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Performance Evaluation of the TFTR  
Gyrotron CTS Diagnostic for Alpha Particles\*†

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### Abstract

A large-angle, 60 GHz collective Thomson scattering (CTS) diagnostic system for localized measurements of DT alpha-particle velocity distribution and fraction is being implemented on TFTR. Calculations of expected CTS spectra, signal to noise ratio per receiver channel, and estimated error in determining the temperature and fraction of alpha-particles are being carried out. The experimental spectra are simulated by adding noise to the theoretical calculation by a Monte Carlo technique. Error analysis is then performed by using a relative intensity calibrated non-linear curve fitting code to calculate the desired plasma parameters ( $T_i, T_\alpha, n_\alpha/n_i$ ). Simulation results indicate that expected background emission of 20 eV during Supershot in TFTR poses no problem to the experiment. Also short integration times ( $< 10$  ms) can be used to resolve the energetic ion features, thus offering a possibility for the study of temporal evolution of energetic ion velocity distribution during a single plasma shot.

## 1 Introduction

A large angle collective Thomson scattering (CTS) diagnostic experiment is currently being installed on TFTR. The primary goal of the diagnostic experiment is to locally measure the velocity distribution and density fraction of fusion alpha particles. A 60 GHz, 500 ms, 200 kW gyrotron will be used as the diagnostic beam source. The heterodyne receiver system will initially have 38 channels of 20 and 80 MHz bandwidth to resolve both the bulk and energetic ion spectra. The gyrotron beam is to be launched from the top port of the TFTR with a steerable wave guide and optics system. The scattered signal from plasma is then collected at the bottom. For more details about the experimental system, see Park et. al.[1]

Scattered power,  $P_s$ , as detected by the receiver system can be expressed as

$$P_s = P_I r_e^2 n_e L d\Omega \Gamma S(\mathbf{k}, \omega) \quad (1)$$

where  $P_I$  is the power of the incident gyrotron beam,  $r_e$  is the classical electron radius,  $n_e$  is the electron density at the scattering volume,  $L d\Omega$  defines the scattering volume, and  $\Gamma$  is the geometric factor that is dependent on plasma dielectric effects.  $S(\mathbf{k}, \omega)$  is the spectral density function that contains information about the particle species in the plasma[2]. By analyzing the  $S(\mathbf{k}, \omega)$  spectrum, velocity distributions and densities of ion species can be obtained.

The goal of this study is to evaluate the effects of background electron cyclotron emission (ECE) on the diagnostic performance through experiment simulations. Performance analysis of measuring both energetic ion temperature and density in tokamak plasma is presented. Previously, similar studies have addressed the performance issue of bulk ion temperature measurements only[3, 4, 5]. Also, a similar study which focuses on the effects of ECE and receiver noise to determine bulk ion parameters in JET is presented by F. Orsitto[6]. In TFTR, a background noise has been measured to be about 20 eV noise equivalent power (NEP) at 60 GHz during a Supershot[7], and the receiver noise is designed to be approximately 1 eV[8]; hence noise is assumed to be dominantly from ECE and receiver noise is not considered in the following analysis. Using this measurement as a basis, error analysis was performed with simulated data generated with a Monte Carlo technique. Simulation results indicate that a 20 eV background level of ECE will not be a problem for the diagnostic. Moreover, temporal analysis of the energetic ion feature may be possible if the signal integration time can be kept to about 10 ms.

## 2 Effect of ECE on signal to noise ratio

How well the plasma parameters may be measured can be determined from an analysis of the detected power spectrum. An ideal (noiseless and infinite duration) experiment would measure  $S(\mathbf{k}, \omega)$  directly. However in practice, the background ECE signal is nearly as strong as the scattered signal in a tokamak, and steps have to be taken to extract  $S(\mathbf{k}, \omega)$  from noisy signals. Also, finite source pulse lengths and small but still significant inherent detector noise have to be considered. Estimates of the required gyrotron power and integration time to measure the desired plasma parameters accurately can be made by considering the signal to noise ratio of spectrum detected by the receiver. For a heterodyne receiver the expression for the signal to noise ratio, the ratio of expectation value of the signal component to the variance of the total signal, is shown to be for each spectral channel[4],

