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Fast Wave Flow Drive In a Two-Component Plasma with Low Minority Ion Concentration

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Abstract

Fast wave flow drive in a two-component plasma with low minority ion concentration is investigated. It is found that strongly sheared poloidal flow can be generated in a small region near the hybrid ion-ion resonance due to the rapid spatial variation of plasma response. The flow drive depends strongly on local plasma parameters such as the plasma temperature and density and is shown to be very effective in the edge region of a tokamak where the plasma temperature is low. The result is applied to a discussion of shear flow suppression of edge turbulence.

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Sheared poloidal flow has been found to play a crucial rule in suppressing plasma turbulence, thereby improving tokamak confinement[1−3]. Finding practical means of generating and control of sheared poloidal flow thus becomes an important and challenging task in current fusion research. Towards this end, several scenarios of flow drive using externally launched radio frequency (RF) waves have been proposed[4,5]. In these scenarios, the externally launched RF waves are either the mode converted low frequency kinetic Alfven wave or the high frequency ion Bernstein wave. The common features of these scenarios are that the RF waves utilized have very short radial wavelengths comparable to the ion Larmor radius, and the flow drive is determined mainly by the radial propagation of the waves. In this letter, we will present a different scenario of flow drive using the fast magnetosonic wave. For fast waves, the radial wavelength is much longer than the ion Larmor radius. The flow drive is determined by the spatial variation of the plasma response and depends strongly on local plasma parameters such as the plasma temperature and density. In particular, the flow drive is shown to be very effective in the edge region of a tokamak where the plasma temperature is low.

We consider a two-component plasma in the low minority ion concentration regime. In such regime, the hybrid ion-ion resonance[6] falls into the minority ion resonance layer so that the fast wave does not mode convert into the ion Bernstein wave. In the high minority ion concentration regime, the fast wave will mode convert into the ion Bernstein wave[7,8] and the physics of flow drive will be similar to that described in Ref.5. For simplicity, we consider a slab model of a tokamak plasma. The equilibrium magnetic field $\vec{B}_0$ is in z (toroidal) direction. Its magnitude B_0 varies only in x (radial) direction and is inversely proportional to the plasma major radius R. The fast wave is assumed to propagate in x direction only. The allowed mode of propagation is determined by the dispersion relation[9]:

$$N_x^2 = K_{xx} - N_z^2 - \frac{K_{xy}^2}{K_{xx} - N_z^2} \tag{1}$$

and the electric field polarization:

$$\frac{E_x}{E_y} = -\frac{iK_{xy}}{K_{xx} - N_z^2} \tag{2}$$

where $N_x = k_x v_A/\omega$, and $N_z = k_z v_A/\omega$ are the radial and parallel component of the Alfven refractive index, k_x and k_z are the radial and parallel wavenumber respectively, $v_A = \sqrt{B^2/4\pi\rho_m^T}$ is the Alfven speed, ω is the wave frequency, $\rho_m^T = \rho_M + \rho_m$ is the total plasma mass density, ρ_M and ρ_m are the majority and minority ion mass density respectively, K_{xx} and K_{xy} are components of a normalized dielectric tensor. For a two-component plasma with minority ion resonance, K_{xx} and K_{xy} are given by[10]:

$$K_{xx} = -\frac{(\Omega_\mathrm{i}^\mathrm{M})^2}{\omega^2 - (\Omega_\mathrm{i}^\mathrm{M})^2} - \frac{1}{2}\frac{\rho_\mathrm{m}}{\rho_\mathrm{M}}\frac{\Omega_\mathrm{i}^\mathrm{m}}{\omega}\left[\frac{\Omega_\mathrm{i}^\mathrm{m}}{\omega + \Omega_\mathrm{i}^\mathrm{m}} - \frac{\Omega_\mathrm{i}^\mathrm{m}}{\omega - \Omega_\mathrm{i}^\mathrm{m}}\alpha_{-1}^\mathrm{m} \mathrm{Z}(\alpha_{-1}^\mathrm{m})\right] \tag{3a}$$

