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DOUBLE STREAM CYCLOTRON MASER

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ABSTRACT

The double stream cyclotron maser is a novel source of millimeter wavelength radiation in which two copropagating electron beams are caused to gyrate in a uniform axial magnetic field. The interaction of the slow cyclotron space charge wave on one beam with the fast cyclotron space charge wave on the other beam leads to high frequency bunching. The desired operating frequency is proportional to the electron cyclotron frequency (or a harmonic thereof) and inversely proportional to the difference in beam velocities, and can be achieved at low beam energies and axial magnetic fields. The linear instability growth rate is calculated from the fully relativistic Vlasov equation for the case of cold electron beams.

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Electrons with relativistic velocities gyrating and drifting in a uniform axial magnetic field $B_{||}$ are known[1] to generate intense, coherent electromagnetic radiation at centimeter and millimeter wavelengths. This radiation is a manifestation of the "negative mass" instability[2] of the copropagating right circularly polarized wave traveling in the magnetic field direction. The instability arises from a relativistic mass shift of the electron gyrofrequency $\Omega = (eB_{||}/m_o\gamma)$ with $\gamma = [1 - (v_{||}/c)^2 - (v_\perp/c)^2]^{-\frac{1}{2}}$. The resulting emission occurs at a radiation frequency

$$\omega_1 \simeq (1 + \beta_{||})\gamma_{||}^2(m\Omega), \quad (m = 1, 2, 3...) \tag{1}$$

where $\beta_{||} = v_{||}/c$ and $\gamma_{||} = [1 - \beta_{||}^2]^{-\frac{1}{2}}$. The gyrotron[3] (with $\gamma_{||} \simeq 1$) and the cyclotron autoresonance maser [4] (with $\gamma_{||} > 1$) are special cases of the above instability.

The double-stream cyclotron maser is a novel source that does not rely on relativistic effects for instability growth, but on the mutual interaction of charge-density perturbations of two copropagating beams traveling at somewhat different speeds. The advantage of the double stream system over the conventional one-beam cyclotron maser is the much higher frequency that can be achieved for the same experimental parameters. As will be shown below, the interaction frequency is now given by,

$$\omega_2 \simeq 2(\beta_{||}\gamma_{||})^2(\gamma_{||}/\Delta\gamma_{||})(m\Omega) \tag{2}$$

where $\Delta\gamma_{||}$ is the energy difference between the two electron beams. Comparing Eqs. (1) and (2), it follows that $\omega_2/\omega_1 \simeq (\gamma_{||}/\Delta\gamma_{||})$ so that, for example, a 20% energy difference can increase the interaction frequency fivefold. Conversely, a desired operating frequency can be now obtained at relatively low beam energies and axial magnetic fields.

The double stream cyclotron maser is illustrated schematically in Fig. 1. Two well intermingled electron beams of different axial velocities $v_{||,1}$ and $v_{||,2}$ are spun up and allowed to gyrate in a uniform axial magnetic field $B_{||}$. The appropriate velocity difference can be obtained, for example, by means of a split-cathode[5] technique in which half of the cathode is connected to the accelerator via a resistive load. The spinning up of the beams can be performed using a magnetron injection[6] gun configuration; however, in order to minimize axial velocity spread, a short section of bifilar helical wiggler[7] is preferable (see Fig. 1). As a result, the electron beams acquire transverse velocities $v_{\perp,1}$ and $v_{\perp,2}$.

The spun-up beams interact in a cylindrical drift tube which also acts as a wave-guide. In this three-dimensional configuration, the wave vector $\vec{k}$ has both axial ($k_{||}$) and transverse ($k_{\perp}$) components, with $k_{\perp}$ being determined by the beam radius and the transverse guide dimensions. Excitation of a space-charge wave ($\vec{E}_{rf} \parallel \vec{k}$), at the cyclotron frequency and its harmonics comes about solely because the transverse component of the rf electric field E_{rf} exerts a tangential force on the electron in its helical motion. This orbit perturbation leads to harmonic generation. In order to achieve significant wave amplitude at a given harmonic, the perpendicular wavelength must be comparable in magnitude to the electron Larmor radius, or, $k_{\perp} v_{\perp}/\Omega \simeq 1$. It should be noted that the type of two-stream instability discussed here differs from the conventional[8] one characterized by frequency oscillations tied to the electron plasma frequency $\omega_p^2 = Ne^2/m_o\epsilon_o\gamma$ rather than the electron cyclotron frequency Ω. Indeed, in such cases, calculations[9] and the resulting microwave amplifier tubes[10] are based on the assumption that $v_\perp \simeq 0$, so that coupling to the electron cyclotron motion is of no concern.