$$S/N = \frac{P_s(\omega)}{P_s(\omega) + P_N(\omega)} \sqrt{1 + \Delta\nu\tau}, \quad (2)$$

where  $\Delta\nu$  is the bandwidth of each spectral channel centered at  $\omega/(2\pi)$ , and  $\tau$  is the sampling integration time. The power observed in each detector channel consists of a signal and a noise

component,  $P_T(\omega) = P_s(\omega) + P_N(\omega)$ .  $P_s(\omega)$  is the desired signal component and  $P_N(\omega)$  is the noise component of the total sampled signal. For the heterodyne detection technique, the ultimate accuracy possible in spectral estimation is governed by statistical considerations of finite receiver intermediate frequency bandwidth,  $\Delta\nu$  and limited integration time,  $\tau$ .

The predetection signal to noise ratio,  $P_s/(P_s + P_N)$ , is the ratio of the signal level to total signal and noise levels. Table 1 lists the corresponding predetection signal to noise ratio values for assumed values of ECE. This calculation assumes that receiver noise is negligible compared to ECE. If the receiver noise needs to be included, the noise values can be viewed as the effective ECE where both receiver and ECE noise are combined. The values listed on the table are calculated on the assumption that gyrotron power is at 200 kW and the geometrical form factor,  $\Gamma$ , is at about 0.1 for X to X mode scattering. The predetection signal to noise ratio is much less than 1 for the alpha feature of the spectrum, and  $P_s$  for the bulk feature of the spectrum is comparable to  $P_N$ . This means that signal from plasma ECE is comparable to or larger than scattered signal from the gyrotron beam. Therefore according to equation (2), long sampling integration time and/or large channel frequency bandwidths are needed to raise S/N to useful levels.

It is apparent that resolving the energetic feature of the spectrum is much more difficult than resolving the bulk feature. To bring the S/N of the energetic ion feature to be greater than 1, the term  $\sqrt{1 + \Delta\nu\tau}$  has to be greater than 1000. With about 2 GHz of the scattered spectrum dominated by the energetic ion feature, the integration time has to be at least 0.5 ms to bring the S/N of the energetic ion feature to about 1. But in order to resolve the scattered spectrum with any accuracy, S/N needs to be much greater than 1. S/N needs to be at least 10 and more likely needs to be on the order of 100. Therefore signal integration time needs to be greater than 50 ms to be able to resolve the energetic ion feature in the scattered spectrum. The ECE background signal at 60 GHz has been measured to be about 20 eV NEP during a Supershot in TFTR with an absolutely calibrated heterodyne receiver. At this level, the predetection S/N for the energetic feature is about 0.02. Hence with  $\Delta\nu$  of 2 GHz,  $\tau$  needs to be about 12 ms to bring the total S/N to about 100.

### 3 Performance analysis in TFTR

Starting with an estimate of background ECE, a statistical analysis of the experiment can be made that will give more detailed information on the expected diagnostic performance. Given an ECE level, the analysis can give information on what the power requirement of the gyrotron source should be and the needed integration time to make the scattering experiment feasible. Also the same analysis can give information on how much ECE noise can be tolerated given the gyrotron power level.

In the simulation, the theoretical spectrum is first calculated for given plasma parameters for each  $N$  discrete frequency values corresponding to the mid point of  $N$  spectral channels. With a constant noise value added to the theoretically calculated signal values in each of the  $N$  channels and randomized statistically with a  $\chi^2$  distribution, these values represent the signal which would be measured in each channel after a long observation period (Figure 1).

