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Abstract

A moment approach to the transport theory of impure tokamak plasmas with strong rotation velocity V_φ in the Pfirsch-Schlüter regime is presented. When $V_\varphi \lesssim c_s$ (sound speed) strong poloidal variations of the impurity density in the flux surface, first order poloidal flows and up-down asymmetric density variations are found. The radial particle flux and the radial flux of toroidal angular momentum are evaluated. Corrections to Braginskii's viscous tensor due to the large rotation are also found.

I. Introduction

The problem of neutral beam produced toroidal rotation of tokamak plasmas has been studied extensively, experimentally[1,2] and theoretically.[3,4] The surprising experimental observations are a momentum confinement time much shorter than corresponds to Braginskii's[5] classical perpendicular viscosity, up-down asymmetries of the main ion and impurity density[6] and a strong dependence of particle confinement time on co- vs. counter-injection.

Theoretically, one approach is a systematic extension of the Larmor radius expansion of the neoclassical drift kinetic equation[7], the other is the fluid approach[8,9] based on the Braginskii's viscosity tensor. Previously, Stacey and Sigmar[10] proposed a gyroviscous damping mechanism in plasmas containing impurity ions (such that $\alpha \equiv \frac{n_I Z^2}{n_i} \sim 0(1)$), depending on an $0(\epsilon)$ up-down asymmetric variation of the particle-density in the flux surface and finite poloidal flow velocities for the ions and impurities. Connor et al[9], focusing essentially on the zeroth order (in Larmor radius expansion) fluid momentum balance, have concluded that to this order the poloidal flow velocity and therefore the up/down density variations must vanish. To first order, however, they considered mainly a pure plasma. Also, Wong[7] has studied the impure plasma system with large rotation and carried out the formal Larmor radius expansion of the transport equations in the banana regime.

With this background in mind, in this work, we study the transport theory of a strongly rotating, impure tokamak plasma in the Pfirsch-Schlüter (P-S) regime, adopting the Larmor radius expansion in conjunction with a moment approach (cf. Grad,[11] Herdan and Liley[12]). The general description of the systematic moment approach to transport theory is given elsewhere.[13] In this approach one (i) derives the set of moment equations for each order in $\delta_{pi} \equiv \frac{\rho_{pi}}{L}$ (where ρ_{pi} is the ion Larmor radius and L the radial length scale); (ii) solves the set of moment equations including the higher rank moments such as the viscous tensor

$$\mathbf{\Pi} \equiv \int d\mathbf{v}\; m(\mathbf{v}\mathbf{v} - \frac{v^2}{3}\mathbf{I})f$$

and the heat flow

$$\mathbf{q} \equiv \int d\mathbf{v} \; m\frac{v^2}{2}\mathbf{v}f \; ,$$

the flow velocity $\mathbf{V}$ and the up-down variations of density n and temperature T in the flux surface; (iii) determines the radial transport fluxes. Here we only remark that, according to the well known H-theorem, unless there exists an $O(1)$ momentum source, the Coulomb collisions will force the $O(1)$ distribution function to be a purely shifted Maxwellian[14]

$$f_M = \frac{n}{\left(\frac{2\pi T}{m}\right)^{3/2}} e^{-\frac{m(\mathbf{v}-\mathbf{V})^2}{2T}} \tag{1}$$

This implies that, in the absence of an $O(1)$ source, all moments (except for n, T, and $\mathbf{V}$) must be $\leq O(\delta_{pi})$.

An additional constraint on the zeroth order flow velocity $\mathbf{V}$ in an axisymmetric system is given briefly as follows. Adopting the usual flux coordinate system (ψ, θ, φ), (cf. Fig. 1) using the lowest order $0(\delta_p^{-1})$ momentum equation

$$Z_j n_j e \nabla \phi^{(-1)} = m_j n_j \Omega_j \mathbf{V}_j^{(0)} \times \mathbf{b}.$$

and the $O(1)$ continuity equation

$$\nabla \cdot n_j^{(0)} \mathbf{V}_j^{(0)} = 0,$$

one finds that in an axisymmetric system

$$\mathbf{V}_j^{(0)} = \frac{K_j^{(0)}(\psi)}{n_j^{(0)}} \mathbf{B} + \omega^{(0)} R^2 \nabla \varphi. \tag{2}$$

Here $\phi^{(-1)} = \phi^{(-1)}(\psi)$,

$$\omega^{(0)} = -\frac{1}{c}\frac{\partial}{\partial \psi}\phi^{(-1)} \tag{3}$$

is the zeroth order toroidal rotation frequency which is a species independent flux quantity, and $\frac{B_p}{n^{(0)}} K_j^{(0)}(\psi)$ is the zeroth order poloidal flow. However, as described in Appendix A, a nonvanishing $K_j^{(0)}(\psi)$ will generate an $O(1)$ parallel viscous force which will then damp $K_j^{(0)}$ at a rate $\sim \frac{\omega_t^2}{\nu}$ (where ω_t is the transit frequency and ν the collision frequency). Therefore, $K_j^{(0)}(\psi)$ must vanish and

$$\mathbf{V}_j^{(0)} = \omega^{(0)} R^2 \nabla \varphi \tag{4}$$

is species independent. That is, the zeroth order flow $V^{(0)} \sim O(1)v_{th}$ in an axisymmetric system must be purely in the toroidal direction and will induce a large radial electric field such that $\frac{e\phi(\psi)}{T} \sim O(\delta_{pi}^{-1})$.

We note that the up-down variations of density and temperature are essential for the neoclassical transport in a toroidally confined plasma because the radial fluxes arise from the radial drift of particles due to the poloidal gradient forces and only up-down symmetric forces (which come from the parallel gradient of up-down asymmetric pieces of n and T) can survive the flux surface average. Since the present work is restricted to plasma systems with radially constant temperature, the remainder of the paper will then concentrate on the derivation of the up-down variations of density, the poloidal flows, and the radial transport fluxes.

Consider a plasma system with electrons, main ions and one impurity species with the impurity charge $Z \sim \frac{m_I}{m_i} \gg 1$. Due to the largeness of Z and the smallness of $\frac{n_I}{n_i}$, it is useful to write the momentum equations for ions and impurities in the following form

$$m_i \mathbf{V}_i \cdot \nabla \mathbf{V}_i + \frac{\nabla \cdot \mathbf{\Pi}_i}{n_i} + T_i \nabla(\ln n_i) = -e\nabla\phi + m_i \Omega_i (\mathbf{V}_i \times \mathbf{b}) + \frac{\mathbf{R}_{iI}}{n_i} \tag{5}$$

$$\frac{m_I}{Z} \mathbf{V}_I \cdot \nabla \mathbf{V}_I + \frac{\nabla \cdot \mathbf{\Pi}_I}{n_I Z} + \frac{T_I}{Z} \nabla(\ln n_I) = -e\nabla\phi + m_i \Omega_i (\mathbf{V}_I \times \mathbf{b}) - \frac{\mathbf{R}_{iI}}{n_I Z} \tag{6}$$

such that the driving centrifugal force and electric force in both equations appear to be comparable, independent of Z. However, it will become clear in the next section that the centrifugal force can only drive the in-out density variation; and the up-down density variation can only be driven through a nonvanishing parallel friction which, in the P-S regime, can be written as (see Appendix A)

$$R_{iI\parallel} \equiv \mathbf{b} \cdot \mathbf{R}_{iI} = m_i n_i \nu_{iI} D_{1i} (V_{I\parallel} - V_{i\parallel}) \ , \tag{7a}$$

where $\nu_{iI} = \sqrt{2}\alpha\nu_{ii}$ is the $i - I$ collision frequency and D_{1i} is given in Eq. (A22). Furthermore, from Eqs. (3) and (4), $V^{(0)}$ is found to be species independent and scales as

$$R_{iI\parallel} \sim m_i n_i \nu_{iI} \delta_{pi} v_{thi} \tag{7b}$$

and

$$\frac{\tilde{n}_{I-}(\theta)}{\overline{n}_I} \sim \Delta \, .$$

Here $\tilde{n}_{I-}(\theta)$ denotes the up-down asymmetric part of impurity density, overbar denotes the flux surface average and

$$\Delta \equiv \frac{\delta_{pi} Z^2 \overline{\nu}_{ii} \sqrt{2}}{\omega_{ti}} \, . \tag{8}$$

Consequently, we have two interesting cases, namely (1) $\Delta \sim \delta_{pi}$ which leads to $\frac{\tilde{n}_{I-}(\theta)}{\overline{n}_I} \sim O(\delta_{pi})$ and will be studied with finite ϵ (the inverse aspect ratio) in Section II; and (2) the case $\Delta \sim 1$ which leads to $\frac{\tilde{n}_{I-}(\theta)}{\overline{n}_I} \sim O(1)$; this strong ordering case will be studied assuming small ϵ for simplicity in Section III. The second case with small rotation has been previously studied[15]). In Section IV, the radial transport fluxes of particles and of toroidal angular momentum will be evaluated. In Section V, conclusions will be given. In Appendix A, we obtain the parallel friction and parallel viscosity, which contains nontrivial corrections to Braginskii's viscosity tensor.[5]

II. Usual δ_{pi} ordering

In this section, we assume that $\Delta \sim \delta_{pi}$, that is, the friction is omitted in the zeroth order equations. Therefore, one needs to first solve the $O(1)$ equations for the in-out density variations, and then solve $O(\delta_{pi})$ equations for the up-down density variations and poloidal flows.

II.A. The Zeroth Order Solution

The zeroth order of Eqs. (5), (6), (A1) can be written as

$$m_j n_j^{(0)} \mathbf{V}_j^{(0)} \cdot \nabla \mathbf{V}_j^{(0)} + \nabla n_j^{(0)} T_j^{(0)} = -n_j^{(0)} Z_j e \nabla \phi^{(0)} + m_j n_j^{(0)} \Omega_j \mathbf{V}_j^{(1)} \times \mathbf{b}, \tag{9}$$

$$n_j^{(0)} T_j^{(0)} \mathbf{W}_2 \left[\nabla \mathbf{V}_j^{(0)} \right] = \Omega_j \mathbf{K}_2 [\mathbf{\Pi}_j^{(1)}] \equiv \Omega_j (\mathbf{\Pi}_j^{(1)} \times \mathbf{b} + \uparrow_2). \tag{10}$$

Here tensor operator $\mathbf{W}_2[\mathbf{A}] \equiv (\mathbf{A} + \uparrow_2) - \frac{2}{3}(Tr\mathbf{A})\mathbf{I}$ and $\uparrow_2$ means transpose, also cf. Eqs. (A2) and (A3). The parallel projection of Eq. (9) yields

$$T_j \mathbf{B} \cdot \nabla \ln n_j^{(0)} = \frac{m_j}{2} \omega^{(0)^2} \mathbf{B} \cdot \nabla R^2 - Z_j e \mathbf{B} \cdot \nabla \phi^{(0)}, \tag{11}$$

and thus

$$n_j^{(0)} = N_j(\psi) \exp \left(\frac{\frac{m_j}{2} \omega^{(0)^2} R^2 - Z_j e \phi^{(0)}}{T_j} \right). \tag{12}$$

By using the small-mass-ratio $\frac{m_e}{m_i}$ and quasineutrality, the electrostatic potential satisfies

$$\mathbf{B} \cdot \nabla \phi^{(0)} = \frac{T_e}{e} \mathbf{B} \cdot \nabla \ln n_i^{(0)} (1 + \frac{\alpha^{(0)}}{Z}), \tag{13}$$

where

$$\alpha^{(0)} \equiv \frac{n_I^{(0)} Z^2}{n_i^{(0)}} .$$

Then, for a two ion species plasma, Eqs. (11)-(13) yield

$$\alpha^{(0)} \left(1 + \frac{\alpha^{(0)}}{Z} \right)^{(Z \frac{T_i}{T_I} - 1) \frac{T_e}{T_i + T_e}} = \alpha(\psi) \exp \left[\frac{m_i Z w^{(0)^2} R^2}{2 T_I} \left(\mu - \frac{T_e + \frac{T_I}{Z}}{T_i + T_e} \right) \right], \tag{14}$$

where $\mu \equiv \frac{m_I}{Zm_i}$ which is $= 2$ for a fully ionized impurity. The densities are thus determined by

$$n_i^{(0)} = n(\psi)\alpha^{(0)\frac{T_I}{ZT_i - T_I}} \exp\left(\frac{m_i(1-\mu)w^{(0)^2}R^2}{2(T_i - \frac{T_I}{Z})}\right), \tag{15}$$

$$n_I^{(0)} = \frac{n(\psi)}{Z^2}\alpha^{(0)\frac{ZT_i}{ZT_i - T_I}} \exp\left(\frac{m_i(1-\mu)w^{(0)^2}R^2}{2(T_i - \frac{T_I}{Z})}\right). \tag{16}$$

The flux functions $n(\psi)$ and $\alpha(\psi)$ can be determined by taking the flux average of Eqs. (15) and (16). Denoting the flux surface average by

$$\overline{f} \equiv < f >_\psi$$

and the impurity strength parameter by

$$\hat{\alpha} \equiv \frac{\overline{n}_I Z^2}{\overline{n}_i}$$

one finds that $\alpha(\psi)$ is determined implicitly through

$$\hat{\alpha} = \frac{\left\langle \alpha^{(0)\frac{ZT_i}{ZT_i - T_I}} \exp\left(\frac{m_i(1-\mu)w^{(0)^2}R^2}{2(T_i - \frac{T_I}{Z})}\right)\right\rangle_\psi}{\left\langle \alpha^{(0)\frac{T_I}{ZT_i - T_I}} \exp\left(\frac{m_i(1-\mu)w^{(0)^2}R^2}{2(T_i - \frac{T_I}{Z})}\right)\right\rangle_\psi} \tag{17}$$

and $n(\psi)$ is determined by

$$n(\psi) = \frac{\overline{n}_i}{\left\langle \alpha^{(0)\frac{T_I}{ZT_i - T_I}} \exp\left(\frac{m_i(1-\mu)w^{(0)^2}R^2}{2(T_i - \frac{T_I}{Z})}\right)\right\rangle_\psi}. \tag{18}$$

Here, $\omega^{(0)}(\psi)$, $\overline{n}_j(\psi)$, and therefore $\hat{\alpha}$, are assumed to be prescribed. Henceforth, we shall drop the subscript ψ on all flux surface averages, for simplicity.

It is important to note that Eqs. (14)-(18) give the exact solution of the zeroth order moment equations without assumptions on the inverse aspect-ratio ϵ or the impurity concentration. When the impurity species is super-sonic, i.e., $V \sim v_{thi} \gg v_{thI}$, one finds, as shown in Figs. 2a-2f, that

$$\max(n_I^{(0)}) - \overline{n}_I \simeq \overline{n}_I.$$

Therefore, the small ϵ expansion

$$n_I^{(0)} = \bar{n}_I + \epsilon \tilde{n}_I \cos\theta$$

is no longer appropriate. Note that Figs. 2a-2f refers to a numerical solution of Eqs. (14)-(18), with

$$w^{(0)} R_0 = (.7v_{thi},\ .9v_{thi}), \quad \epsilon = \frac{1}{6}, \quad Z = 6, \quad \hat{\alpha} = 1, \quad \mu = \frac{16}{6}.$$

The significance of the above result, as shown in Figs. 2, is that the super-sonic impurity is strongly pushed out (in the flux surface toward larger R) and becomes very dilute for a large domain in the surface. The ion density, on the other hand, varies much more slowly in the surface. Therefore, the friction force will not be able to vanish everywhere on the surface no matter how strong the i-I collisional coupling is. Hence, we expect substantial up-down asymmetric density variations, to be shown in the next section.

It is interesting to discuss the hollow profile of the main ion density near the outboard side where $\theta = 0$ (cf. Figs. 1-2). By using Eqs. (11) and (13), we obtain

$$\mathbf{B} \cdot \nabla \ln n_j^{(0)} = \frac{m_j \omega^{(0)^2}}{2T_i} \left(\frac{\left(1 + \frac{T_e}{T_I}\alpha^{(0)} + \frac{\alpha^{(0)}}{Z}\right) - \frac{T_e}{T_I}\alpha^{(0)}\mu}{\left(1 + \frac{\alpha^{(0)}}{Z}\right) T_i + \left(1 + \frac{T_i}{T_I}\alpha^{(0)}\right) T_e} \right) \mathbf{B} \cdot \nabla R^2, \tag{19}$$

which shows that (i) when $\alpha^{(0)}$ peaks strongly near $\theta = 0$, the term in the large parentheses of (19) becomes negative, and (ii) the degree of the hollowness strongly depends on μ. This can be understood from the fact that the density distributions of both species are due to the centrifugal force and ambipolar field while the heavier species has the larger centrifugal force, which is proportional to the mass, and therefore has a larger chance to peak near the outboard side. When this peak becomes so high that it induces an ambipolar field stronger than the centrifugal force on the lighter species, the lighter species will be pushed away from the outboard side and peak at the position where the ambipolar field is balanced with the centrifugal force.

So far, the derivation is general. In particular, radial, and thus parallel temperature gradients which may induce thermal friction, and therefore strongly alter radial transport, have not been ruled out. However presently, in the $0(\delta_p)$ equation, we will neglect these effects and concentrate on the impurity induced transport in a strongly rotating tokamak plasma without temperature gradient effects which will be included in a separate work.

II.B. The First Order Solution: Poloidal Flow and Up-down Asymmetry

First, from Eqs. (9)-(10), the $0(\delta_p)$ perpendicular moments, the diamagnetic flow and the gyroviscous tensor become

$$\mathbf{V}_{\perp j}^{(1)} = \frac{1}{m_j \Omega_j} \mathbf{b} \times \left(T_j \nabla \ln n_j^{(0)} - \frac{m_j}{2} \omega^{(0)^2} \nabla R^2 + Z_j e \nabla \phi^{(0)} \right), \tag{20}$$

$$\begin{aligned}\mathbf{\Pi}_{\perp j}^{(1)} &= \mathbf{K}_2^{(-1)} \left[\frac{n_j^{(0)} T_j}{\Omega_j} \mathbf{W}_2 [\nabla \mathbf{V}_j^{(0)}] \right] \\ &= \frac{n_j^{(0)} T_j}{4 \Omega_j} \frac{\partial \omega^{(0)}}{\partial \psi} \left[\left((I\mathbf{b} - B\mathbf{e}_\varphi)(\mathbf{e}_\varphi + \frac{3I}{B} \mathbf{b}) + \frac{1}{B} \nabla \psi \nabla \psi \right) + \uparrow_2 \right],\end{aligned} \tag{21a}$$

where $\mathbf{e}_\varphi \equiv R^2 \nabla \varphi$ and $\mathbf{W}_2$ is defined in Eq. (A2). Here, $\mathbf{\Pi}_j$ is decomposed into parallel and perpendicular components

$$\Pi_{\| j} \equiv \mathbf{b} \cdot \mathbf{\Pi}_j \cdot \mathbf{b}, \tag{21b}$$

$$\mathbf{\Pi}_{\perp j} \equiv \mathbf{\Pi}_j - \frac{3}{2} \Pi_{\| j} (\mathbf{bb} - \frac{1}{3} \mathbf{I}). \tag{21c}$$

Moreover,

$$\mathbf{K}_2^{(-1)}[\mathbf{A}] = \frac{1}{4} \left[(\mathbf{b} \times \mathbf{A} \cdot (\mathbf{I} + 3\mathbf{bb})) + \uparrow_2 \right] \tag{21d}$$

is the inverse tensor operator[16] of $\mathbf{K}_2$ defined in Eq. (A3). We note that the rank-3 inverse tensor operator $\mathbf{K}_3^{(-1)}$ of

$$\mathbf{K}_3[\mathbf{A}] \equiv \mathbf{A} \times \mathbf{b} + \uparrow_3$$

has also been derived in Ref. [17]. By taking the $\nabla \psi$ projection of Eq. (20) and adopting Eq. (11) one finds

$$\mathbf{V}_j \cdot \nabla \psi \simeq 0(\delta_p^2).$$

As mentioned, the general solution of the continuity equation

$$\nabla \cdot n_j \mathbf{V}_j = 0$$

in an axisymmetric system has the form, to $0(\delta_p)$,

$$\mathbf{V}_j^{(1)} = \frac{K_j^{(1)}(\psi)}{n_j^{(0)}}\mathbf{B} + \omega_j^{(1)} R^2 \nabla\varphi, \tag{22}$$

which, by using Eqs. (12) and (20), leads to

$$\omega_j^{(1)}(\psi,\theta) = -\frac{B}{m_j\Omega_j}\left(T_j\frac{\partial}{\partial\psi}\ln N_j(\psi) + \frac{m_j}{2}R^2\frac{\partial}{\partial\psi}\omega^{(0)^2}\right). \tag{23}$$

Here, $K_j^{(1)}(\psi)$ corresponds to the first order poloidal flow $\mathbf{V}_{pj}^{(1)}$ $(= \frac{K_j^{(1)}(\psi)}{n_j^{(0)}}B_p)$ and is to be determined.

