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Plasma shape control: A general approach and its application to Alcator C-Mod

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Abstract

A general approach to plasma shape control and its application to the tokamak Alcator C-Mod is described. The method is linear in the magnetic measurements but is entirely algorithmic, requiring no fitting of databases. Estimators of the shape parameters are based on a complete vacuum reconstruction of the flux, so that control points can be defined anywhere within the reconstructed region. The conversion of flux differences into flux-surface distances, and the calculation of appropriate coil currents for controlling each parameter, requires a specific reference equilibrium. However, the control is very insensitive to the choice of reference equilibrium provided that the shape parameters are chosen appropriately. Control current combinations that are orthogonal, in the sense of changing one parameter and not the others, are obtained. Experiments with these estimators and controllers show them to be accurate and robust over a wide range of plasma shapes.

1. Introduction

The problem of control of the plasma shape in tokamaks can be considered to consist of two main parts. First the shape must be diagnosed in real time, and second the coil currents must be adjusted to correct it to what is required. In the past, a variety of methods have been used for both the shape diagnosis ("estimation") and its correction ("control"). The purpose of the present work is to describe a unified method of performing both of these tasks and to illustrate its operation by experimental results from Alcator C-Mod [1]. We note that the problem of stability, notably vertical stability, which has been extensively studied elsewhere, is deliberately excluded from consideration here.

The problem of shape estimation can be approached in one of two main ways. Either an algorithmic solution of the flux reconstruction problem is used, or else a large database of equilibria spanning the space of interesting plasmas is precalculated and convenient fits to the database derived. Historically, in early circular cross-section tokamaks, the plasma position (and current) were diagnosed algorithmically, sometimes using specially designed pick-up coils. However, because of the greater complication of magnetic diagnosis of shaped plasmas, and the limited capabilities of real-time calculations, the database fitting approach tended to be favored in many shaped tokamaks. This fitting has been done in a variety of ways: for example, an ad hoc approach guided by an intuitive model was used on DIIID [2], function-parameterization on ASDEX and ASDEX Upgrade [3,4], singular value decomposition (SVD) on PBX-M [5], and neural networks on Compass [6]. In contrast, the method described here and in use on Alcator C-Mod is purely algorithmic yet performed in real-time at speeds easily exceeding those necessary for tokamak control. The trend toward a return to the algorithmic approach is also represented by JET [7], and TCV [8].

A feature of the Alcator C-Mod control system hardware [9,10] is that it is capable of summing real-time signals multiplied by programmable coefficients; that is, it is a linear system. Because of this practical restriction, because of the relative ease of analysis of linear systems, and because an accurate linear system can in fact be developed, the method we describe is linear. (Prior recognition that the problem could be specified linearly permitted the original hardware choice that was made.) Section 2 describes the method and illustrates its accuracy.

With reliable estimators of the required shape parameters in hand, the problem of what current combinations to use to control them arises. Most modern shaped tokamaks have a substantial number of independently controllable poloidal field coils that must be used in concert to control the shape. Here ad hoc approaches have dominated past practice, although systematic methods have been proposed [11,12]. In section 3 we describe a systematic method of deriving "controllers" based on estimators of the type described in section 2.

Section 4 shows examples of the complete shape control system in action on Alcator C-Mod.

2. Shape Estimation

The plasma shape is often described in terms of intuitive generic parameters such as size, position, elongation, triangularity, and so on. However, the customary intuitive definition of these parameters rarely gives the exact quantities that one wishes to control in a practical tokamak. Therefore one needs a more flexible definition of plasma shape parameters. The shape parameters of interest refer mostly to the locus of the last closed flux surface; for a divertor plasma, the separatrix. The control of this shape requires us, therefore, to be able to reconstruct the separatrix from magnetic measurements.

Broadly, tokamak flux surface reconstructions can be classified as belonging to two main categories: those that do not use the fact that the plasma is in force-balance, and those that do. In the former category are reconstructions of the vacuum flux surfaces outside the plasma. These require the representation of the plasma current in the form of sums of current elements, whether multipoles [13,14], filaments [15], surface current expansions [16], or finite elements [17]. The choice of current representation is immaterial to the present discussion. In the present realization a large number of filaments distributed on a surface are used. The major advantage of the vacuum reconstruction technique, which makes it the method of choice for the present purposes, is that it can be formulated as a linear problem. The second type of reconstructions, that do use the force balance to produce solutions of the Grad-Shafranov equation, are non-linear, and generally non-local.

In brief the reconstruction technique for an assumed axisymmetric plasma is as follows, the currents in the problem are denoted by the vector I whose components are all the coil currents, any currents in assumed passive elements such as vessel and structure, and the plasma current elements. Naturally a finite approximation to the distributed currents must be adopted. We find that the method of representation of the passive currents is relatively unimportant provided that the magnetic measurement set is sufficiently complete and that the active coil currents are present as elements in the model. The influence of each current element at any measurement (flux-loop or poloidal field pick-up coil) can readily be calculated from the Green's function so to give a matrix C such that the measurement signals (vector m) is given by

$$m = CI \quad . \tag{1}$$

One can then invert this relation in a least-squares sense to find the currents in terms of the measurements. This may be done most conveniently using a pseudo-inverse. If D_m and D_I are diagonal square matrices whose diagonal elements are respectively the characteristic sizes of the measurements and currents, then we form the matrix

$$C^{-1} \equiv D_I (D_m^{-1} C D_I)^{\dagger} D_m^{-1} \quad , \tag{2}$$

where † denotes a pseudo-inverse[18] obtained by singular value decomposition truncated by discarding those singular values that are smaller than the maximum singular value by some tolerance factor. The truncation procedure amounts to a smoothing. The solution $C^{-1}m$ then gives the set of currents that minimizes the measurement errors (normalized by D_m) and the mean square currents (normalized by D_I) subject to the smoothing given by the truncation. Thus the model currents are expressed as linear combinations of the measurements.

Alcator C-Mod uses 26 flux loops and 26 poloidal field loops distributed approximately uniformly around the vacuum vessel. The truncation tolerance is decided on the basis of considerations of measurement uncertainty and smoothness of reconstruction. We find that a truncation that leaves 5 modes of the plasma and about 14 of the passive structure plus 9 for the active coils is approximately optimal.

Such current moments as the total plasma current, I_p, and I_p times the centroid of the current, $I_p.R_c$, $I_p.Z_c$ can then also be written as linear combinations of the measurements. These are useful as control parameters but are not sufficient for detailed shape control.

The influence of each current element at any other position in the domain of interest can readily be calculated. We use a 65×65 rectangular mesh as a practical basis for interpolating to any other point, although direct Green's function evaluation is also possible. If we denote that influence vector by F, such that the flux at the relevant point is $\psi = F^T I$, then the reconstructed flux can be written as

$$\psi = F^T C^{-1} m \quad , \tag{3}$$

which is completely linear, with F and C^{-1} being precalculated.

General plasma shape estimators can be considered to be flux differences or sets of flux differences between points in the domain of interest. For example, instead of the current centroid, we can define the plasma radial position R_g via the difference in flux between two control positions at the midplane, at $R = R_0 \pm a$, where a is the nominal minor radius. When the flux difference is zero, a flux surface passes through both points, so the deviation of the geometrical major radius from R_0 is then zero. Similar definition of vertical position is also possible. It must be emphasised that the control points do not have to be at places where measurements exist. The reconstruction allows one to put them essentially anywhere. Figure 1 shows the standard control points used for Alcator

C-Mod. For diverted plasmas, the clearance of the separatrix from limiting structures at the inboard and outboard is important. Flux estimators for these clearances are most naturally defined as the difference between the flux at a point located at the x-point and that at a control point near the limiting structure. If the surface of the structure is chosen, then this estimator is zero when a flux surface passes through both control points, i.e. when the separatrix just touches the limiter. The strike-point of the separatrix on the divertor plates can similarly be defined in terms of the flux difference between the x-point and a local control point on the plate.

The position of the x-point itself is the place where both B_R and B_z are zero. Therefore the values of the fields (which are also, of course, linear functions of the fluxes, since they are given by flux derivatives) at a control point close to the expected position of the x-point provides a measure of the distance of the x-point from the control point.

In all these cases, the estimator can, and generally should, be chosen so that its value is small when the parameter is at its nominal value. The limit of this prescription is to adopt the procedure of Hofmann and Jardin [17] whereby the reference point is moved such that the desired estimator value is always exactly zero. It is generally unnecessary to proceed to this limit, however, and even if one does, the question remains of how to interpret the error when it is non-zero. To derive a quantitative measure of the parameter, when it is non-zero, expressed in terms of distance from the control point of the flux surface, requires division by a flux gradient. We may want to specify many shape parameters in terms of the position of the flux surface, not merely the value of the flux. For example we might want to specify the separatrix outboard clearance in meters. To transform from flux-difference to distance requires us to know the flux gradient and obtain $dx = d\psi(d\psi/dx)^{-1}$ The flux gradient is another linear flux function and so the calculation is straightforward. However, for a linear system, an estimate of the calibrating flux gradient must be specified ahead of time. The most accurate way to do this is to derive the gradient from a reference equilibrium, which may be an actual plasma or simply a numerical equilibrium. Once that factor is derived, the shape parameter can be expressed linearly in terms of fluxes. That linearization is very good provided that we recognize that the calibrating flux gradient for a plasma of constant shape is proportional to the plasma current. Therefore the shape error must be considered to be expressed not in distance alone but as the product of distance and plasma current. In other words, we consider the quantities under control to be $I_p x$, governed by:

$$d(I_p x) = d\psi \left(\frac{d\psi}{I_p dx} \right)^{-1} \quad , \tag{4}$$

and the entire calibrating quantity in the brackets is evaluated once and for all for the reference shot. Note that this calibrating factor has the dimensions of magnetic permeability, which for free-space can be considered to be a constant, μ_0.

In the estimators for the clearance of the separatrix the presumption thus far has been that the x-point position is known and so the separatrix flux value is simply equal to the value at that x-point reference. In reality the x-point position may not be exactly at its control point. However, since the x-point is where the flux gradients are zero, the difference of the separatrix value from the reference value will generally be rather small, provided that the reference is not too far from the actual x-point. It would be possible to improve on the estimate of the separatrix flux value by using an expansion of the flux about the x-point. However this extra sophistication is not essential.

