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Flux Limiting Due to Electron Impact Excitation Energy Loss

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Flux Limiting Due to Electron Impact Excitation Energy Loss

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Abstract

Large radiation losses in the electron energy balance equation due to electron impact excitation can cause the parallel scale length to become small enough that a short mean free path expansion becomes questionable for the high speed tail electrons. It then becomes necessary to limit the parallel electron heat flux so that it remains below its free streaming value $nT^{3/2}/m$, where n, T, and m are the electron density, temperature, and mass. We adopt a Boltzmann inelastic scattering collision operator to investigate whether electron impact excitation can lead to a self-consistent flux limit by depleting the electron tail.

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I. Introduction

Local steepening of the parallel electron temperature profile can occur in diverted tokamaks because of the energy loss due to line radiation[1-2]. If the parallel scale length becomes comparable to the Coulomb mean free path of the tail electrons the usual short mean free path treatment[3] must be modified. In the absence of radiation losses, nonlocal modification of the parallel electron heat flux gives a strong flux limit[4-6], as was shown originally in the context of inertial fusion. These nonlocal flux limiting models assume the effective charge state Z of the ions plus impurities is so large that the electron-electron mean free path λ_e can be taken comparable to or larger than the parallel scale length ℓ, while the electron-ion mean free path λ_e/Z remains small compa red to ℓ. We consider an alternate limit in which radiation is retained, but nonlocal effects neglected, to determine if flux limiting due to tail depletion is large enough to significantly reduce the parallel electron heat flux $q_\parallel$.

When a tail electron impacts an impurity and causes it to go to an excited state, the energy of the electron is lowered by the excitation energy. If we assume that the line radiation due to electron excitation is instantaneously radiated away without re-absorption, electron impact excitation can be modeled by including a Boltzmann inelastic scattering collision operator in the electron kinetic equation. The line radiation modifications to the parallel heat flux and parallel electron current can be evaluated by solving the electron drift kinetic equation with inelastic scattering as well as Coulomb collisions. To carry out this calculation we assume the the effective charge state of the ions is high (Z >> 1) so that we need not assume the electron-electron mean free path is short, but can assume the electron-ion mean free path is small compared to the parallel scale length ℓ. The Z >> 1 assumption causes pitch angle scattering to dominate so that the lowest order electron distribution function is isotropic.

In Sec. II we introduce the tail electron model we employ. Section III discusses orderings and uses them to obtain an explicit solution for the electron distribution function to the required order. To find the solution the Boltzmann form of the impact excitation collision operator is expanded to obtain a Fokker-Planck form by assuming the characteristic excitation energy loss is small compared to the electron thermal speed. We also employ a high speed expansion of the electron-electron collision operator in a form which attempts to drive the tail electrons towards a Maxwellian to counteract the non-Maxwellian influence of the inelastic scattering. When excitation loss is strong enough the tail can be depleted by inelastic scattering and the tail contribution to the parallel heat flux and current reduced. The flux limiting factors are evaluated in Sec.IV for a realistic and analytically convenient form of the inelastic scattering excitation cross section. Section V estimates the size of the dimensionless parameter characterizing tail depletion due to impact excitation. These estimates indicate that order unity decreases in the electron heat flux are difficult to obtain for realistic divertor parameters. As a result, flux limiting due to impact excitation depletion of the tail is expected to be weak is expected to be weak in tokamak scrape-off layers. However, for the partially ionized plasmas used in plasma processing metastable states must be considered and significant flux limiting is possible.

II. Model

To generate a steepening of the profile consistent with energy loss by line radiation kinetically we include an inelastic impact excitation operator, as well as Fokker-Planck collisions, in the gyro-averaged electron kinetic equation. Line radiation is modeled as being due to electrons exciting partially stripped impurities which then instantaneously radiate the energy away without re-absorption since the plasma is assumed to be transparent. If the higher energy, longer mean free path electrons can be strongly depleted by this inelastic scattering process then the influence of these electrons on the heat flux and current can be strongly reduced.

In addition to inelastic and Coulomb collisions we retain parallel electron streaming, which results in the parallel heat conduction, and the parallel electric field $E_{||}$. The gyro-averaged electron kinetic equation may then be written as

$$v_{||}\hat{n}\cdot\nabla f - \frac{e}{m}E_{||}\hat{n}\cdot\nabla_v f = C\{f\} + X\{f\} \ , \tag{1}$$

where C{f} and X{f} denote the elastic and inelastic collision operators acting on the electron distribution function f, with $\hat{n} = \vec{B}/B$, $B = |\vec{B}|$, $v_{||} = \hat{n}\cdot\vec{v}$, and e and m the magnitude of the electron charge and mass. Because we are primarily interested in effects on the tail electrons we will normally use the large speed ($v > v_e$) expansion of the electron collision operator which we write in the form

$$C\{f\} = \nabla_v\cdot[f_M\overleftrightarrow{D}\cdot\nabla_v(f/f_M)] \ , \tag{2}$$

where

$$\overleftrightarrow{D} = \frac{\nu_e v_e^3}{2v^3}[(Z+1)(v^2\overleftrightarrow{I} - \vec{v}\vec{v}) + 2v_e^2] \ , \tag{3}$$

$$f_M = (2\pi v_e^2)^{-3/2}\exp(-v^2/2v_e^2) \ , \tag{4}$$

and $v_e^2 = T/m$, $n = \int d^3v f$, and $\nu_e = 4\pi ne^4 \ell n\Lambda/m^2 v_e^3 = 3(\pi/2)^{1/2}/\tau_e$ with τ_e the Braginskii electron-electron collision time.

