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Abstract

To gain insight into divertor operation, we employ similarity techniques to investigate whether model systems of equations plus boundary conditions admit scaling transformations that lead to useful divertor similarity scaling laws. These can be used to perform similarity experiments or fully exploit large computer simulations. We adopt fluid plasma models of the divertor region which ignore anomalous processes, and consider neutral descriptions in both the short and long mean free path limits. As usual, the more approximations we make, the more scaling transformations are allowed, leading to fewer independent dimensionless parameters that need to be considered, thereby imposing fewer divertor similarity constraints. The simplest model considered balances electron heat conduction with impurity radiation and places the fewest constraints on divertor similarity. To be able to model detached divertor operation in short mean free path regimes, a fluid neutral description is employed which balances plasma pressure by neutral pressure. In this model the constraints on divertor similarity are most severe. A less constrained long mean free path or Knudsen neutral model is also considered. It models detachment by balancing plasma pressure with momentum transfered to neutrals randomized by collisions with the deep slot sidewalls. The simpler models have relaxed divertor similarity constraints, but all models remain severely restricted by collisionality and parallel heat flux constraints.

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I. Introduction

Two-dimensional numerical models of divertors employing fluid descriptions of the plasma and either short (fluid) or long (Knudsen) mean free path descriptions of the neutrals contain large numbers of dimensionless parameters that must be varied to investigate all operating regimes expected to be of interest. Given the complexity of the descriptions the task is a rather daunting one. The question arises, therefore, as to whether useful information can be obtained and the number of independent parameters reduced by considering the scaling transformation properties[1,2] of the system of differential equations and boundary conditions. The retention of boundary conditions is a new and necessary feature that must be considered when determining the allowed scaling transformations. If scale transformations can be found for a particular system, then by the invarince principle any quantities evaluated from the same system must satisfy the same scalings. To this end, we consider various divertor models and show that the techniques introduced by Connor and Taylor[1] and reviewed by Connor[2] can be employed on boundary conditions as well as the accompanying differential equations to find the constraints on divertor similarity.

The sections that follow consider fluid and Knudsen neutral models with fluid plasma equations and boundary conditions appropriate for complete recycling. Both the short and long mean free path neutral descriptions adopted are capable of modeling the observed drops in temperature, particle flux, and energy flux at the target. The models assume all perpendicular transport is due to the neutrals since they ignore anomalous transport processes. They are best viewed as models of the divertor between the x-point and target. More sophisticated models retaining anomalous transport are possible only if explicit perpendicular transport models are assumed. In order to avoid such ad hoc assumptions about the anomalous transport coefficients we investigate cases in which they enter only through the scrape off layer width which we assume specified.

Section II presents the fluid neutral and fluid plasma equations and boundary conditions as obtained from Refs.3 and 4. The equations and boundary conditions are made dimensionless in Sec.III using the ionization energy of hydrogen, the peak upstream plasma pressure, and an appropriate neutral penetration length. In Sec.IV the scaling transformations and similarity constraints of various fluid descriptions are considered. The use of the technique of Connor and Taylor[1,2] when boundary conditions must be treated is illustrated in detail in Sec.IV.A for the simple case in which the neutrals are neglected and electron heat conduction balances impurity radiation. The scaling law for the power to the target plates P divided by the major radius R is derived and several possible ways to consider similarity are noted. Section IV.B considers a reduced two-dimensional (2-D) fluid neutral model based on the one-dimensional (1-D) model shown in Ref.5 to exhibit the key features of divertor detachment. In 2-D we recover Lackner's[6] P/R = constant scaling, while in the 1-D limit P/R need not be held constant for similarity. Moreover, for this fluid neutral model it is found that $P/R \propto \Delta_p / \ell_n \gg 1$, with Δ_p and ℓ_n the scrape off layer (SOL) width and neutral penetration length. In Sec.IV.C it is shown that the general 2-D fluid neutral model which does not allow any scaling transformations leads to essentially the same conclusions as the reduced 2-D fluid model, but involves more parameters. Section V describes the Knudsen neutral model and its limitations. It then investigates the scaling transformations for the dimensionless equations and boundary conditions to show that $P/R \propto (\Delta_p / \ell_n)^{3/2} \ll 1$ for the Knudsen model and that it need not be held constant for similar devices. In Sec.V we discuss the implications of our results.

II. Fluid Neutral and Plasma Equations and Boundary Conditions

We adopt a simplied version of the full fluid neutral and plasma equations in the SOL as given in Refs. 3 and 4. Most of the simplifications correspond to those normally employed in 2-D SOL codes and are based in part on the 1-D model solved in Ref. 5. Neglecting recombination here and elsewhere, the steady-state ion and neutral continuity equations are

$$\nabla \cdot \left(N_i \vec{V}_i \right) = \langle \sigma v \rangle_z N_e N_n \tag{1}$$

and

$$\nabla \cdot \left(N_n \vec{V}_n \right) = - \langle \sigma v \rangle_z N_e N_n \; , \tag{2}$$

where the subcripts e, i, and n denote electrons, ions, and neutrals; N_j and $\vec{V}_j$ denote the density and mean velocity of species j; and $\langle \sigma v \rangle_z$ is the rate constant for electron impact ionization. We assume singly charged ions and employ quasi-neutrality and local ambipolarity to obtain

$$N_e = N_i \quad \text{and} \quad \vec{V}_e = \vec{V}_i \; . \tag{3}$$

We assume that the mean ion velocity perpendicular to the magnetic field $V_{i\perp}$ vanishes,

$$\vec{V}_{i\perp} = 0 \; , \tag{4}$$

and for the perpendicular mean neutral velocity $\vec{V}_{n\perp}$ we use the perpendicular neutral momentum balance equation to obtain

$$N_n \vec{V}_{n\perp} = \frac{-\nabla_\perp (N_n T) + 0.24 N_n \nabla_\perp T}{M N_i \langle \sigma v \rangle_x + M N_e \langle \sigma v \rangle_z} \; , \tag{5}$$

where M and T are the mass and temperature of the ions and neutrals and $\langle \sigma v \rangle_x$ is the charge exchange rate constant. The $0.24 N_n \nabla T$ term is the thermal force found in Ref. 4 and the inertial terms are neglected since we assume that the perpendicular mean flows are small. By taking $\vec{V}_{i\perp} = 0$ we are neglecting the anomalous perpendicular particle flux normally retained in the codes. Were explicit expressions available for the anomalous transport coefficients, the boundary conditions could be suitably modified

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and similarity techniques employed on the generalized system of equations. However, divertor simulators and codes need not necessarily model the details of anomalous transport in the SOL. We avoid making ad hoc assumptions about the anomalous transport coefficients in the SOL by specifying the SOL width as an input parameter and by considering only the divertor region between the x-point and the target.

To find the equation for the parallel ion velocity $V_{i\parallel}$ we add the parallel ion and electron momentum equations to eliminate electron-ion friction and obtain the parallel plasma momentum balance equation

$$\nabla \cdot \left[(MN_iV_{i\parallel}^2 + N_iT + N_eT_e + \pi_{i\parallel})\hat{n} \right] = \langle \sigma v \rangle_z MN_eN_nV_{n\parallel}$$
$$- \langle \sigma v \rangle_x MN_nN_i(V_{i\parallel} - V_{n\parallel}) - 0.24N_n\hat{n} \cdot \nabla T , \qquad (6)$$

where $\hat{n} = B/B$ is the unit vector along the magnetic field and T_e is the electron temperature. Because of charge exchange coupling the parallel ion viscosity $\pi_{i\parallel}$ is $(N_n+N_i)/N_i$ times the Braginskii[7] value[3,4] and therefore given by

$$\pi_{i\parallel} = 1.3(N_i + N_n)T_i\tau_i\hat{n} \cdot \nabla V_{i\parallel} , \qquad (7)$$

with τ_i the ion-ion collision time

$$\tau_i = \frac{3M^{1/2}T^{3/2}}{4(\pi)^{1/2}N_ie^4\ell n\Lambda} . \qquad (8)$$

The parallel neutral velocity $V_{n\parallel}$ is found from the parallel neutral momentum equation

$$\nabla \cdot \left[(MN_nV_{n\parallel}^2 + N_nT)\hat{n} + \stackrel{\leftrightarrow}{\pi}_n \cdot \hat{n} \right] = - \langle \sigma v \rangle_z MN_eN_nV_{n\parallel}$$
$$+ \langle \sigma v \rangle_x MN_nN_i(V_{i\parallel} - V_{n\parallel}) + 0.24N_n\hat{n} \cdot \nabla T , \qquad (9)$$

where, in the presence of charge exchange coupling to the ions, the neutral viscosity $\pi_n \cdot \hat{n}$ is is given by[3,4]

$$\stackrel{\leftrightarrow}{\pi}_n \cdot \hat{n} = \left(\frac{N_n}{N_i}\right)\pi_{i\parallel}\hat{n} - \frac{N_nT}{N_i\langle \sigma v \rangle_x}\left[\nabla V_{n\parallel} + \frac{1}{3}\hat{n}\hat{n} \cdot \nabla V_{n\parallel}\right] . \qquad (10)$$

