PFC/JA-90-7

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October, 1989

Paper presented at the IAEA Technical Committee Meeting on ICRH/Edge Physics, October 2-5, 1989, Garching, Federal Repulic of Germany.

To be published in Fusion Engineering and Design.

This work was supported by the U. S. Department of Energy Contract No. DE-AC02-78ET51013. Reproduction, translation, publication, use and disposal, in whole or in part by or for the United States government is permitted.

PARAMETRIC INSTABILITIES IN THE TOKAMAK EDGE PLASMA IN THE ION CYCLOTRON HEATING REGIMES

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Abstract

Parametric instabilities in the scrape-off layer (SOL) of tokamak plasmas are studied in the presence of ICRF and/or IBW rf fields. Growth rates and typical threshold rf fields are deduced for multi-ion species plasmas for pump frequencies in the vicinity of the first few ion cyclotron harmonics. Decay instabilities into ion Bernstein waves and quasi-modes are predicted to occur. Such decay waves may lead to edge heating and impurity generation.

I. Introduction

The possible excitation of parametric decay instabilities in the scrape-off layer of tokamak plasmas (SOL, namely the shadow of the limiter in a limiter defined plasma, or outside the X-point region in a diverted plasma) during ICRF (Ion Cyclotron Range of Frequencies) and/or IBW (Ion Bernstein Wave) heating is studied. Such processes may be particularly important in the vicinity of the rf antenna where theory predicts strong electric fields ($E_\perp \simeq 100 - 400$ V/cm for $P_{RF} \simeq$1 MW) during ICRF fast wave heating experiments [1-4]. In addition, strong rf fields have been measured (of the order of several 100 V/cm for $E_\perp$) during fast wave launching in the SOL at distances several meters from the antenna [3].

In the case of IBW excitation, even stronger fields may exist near the antenna due to the partial electrostatic nature of wave excitation ($E_\perp > 1$ kV/cm, $E_{||} \sim$200 V/cm) which may result in very strong parametric decay instability (PDI) excitation and ponderomotive force effects [5,6]. Particularly rich parametric decay phenomena has been recently observed in the DIII D, IBW Experiments [5]. In this paper we shall consider parametric excitation in both ICRF and IBW regime. Additional important nonlinear phenomena include rectification and harmonic generation in the sheath surrounding the Faraday shields [7]. The harmonics may propagate away as lower-hybrid resonance cones and suffer their own PDI [7]. Typical scrape-off layer plasma parameters are as follows: temperatures $T_e \sim T_i \sim 20$ eV, density $n_e \lesssim 10^{18} m^{-3}$, collision frequency $\nu_{ei} \simeq 4 \times 10^5 \mathrm{sec}^{-1}$; typical fast wave antenna dimensions are $L_z \simeq 20-40$ cm, and $L_y \sim 60-100$ cm, and $L_z \times L_y \simeq$100 cm $\times$ 40 cm for IBW antennas (i.e., DIIID). Thus, the pump wave spatial extent is large and convective thresholds are not prohibitively restrictive. The antenna often consists of two halves which may be fed either in or out of phase, thus further defining k_{oz} or k_{oy} (where sub-zero refers to the pump wave quantities, z is the toroidal direction, and y is the poloidal direction). However, this is not a particularly important restriction for the excitation of PDI. To a good approximation we take $k_{o\perp} \simeq 0$ since the pump wave is electromagnetic for fast wave launching and the decay waves are electrostatic modes with short wave lengths as compared to that of the pump wave. In the case of IBW pump wave excitation, the most important decay region may be the immediate vicinity of the antenna where large evanescent electric fields may exist, of order $E_{o\perp} \sim 1 - 3$ kV/cm , $E_{o||} \simeq 200$ V/cm [2] (where $\perp$ and $||$ refer to the components perpendicular and parallel with respect to B_o, the magnetic field). Therefore, even for IBW near the edge region, the dipole approximation (i.e., $k_o \simeq 0$) may suffice since for the decay waves $k_\perp r_{ci} \sim 1$, and for the pump wave

$k_{o\perp} r_{ci} << 1$. Thus, we take for the selection rules

$$\omega_o = \omega' + \omega'' \ , \ \vec{k}'_\perp + \vec{k}''_\perp \simeq 0 \ , \tag{1(a)}$$

where the prime and double prime refer to the low frequency mode and the lower sideband, respectively. Nevertheless, the finite $k_{o||}$ may, in some cases, be important in optimizing the damping of quasi-modes, hence maximizing the growth rate. Thus, in some cases the condition

$$k'_{||} + k''_{||} = k_{o||} \tag{1(b)}$$

may be necessary to consider the dissipative response terms (where $k_{o||} \simeq k_{oz}$).

The plan of the paper is as follows: In Section II, the parametric instability growth rate is deduced for a multi-ion species plasma. In Section III, the selection rules and conditions for PDI during fast wave excitation are considered. Growth rates and convective thresholds are given. In Section IV, application of this theory to PDI in the IBW regime is summarized (but not evaluated in any detail). Finally, in Section V the conclusions and connection to experimental observations are given.

II. Parametric Decay Instability Theory

The dispersion relationship of PDI in multi-ion species plasma has been given previously [7, 8], and may be summarized as follows:

$$\epsilon(\omega^-, \ k^-)\epsilon(\omega, k) = -\frac{1}{8}\sum_{\sigma,\eta} \left|\mu_\sigma - \mu_\eta\right|^2 \left[(\chi_\sigma - \chi_\sigma^-)(\chi_\eta - \chi_\eta^-)\right] \tag{2}$$

where ϵ is the dielectric constant, χ_σ is the susceptibility of species σ, and μ is the parametric coupling constant to be given later. Here the sideband mode, $\omega^- = \omega - \omega_o$, $\vec{k}^- = \vec{k} - \vec{k}_o \simeq \vec{k}$, is assumed to be an ion Bernstein wave (IBW), the low frequency decay wave $(\omega, \ \vec{k})$ may be another Bernstein wave or a quasi-mode (a type of dissipative mode such that $\omega - \Omega_i \simeq k_{||} v_{ti}$ or $\omega \simeq k_{||} v_{te}$, which exists only in the presence of the pump wave; here $v_{te} = (2T_e/m_e)^{1/2}$, $v_{ti} = (2T_i/m_i)^{1/2}$, $\Omega_i = eZ_iB_o/m_ic$, and σ, η refer to unlike charged particle species, including ions and electrons.