The inelastic collision operator X{f} is given by[7]

$$X\{f\} = \sum_x N_x v^{-1}[(v')^2\sigma_x(v')f(\vec{v}') - v^2\sigma_x(v)f(\vec{v})] \ , \tag{5}$$

with $\frac{1}{2}mv_x^2 = \frac{1}{2}m(v')^2 - \frac{1}{2}mv^2$ the energy loss of an electron caused by exciting the x^{th} state of the impurity which has a excitation cross section of σ_x and a number density of N_x. The first term in X is the increase in the number of electrons of velocity $\vec{v}$ which had velocity $\vec{v}'$ prior to losing energy $\frac{1}{2}mv_x^2$, while the second is the decrease in the number of electrons of velocity $\vec{v}$ due to the inelastic scattering loss.

If we assume a uniform magnetic field, use the full electron collision operator and the number conserving properties of C and X, define the parallel electron heat flux $q_\parallel$ and current $J_\parallel$ by

$$q_\parallel = \int d^3v(\frac{1}{2}mv^2 - \frac{5}{2}T)v_\parallel f \tag{6}$$

and

$$J_\parallel = -e\int d^3v v_\parallel f \ , \tag{7}$$

and note that

$$\int d^3v\frac{1}{2}mv^2 X\{f\} = -N_x\frac{1}{2}mv_x^2\int d^3v v\sigma_x(v)f(\vec{v}) \equiv -N_xL_x(T) \ , \tag{8}$$

then the $(mv^2-5T)/2$ moment of Eq.(1) gives the conservation of energy equation

$$\hat{n}\cdot\nabla q_\parallel - E_\parallel J_\parallel = \frac{3Zmn(T-T_i)}{M\tau_e} - \sum_x N_xL_x(T) \ . \tag{9}$$

Here Z, M, and T_i are the effective charge number, mass, and temperature of the ions, respectively. Note also that the continuity equation associated with Eq.(1) is simply $\hat{n}\cdot\nabla J_\parallel = 0$. From Eq.(9) we can see that when the radiation energy loss rate function $L_x(T)$ is large enough (often the case in the presence of impurities) it can alter the electron energy balance by causing parallel scale lengths to decrease. As the electrons lose energy by radiation it can be resupplied by equilibration with the ions.

III. Solution Technique

When the steepening of the parallel temperature profile causes the parallel scale length ℓ to approach the electron mean free path $\lambda_e = v_e\tau_e$ so that the short mean free path expansion ($\lambda_e << \ell$) becomes questionable, an adhoc flux limit is sometimes introduced to keep the the parallel heat flux below its free streaming value $mnv_e^3 = nT^{3/2}/m$. The question then arises as to whether the radiation losses can ever be large enough to substantially deplete the electron tail at $v > v_e$ so that high order moments of the distribution function such as $q_\parallel$ and $J_\parallel$ can be substantially reduced (or flux limited) by inelastic scattering. To investigate this question we consider a strongly radiating, high Z plasma so that we can adopt the orderings

$$\lambda_e/Z\ell << 1 \sim eE_\parallel \ell/T \sim \nu_x/\nu_e\ , \tag{10}$$

where $X\{f\} \sim \nu_x f$, with ν_x the characteristic frequency associated with inelastic scattering. We will normally employ the $v > v_e$ form of $C\{f\}$, but since $C\{f\} = 0$ gives a lowest order distribution that is Maxwellian below v_e and the ordering procedure that we employ is valid for the general $C\{f\}$, it is usually convenient to view $v \sim v_e$.

We also employ $\hat{n}\cdot\nabla = \partial/\partial s$ and the velocity space variables $v = |\vec{v}|$ and $\xi = v_\parallel/v = \cos\theta$, as well as gyrophase, to write Eqs.(1)-(3) as

$$\xi v\frac{\partial f}{\partial s} - \frac{e}{m}E_\parallel\left[\frac{1}{v^2}\frac{\partial}{\partial v}(\xi v^2 f) + \frac{\partial}{\partial \xi}(\frac{1-\xi^2}{v}f)\right] = C\{f\} + X\{f\} \tag{11}$$

and

$$C\{f\} = \frac{(Z+1)\nu_e v_e^3}{2v^3}\frac{\partial}{\partial\xi}\left[(1-\xi^2)\frac{\partial f}{\partial \xi}\right] + \frac{\nu_e v_e^3}{v^2}\frac{\partial}{\partial v}\left[\frac{Tf_M}{mv}\frac{\partial}{\partial v}\left(\frac{f}{f_M}\right)\right]. \tag{12}$$