$$K_{xy} = -\frac{\omega}{\Omega_\mathrm{i}^\mathrm{M}}\frac{(\Omega_\mathrm{i}^\mathrm{M})^2}{\omega^2 - (\Omega_\mathrm{i}^\mathrm{M})^2} + \frac{1}{2}\frac{\rho_\mathrm{m}}{\rho_\mathrm{M}}\frac{\Omega_\mathrm{i}^\mathrm{m}}{\omega}\left[\frac{\Omega_\mathrm{i}^\mathrm{m}}{\omega + \Omega_\mathrm{i}^\mathrm{m}} + \frac{\Omega_\mathrm{i}^\mathrm{m}}{\omega - \Omega_\mathrm{i}^\mathrm{m}}\alpha_{-1}^\mathrm{m} \mathrm{Z}(\alpha_{-1}^\mathrm{m})\right] \tag{3b}$$

where Ω_i is the ion cyclotron frequency, the superscript 'M' and 'm' refer to majority and minority ions respectively, $\alpha_{-1}^m = \frac{\omega - \Omega_i^m}{k_z v_i^m}$, v_i^m is the minority ion thermal speed, and $Z(\alpha_{-1}^m)$ is the plasma dispersion function.

The physics of RF flow drive is based on the following plasma poloidal momentum balance equation:

$$\rho_m^T\{\frac{\partial}{\partial t}\langle V_y\rangle + \langle\tilde{\vec{V}}\cdot\vec{\nabla}\tilde{V}_y\rangle\} = \langle\tilde{\rho}_q^T\tilde{E}_y\rangle + \frac{1}{c}\langle(\tilde{\vec{J}}\times\tilde{\vec{B}})_y\rangle - \mu_{neo}\rho_m^T\langle V_y\rangle \tag{4}$$

where $\langle V_y\rangle$ is the mean poloidal flow velocity, $\tilde{\vec{V}}$ is the fluctuating velocity, $\tilde{\vec{E}}$ and $\tilde{\vec{B}}$ are the fast wave electric and magnetic field, $\tilde{\rho}_q^T$ and $\tilde{\vec{J}}$ are the total plasma charge and current density, μ_{neo} is the neoclassical poloidal damping rate[11], and the average $\langle\cdots\rangle$ is taken over one fast wave oscillation period.

In Eq.(4), the first two terms on the right hand side are the electric and J×B force directly associated with the fast wave electromagnetic fields, while the second term on the left hand side is the nonlinear inertial force called Reynolds stress. From $\vec{\nabla} \cdot \tilde{\vec{E}} = 4\pi \tilde{\rho}_q^T$, and noting that $E_z \ll E_x$ and $k_x \sim k_z$, the electric force can be expressed as:

$$\langle \tilde{\rho}_q^T \tilde{E}_y \rangle = \frac{1}{4\pi} \langle \tilde{E}_y \frac{\partial}{\partial x} \tilde{E}_x \rangle \tag{5}$$

To calculate the J×B force, we express $\tilde{\vec{J}}$ in terms of $\tilde{\rho}_q^T$ and $\tilde{\vec{B}}$ in terms of $\tilde{\vec{E}}$ using the charge conservation and the Faraday's law respectively. Assuming $\{\tilde{\vec{\mathrm{E}}}, \tilde{\vec{\mathrm{B}}}\} \propto \exp\{\mathrm{i} \int^{\mathrm{x}} \mathrm{k_x dx'} + \mathrm{i k_z z} - \mathrm{i}\omega \mathrm{t}\}$, and noting that $\tilde{J}_z \ll \tilde{J}_x$ and $\tilde{B}_x \ll \tilde{B}_z$, we have: $\tilde{J}_x \simeq \frac{\omega}{k_x} \tilde{\rho}_q^T$, and $\tilde{B}_z = (c/\omega) k_x \tilde{E}_y$. The J×B force can thus be written as:

$$\frac{1}{c} \langle (\tilde{\vec{J}} \times \tilde{\vec{B}})_y \rangle \simeq -\langle \tilde{\rho}_q^T \tilde{E}_y \rangle \tag{6}$$

Comparing Eqs.(5) and (6) shows that the J×B force has the same magnitude but opposite sign as the electric force. The total electromagnetic force which is the sum of the two thus tends to vanish and have little effect on the temporal evolution of the mean poloidal flow.

