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During Lower-Hybrid Current Drive

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I. Introduction

In steady state lower-hybrid current drive (LHCD), the toroidal current in tokamaks is sustained by fast electrons which absorb energy and momentum from externally injected waves. The dynamics of the fast electron population is well described by balancing wave induced quasilinear diffusion with collisional slowing down and pitch angle scattering off of fixed Maxwellian field particles. Here we consider a quasilinear-Fokker-Planck formulation which includes the wave induced radial transport of fast electrons, thus generalizing the radially local, velocity space treatments of LHCD [1,2,3].

The best current drive efficiencies for LHCD experiments are achieved when the wave spectrum launched into the plasma is narrow and close to the accessibility limit. For central electron temperatures up to a few kev, waves launched near the accessibility limit are very weakly damped and there results a significant "spectral gap" which must be filled before the waves can Landau damp on electrons. Because of the weak dissipation, the ray trajectories of the waves can make several toroidal transits and suffer numerous radial reflections. It has been shown that by including toroidal effects in the ray dynamics, the poloidal mode numbers of the rays can upshift [4] and thus fill the spectral gap [5]. The fields required to bridge the spectral gap thus have a significant poloidal component, which will contribute to the radial $E \times B$ drift of resonant electrons.

II. The Fokker-Planck Equation

In the absence of the RF fields, the electron orbits are assumed to follow the magnetic field lines with a constant parallel velocity, u. We consider the evolution of the electron distribution function, $f(u, v_\perp, \rho, t)$, where $\frac{1}{2}v_\perp^2$ is the perpendicular energy and ρ is a radial variable which labels the magnetic flux surfaces. For a small inverse aspect ratio tokamak with circular flux surfaces, we take $\rho = r + \Delta(r) \cos \theta$, where $\Delta(r)$ is the Shafranov shift. The evolution of f is given by

$$\frac{\partial}{\partial t}f = (\frac{\partial}{\partial t})_{ql}f + (\frac{\partial}{\partial t})_c f \quad , \tag{1}$$

which represents a balance between RF quasilinear diffusion and collisional pitch angle scattering and slowing down. Here, $(\partial/\partial t)_c$ is the linearized collision operator for fast electrons slowing down and pitch angle scattering in $(u, v_\perp)$ space, as given in various references (see e.g. [3]). The collisional contribution to radial transport has a negligible effect on the confinement of fast electrons and can be ignored; hence the collision operator does not act on the ρ dependence of f. The quasilinear operator is

$$(\frac{\partial}{\partial t})_{ql}f = \frac{1}{\rho}\frac{\partial}{\partial\rho}\rho[D_{\rho\rho}\frac{\partial}{\partial\rho} + D_{\rho u}\frac{\partial}{\partial u}]f$$
$$+ \frac{\partial}{\partial u}[D_{uu}\frac{\partial}{\partial u} + D_{u\rho}\frac{\partial}{\partial\rho}]f \quad . \tag{2}$$

Assuming that the RF fields remain in the slow wave (electrostatic) polarization, the quasilinear diffusion coefficients are:

$$D_{uu} = \frac{\pi e^2}{2m_e^2} \langle \sum_s |\Phi_s|^2 \delta(u k_\parallel^s - \omega)(k_\parallel^s)^2 \rangle$$

$$D_{\rho u} = \frac{\pi e}{2m_e B_o} \left\langle \sum_s |\Phi_s|^2 \delta(u k_\parallel^s - \omega) k_\parallel^s k_\rho^s \right\rangle$$

$$D_{\rho\rho} = \frac{\pi}{2B_o^2} \left\langle \sum_s |\Phi_s|^2 \delta(u k_\parallel^s - \omega)(k_\rho^s)^2 \right\rangle \quad , \tag{3}$$

where $D_{u\rho} = D_{\rho u}$ and B_o is the toroidal field on axis. The angular brackets in (3) represent a flux surface average. The RF scalar potential, $\Phi_{rf}(\mathbf{x}, t)$, is represented in eikonal form,

$$\Phi_{rf} = \frac{1}{2} \sum_s \Phi_s(\mathbf{x}) \exp i \left(\int^{\mathbf{x}} d\mathbf{x}' \cdot \mathbf{k}_s(\mathbf{x}') - \omega t \right) + \text{c.c.} \quad , \tag{4}$$

where Φ_s and $\mathbf{k}_s$ are assumed to be slowly varying. We have also defined $k_\parallel^s = \mathbf{k}_s \cdot \mathbf{b}$ and $k_\rho^s = \mathbf{b} \times \mathbf{k}_s \cdot \nabla \rho$, where $\mathbf{b}$ is the spatially varying unit vector along the equilibrium magnetic field. The basic physics of (2) is simple; resonant electrons experience a diffusion in u, due to the wave's parallel field, as well as a diffusion in ρ, due to the radial component of the $E_{rf} \times B$ drift. Because these two processes are coupled, the quasilinear operator also includes cross flows that are proportional to $D_{\rho u}$ and $D_{u\rho}$. We use the previously developed ray-tracing model of Bonoli and Englade [5], to determine the fields inside the plasma, thereby allowing us to evaluate the radial flow of fast electrons.