In Eq. 1 we have the following relationships:

$$\vec{E}_o = E_{ox}\hat{x} + E_{oy}\hat{y} + E_{oz}\hat{z} \tag{3(a)}$$

$$\epsilon(\omega, \ k) = 1 + \sum_j \chi_j(\omega, \ k) \tag{3(b)}$$

$$\epsilon^{-}(\omega^{-},\ k^{-}) = 1 + \sum_j \chi_j(\omega - \omega_o,\ k - k_o) \qquad 3(c)$$

$$\chi_j = \frac{1}{k^2\lambda_{Dj}^2}\left(1+\zeta_{oj}\sum_l \Gamma_l Z(\zeta_{lj})\right),\ \Gamma_l = I_l(b_j)e^{-b_j},\ \zeta_{oj} = \omega/k_{\|}v_{tj}\ ,\ b_j = k_\perp^2 r_{cj}^2, \zeta_{lj} = (\omega - l\Omega_j)/k_{\|}v_{tj}$$

where χ_j are the hot plasma linear susceptibility of particle species j, $r_{cj}^2 = v_{tj}^2/2\Omega_j^2, j = 1, 2, e$ refer to ion species 1 and 2 and to electrons (e) (we shall consider only two ion species in the plasma, such as deuterium and hydrogen, etc.). $Z(\zeta_j)$ is the Fried-Conte plasma dispersion function, $I_l(b_j)$ is the modified Bessel function of order l, and $\lambda_{Dj}^2 = v_{tj}^2/2\omega_{pj}^2$ is the Debye length of species j (squared).

The parametric coupling term in the dipole approximation is given by:

$$\mu_\sigma = \frac{eZ_\sigma}{m_\sigma}\left[\left(\frac{\vec{E}_{o\perp}\cdot\vec{k}_\perp}{\omega_o^2 - \Omega_\sigma^2} + \frac{E_{o\|}\ k_{\|}}{\omega_o^2}\right)^2 + \frac{|(\vec{E}_{o\perp}\times\vec{k}_\perp)\cdot\hat{z}|^2\ \Omega_\sigma^2}{(\omega_o^2 - \Omega_\sigma^2)^2\ \omega_o^2}\right]^{1/2} \quad . \qquad 3(d)$$

Here we recognize the first term as that due to polarization drift, the second term as the parallel drift, and the last term as the $\vec{E}_o \times \vec{B}$ drift. Here the upper sideband, $\omega^+ = \omega_o + \omega$, $\vec{k}^+ = \vec{k}_o + \vec{k}$, has been neglected. Such an assumption is valid as long as $\gamma'' < \omega$, $\gamma_0 < \omega$, where γ_0 is the parametric growth rate and γ'' is the linear damping rate at the sideband frequency. In general, these approximations are satisfied since $\gamma'' < \omega \sim \Omega_i$ and $\gamma_o < \Omega_i$ in most cases of interest. We note that in the case of direct IBW launching, the fields may be strong enough for driving lower hybrid soliton and filamentation instabilities (or the oscillating two stream instability). In such a situation, which we will not study here, both upper and lower sidebands must be retained.

Combining Eqs. 2 and 3(d), and assuming that we have resonant excitation of IBW at the sideband (i.e., $\epsilon(\omega - \omega_o) \simeq -i\left(\gamma_o + \gamma''\right)\ |\partial\epsilon/\partial\omega_R^-|$, where $\gamma'' = \epsilon_{IM}(\omega^-)\,/\,|\partial\epsilon/\ \partial\omega_R^-|$ is the damping rate at the sideband), we obtain the following:

$$\left(\gamma_o + \gamma''\right)\left|\frac{\partial\epsilon}{\partial\omega_R^-}\right| = Im\ \frac{1}{4}\left[\frac{e}{m_e}\ \left(\frac{((\vec{E}_{o\perp}\times\vec{k}_\perp)\cdot\hat{z})^2}{\Omega_e^2\ \omega_o^2} + \frac{E_{o\|}^2\ k_{\|}^2}{\omega_o^4}\right)^{1/2}\right.$$

$$\left. + \sum_{j=1,2}\frac{eZ_j}{m_j}\left(\frac{(\vec{E}_{o\perp}\cdot\vec{k}_\perp)^2}{(\omega_o^2 - \Omega_j^2)^2} + \frac{((\vec{E}_{o\perp}\times\vec{k}_\perp)\cdot\hat{z})^2}{(\omega_o^2 - \Omega_j^2)^2}\ \frac{\Omega_j^2}{\omega_o^2}\right)^{1/2}\right]^2\ \left[\frac{(\chi_e - \chi_e^-)\ (\chi_j - \chi_j^-)}{\epsilon(\omega,\ k)}\right]$$

$$+\left[\frac{eZ_1}{m_1}\left(\frac{(\vec{E}_{o\perp}\cdot\vec{k}_\perp)^2}{(\omega_o^2 - \Omega_1^2)^2} + \frac{((\vec{E}_{o\perp}\times\vec{k}_\perp)\cdot\hat{z})^2\Omega_1^2}{(\omega_o^2 - \Omega_1^2)^2\omega_o^2}\right)^{1/2} - \frac{eZ_2}{m_2}\left(\frac{(\vec{E}_{o\perp}\cdot\vec{k}_\perp)^2}{(\omega_o^2 - \Omega_2^2)^2} + \frac{((\vec{E}_{o\perp}\times\vec{k}_\perp)\cdot\hat{z})^2\Omega_2^2}{(\omega_o^2 - \Omega_2^2)^2\ \omega_o^2}\right)^{1/2}\right]^2$$

$$\times\left[\frac{(\chi_1 - \chi_1^-)(\chi_2 - \chi_2^-)}{\epsilon(\omega,\ k)}\right] . \qquad (4)$$

In the above, Im refers to the imaginary part, and R refers to the real part. For IBW excitation $E_{o\perp} \simeq E_{ox}$, and for fast wave replace $E_{ox} \rightarrow E_{oy}$ and $k_x \rightarrow k_y$, $k_y \rightarrow k_x$ (although near the antenna, both E_{ox} and E_{oy} may be comparable (1,2)). Above, in the electron driving term we neglected the polarization drift term, and in the ion driving term we neglected the parallel drift term ($E_{o\|}k_{\|}$). In addition, we assumed $\omega^2 \ll \Omega_e^2$. To proceed, we have to consider specific cases, i.e. we must specify which minority species (if any) is present, whether the driving term is E_{oy} (fast wave) or E_{ox} and E_{oz} (IBW), and what the value of ω_o/Ω_j is. In general, we shall consider the low frequency modes $\epsilon(\omega,\ k)$ to be quasi-modes, i.e., $|\epsilon_{Im}| \sim |\epsilon_R| \gg 1$. In such situations we may assume that $\chi_e \gg |\chi_e^-|$. This amounts to assuming that $1 \gg k^2\lambda_{De}^2\ |1 - k_\|^2\ \omega_{pe}^2\ /\ k^2\ (\omega'')^2|$ (since $\omega_{pe}^2\ /\ \Omega_e^2 \ll 1$ in the SOL) which is usually true since $k^2 \sim k_\perp^2 \sim r_{ji}^{-2}$, $|\omega^-| = \omega'' \sim l\Omega_j$, $|\omega^-| \gg |k_\|^-|v_{te}$. Usually in the first term we may also assume that $\chi_j \gg \chi_j^-$, which is true marginally since $\epsilon(\omega, k)$ corresponds to an ion cyclotron, or an electron Landau damped quasi-mode, ($\omega - l\Omega_j \sim k_\| v_{tj}$ or $\omega \sim k_\| v_{te}$) and the sideband is an IBW so that $|\omega^- - l\Omega_j| \gg k_\|^-\ v_{tj}$.

