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A new instability in laser-plasma interactions is analyzed. The instability arises when a laser pulse propagates through a plasma channel. The ponderomotive force of the head of an initially offset laser pulse distorts the channel so that the back of the laser pulse is pushed further off axis. This wake-like nature of the instability is similar to the beam break-up instabilities of charged particle beams. A dispersion relation is developed and evaluated for different parameter regimes.

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The stable propagation of laser pulses in underdense plasmas is fundamental to the development of laser wake-field accelerators. The laser pulses must be focused to a small spot size in order to generate a large amplitude plasma wave, and, thereby, a high accelerating gradient [1]. The laser will, in free space, be focused only over a diffraction length (Rayleigh range) $Z_r = \pi a^2/\lambda$, where λ is the laser wavelength and a the laser waist at the focus. A homogeneous plasma, which has a dielectric constant $\epsilon = 1 - \omega_p^2/\omega^2$, will only enhance the tendency of the light to diffract. To achieve net acceleration of, say, a few GeV, with present terawatt lasers, will require propagation lengths of order 10-20 Rayleigh ranges, and TeV accelerators hundreds of Rayleigh lengths.

Several schemes have been proposed to overcome diffraction. Relativistic guiding [2–5] relies on the energy dependence of the plasma frequency, $\omega_p^2 = \omega_{p0}^2/\gamma$, where $\gamma = \sqrt{1 + \vec{p}\cdot\vec{p}/m^2c^2}$. The electron momentum $\mid \vec{p} \mid$ will be largest where the laser pulse is most intense, and therefore the plasma frequency will be lower there, and the pulse will generate a nonlinear index of refraction which is larger at the center of the pulse than at the pulse edges. Analysis has shown [2] that in steady-state relativistic guiding can focus the pulse whenever the total power is greater than $P = 16.2(\omega/\omega_p)^2$ GW. For the accelerator application, long-pulse operation is unsatisfactory- pulses much longer than a plasma wavelength lead to small density wakes and, therefore, to low accelerating gradients. Relativistically guided long laser pulses also suffer from Raman sidescatter instabilities [6–8].

For the short pulses (of order a plasma wavelength) required to generate a wakefield, however, relativistic guiding is substantially reduced [9]. This is due to the tendency of the ponderomotive force of the front of the laser pulse to push plasma electrons forward and generate a density increase which balances the relativistic mass increase. The plasma frequency then has no transverse variation and cannot optically guide the laser pulse.

An alternative scheme which has been investigated [10] envisions guiding the laser pulse in a plasma density channel. The channel has a higher density on the outside than on the inside, resulting in an index of refraction of the plasma which decreases from the channel axis (due to the increase in plasma density). A fixed plasma channel is analogous to an optical fiber, and its guiding properties can be similarly analyzed. The plasma channel can be used to guide short pulses, and has been studied using an axisymmetric model [10].

In this work we analyze a new transverse instability which results when the laser pulse is initially not centered on the channel axis. The resulting non-axisymmetric component of the ponderomotive force generates a non-axisymmetric perturbation of the channel. The channel, perturbed by the head of the pulse, will, in turn cause the tail of the laser pulse to deflect further off axis. Thus, the plasma couples different longitudinal slices of the laser pulse. Consistent with causality, a given slice of the pulse is acted on only by those slices ahead of it. The underlying physics is straightforward : the off-centered laser produces a ponderomotive force with a dipole component; this causes the surrounding plasma electrons to try to follow the laser pulse. This results in the "beam-chasing" instability, which makes a small laser-channel misalignment grow exponentially and displace the pulse from its path. For a given initial misalignment, the instability imposes a limit on the laser pulse length and on the distance the pulse can propagate without significant deflection.

The analysis in this paper is limited to the model problem of a quadratic plasma density variation with radius, which has only a discrete spectrum of eigenfunctions, given by Hermite polynomials. An analytical theory of this instability when the density gradient length is much longer than plasma collisionless skin-depth is presented. The present paper deals with the linear evolution of the instability and is, therefore, limited to the transverse displacements smaller than the waist size of the fundamental mode.

The physical model consists of a preformed neutral plasma channel with unperturbed density given by

$$n_0(\vec{x}_\perp) = \bar{n}_0(1 + \frac{r^2}{W^2}), \tag{1}$$

where $r^2 = x^2 + y^2$. Since the duration of the laser pulse is assumed to be short compared to $2\pi/\omega_{pi}$, where $\omega_{pi}^2 = 4\pi e^2 n_o/m_i$, the ions can be considered immobile. Furthermore, the laser frequency is much larger than the plasma frequency so that the evolution of the laser pulse, caused by the electron density wake, occurs on a time-scale much longer than the laser period. Thus, we consider an averaged (over a laser period), slow time-scale, weakly relativistic equation of motion for plasma electrons under the influence of the ponderomotive force of the laser field.

