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KINETIC RAY TRACING IN TOROIDAL GEOMETRY WITH APPLICATION TO MODE-CONVERTED ION-BERNSTEIN WAVES

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ABSTRACT

We discuss results of detailed numerical and analytical studies on the propagation of ion-Bernstein waves (IBW) in a toroidal plasma. Such waves can be excited by mode conversion of an externally launched fast Alfvén wave near the ion-ion hybrid resonance (or near the second harmonic resonance for a single ion species plasma) in a tokamak. We find that there is a significant upshift in the poloidal mode numbers of the IBW over short radial distances of propagation so that the IBW can Landau damp onto the electrons. The numerically obtained results are corroborated by a simple analytical model.

INTRODUCTION

It is well known that, for small $k_{\parallel}$ (the component of the wave vector along the total magnetic field), the fast Alfvén wave can couple to the IBW near the ion-ion hybrid (or second harmonic) resonance layer. We study the propagation of this IBW in a toroidal plasma using a numerical ray trajectory code that evolves the amplitude of the IBW along the ray. Results show that there is a substantial upshift in $k_{\parallel}$ as the IBW propagates essentially radially and that the amplitude of the IBW decreases significantly in the region where there is damping. The energy of the IBW is Landau damped onto the electrons. This could partially explain the electron heating that has been observed in the ICRF heating of various tokamak plasmas. We give details of the numerical and analytical models used for studying the propagation of the IBW and discuss some of the results obtained from these models.

NUMERICAL RAY TRAJECTORY ANALYSIS

A numerical code has been developed which solves for the propagation of rays in three-dimensional toroidal geometry [1]. The local dispersion function, $D(\vec{k}, \omega, \vec{r})$, used for following the rays is obtained from the fully electromagnetic *hermitian* dielectric tensor [2,3] describing a kinetic, hot Maxwellian plasma. Here $\vec{r} = (r, \theta, \phi)$ is the position vector where r is the radius measured from the magnetic axis of the torus, θ is the poloidal angle, and ϕ is the toroidal angle; $\vec{k} = (k_r, m, n)$ is the wave vector with components k_r, m and n in the radial, poloidal and toroidal directions, respectively; and ω is the frequency. The spatial profiles of the density, temperature, and the magnetic field components are included, in a WKB sense, explicitly in D. Besides the usual ray trajectory equations:

$$\frac{d\vec{k}}{dt} = \frac{(\partial D/\partial \vec{r})}{(\partial D/\partial \omega)}, \quad \frac{d\vec{r}}{dt} = - \frac{(\partial D/\partial \vec{k})}{(\partial D/\partial \omega)} \tag{1}$$

the numerical code also solves for the variation of the wave energy density, U, along the rays [2,3]:

$$\frac{\partial U}{\partial t} + \nabla \cdot \left(\frac{\partial \omega}{\partial \vec{k}}\, U\right) + \frac{1}{2}\bar{\bar{\sigma}}^{H} : \vec{a}\vec{a}^{*} = 0 \tag{2}$$

where $\bar{\bar{\sigma}}^{H}$ is the hermitian part of the conductivity tensor and $\vec{a}$ is the slowly varying (complex) part of the electric field of the wave. The relation between U and $\vec{a}$ is given by:

$$U = \frac{1}{16\pi}\frac{\partial(\omega \bar{\bar{D}})}{\partial \omega} : \vec{a}\vec{a}^{*}; \qquad \bar{\bar{D}} = \left(1 - \frac{c^2 k^2}{\omega^2}\right)\bar{\bar{I}} + \frac{c^2 \vec{k}\vec{k}}{\omega^2} + \frac{4\pi i \bar{\bar{\sigma}}^{A}}{\omega} \tag{3}$$

where $\bar{\bar{\sigma}}^{A}$ is the anti-hermitian part of the conductivity tensor, $k^2 = \vec{k} \cdot \vec{k}$, c is the speed of light, $\bar{\bar{I}}$ is the unit tensor, and $\det(\bar{\bar{D}}) = D(\vec{k}, \omega, \vec{r})$. The second term in Eq.(2) describes changes associated with the convergence and divergence of a bundle of rays and the third term describes changes in U due to damping of the wave energy on the particles.