Our calculations of the wave frequency and growth rate are based on a solution[11] of the relativistic Vlasov equation and Maxwell's equations in free space. In place of a fully self-consistent solution of the unneutralized two-beam system, a neutralizing background of infinitely massive ions is assumed. The wave with $\vec{E}_{rf} \parallel \vec{k}$ and an exp $[j(\omega t - \vec{k}\cdot\vec{r})]$ dependence has a linear (small signal) dispersion equation of the form, $K_L(k,\omega) = (\vec{k}\cdot K\cdot\vec{k})/k^2 = 0$, where K is the tensor dielectric coefficient and K_L the effective longitudinal component given by[12],

$$K_L = 1 - \sum_{beams} \int_0^\infty dv_\perp v_\perp \int_{-\infty}^\infty dv_\parallel \frac{\omega_p^2}{k^2} 2\pi f_0 \quad \times$$
$$\sum_{m=-\infty}^{\infty} \left[\frac{m(\frac{k_\perp}{v_\perp})\frac{d}{dp}[J_m^2(p)]}{(\omega - k_\parallel v_\parallel - m\Omega)} + \frac{[k_\parallel^2 - \frac{(k_\parallel v_\parallel + m\Omega)^2}{c^2}]J_m^2(p)}{(\omega - k_\parallel v_\parallel - m\Omega)^2} \right] = 0. \qquad (3)$$

Here $f_0 = f_o(v_\perp, v_\parallel)$ is the unperturbed velocity distribution function for each beam, and $J_m(p)$ is the mth order Bessel function with argument given by $p = k_\perp v_\perp/\Omega$.

We now assume for simplicity that the two beams are completely superimposed and that they fill entirely a cylindrical waveguide of radius a. In that case[8] $k_\perp = u_{l,s}/a$ where $u_{l,s}$ is the sth root of $J_l(u) = 0$. Then,

$$p = u_{l,s} v_\perp/\Omega a \qquad (4)$$

$$= u_{l,s} r_L/a$$

where r_L is the Larmor radius of the electron orbit $(r_L < a)$. The values of l and s select the waveguide mode. For the lowest transverse magnetic $TM_{0,1}$ mode of interest here, $u_{l,s} = 2.405$. For beams with cold electrons, the distribution function has the form,

$$f_o(v_\perp, v_\parallel) = \frac{1}{2\pi v_{0\perp}} \delta(v_\perp - v_{0\perp})\delta(v_\parallel - v_{0\parallel}). \tag{5}$$

Substituting Eq. (5) in Eq. (3) yields the sought-after dispersion equation:

$$K_L = 0 = 1 - \sum_{beams} \sum_{m=-\infty}^{\infty} \frac{\omega_p^2}{k^2} \frac{\left[k_\parallel^2 - \left\{\frac{k_\parallel v_\parallel + m\Omega}{c}\right\}^2\right] J_m^2}{(\omega - k_\parallel v_\parallel - m\Omega)^2} + \frac{k_\perp^2 \{J_{m-1}^2 - J_{m+1}^2\}}{2\Omega(\omega - k_\parallel v_\parallel - m\Omega)}. \tag{6}$$

In the case of one beam alone, the above dispersion equation shows that around each harmonic number $m\Omega = \omega - k_\parallel v_\parallel$, there are two narrow propagation bands, one above and one below each integral multiple of Ω. The propagation bands extend approximately $\pm\omega_p J_m(p)$ on either side of $m\Omega$. The lower, slower branch around each harmonic is a negative energy wave[13], while the upper, faster wave is a positive energy wave. At high plasma densities $(\omega_p > m\Omega)$ the negative energy wave of the mth harmonic can come into phase velocity synchronism with the positive energy wave associated with the $(m-1)$th harmonic, and a high frequency micro-instability can develop[14]. However, this phenomenon is of little interest for unneutralized electron beams considered here since such beams will become macroscopically unstable unless[8] $\omega_p \leq \Omega/\sqrt{2}$.

