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Abstract

The presence of cyclic temperature fluctuations in the plasma, driven by either sawteeth effects, marfes or other non-classical phenomena muddles the process to control the thermal instability. Furthermore, the power fluctuations corresponding to the changes in temperature profiles could adversely affect the lifetime of components in reactors due to thermal cycling. Methods for minimizing the effects, both on the thermal instability control method and the fusion power variations, are discussed. Detailed $1\frac{1}{2}-D$ *calculations are performed, and the results are justified using simplified models.*

1 Introduction

The feedback system of a burn control scheme based on auxiliary power modulation or any other control mechanism,[1,2] is effected by both the real temperature excursions as well as by benign fluctuations such as sawteeth. During sawtooth activity the temperature profile changes (flattens during the sawtooth collapse)[3,4] resulting in changes in the fusion yield changes during the collapse.

Previous analysis of plasma burn control have not considered the effect of instabilities such as sawteeth on the burn control system. The analyses were primarily based on volume averaged (0−D) transport models. These models, even though they provide a good global picture of the burn control problem, do not give any profile information. Recently the investigation of profile effects on burn control has gained increased interest and new $1\frac{1}{2}$−D computer codes are being developed in order to investigate the effects of various diffusion coefficient profiles and of various off axis heating profiles.[5]

In nearly ignited plasmas with auxiliary heating, and in tokamaks with steady state current drive, the characteristics of sawtooth oscillations will be different than in ohmically heated machines. In present fusion experiments it is observed that the sawtooth period increases as auxiliary power in the form of RF is supplied to the plasma.[6,7] The results presented in this paper are independent of the sawtooth repetition rate. Since the sawtooth repetition rate is the parameter that will be most affected by RF heating, the following results are valid regardless of how auxiliary heating affects the sawtooth activity.

Another cyclical perturbation that would interfere with the burn control feedback and could result in fusion power fluctuations is the temperature drop/density rise due to pellet injection. This perturbation is characterized, similarly to the sawteeth crashes, by conservation of plasma pressure. The model developed in this paper can also be used to analyze the effects of pellet injection.

The paper is organized as follows. First, a simple analytical sawtooth model for the temperature profile is developed and the relative change in fusion power production due to sawtooth oscillations is investigated. In section 3 the effects of sawteeth on fusion power production is investigated using the variational $1\frac{1}{2}$−D transport code MITra.[5]

2 Analytical model

In order to gain some insight into the sawtooth effects on plasma behavior with regard to burn control a simple analytical model is presented.

Typical temperature profiles before and after the sawtooth collapse are shown on Fig. 1. The solid line represents the profile before the collapse