It will be shown that this species dependent $\omega_j^{(1)}$, which induces the i-I friction, generates the $0(\delta_p)$ poloidal flow and the up-down asymmetric density variations. On the other hand, near the quasi-static state, $\frac{\partial}{\partial t} \simeq 0(\delta_p^2)$, the super-sonic-impurity poloidal flow becomes much smaller than the ion poloidal flow, in contrast to the small rotation case. This is due to the fact that $\frac{K_j}{n_j}B$ should not be much larger than its driving term $\omega_j^{(1)}R$ anywhere in the surface, while the extremely large variation of n_I, compared with that of n_i and R, keeps $\frac{K_I}{n_I}B$ very small.

We now proceed to study the $0(\delta_p)$ terms of the parallel momentum equation

$$-m_j n_j^{(0)}\omega^{(0)}\omega_j^{(1)}\mathbf{B}\cdot\nabla R^2 - \frac{1}{2}m_j n_j^{(1)}\omega^{(0)^2}\mathbf{B}\cdot\nabla R^2 + \mathbf{B}\cdot\nabla\mathbf{\Pi}_{\perp j}^{(1)} + \mathbf{B}\cdot\nabla\Pi_{\parallel j}^{(1)} - \frac{3}{2}\Pi_{\parallel j}^{(1)}(\mathbf{b}\cdot\nabla B)$$

$$+\mathbf{B}\cdot\nabla n_j^{(1)}T_j = -Z_j e\left(n_j^{(0)}\mathbf{B}\cdot\nabla\phi^{(1)} + n_j^{(1)}\mathbf{B}\cdot\nabla\phi^{(0)}\right) + BR_{\parallel j}. \tag{24}$$

Here, similar to $\phi^{(0)}$, one has

$$\mathbf{B}\cdot\nabla\phi^{(1)} = \frac{T_e}{e}\mathbf{B}\cdot\nabla\left(\ln(n_i + Zn_i)\right)^{(1)}$$

and from Eq. (29)

$$\mathbf{B}\cdot\nabla\cdot\mathbf{\Pi}_{\perp j}^{(1)} = \frac{B}{\Omega_j}T_j I^2\frac{\partial\omega^{(0)}}{\partial\psi}\left(B^2\mathbf{B}\cdot\nabla\frac{n_i^{(0)}}{B^4}\right) + 0\left(\frac{B_p^2}{B^2}\right). \tag{25}$$

By using Eqs. (4), (21), (A12), and (A13), the parallel viscosity, in the P-S regime, becomes

$$\Pi_{\parallel j}^{(1)} = -D_{2j}\frac{n_j^{(0)}}{\nu_{jj}}T_jK_j^{(1)}\left(\frac{\mathbf{b}\cdot\nabla B}{n_j^{(0)}}+\frac{2}{3}\mathbf{B}\cdot\nabla\frac{1}{n_j^{(0)}}\right) \tag{26}$$

and the parallel friction becomes

$$BR_{\parallel i} = -R_{\parallel I}B \simeq +m_i\frac{\overline{\nu}_{iI}}{\overline{n}_I}D_{1i}\left((n_i^{(0)}K_I - n_I^{(0)}K_i)B^2 + n_i^{(0)}n_I^{(0)}(\omega_I^{(1)}-\omega_i^{(1)})I\right) \tag{27}$$

Note that $\Pi_{\parallel j}^{(1)}$ is different from Braginskii's not only in including the i-I collisions but also in carefully including corrections due to the large rotation.

Noticing that $\omega_j^{(1)} = \omega_j^{(1)}(R,\psi)$, $n_j^{(0)} = n_j^{(0)}(R,\psi)$ are both up-down symmetric, we can decompose the first order density $n_j^{(1)}$ into up-down symmetric (even) and up-down asymmetric (odd) portions

$$n_j^{(1)} = n_{j+}^{(1)} + n_{j-}^{(1)}.$$

The even part of Eq. (32), by using Eqs. (19)-(22), yields

$$T_jn_j^{(0)}\mathbf{B}\cdot\nabla\frac{n_{j-}^{(1)}}{n_j^{(0)}} + \mathbf{B}\cdot\nabla\Pi_{\parallel j-}^{(1)} - \frac{3}{2}\Pi_{\parallel j-}^{(1)}(\mathbf{b}\cdot\nabla B)$$

$$= -Z_jen_j^{(0)}\mathbf{B}\cdot\nabla\phi_-^{(1)} + BR_{\parallel j+}^{(1)}. \tag{28}$$

In the P-S regime, by using Eqs. (34)-(35) and performing the flux-surface average,

$$\frac{\overline{n}_j}{m_i\overline{n}_i\overline{\nu}_{iI}B_o}\left\langle\frac{1}{n_j^{(0)}}Eq.\ (36)\right\rangle$$

one obtains the coupled equations for $K_j^{(1)}$

$$\left(\langle Q_i\rangle + \left\langle\frac{\alpha^{(0)}}{\hat{\alpha}}A_i\right\rangle\right)U_i - \langle A_i\rangle\,U_I = \langle E_i\rangle\,, \tag{29a}$$

and

$$\left(\langle Q_I\rangle + \left\langle\frac{\hat{\alpha}}{\alpha^{(0)}}A_i\right\rangle\right)U_I - \langle A_i\rangle\,U_i = -\langle E_I\rangle\,. \tag{29b}$$

Here,

$$\begin{cases} U_j \equiv \frac{K_j^{(1)} B_o}{\overline{n}_j \delta_{pi}}, \\ A_i \equiv \frac{B^2}{B_o^2} D_{1i}, \\ E_j \equiv \frac{I}{B_o} \frac{n_i^{(0)}}{\overline{n}_j} D_{1i} (\omega_I^{(1)} - \omega_i^{(1)}) / \delta_{pi}, \\ Q_j \equiv \frac{3 \overline{n}_j T_j}{2 m_i \overline{n}_i \overline{\nu}_{iI} \overline{\nu}_{jj}} D_{2I} \left(\frac{\overline{n}_j}{n_j^{(0)}} \mathbf{b} \cdot \nabla \frac{B}{B_o} + \frac{2}{3 B_o} \mathbf{B} \cdot \nabla \frac{\overline{n}_j}{n_j^{(0)}} \right)^2, \end{cases} \tag{29c}$$

where the coefficients D_{ij} are defined in App. A. $I(\psi) = RB_\varphi$ is a measure of the toroidal magnetic field, defined through the representation

$$\mathbf{B} = I(\psi) \nabla \varphi + \nabla \varphi \times \nabla \psi \, .$$

The flux surface average is

$$\langle \cdots \rangle \equiv \frac{1}{V'} \oint d\theta J(\cdots)$$

and J is the Jacobian,

$$J \equiv |\nabla \varphi \times \nabla \psi \cdot \nabla \theta| \, .$$

It is now clear that the non-vanishing term $\omega_I^{(1)} - \omega_i^{(1)}$, which can be written as

$$\omega_I^{(1)} - \omega_i^{(1)} = - \frac{B}{m_i \Omega_i} \left((\frac{T_I}{Z} - T_i) \frac{\partial}{\partial \psi} \ln n(\psi) + \frac{m_i (\mu - 1)}{2} R^2 \frac{\partial}{\partial \psi} \omega^{(0)^2} \right), \tag{30}$$

drives the first order **non-zero** poloidal flow even in a up-down symmetric magnetic configuration, in contrast to the pure plasma case.[9] $n(\psi)$ was determined in Eq. (18).

It is interesting to note that, by summing Eq. (28) over species and then flux averaging, we have

$$\sum_j T_j \left\langle n_j^{(0)} \mathbf{B} \cdot \nabla \frac{n_{j-}^{(1)}}{n_j^{(0)}} \right\rangle = \frac{3}{2} \sum_j \left\langle \Pi_{\parallel j}^{(1)} (\mathbf{b} \cdot \nabla B) \right\rangle . \tag{31a}$$

For a slowly rotating, impure plasma, this reduces to the well known relation[18]

$$\sum_j \left\langle \Pi_{\parallel j}^{(1)} (\mathbf{b} \cdot \nabla B) \right\rangle = 0. \tag{31b}$$

Before proceeding to solve for Eq. (28), it is noticed in this equation that for each species there exists an odd function $G_j(\psi, \theta)$ which satisfies

$$\mathbf{B}\cdot\nabla G_j = \frac{1}{Z_j n_j^{(0)}}\left(BR_{\|j+}^{(1)} - \mathbf{B}\cdot\nabla\Pi_{\|j-}^{(1)} + \frac{3}{2}\Pi_{\|j-}^{(1)}(\mathbf{b}\cdot\nabla B)\right). \tag{32}$$

Because it is an odd periodic function, we expand

$$G_j = \sum_{k=1}^{\infty} G_{jk}\sin k\theta,$$

whence

$$G_{jk} = \frac{2V'}{k}\left\langle \frac{BR_{\|j+}^{(1)} - \mathbf{B}\cdot\nabla\Pi_{\|j-}^{(1)} + \frac{3}{2}\Pi_{\|j-}^{(1)}(\mathbf{b}\cdot\nabla B)}{Z_j n_j^{(0)}}\cos k\theta\right\rangle. \tag{33}$$

In the P-S regime, using Eqs. (26)-(30), this becomes

$$G_{ik} = \frac{2V' m_i \bar{n}_i \bar{\nu}_{iI} B_o}{k}\delta_{pi}\Bigg\{\langle E_i\cos k\theta\rangle + \langle A_i\cos k\theta\rangle U_I$$

$$+\left(\langle Q_{si} k\sin k\theta\rangle - \langle Q_i\cos k\theta\rangle - \left\langle\frac{\alpha^{(0)}}{\hat{\alpha}}A_i\cos k\theta\right\rangle\right)U_i\Bigg\}, \tag{34a}$$

and

$$G_{Ik} = \frac{2V' m_i \bar{n}_i \bar{\nu}_{iI} B_o \delta_{pi}}{\bar{n}_I Z_I k}\Bigg\{-\langle E_I\cos k\theta\rangle + \langle A_i\cos k\theta\rangle U_i$$

$$+\left(\langle Q_{sI} k\sin k\theta\rangle - \langle Q_I\cos k\theta\rangle - \left\langle\frac{\hat{\alpha}}{\alpha^{(0)}}A_i\cos k\theta\right\rangle\right)U_I\Bigg\}, \tag{34b}$$

where

$$Q_{sj} \equiv \frac{\bar{n}_j T_j}{m_i \bar{n}_i \bar{\nu}_{iI}\bar{\nu}_{jj}}\frac{D_{2j}}{J}\left(\frac{\bar{n}_j}{n_j^{(0)}}\mathbf{b}\cdot\nabla\frac{B}{B_o} + \frac{2}{3B_o}\mathbf{B}\cdot\nabla\frac{\bar{n}_j}{n_j^{(0)}}\right)\frac{\bar{n}_j}{n_j^{(0)}}. \tag{34c}$$

Now, by defining

$$\hat{n}_j \equiv \frac{n_{j-}^{(1)}}{n_j^{(0)}},$$

Eqs. (28) and (32), after some manipulations, yield

$$T_i\mathbf{B}\cdot\nabla\hat{n}_i - \frac{T_I}{Z}\mathbf{B}\cdot\nabla\hat{n}_I = \mathbf{B}\cdot\nabla(G_i - G_I), \tag{35a}$$

$$\sum_j (n_j^{(0)} T_j \mathbf{B}\cdot\nabla\hat{n}_j) + (n_i^{(0)} + Z n_I^{(0)}) T_e \mathbf{B}\cdot\nabla\left(\frac{\hat{n}_i n_i^{(0)} + Z\hat{n}_I n_I^{(0)}}{n_i^{(0)} + Z n_I^{(0)}}\right)$$

$$= \sum_j (Z_j n_j^{(0)} \mathbf{B}\cdot\nabla G_j). \tag{35b}$$

It is then straightforward to obtain the solutions

$$\hat{n}_i = \frac{\left(1 + \frac{T_e}{T_I}\alpha^{(0)} + \frac{\alpha^{(0)}}{Z}\right) G_i - \frac{T_e}{T_I}\alpha^{(0)} G_I}{\left(1 + \frac{\alpha^{(0)}}{Z}\right) T_i + \left(1 + \frac{T_i}{T_I}\alpha^{(0)}\right) T_e}, \tag{36a}$$

$$\hat{n}_I = Z\frac{\left(\left(1 + \frac{\alpha^{(0)}}{Z}\right)\frac{T_i}{T_I} + \frac{T_e}{T_I}\right) G_I - \frac{T_e}{T_I} G_i}{\left(1 + \frac{\alpha^{(0)}}{Z}\right) T_i + \left(1 + \frac{T_i}{T_I}\alpha^{(0)}\right) T_e}. \tag{36b}$$

Here, we have obtained the solution for K_j from Eqs. (29) and $n_{j-}^{(1)}$ from Eqs. (36), namely, the poloidal flows and up-down asymmetric portions of the densities, which have been shown to be essential for the radial transport[9,10].

To determine them from given values of $\omega^{(0)}(\psi)$, $\overline{n}_j(\psi)$, one needs $n_j^{(0)}$ whose analytic form is highly nonlinear (*cf*. Eqs. (14)-(16)). Nevertheless, it is numerically straightforward to calculate $n_j^{(0)}$ from Eqs. (14)-(16), and therefore $\omega_I^{(1)} - \omega_i^{(1)}$, K_j, $n_{j-}^{(1)}$. The results are shown in Figs. 3-4 with the same parameters as given before. The results shown in Figs. 3 agree with those of Ref. [19] in which a particle and momentum conserving, 1-D (poloidal) time dependent code is used.

The ion poloidal flows for both cases are $0(\delta_p)$ and the impurity poloidal flows are too small to account for. This difference from the rotationless case, in which the ion poloidal flow are usually much smaller than the impurity poloidal flow, can be understood from Eqs. (31). The small increase of the ion poloidal flow for larger rotation is due to the inertial contribution to the diamagnetic flows as shown in Eq. (30).

From Figs. 3b and 3d, one also notices that the odd portion of the impurity density increases with the rotation as expected from Eqs. (36). Moreover, Figs. 3 show that not only the first but also second poloidal harmonic ($\sin 2\theta$) dominates the up-down density modulations. This is a result of the nonlinearity arising from the finite value of $\frac{\omega^{(0)2} R^2}{v_{thI}}\epsilon$.

To estimate the result analytically, we assume a strong supersonic impurity and subsonic main ion, i.e.

$$V_{thI} \ll \omega^{(0)} R_o \leq V_{thi}, \quad Z \gg 1,$$

whence, from Eq. (15)

$$\frac{n_i^{(0)} - \overline{n}_i}{\overline{n}_i} \leq 0(\epsilon).$$

One finds, from Eqs. (14)-(16),

$$\omega_I^{(1)} - \omega_i^{(1)} \simeq -\frac{B}{m_i \Omega_i} \left((\frac{T_I}{Z} - T_i) \frac{\partial}{\partial \psi} \ln \overline{n}_i + \frac{m_i(\mu - 1)}{2} (R^2 - \langle R^2 \rangle) \frac{\partial}{\partial \psi} \omega^{(0)^2} \right), \tag{37}$$

The super-sonic impurity makes the coefficient

$$\left\langle B^2 D_{1i} \frac{\hat{\alpha}}{\alpha^{(0)}} \right\rangle$$

very large. The parallel viscous term, which can be smaller than the friction force in the P-S regime, i.e.

$$\overline{\nu}_{iI} \quad \gg \quad \frac{\epsilon^2 \omega_{ti}^2}{\overline{\nu}_{ii}} \quad \gg \quad \frac{\epsilon^2 \omega_{ti}^2}{Z^2 \overline{\nu}_{II}},$$

can be neglected for simplicity. We then find, neglecting $O(\epsilon)$ terms, for U_i defined in (29c),

$$U_i \sim \frac{BI}{m_i B_o \Omega_i \delta_{pi}} \left((T_i - \frac{T_I}{Z}) \frac{\left\langle \frac{n_I^{(0)}}{\overline{n}_I} D_{1i} \right\rangle}{\left\langle \frac{\alpha^{(0)}}{\hat{\alpha}} A_i \right\rangle} \frac{\partial}{\partial \psi} \ln \overline{n}_i \right) \sim v_{thi}. \tag{38}$$

Note that unless the toroidal velocity profile is much steeper than the density profile

$$\frac{\partial}{\partial \psi} \ln \omega^{(0)} \gg \frac{\partial}{\partial \psi} \ln \overline{n}_i$$

or unless

$$\omega^{(0)} R \gg v_{thi},$$

Eq. (38) gives a fair estimation.

Therefore, we have obtained the poloidal flows and up-down density modulations driven by the relative diamagnetic flow between main ions and impurities ($i - I$), using the δ_{pi} ordering scheme while keeping important nonlinear effects. Nonetheless, since $\frac{n_-(\theta)}{\overline{n}\epsilon} \sim O(\delta_{pi})$, one will not expect very strong transport from the results obtained here. Hence, the strong ordering calculation, which is expected to induce $\frac{n_-(\theta)}{\overline{n}\epsilon} \sim O(1)$, is of particular interest.