To calculate the Reynolds stress, we express the velocity response in terms of a mobility tensor $\overleftrightarrow{M}$: $\vec{V} = (c/B) \overleftrightarrow{M} \cdot \vec{E}$. For a two-component plasma with one minority component, $\overleftrightarrow{M} = \overleftrightarrow{M}_M + (\rho_m/\rho_M) \overleftrightarrow{M}_m$, where $\overleftrightarrow{M}_M$ and $\overleftrightarrow{M}_m$ are the mobility tensor for the majority and minority ions respectively. Since the ion's motion are primarily in the plane perpendicular to the equilibrium magnetic field, the mobility tensor has the form:

$$\overleftrightarrow{M} = \begin{pmatrix} iM_{xx} & M_{xy} \\ -M_{xy} & iM_{xx} \end{pmatrix} \tag{7}$$

where M_{xx} and M_{xy} are given by[10]:

$$M_{xx} = \frac{\omega}{\Omega_i^M}\frac{(\Omega_i^M)^2}{\omega^2 - (\Omega_i^M)^2} + \frac{1}{2}\frac{\rho_m}{\rho_M}\{\frac{\Omega_i^m}{\omega + \Omega_i^m} - \frac{\Omega_i^m}{\omega - \Omega_i^m}\alpha_{-1}^m Z(\alpha_{-1}^m)\} \qquad (8a)$$

$$M_{xy} = -\frac{(\Omega_i^M)^2}{\omega^2 - (\Omega_i^M)^2} + \frac{1}{2}\frac{\rho_m}{\rho_M}\{\frac{\Omega_i^m}{\omega + \Omega_i^m} + \frac{\Omega_i^m}{\omega - \Omega_i^m}\alpha_{-1}^m Z(\alpha_{-1}^m)\} \qquad (8b)$$

Since $|M_{xx}| \sim 1$, and $|M_{xy}| \sim 1$, the magnitude of the Reynolds stress can be estimated:

$$\langle\tilde{\vec{V}} \cdot \vec{\nabla}\tilde{V}_y\rangle = \langle\tilde{V}_x\frac{\partial}{\partial x}\tilde{V}_y\rangle \geq (\frac{c}{B})^2\langle\tilde{E}_y\frac{\partial}{\partial x}\tilde{E}_x\rangle$$

where the $'>'$ sign applies when the spatial variation of $\overleftrightarrow{M}$ is faster than that of the $\tilde{\vec{E}}$ field. The ratio between the Reynolds stress and the electric force is: $\rho_m^T\langle\tilde{\vec{V}} \cdot \vec{\nabla}\tilde{V}_y\rangle/\langle\tilde{\rho}_q^T\tilde{E}_y\rangle \geq c^2/v_A^2 \gg 1$. Therefore, the Reynolds stress is larger than either the electric or the J × B force (much larger than the two combined) and is the dominant force for flow drive! In the following, we will concentrate only on the Reynolds stress.

Using Eq.(2), we can express $\tilde{\vec{V}}$ in terms of $\tilde{E}_y$ only: $\tilde{V}_x = (c/B)S_x\tilde{E}_y$, and $\tilde{V}_y = i(c/B)S_y\tilde{E}_y$, where S_x and S_y are the ion velocity responses:

$$S_x = \frac{M_{xy}(K_{xx} - N_z^2) + M_{xx}K_{xy}}{K_{xx} - N_z^2} \qquad (9a)$$

$$S_y = \frac{M_{xx}(K_{xx} - N_z^2) + M_{xy}K_{xy}}{K_{xx} - N_z^2} \qquad (9b)$$

Carrying out the temporal average, we have:

$$\langle\tilde{\vec{V}} \cdot \vec{\nabla}\tilde{V}_y\rangle = \frac{1}{2}Re\{\tilde{V}_x^*\frac{\partial}{\partial x}\tilde{V}_y\} \qquad (10)$$

