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**Maximizing Absorption in  
Ion-Cyclotron Heating of Tokamak Plasmas**

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# MAXIMIZING ABSORPTION IN ION-CYCLOTRON HEATING OF TOKAMAK PLASMAS

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## I. Introduction

Efficient ion-cyclotron heating in tokamak plasmas is effectively localized in the central region of the plasma, near the ion second-harmonic layer in a single ion species plasma, or near the ion-ion hybrid layer in a plasma containing a minority ion species. The fast Alfvén wave (FAW), which carries the incident rf power, from the low magnetic field side, is generally focused in (by antenna design and propagation) toward this central region on the equatorial plane of the tokamak plasma. There the FAW encounters a “coupling” region and may undergo reflection ( $R$ ), transmission ( $T$ )—to the high magnetic field side, mode conversion ( $C$ )—to an ion-Bernstein wave (IBW), and kinetic dissipation ( $D$ )—due to Doppler-shifted ion-cyclotron damping. (Here we ignore electron TTMP and/or Landau damping; these can be readily included by expanding on our formalism.) To determine these power transfer coefficients ( $R$ ,  $T$ ,  $C$  and  $D$ ) the problem is in general formulated as an integro-partial-differential equation based upon the linearized Vlasov-Maxwell equations for an inhomogeneous plasma. This is however difficult to solve, even numerically, and has been usually approximated by a fourth or sixth-order o.d.e. description which is amenable to numerical integration.

We have attempted to formulate reduced order analyses that can give approximate, closed form analytic results for these coefficients [1]. We have recently completed this and here we only summarize the results. From these we show the regime of parameters that gives maximum energy deposition on the ions in the central plasma region of interest.

## II. Results of Reduced-Order Analyses

Our analytic model describes the central plasma region of interest in slab geometry and neglects the effects of poloidal magnetic fields. Away from the coupling regions we use the Vlasov-Maxwell local dispersion relation expanded to second-order in  $(k_{\perp}\rho_i)^2$  [1]. This gives an eikonal description of the FAW's [2] and IBW's [3].

(a) Transmission. The power transmission coefficient  $T$  for the FAW can be obtained from perturbation of modes that entails the solution of only a first-order o.d.e. [4].

(b) Reflection. Both the power transmission and reflection coefficients ( $T$  and  $R$ ) for the FAW can be obtained from a second-order o.d.e. that describes the modified FAW due to its coupling to the IBW [2]. From this we have recently obtained an approximate analytic description of the power reflection coefficient [5].

(c) Conversion. The power conversion coefficient  $C$  gives the amount of incident FAW power converted to the IBW. We have shown that it can be obtained from an approximate coupled mode formulation that entails solving two, coupled, first o.d.e.'s [6].

(d) Dissipation. The kinetic power dissipation coefficient  $D$  follows from the global power conservation, i.e. integrating the energy flow over a volume containing the coupling, central plasma region. For the FAW incident from the low-field side of the toroidal magnetic field we then have

$$T + R + C + D = 1 \quad (1)$$

With  $T$ ,  $R$ , and  $C$  known, (1) then gives  $D$ . When kinetic dissipation is dominant,  $D$

can also be obtained approximately from a local, kinetic WKB propagation analysis [7].

### III. Maximizing Single-Pass Absorption

Here we focus our discussion to  $D(H)$  plasmas with the FAW incident from the low-field side. From [6] we find that, for small parallel Alfvén indices  $N_{\parallel} = k_{\parallel}c_A/\omega$ ,  $C$  is proportional to  $T$ , as might be expected. Furthermore, from [4] and [5] we have approximately

$$T = e^{-a}; \quad a \approx \frac{\pi}{4} R_A (\eta + \beta_D N_c^2) \quad (2)$$

$$R = (1 - T)^2 e^{-b}; \quad b \approx \frac{4\pi}{9} R_A N_{\parallel}^2 \frac{\beta_H^*}{(\beta_H^* N_c^2 + \eta)} + 3\sqrt{\pi} R_A \eta \delta e^{-\delta^2} \quad (3)$$

where  $R_A = (R_0 \omega / c_A)$ ,  $\eta = n_H / n_D$ ,  $\beta_D = (2T_D / m_D c_A^2)$ ,  $\beta_H^* = 2\beta_D T_H / T_D$ ,  $\delta = [(3/4)(\eta + \beta_D N_c^2) / N_{\parallel} (\beta_H^*)^{1/2}] > 1$ ,  $N_c^2$  is of order unity (weakly dependent on  $\eta$  and  $N_{\parallel}$ ), and other such terms of order unity multiplying  $a$  and  $b$  have been omitted.