III. Strong Impurity Ordering $\Delta \sim 1$

In this section, we consider the strong ordering case $\Delta \sim 1$ (where Δ is defined in Eq. (8)); therefore, the parallel friction term is kept in the zeroth order of the impurity Eq. (6) but neglected in the 0(1) ion Eq. (5). This is because of the smallness of $\frac{n_I}{n_i} = \frac{\alpha}{Z^2} \ll 1$. Note that, even for a light impurity such as C^{+6} or O^{+8}, the ordering $\Delta \sim 1$ in the P-S regime is still appropriate. Now, taking the parallel projection of the 0(1) terms, Eq. (5) and Eq. (13) yield

$$\mathbf{B} \cdot \nabla \ln n_i = \frac{m_i \omega^{(0)2}}{2(T_i + T_e)} \mathbf{B} \cdot \nabla R^2 - \frac{T_e \mathbf{B} \cdot \nabla \ln(1 + \frac{\alpha^{(0)}}{Z})}{T_e + T_i} , \tag{39a}$$

and the subtraction of Eq. (6) from Eq. (5) yields

$$\frac{m_i}{2}(\mu - 1)\omega^{(0)2}\mathbf{B} \cdot \nabla R^2 + (T_i - \frac{T_I}{Z})\mathbf{B} \cdot \nabla \ln n_i^{(0)} - \frac{T_I}{Z}\mathbf{B} \cdot \nabla \ln \alpha^{(0)}$$

$$= \sqrt{2} m_i Z \bar{\nu}_{ii} D_{1i} \left\{ B^2 \left(\frac{K_I^{(1)}}{\bar{n}_I} \frac{\hat{\alpha}}{\alpha^{(0)}} - \frac{K_i^{(1)}}{\bar{n}_i} \right) \right.$$

$$\left. + \frac{BI}{m_i \Omega_i} \frac{n_i^{(0)}}{\bar{n}_i} \left[\frac{m_i}{2}(\mu - 1)\omega^{(0)2} \frac{\partial}{\partial \psi} R^2 + (T_i - \frac{T_I}{Z}) \frac{\partial}{\partial \psi} \ln n_i^{(0)} - \frac{T_I}{Z} \frac{\partial}{\partial \psi} \ln \alpha^{(0)} \right] \right\} . \tag{39b}$$

Here, the right hand side of Eq. (39b) is $\frac{\mathbf{B} \cdot \mathbf{R}_{iI}}{n_I Z}$. It is also useful to combine Eqs. (39a) and (39b) into

$$\frac{m_i}{2} \left(\mu - 1 + \frac{T_i(1 - \frac{T_I}{ZT_i})}{T_i + T_e} \right) \omega^{(0)2} \mathbf{B} \cdot \nabla R^2$$

$$- \frac{T_i}{Z} \mathbf{B} \cdot \nabla \left((1 - \frac{T_I}{ZT_i}) \frac{ZT_e}{T_i + T_e} \ln(1 + \frac{\alpha^{(0)}}{Z}) + \frac{T_I}{T_i} \ln \alpha^{(0)} \right) = \frac{\mathbf{B} \cdot \mathbf{R}_{iI}}{n_I Z} \tag{39c}$$

Note that Eqs. (39) form a closed 2-D nonlinear system, which can be reduced to a 1-D system by assuming that $\frac{\partial}{\partial \theta} \tilde{f}(\theta) \sim r \frac{\partial}{\partial r} \tilde{f}(\theta)$, that is,

$$\left| \frac{\sqrt{2} m_i Z \bar{\nu}_{ii} D_{ii} \frac{BI}{m_i \Omega_i} \frac{\partial}{\partial \psi} \tilde{f}(\theta)}{\mathbf{B} \cdot \nabla \tilde{f}(\theta)} \right| \sim \frac{\Delta}{Z} \ll 1 .$$

Therefore, $\frac{\partial}{\partial \psi} \ln n_i^{(0)}$ and $\frac{\partial}{\partial \psi} \ln \alpha^{(0)}$ on the right hand side of Eq. (39b) can be replaced by $\frac{\partial}{\partial \psi} \ln \bar{n}_i$ and $\frac{\partial}{\partial \psi} \ln \hat{\alpha}$ which become pure driving terms. Then, the 1-D nonlinear system can be solved numerically.

To further proceed with the **analytic** study, however, we linearize the equations by assuming $\epsilon \ll 1$ and neglecting $O(1/Z)$ terms. Equations (39) thus yield, to $O(1)$

$$U_I - U_i = A_d \equiv -\frac{1}{2}\left\{r\frac{\partial}{\partial r}\ln \bar{n}_i - \frac{T_I}{ZT_i} r\frac{\partial}{\partial r}\ln \bar{n}_I\right\} , \tag{40a}$$

and to $O(\epsilon)$

$$\frac{\partial}{\partial \theta} y - a_\Delta (A_d + U_i) y = -b_\omega \sin\theta + a_\Delta \left(2A_d - \frac{(\mu-1)\hat{\omega}^2}{Z}\right)\cos\theta . \tag{40b}$$

Here,

$$\hat{\omega}^2 \equiv \frac{\omega^{(0)2} R_o^2}{v_{thi}^2} Z , \quad y \equiv \frac{\alpha^{(0)} - \hat{\alpha}}{\hat{\alpha}\epsilon} , \quad U_j \equiv \frac{K_j^{(1)} B_o}{\bar{n}_j \delta_{pi} v_{thi}} , \tag{40c}$$

$$a_\Delta \equiv \frac{2\Delta D_{1i}}{(1 - \frac{T_I}{ZT_i})\frac{T_e}{T_i+T_e}\hat{\alpha} + \frac{T_I}{T_i}} , \tag{40d}$$

and

$$b_\omega \equiv \frac{2(\mu - 1 + \frac{T_i - \frac{T_I}{Z}}{T_i+T_e})\hat{\omega}^2}{(1 - \frac{T_I}{ZT_i})\frac{T_e}{T_i+T_e}\hat{\alpha} + \frac{T_I}{T_i}} . \tag{40e}$$

We therefore obtain

$$y = y_c \cos\theta + y_s \sin\theta \tag{41a}$$

with

$$y_c = \frac{b_\omega - a_\Delta^2 (A_d + U_i)(2A_d - \frac{(\mu-1)\hat{\omega}^2}{Z})}{1 + a_\Delta^2 (A_d + U_i)^2} , \tag{41b}$$

$$y_s = \frac{a_\Delta [b_\omega (A_d + U_i) + 2A_d - \frac{(\mu-1)\hat{\omega}^2}{Z}]}{1 + a_\Delta^2 (A_d + U_i)^2} . \tag{41c}$$

Here, y_c measures the in-out density modulation and y_s measures the up-down density modulation. The ion density modulation can then be derived from Eqs. (39a) and (41), and the impurity density modulation can be obtained from

$$\frac{\tilde{n}_I(\theta)}{\epsilon \bar{n}_I} = y + \frac{\tilde{n}_i(\theta)}{\epsilon \bar{n}_i} \simeq y + O(\frac{1}{Z}). \tag{41d}$$

The importance of the parameter Δ, influencing the parallel friction, can be seen from the dependence of y on a_Δ. That is, if $\Delta \sim \delta_{pi} \ll 1$, then the $O(1)$ density modulation, which reduces to the $O(1)$ solution in Section II for small ϵ, is up-down symmetric and is driven purely by the centrifugal force via b_ω. On the other hand, for finite Δ, y_c and y_s are strongly coupled to each other. It is interesting to note here that the strong ordering $\Delta \sim 1$ does not explicitly depend on the impurity concentration, and therefore Eqs. (41) are still valid and useful for impurity species with $\alpha \ll 1$; however, the ion dynamics, e.g., U_i, n_i, reduce to that of a pure plasma system. In this case, one may expect that both y_c and y_s increase with rotation through b_ω, as predicted from a simple minded argument: since y_c is basically driven by centrifugal force and y_s basically by parallel friction, increasing rotation increases y_c, and increasing y_c induces an increasing parallel friction which then drives an increasing y_s. However, this simple minded story will not be relevant for impurities of strength $\alpha \sim 1$, because the ion poloidal flow will be strongly coupled to the impurity flow through friction. In other words, simple mindedly assuming the ion poloidal flow to be driven by the temperature gradient as in a pure plasma system[20] is totally inadequate for an impure plasma. In fact, it will be shown later that by self-consistently evaluating U_i, one finds that y_s decreases with increasing rotation for $\alpha \sim 1$ and $\hat{\omega} > 1$.

To evaluate the ion poloidal flow self-consistently, one needs to utilize the $O(\delta_{pi})$ ion momentum equation Eq. (5) which yields

$$\left\langle \frac{\mathbf{B} \cdot \nabla \cdot \mathbf{\Pi}_i^{(1)}}{n_i^{(0)}} \right\rangle = \left\langle \frac{\mathbf{B} \cdot \mathbf{R}_{iI}^{(1)}}{n_i^{(0)}} \right\rangle , \tag{42}$$

where, using Eq. (39c)

$$\left\langle \frac{\mathbf{B} \cdot \mathbf{R}_{iI}^{(1)}}{n_i} \right\rangle = \frac{m_i {\omega^{(0)}}^2}{2Z} \left(\mu - 1 + \frac{T_i - \frac{T_I}{Z}}{T_i + T_e} \right) \left\langle \alpha^{(0)} \mathbf{B} \cdot \nabla R^2 \right\rangle . \tag{43}$$

Note that, using the fact that Eq. (23) is still valid for ions in the strong ordering case, $\langle \mathbf{B} \cdot (\mathbf{V}_i \cdot \nabla \mathbf{V}_i) \rangle$ has been neglected from Eq. (42). Hence, by using Eqs. (21), (25), (26) and (43), Eq. (42) yields

$$\xi_i a_\Delta U_i \left\{ \left(1 + \frac{4\hat{\omega}^2 T_i}{3Z(T_i + T_e)} - \frac{2\hat{\alpha} T_e}{3Z(T_i + T_e)} y_c \right)^2 + \left(\frac{2\hat{\alpha} T_e}{3Z(T_i + T_e)} \right)^2 y_s^2 \right\}$$
$$= -\hat{\alpha}(g_\omega + b_\omega) y_s \tag{44}$$

where

$$\xi_i \equiv \frac{3}{4\sqrt{2}} \frac{D_{2i}(\hat{\alpha}) \omega_{ti}^2}{D_{1i}(\hat{\alpha}) \overline{\nu}_{ii}^2}$$

represents the parallel viscosity, and

$$g_\omega \equiv \frac{(r\frac{\partial}{\partial r}\ln\omega^{(0)})\delta_{pi}Z^{1/2}\frac{T_e}{T_i+T_e}2\hat{\omega}}{(1-\frac{T_I}{ZT_i})\frac{T_e\hat{\alpha}}{(T_i+T_e)}+\frac{T_I}{T_i}} ,$$

represents the gyroviscosity which can become important only when the rotation is of $O(\delta_{pi})v_{thi}$.

Therefore, the normalized ion poloidal flow U_i and the density modulation y can be determined straightforwardly by solving Eqs. (41) and (44). The results are given in Figs. 4a-4d as functions of parameters $\hat{\omega}, \Delta$, and ξ_i defined after Eqs. (40)and Eq. (44), for $Z = 8$, $\delta_{pi} = 0.05$ and $A_d = 1$. From Eq. (44) and Fig. 4c, it is noticed that the results are sensitive to the parameter ξ_i arising from parallel viscosity. Since our derivation of the parallel viscosity (cf. Appendix A) is based upon the assumption of high collisionality, it is adequate to set $\xi_i = 0.2$ as in Fig. 4d. However, the main ion in a typical tokamak plasma is mainly in the plateau regime. By analogy with neoclassical transport theory in the small rotation case,[18] we conjecture that taking $\xi_i \simeq O(1)$ will give a reasonable estimate for the main ion plateau regime with $\frac{\nu_{ii}}{\omega_{ti}} \simeq O(1)$. The results for $\xi = 1.5$ are given in Figs. 4a-4b.

Although there is no direct validity restriction on the value of rotation in Eqs. (41b,c) and (44), the linearization leading to Eqs. (40) comes from a large aspect ratio assumption relying on the smallness of $\frac{\omega^{(0)^2}R^2}{v_{thI}^2}\epsilon = \mu\hat{\omega}^2\epsilon$. This implies that if $\mu\hat{\omega}^2\epsilon$ becomes finite, Eqs. (41) and (44) are no longer valid, and as indicated in Section II nonlinear effects via $\ln n_i^{(0)}$ and $\ln\alpha^{(0)}$ in Eqs. (39) become significant, and higher poloidal harmonics can be important. In that case, one needs to solve the nonlinear equations numerically. However, this case will not be included in the present work. Also note that for large rotation ($\hat{\omega} > 1$) and finite $\hat{\alpha}$, Eqs. (41) and (44) yield

$$U_i \simeq -A_d + \frac{(\mu-1)\hat{\omega}^2}{b_\omega Z} + O(\frac{1}{\hat{\omega}^2}) , \qquad (45a)$$

$$y_s \simeq \frac{\xi_i a_\Delta(A_d - \frac{(\mu-1)\hat{\omega}^2}{b_\omega Z})}{\hat{\alpha}(g_\omega + b_\omega)}\left(1 + \frac{4}{3}\frac{\hat{\omega}^2}{Z}\frac{T_i}{(T_i+T_e)}\frac{\frac{T_I}{T_i} - (\mu-1)\hat{\alpha}\frac{T_e}{T_i}}{(1-\frac{T_I}{ZT_i})\frac{T_e}{T_i+T_e}\hat{\alpha} + \frac{T_I}{T_i}}\right)^2 . \qquad (45b)$$

This explains the decrease of y_s with increasing $\hat{\omega}^2$ shown in Fig. 4a.

On the other hand, for small rotation, Eqs. (41) and (44) yield

$$y_s \simeq \frac{a_\Delta A_d(2+b_\omega)}{1+a_\Delta^2 A_d^2} , \qquad (46a)$$

$$U_i \simeq -\frac{\hat{\alpha} A_d (2+b_\omega)}{\xi_i (1+a_\Delta^2 A_d^2)} (g_\omega + b_\omega) \; . \tag{46b}$$

That is, y_s increases with $\hat{\omega}^2$ for small rotation. The turning point at which y_s starts to decrease with rotation occurs near the point

$$b_\omega (A_d + U_i) - \frac{(\mu - 1)}{Z} \hat{\omega}^2 = 0 \; .$$

For $A_d \sim 1$ and $Z \gg 1$, this occurs when $U_I = A_d + U_i$ changes sign, i.e., when the impurity poloidal flow reverses direction (cf. Fig. 4a).

Moreover, it is of particular interest to study the case of a heavy test impurity (i.e., $\hat{\alpha} = \frac{\bar{n}_I Z^2}{\bar{n}_i} \ll 1$) using our strong ordering scheme. This is because the ordering parameter $\Delta = \frac{\delta_{pi} Z^2 \sqrt{2} \bar{\nu}_{ii}}{\omega_{ti}}$ is independent of $\hat{\alpha}$ but is proportional to Z^2. It is worth mentioning that although Eqs. (39a) and (44) do not give an adequate description of the ion dynamics for $\alpha \lesssim \delta_{pi}$, they nonetheless provide the correct description to $O(\delta_{pi})$ in the limit $\hat{\alpha} = 0$. One obtains

$$U_i = 0 \quad \text{and} \quad n_i \propto e^{\frac{m_i \omega(0)^2 R^2}{2(T_i + T_e)}} \; , \tag{47a}$$

as predicted by other authors.[3,9] Also, the impurity density modulation can be determined by taking $\hat{\alpha} = 0$ from Eqs. (41) which yields

$$y_s = \frac{a_\Delta [A_d (2 + b_\omega) - \frac{(\mu-1)}{Z} \hat{\omega}^2]}{1 + a_\Delta^2 A_d^2} \; . \tag{47b}$$

With $A_d \sim 1$ and $Z \gg 1$, this implies that y_s increases with $\hat{\omega}^2$ as shown in Fig. 5.

IV. Radial Transport

In this section, we evaluate the radial transport flux using the moment approach and the strong ordering results obtained in Section III. The detailed formalism of the moment approach to the transport theory of magnetized plasma is given elsewhere;[13] here, we only briefly describe the approach and present the results.

Due to the neglection of temperature variations, only the particle diffusion and toroidal momentum damping is considered. We start with the moment equations

$$\frac{\partial}{\partial t} n_j + \nabla \cdot n_j \mathbf{V}_j = N_{sj} \tag{48a}$$

$$\frac{\partial}{\partial t} m_j n_j \mathbf{V}_j + \nabla \cdot (m_j n_j \mathbf{V}_j \mathbf{V}_j + \mathbf{P}_j) = n_j e Z_j \mathbf{E} + m_j n_j \Omega_j \mathbf{V}_j \times \mathbf{b} + \mathbf{R}_j + \mathbf{M}_{sj} \tag{48b}$$

$$\frac{\partial}{\partial t} (\mathbf{P}_j + m_j n_j \mathbf{V}_j \mathbf{V}_j) + \nabla \cdot \mathbf{Q}_j = n_j Z_j e (\mathbf{E V}_j + \mathbf{V}_j \mathbf{E}) + (\mathbf{R}_j \mathbf{V}_j + \mathbf{V}_j \mathbf{R}_j)$$

$$+\Omega_j [(\mathbf{P}_j + m_j n_j \mathbf{V}_j \mathbf{V}_j) \times \mathbf{b} + \text{transpose}] + \mathbf{\Pi}_{cj} \tag{48c}$$

Here, N_{sj} is the particle source, $\mathbf{M}_{sj}$ the momentum source,

$$\mathbf{Q}_j \equiv \int d\mathbf{v} \;\; m_j(\mathbf{v} + \mathbf{V}_j)(\mathbf{v} + \mathbf{V}_j)(\mathbf{v} + \mathbf{V}_j) f_j \tag{48d}$$

(with $\mathbf{v}$ the particle velocity in the rest frame of the fluid moving with velocity V_j)

$$\mathbf{\Pi}_{cj} \equiv \int d\mathbf{v} \;\; m_j(\mathbf{vv} - \frac{v^2}{3}\mathbf{I}) C_j(f_j) \; . \tag{48e}$$

C_j is the collision operator, and $\mathbf{R}_j$ is the collisional interspecies friction, and $\mathbf{P}$ is the pressure tensor. Hence, the particle conservation for species j is

$$< \frac{\partial}{\partial t} n_j > + \frac{1}{V'} \frac{\partial}{\partial \psi} V' \Gamma_j = < N_{sj} > \, , \tag{49a}$$

and the flux surfaced toroidal momentum conservation is

$$\sum_j \left\langle \frac{\partial}{\partial t} (m_j n_j \mathbf{V}_j \cdot e_\varphi) \right\rangle + \frac{1}{V'} \frac{\partial}{\partial \psi} V' \sum_j \Pi_j = \sum_j < \mathbf{M}_{sj} \cdot \mathbf{e}_\varphi > + \frac{1}{c} < \mathbf{J} \cdot \nabla \psi > \; . \tag{49b}$$

Here, $\mathbf{e}_\varphi \equiv R^2 \nabla_\varphi$,

$$\Gamma_j \equiv < n_j \mathbf{V}_j \cdot \nabla \imath \cdot > \tag{50}$$

is the particle flux, and

$$\Pi_j \equiv < \mathbf{e}_\varphi \cdot (m_j n_j \mathbf{V}_j \mathbf{V}_j + \mathbf{\Pi}_j) \cdot \nabla\psi > \tag{51}$$

is the toroidal angular momentum flux. Concerning the radial current in the right hand side of Eq. (49b), it has been shown by many authors[18,9] that

$$\frac{1}{c} < \mathbf{J} \cdot \nabla\psi > \sim \frac{v_A^2}{c^2}\frac{\partial}{\partial t} < m_j n_j \mathbf{V}_j \cdot \mathbf{e}_\varphi >$$

which is therefore negligible in low beta plasmas.