We then solve Eq.(11) by expanding in powers of $\lambda_e/Z\ell$ by writing

$$f = f_0 + f_1 + f_2 + \ldots, \tag{13}$$

where $f_{j+1}/f_i \sim \lambda_e/Z\ell << 1$. To lowest order pitch angle scattering dominates giving

$$\frac{(Z+1)\nu_e v_e^3}{2v^3}\frac{\partial}{\partial\xi}\left[(1-\xi^2)\frac{\partial f_0}{\partial \xi}\right] = 0 \quad \text{or} \quad \partial f_0/\partial\xi = 0\ . \tag{14}$$

The next two orders of the electron kinetic equation are

$$\frac{(Z+1)\nu_e v_e^3}{2v^3}\frac{\partial}{\partial\xi}\left[(1-\xi^2)\frac{\partial f_1}{\partial \xi}\right] = -\frac{\nu_e v_e^3}{v^2}\frac{\partial}{\partial v}\left[\frac{Tf_M}{mv}\frac{\partial}{\partial v}\left(\frac{f_0}{f_M}\right)\right]$$
$$-X\{f_0\} + \xi\left(v\frac{\partial f_0}{\partial s} - \frac{e}{m}E_\parallel\frac{\partial f_0}{\partial v}\right) \tag{15}$$

and

$$\frac{(Z+1)\nu_e v_e^3}{2v^3}\frac{\partial}{\partial\xi}\left[(1-\xi^2)\frac{\partial f_2}{\partial\xi}\right] = -\frac{\nu_e v_e^3}{v^2}\frac{\partial}{\partial v}\left[\frac{Tf_M}{mv}\frac{\partial}{\partial v}\left(\frac{f_1}{f_M}\right)\right] - X\{f_1\}$$

$$+\xi v\frac{\partial f_1}{\partial s} - \frac{e}{m}E_{\|}\left[\frac{1}{v^2}\frac{\partial}{\partial v}(\xi v^2 f_1) + \frac{\partial}{\partial\xi}(\frac{1-\xi^2}{v}f_1)\right]. \tag{16}$$

The equation for f_0 is found by averaging Eq.(15) over all ξ by integrating from $\xi = 1$ to $\xi = -1$ to obtain

$$\frac{\nu_e v_e^3}{v^2}\frac{\partial}{\partial v}\left[\frac{Tf_M}{mv}\frac{\partial}{\partial v}\left(\frac{f_0}{f_M}\right)\right] + X\{f_0\} = 0\ . \tag{17}$$

Our orderings are designed to make inelastic scattering significant when $mv^2/2T > 1$ for $\nu_x/\nu_e < 1$ in order that only the tail is depleted in the lowest order distribution function. To find f_1 we subtract Eq.(17) from Eq.(15) to find

$$\frac{(Z+1)\nu_e v_e^3}{2v^3}\frac{\partial}{\partial\xi}\left[(1-\xi^2)\frac{\partial f_1}{\partial\xi}\right] = \xi\left(v\frac{\partial f_0}{\partial s} - \frac{e}{m}E_{\|}\frac{\partial f_0}{\partial v}\right), \tag{18}$$

which shows that $f_1 \sim f_0\lambda_e/Z\ell$ as desired. The rotational symmetry of the pitch angle scattering operator means the f_1 must be of the form

$$f_1 = \xi H(s,v) + G(s,v)\ . \tag{19}$$

Solving Eq.(18) gives H to be

$$H = \frac{-v^4 f_0 A}{(Z+1)\nu_e v_e^3} \tag{20}$$

with

$$A = \left(\frac{\partial}{\partial s} - \frac{eE_{\|}}{mv}\frac{\partial}{\partial v}\right)\ell n f_0\ . \tag{21}$$

The equation for G is obtained by integrating Eq.(16) over all ξ. Inserting the form for f_1 as given by Eq.(19) gives

$$\frac{3\nu_e v_e^3}{2v^2}\frac{\partial}{\partial v}\left[\frac{Tf_M}{mv}\frac{\partial}{\partial v}\left(\frac{G}{f_M}\right)\right] + \frac{3}{2}X\{G\} = v\frac{\partial H}{\partial s} - \frac{eE_{\|}}{mv^2}\frac{\partial}{\partial v}(v^2 H)\ . \tag{22}$$

Notice that $H \sim f_0\lambda_e/Z\ell$ and $G \sim f_0 Z^{-1}(\lambda_e/\ell)^2$ according to our orderings, so G gives an order $Z^{-1}(\lambda_e/\ell)^2$ correction which is assumed to be negligible. Fortunately, for our purposes it is not necessary to solve for G since are primarily interested in the odd (in $v_{\|}$) moments $q_{\|}$ and $J_{\|}$.