From (2) we note that for  $a > 1$ , we have  $T \approx 0$  essentially independent of  $N_{\parallel}$ . It then follows from [6] that  $C \approx TR \exp(-2D_c)$  will become vanishingly small;  $D_c$  is defined in [6] and does not play an important role in our considerations of low-field incident FAW's. Hence (1) becomes approximately

$$R + D \approx 1 \quad (4)$$

Since  $R$  is a strong function of  $N_{\parallel}^2$ , complete absorption (i.e.,  $D \approx 1$ ) is then obtained for  $N_{\parallel}$  in a range exceeding (by about a factor of two or four) the  $N_{\parallel}$  that makes  $R \approx (1/2) \approx D$ ; we call the latter the "crossover- $N_{\parallel}$ ." As can be seen from (2) and (3), the ion minority species fraction  $\eta$  and its temperature  $T_H$  will also affect the above considerations for making  $D \approx 1$ . Clearly, high-density and large  $R_0$  plasmas are amenable to this optimization.

The results can be conveniently illustrated in a plot of the crossover- $k_{\parallel}$  as a function of  $\eta$  for various values of  $T_H$ . Since this entails calculating only  $R$ , we have used the approximate fast-wave equation [2] to solve numerically for these parameters when  $R \approx (1 - T)/2$  (this allows for nonzero  $T$  when  $\eta \rightarrow 0$  and  $\beta_D$  is not sufficiently large). The approximate analytic calculations in [4-6], viz. (2) and (3) can be used to understand the numerical results. Figure 1 shows such results for parameters relevant to forthcoming ICH in Alcator C-Mod:  $R_0 = 0.67m$ ,  $B_0 = 5.33T$ ,  $D(H)$  plasma,  $n_{e0} = 5 \times 10^{20}/m^3$ ,  $T_D = 2keV$ . Figure 2 shows the results for parameters relevant to plans for CIT/BPX:  $R_0 = 2.8m$ ,  $B_0 = 5.66T$ ,  $DT(H)$  plasma,  $n_{e0} = 2.5 \times 10^{20}/m^3$ ,  $T_D = 3keV$ . In each case: Figure (a) shows a calculation of  $R$ ,  $D$ ,  $T$ , and  $C$  (not visible since it is zero) for a given  $\eta$  and  $T_H$ ; Figure (b) gives the crossover- $k_{\parallel}$  as a function of  $\eta$  with  $T_H$  as a parameter.