Charge exchange causes the ion and neutrals temperatures to be equal to lowest order, as well as coupling the ion and neutral flows via Eq.(6). To close our system of

equations we need separate equations for the ion/neutral and electron temperatures. Ion plus neutral energy conservation with the viscosity and electric field terms neglected gives the equation for T to be

$$\nabla \cdot \left[(\frac{5}{2}T + \frac{1}{2}MV_{\parallel}^2)(N_i + N_n)\vec{V} + \vec{q}_i + \vec{q}_n \right] = \frac{3mN_e(T_e - T)}{M\tau_{ei}} , \qquad (11)$$

where V and $V_{\parallel}$ are mean mass velocities defined by

$$(N_i + N_n)\vec{V} = N_i\vec{V}_i + N_n\vec{V}_n , \qquad (12)$$

$V^2 \approx V_{\parallel}^2$, and τ_{ei} is the electron-ion collision time

$$\tau_{ei} = \frac{3m^{1/2}T_e^{3/2}}{4(2\pi)^{1/2}N_ie^4 \ell n\Lambda} . \qquad (13)$$

Charge exchange makes the parallel ion heat flux $\vec{q}_i$ larger than its Braginskii[7] value by[3,4] $(N_n+N_i)/N_i$ giving

$$\vec{q}_i = -3.9\left[(N_i + N_n)T\tau_i / M\right]\hat{n}\hat{n} \cdot \nabla T . \qquad (14)$$

The neutral heat flux $\vec{q}_n$ in the presence of charge exchange coupling to the ions is given by[3,4]

$$\vec{q}_n = \left(\frac{N_n}{N_i}\right)\vec{q}_i - \frac{2.4\,N_nT}{MN_i\langle\sigma v\rangle_x}\nabla T - 0.24\,N_nT(\vec{V}_n - \vec{V}_i) , \qquad (15)$$

where the last term is of the form referred to as a diffusion thermo-effect according to Chapman and Cowling[8].

The final equation for the electron temperature T_e follows from the electron energy conservation equation

$$\nabla \cdot (\frac{5}{2}T_eN_e\vec{V}_e + \vec{q}_e) = -\frac{3mN_e(T_e - T)}{M\tau_{ei}} - (I\langle\sigma v\rangle_z + E_H\langle\sigma v\rangle_H)N_eN_n - E_I\langle\sigma v\rangle_I N_eN_I , \qquad (16)$$

where I is the ionization potential for hydrogen (13.6 eV), E_H is the excitation energy of hydrogen (10.2 eV for Lyman α) with an excitation rate constant of $\langle\sigma v\rangle_H$, N_I is the impurity density, and E_I is the energy of the relevant excited impurity state with an excitation rate constant of $\langle\sigma v\rangle_I$. The parallel electron heat flux $\vec{q}_e$ is just the Braginskii[7] result

$$\vec{q}_e = -3.2\,(N_e T_e \tau_{ei}\,/\,m)\,\hat{n}\hat{n}\cdot\nabla T_e \ . \tag{17}$$

The preceding system of equations requires ten boundary conditions since it is tenth order in the eight unknowns N_e, N_n, $V_{i\parallel}$, V_n, T_e, and T. For upstream boundary conditions we employ

$$N_n|_{up} = 0 \ , \tag{18}$$

$$N_e(T_e+T)|_{up} = P_{up} \ , \tag{19}$$

$$-3.2(N_e T_e \tau_{ei}/m)\hat{n}\cdot\nabla T_e|_{up} = q^{up}_{e\parallel} \ , \tag{20}$$

and

$$-3.9\,[(N_i+N_n)T\tau_i/M]\hat{n}\cdot\nabla T|_{up} = q^{up}_{i\parallel} \ , \tag{21}$$

where the upstream pressure P_{up}, and upstream parallel electron (ion) heat flux $q^{up}_{e\parallel}$ ($q^{up}_{i\parallel}$) are all specified functions at the upstream entrance to the divertor. The neutral density boundary condition corresponds to considering complete recycling in the divertor region since it, ambipolarity, and the sum of the two continuity equations require that there be no plasma entering through the upstream boundary.

The remaining six boundary conditions are applied downstream at the divertor target plates and sidewalls, where wall values of density, temperature, etc. will be denoted by a subscript w. The outward directed unit vector normal to the wall is defined as $\hat{w}$, where for definiteness we assume $\hat{w}\cdot\hat{n} > 0$ ($\hat{w}\cdot\hat{n} < 0$) when $V_{i\parallel} < 0$ ($V_{i\parallel} > 0$). The parallel ion flow into the walls must satisfy a generalized Bohm sheath criterion,

$$\hat{w}\cdot\hat{n}\,V_{i\parallel}|_w = -\,\hat{w}\cdot\hat{n}\,\alpha(T_w/M)^{1/2}, \tag{22}$$

where $\alpha \sim 0.5\text{-}1$. To maintain a steady state, complete recycling is imposed at the walls by demanding that the outgoing normal neutral flux equal the incoming normal ion flux,

$$\hat{w}\cdot(\hat{n}N_i V_{i\parallel} + N_n V_n)|_w = 0. \tag{23}$$

The parallel plasma momentum flux normal to the wall is specified by

$$\hat{w}\cdot\hat{n}(MN_i V^2_{i\parallel} + N_i T + N_e T_e + \pi_{i\parallel})|_w = \gamma_\parallel\,\hat{w}\cdot\hat{n}\,N_{iw}T_w \ , \tag{24}$$

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where $\gamma_\| \sim 2\text{-}3$ is the plasma momentum transmission coefficient. Similarly, the normal neutral momentum flux normal to the wall is taken to be

$$[\hat{\mathbf{w}} \cdot \hat{\mathbf{n}}(MN_n V_{n\|}^2 + N_n T) + \hat{\mathbf{w}} \cdot \overleftrightarrow{\pi}_{n\|} \cdot \hat{\mathbf{n}})]|_w = \gamma_m N_{nw} T_w , \qquad (25)$$

with $\gamma_m \sim 1\text{-}2$ the neutral momentum transmission coefficient.

The final wall boundary conditions are on the energy fluxes. The normal ion plus neutral energy flux onto the wall is taken as

$$|\hat{\mathbf{w}} \cdot [(5T/2 + MV^2/2)(N_n + N_i)\mathbf{V} + \vec{q}_i + \vec{q}_n)]|_w = -(\alpha|\hat{\mathbf{w}} \cdot \hat{\mathbf{n}}|\gamma_i N_{iw} + \gamma_n N_{nw})T_w(T_w/M)^{1/2}, \qquad (26)$$

where $\gamma_i \sim 2\text{-}3$ and $\gamma_n \sim 0.1\text{-}0.3$ are the ion and neutral heat transmission coefficients, respectively. The $\alpha\hat{\mathbf{w}} \cdot \hat{\mathbf{n}}$ in the ion contribution on the right side accounts for the ion heat flux moving along the magnetic field and ions hitting the wall with a parallel speed satisfying the Bohm condition. A similar expression holds for the normal electron energy flux onto the wall,

$$\hat{\mathbf{w}} \cdot [\hat{\mathbf{n}}(5T_e/2)N_e V_{e\|} + \vec{q}_e)]|_w = -\alpha\hat{\mathbf{w}} \cdot \hat{\mathbf{n}}\gamma_e N_{ew} T_{ew}(T_{ew}/M)^{1/2}, \qquad (27)$$

with $\gamma_e \sim 2\text{-}3$ the electron heat transmission coefficient. The various transmission coefficients are discussed in Refs. 9.

The preceding equations and boundary conditions form the basic system of equations that we will investigate for fluid neutrals. In the following sections we will make further assumptions that allow us to combine the preceding equations to obtain simpler systems. Moreover, In Sec. V we will make modifications that allow us to consider Knudsen neutrals.

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III. Dimensionless Form of the Equations and Boundary Conditions

To determine if the system of equations and boundary conditions (and, therefore, the heat loads on target, etc.) of Sec. II are invariant under particular sets of scaling transformations, it is convenient to first make the entire system dimensionless. To do so, we will adopt a 2-D cartesian model with x and y denoting the radial and poloidal coordinates, and z corresponding to the ignorable toroidal direction ($\partial/\partial z = 0$). Writing the unit vector along the magnetic field as

$$\hat{n} = (B_T/B)\hat{z} + b\hat{y} \ ,$$

with $b = B_p/B$, and B_p (B_T) the poloidal (toroidal) magnetic field components, gives

$$\hat{n} \cdot \nabla = b\partial/\partial y \ , \quad V_{iy} = bV_{i\parallel} \ , \quad V_{iz} = (B_T/B)V_{i\parallel} \quad \text{and} \quad V_{j\parallel} = bV_{iy} + (B_T/B)V_{iz} \ , \quad (28)$$

where $j = i$, n or e and we will assume that $B = |B|$ is a constant.