The noise level, dominated by plasma ECE background radiation, is assumed to be reasonably reproducible and measurable. With the noise subtracted from each channel, a sample spectrum,  $P_s(\omega)$  is obtained for  $N$  theoretical values of  $P_{th}(\omega)$  in the following way[4].

$$P_s(\omega) = \chi_M^2 [P_{th}(\omega) + P_N(\omega)] - \chi_M^2 [P_N(\omega)], \quad (3)$$

where  $\chi^2$  distribution has  $M = 2(\Delta\nu\tau + 1)$  degrees of freedom. As mentioned before,  $\Delta\nu$  is the bandwidth of each spectral channel centered at  $\omega/(2\pi)$  and  $\tau$  is the sampling integration time. Hence for each spectral channel, the  $\chi^2$  randomized value of the sum of the theoretical value of signal and background noise is subtracted from the  $\chi^2$  randomized value of background noise with a different seed for the  $\chi^2$  function. Each value of such a spectrum is a sample from the ensemble of possible simulations corresponding to the same experimental conditions (Figure 1). In an actual experiment, a similar method is used to calculate the actual signal by measuring the total signal for a period with the gyrotron on, and measuring the background noise for the same duration with gyrotron off. The actual signal will then be calculated by subtracting the noise signal from the total signal.

With the simulated spectra in hand, a non-linear least squares fitting routine, is applied to each spectrum to find the best values of desired parameters. The non-linear least squares fitting code used here fits for bulk and energetic  $\alpha$  ion temperatures and normalized  $\alpha$  density

fraction ( $T_D, T_T, T_\alpha, n_\alpha$ ). The code assumes that the signal levels are not calibrated, and bulk ion density fractions are assumed to be known. To permit statistical analysis, a set of 33 or more independent simulations at each set of conditions is performed. This number is arbitrarily chosen, but is at the lower limit for drawing relevant statistical conclusions. The sample mean and variance derived from this procedure provides estimates of the experimental shot to shot variations which would be encountered in an actual scattering experiment.

For the scattering experiment, parameters that describe density and energy of the plasma species are adjusted to fit the theoretical model to experimental data. Bulk ion temperatures and the fusion  $\alpha$  particle density and birth energy are the free parameters that are adjusted in the fitting routine. For simplicity, Maxwellian velocity distributions are assumed for all plasma species. Because the code fits for the  $\alpha$  density fraction with bulk ion density fractions as fixed inputs, quasineutrality is not obeyed in the code. However because the density of  $\alpha$  particles are much less compared to the bulk ions, quasineutrality is still a good assumption. For theoretical calculations, the plasma is assumed to consist of equal parts of deuterium and tritium ions with temperatures of 20 keV, and a small fraction of fusion reaction produced alpha particles (.5%) with a Maxwellian distribution (1 MeV). The electron density and temperature are assumed to be  $1.0 \times 10^{14} \text{ cm}^3$  and 12 keV, respectively, and magnetic field is assumed to be 5 Tesla.

The mean values and variance of the fitted parameters (bulk and  $\alpha$  ion temperatures and  $\alpha$  density fraction) calculated from at least 33 or more independent simulated data are then analyzed. The standard deviation of the fitted parameters from 33 independent simulated data sets are plotted against the predetection signal to noise ratio to analyze the goodness of fit. The expected trend is that the greater the predetection signal to noise ratio, the smaller the standard deviation of the fitted parameter should be. As expected the analysis shows that the standard deviation of the fitted parameters decreases with increasing values of predetection signal to noise ratio. Analysis of these plots can give indications of expected performance level of the scattering diagnostic system. The normalized standard deviation of the mean,  $\sigma_N/F$ , is defined as the ratio of the standard deviation of fitted mean values from the independent simulated data sets to the theoretical value of  $F$ . Figures 2 and 3 show the plots of normalized standard deviation of mean versus predetection S/N values for fits of the alpha features. Hence

on Figure 2, each point on the plot represents a standard deviation of a mean temperature from 33 curve fitted temperature values from 33 independent simulated data sets. This value can be viewed as a figure of merit for the expected performance of the scattering diagnostic for a given noise level and gyrotron power. The predetection S/N values on the abscissa correspond to the equivalent ECE signal levels as given in Table 1. The figures show three lines that corresponds to three signal integration times of 10 ms, 30 ms, and 100 ms. The frequency channel bandwidth is fixed by the multiplexer of the receiver system to be at 2.56 GHz total, which includes bandwidths for both bulk and minority energetic features on the spectrum. Plots show that at ECE of 20 eV, which is expected during a Supershot in TFTR, less than 5 % errors in both density and temperature fit for alpha particles can be expected, even for signal integration time of 10 ms. This result suggests that the estimated calculation with Equation (2) of about 12 ms to achieve S/N of about 100 to resolve the alpha spectrum agrees well with the result from the statistical simulations. At integration time of 100 ms, errors drop to about 1 % levels. With signal integration time of 100 ms, even 100 eV ECE signal can be tolerated to give fit errors of less than 10 %. Results from this simulation analysis indicate that with a typical TFTR plasma shot lasting about one second with flat top lasting about 300 ms, analysis of temporal characteristics of the alpha particles may be possible.