### IV. Acknowledgements

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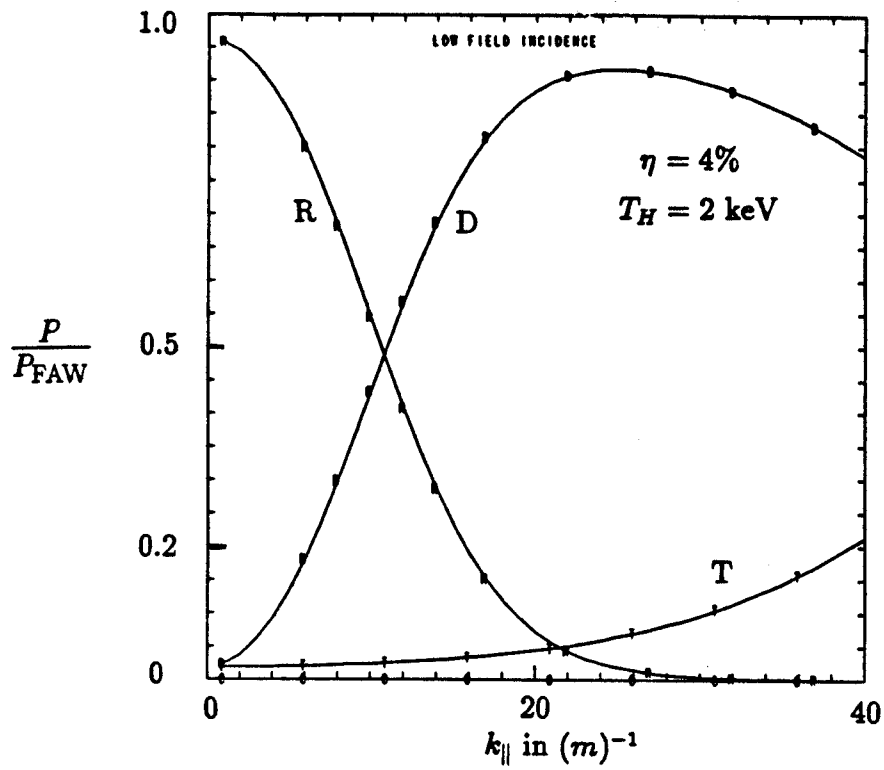