The thresholds are given by the condition

$$\gamma_o > \gamma'' \quad , \tag{5(a)}$$

which is a prerequisite for instability in a uniform, homogeneous, infinite medium. More appropriately, the instability is convective and the finite spatial extent of the pump wave amplitude determines the threshold. The threshold is given by

$$ln\left(\left|E''\right|^2\ /\ \left|E''_{th}\right|^2\right) = \int \frac{2\gamma(x)dx}{|v_{gx}^-(x)|} \simeq \frac{2\gamma_o\Delta x}{v''_{gx}} > 2\delta \tag{5(b)}$$

where v''_{gx} is the group velocity at the sideband IBW, Δx is the width of the unstable region, and where δ is a number of order 3 or greater, depending on whether we require "substantial" growth from thermal fluctuations ($\delta \simeq 3$, usually taken to be π for convenience) or actual pump depletion (in which case δ is larger, of order $\delta \gtrsim 10$). Here $\left|E''_{th}\right|^2$ is the thermal fluctuation level at the sideband, which in thermal equilibrium is deduced from [9, 10]:

$$|E_{th}(\omega'')|^2 = \frac{4\kappa T}{\omega''} \frac{Im\ \epsilon\ (\omega'',\ \vec{k}'')}{\left|\epsilon(\vec{k}'',\ \omega'')\right|^2} \tag{6}$$

which then has to be integrated over all frequencies and $\vec{k}$-space to obtain $|E''_{th}(x)|^2$.

The condition for pump depletion in simple, one dimensional, quasi-mode decay may be written as follows [11, 12]:

$$\frac{2\gamma_o\Delta x}{v''_{gx}} > ln\left(\frac{A_o}{A''}\right) \tag{7}$$

where $A_o/A'' = I_o v_{ox}/I_2'' v_{gx}'' = |E_o|^2 \ (v_{ox}/\omega_o)/|E''|^2(v_{gx}''/\omega'')$, where v_{ox} is the group velocity of the pump wave, and $I_o = |E_o|^2/\omega_o$.

III Parametric Decay Instabilities in the ICRF Fast Wave Regime

Here we consider fast wave launching where $E_o \simeq E_{oy} \simeq 100-400$ V/cm. Typical fast wave resonance conditions for a 3:1 aspect ratio tokamak ($R/a = 3$, where R is the major radius and a is the minor radius, in particular at the position of the antenna surface) are as follows:

(i) 2nd harmonic heating:

$$\omega \simeq 2\Omega_H(o) \simeq 2.66\ \Omega_H(a)$$

(note that with some D^+ ions present, $\omega_o \simeq 5.32\ \Omega_D(a)$ also)

(ii) minority heating:

$$\omega_o \simeq \Omega_H(o) \simeq 2.66\ \Omega_D(a) \quad \text{or}$$

$$\omega_o \simeq \Omega_{He-3}(o) \simeq 1.77\ \Omega_D(a)\ .$$

In the case of an $R/a = 4$ tokamak (such as ASDEX) the first condition changes to $\omega \simeq 2.50\ \Omega_H(a) = 5.0\ \Omega_D(a)$, while the minority regimes change to $\omega = \Omega_H(o) = 2.50\ \Omega_D(a)$ or $\omega = \Omega_{He-3}(o) = 1.56\ \Omega_D(a)$. In these cases, parametric decay into an ion Bernstein wave (IBW) and an ion cyclotron quasi-mode (ICQM), or an IBW and an electron quasi-mode (EQM) may occur. These processes are depicted in Figures (1-3). Here we assumed that $\lambda_{o\perp}$, the rf pump wavelength, is much longer than that of the decay waves so that $k_{0\perp} \ll k_\perp' \simeq k_\perp''$, and $k_\parallel \ll k_\perp$ for all modes of interest. Note that the case of second harmonic heating and minority heating with H^+ ions is highly susceptible to PDI, whereas for the case of $He-3$ minority, only decay into an IBW and an electron quasi-mode is allowed.

The growth rate of the decay instability into an ion cyclotron quasi-mode ($\omega - l\Omega_j \simeq k_\parallel v_{tj}$, $\omega \sim k_\parallel v_{te}$) and an IBW is deduced from Eq. (4). Within reasonable approximations, we obtain the following approximate expression to the growth rate:

(i) 2nd Harmonic Heating

$$\frac{\gamma}{\omega''} = -\frac{\gamma''}{\omega''} + \frac{1}{8}\left(\frac{k_x v_{oy}}{\omega_o}\right)^2 \frac{\pi^{1/2}\zeta_{oe}e^{-\zeta_{oe}^2}}{\left(\frac{n_H}{n_e}P_H + \frac{n_D}{n_e}P_D\right)\left(1 + \chi_{eIm}/\sum_j \chi_{jIm}\right)} +$$

$$+\frac{\theta}{8}\left(\frac{k_y v_{oy}}{\omega_o}\right)^2 \frac{\pi^{1/2}\frac{n_H}{n_e}\left(\frac{T_e}{T_H}\right)^{3/2}\left(\frac{m_H}{m_e}\right)^{1/2}\Gamma_1(b_H)\zeta_{oH}e^{-\zeta_{1H}^2}}{\left(\frac{n_H}{n_e}P_H + \frac{n_D}{n_e}P_D\right)\left(1 + \frac{T_D^{3/2}}{T_H^{3/2}}\frac{n_H}{n_D}\left(\frac{m_H}{m_D}\right)^{1/2}\frac{\Gamma_1(b_H)}{\Gamma_2(b_D)}e^{\zeta_{2D}^2-\zeta_{1H}^2}\right)} \tag{8}$$

where the double prime quantities refer to the sideband ($\omega'' = |\omega - \omega_0|$, etc.), and where $n_e = n_H + n_D$, $v_{oy} = cE_{oy}/B$, $\zeta_{oj} = \omega/k_{\|}v_{tj}$. Here we assumed that $\epsilon_R(\omega - \omega_0) = 0$,

$$P_j = \sum_{l=1}^{\infty} \frac{2l^2\Omega_j^2\Gamma_l(b_j)(\omega-\omega_0)^2}{((\omega-\omega_0)^2 - l^2\Omega_j^2)^2}; \quad \zeta_{lj} = \frac{\omega - l\Omega_j}{k_{\|}v_{tj}} \tag{9}$$

$$\frac{\partial\epsilon}{\partial\omega_R^-} = \frac{2}{\omega''}\frac{1}{k^2\lambda_{De}^2}\left(\frac{n_H}{n_e}P_H + \frac{n_D}{n_e}P_D\right). \tag{10}$$