Using the identities

$$< n_j \mathbf{V}_j \cdot \nabla\psi > = < n_j \mathbf{V}_j \times \mathbf{B} \cdot \mathbf{e}_\varphi >$$

$$< \mathbf{e}_\varphi \cdot \mathbf{\Pi} \cdot \nabla\psi > = \frac{1}{2} < \mathbf{e}_\varphi \mathbf{e}_\varphi : (\mathbf{\Pi} \times \mathbf{B} + \text{transpose}) > ,$$

and the largeness of Ω_i in magnetized plasma, $\frac{B}{m_j\Omega_j} < \mathbf{e}_\varphi \cdot$ Eq. (48a)> yields

$$\Gamma_j = -\frac{B}{m_j\Omega_j} < \mathbf{R}_j \cdot \mathbf{e}_\varphi > = -\frac{IB}{m_j\Omega_j}\left\langle \frac{R_{\parallel j}}{B} \right\rangle - \frac{B}{m_j\Omega_j}\langle \mathbf{R}_{\perp j} \cdot \mathbf{e}_\varphi \rangle \tag{52}$$

and similarly $\frac{B}{2\Omega_j} < \mathbf{e}_\varphi \cdot$ Eq. (48c)> yields

$$\Pi_j = -\frac{IB}{\Omega_j}\left\{ \omega^{(0)} \left\langle \frac{R^2}{B} R_{\parallel j} \right\rangle + \left\langle \frac{3I^2 - B^2 R^2}{4B^2} \Pi_{c\parallel j} \right\rangle \right\}$$

$$-\frac{B}{\Omega_j}\left\{ \omega^{(0)} < R^2 \mathbf{R}_{\perp j} \cdot \mathbf{e}_\varphi > + \frac{1}{2} < \mathbf{e}_\varphi \cdot \mathbf{\Pi}_{c\perp j} \cdot \mathbf{e}_\varphi > \right\} . \tag{53}$$

Here, the terms involving the parallel collisional moments, such as $R_{\parallel j}$ and $\Pi_{c\parallel j}$ correspond to the neoclassical fluxes, and terms such as $\mathbf{R}_{\perp j}$ and $\mathbf{\Pi}_{c\perp j}$ correspond to the classical fluxes. Here,

$$\Pi_{c\parallel j} \equiv \mathbf{b} \cdot \mathbf{\Pi}_{cj} \cdot \mathbf{b} ,$$

$$\mathbf{\Pi}_{c\perp j} \equiv \mathbf{\Pi}_{cj} - \Pi_{c\parallel j}(\mathbf{bb} - \frac{\mathbf{I}}{3}) .$$

The classical fluxes, which are smaller than the neoclassical terms by a factor B_p^2/B^2, can be determined straightforwardly using Eqs. (52)–(53), (A8), (A15), (22), and (21a); that is

$$\Gamma_j^{\text{class}} \equiv \frac{1}{Z_j}\frac{B}{\Omega_i}\left\langle n_i \nu_{iI} \frac{|\nabla\psi|^2}{B^2}(\omega_j^{(1)} - \omega_{j'}^{(1)}) \right\rangle , \tag{54}$$

$$\Pi^{\text{class}} \equiv \sum_{j=i,I} \Pi_j^{\text{class}} = (\mu-1)\frac{B}{\Omega_i}\omega^{(0)}\left\langle m_i n_i \nu_{iI} R^2 \frac{|\nabla\psi|^2}{B^2}(\omega_I^{(1)} - \omega_i^{(1)})\right\rangle$$

$$-\frac{3}{5}\left(\frac{B}{\Omega_i}\right)^2 \frac{\partial\omega^{(0)}}{\partial\psi} T_i \left\langle n_i(\nu_{ii}+\nu_{iI})R^2\frac{|\nabla\psi|^2}{B^2}\right\rangle , \qquad (55)$$

where $\frac{m_I}{m_i}$, $Z \gg 1$ has been used. It is also worth mentioning here that Eqs. (52) and (53) agree with results obtained in the standard δ_p ordering scheme, and all the terms omitted are carefully justified as negligible when considering the strong ordering $\Delta \sim 1$ by noticing that $Z \gg 1$. For instance, in deriving Eq. (52),

$$\left|\frac{\frac{1}{V'}\frac{\partial}{\partial\psi}V' < \mathbf{e}_\varphi \cdot (m_j n_j \mathbf{V}_j \mathbf{V}_j + \mathbf{P}_j)\cdot\nabla\psi >}{< \mathbf{R}_j \cdot \mathbf{e}_\varphi >}\right| \sim \delta_{pi}\frac{\omega^{(0)}R_o}{v_{thi}} \ll 1$$

has been used to drop the non-ambipolar piece of the particle flux.

For neoclassical fluxes, using the parallel collisional moments derived in the Appendix, small ϵ expansion and Eq. (39c), Eqs. (52), (53) yield the neoclassical particle flux

$$\Gamma_i^{neo} = -Z\Gamma_I^{neo} = \frac{Iv_{thi}}{2\Omega_{oi}}\omega_{ti}\bar{n}_I\epsilon^2\left[(1-\frac{T_I}{ZT_i})\frac{T_e\hat{\alpha}}{T_i+T_e}+\frac{T_I}{T_i}\right](1+\frac{b_\omega}{2})y_s , \qquad (56)$$

and the neoclassical momentum flux

$$\Gamma^{neo} = \Pi_i^{neo} + \Pi_I^{neo} =$$

$$= \frac{Iv_{thi}}{2\Omega_{oi}}\omega_{ti}m_i\bar{n}_I\omega^{(0)}R_o^2\epsilon^2\left\{(\mu-1)\left[(1-\frac{T_I}{ZT_i})\frac{T_e\hat{\alpha}}{T_i+T_e}+\frac{T_I}{T_i}\right](1+\frac{b_\omega}{4})+\frac{1}{3}\frac{T_e}{T_i+T_e}\delta_{pi}Z^{3/2}\frac{U_i}{\hat{\omega}}\right\}y_s . \qquad (57)$$

Thus, the neoclassical fluxes appear to be driven explicitly by the up-down density modulation y_s, as expected.

One can then estimate the particle confinement time τ_n and angular momentum confinement time τ_φ by

$$\frac{1}{\tau_{nj}} \equiv \left|\frac{\frac{1}{V'}\frac{\partial}{\partial\psi}V'\Gamma_j}{\bar{n}_j}\right| \simeq \frac{\bar{n}_I}{Z_j n_j}\delta_{pi}\epsilon^2 L_{nj}\omega_{ti}\left[(1-\frac{T_I}{ZT_i})\frac{T_e\hat{\alpha}}{T_i+T_e}+\frac{T_I}{T_i}\right](1+\frac{b_\omega}{2})y_s , \qquad (58)$$

and

$$\frac{1}{\tau_\varphi} \equiv \frac{\left|\frac{1}{V'}\frac{\partial}{\partial\psi}V'\Pi\right|}{m_i\bar{n}_i\omega^{(0)}R_o^2} \simeq \frac{\bar{n}_I}{n_i}\delta_{pi}\epsilon^2 L_\varphi\omega_{ti}\left\{(\mu-1)\left[(1-\frac{T_I}{ZT_i})\frac{T_e\hat{\alpha}}{T_i+T_e}+\frac{T_I}{T_i}\right](2+\frac{b_\omega}{2})\right.$$

$$+\frac{2}{3}\frac{T_e}{T_i+T_e}\delta_{pi}Z^{3/2}\frac{U_i}{\hat{\omega}}\Bigg\}y_s\ , \tag{59}$$

with the profile factors

$$L_{nj} \equiv \left|\frac{1}{2}r\frac{\partial}{\partial r}\ln(V'\Gamma_j)\right|\ ,$$

and

$$L_{\varphi} \equiv \left|\frac{1}{2}r\frac{\partial}{\partial r}\ln(V'\Pi)\right|\ .$$

Note that the classical contributions are neglected from Eqs. (58) and (59) for simplicity.

By noticing that both $\frac{1}{\tau_{ni}}$ and $\frac{1}{\tau_{\varphi}}$ scale as $\frac{\overline{n}_I}{n_i}\delta_{pi}\epsilon^2\omega_{ti}$, one finds two interesting features: (i) For finite $\hat{\omega}$, Δ, and ξ_i the momentum damping rate is of the same order of magnitude as the ion particle diffusion rate which agrees with the experimental observation; (ii) The damping rate scaling $\frac{\overline{n}_I}{n_i}\delta_{pi}\epsilon^2\omega_{ti}$ coincides with that of the "gyroviscous theory"[10] (if it was assumed that impurity gyroviscosity prevails over ion gyroviscosity owing to the strong impurity density modulation). However, it is obvious that the physical origins in the present work are of collisional nature.

In Figs. 6a-6c, the normalized damping rates $(\tau_{ni}\omega_{ti}\frac{\overline{n}_I}{n_i}\delta_{pi}\epsilon^2 L_{ni})^{-1}$ and $(\tau_{\varphi}\omega_{ti}\frac{\overline{n}_I}{n_i}\delta_{pi}\epsilon^2 L_{\varphi})^{-1}$ are presented as functions of parameters $\hat{\omega}$, Δ, and ξ_i. It is also interesting to note here, from the observation of Fig. 5b, that $(\tau_{\varphi}\omega_{ti}\frac{\overline{n}_I}{n_i}\delta_{pi}\epsilon^2 L_{\varphi})^{-1} \simeq 4\Delta$ which leads to a familiar scaling $\frac{1}{\tau_{\varphi}} \simeq 4\delta_{pi}^2\epsilon^2 L_{\varphi}\overline{\nu}_{iI}$. Thus, if one takes $\overline{\nu}_{iI} = \sqrt{2}\hat{\alpha}\overline{\nu}_{ii} \simeq \overline{\nu}_{ii}$ and $q^2 \simeq 10$, then an enhancement of the momentum damping rate by two orders of magnitude over the previously derived classical damping rate,[9] (using Braginskii's perpendicular viscosity tensor in a pure plasma), is not difficult to find.

Since the radial fluxes are explicitly proportional to y_s, their limiting behaviors at large or small rotation are similar to those of y_s discussed in Section III. In addition the particle diffusion flux for a test impurity also behaves quite similar to the behavior of y_s vs $\hat{\omega}^2$, as shown in Fig. 5.

Finally, it is of considerable interest to discuss the difference of radial transport due to the direction of momentum input occurring in Neutral Beam Injection (NBI) experiments; namely, the co- vs counter-injection. It is observed[2] that the toroidal rotation $V_{\varphi i}$ changes sign when the beam direction is changed from co- to counter-going with respect to the plasma current. However, noting that

$$\mathbf{V}_i = \frac{K_i}{n_i}\mathbf{B} + \omega_i^{(1)} R^2 \nabla\varphi + \omega^{(0)} R \nabla\varphi$$

one has

$$V_{\varphi i} \simeq \frac{K_i}{n_i} B_o + \omega_i^{(1)} R_o + \omega^{(0)} R_o + O(\epsilon) \; .$$

That is

$$V_{\varphi i} \simeq \left[\delta_{pi}(U_i + A_d - \frac{1}{2}\frac{T_I}{ZT_i} r \frac{\partial}{\partial r} \ln \bar{n}_1) + \frac{\hat{\omega}}{\sqrt{Z}}\right] v_{thi} \tag{60}$$

In Figs. 7a -7b, we present the radial fluxes vs $\sqrt{Z}\frac{|V_{\varphi i}|}{v_{thi}}$ for co-injection ($V_\varphi > 0$) and counter-injection ($V_{\varphi i} < 0$), using Eqs. (60), (41), and (44).

To give an analytic estimation of the difference between radial fluxes due to co- and counter-injection, we take the case of a test impurity ($\hat{\alpha} \to 0$). Assuming the radial profile remains the same for co- and counter-injection and using Eqs. (47a), (47b), (60), and (56), one finds

$$(\Gamma_I^{neo})_{count} - (\Gamma_I^{neo})_{co} = -\frac{I v_{thi}}{\Omega_{oi}} \omega_{ti} \bar{n}_I \epsilon^2 \frac{T_I}{ZT_i} 2\delta_{pi} \sqrt{Z} A_{di} \hat{V}_\varphi$$

$$\left[4A_d b_o - \frac{\mu - 1}{Z} + 2b_o(\hat{V}_\varphi^2 + \delta_{pi}^2 Z A_{di}^2)(2A_d b_o - \frac{\mu - 1}{Z})\right] \; . \tag{61}$$

Here, $A_{di} \equiv -\frac{1}{2} r \frac{\partial}{\partial r} \ln \bar{n}_i$, $\hat{V}_\varphi \equiv \sqrt{Z}\frac{|V_{\varphi i}|}{v_{thi}}$, and

$$b_o \equiv \frac{b_\omega(\hat{\alpha} = 0)}{2\hat{\omega}^2} = \frac{T_i}{T_I}\left(\mu - 1 + \frac{T_i - \frac{T_I}{Z}}{T_i + T_e}\right) \; .$$

That is, for both co- and counter-injection the impurity has an inward flux, but much less for co-injection than for counter-injection. The difference is given in Eq. (61) and is increasing with increasing δ_{pi}.

V. Conclusions

The transport phenomena induced by ion-impurity friction in strongly rotating tokamak plasmas have been studied, assuming constant temperature profile. First, using the standard ordering scheme, up-down density variations which drive the neoclassical transport have been found driven by parallel $i - I$ friction. These variations are of order $\delta_{pi}\epsilon$, even for up-down symmetric equilibrium configurations. Then, by recognizing that $\delta_{pi}Z^2$ is of lower order than δ_{pi} itself and therefore implementing a strong ordering

$$\Delta \equiv \frac{\delta_{pi} Z^2 \sqrt{2}\bar{\nu}_{ii}}{\omega_{ti}} \sim 1 ,$$

strong up-down density variations of order ϵ have been found in Eqs. (41). One also finds that for plasmas with $\hat{\alpha} \equiv \frac{\bar{n}_I Z^2}{\bar{n}_i} \gg \delta_{pi}$, the main ion dynamics is strongly coupled with the impurities through the $i - I$ friction. Hence, the ion poloidal flows are determined self-consistently via Eq. (44). Moreover, the transport fluxes driven by the strong up-down density variations of order ϵ derived in Section III are calculated.

The strong ordering calculation presented in this work allows for arbitrary rotation as long as $\frac{\omega^{(0)2} R^2}{v_{thi}^2}\epsilon \ll 1$ and $\epsilon \ll 1$ are satisfied. We then find that for large rotation, the up-down density variation (and thus the radial transport) decreases with rotation (cf. Eq. (45b)) while for small rotation, it increases with rotation (as one would expect).

In addition, a brief discussion on the effects of co- and counter-injection on radial transport is given. One concludes that for both injections the impurity species diffuses inwardly, but with co-injection the inward diffusion flux is smaller than that in the counter-injection. (This tendency has been observed in the NBI experiments,[2] but a detailed comparison is beyond the objectives of this paper.)

Finally, remarks can be made on the two restrictions imposed in the present work:

(i) uniform temperature profile; and

(ii) Pfirsch-Schlüter regime for main ions and impurities.

First, since the radial transport obtained here is induced by the parallel friction, when temperature variations are included, a "thermal friction" term can enter which is expected to enhance the up-down variations and thus the radial transport. (A calculation extending the present work to include the temperature variations is in progress.) Secondly, although the present work is developed in the collisional regime, by analogy from the neoclassical transport theory in the small rotation case,[18] one can expect the present calculation to be qualitatively adequate for the main ion plateau regime except for the Pfirsch-Schlüter definition of $\xi_i < 1$ in Eq. (44) due to the parallel viscosity. In the plateau regime one can take $\xi_i \simeq O(1)$ to roughly estimate the radial transport. The results for $\xi_i = 1.5$ are shown in Figs. 4a, 4b, 6a and 6b.

Appendix A

In this Appendix we present the derivation of the first order moments $\Pi_{\|}^{(1)}$ and $R_{\|iI}^{(1)}$ in terms of measurable quantities such as n, T, and $\mathbf{V}$, using the moment approach developed elsewhere. Particularly, the effects of strong rotation and i-I collision have been included in the parallel viscosity $\Pi_{\|}$. As a consequence, corrections to Braginskii's parallel viscosity owing to the inclusion of zeroth order rotation are found. In addition, a general form of parallel friction for arbitrary $\alpha \equiv \frac{n_I Z^2}{n_i} = \frac{\nu_{iI}}{\sqrt{2}\nu_{ii}}$ is obtained and agrees with the result in Table I. of Ref. 21. In the end, a brief discussion on $\Pi_{\|j}^{(0)}$ in a system with fast time evolution in the P-S regime is given for instructive purpose.

First, the equation of moment $\mathbf{\Pi}_j$

$$\frac{d_j}{dt}\mathbf{\Pi}_j + \mathbf{\Pi}_j(\nabla \cdot \mathbf{V}_j) + \nabla \cdot \mathbf{\Theta}_j + \mathbf{W}_2\left[n_j T_j \nabla \mathbf{V}_j + \mathbf{\Pi}_j \cdot \nabla \mathbf{V}_j + \frac{2}{5}\nabla \mathbf{q}_j\right]$$
$$= \Omega_j \mathbf{K}_2[\mathbf{\Pi}] + \mathbf{\Pi}_{cj} + \mathbf{\Pi}_{sj}, \qquad (A1)$$

is obtained by taking $\int d\mathbf{v}\ m(\mathbf{vv} - \frac{v^2}{3}\mathbf{I})$ over the kinetic equation. Here, $\frac{d_j}{dt} \equiv \frac{\partial}{\partial t} + \mathbf{V}_j \cdot \nabla$,

$$\mathbf{\Theta} \equiv \int d\mathbf{v}\ m\left(\mathbf{vvv} - \frac{v^2}{5}(\mathbf{vI} + \uparrow_3)\right) f(\mathbf{v}),$$

$\uparrow_k$ denotes the cyclic permutation components (without repeating) that make the rank-k tensor $(\mathbf{A} + \uparrow_k)$ cyclically symmetric. Note that the rank-3 tensor Θ corresponds to the order-3 Legendre polynomial of pitch angle and is important only with the inclusion of particle trapping effects which is outside the scope of the present work. Moreover, for any rank-2 tensor $\mathbf{A}$, define

$$\mathbf{W}_2[\mathbf{A}] \equiv (\mathbf{A} + \uparrow_2) - \frac{2}{3}(\mathbf{A} : \mathbf{I})\mathbf{I}. \qquad (A2)$$

and

$$\mathbf{K}_2[\mathbf{A}] \equiv \mathbf{A} \times \mathbf{b} + \uparrow_2 . \qquad (A3)$$

For two ion species, the main ion collisional moment $\mathbf{\Pi}_{ci}$ defined in Eq. (48e) can be expressed as

$$\mathbf{\Pi}_{ci} = -\nu_{ii} \int d\mathbf{v}\ H(x) m \left(\mathbf{vv} - \frac{v^2}{3}\mathbf{I}\right) f_i, \qquad (A4)$$

where

$$H(x) \equiv \frac{9\sqrt{2\pi}}{4} \left\{ \frac{\alpha + 2\phi(x) + 3\frac{d}{dx}\left(\frac{\phi(x)}{x}\right)}{x^3} \right\},$$

$$\phi(x) \equiv \frac{2}{\sqrt{\pi}} \int_0^x dy\ e^{-y^2}$$

is the error function, and $x \equiv \frac{v}{v_{thi}}$. Here, the unlike collision operator[21]

$$C_{iI} \simeq \frac{3\sqrt{\pi}}{8} v_{thi}^3 \nu_{iI} \left\{ \frac{\partial}{\partial \mathbf{v}} \cdot \mathbf{U}(\mathbf{v}) \cdot \frac{\partial}{\partial \mathbf{v}} f_i + \frac{4\mathbf{v} \cdot (\mathbf{V}_I - \mathbf{V}_i)}{v^3 v_{thi}^2} f_{Mi} \right\}$$

(deduced by assuming both species are nearly Maxwellian, $\frac{m_I}{m_i} \sim Z \gg 1$, $T_I \sim T_i$, and $|\mathbf{V}_I - \mathbf{V}_i| \ll v_{thi}$), and the linearized like collision operator C_{ii} are used; where

$$\mathbf{U}(\mathbf{v}) \equiv \frac{v^2 \mathbf{I} - \mathbf{v}\mathbf{v}}{v^3}.$$

The fact that the integrand in Eq. (A4) is $\propto \left(\mathbf{v}\mathbf{v} - \frac{v^2}{3}\mathbf{I}\right)$ correspond to the rotational symmetry property of the Fokker Planck collision operator[22].

Letting

$$H(x) = \sum_{\ell=0}^{\infty} c_{0\ell} \frac{(\frac{5}{2})!\ell!}{(\ell + \frac{5}{2})!} L_\ell^{\frac{5}{2}}(x^2)$$

and defining

$$\mathbf{\Pi}_i^\ell \equiv \frac{(\frac{5}{2})!\ell!}{(\ell + \frac{5}{2})!} \int d\mathbf{v}\ m_i \left(\mathbf{v}\mathbf{v} - \frac{v^2}{3}\mathbf{I}\right) L_\ell^{\frac{5}{2}}(x^2) f_i(\mathbf{v}) \qquad (A5)$$

Eq. (A4) becomes

$$\mathbf{\Pi}_{ci} = -\nu_{ii} \sum_{\ell=0}^{\infty} c_{0\ell} \mathbf{\Pi}_i^\ell. \qquad (A6)$$

Here "!"denotes the factorial and $L_\ell^{\frac{5}{2}}(x^2)$ the Sonine polynomials. Using the orthogonality of Sonine polynomials, $c_{0\ell}$ can be determined from

$$c_{0\ell} = \frac{2}{(\frac{5}{2})!} \int_0^\infty dx\ x^6 H(x) L_\ell^{\frac{5}{2}}(x^2) e^{-x^2}. \qquad (A7)$$

Moreover, from the fact that the distribution function f is reasonably smooth in speed v owing to the energy diffusion piece of the collision operator, only the first two terms in ℓ are worth keeping. Actually, as also pointed out in Ref. 5, very small correction but high complexity will arise from keeping higher order terms in ℓ. Hence, Eqs. (A6) and (A7) yield

$$\mathbf{\Pi}_{ci} \simeq -\frac{6}{5}\left[(\nu_{ii}+\nu_{iI})\mathbf{\Pi}_i + \frac{3}{4}(\nu_{ii}+2\nu_{iI})\mathbf{\Pi}_i^*\right], \tag{A8}$$

where $\mathbf{\Pi}_i^* \equiv \mathbf{\Pi}_i^{\ell=1}$ defined in Eq. (A5).