The x-point position is a quantity that it is generally useful to estimate and control. A slightly more subtle algorithm is required for this purpose, as follows. The magnetic field (equivalent to the flux gradient) at a reference point in the vicinity of the x-point can be expressed linearly as:

$$\begin{aligned} B_R &\approx \frac{dB_R}{dR}(R - R_x) + \frac{dB_R}{dz}(z - z_x) \\ B_z &\approx \frac{dB_z}{dR}(R - R_x) + \frac{dB_z}{dz}(z - z_x) \quad . \end{aligned} \tag{5}$$

These simultaneous equations may be solved for $R - R_x$ and $z - z_x$ as

$$\begin{aligned} R - R_x &= \frac{dB_z}{dz}\frac{1}{\Delta}B_R - \frac{dB_R}{dz}\frac{1}{\Delta}B_z \\ z - z_x &= -\frac{dB_z}{dR}\frac{1}{\Delta}B_R + \frac{dB_R}{dR}\frac{1}{\Delta}B_z \quad , \end{aligned} \tag{6}$$

where the determinant is $\Delta = (dB_R/dR)(dB_z/dz) - (dB_R/dz)(dB_z/dR)$. The x-point position (R_x, z_x) can thus be obtained by a linear process if the coefficients of B_R and B_z are precalculated constants. This precalculation is performed based on a reference equilibrium. In addition, both sides of the above equation are considered to be multiplied by the plasma current so that the calibration coefficients for the linear estimator of $I_p.(R - R_x)$ and $I_p.(z - z_x)$ are $(dB_z/dz)(I_p/\Delta)$ etc., which are independent of the current scaling factor.

In the case of strike-point estimation, the difference in flux between the x-point and the reference point must be divided by a flux gradient to give the separatrix position. If we use the flux gradient *at the midplane* rather than along the plate for this transformation, the result is a strike-point estimator in units of $\rho \equiv (\psi - \psi_x)/(d\psi/dR)$ the equivalent flux-surface position at the miplane. This is convenient because this midplane coordinate is conventionally used to describe the scrape-off layer.

Section 4 illustrates the accuracy of the estimators in practice on Alcator C-Mod.

3. Controllers

Having decided what shape parameters it is desirable to control, and given reliable estimators of their values, the problem still remains of what currents and voltages to apply to the coils to perform that control. Ad hoc combinations of currents can be devised intuitively that will serve to control the various parameters. However, a more systematic approach is desirable. In particular, it is useful to obtain combinations of currents (or voltages), which we refer to as "controllers", that are capable of changing particular shape parameters without affecting the other parameters. Such controllers are called "orthogonal".

Recall that we have the flux influence of the coils such that $\psi = F^T I$. Since the shape parameters are combinations of flux at various control points, the influence of the coil currents on the shape parameters can also be written $s = F_s I$, where F_s is a matrix whose rows are the corresponding influence coefficients, F. The parameters s may be considered to have units of flux or alternatively amp-meters: the product of plasma current times distance, as described above. The current vector here includes all the passive elements and the plasma current elements as well as the active coils over which we have direct control. In general both the passive currents and the plasma currents will change in response to any changes in the active coil currents. To calculate these effects requires a model of the plasma equilibrium and circuit equations for the passive currents. A full solution to this problem is then a major undertaking, which is necessary for addressing problems such as the vertical stability of an elongated plasma but is not so essential for shape control. Therefore, as an approximation to be justified later on the basis of its results, we shall *ignore* the changes in plasma and passive element currents and regard the influence of the controlled current on the shape parameters as due to their direct flux influence alone. Then change in the shape vector is related to changes in the coil currents by $\delta s = F_c \delta I_c$ where δI_c is the reduced dimension vector of changes of active coil currents, and F_c is the appropriate submatrix of F_s.

Now consider a set of shape parameters equal in number to the number of active coils, so that F_s is a square matrix. Its inverse, $F_s^{-1} \equiv E$, is, of course, such that the jth column of E (denoted E_j) when multiplied by F_s gives a vector with unity in its jth element and zero everywhere else. Therefore, a set of currents equal to E_j gives rise to unity change in the jth shape parameter and zero change in all the others: E_j is an 'orthogonal' controller. (Actually 'orthonormal' but note that there are two vector spaces here: currents and shape parameters, so the controllers are orthogonal to parameters not to other controllers, and the normalization is not to unit length of controller but to unit change of the parameter.)

On the face of it, this is the solution to the controller problem. However, it proves not to be a fully satisfactory solution, as we will illustrate by a specific example based on Alcator C-Mod. There are nine free poloidal field currents in Alcator C-Mod (not counting the fast vertical stability power-supply whose current is very limited). Figure 2 shows the location and nomenclature of the coils to which they are attached. The upper and lower coils are in series for EF3 and EF4, while the EFCs are in anti-series and controlled by the vertical stability power supply. A reasonable choice of nine shape parameters, with reference to figure 1, is as follows. The flux on axis ψ_0 (which is used to control the plasma current), the position R_g and z_c, the inboard clearance c_i, the position of the lower and upper x-points $(R_{xl}, z_{xl}), (R_{xu}, z_{xu})$ and the current in the EF4 coil I_{EF4}. (This last is appropriate because the power-supply provides limited flexibility in the EF4 current which is outside the thick cylindrical structure. Its inclusion as a shape parameter is the natural method of accounting for this constraint.)

We can calculate the set of completely orthogonal controllers for this set of parameters by direct matrix inversion. The matrix F_s is non-singular because the shape parameter set is reasonably well chosen. The effect of each controller may be illustrated by plotting the flux pattern due to the control currents and the sum of these and the original equilibrium (hence giving the effect of the controller on the plasma, to the extent that our approach is a valid approximation). Figure 3 shows the ψ_0 controller. It is naturally a flat field null, with the consequence that the plasma shape is unperturbed; so the right hand frame illustrates the unperturbed equilibrium. Figure 4, on the other hand, shows the z_c controller influence. An intuitive controller for z_c would be simply a horizontal field. However, this orthogonal controller gives a much more convoluted shape. The reason is straightforward. The controller has been required, for example, not to move the x-points. Therefore the perturbed field at the x-points is zero. The controller has contrived this by a rather peculiar combination of coil currents with considerable opposition of adjacent currents. It is far from obvious that this is a good vertical position controller. It seems likely that the demand for large opposing currents will unreasonably stretch the capabilities of power supplies and possibly lead to unacceptable forces on the coils. If the plasma vertical position is moved without the x-points moving, a combination of currents like this must be involved. However, if we demand that the plasma move vertically regardless of the x-points, the currents required would be much lower and the field simpler. It is probably more important to control the plasma position than the x-point position; therefore a hierarchy of controllers is desirable.

If we consider a set of controllers fewer in number than the number of control currents, then finding orthogonal controllers is an underdetermined problem. We may consider the pseudo-inverse of the then rectangular matrix F_s (suitably weighted) as providing orthogonal controllers that minimize the (weighted) sum of squares of the control currents. If the plasma position and current controllers are considered to be of higher priority than the other shape parameters, then we can obtain better controllers for them by using a restricted set of plasma parameters. For example, we use the set ψ_0, R_g, z_c and the elongation κ, and obtain controllers for R_g and z_c that give essentially pure vertical and horizontal field, in accord with our intuition. Figure 5 illustrates these. The controllers thus obtained place far less demands on the coil currents. For example the largest current element in the z_c controller is reduced by a factor of six (for the same z_c perturbation) compared with the fully orthogonal controller. The penalty is that plasma position changes also change the x-points etc, although if the x-points have other controllers applied to them, this effect will be compensated, albeit possibly on a longer timescale.

The current changes required for the controllers we calculate must be induced by changes in the coil voltages. Therefore a means for transforming the 'current controllers' into 'voltage controllers' is needed. We approximate this tranformation by ignoring all but the mutual and self inductances of the active coils. If this inductance matrix is M_c then application of voltages $M_c^{-1}E_j$ will, in this approximation, give rise to current derivatives such that the shape parameter s_j has 1 Am/s time derivative. For a restricted set of orthogonal parameters it may be advantageous to minimize not the mean-square currents alone but some combination of currents and voltages. This leads to slightly different optimizations depending on the relative weight of current and voltage. The R and z-controllers of figure 4 were obtained using a relative voltage and current weighting appropriate for a situation where the time-constant of evolution is 0.1s. This tends to put significant weight on the voltage optimization. That explains why the z-controller relies strongly on OH2, which has relatively few turns so that its inductance is lower than, for example, EF2.

In a full feedback control loop, the estimator (error) signal is multiplied by a gain, G_j, (with possibly derivative and integral components – PID – but we ignore that complication here) and the resultant multiplies the controller $(M_c^{-1}E_j)$, applying feedback voltages to the coils. Since we have orthonormal controllers, we can immediately derive the approximate behaviour of the system. In so far as our approximations are valid, there results a decoupled eigenmode of the system corresponding to each shape parameter. The decay time-constant of the error in shape parameter s_j is $1/G_j$ s. It should be noted that this extremely simple result is not affected by any uncertainty introduced by the use of flux gradient calibration factors obtained from a reference equilibrium. This is because the *same* factor applies to both the estimator and the controller and so any error in the factor cancels out from the time-constant.

Of course, our controller approximations are not accurate for the parameters R_g and z_c because the assumption that the plasma does not move in response to the currents is obviously incorrect for them. For the other shape parameters, however, the approximations are quite good, and the time-constants derived can be expected to be realistic.

For comparison, controllers have also been derived on the basis of the neighbouring-equilibrium approach of Humphreys and Hutchinson [12], which gives a linearized flux response to coil currents *including* the response of the plasma. We find that the controllers obtained for the plasma position are completely different, as might be expected. However the controllers for the other shape parameters are highly similar to those derived on the present basis. (Within 20% for their largest elements). Moreover, the R and z controllers derived from the neighbouring equilibrium response are less plausible than the present ones. For example, the z controller has inverted sign because the equilibrium position moves in a direction opposite to the applied force for an unstable (i.e. elongated) equilibrium. Therefore it appears that there is no significant advantage for deriving controllers in using the more complicated neighbouring-equilibrium treatment.

Table 1 shows an example of a set of controllers in routine use for Alcator C-Mod. The first three, ψ_0, R_g, z_c, are based on the restricted orthogonality, described above (including orthogonality to κ). The others are fully orthogonal to each other and to the first three. This combination has proven to be a good compromise in practice, as the next section illustrates.

4. Alcator C-Mod Experimental Results

Estimators and controllers of the type described have been implemented on Alcator C-Mod. The entire duration of the plasma shot after the first 0.1 s is routinely controlled by feedback purely upon these shape parameters. (The initiation uses partially preprogrammed currents and voltages because of the uncertainties in reconstructing the very small plasma current.)