The situation is different when two beams with different axial velocities interact with one another as is illustrated in Fig. 2. Now, phase velocity synchronism can be achieved at low plasma densities $\omega_p \ll \Omega$. For the case of almost paraxial propagation, $k_{||} \gg k_{\perp}$, the second term of Eq. (6) can be neglected; it is identically equal to zero when p is so chosen as to maximize $J_m(p)$. Then, the dispersion characteristics for each beam-cyclotron mode can be approximated by

$$\begin{aligned} \omega &\simeq k_{||}v_1 + m\Omega_1 \\ \omega &\simeq k_{||}v_2 + n\Omega_2. \end{aligned} \tag{7}$$

with n, m equal $\pm 1, \pm 2....$ For, say, $n = -m$, and beams whose velocities are not too different, Eqs. (7) yield

$$\begin{aligned} \omega &\simeq m\Omega \frac{v_2 + v_1}{v_2 - v_1} \\ &= 2(\beta_{||}\gamma_{||})^2(\gamma_{||}/\Delta\gamma_{||})(m\Omega) \end{aligned} \tag{8}$$

where now $\Omega, \beta_{||}$ and $\gamma_{||}$ represent averages over the two beams. We note that in the vicinity of the above frequency, $\omega \simeq k_{||}(v_1 + v_2)/2$, the wave is dispersionless, and has a phase velocity equal to the average beam velocity.

The accompanying spatial growth rate, $Im(k)$, of the convective instability (the system is an amplifier rather than an oscillator) is derived from Eq. (6). This is most readily accomplished by noting that for sufficiently small growth rates, $Im(k) \simeq Im(\omega)/v_g$, where v_g is the group velocity of the unstable wave. Thus, solving for complex ω and real k one then obtains,

$$Im(\omega) \simeq \omega_p J_m(p)/2\gamma_{||}. \tag{9}$$

In deriving the growth rate, it is assumed that the two beams have equal plasma frequencies, or more precisely that $\omega_{p,1}J_m(p_1)/\gamma_{||,1} = \omega_{p,2}J_m(p_2)/\gamma_{||,2} = \omega_p J_m(p)/\gamma_{||}$.

For purposes of illustration we consider two cases: the interaction of two slow beams where relativistic effects are very small, and the case of two beams with mildly relativistic energies. In both cases the energy differences $\Delta\gamma_{||}$ are taken to be considerably smaller that the mean beam energies, but much larger than the parallel thermal energy spreads expected in each beam. The results of the calculations are summarized in Table I. The conversion efficiency η (at saturation) quoted in the Table is estimated from[15] $\eta \approx \Delta\gamma_{||}/(\gamma - 1)$.

We see that high frequency operation can be achieved at low beam energies and low magnetic field strengths. The instability growth rate is appreciable, but is limited (see Eq. (9)) by the need to maintain the ratio $\omega_p/\Omega \leq 1\sqrt{2}$ (Brillouin condition[8]), and thereby prevent beam break-up in the unneutralized system. Introduction[16] of a cold background plasma to provide space-charge neutralization offers an interesting option. It allows an increase in ω_p which increases the growth rate, the beam current density and the rf power. In addition, by properly tuning the plasma density of the background plasma, the associated space charge wave can be made synchronous[17] with the unstable beam cyclotron wave discussed above. This is readily seen by deriving the wave phase velocity (as found by setting $v_\perp = v_{||} = 0$ in Eq. (6)):

$$(\frac{\omega}{k_{||}})^2_{plasma} = \frac{(\omega^2 - \omega_p^2)(\omega^2 - \Omega^2)}{k_\perp^2(\Omega^2 - \omega^2 + \omega_p^2)}. \tag{10}$$

Setting $(\omega/k_{||})_{plasma} \simeq (v_1 + v_2)/2$ assures phase velocity synchronism with the beam cyclotron wave of Eq. (8). This three wave interaction may enhance[16,17] the bunching process.

In conclusion, we have described the characteristics of a novel double-stream cyclotron maser that can achieve high oscillation frequencies using two mildly relativistic beams propagating in weak magnetic fields. To interact with the beam-cyclotron modes, the electrons must first acquire perpendicular energy just as in gyrotrons and cyclotron autoresonance masers. The proposed interaction with $n = -m$ results in a slow wave $(\omega/k_{\|} < c)$, and in this respect suffers from the same disadvantages as other Čerenkov type systems. Thus, generation of electromagnetic radiation requires the presence of a slow-wave structure such as a dielectric loaded, or corrugated waveguide. Alternately, our two-beam system could be incorporated in a klystron[18] geometry, and the rf power in the growing space-charge bunches extracted in a resonant cavity. However, a different choice of mode numbers m and n can lead to a fast wave interaction. In the general case, the interaction frequency and phase velocity are found to be $\omega \simeq \Omega(m\beta_2 - n\beta_1)/(\beta_2 - \beta_1)$ and $(\omega/k_{\|}c) \simeq (m\beta_2 - n\beta_1)/(m-n)$. For, $m > 0, n > 0, (m \neq n)$ and for sufficiently large beam energies, $\omega/k_{\|}c$ can be equal to, or greater than, unity, and interaction with an electromagnetic wave can take place. Now, however, the validity of the quasi-static approximation $(\vec{E}_{rf} \parallel \vec{k})$, on which Eq. 3 is based, is open to question.