We restrict the analysis, because of the assumption of a short pulse duration, to a fluid model, which is valid before the wave-breaking [11] has occurred. Let $\vec{\eta}(\vec{x},t)$ be a Lagrangian coordinate of a plasma electron. With these assumptions,

$$\delta n = -\vec{\nabla}(n_0(\vec{x})\vec{\eta}(\vec{x},t)) \tag{2}$$

and

$$\nabla \cdot \vec{E} = -4\pi e \nabla \cdot (n_0(\vec{x})\vec{\eta}(\vec{x})). \tag{3}$$

From Eq.(3), it can be shown that

$$\vec{E} = -4\pi e n_0(\vec{x})\vec{\eta}(\vec{x},t) + \nabla \times \vec{P}. \tag{4}$$

Assuming that the radiation is circularly polarized in the (x, y) plane with a dimensionless amplitude $\vec{a} = e\vec{A}/mc^2$ results in an equation for $\vec{\eta}$:

$$(\frac{\partial^2}{\partial t^2} + \omega_{p0}^2(x))\vec{\eta}(\vec{x},t) = -c^2\vec{\nabla}(\frac{|\vec{a}^2|}{2}) + \frac{e}{m}\nabla \times \vec{P}, \tag{5}$$

where c is the speed of light and $\omega_{p0}^2(\vec{x}_\perp) = 4\pi e^2 n_0(\vec{x}_\perp)/m_e$. The channel density is taken to vary over a distance much larger than collisionless plasma skin-depth, so that $K = c/\omega_p W \ll 1$. Then, as shown below, the laser pulse is focused to a spotsize $w = \sqrt{Wc/\omega_p} \ll W$, so that the density does not vary appreciably in the region where η is nonzero. Under these assumptions,

$$\delta n = -n_0 \vec{\nabla} \cdot \vec{\eta}(\vec{x}).$$

Then, from Eq.(5)

$$\left(\frac{\partial^2}{\partial t^2} + \omega_{p0}^2\right)\frac{\delta n}{n_0} = c^2\nabla^2\left(a_0^* a_1 + a_0 a_1^*\right). \tag{6}$$

We assume a total laser field of the form

$$\vec{a} = (a_0(x_\perp) + a_1(x_\perp, z, t))(\hat{e}_x + i\hat{e}_y)\exp\left(i(k_0 z - \omega_0 t)\right) + cc. \tag{7}$$

and use the wave equation to find the appropriate form for a_0 and a_1.

The field can be described by weakly relativistic wave equation, valid for $|\vec{a}|^2 < 1$:

$$\left(-\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial z^2} + \nabla_\perp^2 - \frac{\omega_p^2(x)}{c^2}\left(1 - \frac{|a|^2}{2}\right)\right)\left(a \exp\left(i(k_0 z - \omega_0 t)\right)\right) = 0. \tag{8}$$

The eikonal approximation and $\omega_p^2 \ll \omega^2$ can be used to obtain

$$\left(\frac{\omega^2}{c^2} - k^2 + \nabla_\perp^2 - \frac{\omega_p^2(x)}{c^2}\left(1 - \frac{|a|^2}{2}\right) + 2ik\frac{\partial}{\partial z}\right) a = 0, \tag{9}$$

where we have introduced the new variables

$$s = t - z/v_{g0}, \quad z = z, \tag{10}$$

with v_{g0} the group velocity of the fundamental mode:

$$\frac{v_{g0}}{c} = \frac{k_0 c}{\omega_0}. \tag{11}$$

Inserting Eq.(7) into Eq.(9) and keeping the leading order terms results in

$$(-\bar{\nabla}_\perp^2 + \bar{r}^2) a_0(s, \vec{\bar{x}}_\perp) = w^2 \left(\frac{\omega_0^2}{c^2} - k_0^2 - \frac{\bar{\omega}_p^2}{c^2}\left(1 - \frac{|a_0|^2}{2}\right)\right) a_0(s, \vec{\bar{x}}_\perp), \tag{12}$$

where the dimensionless transverse coordinates are

$$\bar{x} = x/w, \bar{y} = y/w,$$

with the length scale $w = \sqrt{Wc/\omega_p}$. Note that w is just the spotsize of the guided radiation. The previous inequality $K \ll 1$ is equivalent to requiring that the spotsize be much smaller than the distance over which the plasma density changes. Since the primary concern in this paper is guiding by a channel and not relativistic self-focusing, we will assume that the laser power is below the self-focusing threshold [2]. In our notation, this is equivalent to requiring

$$|a_0^2|/K = |a_0^2| w^2 \omega_p^2/c^2 \ll 1.$$

The solutions to Eq.(12), are, after dropping nonlinear terms, the well-known Hermite polynomials. The spectrum is given by

$$\left(\frac{\omega_0^2}{c^2} - k_0^2 - \frac{\bar{\omega}_p^2}{c^2}\right) = \frac{2n + 2m + 1}{w^2}, \tag{13}$$

with eigenfunctions

$$a(\bar{x}, \bar{y}) = \exp(-\bar{r}^2/2) H_n(\bar{x}) H_m(\bar{y}). \tag{14}$$

The fundamental mode corresponds to $(m,n)=(0,0)$.