The ray trajectory code can describe the propagation of waves of different frequencies in a variety of plasma equilibria. We have used it to study the propagation of the IBW in an axisymmetric plasma with no equilibrium radial magnetic field. For the results obtained in figs. (1-4) we have used JET-type parameters assuming a deuterium plasma with a 4% hydrogen minority (i.e. the ratio of the hydrogen to deuterium densities is 0.4), peak electron density of 2.8×10^{13} $cm.^{-3}$, peak electron temperature of 1.8 keV, peak ion temperature of 1.7 keV, toroidal current of 2 MA, minor radius of 125 $cm.$, major radius of $300 cm.$, a toroidal magnetic filed on axis of 2 $Tesla$, the $q = 1$ surface at $r = 50$ $cm.$, and $\omega = 1.93 \times 10^8$ sec^{-1}. The electron temperature profile is as given in [4], and the ion temperature and the electron and ion densities are assumed to vary like $[1 - (r/a)^2]^{1.5}$ (a being the minor radius) with their peak values being on the magnetic axis. Fig. 1 shows the poloidal projection of two IBW trajectories starting off at $r = 16.9$ $cm.$, $\theta = 2.58$ $radians$ (ray 1) and $r = 17.6$ $cm.$, $\theta = 2.51$ $radians$ (ray 2). These initial points are on the high field side and very close to the mode conversion region. Both rays at the starting point have $m = 0$. The rays start propagating essentially in the radial direction before turning into the poloidal direction. The intersection of the rays is an artifact of the poloidal projection as the rays are separated toroidally at this point. The distances of propagation shown are very small compared to the minor radius. Fig. 2 shows the change in m as a function of ωt (the normalized time of propagation of the rays). (The time period shown is exactly that over which the ray trajectories in fig. 1 are plotted.) There is a significant enhancement in $|m|$ for short distances of propagation. The evolution of the amplitude of the electric field, $|\vec{a}|$, (assumed to be unity at $t = 0$) is shown in fig. 3. The initial increment in $|\vec{a}|$ is due to the convergence of the nearby rays. Thereafter, the rays diverge toroidally leading to a decrease in $|\vec{a}|$. The sudden drop in $|\vec{a}|$ after $\omega t \approx 350$ is primarily due to the damping of the waves. This can be observed in fig. 4 where we have plotted the electron Landau resonance parameter ($y_{0e} = \omega/[\sqrt{2}|k_{\parallel}|v_{te}]$ where v_{te} is the electron thermal velocity) along the rays. Prior to $\omega t \approx 150$, y_{0e} is much larger than three. The sudden drop in $|\vec{a}|$ is closely associated with $y_{0e} \approx 1$. Thus, the IBW Landau damps onto the electrons after a short distance of propagation. The damping is due to a considerable enhancement of the poloidal mode numbers.

ANALYTICAL MODEL FOR THE UPSHIFT OF POLOIDAL MODE NUMBERS

In the vicinity of the mode-conversion region where $k_\perp \rho_i < 1$ (ρ_i is the ion Larmor radius, and $k_\perp$ is the component of $\vec{k}$ perpendicular to the total magnetic field) an approximate form for the local dispersion function, D^{app}, is obtained by expanding the full D to fourth order in $k_\perp \rho_i$. Since $k_\perp \rho_e \ll 1$, D^{app} contains only the cold electron contributions. For a deuterium-hydrogen plasma, we obtain:

$$D^{app} = \alpha_0 \, k_\perp^4 + \alpha_1 \, k_\perp^2 + \alpha_2 \tag{4a}$$

where:

$$\alpha_0 = -\frac{1}{y_0}\left(\frac{4}{3}\frac{\omega}{\omega_{cd}} + \frac{\omega^2}{\epsilon\omega_{pd}^2}\right) Z(y_2) + \frac{1}{\sqrt{2}y_0}\frac{\eta}{\epsilon}\frac{\omega^2}{\omega_{pd}^2} Z\left(\frac{y_2}{\sqrt{2}}\right) + \frac{4}{3y_0^2}\frac{\omega^2}{\omega_{cd}^2}\left(2 - \frac{\omega^2}{\epsilon\omega_{pd}^2}\frac{\omega_{cd}}{\omega}\right) \tag{4b}$$