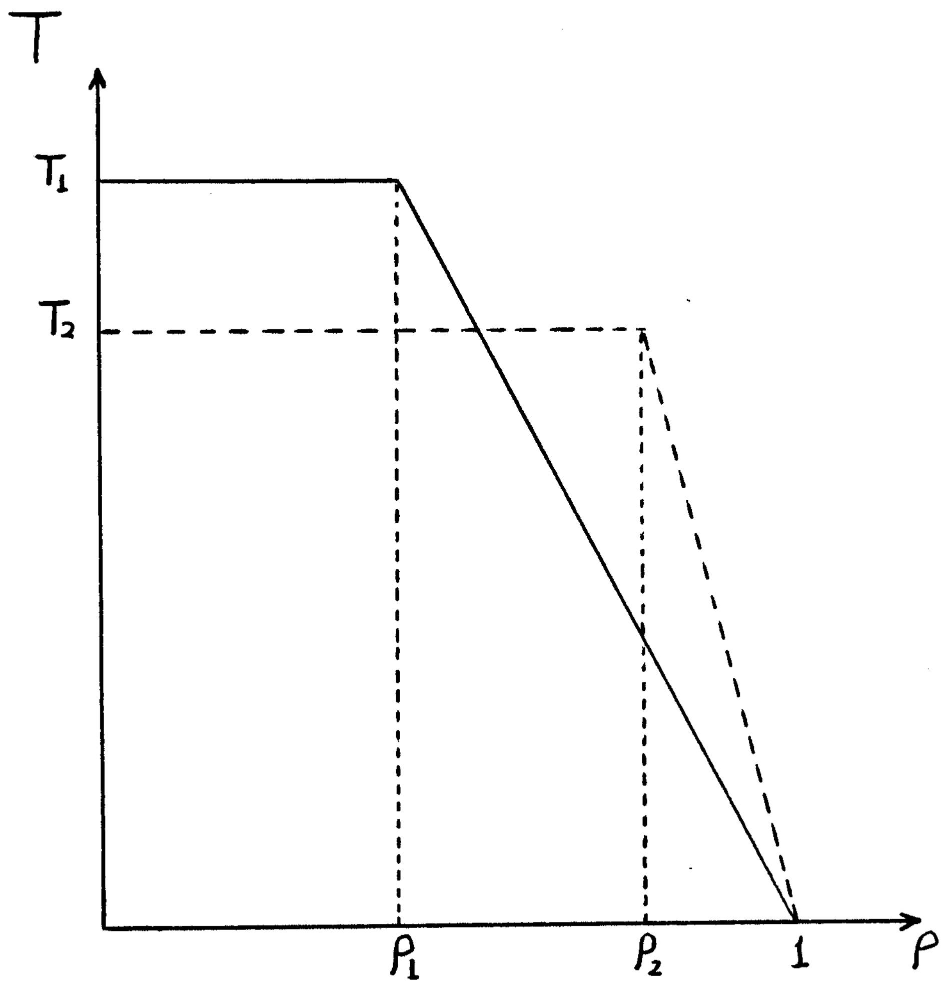


Figure 1: The simplified temperature profile before and after the sawtooth collapse used in the analytical model.

and the dotted line corresponds to the profile after the collapse. The peak temperature before the collapse is T_1, and the peak temperature after the collapse is T_2. In order to perform the calculations analytically the temperature profiles are characterized by a trapezoidal shape as shown on Fig. 1. In this analytical model it is assumed that the plasma density remains constant during the collapse.

The change in the profile shape is characterized by certain conservation quantities in the plasma. In particular the plasma energy $\int nTdV$ is conserved during the collapse.

By specifying the profile shape prior to the sawtooth collapse (i.e. by giving the value of the parameter ρ_1 in Fig. 1) and by imposing the condition that the plasma energy is conserved during the collapse, the value of the parameter ρ_2 (see Fig. 1) can be determined as follows.

The conservation of energy during the collapse may be expressed as

$$\int_V (nKT)_{t_-}\, dV = \int_V (nkT)_{t_+} dV \tag{1}$$

where $(nkT)_{t_-}$ is the energy density right before the collapse and $(nkT)_{t_+}$ is the energy density right after the collapse. In cylindrical geometry the above Eq. 1 can be written as

$$T_1\rho_1^2 + \int_{\rho_1}^{1} \left(\frac{T_1}{\rho_1 - 1}\right)(\rho - 1)\rho d\rho = T_2\rho_2^2 + \int_{\rho_2}^{1} \left(\frac{T_2}{\rho_2 - 1}\right)(\rho - 1)\rho d\rho \tag{2}$$

By performing the integrals indicated in Eq. 2 it is obtained

$$\frac{2}{3}\left(T_1\rho_1^2 - T_2\rho_2^2\right) + \frac{1}{6}\left(T_1\rho_1 - T_2\rho_2\right) + \frac{1}{6}\left(T_1 - T_2\right) = 0 \tag{3}$$

which is a quadratic equation in ρ_2 with the solution

$$\rho_2 = \left(-\frac{1}{2}T_2 + \left(\frac{1}{4}T_2^2 - 8T_2C\right)^{1/2}\right)/4T_2 \tag{4}$$

where

$$C = -2T_1\rho_1^2 - \frac{1}{2}T_1\rho_1 - \frac{1}{2}(T_1 - T_2) \tag{5}$$

In summary, this simple sawtooth model determines the change in the temperature profile shape during the sawtooth collapse by giving the shape prior to the collapse, and the drop in the central temperature.