Studies are in progress concerning effects of electron beam temperature, lack of complete spatial overlapping of the two beams, and nonlinear saturation. In the nonlinear regime, loss of synchronism will occur as a result of changes in γ_1, γ_2 and $\Delta\gamma_{\|}$. This limits the degree of bunching that can be achieved. Equations (8) suggests that tapering the magnetic field as a means of "efficiency enhancement" may be possible.

The calculations in this paper are applicable to beams propagating in an assumed neutralizing background of stationary plasma ions. In the absence of the neutralizing plasma, the results are a good approximation provided that the beam plasma densities are sufficiently low, such that $(\omega_p/\Omega)^2 \ll 1$. As the density increases and the Brillouin condition $(\omega_p/\Omega)^2 = \frac{1}{2}$ is approached, a self-consistent calculation is mandatory.

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TABLE I. Summary of Operating Parameters of a Double Stream Cyclotron Maser for $(\omega_p/\Omega)^2=(1/9)$ and $(k_\perp v_\perp/\Omega)=1.841$.

PARAMETER	I	II
V_{mean} (kV)	87.5	275
ΔV(kV)	25.0	50
J(A/cm^2)	23	100
B_z(kG)	1.0	2.1
$\omega/2\pi$ (GHz)	50	150
Gain (dB/m)	80	68
efficiency η (%)	29	18
harmonic: m,n	1,-1	1,-1
waveguide mode	TM_{01}	TM_{01}
a(cm)	0.5	0.5
$v_\perp/c$	0.2	0.3

FIGURE CAPTIONS

Figure 1: Schematic drawing of a double-stream cyclotron maser.

Figure 2: Sketch of the dispension characteristics for cyclotron modes, $n = -m$ (see Eqs. (7)). The heavy lines denote the slow, negative energy waves on each beam. The dashed bell-shaped curve shows the growth rate $Im(\omega)$.