We normalize temperatures and velocities to the hydrogen ionization potential I by defining

$$\tau = T/I, \quad \tau_e = T_e/I, \quad v_\parallel = (M/I)^{1/2}V_{i\parallel} \ ,$$

$$u = (M/I)^{1/2}V_{ny} \ , \quad w = (M/I)^{1/2}V_{nx} \quad \text{and} \quad v = (M/I)^{1/2}V_{n\parallel} \ . \quad (29)$$

Densities are normalized by introducing the peak upstream plasma pressure P_u so that the known function P_{up} in the upstream pressure boundary condition may be written as

$$P_{up} = P_u S_p(x/\Delta_p) \ , \quad (30)$$

where $S_p(x/\Delta_p)$ is a specified order unity SOL shape function. Using P_u and I the normalized plasma and neutral densities are then defined as

$$n = IN_e/P_u = IN_e/P_u \quad \text{and} \quad \eta = IN_n/P_u \ . \quad (31)$$

We desire to normalize lengths to the neutral penetration scale length ℓ_n at the temperature I and density P_u/I. As a result, it is convenient to introduce another set of shape functions $S_x(T)$ and $S_z(T_e)$ by defining

$$<\sigma v>_x = K_x S_x(T) \quad \text{and} \quad <\sigma v>_z = K_z S_z(T_e) \ , \quad (32)$$

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where K_x and K_z are true constants equal to the appropriate peak values of the charge exchange and ionization rate constants so that the functions $S_x(T)$ and $S_z(T_e)$ are of order unity. Using the preceding definitions, ℓ_n is defined as

$$\ell_n = \frac{I(I/M)^{1/2}}{P_u(K_xK_z)^{1/2}} , \tag{33}$$

and the normalized poloidal (β) and radial (ρ) variables may be defined as

$$\beta = y/\ell_n \quad \text{and} \quad \rho = x/\ell_n . \tag{34}$$

Using the preceding definitions and introducing the definition

$$\sigma = (K_z/K_x)^{1/2} , \tag{35}$$

the continuity equations and the perpendicular neutral flux equations become

$$b\frac{\partial}{\partial\beta}(\eta v_{\parallel}) = \sigma n\eta S_z(\tau) , \tag{36}$$

$$\frac{\partial}{\partial\beta}(\eta u) + \frac{\partial}{\partial\rho}(\eta w) = -\sigma n\eta S_z(\tau_e) , \tag{37}$$

$$\eta(u - bv) = \frac{-\dfrac{\partial}{\partial\beta}(\eta\tau) + 0.24\eta\dfrac{\partial\tau}{\partial\beta}}{n[\sigma S_z(\tau_e) + \sigma^{-1}S_x(\tau)]} , \tag{38}$$

and

$$\eta w = \frac{-\dfrac{\partial}{\partial\rho}(\eta\tau) + 0.24\eta\dfrac{\partial\tau}{\partial\rho}}{n[\sigma S_z(\tau_e) + \sigma^{-1}S_x(\tau)]} . \tag{39}$$

To make the parallel plasma momentum equation dimensionless we need to introduce a new dimensionless parameter μ which is the Coulomb mean free path λ divided by the neutral penetration length at temperature I,

$$\mu = \frac{0.55(MK_xK_z)^{1/2}I^{3/2}}{e^4\ell n\Lambda} \approx \frac{\lambda}{\ell_n}\bigg|_{T=I} . \tag{40}$$

Using this definition in $\pi_{i\parallel}$, parallel plasma momentum balance becomes

$$b\frac{\partial}{\partial\beta}\left[nv_i^2 + n(\tau + \tau_e) + \mu b\tau^{5/2}\frac{(n + \eta)}{n}\frac{\partial v_i}{\partial\beta}\right] + 0.24bn\frac{\partial\tau}{\partial\beta}$$
$$= n\eta\left[\sigma v S_z(\tau_e) - \sigma^{-1}(v_i - v)S_x(\tau)\right] . \tag{41}$$

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No further new parameters are introduced by neutral momentum balance which upon inserting the neutral viscosity may be written as

$$b\frac{\partial}{\partial\beta}\left[\eta v^2 + \eta\tau + \mu b\tau^{5/2}\frac{\eta(n+\eta)}{n^2}\frac{\partial v_{\parallel}}{\partial\beta}\right] - 0.24 b\eta\frac{\partial\tau}{\partial\beta} - \frac{\partial}{\partial\beta}\left[\frac{\sigma(1+\frac{1}{3}b^2)\eta\tau}{nS_x(\tau)}\frac{\partial v}{\partial\beta}\right]$$

$$- \frac{\partial}{\partial\rho}\left[\frac{\sigma\eta\tau}{nS_x(\tau)}\frac{\partial v}{\partial\rho}\right] = -n\eta\left[\sigma v S_z(\tau_e) - \sigma^{-1}(v_{\parallel}-v)S_x(\tau)\right] . \tag{42}$$

To write electron energy balance in dimensionless form it is convenient to first introduce two new shape functions $S_H(T_e)$ and $S_I(T_e)$ for the hydrogen and impurity radiation by letting

$$<\sigma v>_H = K_H S_H(T_e) \quad \text{and} \quad <\sigma v>_I = K_I S_I(T_e) , \tag{43}$$

where K_H and K_I are true constants equal to the appropriate peak values of the hydrogen and impurity rate constants such that the functions $S_H(T_e)$ and $S_I(T_e)$ are order unity functions. Next, we define the dimensionless impurity density n_I and dimensionless constants σ_H and σ_I corresponding respectively to energy and impurity density times energy weighted ratios of radiation to neutral penetration scale lengths,

$$n_I = IN_I/P_u, \quad \sigma_H = E_H K_H/I(K_x K_z)^{1/2} \quad \text{and} \quad \sigma_I = n_I E_I K_I/I(K_x K_z)^{1/2} . \tag{44}$$

Notice that we have implicitly assumed that the shape function for n_I depends only on T_e, but we could introduce an additional shape function to remove this restriction. A final dimensionless constant parameter k, proportional to $(M/m)^{1/2}$ times the Coulomb mean free path λ divided by the neutral penetration length at temperature I, is defined by

$$k = \frac{0.96 M (K_x K_z)^{1/2} I^{3/2}}{m^{1/2} e^4 \ell n\Lambda} \approx \left(\frac{M}{m}\right)^{1/2}\frac{\lambda}{\ell_n}\bigg|_{T_e=I} . \tag{45}$$

Notice that k is also a measure of the temperature equilibration length divided by the neutral penetration length. In terms of the preceding parameters, the dimensionless electron energy equation becomes

$$b\frac{\partial}{\partial\beta}\left[\frac{5}{2}\tau_e nv_{\parallel} - kb\tau_e^{5/2}\frac{\partial\tau_e}{\partial\beta}\right] = -\frac{9.6n^2(\tau_e-\tau)}{k\tau_e^{3/2}} - \sigma n\eta S_z(\tau_e) - \sigma_H n\eta S_H(\tau_e) - \sigma_I n S_I(\tau_e) . \tag{46}$$

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The ion plus neutral energy balance introduces no new parameters. With the heat fluxes inserted it becomes the rather complicated expression

$$\frac{\partial}{\partial\beta}\left\{\left[\frac{5}{2}\tau+\frac{1}{2}\left(\frac{nv_{\parallel}+\eta v}{n+\eta}\right)^{2}\right](bnv_{I}+\eta u)-3\mu b^{2}\tau^{5/2}\frac{(n+\eta)^{2}}{n^{2}}\frac{\partial\tau}{\partial\beta}-0.24\,\eta\tau(u-bv_{\parallel})\right\}$$

$$+\frac{\partial}{\partial\rho}\left\{\left[\frac{5}{2}\tau+\frac{1}{2}\left(\frac{nv_{\parallel}+\eta v}{n+\eta}\right)^{2}\right]\eta w-0.24\eta\tau w\right\}-\frac{\partial}{\partial\beta}\left[\frac{2.4\sigma\eta\tau}{nS_{x}(\tau)}\frac{\partial\tau}{\partial\beta}\right]$$

$$-\frac{\partial}{\partial\rho}\left[\frac{2.4\sigma\eta\tau}{nS_{x}(\tau)}\frac{\partial\tau}{\partial\rho}\right]=\frac{9.6n^{2}(\tau_{e}-\tau)}{k\tau_{e}^{3/2}}\ , \tag{47}$$

where to simplify this result we have assumed for the first time that Bp (and, therefore, b) is a constant.