2.1 Sawtooth effects on fusion power production

The change in the temperature profile due to sawteeth results in changes in the fusion power production. In general the relative change in fusion power production is a function of the operating temperature . The primary reason for this change is the temperature dependance of the fusion reactivity σv. By assuming that the fusion reactivity has the form

$$\langle \sigma v \rangle \sim T^{\overline{\nu}} \tag{6}$$

where the exponent $\overline{\nu}$ is temperature averaged

$$\overline{\nu} = \frac{1}{\int_A T dA} \int_A \nu(T) T dA \tag{7}$$

with $\langle Q \rangle \equiv \frac{1}{A} \int_A Q dA$ Eq. 7 becomes

$$\overline{\nu} = \frac{1}{\langle T \rangle A} \int_A \nu(T) T dA \tag{8}$$

With the expression for $\langle \sigma v \rangle$ given by Eq. 6 the relative change in fusion power production is given by

$$\Delta P = \frac{P_1 - P_2}{P_1} \tag{9}$$

$$P_1 = T_1^{\overline{\nu}} \rho_1^2 + \int_{\rho_1}^{1} \left(\frac{T_1}{\rho_1 - 1} \right)^{\overline{\nu}} (\rho - 1)^{\overline{\nu}} \rho d\rho \tag{10}$$

$$P_2 = T_2^{\overline{\nu}} \rho_2^2 + \int_{\rho_2}^{1} \left(\frac{T_2}{\rho_2 - 1} \right)^{\overline{\nu}} (\rho - 1)^{\overline{\nu}} \rho d\rho \tag{11}$$

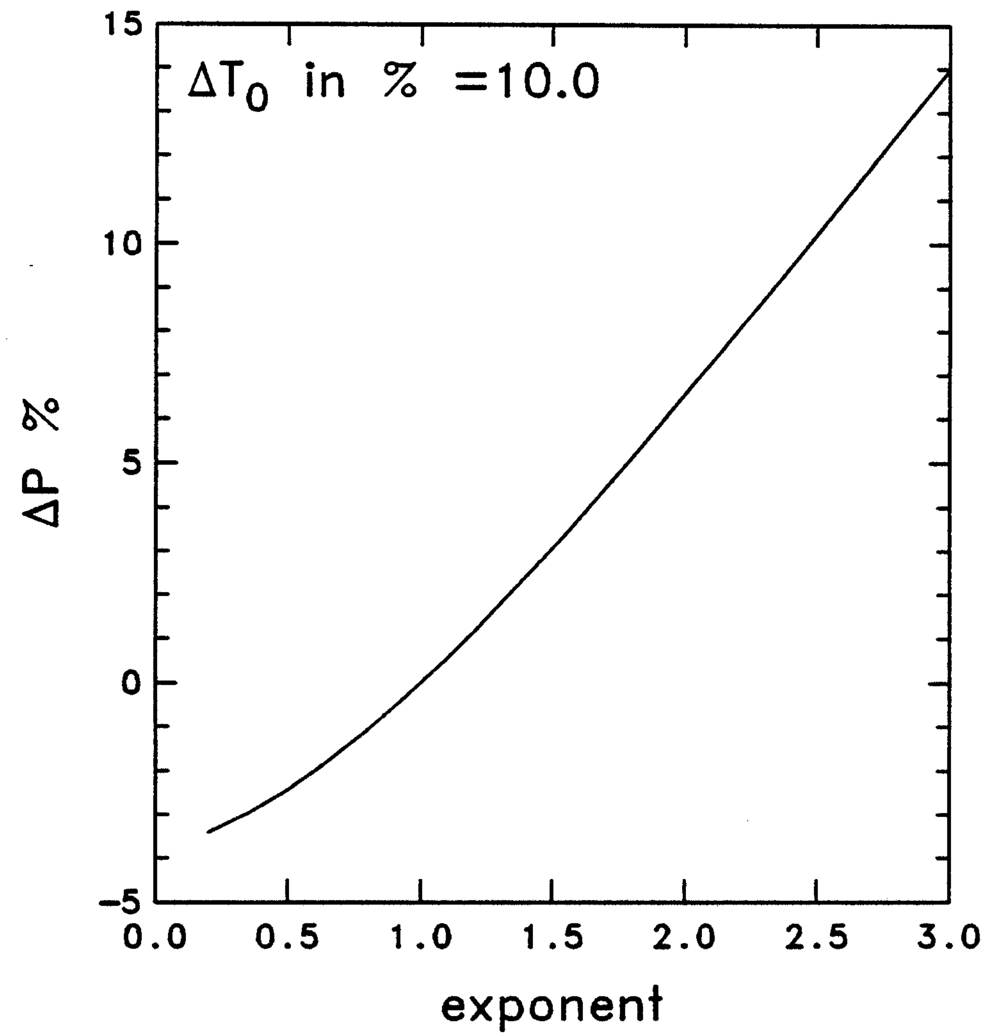


Figure 2: Relative change in fusion power during a sawtooth crash (10% drop in the central temperature) as a function of the exponent $\overline{\nu}$ which characterizes the σv dependance on temperature.