It is not possible to give an explicit solution to Eq.(17) without making further approximations. If the tail is fully depleted then for v just below the depletion point, we can neglect the first term in X{f} since there are no high energy electrons to lose energy. In this case

$$X\{f\} \approx -\sum_x N_x v\sigma_x(v) f(\vec{v}) , \tag{23}$$

but since it turns out to be very difficult to fully deplete the tail in magnetic fusion applications in general and in divertor applications in particular we will use an alternate approximation. We will assume the change in energy during the inelastic scattering are small compared to the velocity space variations of interest,

$$v_x^2 \frac{\partial}{\partial v^2} \ell n(v^2 \sigma_x f) << 1 , \tag{24}$$

so that we can Taylor expand to obtain

$$X\{f\} \approx \frac{1}{2v^2}\frac{\partial}{\partial v}\left(v^2 f \sum_x N_x v_x^2 \sigma_x \right) . \tag{25}$$

Using this form for $X\{f_0\}$ in Eq.(17), integrating from v to ∞, demanding that $f_0/f_M \rightarrow$ constant or zero (because of tail depletion) as $v \rightarrow \infty$, and defining

$$\nu_x(v) = \sum_x N_x v_x^2 \sigma_x / 2v_e , \tag{26}$$

gives

$$\frac{Tf_M}{mv}\frac{\partial}{\partial v}\left(\frac{f_0}{f_M}\right) + \frac{mv^2 \nu_x(v)}{2Tv_e} f_0 = 0 .$$

Integrating again,we find the lowest order distribution function to be

$$f_0 = \eta f_M \exp[-\psi(s,v)] , \tag{27}$$

with

$$\psi(s,v) = \frac{m^2}{2T^2 \nu_e} \int_0^v dv\, v^3 \nu_x(v) \ , \tag{28}$$

where η is the normalization factor which is chosen to make $n = \int d^3v f_0$ to lowest order. Notice that significant tail depletion occurs if $\nu_x \sim \nu_e$. Using ψ we may write A in f_1 as

$$A = \left[\frac{\partial}{\partial s}\ell n(\eta T) + \left(\frac{mv^2}{2T} - \frac{5}{2}\right)\frac{\partial}{\partial s}\ell n T - \frac{\partial \psi}{\partial s} + \frac{eE_\perp}{T}\left(1 + \frac{mv^2 \nu_x}{2T\nu_e}\right)\right] , \tag{29}$$

where the s derivatives are taken at fixed v.

VI. An Explicit Inelastic Scattering Model

To obtain a fully explicit form for f_0 we must choose a reasonable, analytically tractable form for ν_x that we can integrate. A particularly convenient choice, which has a step-like behavior (controlled by the parameter $w \gtrsim 1$) with $\nu_x \approx 0$ below a critical energy $\frac{1}{2}mv^2 = rT$ and fails off as $1/v^2$ at large energy, is

$$\nu_x(v) = \frac{\gamma \nu_e T}{mv^2}\left\{1 + \tanh\left[w\left(\frac{mv^2}{2T} - r\right)\right]\right\} \xrightarrow[w \to \infty]{} \frac{2\gamma \nu_e T}{mv^2} H\left(\frac{mv^2}{2T} - r\right) . \tag{30}$$

where the step function H(z) vanishes for $z \leqq 0$ and is one for $x > 0$. Letting $x = mv^2/2T$ and integrating gives

$$\psi(s,v) = \frac{\gamma}{2}\left[x + \frac{1}{w}\ell n\left\{\frac{\cosh[w(x-r)]}{\cosh(wr)}\right\}\right] \xrightarrow[w \to \infty]{} \gamma \begin{cases} 0 & x \le r \\ x - r & x > r \end{cases} = \gamma(x-r)H(x-r) \tag{31}$$

with the step location $r > 1$ for the tail electrons of interest and $\gamma \sim 1$ for significant depletion. For this form, condition (24) implies $w v_x^2 << v_e^2$, so that strictly speaking we are not allowed to let $w \to \infty$. However, the step approximation has been checked and found to be an extremely good approximation for evaluating moments as long as $w \gtrsim 1$. As a result, we will normally employ the step function approximation to ψ in order to avoid keeping w as an additional parameter and we can assume that the diffusive model with the step function approximation is good as long as $v_x^2 < v_e^2$.

In terms of ψ the density n and pressure p are

$$n = \int d^3 v f_0 = \frac{2\eta}{\pi^{1/2}} \int_0^\infty dx x^{1/2} \exp(-x - \psi) \tag{32}$$

and

$$p = \frac{m}{3} \int d^3 v v^2 f_0 = \frac{4\eta T}{3\pi^{1/2}} \int_0^\infty dx x^{3/2} \exp(-x - \psi)\ . \tag{33}$$

Equation (32) relates η to n, however, we are restricted by our model to situations in which only the tail electrons are affected by radiation loss. If the radiation loss were large enough to affect the bulk electrons then we would need an to retain an explicit energy source term and the full electron-electron collision operator in the equation for f_0, as well as the inelastic sink term represented by ψ. As a result, $\eta \approx n$ and $p \approx nT$, while $q_\parallel$ (and perhaps $J_\parallel$) is allowed to depart significantly from there radiation loss free counterparts because it involves higher moments of f_0 and so is more heavily weighted by the tail electrons with energies of the order of 5T.