The spatial derivative of $\tilde{V}_y$ in the above equation has two contributions:

$$\frac{\partial}{\partial x}\tilde{V}_y = i\frac{c}{B}[(\frac{\partial}{\partial x}S_y) + ik_xS_y]\tilde{E}_y \qquad (11)$$

where the first term is due to the radial variation of the ion response S_y, and the second term is due to the radial propagation of the fast wave. We note that S_y changes rapidly near $K_{xx} \sim N_z^2$, the hybrid ion-ion resonance. In the low minority ion concentration regime, this hybrid ion-ion resonance falls into the minority ion resonance layer, and we can approximate $\partial S_y/\partial x \sim S_y/\delta x$ where $\delta x = R(k_\parallel v_i^m/\omega)$ is the minority ion resonance layer width. If we take $k_\parallel \simeq 1/R$, the ratio between the two terms in Eq.(11) is: $(k_x S_y)/\frac{\partial}{\partial x} S_y \simeq k_x \rho_i \ll 1$. Therefore, the fast wave flow drive is determined by the spatial variation of the plasma response rather than the radial propagation of the waves. As a result, the flow drive depends strongly on local plasma parameters such as temperature and density. In particular, the flow drive is expected to be more effective at lower plasma temperatures because at lower plasma temperatures, the minority ion resonance layer is narrower and the plasma response varies faster.

To proceed, we substitute Eq.(11) into Eq.(10) and have:

$$\langle \tilde{\vec{V}} \cdot \vec{\nabla} \tilde{V}_y \rangle = -\frac{1}{2}(\frac{c}{B})^2 \{Im(S_x^* \frac{\partial}{\partial x} S_y) + [k_x^R Re(S_x^* S_y) - k_x^I Im(S_x^* S_y)]\} |\tilde{E}_y|^2(x) \tag{12}$$

where k_x^R and k_x^I are the real and imaginary part of k_x respectively, and the spatial variation of the fast wave E field amplitude is taken into account through:

$$|\tilde{E}_y|^2(x) = |\tilde{E}_y|^2(x_1) \exp\{-2 \int_{x_1}^{x} k_x^I(x') dx'\} \tag{13}$$

where x_1 is some initial spatial position. To simplify the algebra, we define:

$$A = M_{xy}(K_{xx} - N_z^2) + M_{xx} K_{xy} \tag{14a}$$

$$B = K_{xx} - N_z^2 \tag{14b}$$

$$C = M_{xx}(K_{xx} - N_z^2) + M_{xy} K_{xy} \tag{14c}$$

so that $S_x = A/B$, and $S_y = C/B$. The last two terms in Eq.(12) are given by:

$$Re(S_x^* S_y) = \frac{1}{|B|^2}\{Re(A)Re(C) + Im(A)Im(C)\} \tag{15a}$$

$$Im(S_x^* S_y) = \frac{1}{|B|^2}\{Re(A)Im(C) - Im(A)Re(C)\} \tag{15b}$$

After lengthy but straight forward calculation, the first term in Eq.(12) is:

$$Im(S_x^* \frac{\partial}{\partial x} S_y) = \frac{1}{R} \frac{\rho_m}{\rho_M} (\frac{\Omega_i^m}{k_z v_i^m})^2 \frac{(B - K_{xy})(B - \frac{\Omega_i^m}{\omega} M_{xy})}{|B|^4}$$
$$\times \{[Re(A)Re(B) - Im(A)Im(B)][\alpha_{-1}^m ImZ(\alpha_{-1}^m)]$$
$$-[Re(A)Im(B) + Im(A)Re(B)][1 + \alpha_{-1}^m ReZ(\alpha_{-1}^m)]\} \quad (16)$$

with

$$B - K_{xy} = \frac{\Omega_i^M}{\omega + \Omega_i^M} - \frac{\rho_m}{\rho_M} \frac{\Omega_i^m}{\omega} \frac{\Omega_i^m}{\omega + \Omega_i^m} - N_z^2 \quad (17a)$$

$$B - \frac{\Omega_i^m}{\omega} M_{xy} = (\frac{\Omega_i^m}{\omega} - 1) \frac{(\Omega_i^M)^2}{\omega^2 - (\Omega_i^M)^2} - \frac{\rho_m}{\rho_M} \frac{\Omega_i^m}{\omega} \frac{\Omega_i^m}{\omega + \Omega_i^m} - N_z^2 \quad (17b)$$