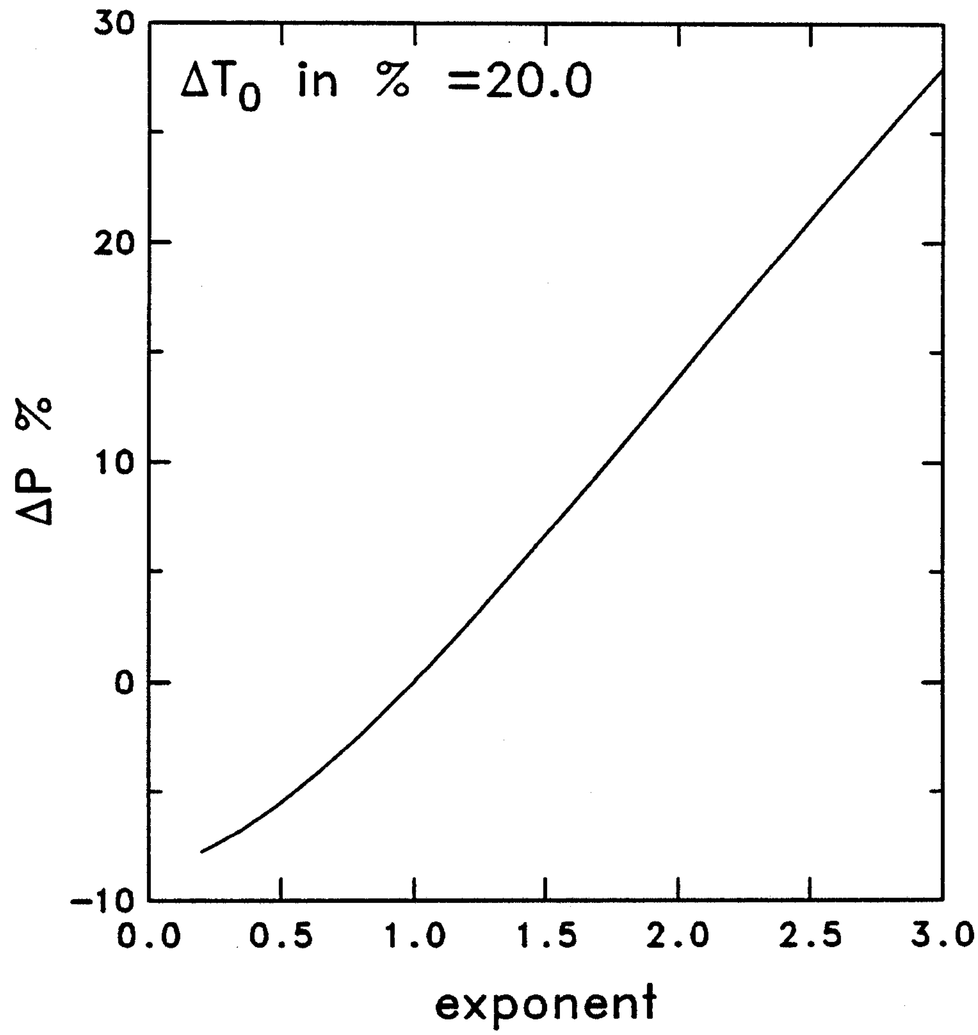


Figure 3: Relative change in fusion power during a sawtooth crash (20% drop in the central temperature) as a function of the exponent $\bar{\nu}$ which characterizes the σv dependance on temperature.