To see that $q_\parallel$ and $J_\parallel$ can be more strongly affected by radiation than n and p we evaluate them by using $d^3v = 2\pi v^2 dv d\xi$. To lowest significant order only the ξH portion of f contributes. Upon carrying out the ξ integrals and using $\eta \approx n$ and $p \approx nT$ we are left with

$$q_\parallel = \frac{200 p T \tau_e}{3\pi(Z+1)m}\left[F\frac{\partial}{\partial s}\ell nT + \frac{6K}{25}\left(\frac{\partial}{\partial s}\ell np + \frac{eE_\parallel}{T}\right) + \frac{6}{25}\left(\frac{K_v eE_\parallel}{T} + \frac{\partial K}{\partial s}\right)\right] \tag{34}$$

and

$$J_\parallel = \frac{32 e^2 n \tau_e}{3\pi(Z+1)m}\left[L\left(E_\parallel + \frac{1}{en}\frac{\partial p}{\partial s}\right) + \frac{3K}{2e}\frac{\partial T}{\partial s} + L_v E_\parallel + \frac{T}{e}\frac{\partial L}{\partial s}\right] \tag{35}$$

where the flux limiting factors F, K, and L, and the flux modification factors F_v and L_v, are defined as follows:

$$F = \frac{2}{75}\int_0^\infty dx x^3 (x - \frac{5}{2})^2 \exp(-x - \psi),\quad L = \frac{1}{6}\int_0^\infty dx x^3 \exp(-x - \psi),$$

and $$K = \frac{1}{9}\int_0^\infty dxx^3(x - \frac{5}{2})\exp(-x - \psi),$$ (36)

and

$$K_v = \frac{1}{9}\int_0^\infty dxx^3(x - \frac{5}{2})\frac{\partial\psi}{\partial x}\exp(-x - \psi), \quad \text{and} \quad L_v = \frac{1}{6}\int_0^\infty dxx^3\frac{\partial\psi}{\partial x}\exp(-x - \psi). \quad (37)$$

In the preceding equations F, L, and K (K_v, $\partial K/\partial s$, L_v, and $\partial L/\partial s$) go to one (zero) as $\psi \to 0$. The large Z, $\psi = 0$ coefficients of $q_\parallel$ and $J_\parallel$ are explicitly displayed[8] so that we need only investigate whether tail depletion due to inelastic scattering can significantly reduce the flux limit factors F, L, and K or introduce significant flux modification factors K_v, $\partial K/\partial s$, L_v, or $\partial L/\partial s$.

The final moment of interest is the radiation loss term in Eq.(9) which can be written as

$$\sum_x N_x L_x(T) = \left(\frac{2}{\pi}\right)^{1/2} p\int_0^\infty dxxv_x \exp(-x - \psi) = \left(\frac{2}{\pi}\right)^{1/2} v_e p\left[1 - \int_0^\infty dx\exp(-x - \psi)\right], \quad (37)$$

where we have used $d\psi/dx = x\nu_x/\nu_e$. Notice that $\Sigma N_x L_x \sim p\nu_x$. Comparing radiation losses with parallel heat conduction ($\Sigma N_x L_x \sim q_\parallel/\ell$) gives $(\nu_x/\nu_e)\exp(-\tau) \sim Z^{-1}(\lambda_e/\ell)^2 << 1$, which for $\nu_x/\nu_e \sim 1$ allows long electron-electron mean free paths [$\lambda_e/\ell \sim Z^{1/2}\exp(-\tau/2) > 1$] and short electron-ion mean free paths [$\lambda_e/Z\ell \sim Z^{-1/2}\exp(-\tau/2) < 1$].

All the preceding integrals to be evaluated are of the form

$$\Lambda_p = \int_0^\infty dxx^p \exp(-x - \psi) \xrightarrow[\psi \to 0]{} \Gamma(p+1), \quad (38)$$

where $\Gamma(p+1)$ is a gamma function. In terms of Λ_p we have

$$F = \frac{2}{75}\left(\Lambda_5 - 5\Lambda_4 - \frac{25}{4}\Lambda_3\right), \quad L = \frac{1}{6}\Lambda_3, \quad K = \frac{1}{9}\left(\Lambda_4 - \frac{5}{2}\Lambda_3\right),$$
$$L_v = \frac{1}{6}(3\Lambda_2 - \Lambda_3), \quad \text{and} \quad K_v = \frac{1}{18}(13\Lambda_3 - 15\Lambda_2 - 2\Lambda_4), \quad (39)$$

as well as

$$\frac{n}{\eta}=\frac{2}{\pi^{1/2}}\Lambda_{1/2}\ ,\quad \frac{p}{nT}=\frac{2\Lambda_{3/2}}{\Lambda_{1/2}}\ ,\quad \text{and}\quad \sum_x N_x L_x=\left(\frac{2}{\pi}\right)^{1/2}\nu_e p(1-\Lambda_0)\ . \tag{40}$$

Inelastic collisions result in significant flux limiting if they can make F (and perhaps L and K) small compared to unity, while keeping $n \approx \eta$ and $p \approx nT$. Notice that for $\psi = 0$, the most important contributions to the integral in F come from the region about $x \sim 6$ [where $x^3(x-5/2)^2\exp(-x) \sim 6$], while for n/η and p/nT it is the regions about $x \sim 1/2$ and $x \sim 3/2$, respectively, that matter most. As a result, it is possible for tail depletion to significantly alter $q_\parallel$ without significantly affecting n and p.