As expected from Table 1, because of higher predetection S/N, errors in fits for bulk ion temperatures are about an order of magnitude better than fits for the fusion  $\alpha$  temperature. At 20 eV ECE with 10 ms integration time, error to the fit is less than 2 %. And even at 100 eV ECE with  $\tau$  of 10 ms, error is still less than 10 %. In summary, the curve fit errors associated with all ion species are still less than 5 % for ECE of 20 eV and integration time of 10 ms.

## 4 Summary

With a 60 GHz 200 kW, 300 ms gyrotron source and about 20 eV ECE signal at 60 GHz from a TFTR plasma, statistical analysis indicate that both the bulk feature and alpha feature in the scattered spectrum can be resolved. With 10 ms integration time at 20 eV ECE, errors due to ECE noise in the curve fitting process as indicated by standard deviation of fitted mean

from many independent simulated data samples are less than 5 %. Even at a high ECE signal of 100 eV, the errors are at about 40 % for fit to alpha particle temperature and at less than 10 % for fits to alpha particle density and bulk ion temperatures. With an integration time of 100 ms, the errors drop to below 10 % for all parameters even for ECE signal of 100 eV.



**References**

- [1] H.K. Park, N. Bretz, R. Ellis, P.C. Efthimion, R. Marsala, M. McCarthy, G. Renda, K.M. Young, D.R. Cohn, J.S. Machuzak, D.Y. Rhee, P. Woskov (to be published in Review of Scientific Instruments) (1992)
- [2] John Sheffield, Plasma Scattering of Electromagnetic Radiation, Academic Press, New York (1975)
- [3] P.A. Krug, R. Behn, S.A. Salito, M.R. Siegrist, Plasma Physics and Controlled Fusion, **32** 713-727 (1990)
- [4] R.L. Watterson, M.R. Siegrist, M.A. Dupertuis, P.D. Morgan, M.R. Green, J. Appl. Phys. **52** 3249-3254 (1981)
- [5] L.E. Sharp, A.D. Sanderson, D.E. Evans, Plasma Physics **23** 357-360 (1981)
- [6] F. Orsitto (to be published in Review of Scientific Instruments) (1992)
- [7] J.S. Machuzak, private communications (1990)
- [8] P.P. Woskov, Workshop on TFTR alpha particle Gyrotron Scattering experiment, Princeton University (1991)

ECE(eV)	$P_s/(P_s + P_N)$ bulk	$P_s/(P_s + P_N)$ Energetic
20	0.740	2.03E-2
40	0.587	1.03E-2
80	0.416	5.17E-3
160	0.262	2.59E-3

Table 1: TFTR predetection signal to noise values for corresponding ECE levels. The calculations assumed that the gyrotron power is 200 kW, 0.5 % $n_\alpha/n_i$ , 49.75 % $n_D/n_i = n_T/n_i$ ,  $n_e=1.0e14cm^3$ , 12 keV  $T_e$ ,  $T_D = T_T= 20$  keV,  $T_\alpha = 1$  MeV,  $55^\circ\mathbf{k}_s$  to  $\mathbf{k}_i$  angle, and  $60^\circ\mathbf{k}$  to  $\mathbf{B}$  angle. The  $P_s$  for bulk ion is the sum of signals from the inner receiver channels where the bulk ion features dominate, and  $P_s$  for energetic ion is the sum of signals from the outer receiver channels. Angular frequency at which the spectrum exhibits transition from bulk to minority feature is defined to be  $2kv_{th}$ .