To obtain the boundary conditions in dimensionless form it is convenient to introduce the shape functions $S_{e}(x/\Delta_{p})$ and $S_{i}(x/\Delta_{p})$ for known upstream poloidal electron ($bq_{e\parallel}^{up}$) and ion ($bq_{i\parallel}^{up}$) heat fluxes by defining

$$bq_{e\parallel}^{up}=-Q_{e}P_{u}(I\,/\,M)^{1/2}S_{e}(x\,/\,\Delta_{p})\quad\text{and}\quad bq_{i\parallel}^{up}=-Q_{i}P_{u}(I\,/\,M)^{1/2}S_{i}(x\,/\,\Delta_{p})\ . \tag{48}$$

The dimensionless parameters Q_{e} and Q_{i} times $P_{u}(I/M)^{1/2}$ are the peak upstream values of the poloidal electron and ion heat fluxes, respectively. Letting $y = 0$ and $y = L$ denote the target and the location at which the upstream boundary conditions are applied, respectively, and employing the preceding definitions, the dimensionless forms of the upstream boundary conditions for the neutral density, plasma pressure, and electron and ion heat fluxes are as follows:

$$\eta(\beta=L/\ell_{n})=0\ , \tag{49}$$

$$n(\tau+\tau_{e})\big|_{\beta=L/\ell_{n}}=S_{p}(\rho\ell_{n}\,/\,\Delta_{p})\ , \tag{50}$$

$$kb^{2}\tau_{e}^{5/2}\frac{\partial\tau_{e}}{\partial\beta}\bigg|_{\beta=L/\ell_{n}}=Q_{e}S_{e}(\rho\ell_{n}\,/\,\Delta_{p})\ , \tag{51}$$

and

$$3\mu b^{2}\tau^{5/2}\frac{\partial\tau}{\partial\beta}\bigg|_{\beta=L/\ell_{n}}=Q_{i}S_{i}(\rho\ell_{n}\,/\,\Delta_{p})\ . \tag{52}$$

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We assume that the magnetic field is parallel to the sidewalls located at x = ±Δ and denote by a subscript d the target plate (y = 0). Then, the dimensionless forms of the boundary conditions for the parallel ion flow, parallel plasma momentum, and parallel electron energy flow (which do not involve the sidewalls) follow:

$$v_{\|d} = -\alpha\tau_d^{1/2} , \tag{53}$$

$$\alpha^2 n_d\tau_d + n_d(\tau_d + \tau_{ed}) + \mu b\tau_d^{5/2}\frac{(n_d + \eta_d)}{n_d}\left.\frac{\partial v_\|}{\partial\beta}\right|_d = \gamma_\| n_d\tau_d , \tag{54}$$

and

$$\frac{5}{2}\alpha n_d\tau_{ed}\tau_d^{1/2} + kb\tau_{ed}^{5/2}\left.\frac{\partial\tau_e}{\partial\beta}\right|_d = \alpha\gamma_e n_d\tau_{ed}^{3/2} , \tag{55}$$

The remaining boundary conditions involve the neutrals so we must distinguish between the sidewalls (at which $\hat{w}\cdot\hat{n} = \hat{x}\cdot\hat{n} = 0$ and whose location we denote by subscript s meaning x = ±Δ/2) and target (at which $\hat{w}\cdot\hat{n} = \hat{y}\cdot\hat{n} = b$). As a result, for the remaining dimensionless boundary conditions for the ion plus neutral flows, the neutral momentum, and the ion plus neutral energy we find the following forms:

$$\eta_d u_d = \alpha b n_d\tau_d^{1/2} , \qquad \eta_s w_s = 0, \tag{56}$$

$$b\eta_d(v_d^2 + \tau_d) + \mu b^2\tau_d^{5/2}\frac{\eta_d(n_d + \eta_d)}{n_d^2}\left.\frac{\partial v_\|}{\partial\beta}\right|_d - \frac{\sigma(1 + \frac{1}{3}b^2)\eta_d\tau_d}{n_d S_x(\tau_d)}\left.\frac{\partial v}{\partial\beta}\right|_d = \gamma_m\eta_d\tau_d , \tag{57a}$$

$$-\frac{\sigma\eta_s\tau_s}{n_s S_x(\tau_s)}\left.\frac{\partial v}{\partial\rho}\right|_s = \gamma_m\eta_s\tau_s , \tag{57b}$$

and

$$\left[\frac{2.4\sigma\eta_d\tau_d}{n_d S_x(\tau_d)} + 3\mu b^2\tau_d^{5/2}\frac{(n_d + \eta_d)^2}{n_d^2}\right]\left.\frac{\partial\tau}{\partial\beta}\right|_d + 0.24\eta_d\tau_d(u_d - bv_{|d}) = (\alpha b\gamma_i n_d + \gamma_n\eta_d)\tau_d^{3/2} , \tag{58a}$$

$$-\frac{2.4\sigma\eta_s\tau_s}{n_s S_x(\tau_s)}\left.\frac{\partial\tau}{\partial\rho}\right|_s = \gamma_n\eta_s\tau_s^{3/2} . \tag{58b}$$

Our fluid description in its most general form is now complete. In the next section we will consider the properties and simplifications of our system of equations.

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IV. Scaling Transformations and Similarity for Fluid Descriptions

The full 2-D system of differential equations and boundary conditions as given by Eqs.(36-39), (41), (42), (46), (47), (49-52), (53-55), and (56-58) involves the eight unknowns n, η, $v_\parallel$, w, u, v, τ, and τ_e and the seventeen dimensionless parameters σ, b, μ, k, σ_H, σ_I, α, Q_e, Q_i, $\gamma_\parallel$, γ_m, γ_e, γ_i, γ_n, L/ℓ_n, Δ_p/ℓ_n, and Δ/ℓ_n. It is important to notice that P_u only appears in the definitions n, η, ℓ_n, σ_I, and through the normalizations for the upstream heat fluxes as given by Eq.(48). As a result, for similar configurations the densities (recall $\sigma_I \propto n_I \propto N_I/P_u$) must scale linearly with P_u, while the depth (L) and width (Δ_p and Δ) scaling must be inversely proportional to P_u. Recall also that $Q_e \propto b q_{eI}^{up}/P_u$ because of the normalization of the upstream heat fluxes. Notice that the usual collisionality scaling of density times scale length is embedded in L/ℓ_n, Δ_p/ℓ_n, and Δ/ℓ_n which are proportional to density through P_u.

Fortunately, most of the dimensionless parameters are reasonably well known, but rather wide ranges of b, σ_I, Q_e, Q_i, L/ℓ_n, Δ_p/ℓ_n, and Δ/ℓ_n are of interest since they depend on the upstream plasma pressure and heat fluxes, the impurities, and the divertor configuration. Typically $\sigma \sim 0.1\text{-}1$, $\sigma_I \sim 1$, and for a tokamak $b \sim 0.05$; while for $P_u \sim 10^{14}\,\text{cm}^{-3} \times 100\text{eV}$, $\lambda \sim \ell_n \sim 0.3\text{cm}$, giving $\mu \sim 1$ and $k \sim 50$.

To reduce the number of dimensionless parameters we can seek scale transformations of this system of equations. It will turn out that the full system does not allow any scale transformations, but that by simplifying the system, scale transformations can be found which allow us to reduce the number of dimensionless parameters that need be considered.

A. Model Without Neutrals

We first employ the Connor and Taylor[1,2] procedure on a simple limit to illustrate the need to retain boundary conditions and how they are easily included by their technique. If we adopt the simplest model possible by neglecting the neutrals completely ($\eta \to 0$), letting $v_\parallel \to 0$ and $\tau = \tau_e$, adding the energy equations to eliminate

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electron-ion energy exchange, and neglecting the ion heat conduction compared to that of the electrons, we find that plasma pressure balance becomes $2n\tau_e = S_p(\rho\ell_n/\Delta_p)$, so that we need only consider

$$2b\frac{\partial}{\partial\beta}\left[kb\tau_e^{5/2}\frac{\partial\tau_e}{\partial\beta}\right] = \sigma_I\tau_e^{-1}S_I(\tau_e)S_p(\rho\ell_n/\Delta_p) . \tag{59}$$

In this limit the upstream boundary condition is given by Eq. (51), and the target boundary condition is given by Eq. (55) with the convection term neglected,

$$2kb\tau_{ed}^{5/2}\frac{\partial\tau_e}{\partial\beta}\bigg|_d = \alpha\gamma_e\tau_{ed}^{1/2}S_p(\rho\ell_n/\Delta_p) . \tag{60}$$

In this model the parameter Δ/ℓ_n does not enter because sidewall boundary conditions are not needed since $\Delta_p < \Delta$.

We seek scale transformations by scaling the dependent variable τ_e, the two independent variables ρ and β, and the seven dimensionless parameters b, k, σ_I, Q_e, $\alpha\gamma_e$, L/ℓ_n, and Δ_p (but only the arguments of shape functions since their coefficients are accounted for by scaling the dimensionless parameters). Letting $\tau_e \rightarrow \omega_1\tau_e$, $\beta \rightarrow \omega_2\beta$, $\rho \rightarrow \omega_3\rho$, $b \rightarrow \omega_4 b$, $k \rightarrow \omega_5 k$, $\sigma_I \rightarrow \omega_6\sigma_I$, $Q_e \rightarrow \omega_7 Q_e$, $\alpha\gamma_e \rightarrow \omega_8\alpha\gamma_e$, $L/\ell_n \rightarrow \omega_9 L/\ell_n$, and $\Delta_p/\ell_n \rightarrow \omega_{10}\Delta_p/\ell_n$ we find four independent scale transformations:

(i) $\beta \rightarrow \omega_2\beta$, $L/\ell_n \rightarrow \omega_2 L/\ell_n$, $k \rightarrow \omega_2 k$, $\sigma_I \rightarrow \sigma_I/\omega_2$;

(ii) $\rho \rightarrow \omega_3\rho$, $\Delta_p/\ell_n \rightarrow \omega_3\Delta_p/\ell_n$;

(iii) $b \rightarrow \omega_4 b$, $\sigma_I \rightarrow \omega_4\sigma_I$, $k \rightarrow k/\omega_4$, $Q_e \rightarrow \omega_4 Q_e$; and

(iv) $Q_e \rightarrow \omega_7 Q_e$, $\sigma_I \rightarrow \omega_7\sigma_I$, $k \rightarrow \omega_7 k$, $\alpha\gamma_e \rightarrow \omega_7\alpha\gamma_e$; \hfill (61)

where ω_j denotes the scaling constants and for each transformation we do not indicate quantities not scaled (notice that τ_e can never be scaled because of the complicated dependence of S_I). The second transformation is needed to keep the argument of ρ dependent shape functions fixed.