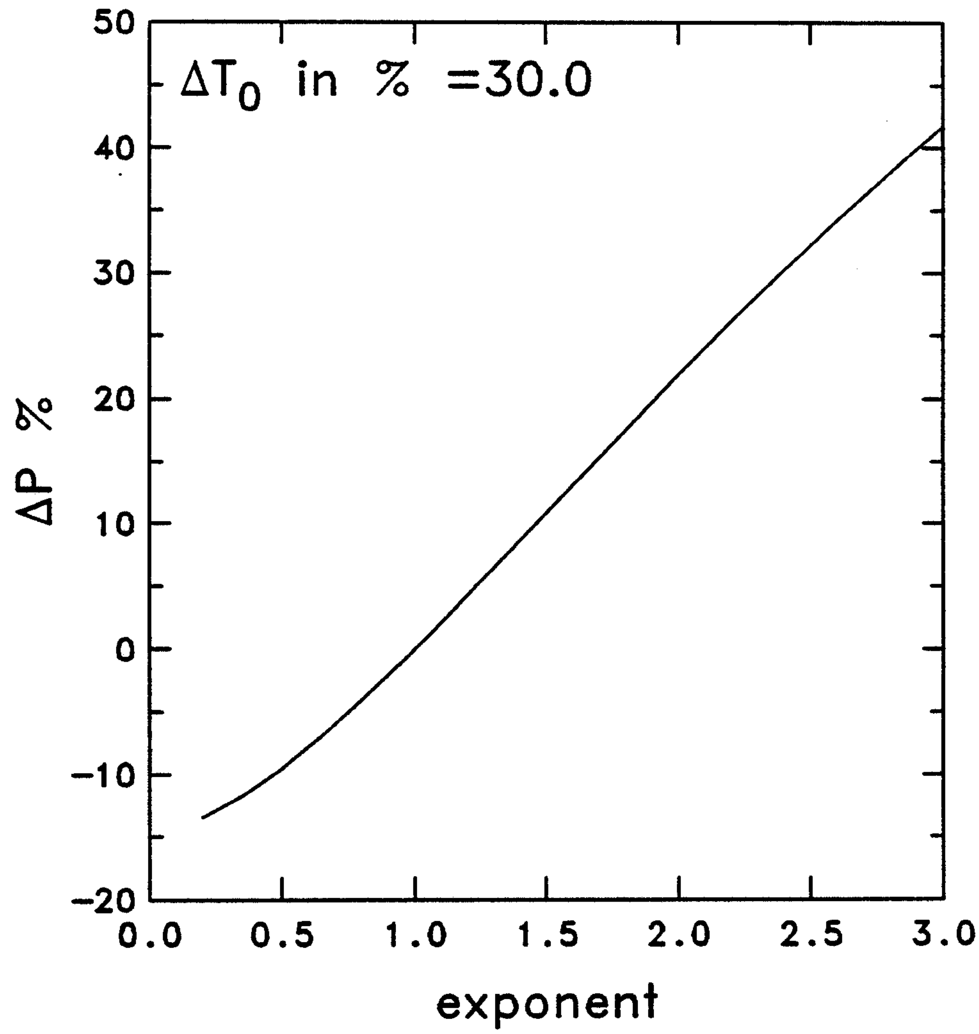


Figure 4: Relative change in fusion power during a sawtooth crash (30% drop in the central temperature) as a function of the exponent $\bar{\nu}$ which characterizes the σv dependance on temperature.

By assigning values to T_1, T_2 and ρ_1, Eq. 4 can be solved for ρ_2. In turn ΔP can be calculated from Eq. 9 as a function of the exponent $\overline{\nu}$. Figures 2,3,4 give the values of ΔP in % as a function of the exponent $\overline{\nu}$ for a 10%, 20%, and 30% central temperature drop (where $\Delta T = (T_1 - T_2)/T_1$). Note that the change in the fusion power due to sawteeth becomes zero for $\overline{\nu} = 1$. With the assumed temperature dependance of $\sigma v \sim T^{\overline{\nu}}$ the temperature at which $\overline{\nu} = 1$ is $\sim$ 26keV [8].

This model, even though it is very simplified, brings to light some very important issues regarding burn control. First, it is observed that operation in a temperature regime in which the fusion reactivity σv has an approximate linear dependance on temperature is advantageous since the fusion power fluctuations due to sawteeth activity are eliminated. The elimination of fusion power fluctuations due to sawteeth is particularly important in long pulse machines in which thermal fatigue of the first wall and blanket components can be a problem. From the point of view of burn control, operation in the regime in which $\Delta P = 0$ is advantageous since a feedback system which is based on fusion power (neutron) detection will not be affected by the benign sawtooth fluctuations. However, since the temperature at which $\sigma v \sim T$ is high, it is certain that such an operating point will be stable and thus it will not require active burn control. Second, and somewhat parallel to the first observation is the characterization of the feedback control system as a function of the relation between the relative change in temperature and the relative change in fusion power (neutrons). By investigating Figs. 2, 3, 4 it is apparent that the relative change in the fusion power production equals the relative change in cental temperature due to sawteeth at an exponent value of $\sim$ 2.5.