The Reynolds stress is completely determined by Eqs.(12)-(17). From Eq.(4), the steady state mean poloidal flow is obtained by setting $\partial\langle V_y \rangle / \partial t = 0$:

$$\langle V_y \rangle = -\frac{\langle \tilde{\vec{V}} \cdot \vec{\nabla} \tilde{V}_y \rangle}{\mu_{neo}} \quad (18)$$

To illustrate the results, we solve Eqs.(12)-(18) numerically. To be specific, we assume a two-component plasma with tritium majority and deuterium minority. The following parameters relevant to edge plasma are used: R=150cm, a=50cm, B_0=2T, T_i=100eV, $n_T = 10^{13}\text{cm}^{-3}$, $n_D/n_T = 0.005$, ω=96.13MHz, and the parallel wavelength $\lambda_z = 130$cm. The minority ion resonance position is chosen at $R_0 = 190cm$. In figure 1, we show the spatial profile of the flow velocity $\langle V_y \rangle(x)$. More preciously, what we show in Fig.1 is the spatially varying part of $\langle V_y \rangle(x)$: $F(x, x_1)$, which is defined through $\langle V_y \rangle(x) = (1/2\mu_{neo})(c/B)^2 |\tilde{E}_y|^2(x_1) F(x, x_1)$. The rapid spatial variation of $\langle V_y \rangle(x)$ occurs in a small region near the hybrid ion-ion resonance, which, in this case, is a little inside the minority ion resonance position R_0. The strong spatial variation of the flow is due to the strong spatial variation of the plasma response S_y. The width of the variation (shear layer width) is approximately 2.5cm. Outside this region, the small and constant (shearless) flow is due to the radial propagation of the wave. In Figure 2, we show the spatial profile of the flow shear $d\langle V_y\rangle/dx$. The localization of the flow shear near the hybrid ion-ion resonance is evident. This figure also shows that the mean poloidal flow is not only strongly sheared, but also strongly curved (large $d^2\langle V_y\rangle/dx^2$).

As an application, we consider turbulence suppression by fast wave induced shear flow. The physics of shear flow suppression of turbulence has been discussed extensively in Ref.3. The criterion is that the shearing rate of the poloidal flow exceeds the turbulence decorrelation rate $\Delta\omega_{\vec{k}}$: $\bar{k}_\theta(\Delta x_{\vec{k}} d\langle V_y\rangle/\partial x) > \Delta\omega_{\vec{k}}$, where $\bar{k}_\theta$ is the spectral averaged poloidal wavenumber and $\Delta x_{\vec{k}}$ is the radial correlation length of the turbulence. For the particular example given above, the flow shear can be inferred from Fig.2: $d\langle V_y\rangle/dx \simeq 6.0 \times 10^6 |\tilde{E}_y|^2(x_1)s^{-1}$. Assuming drift wave characteristic of edge turbulence, namely, $\bar{k}_\theta\rho_s \simeq 0.2$, $\bar{k}_\theta\Delta x_{\vec{k}} \simeq 1$, $\Delta\omega_{\vec{k}} \simeq \omega_e^* = 2.3 \times 10^4/s$, and the plateau collisional regime for the ions[11] $\mu_{neo} = \frac{\sqrt{\pi}}{2}\omega_{ti} = 8.3 \times 10^3/s$, where ρ_s is the ion Larmor radius at electron temperature, $\omega_e^* = (k_\theta\rho_s)(c_s/L_n)$ is the electron diamagnetic drift frequency, c_s is the ion sound speed, $L_n = a$ is the plasma density scale length, and $\omega_{ti} = v_i/R$ is the ion transient frequency, we obtain $\tilde{E}_y \geq 2.0kV/m$ for turbulence suppression. At this magnitude of the electric field, we can estimate the total heating power absorbed by the plasma (RF power threshold). The total heating power P_t absorbed by the plasma (assuming minority ion resonance only) is given by[9]: $P_{abs} = \pi^2 aR^2(\rho_m\Omega_i^m)(c/B)^2|\tilde{E}_+|^2$, where $\tilde{E}_+ = \tilde{E}_x + i\tilde{E}_y$ is the left-hand component of fast wave electric field. Using Eq.(2) and the parameters given above, we obtain $P_t \geq 7.5kW$, a number substantially smaller than those currently used in plasma heating experiments. The negligible heating effect at lower plasma temperatures is an important feature of the fast wave flow drive. Such feature can be utilized to drive a strongly sheared poloidal flow in the edge region of a tokamak, where less plasma heating is desirable because it leads to less wall sputtering and thus less impurity influx. The physical reason for the decoupling between the flow drive and power absorption at lower plasma temperatures is that the fast wave flow drive is determined by the spatial variation of the plasma response across the minority ion resonance layer. At lower plasma temperatures, the minority ion resonance layer is narrower, the plasma response varies faster, the flow drive is stronger, and thus less RF power is needed.