The estimators are found to be extremely accurate as evidenced by their agreement with calculations of the same quantity using the full equilbrium code EFIT[19]. As a quantitative test of the accuracy of the vacuum reconstruction, the last closed flux surface contour it finds has been compared with that of EFIT. The (signed) perpendicular distance was obtained between the two boundary contours at a large number of points around the boundary. The mean and standard deviation of the set of distances was then evaluated for a variety of different plasma shapes, limited and diverted. The worst case found, for plasmas reasonably centered in position, gave a mean distance of 2.3 mm and a standard deviation of 2.8 mm. Some shapes are somewhat better, and the reconstruction develops major errors if the plasma is moved by a large fraction ($\gtrsim 1/4$) of the minor radius without compensating adjustments to the plasma representation, but these values may be taken as representative. Two millimeters is about 1 percent of the minor radius.

Figure 6 illustrates a particular estimator, the inner gap between the bumper limiter and the separatrix. The shape parameters in this and subsequent figures have been divided by the plasma current so as to display them in units of meters. The gap as determined by the linear estimator and by EFIT agree to within about 2 mm when the plasma is diverted. When the plasma is limited, at the beginning and end of the shot, EFIT sets the gap to zero. The linear estimator derives a negative value at that time, entirely consistent with its definition in terms of the flux value at the wall relative to the x-point. It says that if the separatrix were followed round to the inboard position, it would be beyond the wall by the indicated distance. It should be emphasized that this is a *better* quantity to use for feedback control than the gap as determined by EFIT (based on the last closed flux surface) because the linear estimator has a consistent meaning and remains controllable even when limited, whereas the gap has a slope discontinuity and becomes zero by definition when limited.

Of course, both EFIT and the linear estimators depend on the accuracy of the same magnetics measurements. However, independent comparisons of the reconstructions with, for example, probe data, indicate that the separatrix strike-point is obtained correctly to within two or three millimeters.

Figures 7 (a) and (b) display time histories of the controlled quantities for a typical elongated, lower diverted shot, overlaid with their programmed values. The four quantities, I_p, R_g, z_c and c_i, in (a) must be controlled the most accurately. Together they determine the proximity of the separatrix to the main chamber wall and RF limiter. As is evident from the figure, these quantities are in fact well controlled. During the plasma ramp-up and ramp-down the control is not quite so good because of power supply limitations, for example at about 0.35s where cross-over of the current polarity of the OH power-supply renders its control inactive for about 50ms, causing a perturbation on the parameters. The use of controllers for I_p, R_g, and z_c that are not orthogonal to the other parameters is one reason why their control is so accurate. (The vertical position control is also assisted by the fast stabilization supply which is not part of the present analysis.) Good accuracy is also achieved on the inboard clearance, c_i, by the use of integral gain in the PID controller. This is the cause of the apparent delay in the c_i trace relative to its demand.

Control of the four x-point parameters (b) is important but not as critical for plasma shape control. For a lower diverted shot, the upper x-point location is used to control the poloidal field at the top of the plasma. As long as the field is sufficiently high that the plasma is far from being double-diverted, the exact upper x-point position is not important. The lower x-point position control requires higher precision. The radial position is controlled within perhaps 2 mm; the vertical position is allowed to run with a constant offset of perhaps 5 mm. This offset is a result of various factors including coil resistance and finite loop gain. We could use integral gain here (as we did on c_i) to remove the offset but we find the present configuration acceptable in practice. After time 1 s, the current is ramped down with the plasma limited on the inner wall: negative c_i. During this time, the lower and upper x-points are allowed to evolve freely by lowering their feedback gain.

Figure 8 illustrates the extent to which independent x-point control has been obtained. In two different shots (but with the same feedback control settings) the demanded x-point position was subjected to steps in R and z. We find that not only does the x-point follow the demand and equilibrate at the new setting, but also it does so in accord with the expectations of an independent eigenmode of the system. The other parameters are not significantly perturbed by the motion of the x-point, and the approach of the x-point position to its new demand is via an exponential decay whose time constant is equal to the theoretical value (28 ms for both R and z) within experimental uncertainty.

It is useful in some circumstances to employ a control configuration which is overdetermined, i.e. in which one is nominally controlling more parameters than there are degrees of freedom (power supplies). Such a situation arises on Alcator C-Mod for the divertor configuration. It is often advantageous to control not the x-point position but the position of the strike-point on (for example) the outer divertor plate. There is in general a close relationship between the strike-point and x-point positions but, because of the evolution of the overall magnetic configuration, the relationship is not unique. Figure 9 shows an example of the transition between x-point and strike-point control. A controller that is orthogonal to all parameters except R_{xl} and z_{xl} is used *in addition* to the x-point controllers. In (a) its gain is zero, so no added control is being attempted. There is a significant drift of the strike-point, which is normalized in the estimator itself to read in terms of flux-surface distance at the midplane (ρ).

Figure 9(b) shows a subsequent shot, identical except for applying significant gain ($100\ \mathrm{s}^{-1}$) on the strike-point control and reduced gain on the x-point. The result, for these settings, is extremely good strike-point control at the expense of slightly worse x-point control. A continuously variable trade-off of the accuracies of these degenerate parameters is possible via different gain settings.

Figure 10shows examples of a few of the shapes that can be produced and controlled using the present approach. A 'Slot' divertor, where the separatrix runs almost parallel to the vertical divertor plate, is most accurately produced using the strike-point controller. Diversion on to the top face of the shaped divertor can be produced using lower x-point position control or strike-point control. In the latter case, moving the strike-point reference point to the vicinity of the required position is found to be advantageous. Plasmas that are limited on the inner wall are also easily programmed by using negative values of the inner clearance, c_i; their shape is still controlled by the same parameters.

5. Conclusions

A systematic linear approach to plasma shape control has been described. The real-time estimators are found to have accuracy essentially as good as that of a full Grad-Shafranov reconstruction. A method of deriving orthogonal controllers has been employed but it proves disadvantageous to use an orthogonal system of full rank (i.e. with the number of shape parameters equal to the number of power supplies). Instead, lower rank controllers are used for the major parameters, plasma current and position. Fully orthogonal controllers are used for the remaining shape parameters. In some cases, an overdetermined control system is useful (i.e. one in which the nominal number of parameters under control exceeds the number of power supplies). Smooth transitions between different dominantly controlled parameters are then possible. The system is in routine operation on Alcator C-Mod and provides accurate and reliable control of a variety of plasma shapes. The approach developed here appears to have widespread applicability for existing and future tokamaks.

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References

1. I.H. HUTCHINSON et al., "First Results from Alcator C-Mod," *Phys. Plasmas*, **1**, 1511 (1994).

2. S. KINOSHITA, H. FUKUMOTO, A.G. KELLMAN, et al., "Independent Control of Gaps in Single-Null Divertor Discharges on the DIII-D Tokamak," General Atomics Report GA-A19584 (1990).

3. B.J. BRAAMS, W. JILGE, K. LACKNER, "Fast Determination of Plasma Parameters Through Function Parametrization," *Nucl. Fusion*, **26**, 699 (1986).

4. MCCARTHY, P.J., SCHNEIDER, W. et al., "MHD Equilibrium Indentification on ASDEX-Upgrade," Proc. 19th EPS Conf. on Controlled Fusion and Plasma Physics, (Innsbruck, Austria) 16C, I, 459 (1992).

5. R.E. BELL, N. ASAKURA, S. BERNABEI, et al., "Control of Plasma Shape and Performance of the PBX-M Tokamak Experiment in high-β_t/ high-β_p Regimes," *Phys. Fluids B*, **2**, 1271 (1990).

6. T.N. TODD and the COMPASS TEAM, "MHD Control and ECCD in Compass-D," *Plasma Phys. and Controlled Fusion*, **35**, B231 (1993).

7. D.P. O'BRIEN, J.J. ELLIS, J. LINGERTAT, "Local Expansion Method for Fast Plasma

Boundary Identification in JET," *Nucl. Fusion*, **33**, 467 (1993).

8. F. HOFMANN, J.B. LISTER, M. ANTON, et al., "Creation and Control of Variably Shaped Plasmas in TCV," *Plasma Phys. Contr. Fusion*, **36**, B277 (1994).

9. S. HORNE, M. GREENWALD, I. HUTCHINSON, et al., "Performance of the C-Mod Shape Control System," *Proc. of IEEE 15th Symposium on Fusion Engineering, Hyannis, MA, Oct. 1993*, **1**, 242 (1994).

10. P.F. ISOZ, J.B. LISTER, Ph. MARMILLOD, "A Hybrid Matrix Multiplier for Control of the TCV Tokamak", Proc. of 16th Symposium on Fusion Technology, London, Sep 1990, **2**, p 1264 (North Holland, 1990)

11. D.J. STRICKLER, N. POMPHREY, AND S.C.JARDIN, "Equilibrium Shape Control in CIT PF Design," *Proc. of IEEE 13th Symposium on Fusion Engineering, Knoxville, TN, Oct. 1989*, **2**, 898 (1990).

12. D.A. HUMPHREYS and I.H. HUTCHINSON, "Axisymmetric Magnetic Control Design in Tokamaks using Perturbed Equilibrium Plasma Response Modeling," *Fusion Technol.*, **23**, 167 (1993).

19. L.L. LAO, H. St. JOHN, R.D. STAMBAUGH, A.G. KELLMAN, and W.PFEIFFER, "Reconstruction of Current Profile Parameters and Plasma Shapes in Tokamaks", *Nucl. Fusion*, **25**, 1611, (1985).

13. A.J. WOOTTON, "Measurement of Plasma Shape in a Tokamak," *Nucl. Fusion*, **19**, 987 (1979).

14. G.N. DESHKO, T.G. KILOVATAYA, YU.K. KUZNETSOV, et al., "Determination of the Plasma Column Shape in a Tokamak from Magnetic Measurements," *Nucl. Fusion*, **23**, 1309 (1983).

15. D.W. SWAIN, G.H. NEILSON, "An Efficient Technique for Magnetic Analysis of Non-Circular, High-Beta Tokamak Equilibria," *Nucl. Fusion*, **22**, 1015 (1982).

16. W. FENEBERG, K. LACKNER, and P. MARTIN, "Fast Control of the Plasma Surface," *Comp. Phys. Commun.*, **31**, 143 (1984).

17. F. HOFMANN, S.C. JARDIN, "Plasma Shape and Position Control in Highly Elongated Tokamaks," *Nucl. Fusion*, **30**, 2013 (1990).