By assuming the transport ordering (whence both the time evolution term and the source term are negligible) and high collisionality ($\nu \gg \omega_t$), and neglecting the effects of particle trapping and temperature variations, to $O(1)$, Eq. (A1) reduces to Eq. (10), while to $O(\delta_p)$ its parallel component becomes

$$\mathbf{bb} : \left\{\mathbf{V}_j^{(0)} \cdot \nabla\mathbf{\Pi}_{\perp j}^{(1)} + \mathbf{\Pi}_{\perp j}^{(1)}(\nabla \cdot \mathbf{V}_j^{(0)}) + \mathbf{W}_2\left[n_j^{(0)}T_j\nabla\mathbf{V}_j^{(1)} + \mathbf{\Pi}_{\perp j}^{(1)} \cdot \nabla\mathbf{V}_j^{(0)}\right]\right\} = \Pi_{c\|j}^{(1)}. \tag{A9}$$

The neglect of all $\Pi_\|$ terms on the left hand side of Eq. (A4) is justified if the collision frequency is larger than the transit frequency as in the case in the collisional regime. However, it is important to emphasize that terms with $\mathbf{\Pi}_\perp^{(1)}$, deduced in Eq. (21a) from Eq. (10), must be kept to retain the effects of large rotation.

To evaluate the parallel viscosity from Eqs. (A8)-(A9) the equation for the moment $\mathbf{\Pi}_i^*$ is needed. To $O(\delta_{pi})$, this equation has the form

$$0 = \Pi_{c\|i}^{*(1)}, \tag{A10}$$

where

$$\mathbf{\Pi}_{ci}^* \equiv \mathbf{\Pi}_{ci}^{\ell=1},$$

and

$$\mathbf{\Pi}_{ci}^\ell \equiv \frac{(\frac{5}{2})!\ell!}{(\ell+\frac{5}{2})!}\int d\mathbf{v}\; m\left(\mathbf{vv} - \frac{v^2}{3}\mathbf{I}\right) L_\ell^{\frac{5}{2}}(x^2)C_i(f_i).$$

Using the same procedures yielding $\mathbf{\Pi}_{ci}$ except for an extra weighting function $L_{\ell=1}^{\frac{5}{2}}(x^2)$ in the integrand of Eq. (A4), $\mathbf{\Pi}_{ci}^*$ is obtained as

$$\mathbf{\Pi}_{ci}^* \simeq -\frac{9}{35}\left[(\nu_{ii}+2\nu_{iI})\mathbf{\Pi}_i + \frac{205}{36}(\nu_{ii}+\frac{204}{205}\nu_{iI})\mathbf{\Pi}_i^*\right]. \tag{A11}$$

Eqs. (A8)-(A11) thus yield

$$\Pi_{\parallel i}^{(1)} = -\frac{D_{2i}}{2\nu_{ii}}\mathbf{bb}:\left\{\mathbf{V}_i^{(0)}\cdot\nabla\mathbf{\Pi}_{\perp i}^{(1)} + \mathbf{\Pi}_{\perp i}^{(1)}(\nabla\cdot\mathbf{V}_i^{(0)}) + \mathbf{W}_2\left[n_i^{(0)}T_i\nabla\mathbf{V}_i^{(1)} + \mathbf{\Pi}_{\perp i}^{(1)}\cdot\nabla\mathbf{V}_i^{(0)}\right]\right\}, \tag{A12}$$

where

$$D_{2i} = \frac{1025 + 1020\sqrt{2}\alpha}{534 + 903\sqrt{2}\alpha + 576\alpha^2}. \tag{A13}$$

Note that the limiting values

$$\frac{D_{2i}}{2} = 0.96 \qquad \text{for } \alpha = 0$$

and

$$\frac{D_{2i}}{2\nu_{ii}} = \frac{0.73}{\nu_{iI}} \qquad \text{for } \alpha = 1$$

agree with Braginskii's ion viscosity in a pure plasma and electron viscosity (with the analogy of the small mass ratio between e-i and i-I), respectively. Moreover, the terms involving the perpendicular viscosity $\mathbf{\Pi}_{\perp i}^{(1)}$ on the right hand side of Eq. (A12), owing to the effects of large rotation, give extra corrections to Braginskii's viscosity. As a consequence, as shown in Eq. (26), the first order parallel viscosity is driven only by the poloidal component, but not the toroidal component, of the flow velocity.

Similarly, the impurity parallel viscosity is obtained

$$\Pi_{\parallel I}^{(1)} = -\frac{D_{2I}}{2\nu_{II}}\mathbf{bb}:\left\{\mathbf{V}_I^{(0)}\cdot\nabla\mathbf{\Pi}_{\perp I}^{(1)} + \mathbf{\Pi}_{\perp I}^{(1)}(\nabla\cdot\mathbf{V}_I^{(0)}) + \mathbf{W}_2\left[n_I^{(0)}T_I\nabla\mathbf{V}_I^{(1)} + \mathbf{\Pi}_{\perp I}^{(1)}\cdot\nabla\mathbf{V}_I^{(0)}\right]\right\}. \tag{A14}$$

Here, $D_{2I} = \frac{1025}{534} \simeq 1.92$ because of the largeness of $\frac{m_I}{m_i}$.

Using a procedure similar to the one for deriving Eqs. (A7)-(A8) (except that order-$\frac{3}{2}$ Sonine polynomials $L_\ell^{\frac{3}{2}}$ are used to generate the coefficients) the parallel friction can be expressed as

$$\mathbf{R}_i \simeq m_i n_i \nu_{iI}\left((\mathbf{V}_I - \mathbf{V}_i) + \frac{3}{5n_iT_i}(\mathbf{q}_i + \frac{5}{4}\mathbf{q}_i^*)\right), \tag{A15}$$

where $\mathbf{q}_i \equiv \mathbf{q}_i^{\ell=1}$, $\mathbf{q}_i^* \equiv \mathbf{q}_i^{\ell=2}$, and

$$\mathbf{q}_i^\ell \equiv -T_i\frac{(\frac{5}{2})!\ell!}{(\ell+\frac{3}{2})!}\int d\mathbf{v}\ \mathbf{v}L_\ell^{\frac{3}{2}}(x^2)f_i(\mathbf{v}). \tag{A16}$$

Thus two more moment equations for $\mathbf{q}_i$ and $\mathbf{q}_i^*$ to $O(\delta_{pi})$ are needed to determine the parallel friction, i.e.,

$$0 = q_{c\|j}^{(1)}, \qquad \text{and} \qquad 0 = q_{c\|j}^{*(1)}. \tag{A17}$$

Here, the collisional moments are defined, similar to Eqs. (16), as $\mathbf{q}_{ci} \equiv \mathbf{q}_{ci}^{\ell}$, $\mathbf{q}_{ci}^{*} \equiv \mathbf{q}_{ci}^{\ell=2}$, and

$$\mathbf{q}_{ci}^{\ell=1} \equiv -T_i \frac{(\frac{5}{2})!\ell!}{(\ell+\frac{3}{2})!} \int d\mathbf{v}\ \mathbf{v} L_\ell^{\frac{3}{2}}(x^2) C_i(f_i). \tag{A18}$$

Following the same derivation leading to Eq. (A15) Eq. (18) yields

$$\mathbf{q}_{ci} \simeq \frac{3}{2}\nu_{iI} n_i T_i (\mathbf{V}_i - \mathbf{V}_I) - \frac{4}{5}(\nu_{ii} + \frac{13}{8}\nu_{iI})\mathbf{q}_i - \frac{3}{5}(\nu_{ii} + \frac{23}{8}\nu_{iI})\mathbf{q}_i^*, \tag{A19}$$

$$\mathbf{q}_{ci}^* \simeq \frac{15}{14}\nu_{iI} n_i T_i (\mathbf{V}_i - \mathbf{V}_I) - \frac{12}{35}(\nu_{ii} + \frac{23}{8}\nu_{iI})\mathbf{q}_i - \frac{9}{7}(\nu_{ii} + \frac{433}{360}\nu_{iI})\mathbf{q}_i^*, \tag{A20}$$

It is then straightforward to obtain

$$R_{\|i}^{(1)} = m_i n_i^{(0)} \nu_{iI} D_{i1}(V_{\|I}^{(1)} - V_{\|i}^{(0)}), \tag{A21}$$

where,

$$D_{1i} = \frac{576 + 488\sqrt{2}\alpha + 128\alpha^2}{576 + 1208\sqrt{2}\alpha + 434\alpha^2}, \tag{A22}$$

Note that Eq. (A22) is general for arbitrary α and agrees with the results given in Table I of Ref. 21.

It is also of interest to study the relaxation of poloidal flow due to the parallel viscosity in the P-S regime. Assuming that there exists a zeroth order poloidal flow, the $O(1)$ parallel projection of Eq. (A1) becomes

$$\frac{\partial}{\partial t}\Pi_{\|j}^{(0)} \ - \ \Pi_{c\|j}^{(0)} = -\mathbf{bb} : \mathbf{W}_2\left[n_j^{(0)} T_j \nabla \mathbf{V}_j^{(0)}\right]. \tag{A23}$$

Eqs. (A8), (A23), and (2) then imply that for a given $K^{(0)}$ the parallel viscosity will relax to be constant with respect to time in a short time of $O(\frac{1}{\nu})$. This relaxed zeroth order parallel viscosity is thus obtained as

$$\Pi_{\|j}^{(0)} = -\frac{D_{2j}}{2\nu_{jj}}\mathbf{bb} : \mathbf{W}_2\left[n_j^{(0)} T_j \nabla \mathbf{V}_j^{(0)}\right] \tag{A24}$$

by further imposing $\frac{\partial}{\partial t} \ll \nu$ and using Eqs. (A8), (A11), and (A23). Eqs. (A24) and (48b) indicate that the poloidal flow will damp due to the parallel viscosity at a rate $\sim \frac{\omega_t^2}{\nu}$ for tokamak plasma in the P-S regime. A recent calculation[23] shows that this relaxation rate for poloidal flow is still basically true in the banana-plateau regime.

Figure Captions

Fig. 1: Illustration of the flux coordinates system (ψ, θ, φ).

Figs. 2a-2f: $O(1)$ solutions for $\alpha = \frac{n_I Z^2}{n_i}$, n_i and n_I (with $\epsilon = 1/6$, $Z = 6$, $\hat{\alpha} = 1$, $\mu = 16/6$) vs the poloidal angle θ (cf. Eqs. (14)-(18)). In Figs. 2a-2c $\frac{\omega R}{v_{thi}} = 0.7$, and in Figs. 2d-2f, $\frac{\omega R}{v_{thi}} = 0.9$.

Figs. 3a-3d: $O(\delta_{pi})$ up-down asymmetric modulation of n_i and n_I vs θ (cf. Eqs. (33)-(36)), for $\frac{\omega R}{v_{thi}} = 0.7$, (Figs. 3a-3b) and $\frac{\omega R}{v_{thi}} = 0.9$ (Figs. 3c-3d). Also, the magnitudes of the normalized poloidal flows U_j are given (cf. Eqs. (29)). Other parameters are the same as in Figs. 2.

Figs. 4a-4d: Normalized density modulation and poloidal flow (cf. Eqs. (40)-(44)) for $\delta_{pi} = 0.05$, $Z = 8$, $A_d = 1, \hat{\alpha} = 1$ plotted vs $\hat{\omega}^2$ (with $\xi_i = 1.5$ in Fig. 4a and with $\xi_i = 0.2$ in Fig. 4d), vs Δ (with $\xi_i = 1.5$ in Fig. 4b), and vs ξ_i (in Fig. 4c).

Fig. 5: Normalized impurity density modulation and diffusion flux in test impurity case ($\hat{\alpha} = 0$) plotted as functions of $\hat{\omega}^2$.

Figs. 6a-6c: Normalized diffusion rates (cf. Eqs. (58)-(59)) plotted vs $\hat{\omega}^2, \Delta,$ and ξ_i. Other parameters are the same as in Figs. 4a-4c.

Figs. 7a-7b: Normalized diffusion rates plotted vs $\sqrt{Z}\frac{v_{\varphi i}}{v_{thi}}$ (cf. Eq. (60)) for co- and counter-injection. In Fig. 7a, $\hat{\alpha} = 1$, in Fig. 7b, $\hat{\alpha} = 0$.

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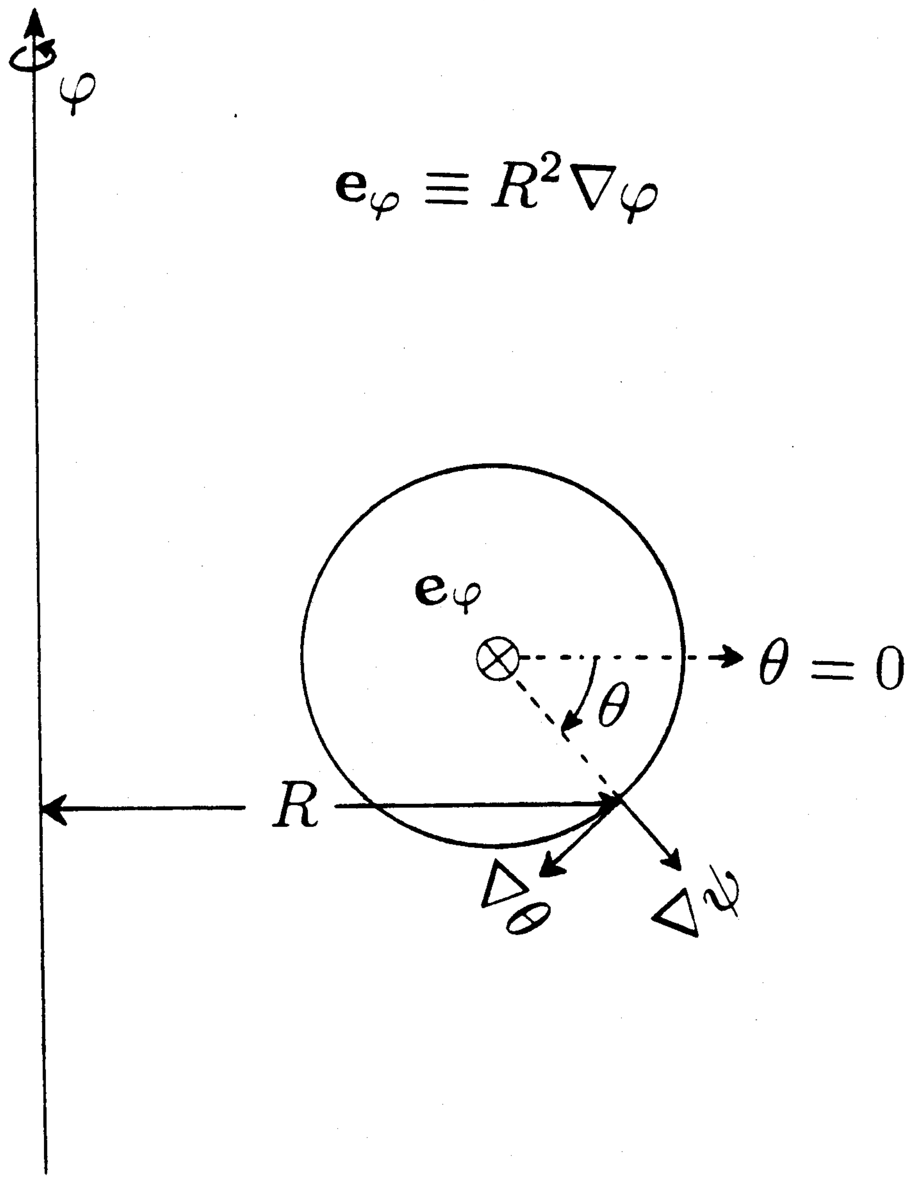

φ
$\mathbf{e}_\varphi \equiv R^2 \nabla \varphi$
$\mathbf{e}_\varphi$
$\theta = 0$
θ
R
$\nabla\theta$
$\nabla\psi$