$$\alpha_1 = \frac{1}{\epsilon y_0}\frac{\omega}{\omega_{cd}}\left[\frac{4}{3}Z(y_2) - \sqrt{2}\eta\left(2 + \frac{c^2}{\epsilon\omega_{pd}^2}\frac{\omega_{cd}}{\omega}\right) Z\left(\frac{y_2}{\sqrt{2}}\right) + \frac{16}{9y_0}\frac{\omega}{\omega_{cd}}\left(1 + \frac{3}{4\epsilon}\frac{c^2}{\omega_{pd}^2}\frac{\omega_{cd}}{\omega}\right)\right] \tag{4c}$$

$$\alpha_2 = -\frac{8}{3\epsilon^2 y_0^2}\frac{\omega^2}{\omega_{cd}^2}\left[1 - \frac{\eta}{\sqrt{2}}\frac{\omega_{cd}}{\omega} y_0 Z\left(\frac{y_2}{\sqrt{2}}\right)\right] \tag{4d}$$

and, $y_n = (\omega - n\omega_{cd})/(\sqrt{2}|k_\parallel| v_{td})$ for $n = 0, 2$; ω_{cd} (ω_{pd}) is the local deuterium cyclotron (plasma) frequency, Z is plasma dispersion function, $\epsilon = (v_{td}^2/\omega_{cd}^2)$ with v_{td} as the deuterium thermal velocity, and η is the ratio of hydrogen to deuterium ion densities. Here we have assumed that the two ion species are at the same bulk temperature. Eqn.(4a) can be written in a coupled mode form and the propagating IBW is then given by $(k_\perp)_{IB}^2 = -(\alpha_1/\alpha_0)$ away from the mode-conversion region. We use this form of $(k_\perp)_{IB}$ with the added assumption that $k_\parallel$ is small (so that the asymptotic form of Z-functions can be used). Since the numerical results show that the IB ray propagates for short radial distances we ignore the variation in temperatures and density along the ray and include only the poloidal and radial variations of the magnetic field. The rate of change of m is related to the poloidal variations of D^{app} so that the toroidicity included in the magnetic fields is important. The resulting equation relating the change in m to a change in r is given by:

$$\Delta m \approx \frac{2}{3}\frac{\omega_{cd}^2}{k_r v_{td}^2}\left(2 + 11\,\frac{v_{td}^2}{c^2}\,\frac{\omega_{pd}^2}{\omega_{cd}^2}\right)\frac{r\sin\theta}{(R + r\cos\theta)}\;\Delta r \tag{5}$$

where we assume that $k_r = (k_\perp)_{IB}$. For the parameters shown in figs. (1-4) we find that $\Delta m \approx -0.82\Delta r$ for ray 1, $\Delta m \approx -0.92\Delta r$ for ray 2. With $\Delta r \approx 20cm$. this gives $\Delta m \approx -16.4$ and -18.4 for the two rays, respectively. This is in good agreement with the numerically obtained results in fig. 2.

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REFERENCES

1. A. K. Ram and A. Bers in *Applications of Radio-Frequency Power to Plasmas*, Eds. S. Bernabei and R. W. Motley, AIP Conference Proceedings 159 (American Institute of Physics, New York, 1987).

2. A. Bers in *Plasma Physics - Les Houches 1972*, Eds. C. DeWitt and J. Peyraud (Gordon & Breach Science, 1975).
3. I. B. Bernstein and L. Friedland in *Handbook of Plasma Physics*, vol. 1, Eds. M. N. Rosenbluth and R. Z. Sagdeev (North Holland, 1983).
4. W. M. Tang, Nuclear Fusion 26, 1619 (1986).