3 Results from an $1\frac{1}{2}$–D code

In order to investigate the relation between temperature change and fusion power change we impose sawtooth fluctuations that are characterized by a prescribed central temperature drop and then observe quantities such as the change in fusion power, and the plasma beta.

In order to investigate these effects we use the new $1\frac{1}{2}$−D plasma transport code MITra whose thermodynamic and equilibrium calculations are based variational techniques [5,9] The sawtooth model in MITra requires the characterization of the mixing radius. However, in this analysis we assume that the profiles after the sawtooth collapse changes up to $\rho = 1$ (i.e. the mixing radius is at $\rho = 1$). Typical before and after profiles during a sawtooth collapse are shown on Fig. 5. Note that the central temperature drops and the profile broadens in order to conserve the total plasma energy.

The procedure by which the results will be obtained is as follows. First the plasma (temperature, density) evolves to an equilibrium and then a sawtooth is imposed (see Fig. 6) The fusion power is calculated at position 1 (before collapse) and position 2 (after collapse)

In designing a burn control system for an ignited fusion reactor it is important to chose the appropriate observable(s). In a fusion reactor observables such as the fusion power (neutron) production , and the temperature can be used. The decision as to which observable is better suited for supplying information to the feedback system is complicated by factor such as signal error and diagnostic signal delay. However, in the presence of fluctuations such as sawteeth a new piece of information is supplied by the relation between the characteristics of the various observables. For example, in the case of sawtooth fluctuations it is possible to relate the change in the central temperature due to the sawtooth collapse to the change in fusion power due to the alteration of the temperature and density profiles.

In Fig. 7 the change in the fusion power production as a result of a sawtooth crash is plotted as a function of the volume averaged operating temperature for sawteeth characterized by a 10% and a 20% drop of the central temperature. If a burn control system is to be designed for operation at the temperatures indicated in Fig. 7 then the optimum observable is a function of the operating temperature. For example, if operation at $\langle T \rangle =$