Figure 1a

ALCATOR C-MOD TYPE PLASMA

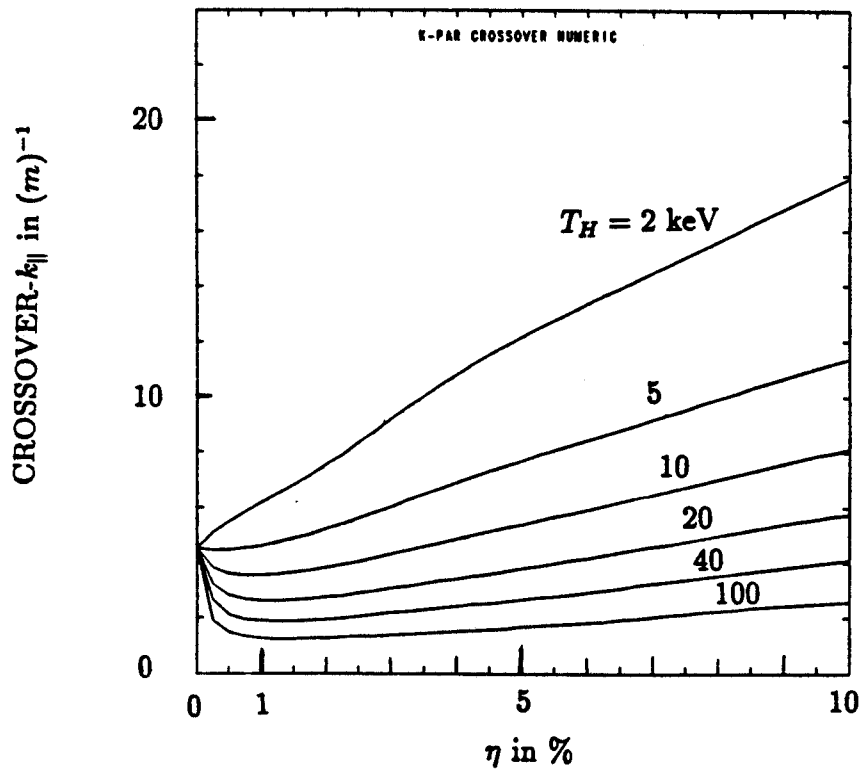


Figure 1b

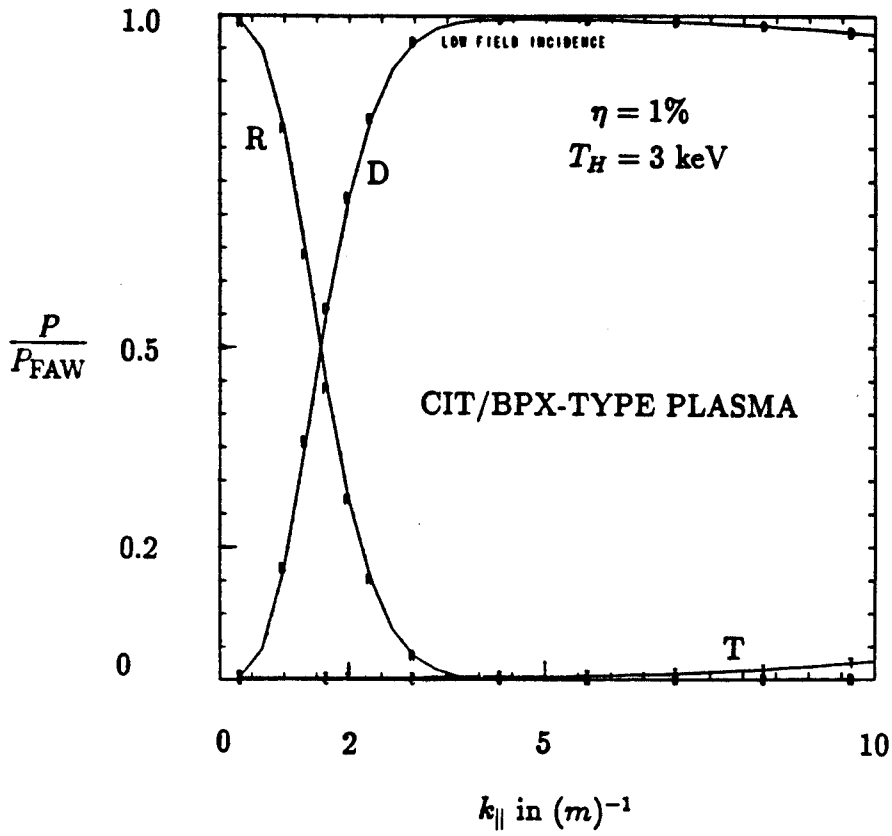


Figure 2a

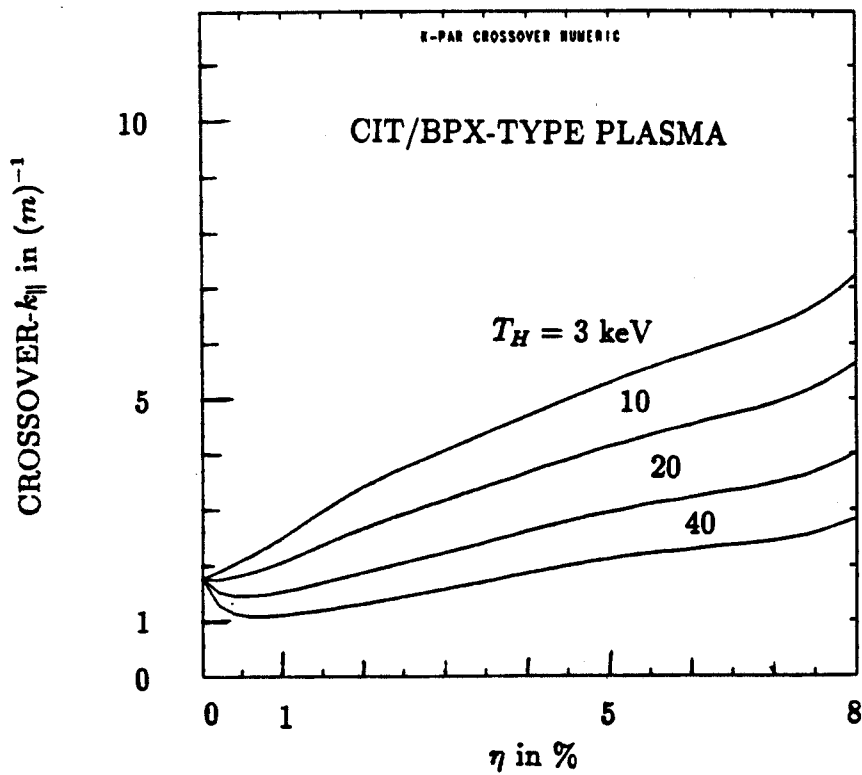


Figure 2b