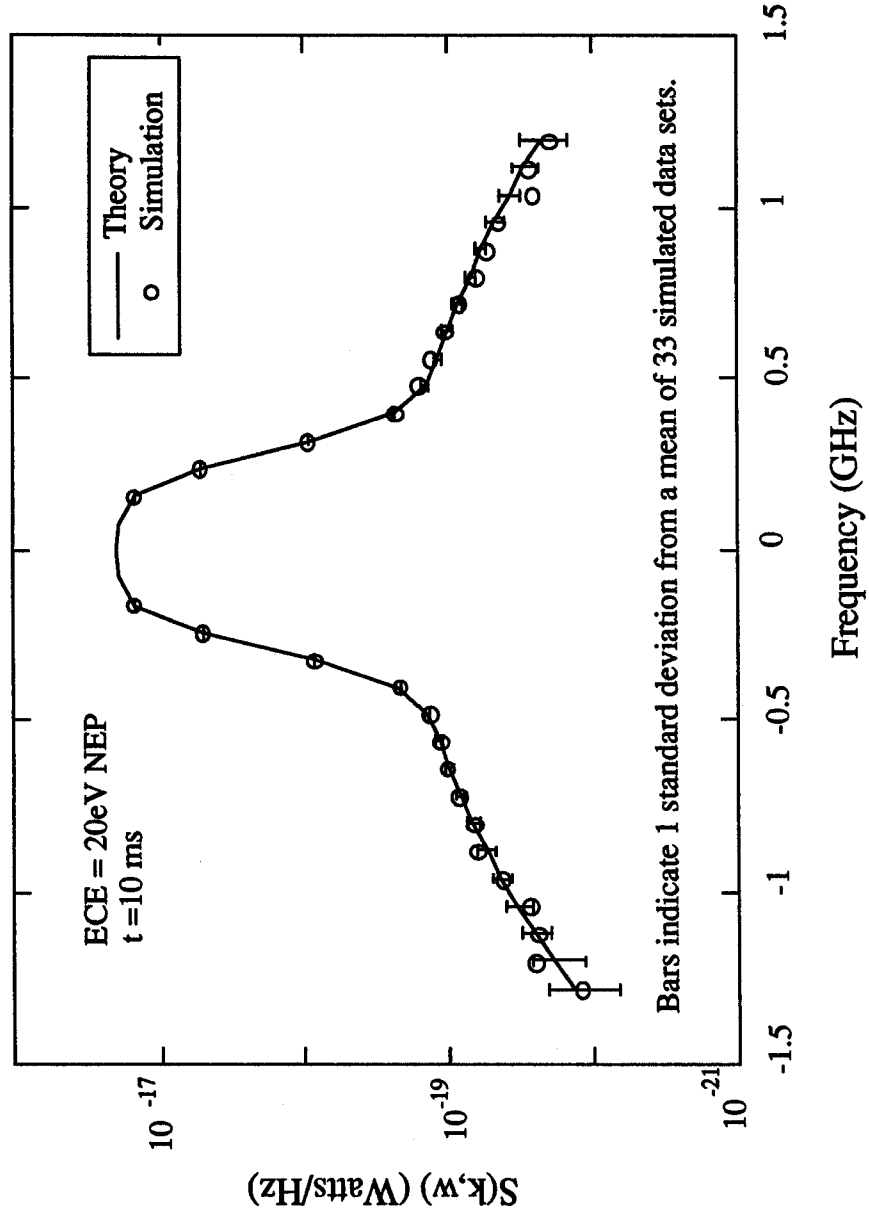


Figure 1: Scattered power spectrum with the theory represented by the solid line and open circle indicating a simulated data. Bars are the standard deviation of a mean calculated from 33 simulated data sets. For this example, ECE noise is set at 20 eV noise equivalent power (NEP), and integration time is 10 ms. Again, the calculations assumed that the gyrotron power is 200 kW,  $0.5 \% n_{\alpha}/n_i$ ,  $49.75 \% n_D/n_i = n_T/n_i$ ,  $n_e = 1.0 \times 10^{14} \text{ cm}^{-3}$ , 12 keV  $T_e$ ,  $T_D = T_T = 20 \text{ keV}$ ,  $T_{\alpha} = 1 \text{ MeV}$ ,  $55^{\circ} \mathbf{k}_s$  to  $\mathbf{k}_i$  angle, and  $60^{\circ} \mathbf{k}$  to  $\mathbf{B}$  angle.