In summary, we have shown that strongly sheared poloidal flow can be generated in a small region near the minority ion resonance using externally launched fast waves. The flow drive is due to the spatial variation of the plasma response and depends strongly on local plasma parameters such as temperature and density. In particular, the flow drive is stronger at lower plasma temperatures (as in the edge region of a tokamak).

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References

1. R. J. Groebner, K. H. Burrell, and R. P. Seraydarian, Phys. Rev. lett. **64**, 3105 (1990).
2. Ch. P. Ritz, et al. Phys. Rev. Lett. **65**, 2543 (1990).
3. H. Biglari, P. H. Diamond, and P. W. Terry, Phys. Fluids B **2**, 1 (1990).
4. G. G. Craddock, and P. H. Diamond, Phys. Rev. Lett. **67**, 1535 (1991).
5. G. G. Craddock, P. H. Diamond, H. Biglari, M. Ono, and Y. L. Ho, Bull. Am. Phys. Soc. **36**, 2411 (1991).
6. S. J. Buchsbaum, Phys. Fluids **3**, 418 (1960).
7. F. W. Perkins, Nucl. Fusion **17**, 1197 (1977).
8. V. P. Bhatnagar, J. Jacquinot, D. F. H. Start, and B. J. D. Tubbing, Nucl. Fusion **33**, 83 (1993).
9. T. H. Stix, Nucl. Fusion **15**, (1975).
10. T. H. Stix, *Theory of Plasma Waves* (McGraw-Hill, New York, 1962).
11. S. P. Hirshman, Nucl. Fusion **18**, 917 (1978).

Figure Captions

1. Figure 1. Spatial profile of the flow velocity $\langle V_y \rangle(x)$. The minority ion resonance location is chosen at R_0=190cm. Note the rapid spatial variation of the flow velocity (flow shear) near the hybrid ion-ion resonance, which is a little inside R_0.
2. Figure 2. Spatial profile of the flow shear $d\langle V_y \rangle(x)$, localized near the hybrid ion-ion resonance where the rapid spatial variation of plasma response occurs.