We started with seven dimensionless parameters and two dimensionless independent variables and have found four transformations. Any physical quantity of

interest derivable from this system of equations must be invariant under the four scaling transformations. In particular, we can find the five invariant combinations of the seven dimensionless parameters and the two dimensionless variables by considing the scaling properties of the invariant product

$$Z_{ABC..} = b^A k^B \sigma_I^C Q_e^D (\alpha \gamma_e)^E (L / \ell_n)^F (\Delta_p / \ell_n)^G \rho^H \beta^I .$$ (62)

For both sides of Eq.(62) to be invariant $(Z_{ABC...} \rightarrow Z_{ABC...})$ under the scale transformations of Eq.(61) requires

$$1 = \omega_2^{B-C+F+I} \omega_3^{G+H} \omega_4^{A-B+C+D} \omega_7^{B+C+D+E}$$

or

$$F = C - B - I, G = - H, A = B - C - D, \text{ and } E = - B - C - D ,$$

which when used in Eq.(62) gives

$$Z_{BCDHI} = (bk\ell_n / L\alpha\gamma_e)^B (\sigma_I L / \ell_n b\alpha\gamma_e)^C (Q_e / b\alpha\gamma_e)^D (\rho\ell_n / \Delta_p)^H (\beta\ell_n / L)^I .$$ (63)

As a result, only the three independent dimensionless parameters $bk\ell_n / L\alpha\gamma_e$, $\sigma_I L / \ell_n b\alpha\gamma_e$, and $Q_e / b\alpha\gamma_e$, as well as the independent variable combinations $\rho\ell_n / \Delta_p$ and $\beta \ell_n / L$ satisfy the four scaling transformations, reducing the number of dimensionless quantities by four.

We are particularly interested in the form of the normalized poloidal energy flux on the target plates $Q_t \equiv q_t / P_u (I/M)^{1/2}$, which depends on $\rho\ell_n / \Delta_p$ but not $\beta \ell_n / L$ since $\beta = 0$. We first note that Q_t must contain a coefficient having the same scaling as Q_e $(Q_t \rightarrow \omega_4 \omega_7 Q_t)$ times an unknown function of these three independent parameters and $\rho\ell_n / \Delta_p$:

$$q_t = Q_e P_u (I/M)^{1/2} f(bk\ell_n / L\alpha\gamma_e, \sigma_I L / \ell_n b\alpha\gamma_e, Q_e / b\alpha\gamma_e, \rho\ell_n / \Delta_p) .$$ (64)

Here and elsewhere f is used to denote an unknown function of the arguments listed. The arguments of f can be written in various equivalent ways since products of the invariant dimensionless quantities are invariant. A convenient form for our purposes is

$$q_t = b\alpha\gamma_e P_u (I/M)^{1/2} f(bk\ell_n/L\alpha\gamma_e, k\sigma_I/\alpha^2\gamma_e^2, Q_e/b\alpha\gamma_e, \rho\ell_n/\Delta_p) ,\qquad (65)$$

where Q_e and $b\alpha\gamma_e$ scale the same way.

Since the hydrogen ionization potential does not enter this simple model we can make the replacement $I \rightarrow E_I$ in all quantities to replace I by the excitation energy of interest. Notice that none of the dimensionless parameters that enter Eqs.(64) or (65) depend on $K_x K_z$ since the neutral penetration length, which no longer enters, is replaced by b times an appropriate Coulomb mean free path $(0.96 E_I^3 / P_u e^4 \ell n\Lambda)$. In this simple model, these same dimensionless parameters can be found by direct integration and application of the boundary conditions. Then, it can be seen that for a sufficiently localized shape function $S_I(\tau_e)$ with a τ_e dependence permitting no impurity radiation losses for $y > L$, the function f in Eq.(65) will be insensitive to the first parameter which describes the Coulomb collisionality. Moreover, for a sufficiently low impurity density, f will be insensitive to the second parameter.

If we form the power to the plates P by integrating x over the SOL width Δ_p, then for a single null divertor $P = 4\pi R\int dxq_t$ gives

$$P/R = b\alpha\gamma_e\Delta_p P_u (I/M)^{1/2} f(bk\ell_n/L\alpha\gamma_e, k\sigma_I/\alpha^2\gamma_e^2, Q_e/b\alpha\gamma_e) ,\qquad (66)$$

where R is the major radius of the tokamak and, of course, f is a different unknown function. Notice that the only dependence on the SOL width enters as an explicit multiplier in Eq.(66). For similar divertors we must keep the unknown function f fixed. Since $bk\ell_n/L\alpha\gamma_e \propto b/\alpha\gamma_e L P_u$, we may use $P_u \propto b/\alpha\gamma_e L$ in Eq.(66) to obtain

$$P/R \propto b\alpha\gamma_e P_u\Delta_p \propto b^2\Delta_p/L\qquad (67)$$

for similar devices in which $bk\ell_n/L\alpha\gamma_e$, $k\sigma_I/\alpha^2\gamma_e^2$, and $Q_e/b\alpha\gamma_e$ are held constant to keep the unknown function f constant. If f is independent of its first argument (localized S_I) then the final form of Eq.(67) is not relevant.

In Lackner's[6] treatment all lengths and magnetic fields in the divertor must scale the same way for similar tokamaks or similar divertors so that Δ_p/L and b, and, therefore, P/R are constants. It should be mentioned that Lackner actually takes P to be the power from the core that crosses the separatrix, rather than the power entering the divertor. In our restricted model Δ_p, P_u, b, Q_e, and $k\sigma_I \propto n_I E_I K_I$, are viewed as independent control parameters (k is a constant and $\alpha\gamma_e$ does not vary significantly), so for a given L we can adjust P_u, $k\sigma_I \propto n_I E_I K_I$, and Q_e to keep the arguments of f fixed and still be free to adjust Δ_p and b. The constraints and P/R scaling for this model are summarized in the first column of Table I.

Similarity constraints are significantly relaxed by considering this substantially reduced description with no neutrals. In ITER[10] P~300MW, R~8m, b ~ 1/6, Δ_p ~ 1cm, and P_u ~ 10^{14}cm^{-3}×500eV, so using Eq.(67), a similar device with P ~ 30MW and R ~ 4m gives P_{ITER}/R_{ITER} ~ 5P/R and is possible for a P_u about five times smaller (but L five times larger) than the ITER value, assuming roughly the same b, Δ_p, and $\alpha\gamma_e$ (if Δ_p in ITER turns out to be larger than 1cm then the power of the similar device could be lowered by the same amount). If we employ the anticipated Alcator C-MOD numbers (P ~ 8MW, R ~ 0.7m, Δ_p ~ 1cm, and P_u ~ 10^{14}cm^{-3}×100eV), we get a value of $P/RP_u\Delta_p$ larger than the ITER value by almost a factor of two so that it could provide good similarity to ITER in the fluid neutral limit if the collisionality constraint ($bk\ell_n/L\alpha\gamma_e \propto$ $b/\alpha\gamma_e LP_u$ =constant) did not need to be satisfied because of sufficiently localized impurity radiation losses such as those that may be occuring when divertor detachment is observed[11-14].

In addition to Δ_p and b, we are also free to adjust B (since only the ratio b enters in our equations). These three adjustments can be used to satisfy other constraints, such as gyroradius over scale length (for example, constant $B\Delta_p$ or $B_p\Delta_p$) and/or plasma beta (constant $\beta \propto P_u/B^2$) constraints. Some possibilities are shown in Table II, and include a P/R$\propto$b scaling similar to that found In Ref.15.