Fig. 1

O(1) SOLUTION

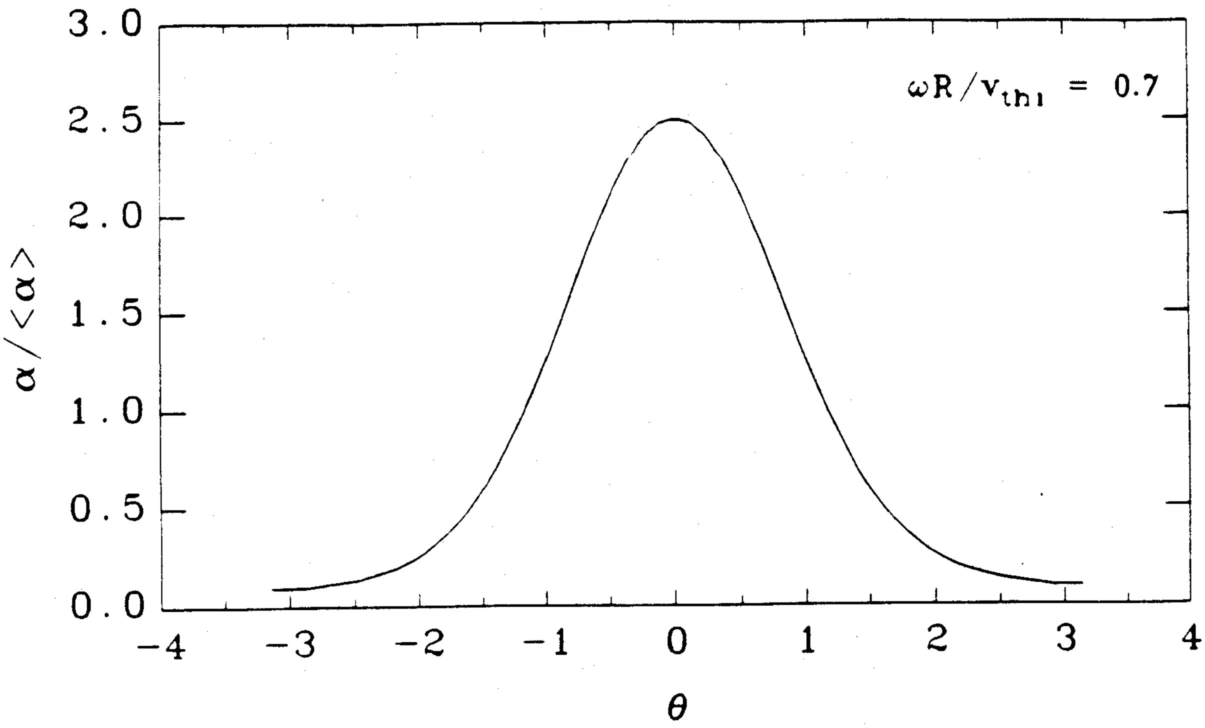


Fig. 2a

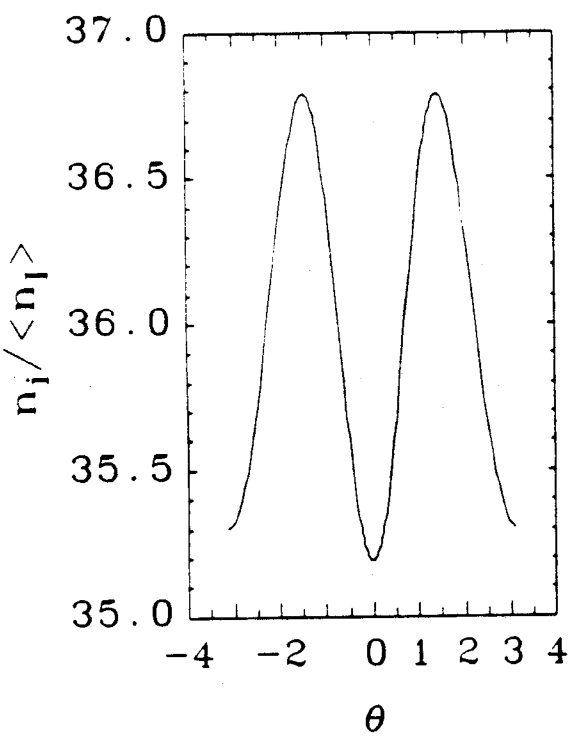


Fig. 2b

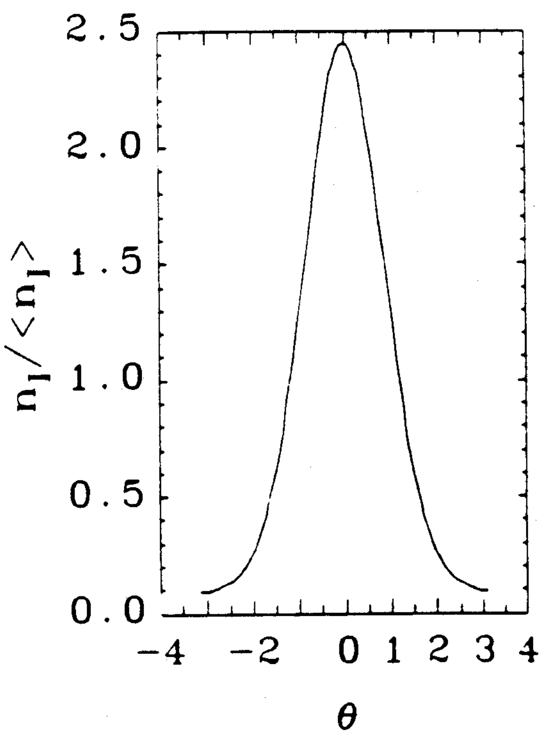


Fig. 2c

O(1) SOLUTION

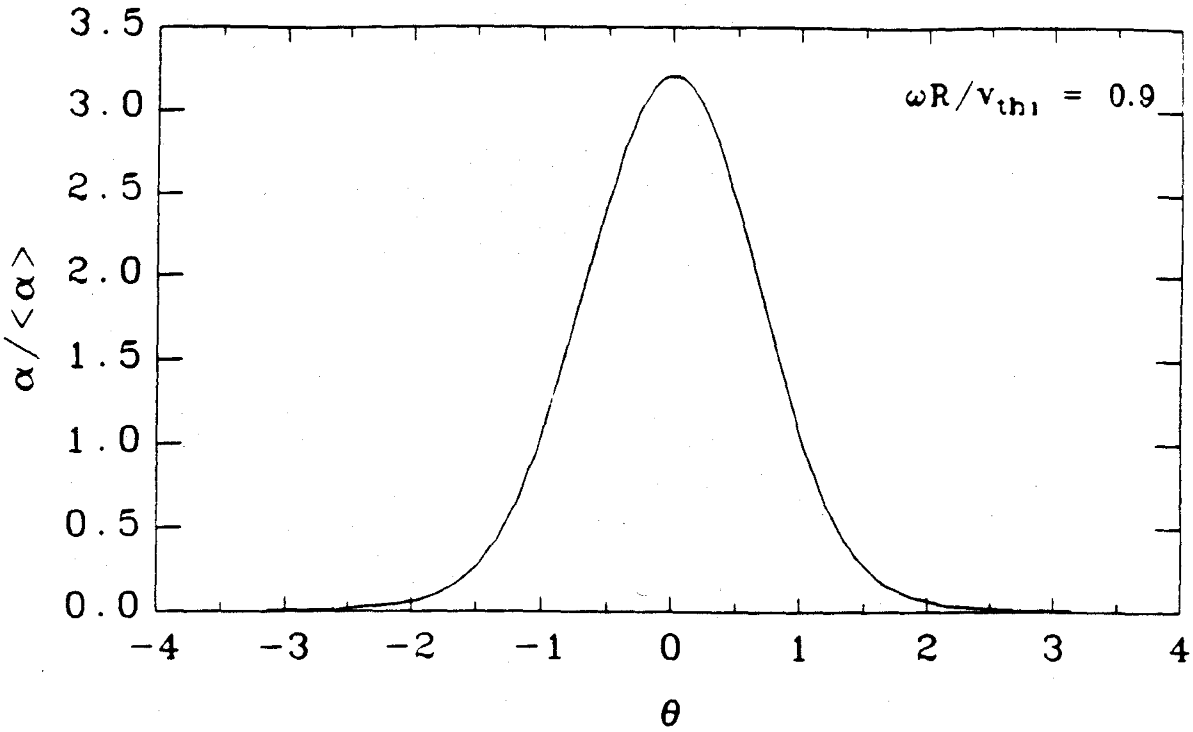


Fig. 2d

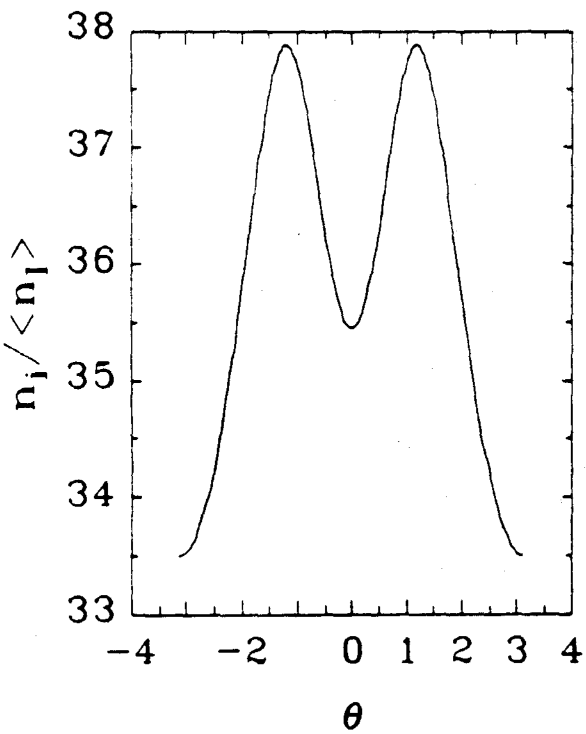


Fig. 2e

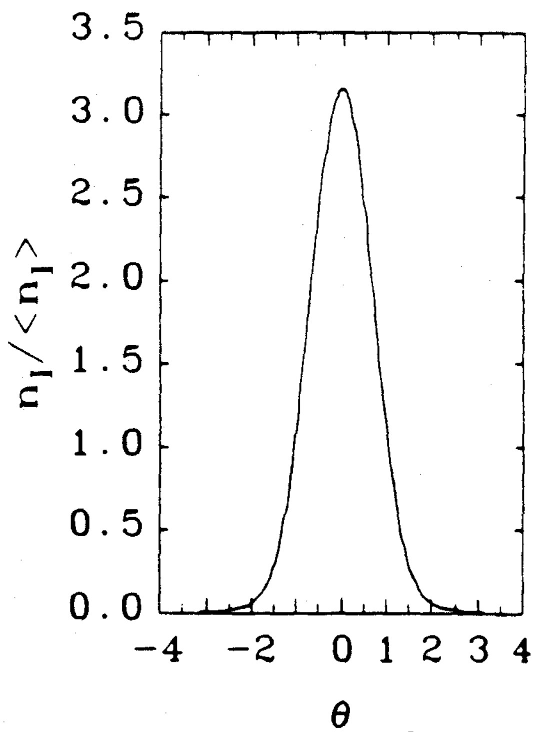


Fig. 2f

FIRST ORDER SOLUTION

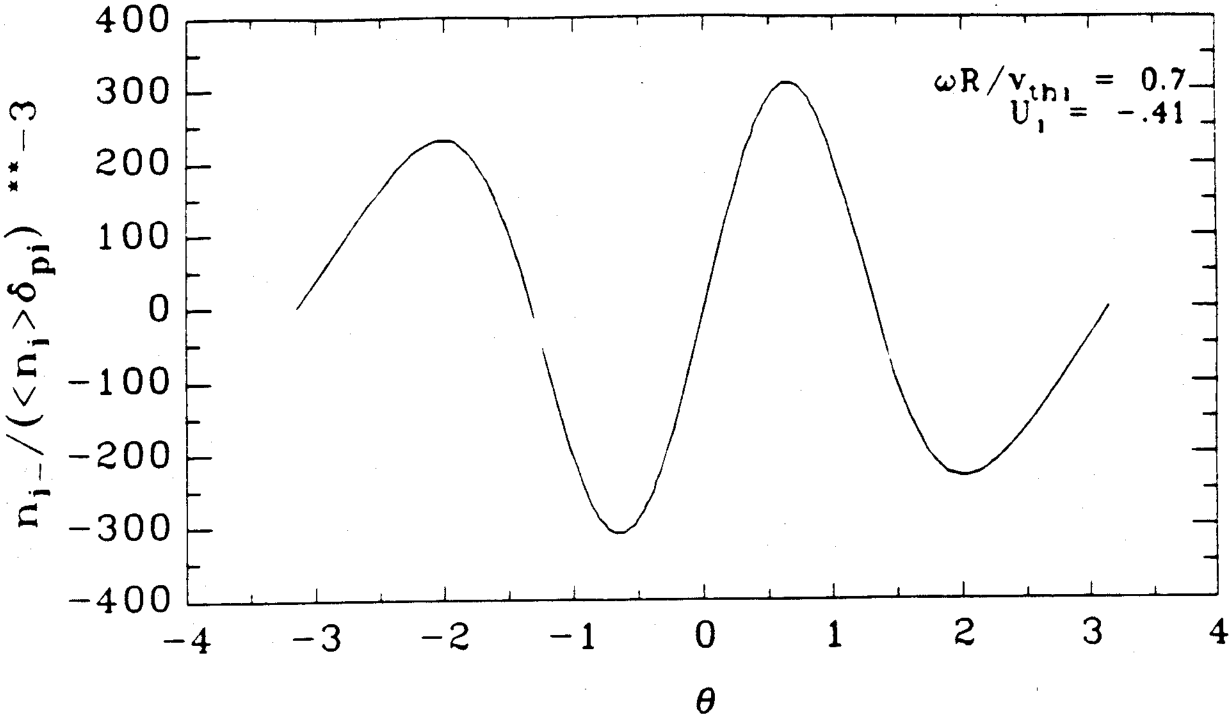


Fig. 3a

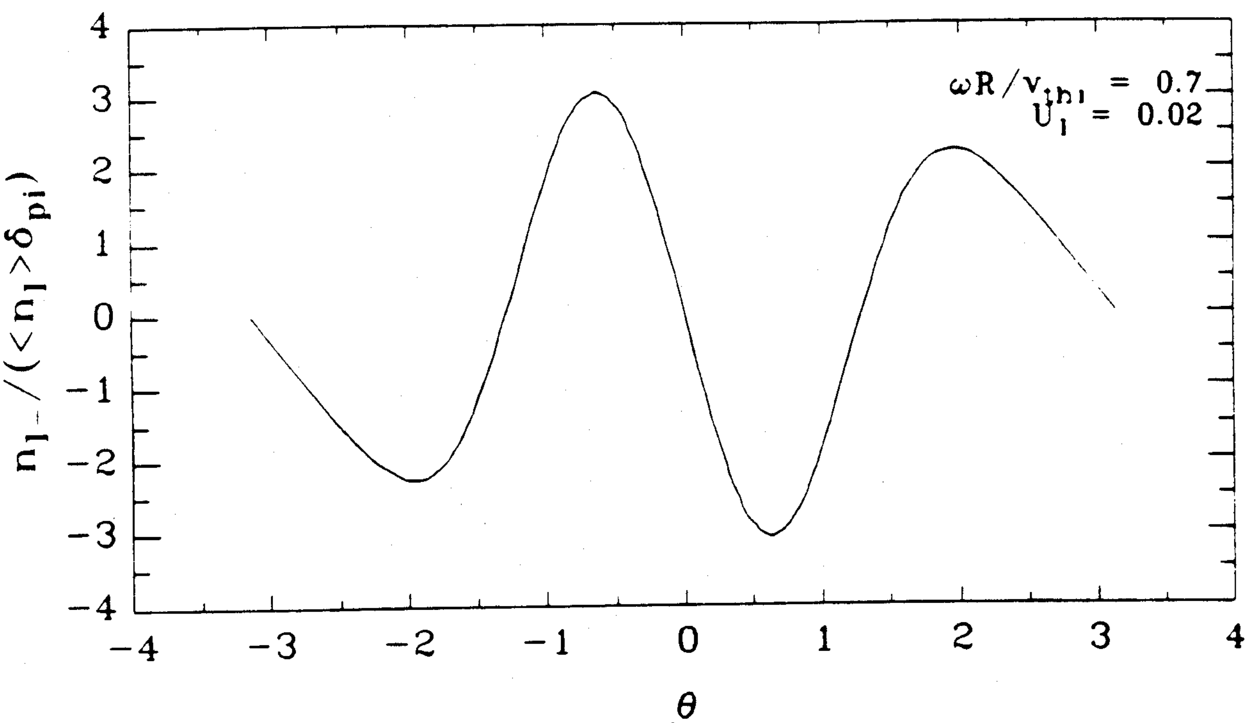


Fig. 3b

FIRST ORDER SOLUTION

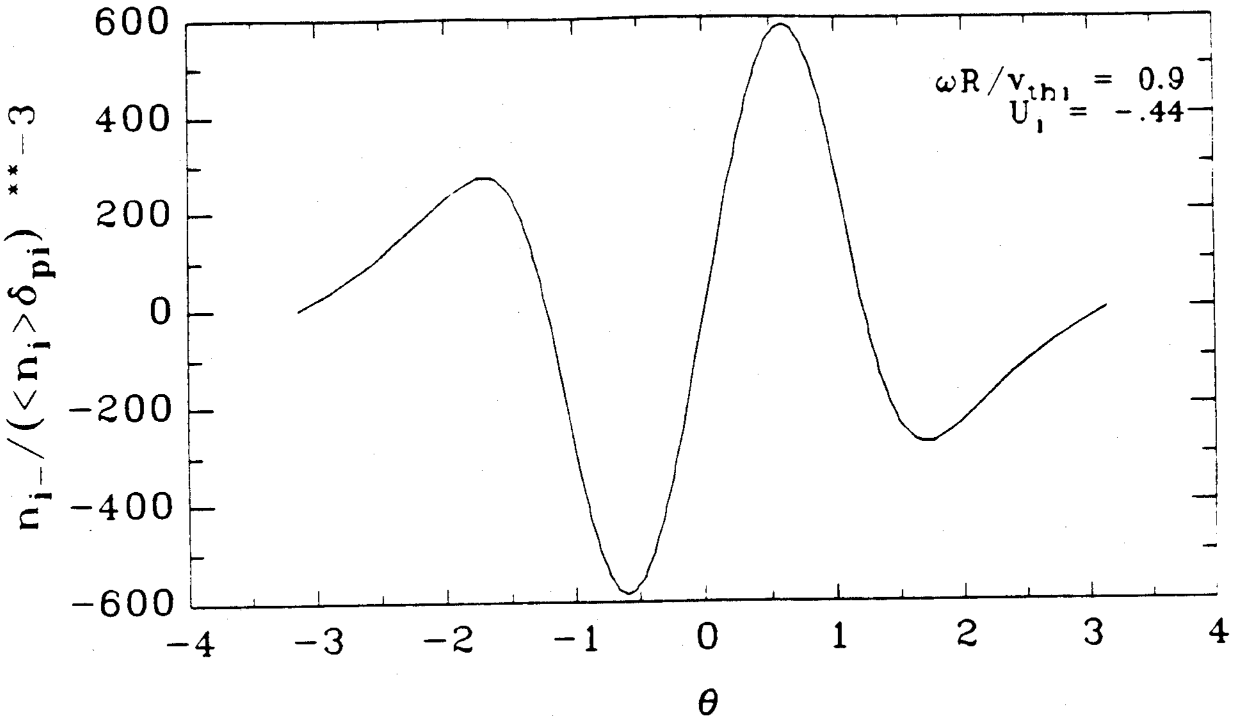


Fig. 3c

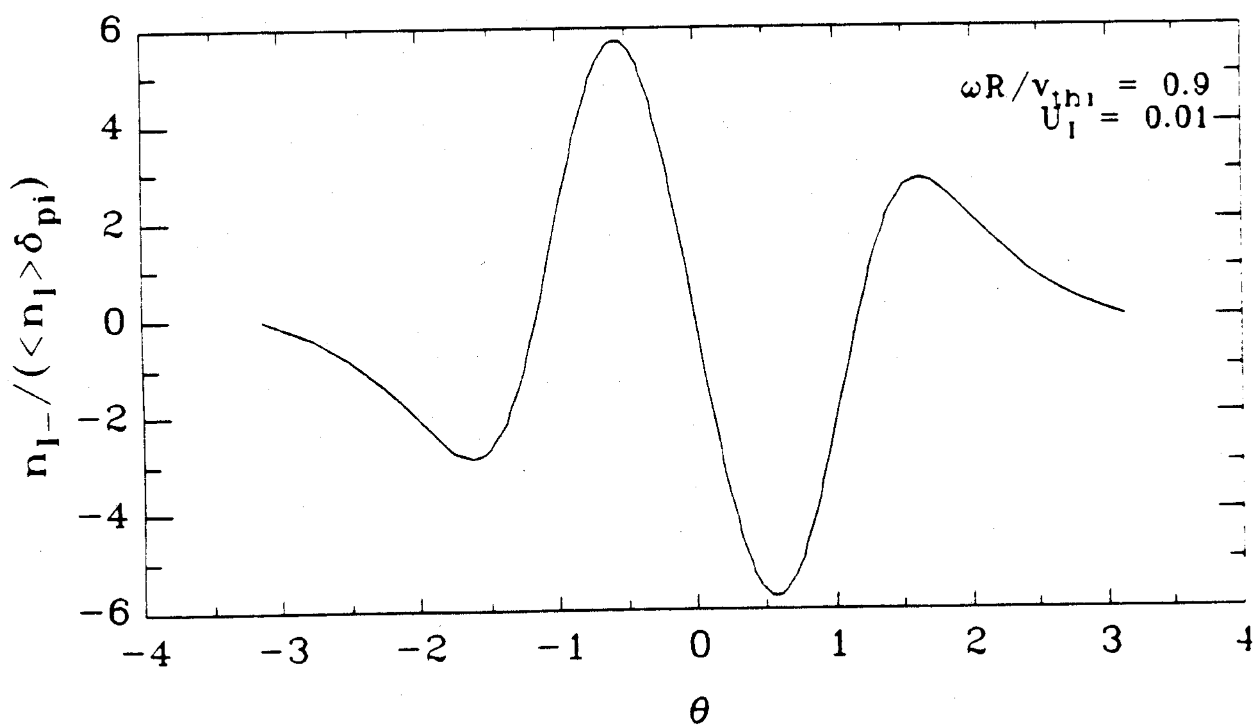
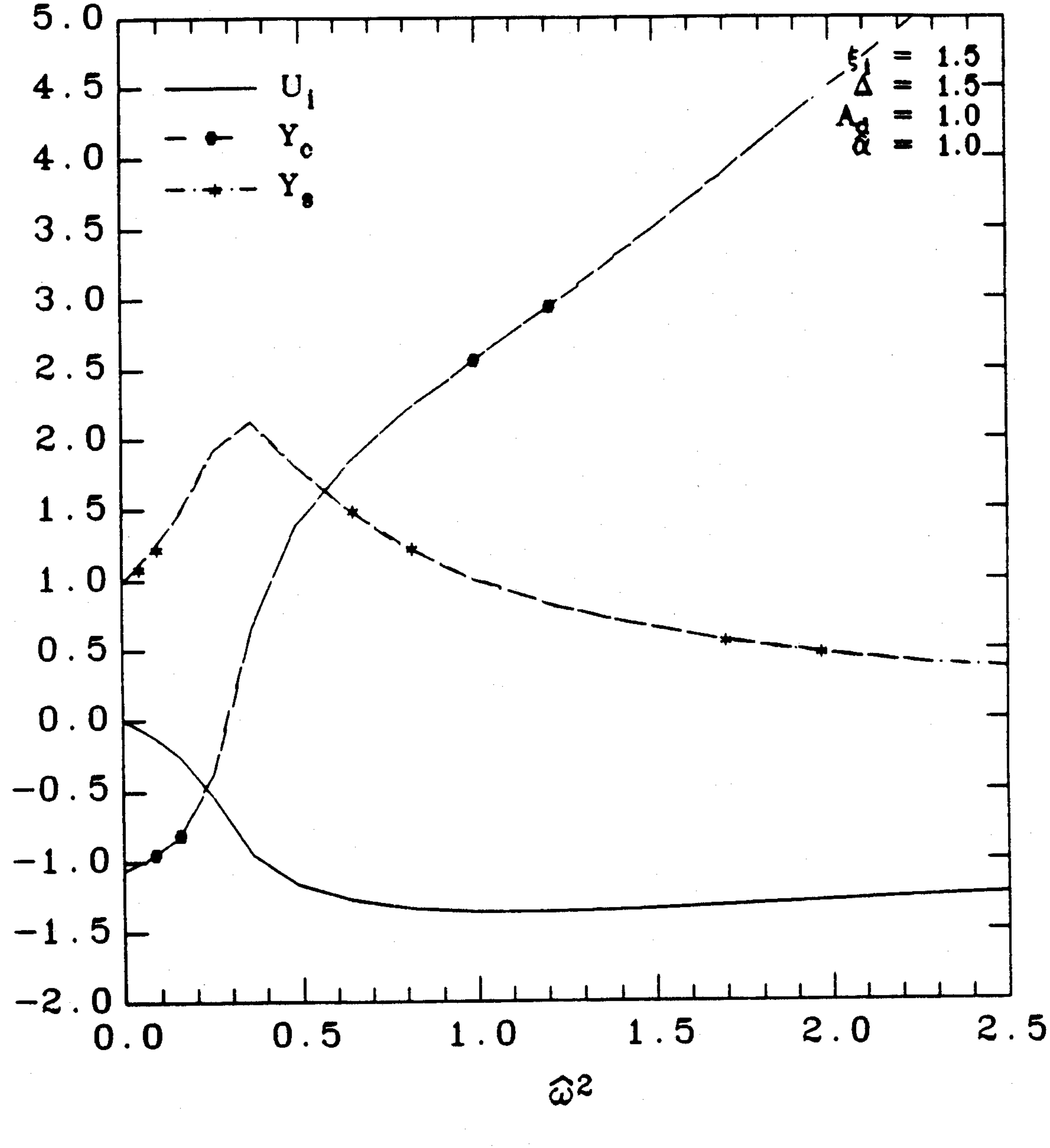