III. The Radial Flux

The radial flux of fast electrons, Γ_{rf}, is obtained by integrating (1) over the velocity space coordinates,

$$\Gamma_{rf} = - \int 2\pi v_\perp dv_\perp du \left(D_{\rho\rho} \frac{\partial}{\partial \rho} + D_{\rho u} \frac{\partial}{\partial u} \right) f \quad . \tag{5}$$

To calculate Γ_{rf} we ignore the enhancement of the perpendicular energy of f due to the pitch angle scattering of resonant electrons with large parallel energy [2,3]. One then finds that

$$\Gamma_{rf} = \int du [V_\rho F(u, \rho) - D_\rho \frac{\partial}{\partial \rho} F(u, \rho)] \quad , \tag{6}$$

where

$$V_\rho(u, \rho) = \frac{u D_c D_{\rho u}}{v_e^2 (D_c + D_{uu})} \tag{7}$$

$$D_\rho(u, \rho) = D_{\rho\rho} - \frac{D_{\rho u}^2}{D_{uu} + D_c} \quad . \tag{8}$$

Here $D_c = (Z_i + 2)\nu_o v_e^5 / u^3$, $\nu_o = \omega_{pe}^4 \ln \Lambda / (4\pi n_e v_e^3)$, Z_i is the ion charge state, $v_e = \sqrt{T_e(\rho)/m}$

$$F(u, \rho) = N \exp \left(\int^u \frac{-D_c u du}{v_e^2 (D_c + D_{uu})} \right) \quad . \tag{9}$$

Since there are relatively few electrons in the tail, we may take $N = n_e / (v_e \sqrt{2\pi})$, where n_e is the local electron density. Because D_{uu} is so large, electrons are diffused rapidly in the resonant region, where $F(u, \rho)$ develops a tail. It is therefore appropriate to consider the following velocity averages of V_ρ and D_ρ:

$$\bar{V}_\rho(\rho) = N_T^{-1} \int du V_\rho(u, \rho) F_T(u, \rho) \tag{10}$$

$$\bar{D}_\rho(\rho) = N_T^{-1} \int du D_\rho(u, \rho) F_T(u, \rho) \tag{11}$$

where $F_T(u, \rho) = F(u, \rho) - (n_e / v_e \sqrt{2\pi}) \exp(-u^2 / 2v_e^2)$ and $N_T(\rho) = \int du F_T(u, \rho)$.

The quantity $\bar{V}_\rho$ is the average convection velocity of resonant electrons and, similarly, $\bar{D}_\rho$ is their average radial diffusion coefficient. The radial convection depends on the amount of momentum absorbed from the waves and the ratio of $k_p/k_\parallel$ at resonance; since the momentum absorbed from the waves is balanced by the momentum destroyed in collisions with the bulk plasma, the radial convection is proportional to D_c and independent of the wave amplitude. There is generally a large asymmetry in the k_p spectrum, because toroidal effects on the wave propagation tend to fill in the spectral gap. In general, the upshifted spectrum, which fills the spectral gap, drives an outward radial convection.

IV. Numerical Results

Let us consider the Alcator C experiment, previously simulated by [5]. The parameters are $n_{eo} = 7.5 \times 10^{13}$ cm^{-3}, $T_{eo} = 1.5$ kev, $T_{io} = 0.7$ kev, $a = 16.5$ cm, $R_o = 64$ cm, $I_p = 170$ kA, and $B_o = 10$T. Here n_{eo}, T_{eo}, and T_{io} are the peak values of the electron density, electron temperature, and ion temperature, I_p is the toroidal plasma current, and a is the minor radius of the plasma. The assumed profiles are described in [5]. The RF frequency is 4.6 GHz (i.e. $\omega/2\pi$). The Brambilla power spectrum is modeled by

$$S(n_\parallel) = \begin{cases} S_a & \text{if } n_a < n_\parallel < n_b \\ S_b & \text{if } n_b < n_\parallel < n_c \\ 0 & \text{otherwise} \end{cases} \qquad (12)$$

where $n_\parallel \equiv ck_\parallel/\omega$. For Alcator C, we take $n_a = 1.25$, $n_b = 2.0$, and $n_c = 7.0$; S_a and S_b are determined so that 70 percent of P_{in} is launched between n_a and n_b. (Note, we have ignored any power which couples into the plasma at negative $n_\parallel$.) For $P_{in} = 440$ kW, we found that the total power resonantly absorbed by electrons was $P_{rf} = 390$ kW, with the remaining 50 kW being damped non-resonantly through electron-ion collisions in the plasma periphery.