and further,

$$\frac{\chi_{eIm}}{\sum_i \chi_{iIm}} = \frac{1}{\Gamma_1(b_H)\frac{T_e^{3/2}}{T_H^{3/2}}\frac{n_H}{n_e}\left(\frac{m_H}{m_e}\right)^{1/2}e^{-\zeta_{1H}^2+\zeta_{oe}^2} + \Gamma_2(b_D)\frac{T_e^{3/2}}{T_D^{3/2}}\frac{n_D}{n_e}\left(\frac{m_D}{m_e}\right)^{1/2}e^{-\zeta_{2D}^2+\zeta_{oe}^2}} \tag{11}$$

and

$$\theta^{1/2} = \frac{(\Omega_1 - \Omega_2)(\omega_o^2 + \Omega_1\Omega_2)\omega_o}{(\omega_o^2 - \Omega_1^2)(\omega_o^2 - \Omega_2^2)}, \tag{12}$$

where we assumed $(k_x^2\Omega_j^2/k_y^2\omega_o^2) << 1$. Such an assumption was also adopted in the first term (the electron $\vec{E}_o \times \vec{B}$ term) which might reduce the growth rates by at most ~25% in typical cases. However, as a result, the algebra is greatly simplified. We note that the second growth term is due to the relative drift of ions, and $\theta \rightarrow 0$ if only one ion species is present ($\Omega_1 = \Omega_2$). In that case only the electron $\vec{E}_o \times \vec{B}$ term drives the instability. Considering case (i), $\omega_o \simeq 2.66\Omega_H = 5.33\Omega_D$, we have $\theta^{1/2} \simeq 0.23$ and $\theta \simeq 0.052 \simeq 1/20$.

For case (ii), we get $\theta^{1/2} \simeq 1.3$ or $\theta \simeq 1.7$, and in principle, an important contribution from the relative ion drift may result in the case of minority heating.

Further evaluation of the terms gives $\Gamma_1(b_H)/\Gamma_2(b_D) \simeq 4$, and taking $T_H \simeq T_D \simeq T_e$, we obtain $k_\perp r_H \simeq 0.7$, $k_\perp r_D \simeq 1.0$, $\omega/\Omega_H = 1$, $\omega''/\Omega_H \simeq 1.75$, $\omega_o/\Omega_H \simeq 2.75$, $P_D = 0.07$, $P_H = 0.6$.

After some algebra, the growth rate is given by:

$$\frac{\gamma}{\omega''} \simeq -\frac{\gamma''}{\omega''} + \frac{k_y^2 v_{oy}^2}{8\omega_o^2} \frac{10\theta\pi^{1/2} e^{-\zeta_{1H}^2}}{\left(P_H + \frac{n_D}{n_H} P_D\right)\left(1 + 3\frac{n_H}{n_D} e^{\zeta_{2D}^2 - \zeta_{1H}^2}\right)}$$
$$+ \frac{k_x^2\, v_{oy}^2}{8\omega_o^2} \frac{\pi^{1/2}\zeta_{oe} e^{-\zeta_{oe}^2}}{\left(\frac{n_H}{n_e} P_H + \frac{n_D}{n_e} P_D\right)\left(1 + \chi_{eIm}/\sum_{j=1,2} \chi_{jIm}\right)} \tag{13}$$

Since $|\chi_{eIm}/\sum_i \chi_{iIm}| < 1$ for $\zeta_{1H} \sim \zeta_{2D} \sim 1$, and $\zeta_{oe} \simeq 1$, the growth rate maximizes for the quasi-mode at the ion cyclotron frequency and its harmonics. Hence, we expect that these modes would have characteristically low frequencies near Ω_H and Ω_D, whereas the sideband would be an ion Bernstein wave. Note that in the present case, since $10\theta \simeq 0.5$, and if $n_D/n_H >> 1$, the electron $\vec{E} \times \vec{B}$ term (the last term in Eq. 13) dominates. However, for $n_H \sim n_D$, $k_x \sim k_y$, the contributions to the growth rate from the electron and ion drift terms are comparable (assuming $\zeta_{oe} \lesssim 1$, $\zeta_{1H} \sim \zeta_{2D} < 1$). For typical cases such as in ASDEX, where $n_H/n_D \sim 0.3$, $E_y \simeq 300$ V/cm, $T_i \sim 20$ ev, we obtain $\gamma \simeq 2\text{x}10^6$ sec^{-1}.

For decay into $\omega \sim \Omega_D$, and the sideband being a Bernstein wave ($\omega'' \simeq 4.5\ \Omega_D$, $\omega_o \simeq 5.5\Omega_D$), the relative ion-ion drift term is not important, and the growth rate is given mainly by the electron $\vec{E} \times \vec{B}$ drift. In this case for $T_e = T_D = T_H$,

$$\left(\chi_{eIm}/\sum_{j=1,2} \chi_{iIm}\right)^{-1} \simeq \Gamma_1(b_D)(n_D/n_e)(m_D/m_e)^{1/2} e^{-\zeta_{1D}^2 + \zeta_{oe}^2} \ ,$$

which is typically larger than unity if $\zeta_{1D}^2 \lesssim 1$, $\zeta_{oe} \simeq 1$, and hence $\chi_{eIm}/\sum_{j=1,2} \chi_{jIm}$ may be neglected as compared to unity. Thus, the growth rate is given by the relatively simple form of the electron $\vec{E}_0 \times \vec{B}$ term,

$$\frac{\gamma}{\omega''} \simeq -\frac{\gamma''}{\omega''} + \frac{1}{8}\frac{k_x^2 v_{oy}^2}{\omega_o^2} \frac{\pi^{1/2}\zeta_{oe} exp(-\zeta_{oe}^2)}{\left(\frac{n_H}{n_e} P_H + \frac{n_D}{n_e} P_D\right)} \tag{14}$$

which is comparable growth rate to decay into the hydrogen ion cyclotron quasi-mode. In particular, for the above parameters, we obtain $\gamma \simeq 1.5 \times 10^6$ sec^{-1}. Both of the above growth rates easily exceed the dissipative threshold ($\gamma_o > \gamma''$). Regarding convective thresholds, we find that in the first case (decay into hydrogen ion cyclotron quasi-mode) $v''_{gx} \simeq v_{tH} \simeq 6 \times 10^6$ cm/sec., the threshold is met marginally for $\Delta x \simeq$ 10cm, namely:

$$\frac{\gamma_o \Delta x}{v_{gx}} \simeq 3 \ . \tag{15}$$

For the second case ($\omega/\Omega_D \simeq 1$) the convective threshold is significantly lower since $v_{gx} \sim O(0.1 v_{tD})$ due to the flat slope of the dispersion curves at the higher harmonic (here $k_\perp r_{cD} \simeq 3$). However, a careful optimization of $k_\|, k''_\|$ and $k_{o\|}$ may be necessary to avoid cyclotron damping at the sideband. Note that here $\Delta x \sim 2Rk_\| v_{tD}/\Omega_D$, the width of the ion cyclotron resonance in the inhomogeneous magnetic field with characteristic scale length R, the major radius. If we assume that for the pump wave $E_{ox} \sim E_{oy}$, we may pick up another factor of $\sqrt{2}$ in Δx, in which case the decay waves would also propagate at $\sim 45^o$ angle to the plane of the antenna, namely $k_x \sim k_y$.