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B. Reduced Fluid Neutral Model

If we assume that $\sigma \ll 1$ and $b \ll 1$, and neglect inertial $(v^2 \approx v_\parallel^2 \ll \tau \sim 1)$ and ion $(\sigma \mu b^2 \ll 1)$ and neutral viscosity $(\sigma \ll 1)$ effects, then the ionization terms in Eqs. (38) and (39) may be neglected and Eq. (41) or (42) employed to find $v \approx v_\parallel$. As a result, the perpendicular neutral flux equations, Eqs. (38) and (39), become

$$n\eta(u - bv_I)S_x(\tau) = \sigma\left[-\frac{\partial}{\partial\beta}(\eta\tau) + 0.24\eta\frac{\partial\tau}{\partial\beta}\right] \tag{68}$$

and

$$n\eta wS_x(\tau) = \sigma\left[-\frac{\partial}{\partial\rho}(\eta\tau) + 0.24\eta\frac{\partial\tau}{\partial\rho}\right]. \tag{69}$$

If we also assume that the ions and neutrals have equilibrated with the electrons, the sum of Eqs. (41) and (42) with inertial and viscous terms neglected gives total parallel momentum balance to be

$$\frac{\partial}{\partial\beta}[(2n + \eta)\tau] = 0. \tag{70}$$

Equilibration between electrons and ions and neutrals follows from Eq. (47) if convection and conduction are made small by taking $k\sigma \ll 1$ since we have already assumed perpendicular flows to be small compared to parallel flows and $b \ll 1$. We obtain the equation for τ under the same assumptions, by adding Eqs. (46) and (47) and using $k \gg \mu$ to form the total energy conservation equation

$$b\frac{\partial}{\partial\beta}\left(5\tau nv_I + 0.24\tau\eta v_\parallel - kb\tau^{5/2}\frac{\partial\tau}{\partial\beta}\right) + \frac{\partial}{\partial\beta}(2.3\tau\eta u) + \frac{\partial}{\partial\rho}(2.3\tau\eta w)$$

$$-\frac{\partial}{\partial\beta}\left[\frac{2.4\sigma\eta\tau}{nS_x(\tau)}\frac{\partial\tau}{\partial\beta}\right] - \frac{\partial}{\partial\rho}\left[\frac{2.4\sigma\eta\tau}{nS_x(\tau)}\frac{\partial\tau}{\partial\rho}\right] = -\sigma n\eta S_z(\tau) - \sigma_H n\eta S_H(\tau) - \sigma_I nS_I(\tau). \tag{71}$$

Equations (36), (37), and (68)-(71) are the reduced fluid neutral equations which consist of a fifth order system of six equations for the six unknowns n, η, $v_\parallel$, u, w, and τ. For the orderings $\partial/\partial\beta \sim 1 \sim \partial/\partial\rho$, $n \sim 1 \sim \eta$, $v_\parallel^2 \ll \tau \sim 1$, $\sigma \ll 1$, $b \ll 1$, $k\sigma \ll 1$, and $\mu \sim 1$,

we see that $bv_{\parallel} \sim u \sim w \sim kb^2 \sim \sigma_H \sim \sigma_I \ll 1$. The 1-D version of this reduced system of equations with the thermal force and impurity radiation neglected is investigated in Ref.5. To complete the reduced fluid neutral description we need the five boundary conditions given by the Bohm sheath criterion, Eq.(53); complete recycling, Eq.(56); the upstream pressure and heat flux,

$$2n\tau\big|_{\beta=L/\ell_n} = S_p(\rho\ell_n / \Delta_p) , \tag{72}$$

$$kb^2\tau^{5/2}\frac{\partial\tau}{\partial\beta}\bigg|_{\beta=L/\ell_n} = Q_e S_e(\rho\ell_n / \Delta_p) ; \tag{73}$$

and the energy flux into the walls,

$$\frac{5}{2}\alpha b n_d \tau_d^{3/2} + \left[kb^2\tau_d^{5/2} + \frac{2.4\sigma\eta_d\tau_d}{n_d S_x(\tau_d)} \right]\frac{\partial\tau}{\partial\beta}\bigg|_d + 0.24\eta_d\tau_d(u_d - bv_{\parallel d})$$

$$= \alpha b(\gamma_e + \gamma_i)n_d\tau_d^{3/2} + \gamma_n\eta_d\tau_d^{3/2} \tag{74a}$$

$$-\frac{2.4\sigma\eta_s\tau_s}{n_s S_x(\tau_s)}\frac{\partial\tau}{\partial\rho}\bigg|_s = \gamma_n\eta_s\tau_s^{3/2} . \tag{74b}$$

Notice that Eqs.(72) and (73) follow from (50) and (51), while Eq.(74) follows from (55) and (58). Equation (49) is no longer needed since it follows from integrating (70) and using (72), and Eqs.(54) and (57) are not needed because of the neglect of viscous effects.

Equations (36), (37), (68)-(71), (53), (56), and (72)-(74) permit only two independent scale transformations:

(i) $\sigma \to \omega_1\sigma$, $b \to \omega_1 b$, $u \to \omega_1 u$, $w \to \omega_1 w$, $k \to k/\omega_1$, $\sigma_H \to \omega_1\sigma_H$, $\sigma_I \to \omega_1\sigma_I$,

$Q_e \to \omega_1 Q_e$, $\gamma_n \to \omega_1\gamma_n$; and

(ii) $v_{\parallel} \to \omega_2 v_{\parallel}$, $b \to b/\omega_2$, $k \to \omega_2^2 k$, $\alpha \to \omega_2\alpha$. $\tag{75}$

In this case the poloidal energy flux onto the plates, q_t, depends on ρ and the twelve parameters b, k, σ, Q_e, ℓ_n/L, σ_H, σ_I, α, γ_n, $\gamma_e + \gamma_i$, Δ_p/ℓ_n and Δ/ℓ_n. The parameter

Δ/ℓ_n enters because of the sidewall boundary condition of Eq.(74b). Recalling that q_t scales as Q_e and proceding as before, gives

$$q_t = b\alpha P_u (I/M)^{1/2} f(\ell_n/L, bk/\alpha, Q_e/b\alpha, \sigma_I/\sigma, \sigma/b\alpha, \sigma_H/\sigma, \gamma_n/b\alpha, \gamma_e + \gamma_i, \Delta/L, \Delta/\Delta_p, \rho\ell_n/\Delta_p)$$
(76)

The dimensionless parameters that enter q_t are consistent (apart from notation) with those found in the one-dimensional model of Ref.5 which neglects thermal force effects (and modifications required by Onsager symmetry), impurity radiation in its recycling region, and the parameters ℓ_n/L, Δ/L, and Δ/Δ_p that depend on the depth and width of the divertor channel and scrape off layer to find

$$\tau_d = f(b^2 k/\sigma, Q_e/b\alpha, \sigma/b\alpha, \sigma_H/\sigma, \gamma_n/b\alpha, \gamma_e + \gamma_i) ,$$
(77a)

which is then solved to obtain the following form of the global energy balance equation:

$$Q_e = b\alpha f(\tau_d, b^2 k/\sigma, \sigma/b\alpha, \sigma_H/\sigma, \gamma_n/b\alpha, \gamma_e + \gamma_i) .$$
(77b)

Forming the power to the plates P for a single null divertor using $P = 4\pi R \int dx q_t$ to integrate Eq.(76) over the scrape off layer width Δ_p gives

$$P/R = b\alpha\Delta_p P_u (I/M)^{1/2} f(\ell_n/L, bk/\alpha, Q_e/b\alpha, \sigma_I/\sigma, \sigma/b\alpha, \sigma_H/\sigma, \gamma_n/b\alpha, \gamma_e + \gamma_i, \Delta/L, \Delta_p/\Delta) .$$
(78)

For similar devices described by this reduced fluid neutral model we need to keep f fixed by keeping each of the dimensionless parameters constant. To see to what extent this is possible we note that the parameters k, σ, σ_H, and $\ell_n P_u$ are constants and that α, γ_n/α, and $\gamma_e + \gamma_i$ do not vary significantly. For a specified depth L we adjust P_u, b, Q_e, and σ_I to hold $\ell_n/L \propto 1/LP_u$, bk/α, $Q_e/b\alpha$, and σ_I/σ fixed, then we must adjust Δ and Δ_p to keep Δ/L and Δ/Δ_p constant. Therefore, we must keep

$$P/R \propto b\alpha\Delta_p P_u \propto \Delta_p/\ell_n = \text{constant.}$$
(79)

Consequently, similarity for this fluid neutral model recovers Lackner's[6] result and requires the same b, Q_e (recall that $Q_e \propto q_{e\parallel}^{up}/P_u$), $\ell_n/L \propto 1/LP_u$, σ_I, Δ/L, and Δ/Δ_p, as

well as the same α, k, σ, σ_H, γ_n, and $\gamma_e+\gamma_i$. For the fluid neutral model, B (or B_p) is the only adjustable quantity that can be used to satisfy either a gyroradius or beta scaling. The second column of Table I summarizes the constraints and P/R scaling (which, of course, can be re-expressed using the constraints) for this model.

It is important to note that the constant Δ_p/ℓ_n must be much larger than unity for the fluid neutral model since the SOL thickness must be small compared to the neutral penetration scale length. As a result, the larger P/R, the more the neutrals behave like a fluid. Except for Alcator-C-MOD[11], current machines[12-14] are in the opposite limit of Knudsen neutrals. However, there is recent experimental evidence from Alcator-C-MOD that the detached divertor operation observed is insensitive to the depth[16] L. If this observation is confirmed it would mean that f is insensitive to L in detached regimes (as in the 1-D model of Ref.5), then the parameters ℓ_n/L and Δ/L in f could be replaced by the single L independent parameter Δ/ℓ_n or Δ_p/ℓ_n. Recall that in the model without neutrals the insensitivity to L occurred whenever the radiation losses were sufficiently localized.