18. S.L. CAMPBELL, AND C.D. MEYER, "Generalized Inverses of Linear Transformations" Dover, New York, (1979).

Table 1. The controller matrix of power supply voltages in volts that are calculated to give a rate of change of 10^3 Am/s in the indicated shape parameter (times plasma current) and no change in the orthogonal parameters. (The axial flux, ψ_0, is normalized by dividing by $2\mu_0 R/a$ to render it into Am.) The EF4 current controller is not used in practice; a controller that uses just EF4 voltage, ignoring inductive coupling is more convenient.

	ψ_0	R_g	z_c	c_i	R_{xl}	z_{xl}	R_{xu}	z_{xu}	EF4
EF1U	0.326	-0.197	0.393	-0.124	-0.011	0.073	0.112	-0.531	-1.244
EF2L	0.326	-0.197	-0.393	-0.125	0.126	0.670	0.009	-0.015	-1.244
EF2U	0.175	-0.045	0.284	1.451	0.311	-0.806	0.381	0.671	11.046
EF2L	0.175	-0.046	-0.284	1.451	0.667	-1.471	0.056	0.121	11.046
EF3	1.871	2.846	0.000	-1.083	-0.197	0.560	-0.010	-0.057	-28.101
EF4	-0.357	0.498	0.000	-0.123	0.043	-0.021	0.058	0.063	87.877
OH1	1.017	-0.592	-0.002	0.057	-0.049	-0.011	-0.051	-0.010	0.859
OH2U	0.275	-0.160	0.282	0.096	0.019	-0.047	-0.078	-0.018	0.269
OH2L	0.275	-0.160	-0.283	0.096	-0.072	-0.018	0.003	0.002	0.269

Figure Captions

Fig. 1 Standard reference points for the estimators and controllers in Alcator C-Mod. Estimators are based on the combinations: $R_g : \psi_2 - \psi_1$; $c_i : \psi_{xl} - \psi_1$; $R_x, z_x : B_{rx}, B_{zx}$; $\rho_s : \psi_s - \psi_{xl}$.

Fig. 2 The poloidal field coils of Alcator C-Mod.

Fig. 3 Form of the ψ_0 flux controller, used for current control. For a chosen amplitude, the flux perturbation and the resulting perturbed flux surfaces are shown. In this case, the flux surfaces are the same as the unperturbed equilibrium, because the ccontroller is a field null.

Fig. 4 Form of the z_c controller orthogonal to all other parameters. The strange shape is caused by the requirement of orthogonality. The perturbed plasma in the right frame should be compared with the unperturbed case of Fig. 3.

Fig. 5 The R_g and z_c controllers obtained when orthogonality only to ψ_0 and κ is required. These are the controllers actually used.

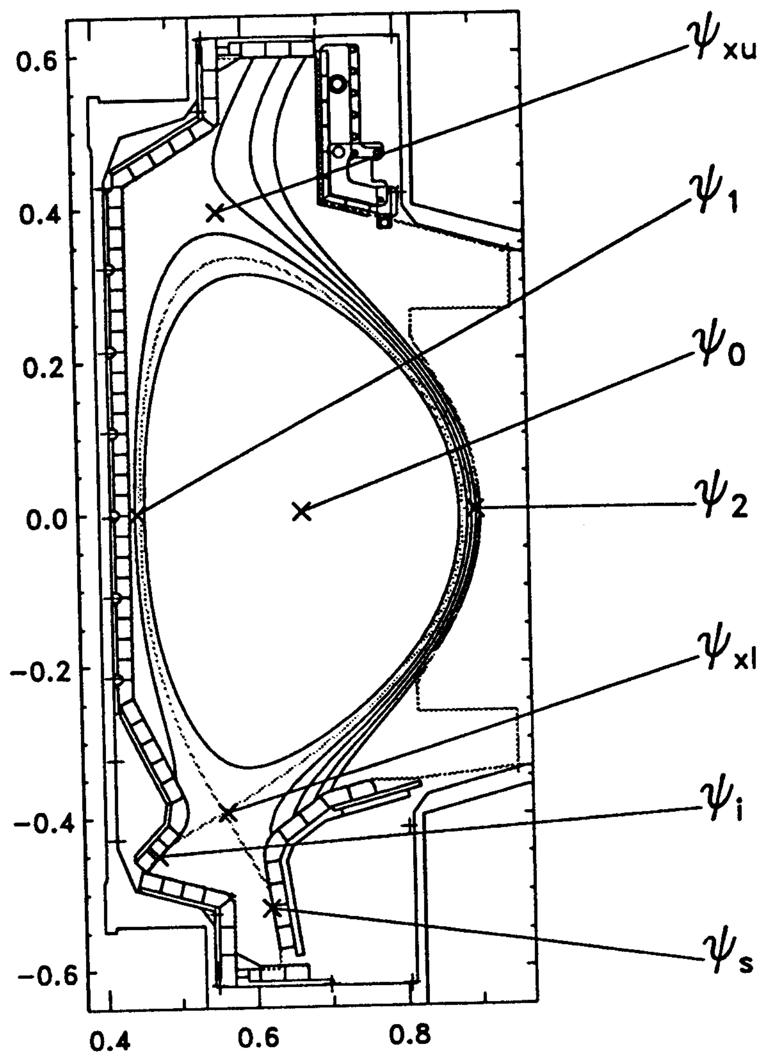
Fig. 6 Comparison of the real-time estimator for the inner clearance, expressed here in distance of the separatrix from the wall, with the inner gap calculated by EFIT.

Fig. 7 Evolution of the shape parameters under control (solid) and their demand signals (dashed) for a typical diverted shot. Values are in meters except for I_p.

Fig. 8 Illustration of independent control of lower x-point radius (a) and height (b). Values in meters.

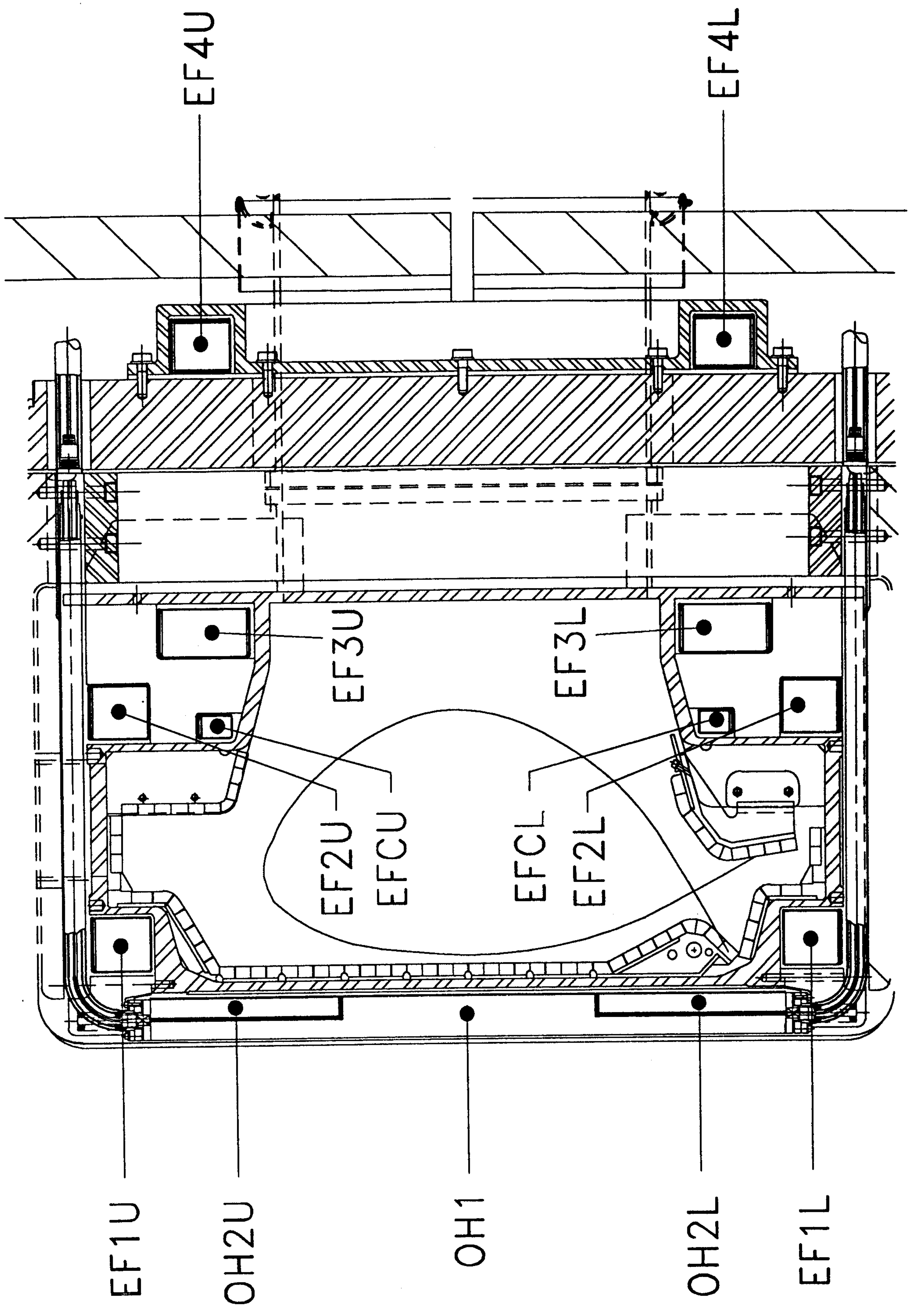
Fig. 9 Using an overdetermined system to control the strike-point. (a) Strike-point ρ_s uncontrolled. (b) ρ_s controlled at the expense of R_{xl} and z_{xl}. The values shown are plotted relative to the x-point reference at $R = 0.56$, $z = -0.39$ m.

Fig. 10 Some equilibria that have been obtained using the present control scheme. (a) a 'Slot' divertor. (b) a 'Flat-plate' divertor. (c) a limited plasma.