The perturbation is assumed to be of the form

$$a_1(\vec{\bar{x}}_\perp, z, s) = \bar{a}_1(z,t)H_1(\bar{x})H_0(\bar{y})exp(-\bar{r}^2/2)\,, \tag{15}$$

with

$$H_0(p) = 1, H_1(p) = 2p\,. \tag{16}$$

This perturbation corresponds to a dipole mode in which the laser beam is displaced from the axis of the channel, in the x direction, by a distance proportional to $\bar{a}_1/\mid a_0\mid$. The wave equation for the perturbed field amplitude is

$$[(1+\bar{\nabla}_\perp^2-\bar{r}^2)+2ikw^2\frac{\partial}{\partial z}]a_1 = \frac{\omega_{p0}^2}{c^2}w^2\left(\frac{\delta n}{n_0}-a_0^*a_1-a_0a_1^*\right)a_0. \tag{17}$$

The integral expression for $\delta n/n_0$ is obtained from Eq.(6), where the Laplacian operator is broken into a longitudinal and a transverse part and, because of the quasistatic approximation,

$$\nabla_\parallel^2 \approx \frac{1}{c^2}\frac{\partial^2}{\partial s^2}.$$

Integrating by parts twice, and inserting the expression for $\left(\frac{\delta n}{n_0}\right)$ into Eq.(17), yields

$$\left(1+\bar{\nabla}_\perp^2-\bar{r}^2+2ikw^2\frac{\partial}{\partial z}\right)a_1(s) =$$

$$= a_0(s)\int_{-\infty}^{s}\omega_{p0}ds'\sin\omega_{p0}(s-s')\left(\bar{\nabla}_\perp^2-\frac{1}{K}\right)(a_0^*a_1+a_0a_1^*), \tag{18}$$

where both a_0 and a_1 are implicitly assumed to be dependent on $\vec{\bar{x}}_\perp$ and on s' under the integral. Formal similarity between the laser-hose and electron-hose [12] instability can be seen by treating the expression $(a_0^*a_1+a_0a_1^*)$ as a beam transverse displacement. Note that both the transverse and the longitudinal dynamics of the electrons need to be included. This is an important difference between our instability and the electron-hose instability. For the latter, only the transverse motion of the electrons is important (which is not surprising, since the fields created by a charge moving with the speed of light are purely transverse).

It is convenient to introduce dimensionless time and space coordinates, normalizing them to a plasma period and Rayleigh length, respectively ($s \rightarrow \omega_{p0}s,\ z \rightarrow z/kw^2$). The fields can be written in the form

$$\begin{aligned} a_0(\bar{s},\vec{\bar{x}}_\perp) &= \bar{a}_0(\bar{s})H_0(\bar{x})H_0(\bar{y})e^{-\bar{r}^2/2}, \\ a_1(\bar{s},\bar{z},\vec{\bar{x}}_\perp) &= \bar{a}_1(\bar{s},\bar{z})H_1(\bar{x})H_0(\bar{y})e^{-\bar{r}^2/2}. \end{aligned} \tag{19}$$

Multiplying Eq.(18) and its complex conjugate by $H_1(\bar{x})H_0(\bar{y})e^{-\bar{r}^2/2}$, and integrating over the transverse dimensions results in

$$\left(-2+2i\frac{\partial}{\partial\bar{z}}\right)\bar{a}_1(\bar{s},\bar{z}) - C\int_{-\infty}^{\bar{s}} d\bar{s}' \sin(\bar{s}-\bar{s}')\bar{a}_0(\bar{s})\bar{a}_0^*(\bar{s}')\bar{a}_1(\bar{s}',\bar{z}) =$$

$$= C\int_{-\infty}^{\bar{s}} d\bar{s}' \sin(\bar{s}-\bar{s}')\bar{a}_1^*(\bar{s}',\bar{z})\bar{a}_0(\bar{s})\bar{a}_0(\bar{s}') \tag{20}$$

and its complex conjugate. Here,

$$C = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \quad H_0^2(y)H_0(x)H_1(x)e^{-x^2-y^2}\left(\nabla_\perp^2 - 1/K\right) \tag{21}$$