We can relax the P/R scaling of this neutral fluid model by considering the one-dimensional limit of Eqs.(36), (37), (68)-(71), (53), (56), and (72)-(74) in which $\partial/\partial\rho = 0 = w$ in Eqs.(37), (69), and (71) for the SOL region $|\rho|<\Delta_p/2\ell_n$ ($|x|<\Delta_p/2$) and the sidewalls and the sidewall boundary condition (74b) do not enter. In this case, which includes the model of Ref.5, Δ cannot enter q_t and P/R. Moreover, the scaling transformation (ii) of Eq.(61) is allowed since ρ enters only through shape functions, which means that Δ_p does not enter the unknown function in P/R and only enters q_t via the combination $\rho\ell_n/\Delta_p$. In addition, the divertor is asssumed infinitely deep (L$\rightarrow\infty$) and impurity radiation is assumed to occur upstream so that L and σ_I do not enter. As a result, for the one-dimensional neutral fluid case

$$q_t=b\alpha P_u(I/M)^{1/2}f(bk/\alpha,Q_e/b\alpha,\sigma/b\alpha,\sigma_H/\sigma,\gamma_n/b\alpha,\gamma_e+\gamma_i,\rho\ell_n/\Delta_p) , \qquad (80a)$$

$$P/R = b\alpha\Delta_p P_u (I/M)^{1/2} f(bk/\alpha, Q_e/b\alpha, \sigma/b\alpha, \sigma_H/\sigma, \gamma_n/b\alpha, \gamma_e + \gamma_i) \ , \qquad (80b)$$

and, upon choosing b and Q_e to hold the unknown function f fixed (recall k, σ, σ_H, and $\ell_n P_u$ are constants, and α and the γ's do not vary significantly),

$$P/R \propto \Delta_p P_u \propto \Delta_p/\ell_n \ . \qquad (80c)$$

Unlike the 2-D case, Δ_p/ℓ_n need not be a constant for similar devices in the 1-D case. Therefore, a 1-D model of a SOL of width Δ_p allows a device similar to ITER to have a much smaller P/R and $\Delta_p P_u \propto \Delta_p/\ell_n$ as in the discussion following Eq.(67).

If we include the inertial terms in our reduced neutral fluid model then we lose the second scaling transformation in Eq.(75) since $v_{\parallel}^2$ and τ must scale in the same way. As a result, α must be held constant for similarity since it appears as a separate parameter in the argument of f. Since $\alpha \approx$ constant to satisfy the Bohm sheath criterion, the modified f is effectively the same as Eq. (78) and, therefore, leads to the same conclusions.

If, in addition, we keep ion heat conduction and neutral and ion viscosity, but assume $b^2 \ll 1$, we need only supplement the first scaling transformation in Eq.(75) by $\mu \rightarrow \mu/\omega_1$, $\gamma_m \rightarrow \omega_1\gamma_m$ and $Q_i \rightarrow \omega_1 Q_i$. Then the only allowed scaling transformation gives Eq.(78) with the additional dimensionless parameters α, $b\mu$, γ_m/b, $\gamma_{\parallel}$ and Q_i/b appearing in the argument of the unknown function. Since μ is a constant and γ_m and $\gamma_{\parallel}$ do not vary significantly, and b had to be held constant to keep f fixed (note that bk is one of the arguments of f and k is a constant), viscosity and ion heat conduction only alter our conclusions from Eqs.(76) and (78) by requiring that each of the dimensionless upstream electron and ion heat fluxes, Q_e and Q_i, respectively, be held fixed in similar devices.

Finally, if we attempt to keep the ionization terms as well as the charge exchange terms in the perpendicular neutral momentum balance equations, (38) and (39), we will also lose the first scaling transformation in Eq.(75) because $\sigma \rightarrow \sigma$. Then

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the unknown function in q_t will depend on the sixteen dimensionless parameters σ, b, μ, k, σ_H, σ_I, α, Q_e, Q_i, $\gamma_\parallel$, γ_m, $\gamma_e + \gamma_i$, γ_n, Δ/L, Δ/Δ_p, and L/ℓ_n, as well as $\rho\ell_n/\Delta_p$. Since σ, μ, k, σ_H, α, $\gamma_\parallel$, γ_m, $\gamma_e + \gamma_i$, and γ_n are either constants or unable to vary significantly, the additional ionization terms have no significant impact on similarity since only the same b, σ_I, Q_e, Q_i, Δ/L, Δ/Δ_p, and L/ℓ_n are required.

C. General Fluid Neutral Model

The general fluid neutral model system of equations described in Sec.III cannot allow any scaling transformations since none are permitted for the reduced system mentioned at the end of Sec.IV.B. Neglecting ion and neutral viscosity, inertial terms, and ionization in the perpendicular neutral flow equations does not help because of the neutral-ion momentum exchange due to charge exchange and the ion-electron energy equilibration terms. Consequently, for the general neutral fluid system no simplification occurs so

$$P/R = \Delta_p P_u (I/M)^{1/2} f(\sigma, b, \mu, k, \sigma_H, \sigma_I, \alpha, Q_e, Q_i, \gamma_\parallel, \gamma_m, \gamma_e, \gamma_i, \gamma_n, \Delta/L, \Delta/\Delta_p, L/\ell_n) , \quad (81)$$

with Q_e and Q_i and γ_e and γ_i appearing separately rather than as sums. In this case similarity requires the same b, σ_I, Δ/L, Δ/Δ_p, $L/\ell_n \propto P_u L$, Q_e, and Q_i (with σ, μ, k, σ_H, α, $\gamma_\parallel$, γ_m, γ_e, γ_i, and γ_n either constants or unable to vary significantly), and, not surprisingly, leads to Lackner's[8] $P/R = $ constant result. Therefore, the general fluid neutral model introduces additional parameters, but leads to essentially the same conclusions as the reduced fluid neutral model.

V. Scaling Transformations and Similarity for Knudsen Neutrals

Reference 5 also considers a deep divertor slot geometry with a complete recycling model of the neutrals in which the neutral mean free path is long compared to the divertor width and the Coulomb mean free path is assumed small compared to parallel scale lengths. This idealized limit, in which charge exchange is retained and the

long mean free path neutrals are randomized by collisions with the walls, is referred to as the Knudsen neutral model or Knudsen flow approximation in Ref.5. For this model the depth of the narrow divertor slot L must be much larger than its width Δ so that nearly all of the neutrals created at the target by the recombining ions can stream to the sidewalls to be randomized within a few Δ's of the target and well before they reach the upstream divertor entrance. As a result, the neutrals are assumed to be uniformly distributed in x (as well as z) for $\Delta \ll y < L$ with no poloidal or toroidal flow. The randomizing wall collisions are assumed to result in a diffusive poloidal neutral flow

$$N_n \vec{V}_n = -\hat{y} D \partial N_n / \partial y , \qquad (82a)$$

where the diffusion coefficient D is given by

$$D = cV\Delta \qquad (82b)$$

with Δ and V the characteristic step in y and speed between randomizing collisions at the walls and c an order unity numerical coefficient which depends on the properties and conditioning of the walls. The speed V is set by the neutral temperature which in this model is small compared to the ion and electron temperatures. Equation (82) is then inserted into Eq.(2) to obtain the neutral continuity equation. To make D dimensionless we introduce the dimensionless diffusivity d defined by

$$d = \frac{cV\Delta}{\ell_n (I/M)^{1/2}} . \qquad (82c)$$

The remaining equations for the Knudsen neutral model are ion continuity, Eq.(1) with Eq.(4) inserted; parallel plasma momentum balance as obtained from Eq.(6) by setting $V_{nI} = 0$, neglecting inertia, and dropping the thermal force term; and total plasma energy conservation as obtained by assuming $T_e \approx T$, neglecting inertia, and adding Eqs.(11) and (16) together with the neutral flow and neutral heat flux terms ignored. If we neglect ion heat conduction and viscosity the dimensionless form of the Knudsen neutral model equations is as follows:

$$d \frac{\partial^2 \eta}{\partial \beta^2} = \sigma n \eta S_z(\tau) \, , \tag{83}$$

$$b \frac{\partial}{\partial \beta}(n v_{||}) = \sigma n \eta S_z(\tau) \, , \tag{84}$$

$$\sigma b \frac{\partial}{\partial \beta}(2 n \tau) = -n \eta v_{||} S_x(\tau) \, , \tag{85}$$

$$b \frac{\partial}{\partial \beta}\left(5 \tau n v_{||} - k b \tau^{5/2} \frac{\partial \tau}{\partial \beta}\right) = -\sigma n \eta S_z(\tau) - \sigma_H n \eta S_H(\tau) - \sigma_I n S_I(\tau) \, . \tag{86}$$

This sixth order system of equations in the four unknowns n, η, $v_{||}$, and τ is employed with the three upstream boundary conditions given by Eqs.(49), (72), and (73), and the three downstream boundary conditions given by Eq.(53) and the appropriately modified versions of (56) and (74a):

$$-d \frac{\partial \eta}{\partial \beta}\bigg|_d \Delta = \alpha b \int_0^{\Delta_p} dx \, n_d \tau_d^{1/2} = \alpha b \Delta_p \int_0^1 d(\rho \ell_n / \Delta_p) \, n_d \tau_d^{1/2} \, , \tag{87}$$

$$5 \alpha b n_d \tau_d^{3/2} + k b^2 \tau_d^{5/2} \frac{\partial \tau}{\partial \beta}\bigg|_d = \alpha b (\gamma_e + \gamma_i) n_d \tau_d^{3/2} \, . \tag{88}$$

The three downstream boundary conditions are applied a few Δ's from the wall in order for the diffusive model of Eqs.(82) to be valid. Equation (87) is obtained by demanding complete recycling, and recalling that the neutral distribution is uniform across the entire divertor channel of width Δ, while the plasma is localized to the SOL of width Δ_p. Notice that Δ enters through d, as well as Eq.(87), but not through any sidewall boundary conditions, and recall that $\beta = L / \ell_n$ at the upstream entrance.