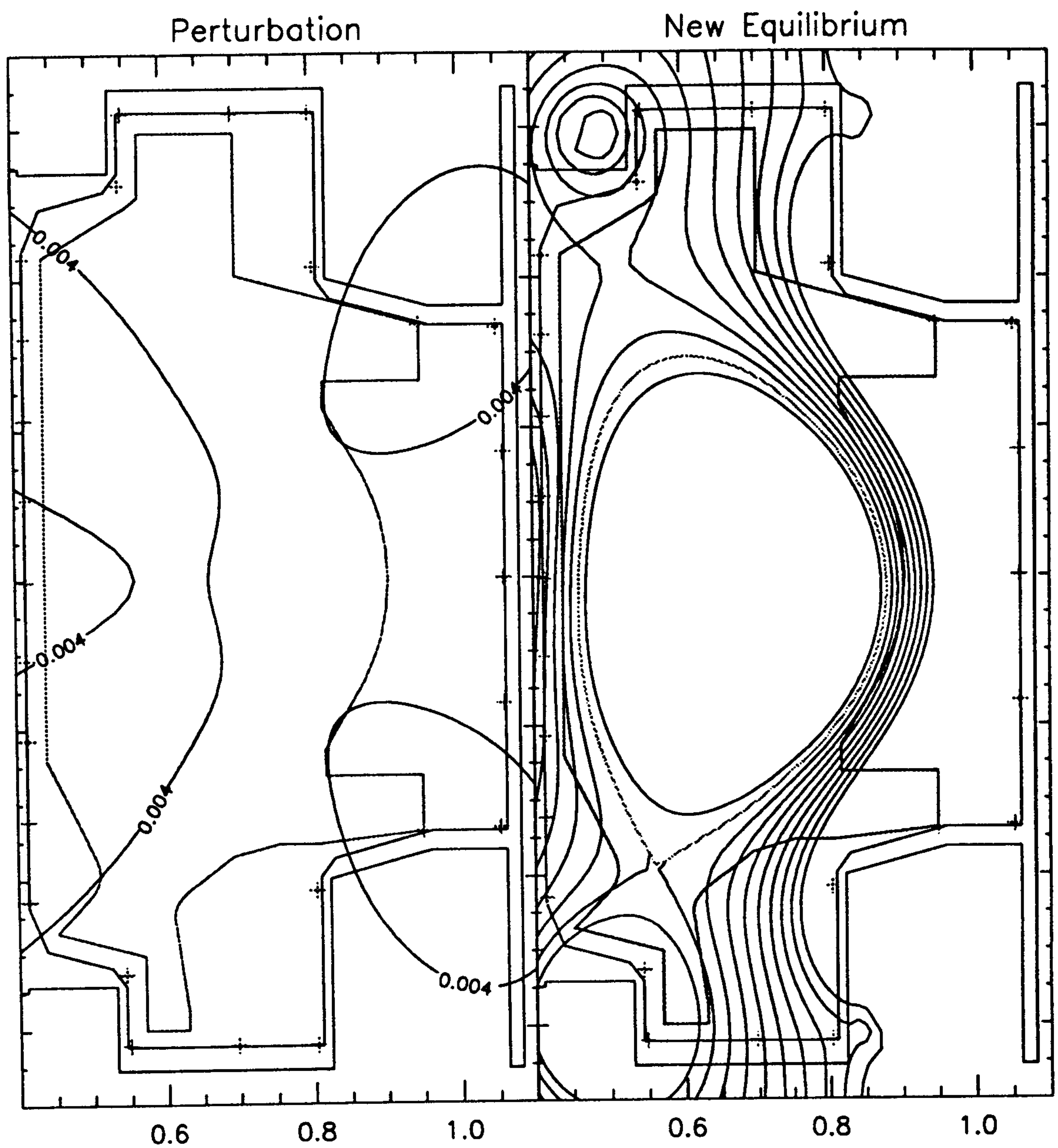
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
0.4
0.6
0.8
ψ_{xu}
ψ_1
ψ_0
ψ_2
ψ_{xl}
ψ_i
ψ_s

Figure 1



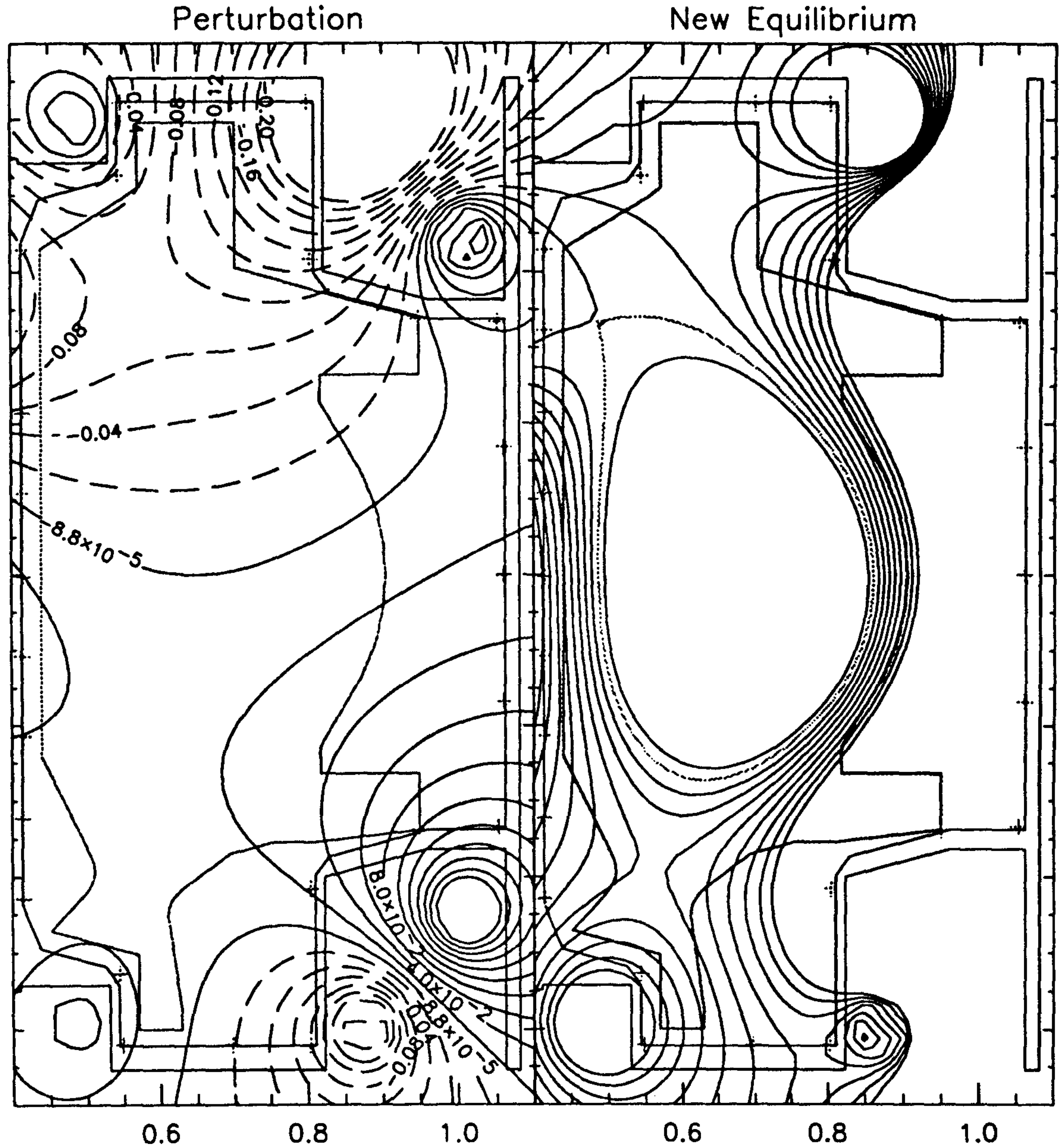
EF4U
EF4L
EF3U
EF3L
EF2U
EFCU
EFCL
EF2L
EF1U
OH2U
OH1
OH2L
EF1L

Figure 2



Perturbation
New Equilibrium
0.004
0.004
0.004
0.004
0.004
0.6
0.8
1.0
0.6
0.8
1.0

Figure 3



Perturbation
New Equilibrium
-0.04
-0.08
-0.12
-0.20
-0.16
-0.08
-0.04
8.8x10^-5
8.0x10^-2
4.0x10^-2
8.8x10^-5
0.04
0.08
0.6
0.8
1.0
0.6
0.8
1.0

Figure 4

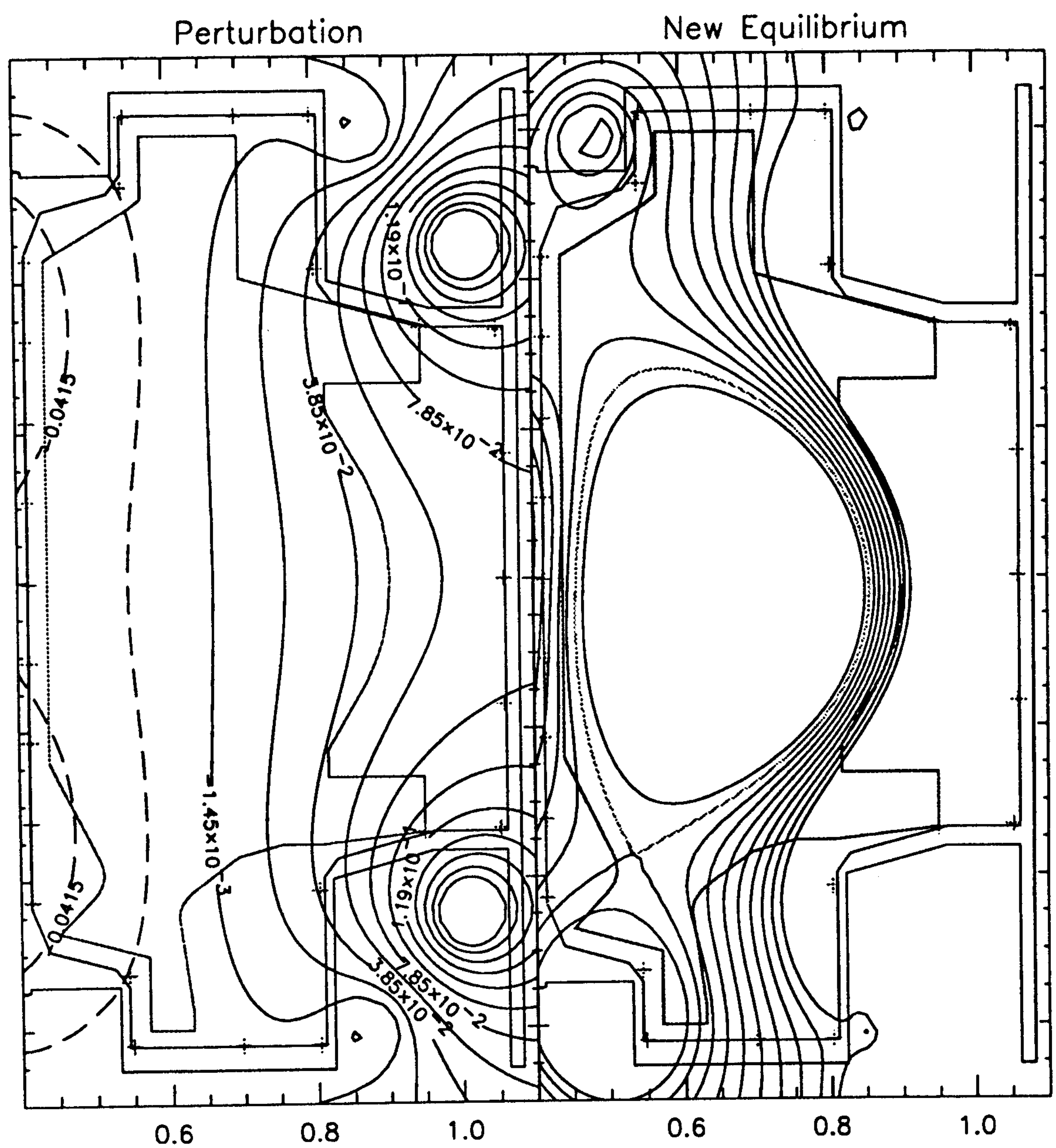

Perturbation
New Equilibrium
0.0415
0.0415
1.45x10^-3
3.85x10^-2
7.85x10^-2
1.19x10^-1
1.19x10^-1
7.85x10^-2
3.85x10^-2
0.6
0.8
1.0
0.6
0.8
1.0