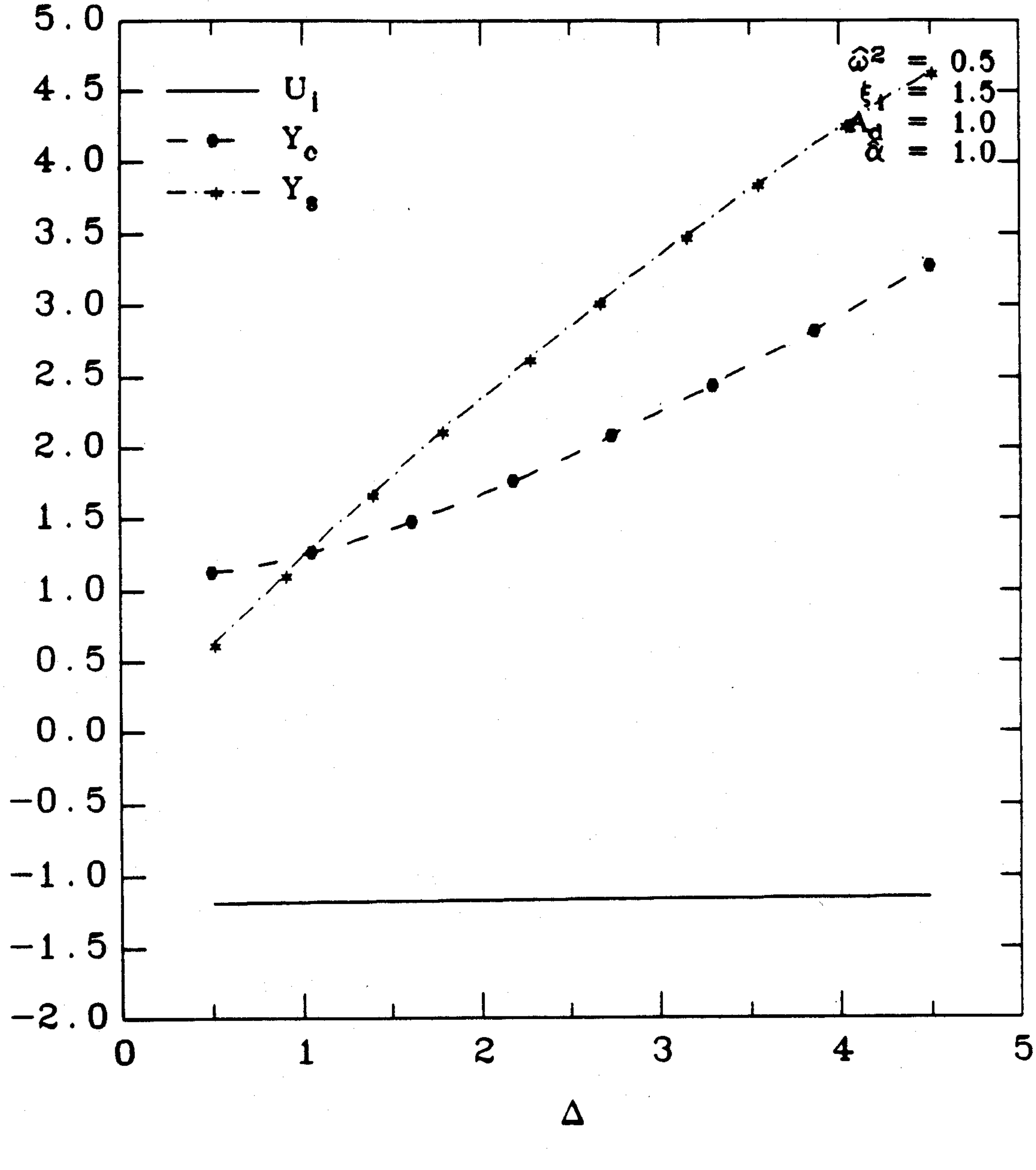


Fig. 3d



5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
0.0
0.5
1.0
1.5
2.0
2.5
U_l
Y_c
Y_s
ξ_l = 1.5
Δ = 1.5
A_d = 1.0
α = 1.0
$\hat{\omega}^2$

Fig. 4a



5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
0
1
2
3
4
5
Δ
U_l
Y_c
Y_s
$\hat{\omega}^2$ = 0.5
ξ_f = 1.5
A_d = 1.0
$\hat{\alpha}$ = 1.0

Fig. 4b

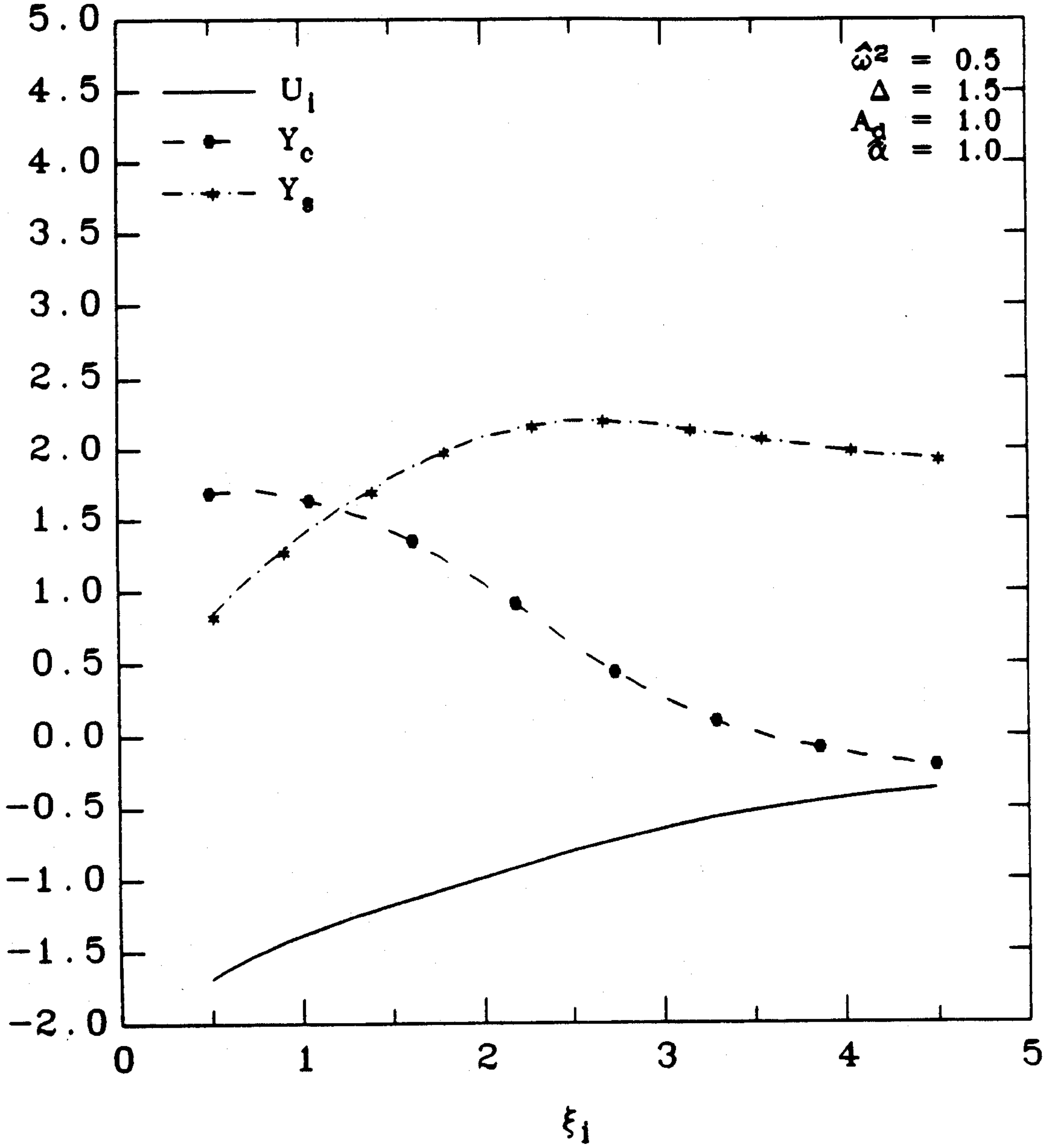
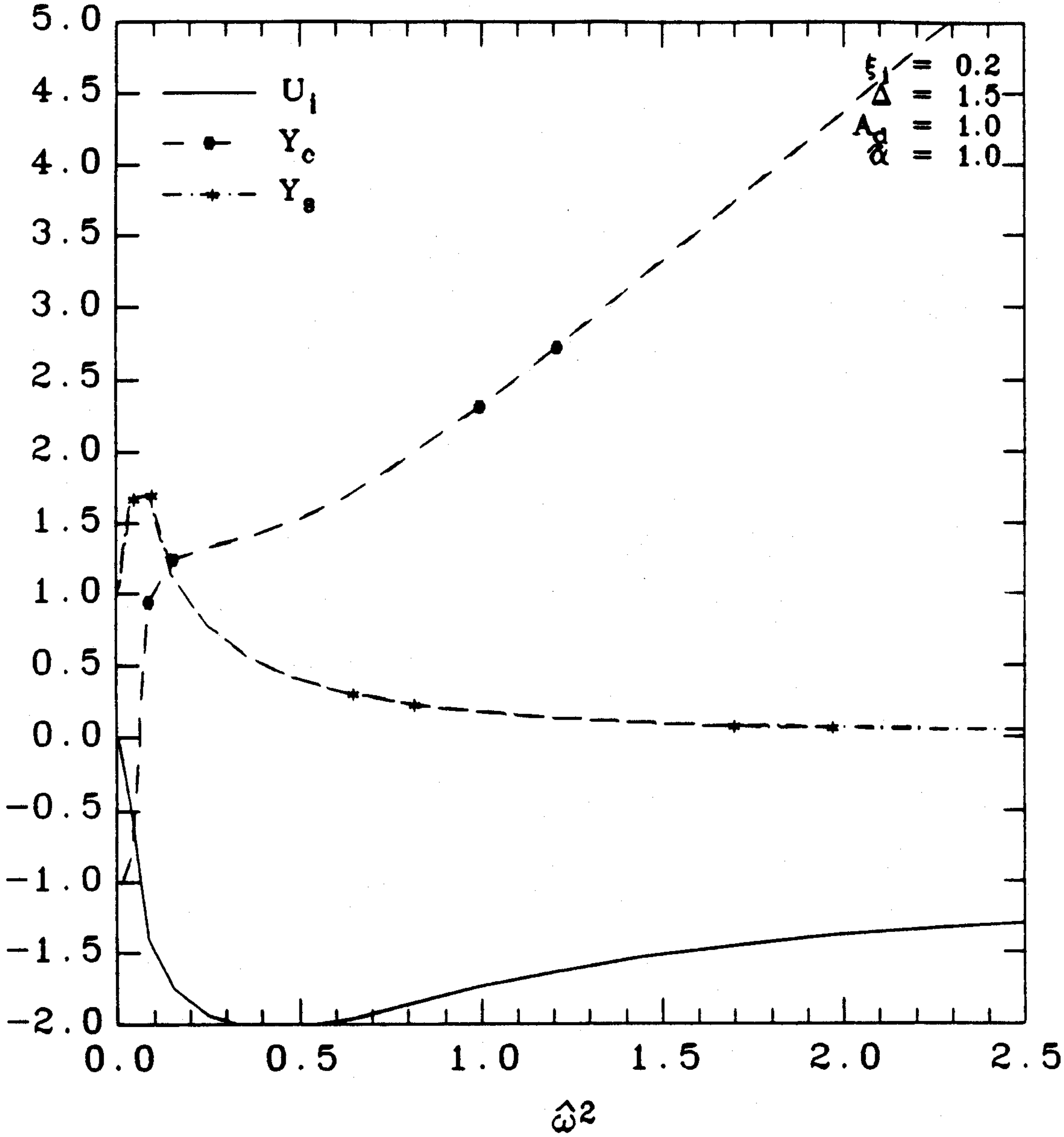
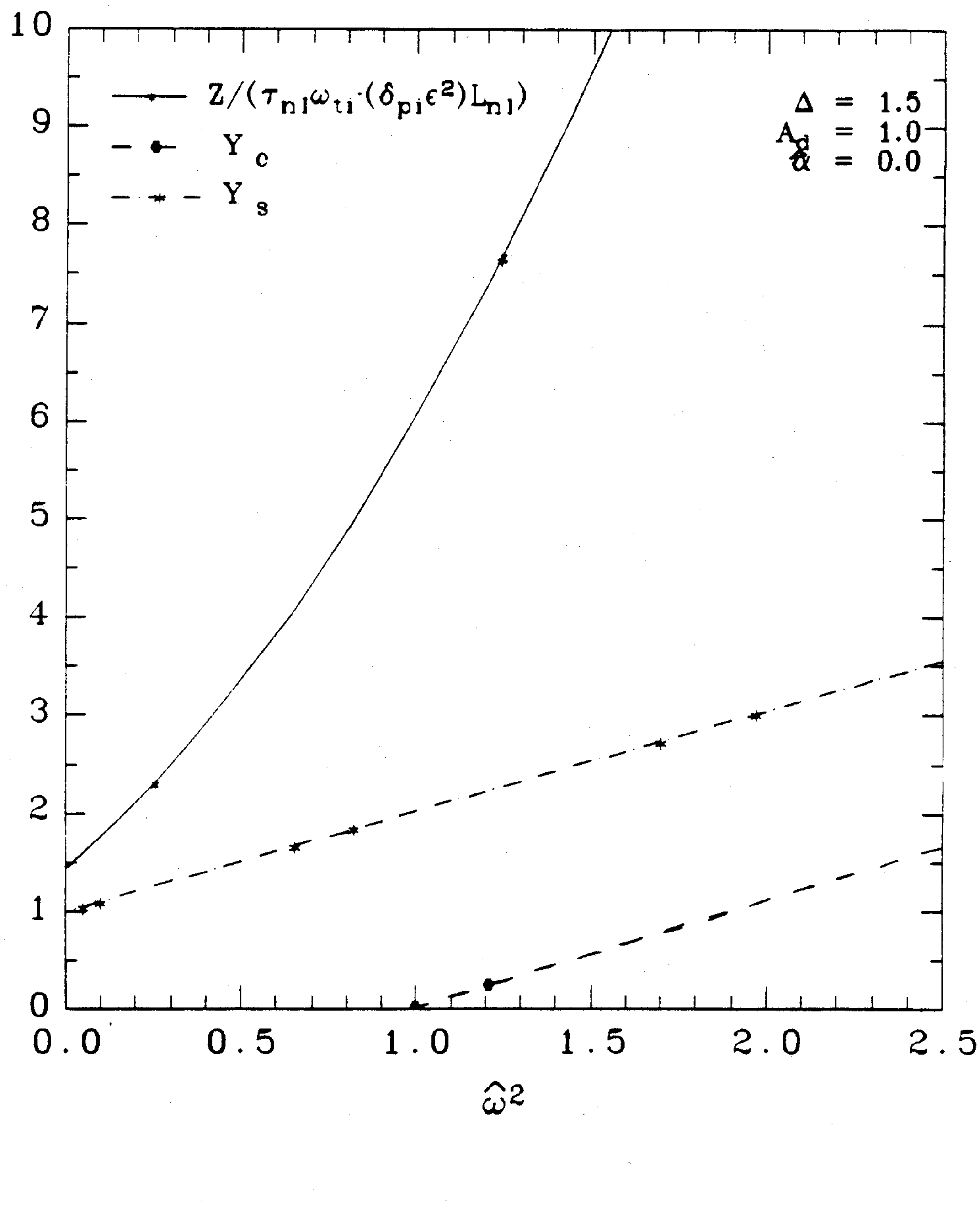

5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
0
1
2
3
4
5
U_i
Y_c
Y_s
$\hat{\omega}^2 = 0.5$
$\Delta = 1.5$
$A_d = 1.0$
$\tilde{\alpha} = 1.0$
ξ_i