If we consider p to be an integer and employ the step function approximation to ψ from Eq.(31), we obtain the alternate representation for Λ_p of

$$\Lambda_p=\Gamma(p+1)\left\{1-\exp(-r)\sum_{k=0}^{p}\left[1-\frac{(1+\gamma)^k}{(1+\gamma)^{p+1}}\right]\frac{r^k}{k!}\right\}\ . \tag{41}$$

From Eq.(41) we can see that strong flux limiting $[\Lambda_p/\Gamma(p+1) \ll 1]$ occurs for a fully depleted tail ($\gamma \gg 1$) if $p \gg r > 1$ since $r^p/p! \ll 1$ gives the γ independent result

$$\Lambda_p \to \Gamma(p+1)\left\{1-\exp(-r)\sum_{k=0}^{p}\frac{r^k}{k!}\right\} \to 0\ . \tag{42}$$

However, for $\gamma \gg 1$ the diffusive approximation of Eq.(25) should be replaced by the fully depleted tail approximation of Eq.(23) (which gives the same γ independent result since it roughly replaces f_0 by a truncated Maxwellian which vanishes for $x > r$). For a moderately ($\gamma \sim 1$) or significantly ($\gamma p \sim 1$) depleted tail and $p \gg r > 1$ the cancellation is less complete and the full expression (41) must be employed to evaluate the γ dependent, typically order unity reductions in $\Lambda_p/\Gamma(p+1)$. The lower order moments are not significantly changed in either case so that $\Lambda_p \approx \Gamma(p+1)$ provided $r > 1 \sim p$, with the approximation being better for smaller γ. When the step function approximation to ψ is used, K_v and L_v can be explicitly evaluated from Eq.(37) and shown to be of order γ.

The flux limiting factors F, K, and L as well as n/η and $p/\eta T$, have been evaluated using Mathematica[9], which was also used to check the insensitivity of the results to the width parameter w. The results are shown in Figs. 1 for the step function approximation to the excitation cross section and ψ as given in Eqs.(34) and (35). Figures 1 (a)-(e) are plots of F, K, L, n/η, and $p/\eta T$, respectively, versus γ for $r = 2$, while Fig. 1 (f) shows F vs. γ for $r = 1$. For $\gamma = 0.2$ ($\gamma p \sim 1$) the figures show more than a 50% reduction in the flux limit factors with only about a 5% in n/η and p/nT. The largest flux limiting is for the parallel heat flux factor F ($p \sim 5$-6) and the cross flux factor K ($p \sim 4$-5) and the smallest for the parallel current factor L ($p = 3$). Decreasing r from 2 to 1 results in stronger flux limiting as can be seen by comparing Figs. 1 (a) and (f) and as would be expected from the form of Λ_p.

V. Discussion

To estimate a typical γ for tokamak edge plasma conditions we note from Eq.(30) that $\gamma = r\nu_x/\nu_e$ where $rT = E_r$ is the energy threshold of the excitation cross section below which no significant radiation occurs. Defining $E_x = \frac{1}{2}m v_x^2$ (which is approximately equal to E_r unless metastable states are important), using $\ell n\Lambda = 10$, and writing E_r and E_x in eV gives

$$\gamma = \frac{r\nu_x}{\nu_e} \approx 4 \times 10^{-5}\left(\frac{\sigma_x}{10^{-16}\text{cm}^2}\right)\left(\frac{N_x}{n_e}\right)E_r E_x \ . \tag{43a}$$

Typically $\sigma_x \lesssim 5\times10^{-16}\,\text{cm}^2$, $N_x/n_e \lesssim 0.1$, and $E_r \sim E_x \lesssim 25$ eV, so that $\gamma \lesssim 10^{-2}$ and for $p \sim 5$, $\gamma p \lesssim 0.05$. As a result, tail depletion due to excitation losses is expected to be a weak effect, which cannot be responsible for strong flux limiting in tokamak edge plasmas. Because the step function approximation to the excitation cross section is extremely good for $w \gtrsim 1$, only the $E_x/T << 1$ assumption is needed to justify the diffusive approximation to inelastic scattering. We could be mis-estimating the size of line radiation losses by using the diffusive approximation. For the tail electrons $E_x > T$ (recall $E_x \approx E_r = rT > T$) the diffusive model overestimates γ by E_x/T as can be seen by considering the fully depleted model of Eq.(23). The fully depleted model, which is valid when $E_x/T > 1$ and overestimates losses by neglecting the re-supply term from higher energies, gives a γ that is $(2v_e/v_x)^2 = 2T/E_x$ times smaller than the diffusive model. As a result, a more realistic estimate for the effective γ is

$$\gamma_{eff} \approx 4 \times 10^{-5}\left(\frac{\sigma_x}{10^{-16}\text{cm}^2}\right)\left(\frac{N_x}{n_e}\right)\left(\frac{2E_r E_x T}{E_x + 2T}\right). \tag{43b}$$