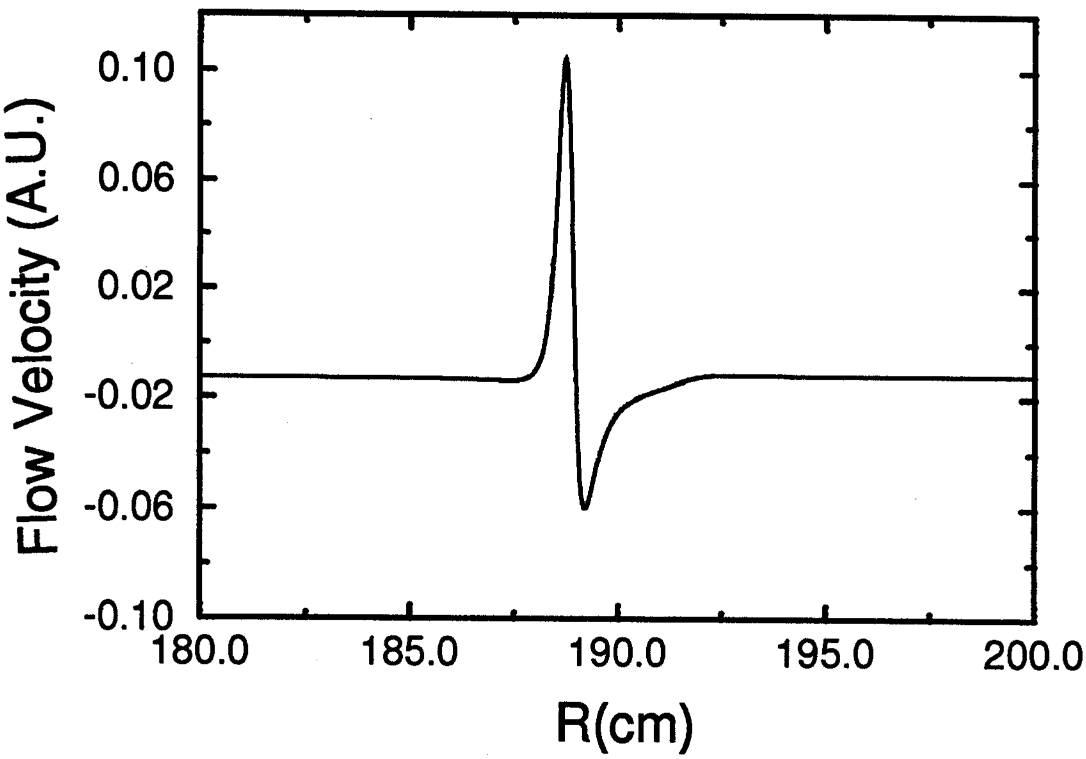

Figure 1

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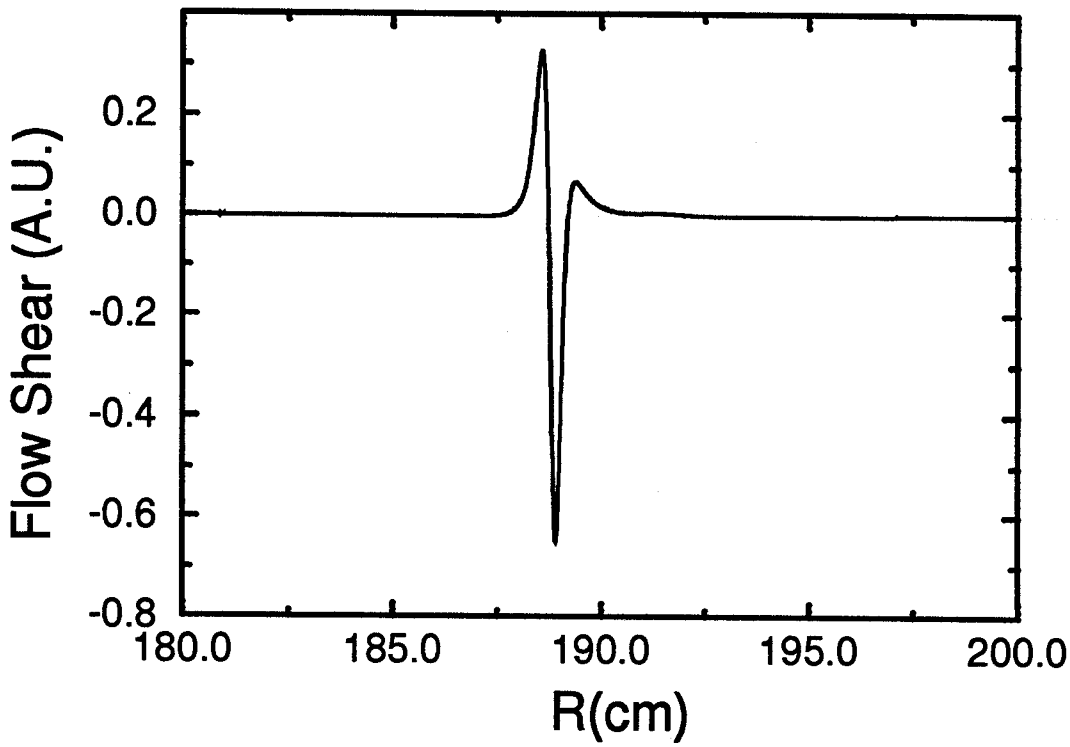

Figure 2

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