The parameters for the JT60 current drive experiments are [6]: $n_{eo} = 3.0 \times 10^{13}$ cm^{-3}, $T_{eo} = 3.0$ kev, $T_{io} = 3.0$ kev, $a = 70$ cm, $R_o = 310$ cm, $I_p = 1.5$ MA, and $B_o = 4.5$T. The RF frequency is 2 GHz. The power spectrum of the multi-junction waveguide array is very narrow and is modeled by (12), where $n_a = 1.25$, $n_b = 1.75$, and $n_c = 4.75$; S_a and S_b are determined so that 80 percent of P_{in} is launched between n_a and n_b. For $P_{in} = 4.6$ MW, the total power resonantly absorbed by electrons was $P_{rf} = 4.2$ MW, the remainder being absorbed non-resonantly through collisions.

The results of our numerical simulations are given in Figures 1 and 2. For both Alcator C and JT60 the radial convection velocity is outward across most of the plasma and has a broad maximum of nearly 1 m/sec. In both cases, the radial diffusion is found to be highly localized near the magnetic axis. For JT60, the peak of the diffusion profile can be quite large, nearly 1 m^2/sec.

V. Acknowledgements

VI. References

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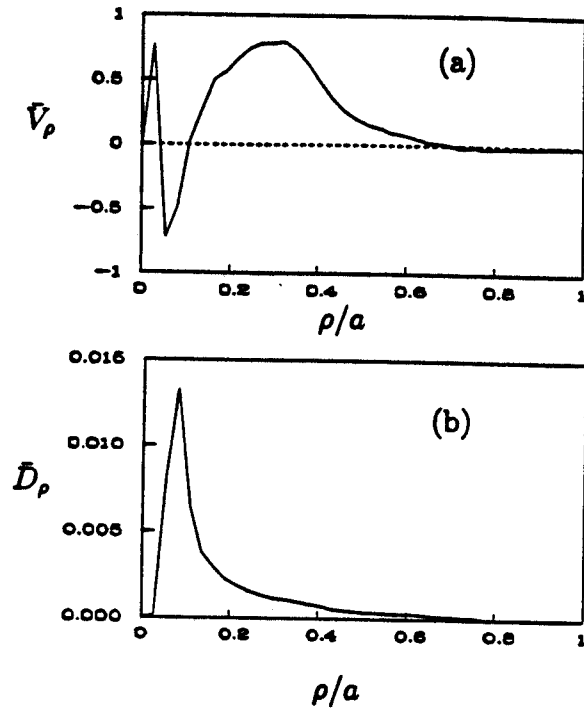[6] Y. Ikeda et al., Nucl. Fusion 29, 1815 (1989).

Figure 1: Model results for Alcator C; radial profiles of (a) average radial convection velocity, $\bar{V}_\rho$ (m/sec) and (b) average radial diffusion coefficient, $\bar{D}_\rho$ (m^2/sec).
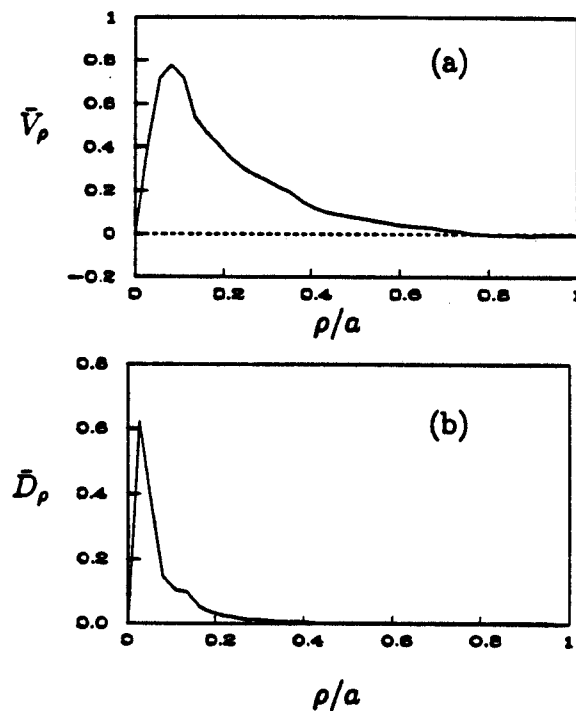

Figure 2: Model results for JT60; radial profiles of (a) average radial convection velocity, $\bar{V}_\rho$ (m/sec) and (b) average radial diffusion coefficient, $\bar{D}_\rho$ (m^2/sec).