We have also examined the growth rate in a single ion species plasma, and in such a case $\gamma_o \sim 5 \times 10^5 s^{-1}$, and the convective threshold is met only marginally ($\Delta x \gamma_o / v''_{gx} \sim 1$).

The case of minority heating (case ii) was also examined for decay into ion cyclotron quasi-modes. In general, by ignoring $k_{o\|}$, decay instability is difficult to obtain since the sideband ($\omega^-/\Omega_D \sim 1.66$, for $\omega_o/\omega_D \sim 2.66$ and $\omega/\Omega_D \sim 1$) is strongly cyclotron damped if we require $(\omega - \Omega_D)/k_\| v_{ti} \simeq 1$, unless we can make $|k_\|^-| = |k_\| - k_{o\|}| << k_\|$. Similar arguments hold for both H^+ minority and He-3 minority cases. Thus, with the exception of particular values of $k_{o\|}$, we do not expect significant PDI into ion cyclotron quasi-modes. This lead us into looking for low frequency electron Landau damped quasi-modes.

(iii) Electron Landau Damped Low Frequency Quasi-Modes

This kind of PDI may occur in both single and multiple ion species plasma for ω_o/Ω_i being in either regimes (i) or (ii) (i.e., minority heating or second harmonic heating). The following conditions apply:

$$\omega << \Omega_j \quad , \quad \zeta_{oe} << 1. \tag{16}$$

Here the response of electrons is simple to obtain, the dielectric constant corresponds to a damped electron quasi-mode,

$$\epsilon(\omega, k) \simeq (1 + i\sqrt{\pi}\zeta_{oe} e^{-\zeta_{oe}^2})/k^2\lambda_{De}^2 \ , \tag{17}$$

and the sideband is an ion Bernstein wave. Typical values are found to be $\zeta_{oe} \sim 0.2$, $\omega \sim (0.1 - 0.2)\Omega_j$. The dominant terms in the growth rate originate from the relative electron-ion drift, which may be written as:

$$(\gamma + \gamma'')|\frac{\partial \epsilon_R}{\partial \omega^-}| \simeq$$

$$\sum_j \frac{1}{4}\left[\frac{c|(E_{o\perp} \times k_\perp)\cdot \hat{z}|}{B\omega_o} + \frac{e}{m_j}\left(\frac{|E_{o\perp}\cdot k_\perp|^2}{(\omega_o^2 - \Omega_j)^2} + \frac{|(\vec{E}_{o\perp} \times \vec{k}_\perp)\cdot \hat{z}|^2 \Omega_j^2}{(\omega_o^2 - \Omega_j^2)^2 \omega_o^2}\right)^{1/2}\right]^2 \times$$

$$\times Im\left(\frac{\chi_e \chi_j}{\epsilon}\right). \tag{18}$$

Here the relative ion-ion drift terms, being small relative to the electron-ion terms, were neglected. For fast waves, we consider $v_o \simeq cE_{oy}/B$. Evaluating $Im(\chi_e\chi_j/\epsilon)$, we find the following result for $E_o \simeq E_{oy}$, in a single ion species plasma:

$$\frac{\gamma}{\omega''} \simeq -\frac{\gamma''}{\omega''} + \frac{\pi^{1/2}\Delta^2 (T_e/T_i)(1 - \Gamma_o(b_j))^2 \zeta_{oe} e^{-\zeta_{oe}^2}}{8P_j\left[\left(1 - \Gamma_o(b_j) + \frac{T_i}{T_e}\right)^2 + \pi^2 \frac{T_i^2}{T_e^2}\zeta_{oe}^2 e^{-2\zeta_{oe}^2}\right]} \tag{19}$$

where $\Gamma_o(b_j), P_j$ have been defined before (see Eq. 9) and we defined the relative drift parameter as:

$$\Delta^2 = \frac{k_z^2 v_{oy}^2}{\omega_o^2}\left(1 + \frac{\omega_o \Omega_j}{(\omega_o^2 - \Omega_j^2)}\left(\frac{k_y^2}{k_z^2} + \frac{\Omega_j^2}{\omega_o^2}\right)^{1/2}\right)^2 . \tag{20}$$

The convective threshold is satisfied since $\gamma_o \gtrsim 3 \times 10^5 s^{-1}$ for $\omega''/\Omega_H \simeq 2.4$, $k_\perp r_H \simeq 3$, $\omega_o/\Omega_H \simeq 2.6$, $\omega/\Omega_H \simeq 0.2$, $T_e \sim T_i \sim 20$ eV, $E_{oy} \simeq 300$ v/cm, $\Delta x \sim 10$ cm, $\gamma'' \simeq 1 \times 10^5 s^{-1}$, and we find $\gamma_o \Delta x / v''_{gx} \simeq 5$ due to the relatively low group velocity (since $k_\perp^2 r_H^2 >> 1$). Including multiple ion species does not significantly alter these results. Thus, electron quasi-mode excitation is expected to occur under most conditions.

IV. Parametric Decay Instabilities During Ion Bernstein Wave Launching

Another regime of interest is IBW launching. In this case, the dominant pump wave field is $E_{ox}, E_{o\parallel}$. The experiments are usually done in a hydrogen-deuterium mixture, with the hydrogen often being the majority species. In this case, rich nonlinear phenomena can occur, including the previously discussed decay processes into ion cyclotron and electron Landau damped quasi-modes (just replace $E_{oy} \rightarrow E_{ox}$, $k_y \rightarrow k_x$, $k_z \rightarrow k_y$) [5].

A few of these processes are shown in Fig. 4. In addition, self interaction of the waves may occur inside the plasma interior [13]. More complicated edge phenomena may include ponderomotive force effects, such as filamentation instabilities. In some cases the growth rates are sufficiently large that pump depletion may be expected. In such cases, however, some of the decay waves may propagate into the plasma interior and actually heat the plasma by minority, or nonlinear self-absorption [14]. In some cases, the thermal fluctuation level has been evaluated and pump depletion is found for an $\gamma_o \Delta x / v_g \sim 10$ growth factor. These results will be discussed in a later publication after a more thorough study. The likely importance of such phenomena is indicated by the anomalously high loading resistance observed in essentially all IBW experiments [5, 15]. However, interpretation of loading resistence is complicated by the importance of rf sheath effects, observed previously [7]. Finally, the importance of scattering IBW by background low frequency fluctuations must be assessed in individual cases [15]. Therefore, it is fair to say that at present the anomalously high loading resistance observed in IBW experiments is not understood.