Figure 5(a)

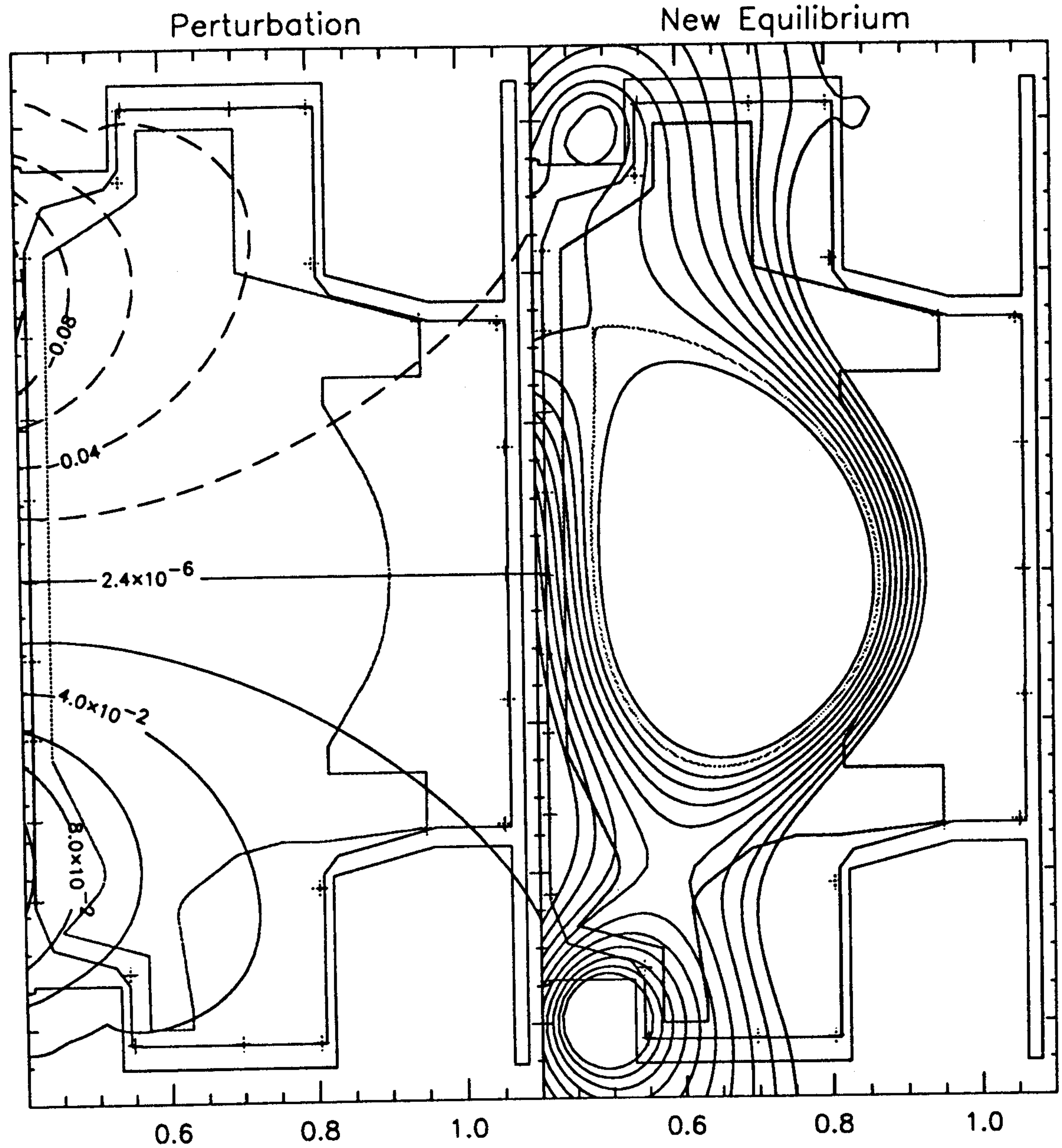

Perturbation
New Equilibrium
-0.08
-0.04
2.4×10^{-6}
4.0×10^{-2}
8.0×10^{-2}
0.6
0.8
1.0
0.6
0.8
1.0

Figure 5(b)

