{^3He}/n_e = 0.15$, $B_0 = 4$ T, $R_0 = 2.25$ m, $a = 0.5$ m, density profile (normalized to the peak density) of the form: $0.2 + 0.8(1 - x^2/a^2)^{0.4}$.

Figure 2: The plasma impedance $|F(k_{\|})/Y_p(x_0)|$ as a function of $k_{\|}$ for Tore Supra parameters [11]: $n_{e0} = 6.2 \times 10^{19}$ m^{-3}, $B_0 = 3.69$ T, H-^{3}He plasma with $n_H/n_e = 0.4$ and $n_{^3He}/n_e = 0.3$, $f_{ICRF} = 48.3$ MHz, $R_0 = 2.365$ m, and $a = 0.715$ m. F is the imposed antenna spectrum and $Y_p(x_0)$ is the plasma admittance calculated at the LFS edge of the plasma.

Figure 3: $|y|$ versus x, where y is the solution to Eqs. (2) and (8) and x is the normalized position over the plasma cross-section. The parameters are the same as for Fig. 2 with the addition that $k_{\|} = 10.5$ m^{-1}.

Figure 4: The spatial damping rate for the IBW's and FAW's as function of the phase velocity of the wave normalized to the electron thermal velocity. The parameters are the same as for Fig. 1. Here the peak temperature is assumed to be 10 keV for all the species and the temperature profile (normalized to the peak temperature) is: $0.01 + 0.99(1 - x^2/a^2)^{1.2}$. The spatial damping rate is evaluated at $x = 0.391$ m on the HFS.

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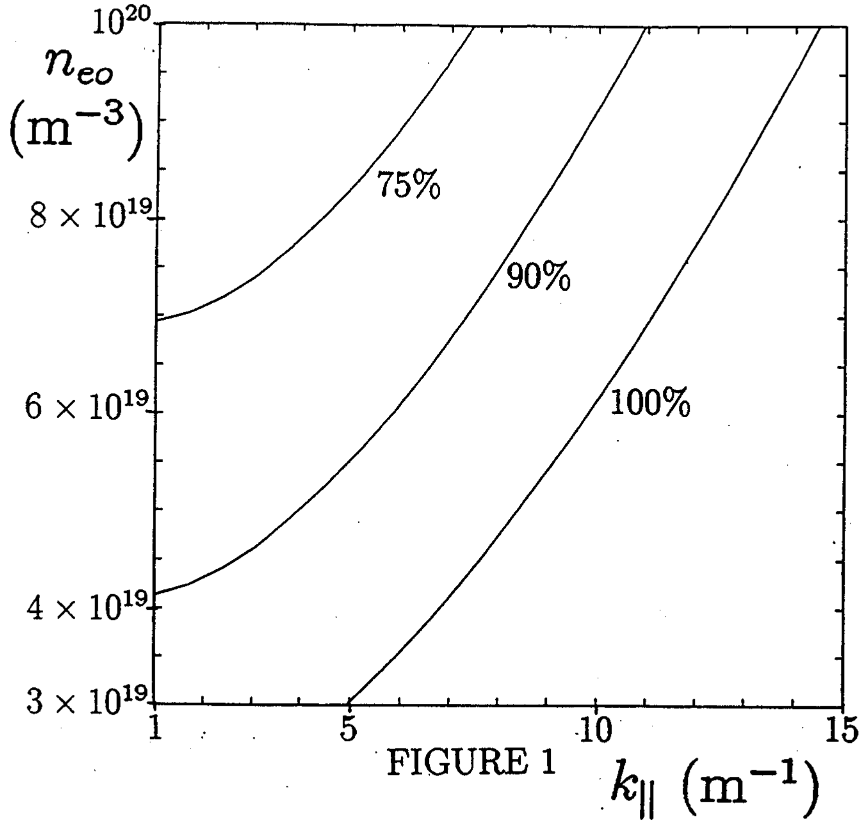
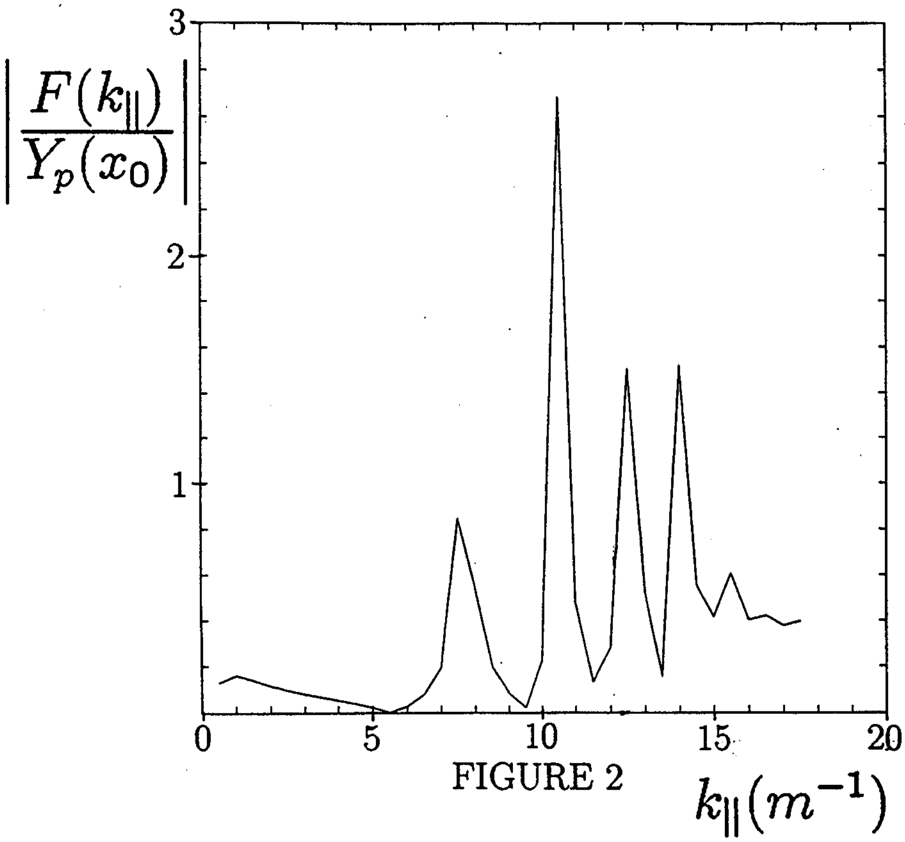