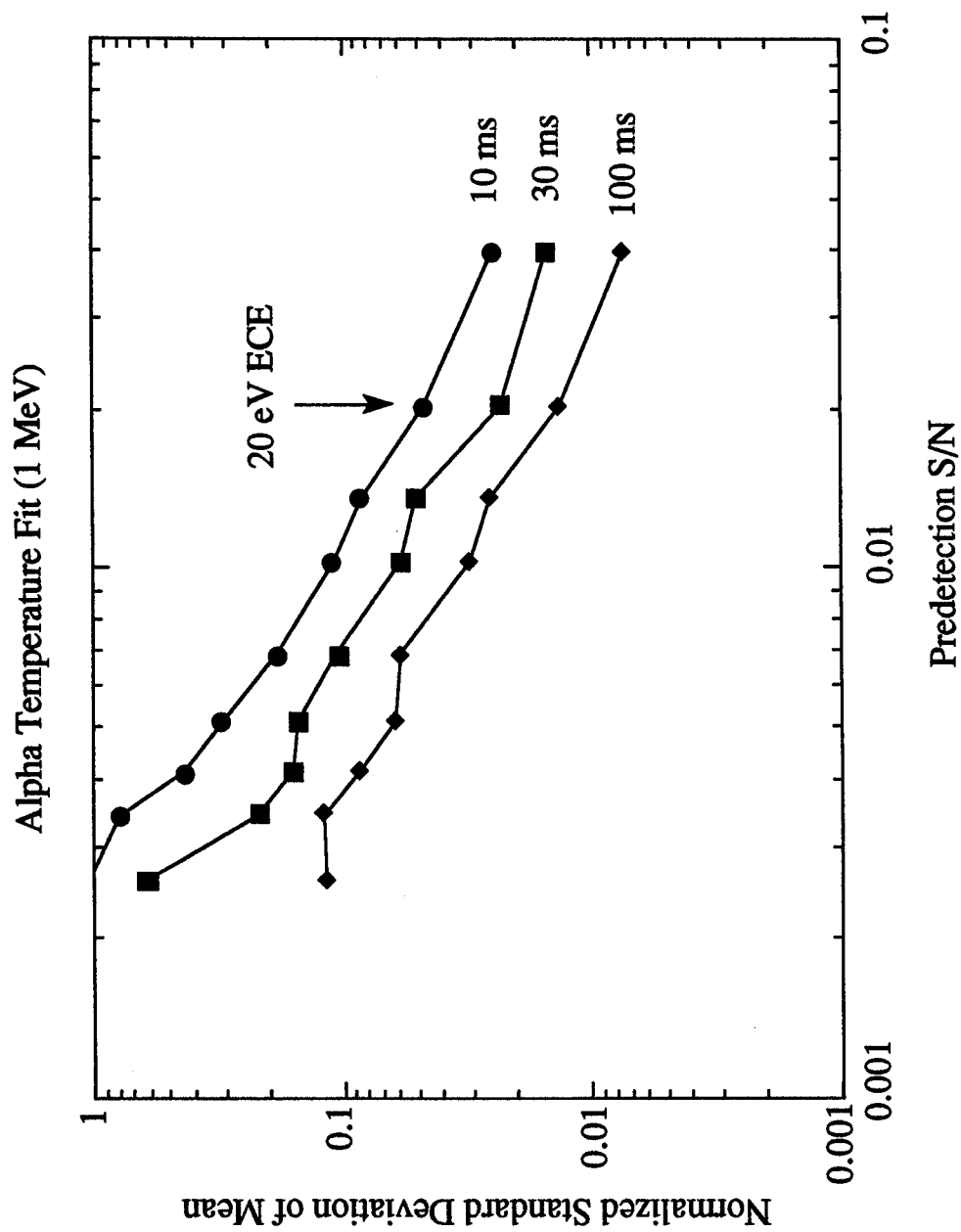


Figure 2: TFTR: Normalized standard deviation of Means from 33 Independent Fits. Fusion  $\alpha$  temperature fit. Theoretical alpha particle temperature is 1 MeV Maxwellian. Other scattering parameters are listed in the Figure 1 caption.