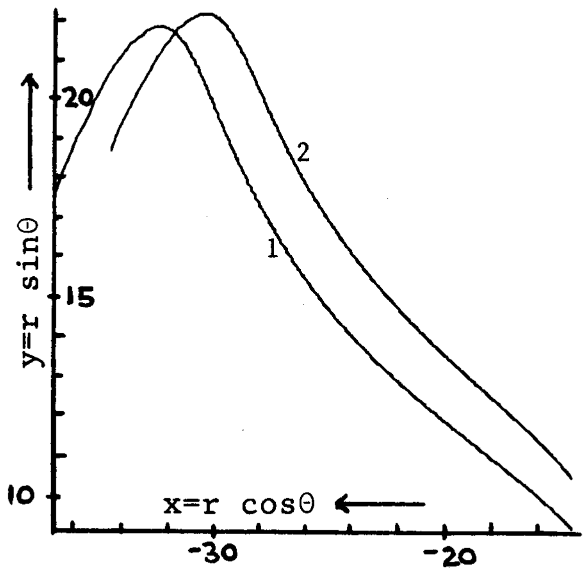

Figure 1: Ray trajectories in the poloidal plane.

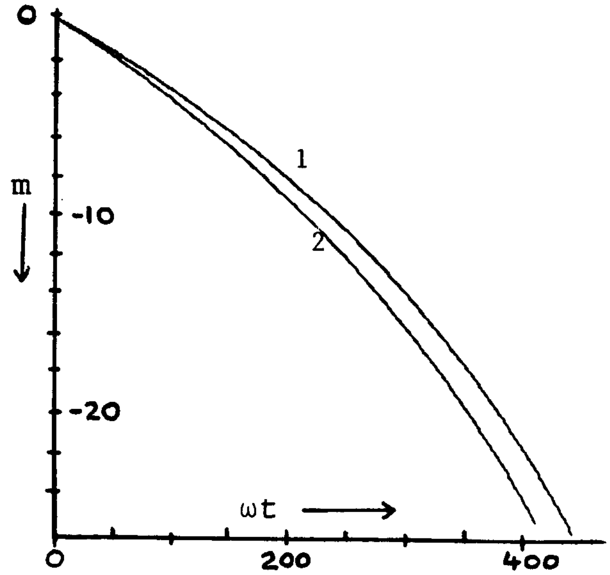

Figure 2: Poloidal mode numbers (m) along the rays.

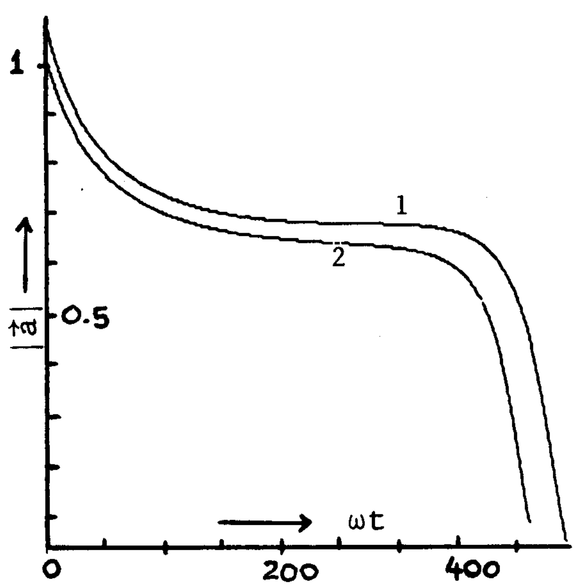

Figure 3: Amplitude of the electric field along the rays.

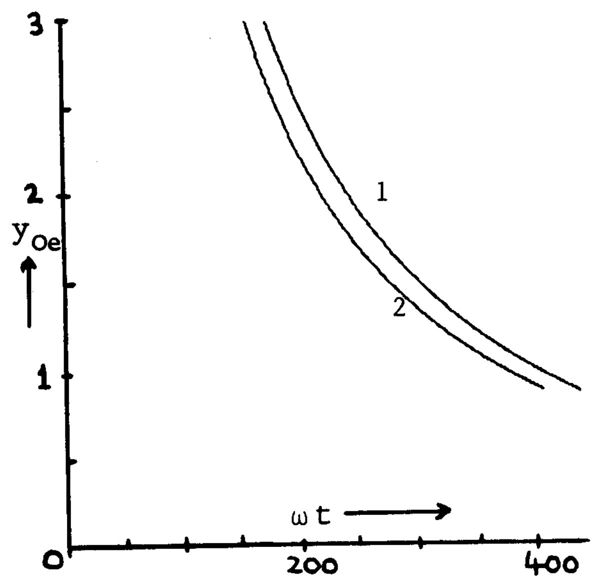

Figure 4: Electron Landau resonance parameter along the rays.