The Knudsen neutral model, consisting of Eqs.(49), (53), (72), (73), and (83)-(88), contains the twelve dimensionless parameters d, σ, b, k, σ_H, σ_I, L/ℓ_n, Q_e, α, $\gamma_e + \gamma_i$, Δ_p / ℓ_n, and Δ_p / Δ, and permits the following three scale transformations:

(i) $\sigma \to \omega_1 \sigma$, $v_{||} \to \omega_1 v_{||}$, $k \to \omega_1 k$, $\sigma_H \to \omega_1 \sigma_H$, $\sigma_I \to \omega_1 \sigma_I$, $d \to \omega_1 d$

$\quad Q_e \to \omega_1 Q_e$, $\alpha \to \omega_1 \alpha$;

(ii) $b \to \omega_2 b$, $\beta \to \omega_2 \beta$, $d \to \omega_2^2 d$, $Q_e \to \omega_2 Q_e$, $L/\ell_n \to \omega_2 L/\ell_n$; and

(iii) $\rho \to \omega_3 \rho$, $\Delta_p / \ell_n \to \omega_3 \Delta_p / \ell_n$. \hfill (89)

These transformations reduce the number of independent dimensionless parameters by three and the procedure of Sec.IV.A gives

$$q_t = b\alpha P_u (I/M)^{1/2} f(b\ell_n/L, k/\alpha, Q_e/b\alpha, \sigma_I/\sigma, \sigma/\alpha, \sigma_H/\sigma, d/\alpha b^2, \gamma_e + \gamma_i, \Delta_p/\Delta, \rho\ell_n/\Delta_p) \quad (90)$$

and

$$P/R = b\alpha\Delta_p P_u (I/M)^{1/2} f(b\ell_n/L, k/\alpha, Q_e/b\alpha, \sigma_I/\sigma, \sigma/\alpha, \sigma_H/\sigma, d/\alpha b^2, \gamma_e + \gamma_i, \Delta_p/\Delta) \quad (91)$$

The new parameter in the unknown function is the effective diffusivity $d/\alpha b^2$ (or $\ell_n d/\alpha bL \propto cV\Delta/\alpha bL$) which replaces the $\gamma_n/b\alpha$ (or $\gamma_n\Delta/b\alpha L$) parameter of the fluid neutral model (cV replaces γ_n). Moreover, for the deep slot Knudsen model Δ/L no longer enters and the b dependence is altered (β scales the same way as b) from the fluid neutral model since the neutrals are no longer strongly coupled to the ions by charge exchange. The other parameters σ/α, σ_H/σ, k/α, and $\gamma_e + \gamma_i$ in the argument of f for P/R must also be held constant. Therefore, if we adjust P_u, Δ, Δ_p, Q_e, and σ_I to keep $b\ell_n/L$, $d/\alpha b^2 \propto (cV\Delta P_u/\alpha b^2)$, Δ_p/Δ, $Q_e/b\alpha$, and σ_I/α fixed for a specified L, we obtain the scaling

$$P/R \propto b\alpha\Delta_p P_u \propto b^2\Delta_p/L \propto (cV)^2(\Delta_p/L)^3 \propto (cV)^{1/2}(\Delta_p/\ell_n)^{3/2}, \quad (92)$$

where we have used $P_u \propto 1/\ell_n \propto b/L$ and $b \propto (cV\Delta_p/\alpha\ell_n)^{1/2}$, and α must be kept constant since σ and k are constants. As a result, P/R is not a constant for the Knudsen fluid model, which requires $\Delta_p/\ell_n \ll 1$, and we are still free to adjust cV and b, as well as B to satisfy other constraints. The constraints and P/R scaling for the Knudsen model are summarized in the third column of Table I. Keeping the inertial corrections in the Knudsen neutral model results in the loss of scaling transformation (ii) and makes it necessary to keep α fixed for similarity, so results in no significant change.

V. Discussion

Based on the model without neutrals and the fluid neutral models considered here, the collisionality, upstream parallel heat flux, and P/R constraints on ITER divertor similarity are all difficult to satisfy in present tokamaks. The model without

neutrals has the least restrictive P/R scaling and the fluid neutral model the most constraints. The less restrictive and more favorable scaling of the Knudsen model means that the constant P/R constraint found for fluid neutrals is relaxed as the neutrals make the transition from $\Delta_p/\ell_n \gg 1$ to $\Delta_p/\ell_n \ll 1$, where Δ_p and ℓ_n are the scrape off layer width and the characteristic neutral penetration depth [recall Eq.(33)], respectively. Consequently, lower density machines may have a less constrained P/R scaling than higher density ones. In either limit, however, severe similarity constraints arise from the upstream parallel heat flux and collisionality. For a given $b=B_p/B$, similar collisionality requires $P_uL=$constant and therefore deep divertor chambers at the lower upstream pressures of present tokamaks. However, in light of recent experimental results from Alcator-C-MOD[16] indicating that divertor operation during detachment is insensitive to the divertor depth L, it is tempting to speculate that it may be possible to ignore the L dependence of f in Eq.(91) and replace the ℓ_n/L and Δ/L dependences of the unknown function f in Eq.(78) by Δ/ℓ_n. Even if this is not the case, the data base from present machines coupled with the key parameters found here and in divertor modeling codes, might be used to construct a power law form for the unknown function in the P/R scaling law. The techniques employed herein can be used to determine the key parameters for the equations and boundary conditions solved in the modeling codes of interest.

The upstream heat flux entering the divertor also places a severe constraint on the similarity of present tokamak divertors to ITER. However, for the various models considered only the parallel heat flux must be matched. Since a divertor simulator need not be a conventional tokamak the parallel heat flux might be matched by adjusting the field line angle in a toroidal device to make $b=B_p/B$ smaller for a fixed poloidal heat flux. For a non-conventional tokamak simulator there is much more flexibility in making the collisionality and other divertor and geometrical parameters similar (for a simulator of SOL length $\mathcal{L}$ the replacement $R \rightarrow \mathcal{L}/4\pi$ is made in P/R).

Acknowledgments

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Table 1. Divertor Similarity: Constraints (key arguments of f) and P/R Scaling (coefficient of f)

Similarity Constraint	No Neutrals (3 arguments)	Fluid Neutrals (6 key arguments)	Knudsen Neutrals (5 key arguments)
parallel heat flux (determines Q_e)	$Q_e/b\alpha\gamma_e$ ($\alpha\gamma_e \approx$ constant)	Q_e/b	Q_e/b
impurity radiation (determines σ_I)	$\sigma_I/(\alpha\gamma_e)^2$ ($\alpha\gamma_e \approx$ constant)	σ_I	σ_I
collisionality (determines L)	$b\ell_n/\alpha\gamma_e L$ ($\alpha\gamma_e \approx$ constant)	$b\ell_n/L$	$b\ell_n/L$
chamber width (determines Δ)	none	Δ/L	d/b^2 ($d \propto cV\Delta/\ell_n$)
B field ratio (determines b)	none	b	none
SOL width (determines Δ_p)	none	Δ_p/Δ	Δ_p/Δ
other constants	k	$k,\sigma,\alpha,\sigma_H,\gamma_n,\gamma_e+\gamma_i$	$k,\sigma,\alpha,\sigma_H,\gamma_e+\gamma_i$
P/R scaling (free to adjust)	$b^2\Delta_p/L$ (Δ_p, b, B)	constant (B)	$b^2\Delta_p/L$ (cV, b, B)

Table 2. Some Additional Constraints and Resulting Scalings for No Neutral Model

Case	Additional Constraints	Resulting P/R Scaling
(a)	b = constant Δ_p/L = constant	constant
(b)	$b\Delta_p/L$ = constant $B\Delta_p$ = constant	b
(c)	$\beta \propto P_u/B^2$ = constant $B\Delta_p$ = constant	$\alpha\gamma_e B_p$
(d)	b = constant $B\Delta_p$ = constant	$1/BL$
(e)	b = constant $\beta \propto P_u/B^2$ = constant	$\alpha\gamma_e B^2\Delta_p$