Since $\gamma_{eff} \lesssim \gamma$ so our conclusion remains that tail depletion due to inelastic scattering caused by electron impact excitation energy loss is weak and cannot lead to substantial flux limiting in tokamak plasmas.

Partially ionized plasmas, such as those of interest for plasma processing, can have $N_x/n_e \lesssim 10^2$ (so that electron-neutral collisions can still be neglected). In addition, metastable states can be excited which decay to vibrationally excited states[10] (for example, molecular hydrogen or nitrogen). Cross sections and thresholds of $\sigma_x \sim 10^{-15}$ cm^2 and $E_r \sim 5$ eV can occur that result in a net electron energy loss of $E_x \sim 1$ eV. For these partially ionized plasmas $\gamma \sim 0.2$ are possible so that 50% or more reductions in the parallel heat flux are possible.

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Figure Caption

Figures 1 (a)-(e) are plots of the flux limiting factors F, K, and L, as well as the density and pressure ratios, n/η and $p/\eta T$, respectively, versus γ for $r=2$. Figure 1 (f) is a plot of F vs. γ for $r=2$.

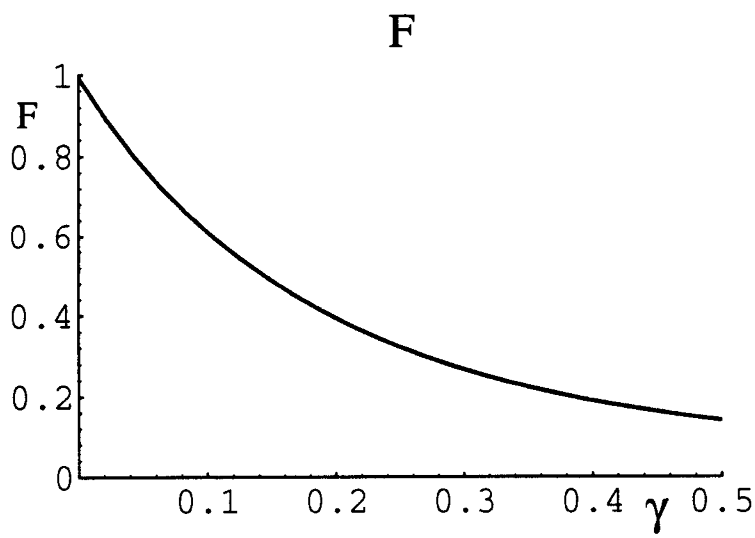

Fig. 1(a)

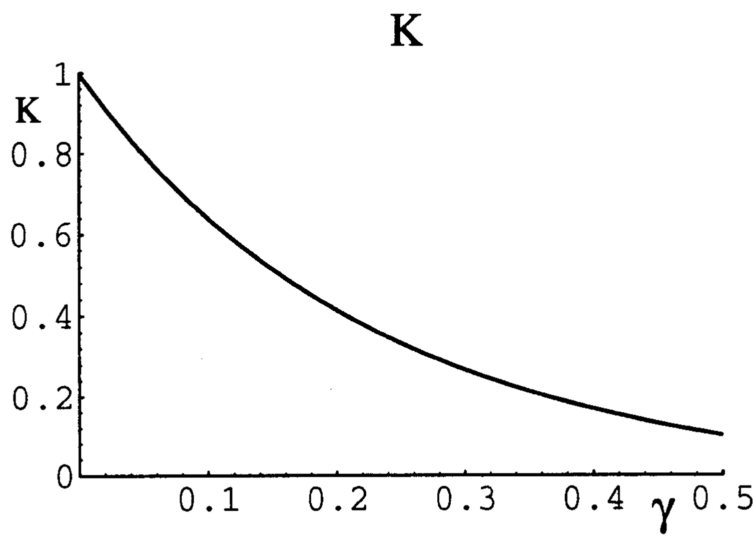

Fig. 1(b)