Fig. 4c



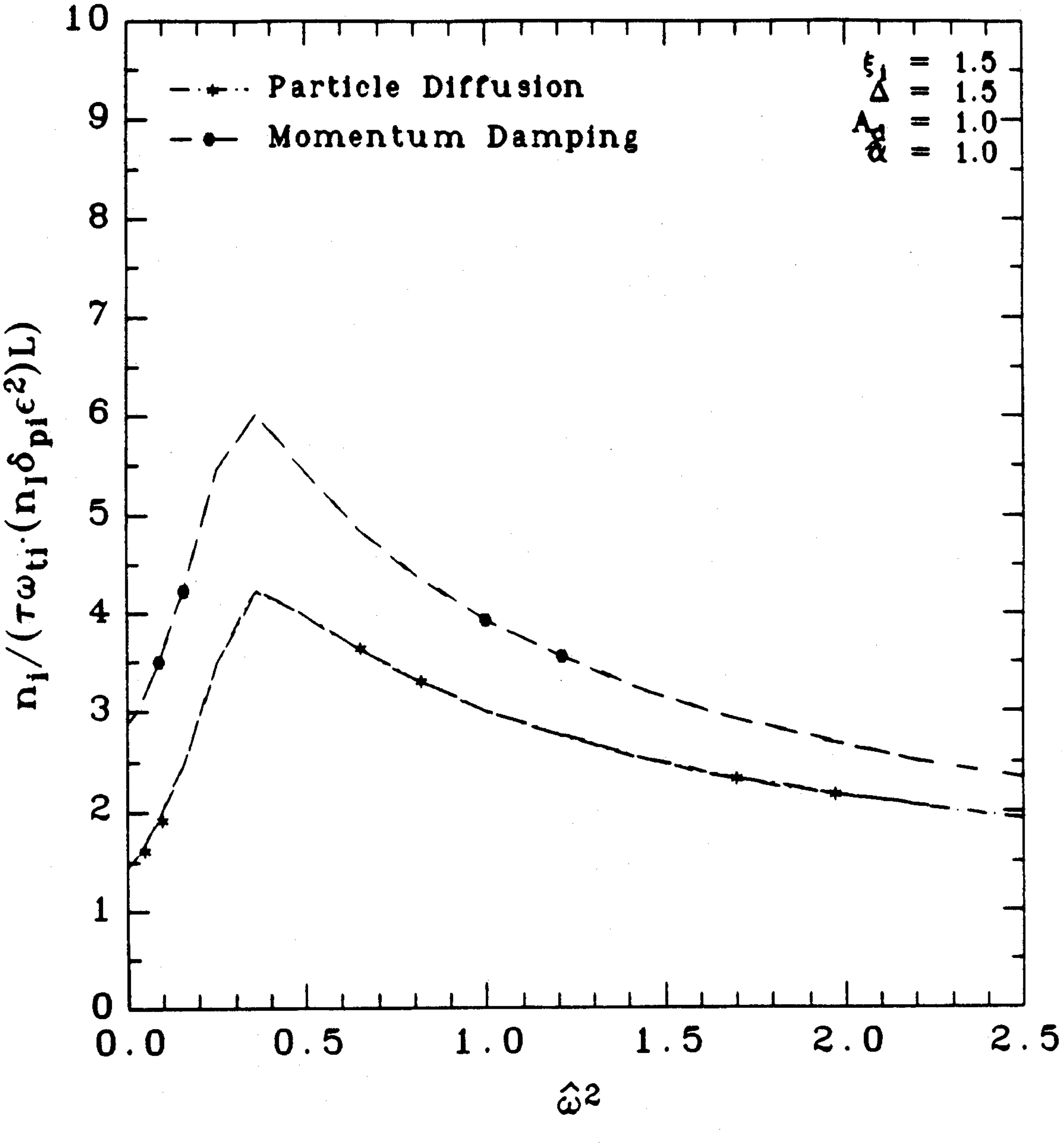
5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
0.0
0.5
1.0
1.5
2.0
2.5
U_i
Y_c
Y_s
ξ_i = 0.2
Δ = 1.5
A_d = 1.0
$\hat{\alpha}$ = 1.0
$\hat{\omega}^2$

Fig. 4d



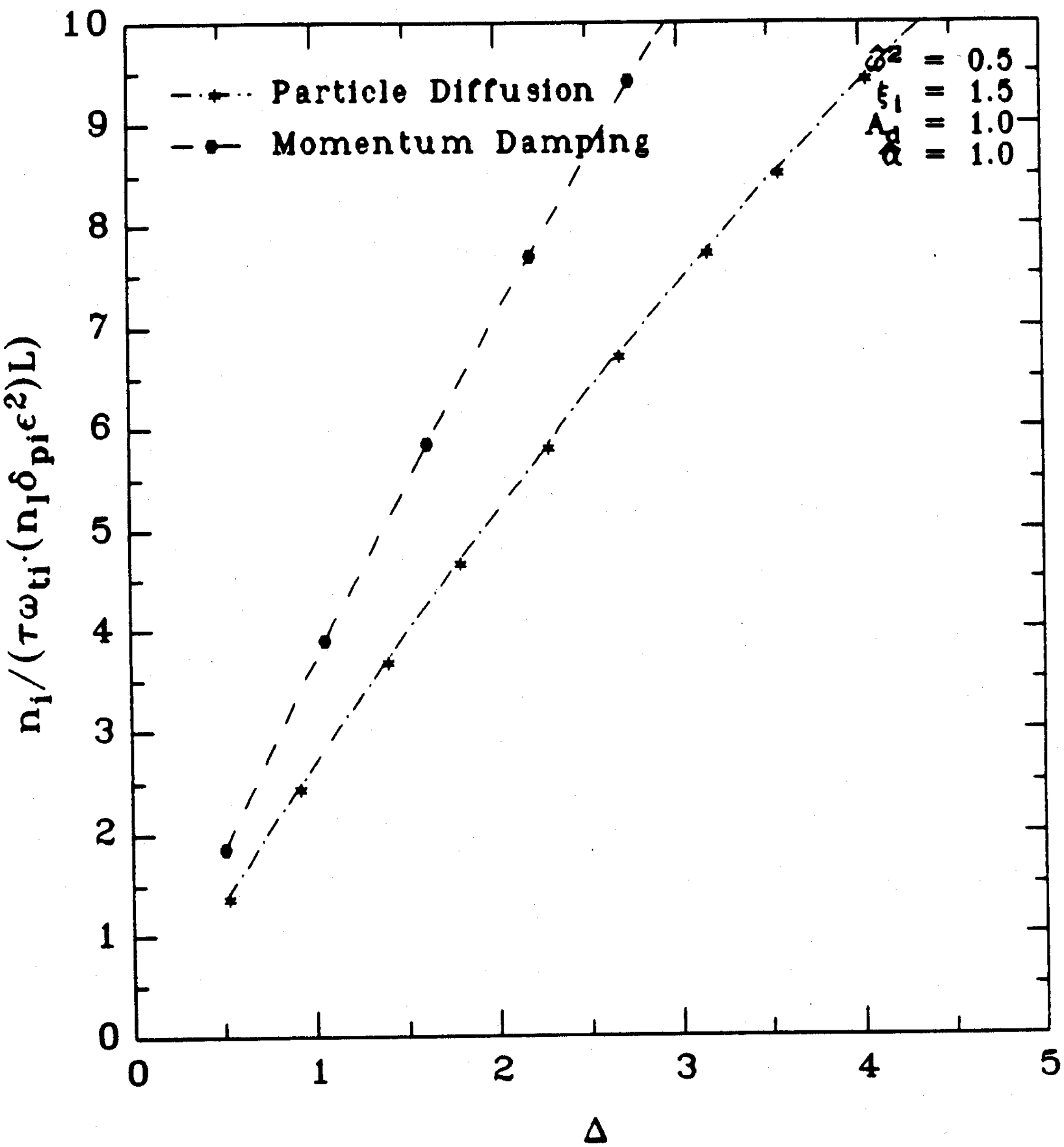
Z/(τ_nl ω_ti·(δ_pi ε²)L_nl)
Y c
Y s
Δ = 1.5
A_d = 1.0
α = 0.0
0.0
0.5
1.0
1.5
2.0
2.5
0
1
2
3
4
5
6
7
8
9
10
ω̂²

Fig. 5



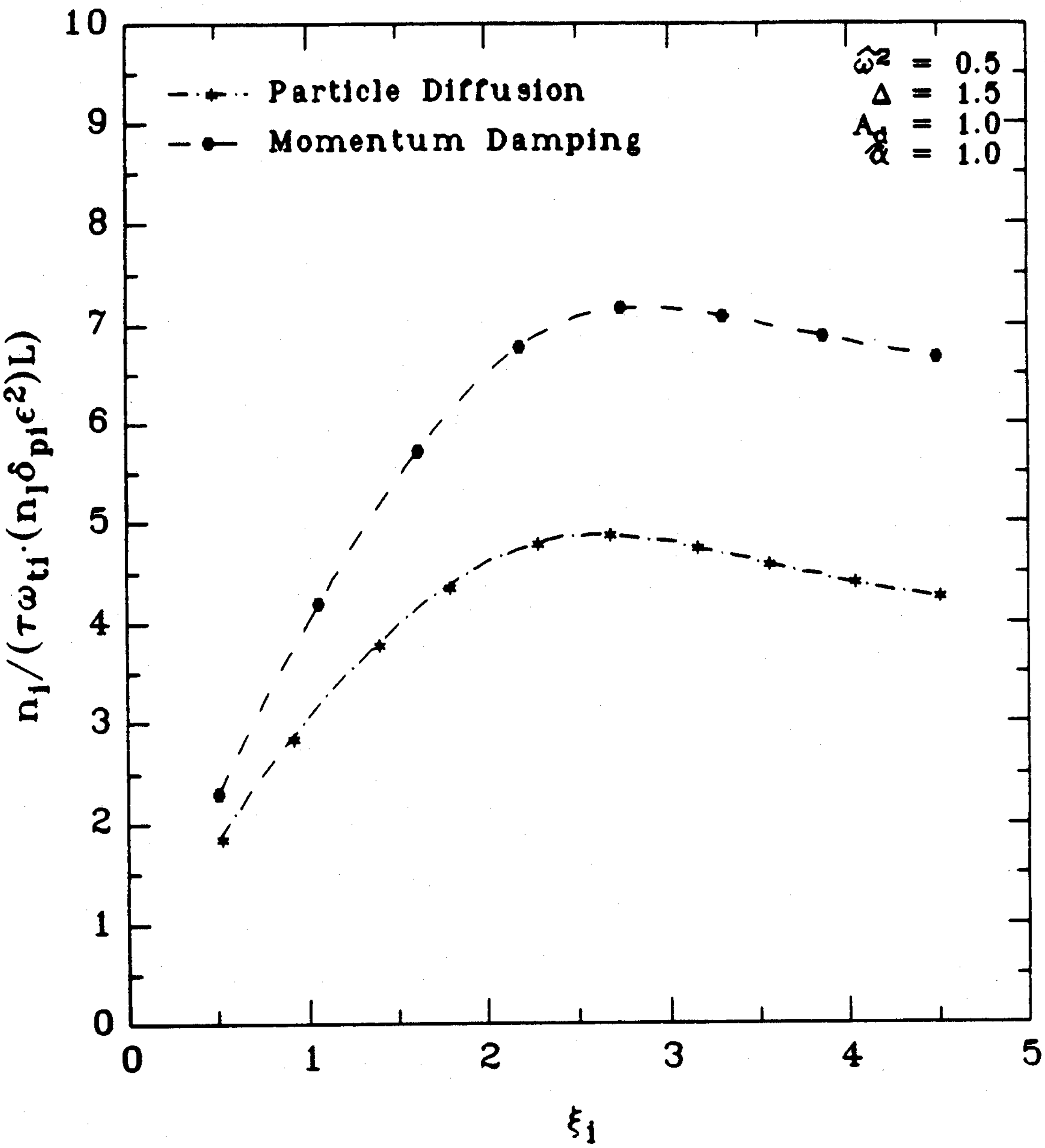
Particle Diffusion
Momentum Damping
ξ_I = 1.5
Δ = 1.5
A_d = 1.0
$\hat{\alpha}$ = 1.0
$n_I/(\tau\omega_{tI}\cdot(n_I\delta_{pI}\epsilon^2)L)$
$\hat{\omega}^2$
0.0
0.5
1.0
1.5
2.0
2.5
0
1
2
3
4
5
6
7
8
9
10

Fig. 6a



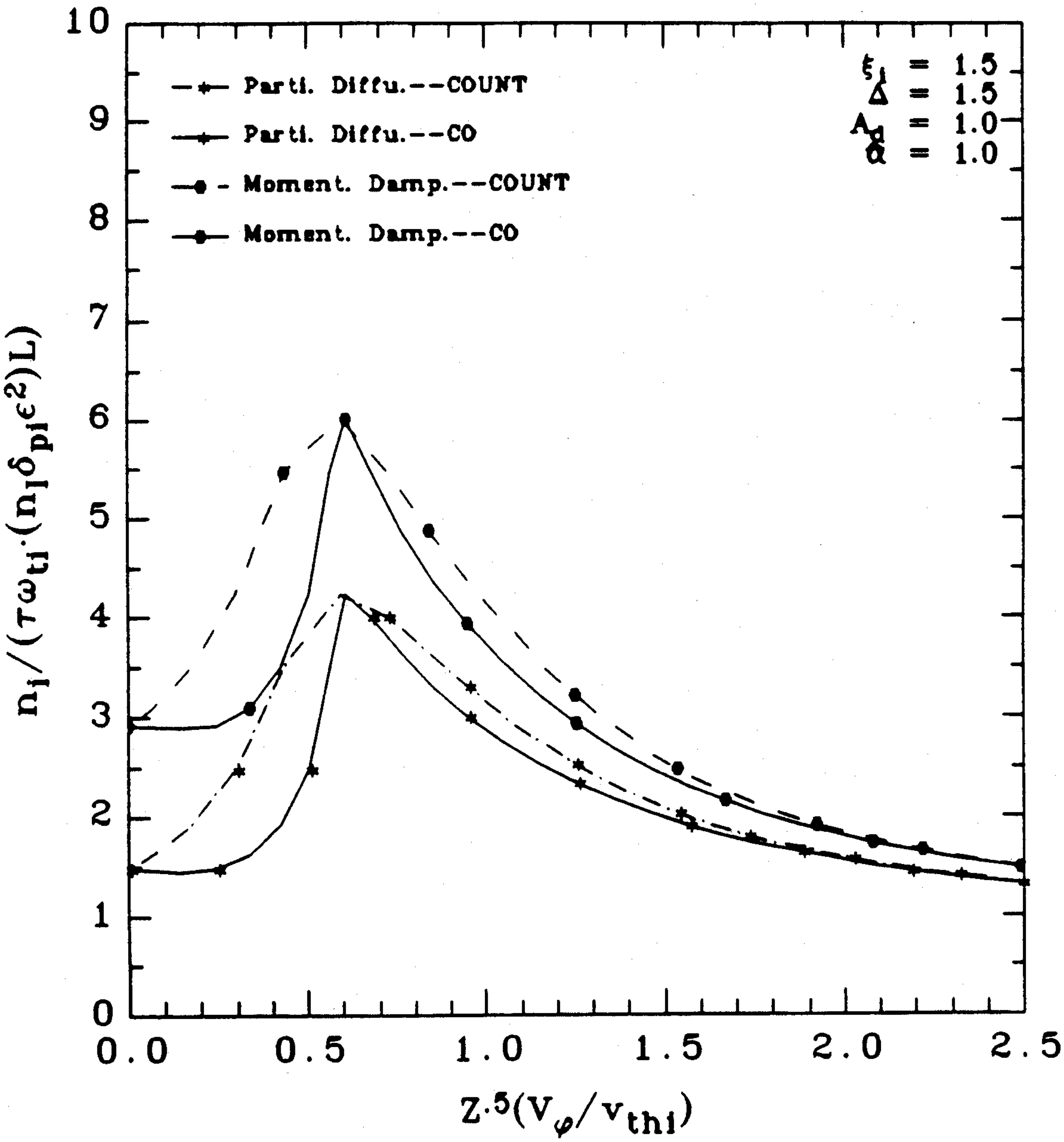
Particle Diffusion
Momentum Damping
$\hat{\omega}^2$ = 0.5
ξ_i = 1.5
A_d = 1.0
α = 1.0
$n_i/(\tau\omega_{ti}\cdot(n_I\delta_{pi}\epsilon^2)L)$
Δ
0
1
2
3
4
5
6
7
8
9
10

Fig. 6b



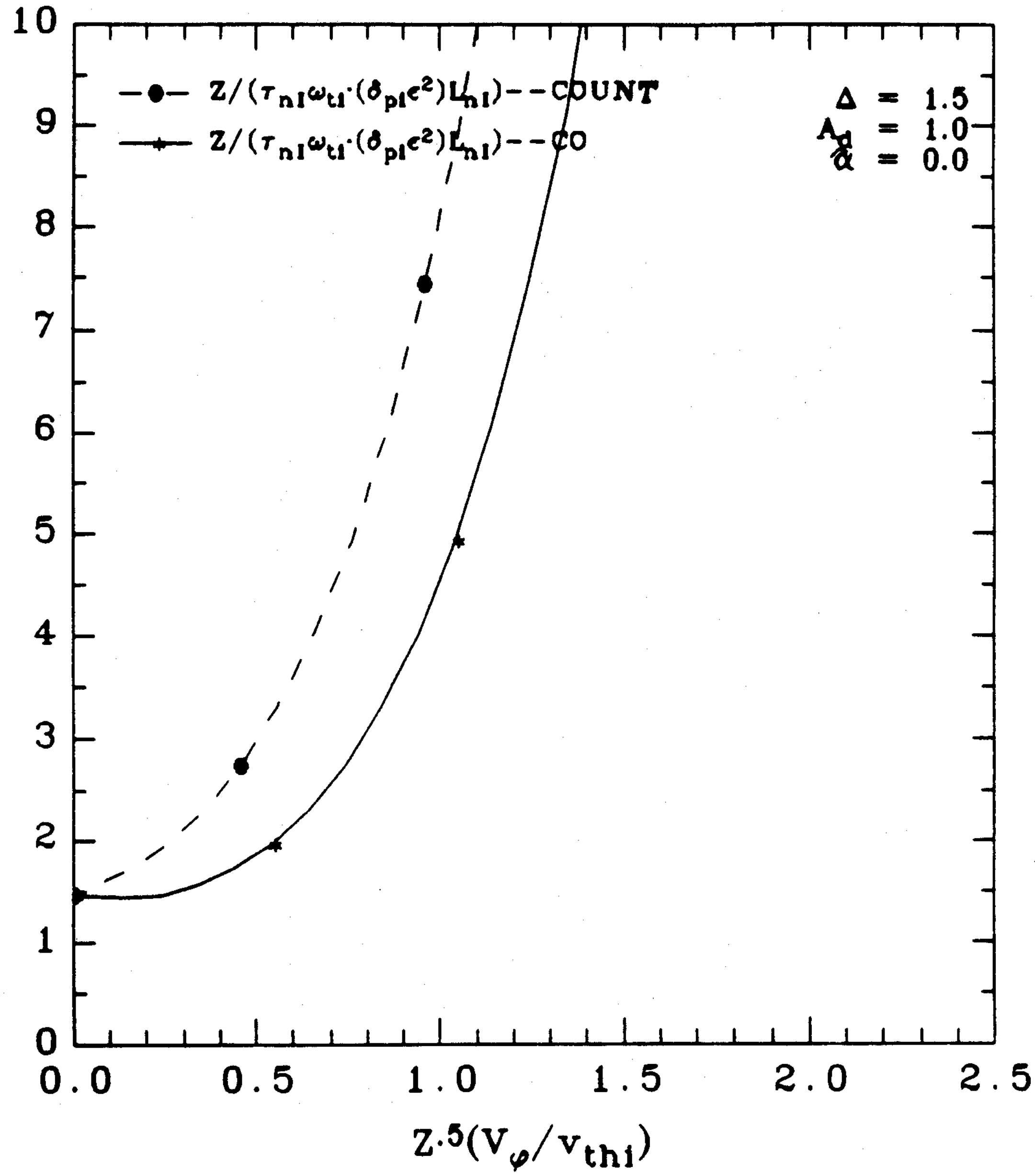
Particle Diffusion
Momentum Damping
$\hat{\omega}^2$ = 0.5
Δ = 1.5
A_d = 1.0
$\hat{\alpha}$ = 1.0
$n_i/(\tau\omega_{ti}\cdot(n_I\delta_{pI}\epsilon^2)L)$
ξ_i

Fig. 6c



Parti. Diffu.---COUNT
Parti. Diffu.---CO
Moment. Damp.---COUNT
Moment. Damp.---CO
ξ_i = 1.5
Δ = 1.5
A_d = 1.0
α = 1.0
$n_i/(\tau\omega_{ti}\cdot(n_I\delta_{pi}\epsilon^2)L)$
$Z^{.5}(V_\varphi/v_{thi})$

Fig. 7a



Z/(τ_nl ω_tl·(δ_pl ε²)L_nl)--COUNT
Z/(τ_nl ω_tl·(δ_pl ε²)L_nl)--CO
Δ = 1.5
A_d = 1.0
α = 0.0
10
9
8
7
6
5
4
3
2
1
0
0.0
0.5
1.0
1.5
2.0
2.5
Z·5(V_φ/v_thi)

Fig. 7b