V. Discussion and Conclusions

We have examined the possibility of PDI in the presence of ICRF fields in the SOL. The strongest instability excitation is found for $\omega_o > 2\Omega_i$, which is the case in 2nd harmonic fast wave heating experiments, as well as during IBW launching experiments. Experimental evidence for such instability activity has been reported in many experiments. For example, PDI of the kinds discussed here have been observed in the ASDEX(4) and Textor(3) tokamaks. Pairs of daughter wave frequencies have been observed at the deuteron and proton cyclotron frequencies, and at the respective difference frequencies $\omega_o - l\Omega_j$, which correspond to Ion Bernstein waves at the sidebands. The cyclotron frequencies have corresponded to the magnetic field at the edge, in the SOL region. The threshold electric fields are in reasonable agreement with the estimates presented here, namely $E_{ox} \sim E_{oy} \sim 300$ volt/cm fields have been estimated (ASDEX) and measured (Textor). Of course, without understanding the pump wave electric field structure, it is hard to estimate the threshold fields accurately. For example, if the unabsorbed wave fields fill the SOL region uniformly, the thresholds would decrease. Thus, the understanding of surface wave generation is a key issue in determining the PDI thresholds. We note that decay into IBW and the low frequency electron quasi mode has also been observed in both the ASDEX and TEXTOR experiments, at least the appropriate frequency spectra has been observed. In addition, decay into half of the pump frequency has also been observed. This may correspond to decay into two Bernstein waves, or decay into electromagnetic ion cyclotron waves (16).

Further evidence of PDI of the kinds desribed here have also been observed in the DIIID IBW launching experiment (5), where decay into Ω_D, Ω_H and appropriate sidebands has been observed. These experimental results are being analyzed now, and details will be presented in a later publication. Finally, we note the strong PDI activity reported at this meeting from the JT-60 tokamak ICRF experiments. Here, the edge heating is believed to be a consequence of PDI. The consequence of such instabilities is electron, and possibly ion heating in the SOL due to absorption of the quasi-mode by thermal particles. At present, no direct correlation between PDI and impurity generation has been reported with the possible exception of JT 60. However, edge electron heating has been observed in most ICRF and IBW experiments. The natural consequence of such edge heating is sputtering by accelerated ions in the sheath. The present theoretical analysis indicates that the He-3 minority regime should be most benign with respect to PDI excitation, especially if the Faraday shield is aligned with the magnetic field so as to eliminate the electrostatic component of the electric field. On the other hand, the IBW and the 2nd harmonic fast wave heating regimes are particularly susceptible to PDI. Excitation of PDI in the SOL region offers one of the very few mechanisms for edge electron heating. Thus, a thorough investigation of PDI excitation and its relation to edge heating is warranted.

Acknowledgements

This work benefited from numerous discussions by many colleagues, including Drs. M. Mayberry, J. Moody, J. M. Noterdaeme, R. Pinsker, and G. VanOost. The author is grateful to the above colleagues for sharing unpublished data from their experiments.

This work was supported by the U.S. Department of Energy under Contract DE-ACO2-78ET51013.

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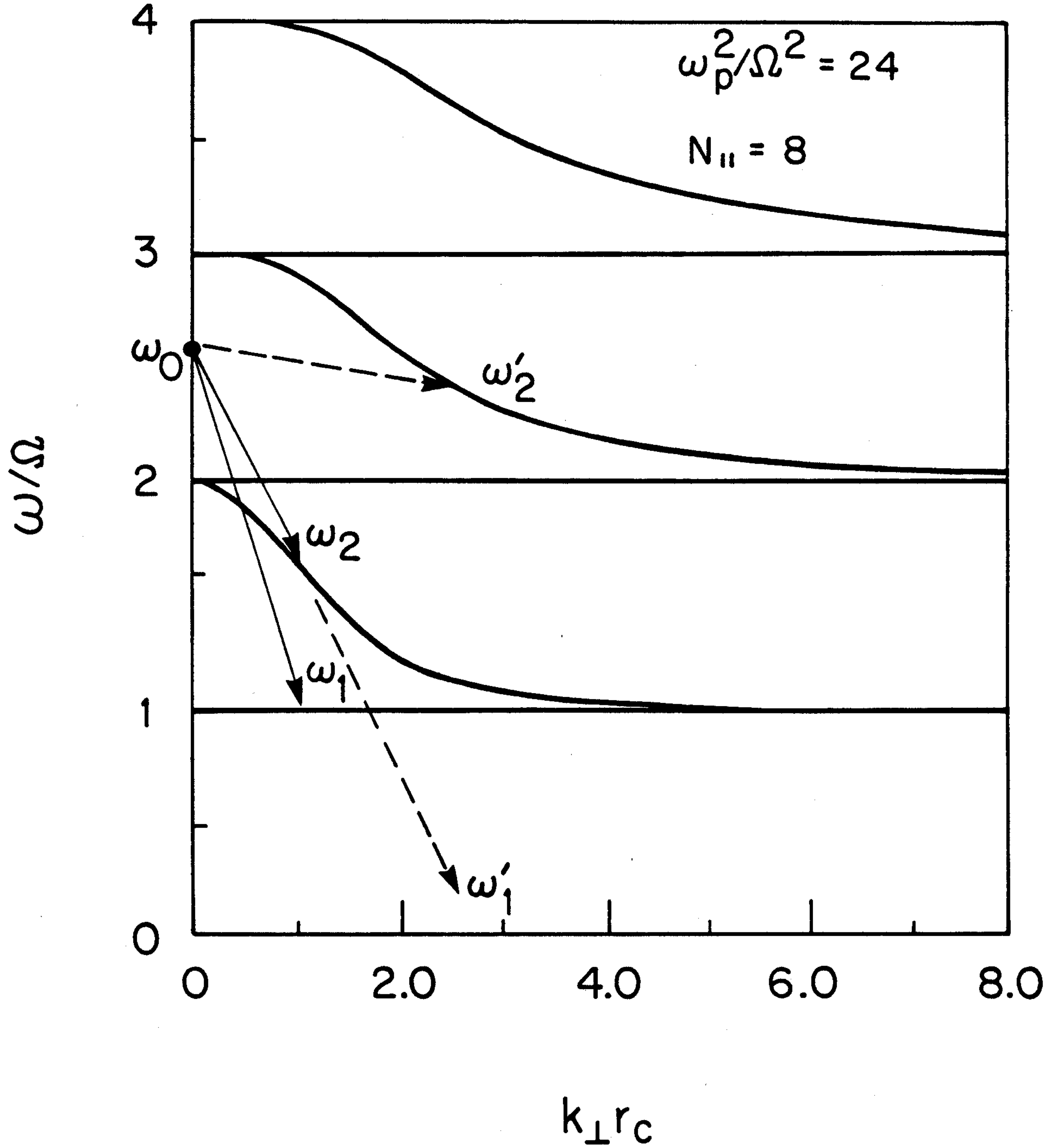
FIGURE CAPTIONS

1. Parametric decay into pairs of ion cyclotron - quasi-mode (ICQM, ω_1) and Ion Bernstein wave (IBW, ω_2), and electron quasi-mode (EQM, ω_1') and (IBW, ω_2'), in the second harmonic heating regime.