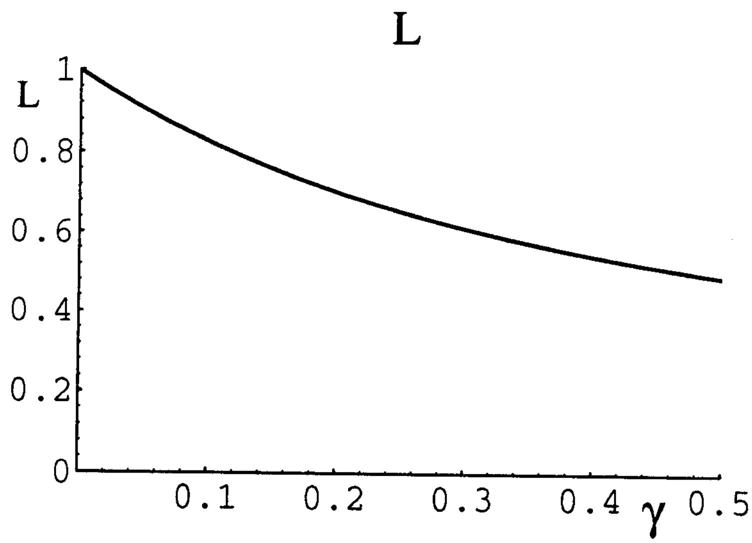

Fig. 1(c)

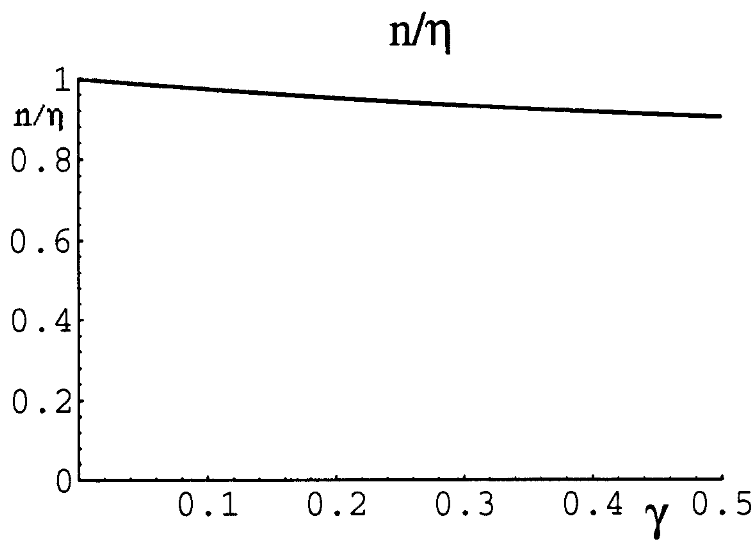

Fig. 1(d)

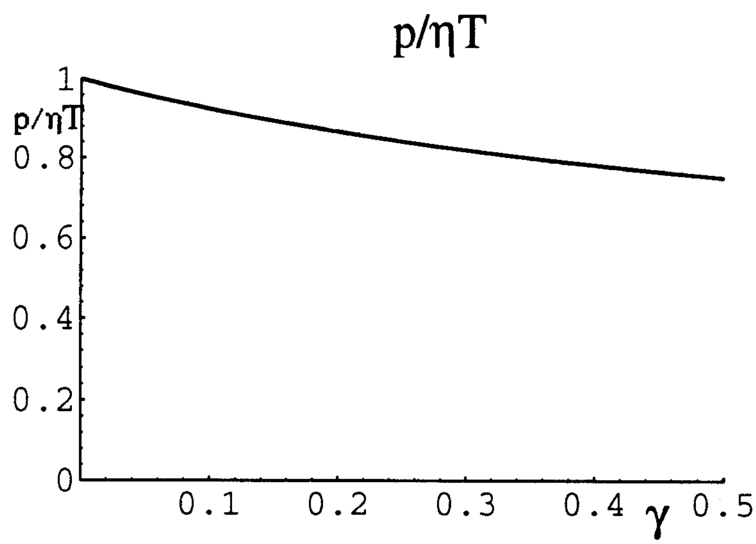

Fig. 1(e)

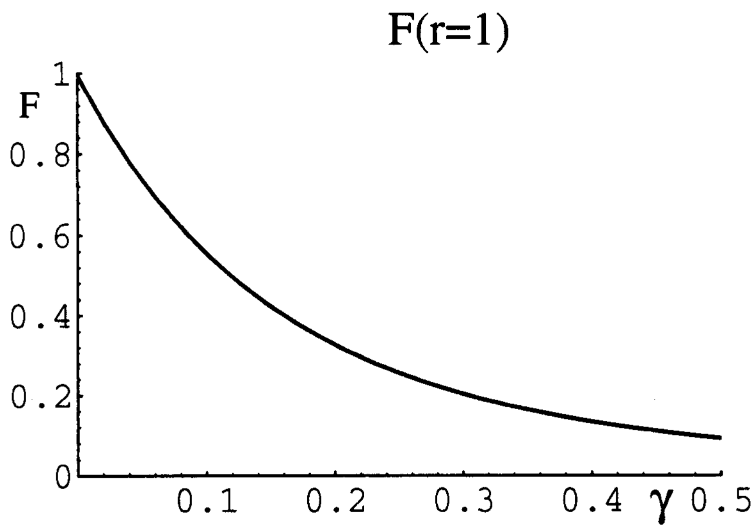

Fig. 1(f)