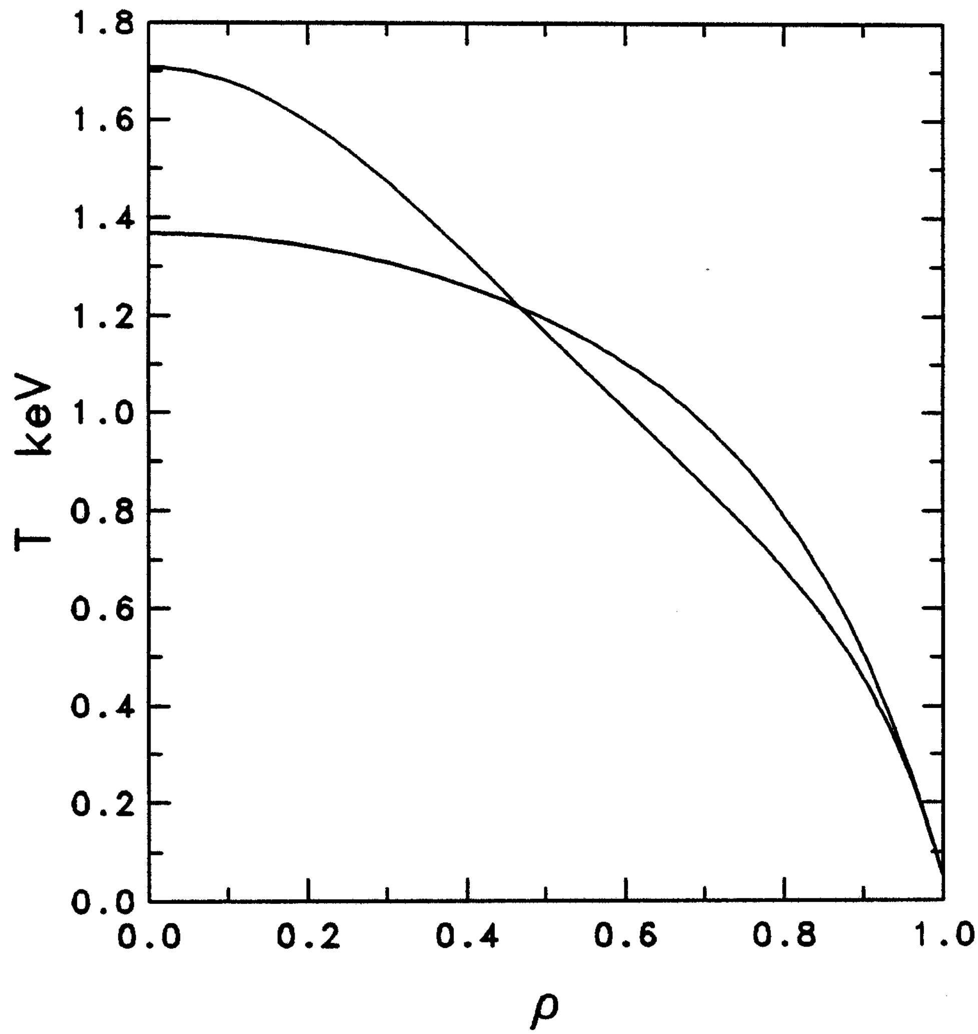
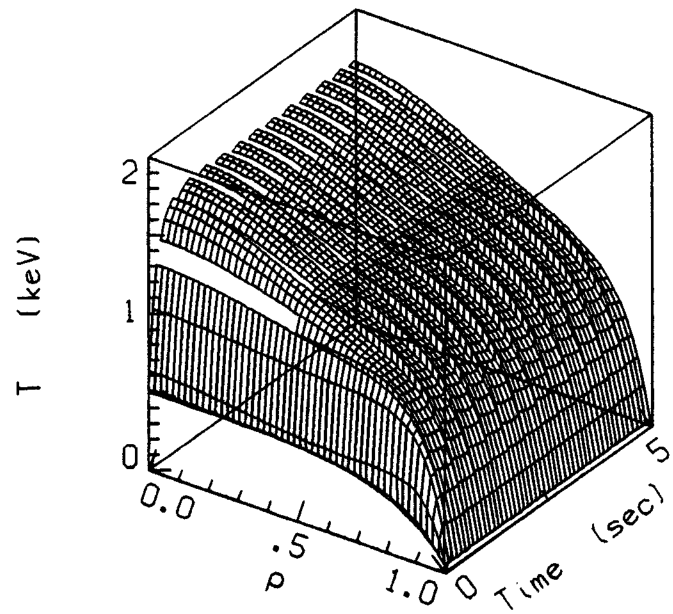


Figure 5: Typical change in profiles due to a sawtooth crash.



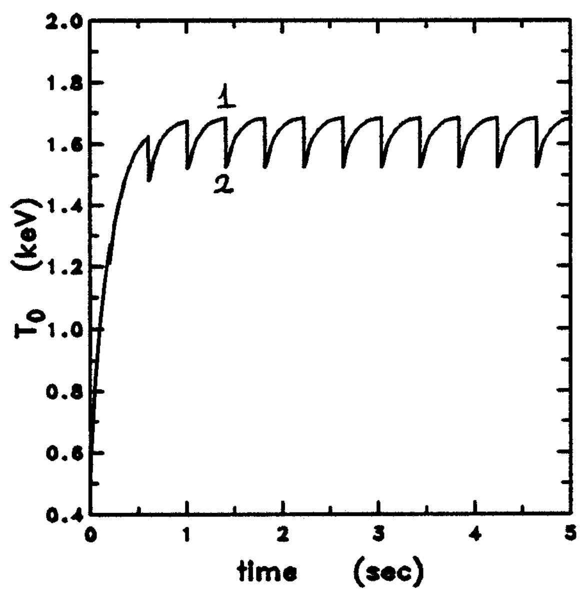


Figure 6: Evolution of the plasma temperature profile (top figure) and of the central temperature (bottom figure) in a sawtoothing discharge.