FIGURE 1



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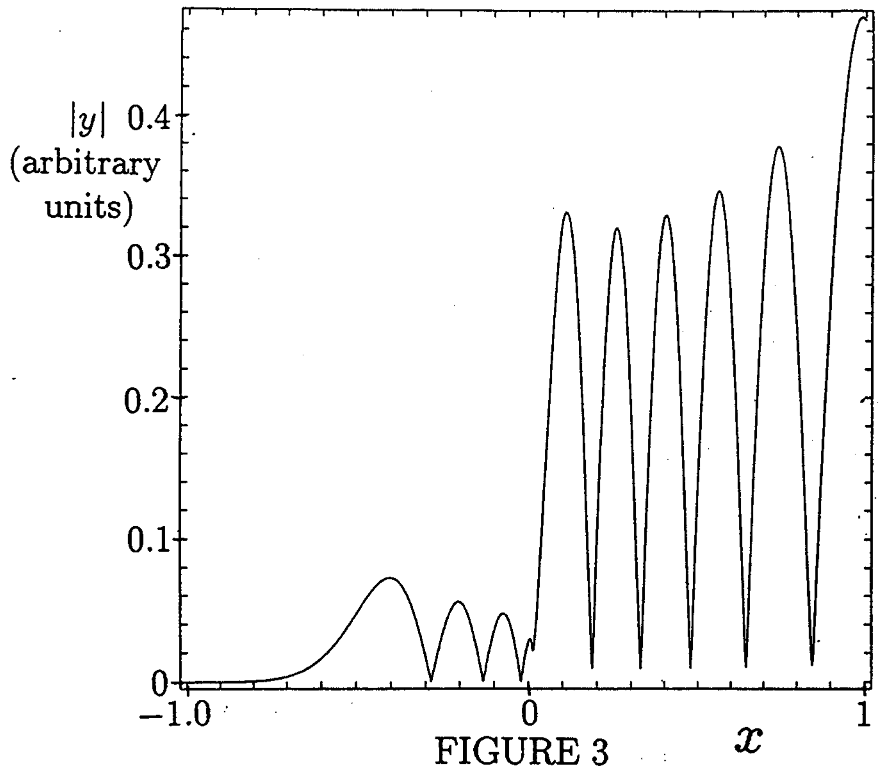

FIGURE 3

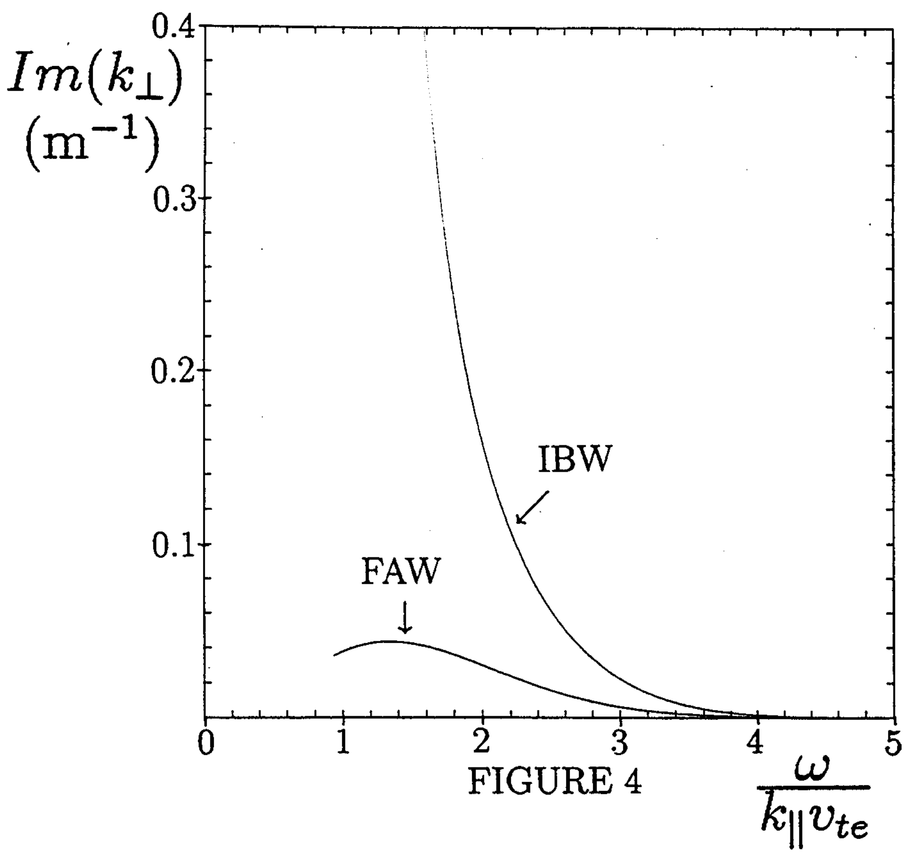

FIGURE 4