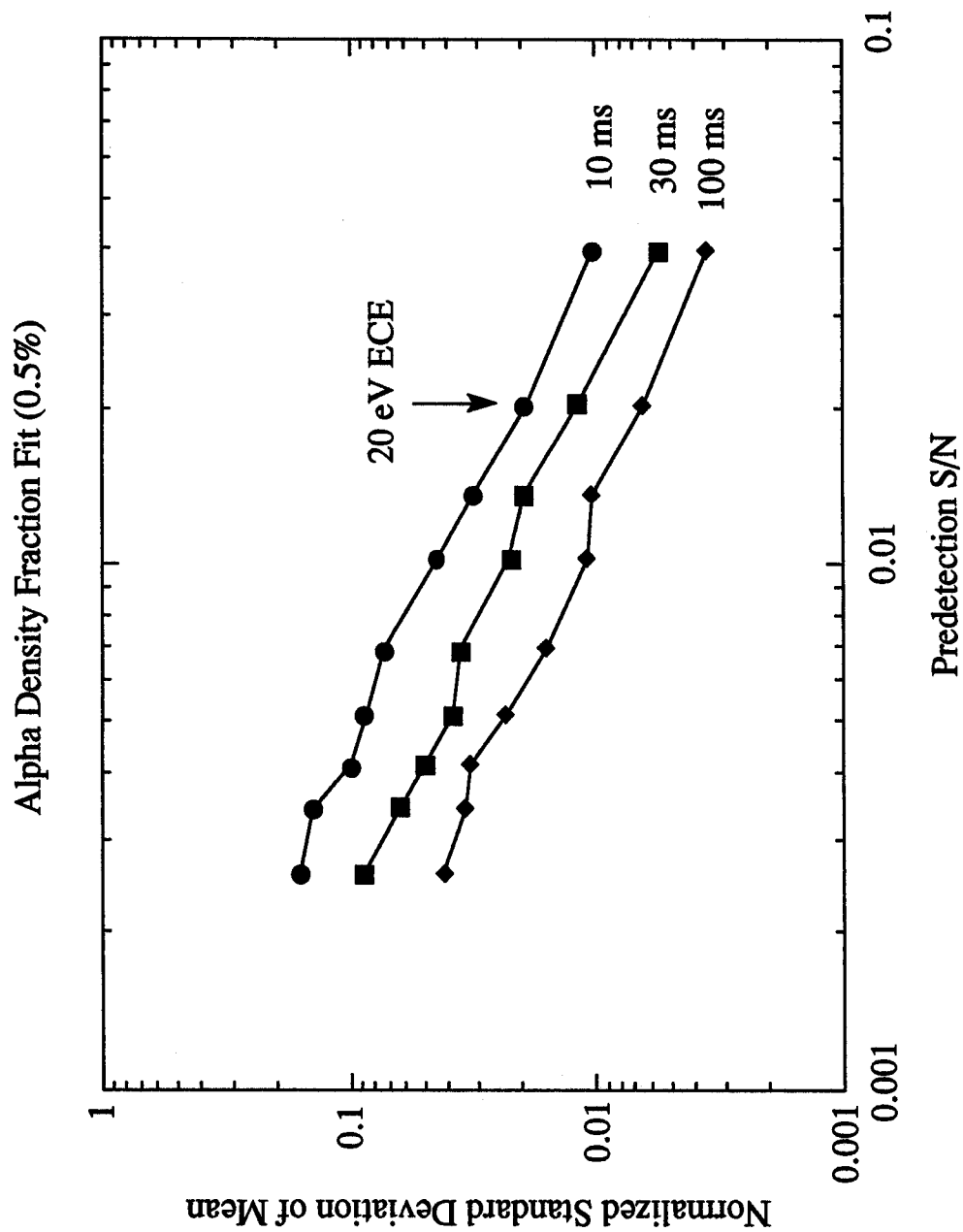


Figure 3: TFTR: Fusion  $\alpha$  density fraction fit. Theoretical  $\alpha$  particle density is 0.5 %.