$$\left(H_0^2(y)H_0(x)H_1(x)e^{-x^2-y^2}\right). \tag{22}$$

Eq.(20) can be used to analyze the growth of the laser-hose instability for a finite duration pulse. Further analitycal progress may be achieved by assuming $\bar{a}_0$ to vary slowly on a time-scale of a plasma oscillation, or that $\bar{a}_0$ has a flat-top profile. Equation (20) can then be solved by Fourier transform in s:

$$\begin{aligned} \left(-2-\frac{C\mid\bar{a}_0^2\mid}{1-\omega^2}+2i\frac{\partial}{\partial\bar{z}}\right)\bar{a}_1 &= \frac{C\bar{a}_0^2}{1-\omega^2}\bar{a}_1^* \\ \left(-2-\frac{C\mid\bar{a}_0^2\mid}{1-\omega^2}-2i\frac{\partial}{\partial\bar{z}}\right)\bar{a}_1^* &= \frac{C\bar{a}_0^{2*}}{1-\omega^2}\bar{a}_1. \end{aligned} \tag{23}$$

The dispersion relation is:

$$k^2 = \left(1-\frac{\mu}{1-\omega^2}\right), \tag{24}$$

where $\mu = \mid \bar{a}_0^2 \mid \left(1+\frac{1}{4K}\right)$.

Asymptotic behavior of the solution for $\bar{z} \geq 1, \bar{s} \geq 1$ is obtained in different regimes, delineated by relations between the length of the pulse s, the interaction length z and the coupling parameter μ. For the analysis presented above to be valid, we have assumed $\mu \ll 1$. Introducing $z_R = k_0 w^2$ and returning to dimensional variables, the different regimes are:

$\mu < 1$

(i) Long pulses:

$$\omega_{p0} s \gg 1/\mu(z/z_R)$$

$$\bar{a}_1 \sim \bar{a}_{10} \exp\left[\frac{\sqrt{3}}{4} \mid a_0^2 \mid^{1/3} \left(1 + \frac{1}{4K}\right)^{1/3} (z/z_R)^{\frac{2}{3}} (\omega_{p0} s)^{\frac{1}{3}}\right] \tag{25}$$

(ii) Intermediate pulses:

$$\mu(z/z_r) \ll \omega_{p0} s \ll 1/\mu(z/z_R)$$

$$\bar{a}_1 \sim \bar{a}_{10} \exp\left[\mid a_0^2 \mid^{1/2} \left(1 + \frac{1}{4K}\right)^{1/2} (z/z_R)^{\frac{1}{2}} (\omega_{p0} s)^{\frac{1}{2}}\right] \tag{26}$$

(iii) Short pulses:

$$\omega_{p0} s \ll \mu(z/z_R)$$

$$\bar{a}_1 \sim \bar{a}_{10} \exp\left[\frac{3\sqrt{3}}{4} \mid a_0^2 \mid^{1/3} \left(1 + \frac{1}{4K}\right)^{1/3} (z/z_R)^{\frac{1}{3}} (\omega_{p0} s)^{\frac{2}{3}}\right] \tag{27}$$

Similar expressions can be derived for $\mu > 1$; these are not given here because this regime violates the approximations assumed in the analysis. Even short pulses, of order of a plasma wavelength, are susceptible to this instability, which manifests itself after a propagation distance of the order of $10 z_R$. For a 10μ radiation focused to a $0.1mm$ spotsize this gives $10 z_R \approx 6cm$.

There is an important distinction between the small-angle Raman scattering [8] instability and the laser-hose instability. The Raman instability is present in an axisymmetric long pulse, and can lead to diffraction and longitudinal pulse break-up. The instability examined here is driven by an initial offset between the axis of symmetry of the injected pulse and an axis of the preformed channel. It is fundamentally non-axisymmetric in nature. Within the context of the quadratic channel model, coupling to higher order symmetric Hermite polynomials is analogous to the Raman scattering discussed in Ref [8].

Some of the conclusions drawn in this paper rely on the quadratic radial variation of the plasma density. Another simple model is an inverted step-function radial dependence, which, due to finite variation in the plasma density, will have only a finite number of discrete eigenmodes. By a careful choice of parameters, the unstable dipole mode can be pushed into the continuum, which may reduce the instability.

In summary, a new instability of the propagation of a laser pulse in a plasma channel has been analyzed for the case of a quadratic density variation. The instability is significant when pulses need to be propagated many Rayleigh lengths, as is envisioned in laser wakefield accelerators. Explicit growth rates have been obtained for a wide range of parameter regimes. The growth rate is seen to be nonzero even for short pulses, of order a plasma wavelength. Future investigations need to examine the influence of the nonlinearities in the unperturbed equation, relativistic electron velocities, coupling to higher order modes, and different density profiles and pulse shapes.

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