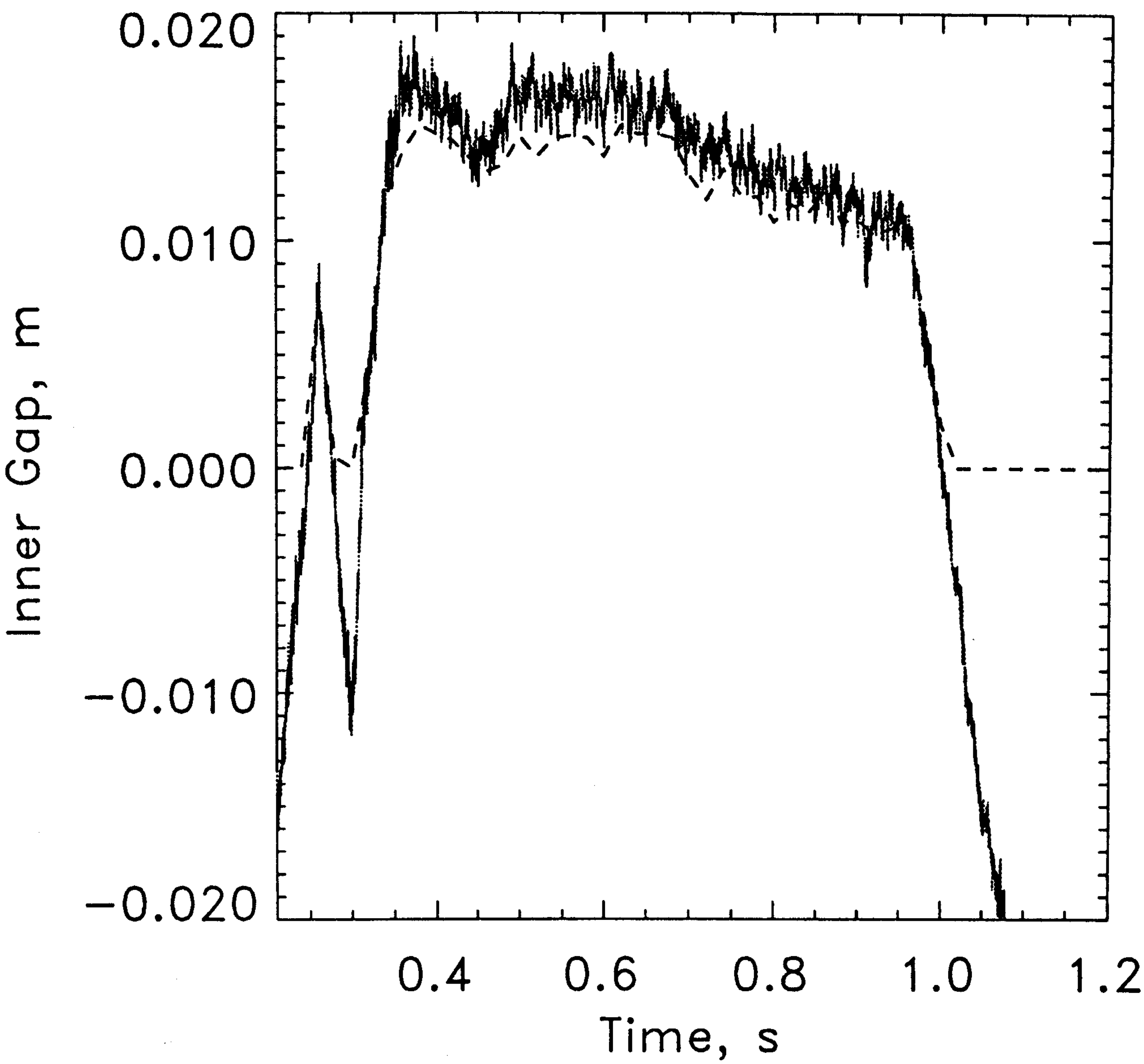


Figure 6

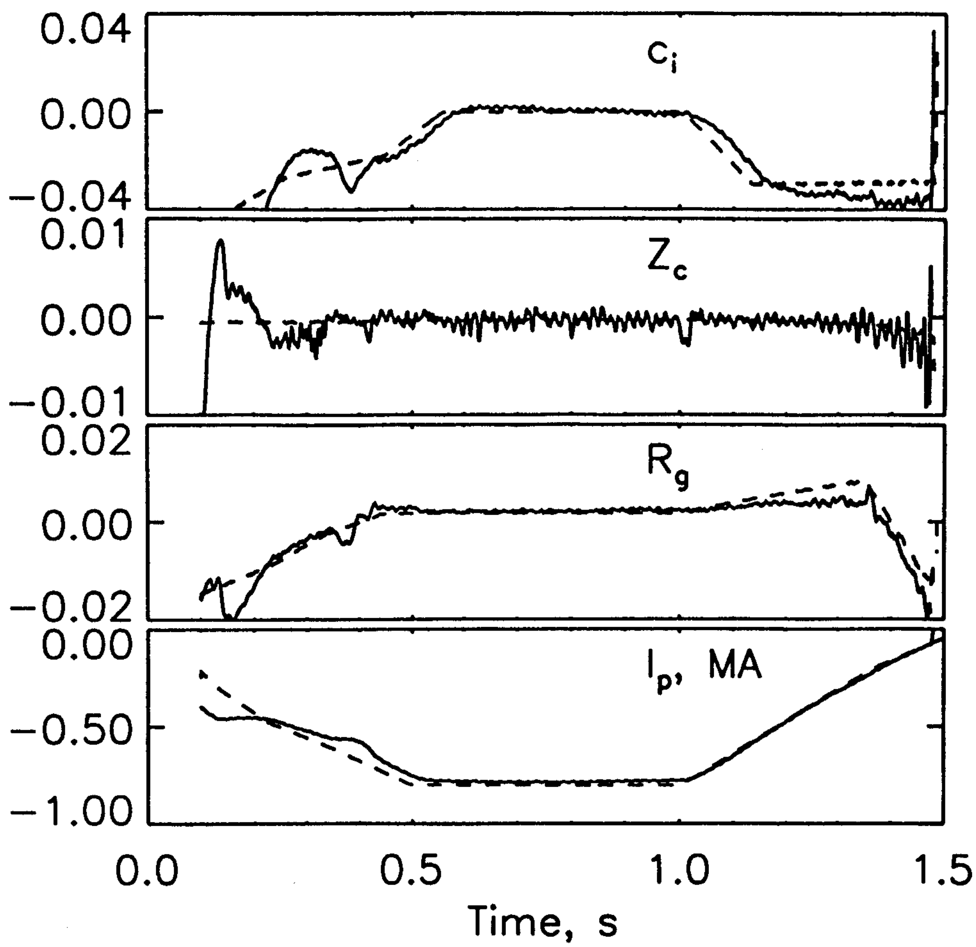
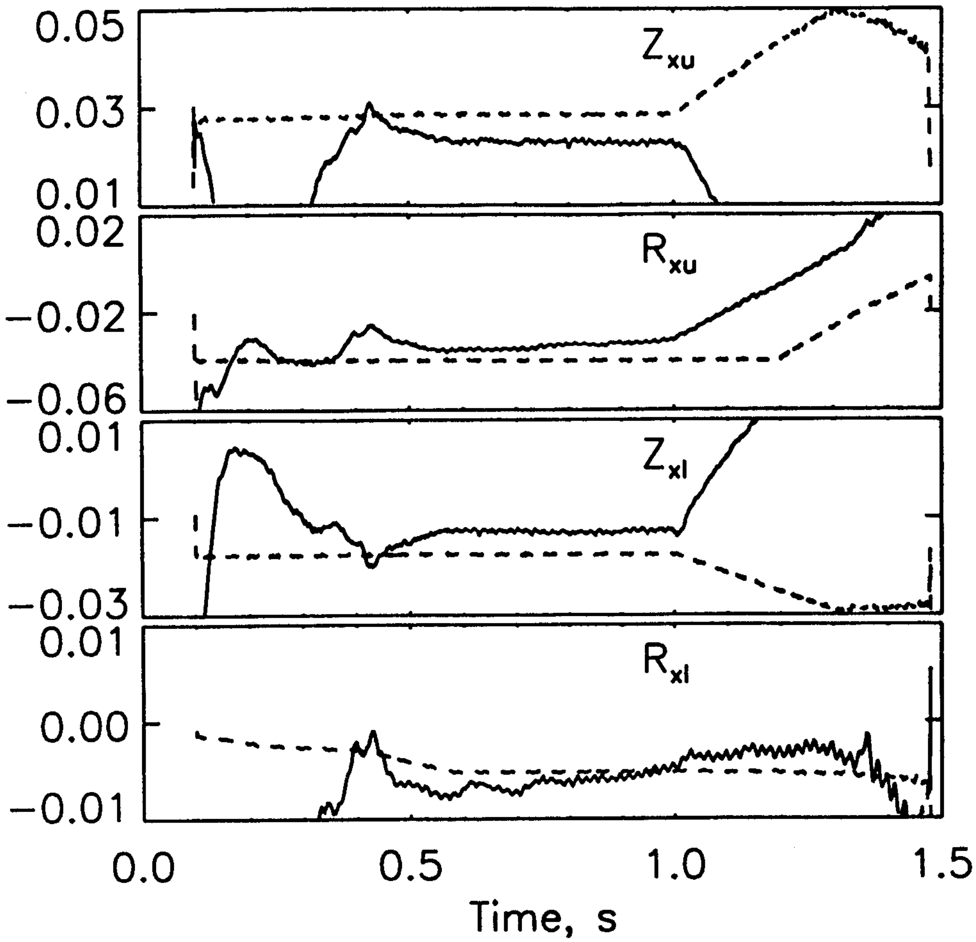
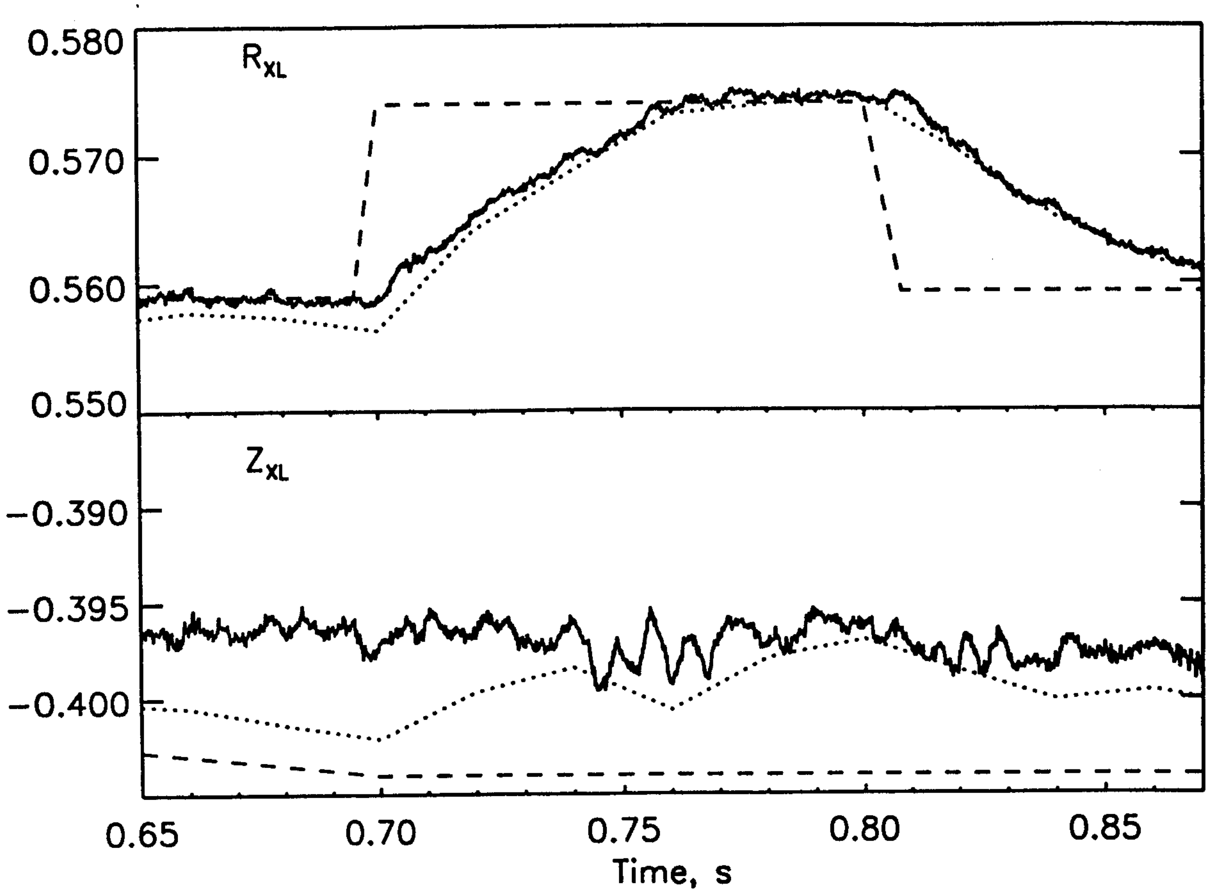

0.04
0.00
−0.04
c_i
0.01
0.00
−0.01
Z_c
0.02
0.00
−0.02
R_g
0.00
−0.50
−1.00
I_p, MA
0.0
0.5
1.0
1.5
Time, s

Figure 7(a)



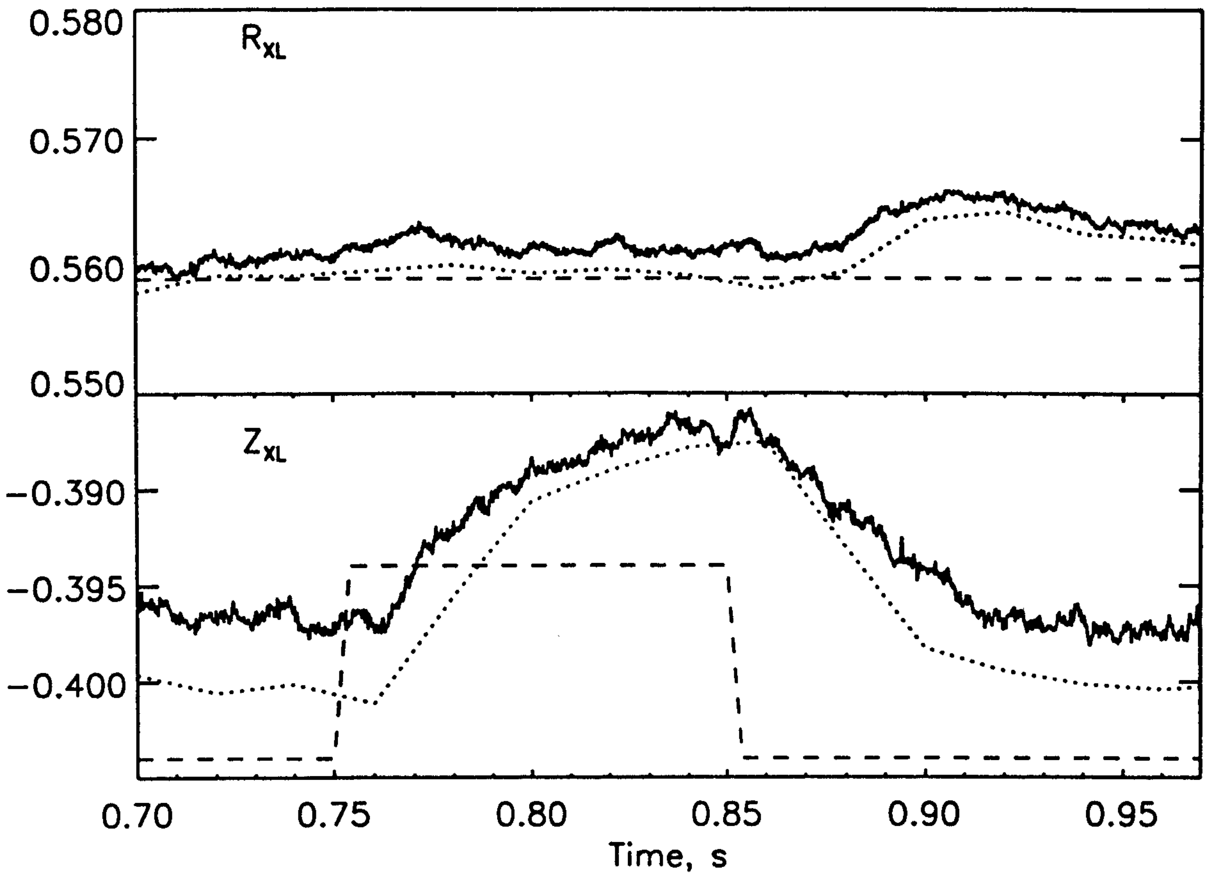
0.05
0.03
0.01
Z_xu
0.02
−0.02
−0.06
R_xu
0.01
−0.01
−0.03
Z_xl
0.01
0.00
−0.01
R_xl
0.0
0.5
1.0
1.5
Time, s