2. Parametric decay into pairs of ICQM and IBW (ω_1 and ω_2, and ω_1' and ω_2', respectively), and EQM and IBW (ω_1'' and ω_2'') in the $2\Omega_H$ regime with substantial deuterium present in the ASDEX tokamak (after Ref. 4).

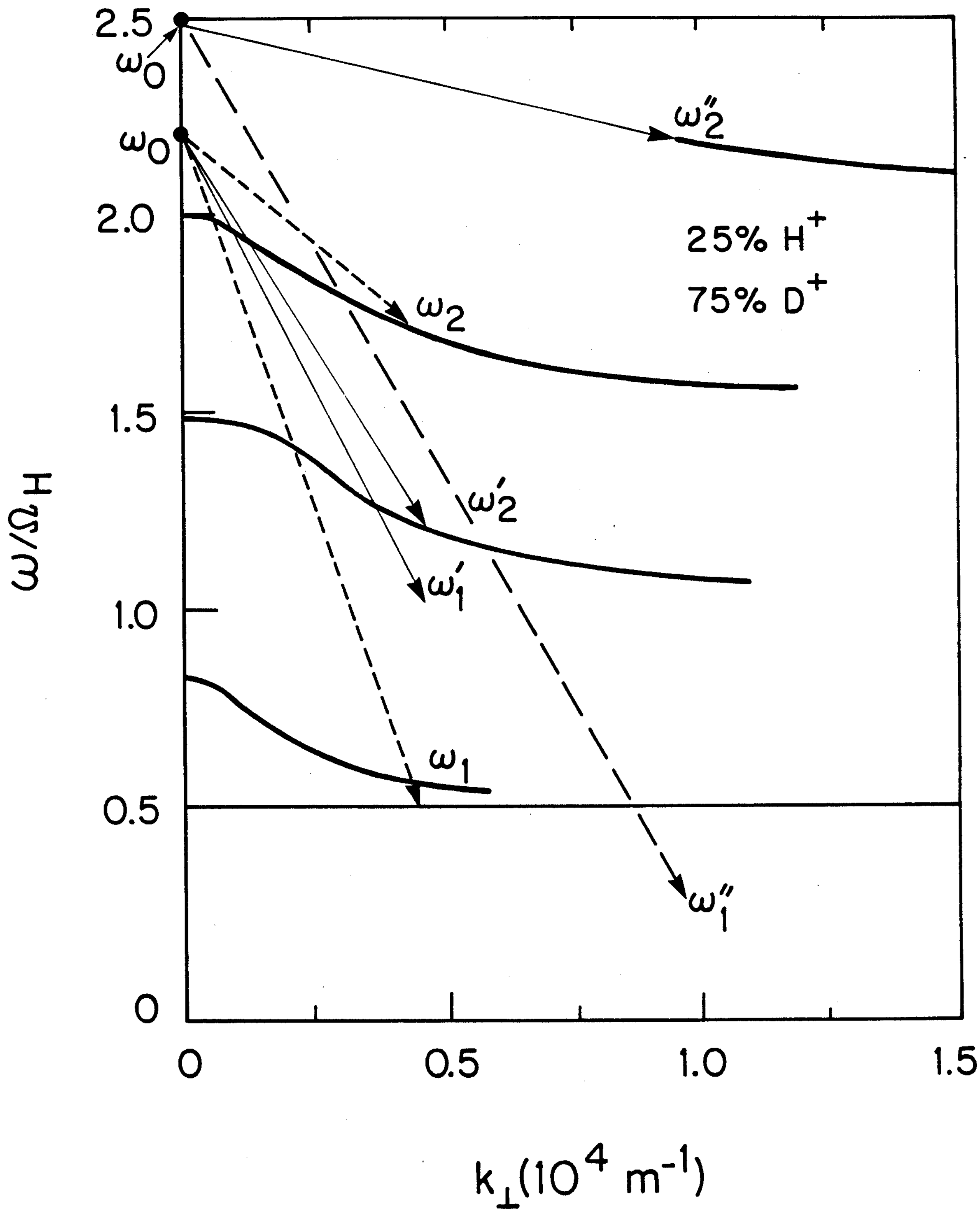
3. Parametric decay into EQM and IBW (ω_1 and ω_2, respectively) in the D majority, He-3 minority regime.

4. Parametric decay into pairs of ICQM and IBW (ω_1 and ω_2, and ω_1' and ω_2', respectively) and EQM and IBW (ω_1'' and ω_2'', respectively) in the IBW launching regime. Notice the possible "jump" in the dispersion branches as ω_o/ω_D crosses the integer 4.



$\omega_p^2/\Omega^2 = 24$
$N_\parallel = 8$
ω_0
ω_2'
ω_2
ω_1
ω_1'
4
3
2
1
0
0
2.0
4.0
6.0
8.0
ω/Ω
$k_\perp r_c$

Figure 1



ω_0
ω_0
ω_2''
ω_2
ω_2'
ω_1'
ω_1
ω_1''
25% H^+
75% D^+
2.5
2.0
1.5
1.0
0.5
0
ω/Ω_H
0
0.5
1.0
1.5
$k_\perp (10^4\ m^{-1})$