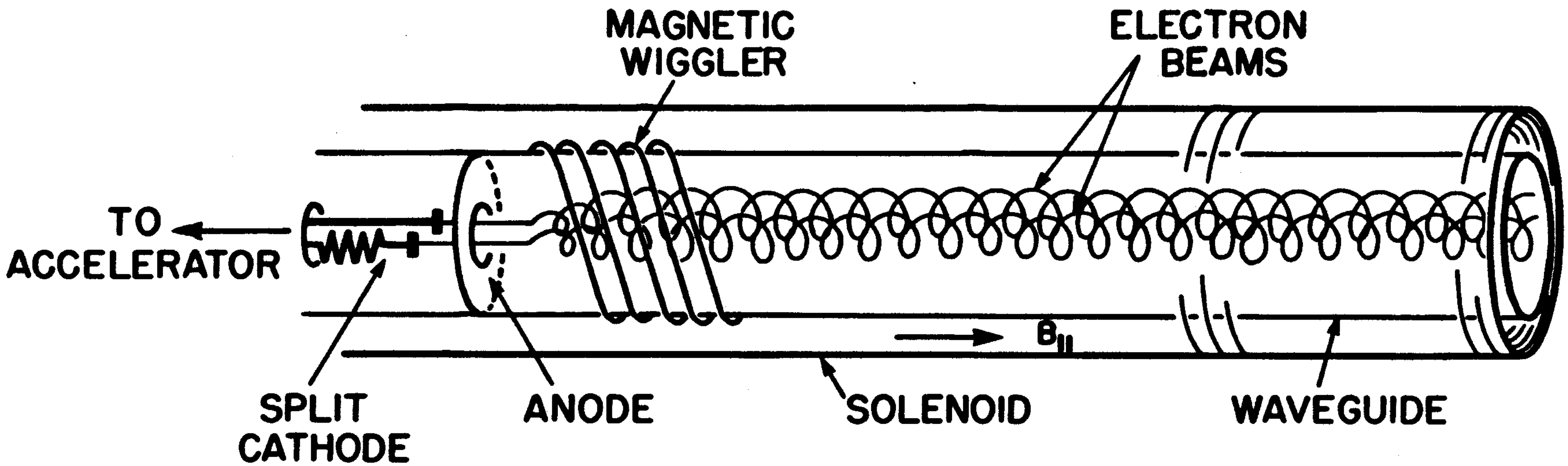

Fig.1. G.Bekefi

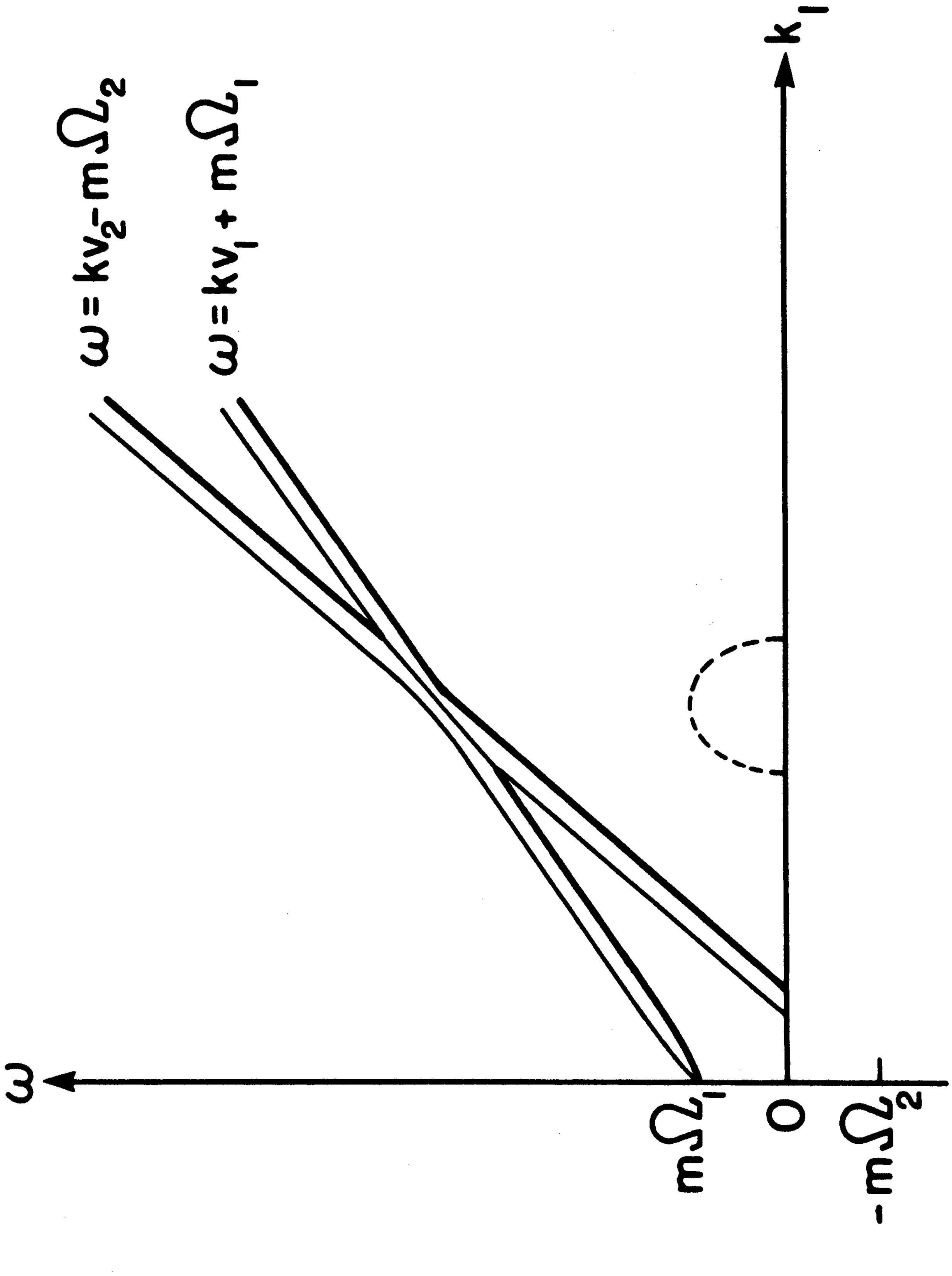

Fig.2. G.Bekefi