Figure 7(b)



0.580
0.570
0.560
0.550
R_{XL}
Z_{XL}
−0.390
−0.395
−0.400
0.65
0.70
0.75
0.80
0.85
Time, s

Figure 8(a)



0.580
0.570
0.560
0.550
R_{XL}
Z_{XL}
−0.390
−0.395
−0.400
0.70
0.75
0.80
0.85
0.90
0.95
Time, s

Figure 8(b)

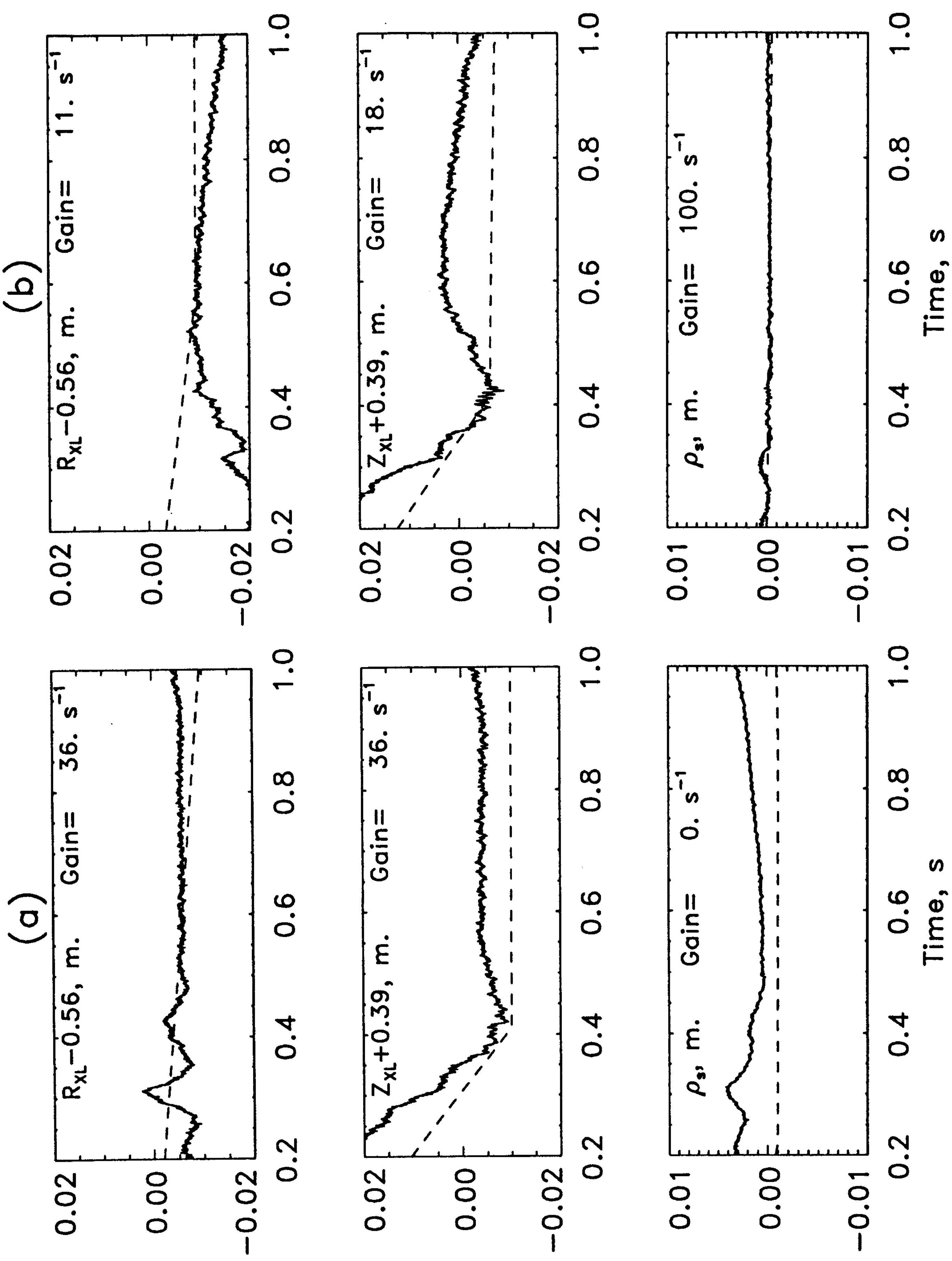
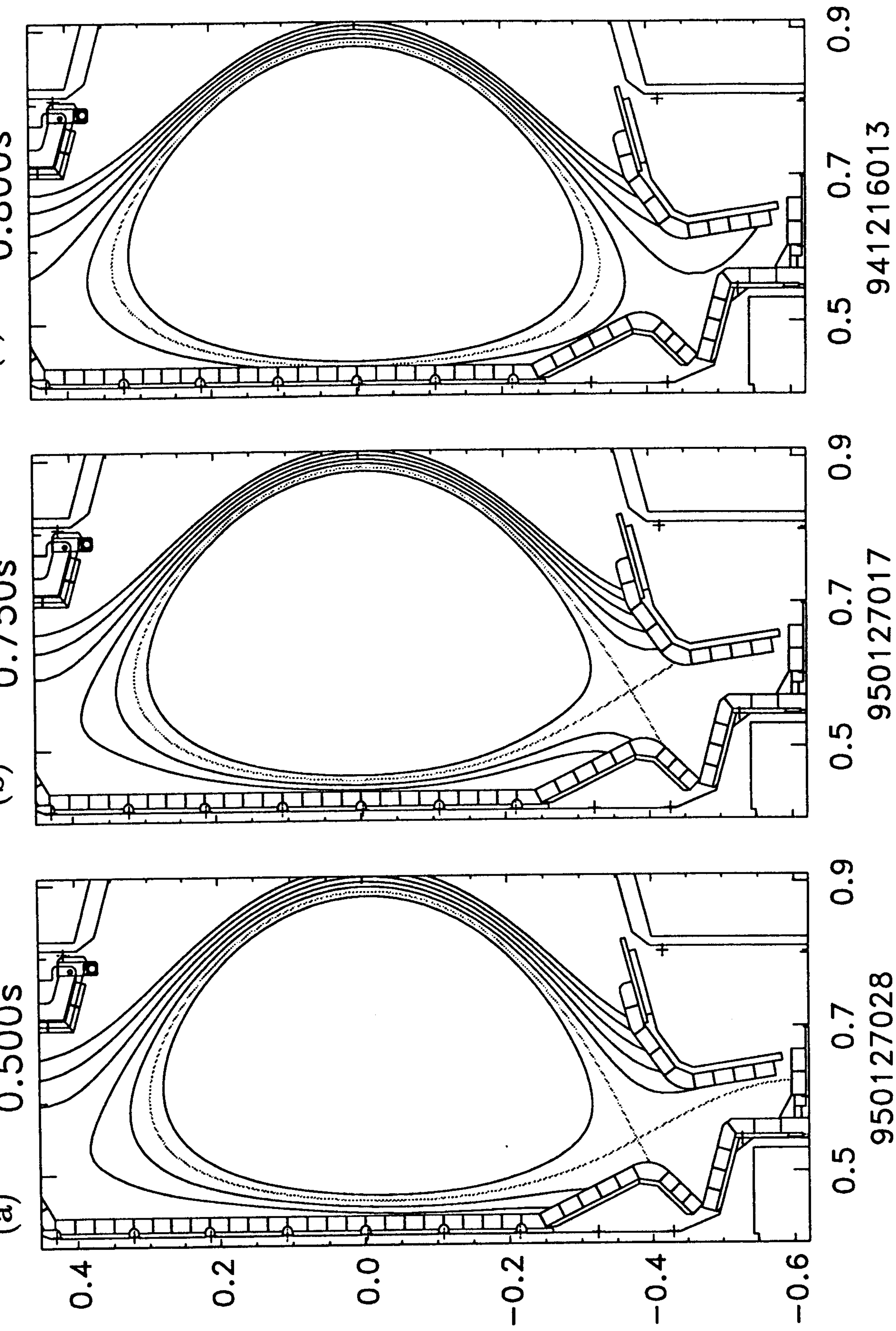

(a)
$R_{XL}-0.56$, m. Gain= 36. s^{-1}
$Z_{XL}+0.39$, m. Gain= 36. s^{-1}
ρ_s, m. Gain= 0. s^{-1}
(b)
$R_{XL}-0.56$, m. Gain= 11. s^{-1}
$Z_{XL}+0.39$, m. Gain= 18. s^{-1}
ρ_s, m. Gain= 100. s^{-1}
0.02
0.00
−0.02
0.01
0.00
−0.01
0.2
0.4
0.6
0.8
1.0
Time, s

Figure 9



(a) 0.500s
(b) 0.750s
(c) 0.800s
0.4
0.2
0.0
−0.2
−0.4
−0.6
0.5
0.7
0.9
950127028
950127017
941216013

Figure 10