Figure 2

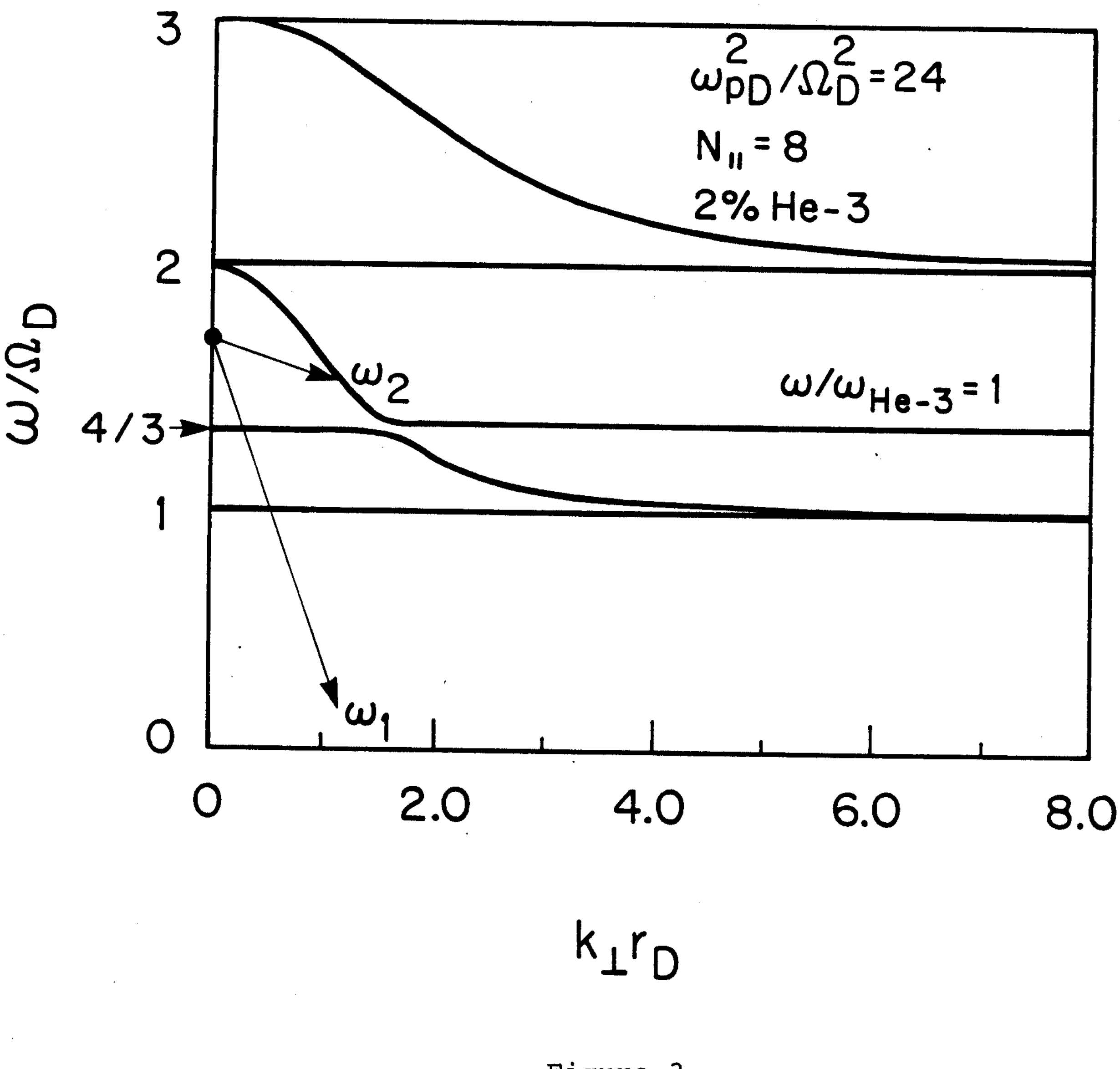

3
2
1
0
4/3
ω/Ω_D
$\omega_{pD}^2/\Omega_D^2 = 24$
$N_\parallel = 8$
2% He-3
ω_2
$\omega/\omega_{He-3} = 1$
ω_1
0
2.0
4.0
6.0
8.0
$k_\perp r_D$

Figure 3

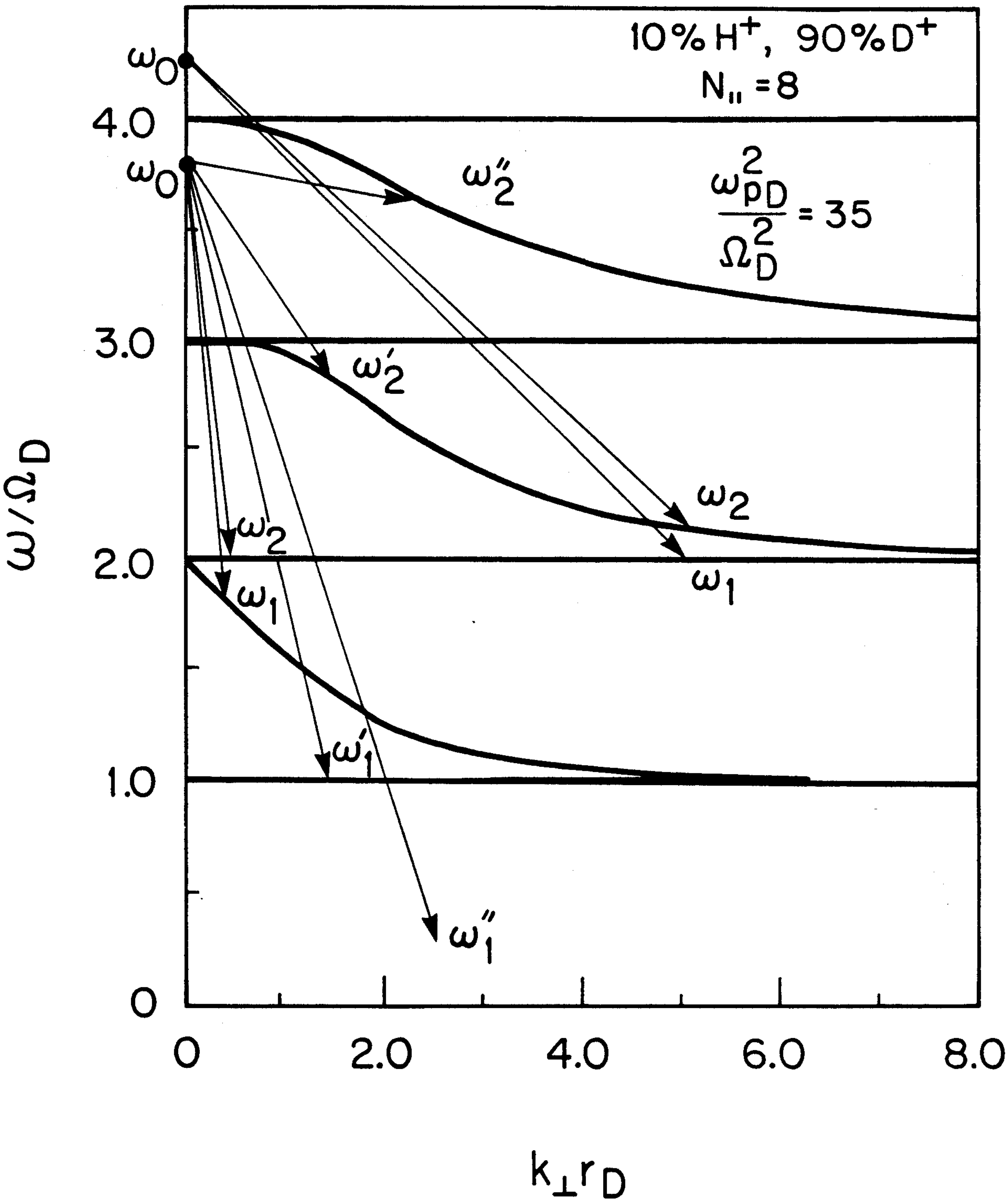

10% H+, 90% D+
N∥ = 8
$\frac{\omega_{pD}^2}{\Omega_D^2} = 35$
ω_0
ω_0
ω_2''
ω_2'
ω_2
ω_1
ω_2
ω_1
ω_1'
ω_1''
ω/Ω_D
0
1.0
2.0
3.0
4.0
0
2.0
4.0
6.0
8.0
$k_\perp r_D$

Figure 4