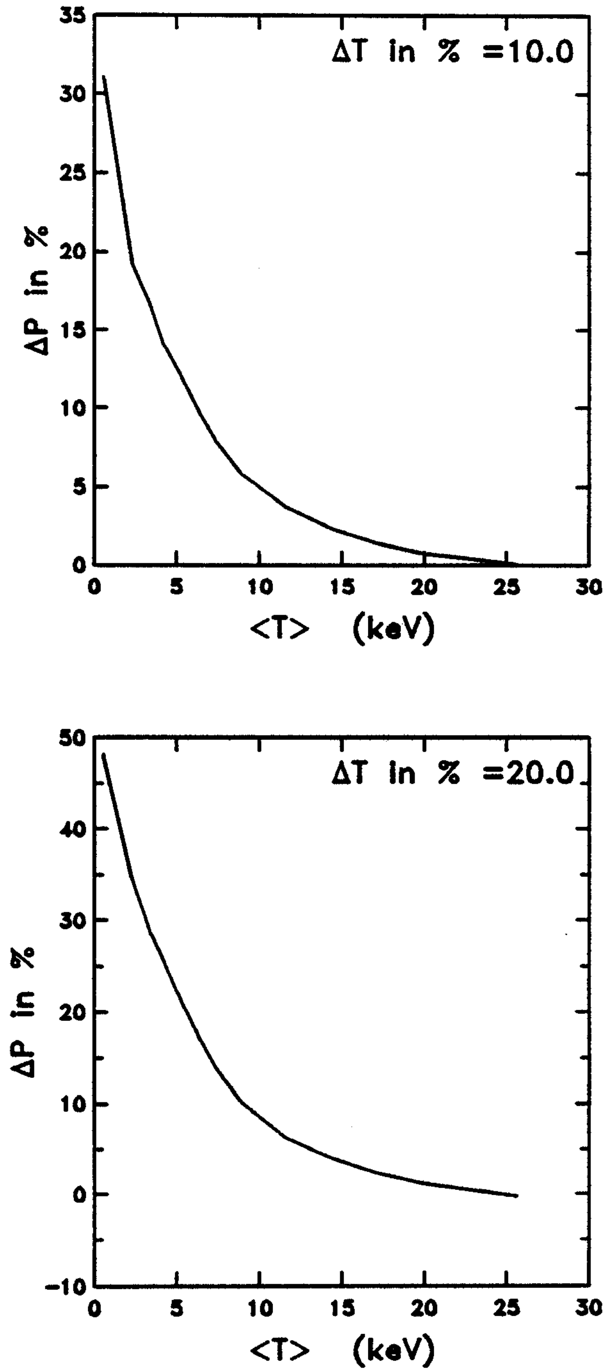


Figure 7: The percent change in fusion power production during a sawtooth crash as a function of the volume averaged plasma temperature. The top figure corresponds to a 10% drop of the central temperature during the sawtooth crash. The bottom figure corresponds to a 20% drop in the central temperature during the sawtooth.

4 keV is desired then it is advantageous to use a feedback system with temperature feedback. However, if operation at higher temperatures is desired it is advantageous to control the burn with fusion power (neutron) feedback. Note that the relative change in fusion power during a sawtooth crash decreases as the operating temperature increases and it becomes zero at $\sim$ 25 keV. This is in agreement with the results obtained from the simple analytic model presented in section 2 since the fusion reactivity $\langle \sigma v \rangle \sim T$ at approximately 25 keV.

In a fusion reactor that is characterized by sawtooth fluctuations it is important to operate in the regime where the periodic sawtooth fluctuations do not introduce thermal cyclic heating of the first wall and blanket components. From Fig. 7 it is seen that such a regime is characterized by high temperatures. However, these high temperatures might not be accessible due to violations of the beta and the wall loading limits. In particular the Troyon β limit is violated for average temperatures above 15 keV or for peak temperatures above $\sim$ 30 keV. This point is another clear demonstration of the relation between the choice of the operating point, the burn control system, and the technological issues that must be addressed.

4 Conclusions

It has been shown that by operating at relatively high plasma temperature it is possible to avoid the problem of interference with the feedback system to control the plasma, including the thermal instability. It has been determined that at relatively high temperatures using the fusion power measurement for feedback control is the preferred control method to minimize the effect of the cyclic fluctuations, while at lower temperatures (below 4-5 keV) the temperature is the preferred measurement. Furthermore, operation at relatively high temperature minimizes the cyclic changes in fusion power, which, if relatively frequent, could limit the lifetime of reactor components, such as the first wall and the blanket.

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