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Simplified Models of Axisymmetric MHD Instabilities

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Abstract

It is shown that simplified models of the axisymmetric vertical instability in elongated plasmas based on uniform shifts of current or of ideal flux surfaces are not equivalent. In comparison with the ideal MHD eigenmode that minimizes the magnetic energy change, δW, the simplified models are shown to satisfy the inequalities $\delta W_i \geq \delta W_c \gtrsim \delta W_m$, where subscripts i, c and m refer to the uniform ideal flux-shift, uniform current-shift and minimizing eigenmode respectively. Thus, of the simplified models, the current-shift is always a better estimate than the flux-shift.

1. Introduction

Recent Tokamak designs almost all take advantage of the substantial enhancements of plasma current and beta that are made possible by vertical elongation. The consequent axisymmetric instability, and its stabilization by a combination of conducting walls and feedback, is then a very important problem in the MHD design and operation.

There is a range of techniques for the analysis of the axisymmetric instability. Extensive numerical codes exist that can investigate the full stability problem on the ideal timescale (e.g. ERATO, GATO [1]) and, more importantly perhaps, on the resistive timescale (e.g. TSC [2]). However, the full simulation of the plasma evolution that a code like TSC can provide is extremely expensive of computer time, and cumbersome to use in investigating a variety of possible practical configurations. Thus, there remains a need for simpler plasma models, particularly for design and analysis of the feedback control system. Simplified models, assuming that the perturbation consists of a rigid vertical shift of the plasma [3-7], can, under some circumstances, provide an accurate assessment of the stability properties. However, its is known that there are experimentally significant situations (e.g.[8]) in which the stability to arbitrary axisymmetric perturbations is appreciably different from the rigid-shift stability.

The purpose of the present work is to prove a theorem about the relative stability properties of simplified models so as to establish general expectations about the applicability of their results.

It is important to realize that models consisting of a uniform vertical shift are actually of two types (1) rigid ideal MHD shifts and (2) rigid constant current shifts. In type (1), which we shall refer to as "ideal shifts", the plasma is assumed to move rigidly, conserving poloidal flux. This is what would happen if the plasma were a perfectly conducting solid. Many of the early theoretical studies of the axisymmetric instability used this model [3-5]. This was justified for the simplest (constant-current high aspect ratio ellipse) analytical equilibria studied because it turns out that the most unstable perturbation for these equilibria is indeed the ideal shift [9,10]. However it was found that for finite aspect ratio, and especially when the equilibrium has triangularity or rectangularity, considerable discrepancies exist between the ideal shift and the full eigenmode analysis [11-13].

In type (2), which will be referred to as "current shifts", the plasma is assumed to move conserving toroidal current density. This is often modelled by regarding the plasma as a set of filaments whose currents, as well as relative positions, are fixed during the motion. This model lends itself readily to simple circuit analysis, and often the plasma has been reduced to a single filament [14,15]. More complex plasmas, modelled as multiple filaments can also be accomodated [7,16,17].

The difference between the two models, though not always recognised, has been known since some of the earliest studies. In their numerical investigations, Lackner and McMahon [3], like Okabayashi and Sheffield [18], used a filament model of the plasma in which the flux was fixed (ideal shift). They showed, however, that the energy perturbation could be divided into two terms, one of which was due to shifting fixed currents and the other due to the current changes induced in the filaments by the motion. The second term was found to have a substantial stabilizing influence in some cases.

In a continuum plasma the difference between the two shift models consists of a perturbed sheet current, flowing on the surface of the plasma for the ideal shift but (obviously) not for the current shift. This is made immediately apparent by consideration of the fact that the poloidal flux function, ψ, satisfies:

$$\Delta^{*}\psi = \frac{\partial^2\psi}{\partial R^2} + \frac{\partial^2\psi}{\partial z^2} - \frac{1}{R}\frac{\partial\psi}{\partial R} = \mu_o j_\phi \quad . \tag{1}$$

The ideal shift rigidly shifts ψ and hence $\Delta^{*}\psi$ inside the plasma; therefore, j_ϕ is also rigidly shifted, as in the current shift. Also, outside the plasma the (vacuum) current is zero in both cases. Thus the only difference can be a surface current. The Lackner and McMahon study can be regarded as a demonstration that this surface current is not in general zero. Note again that the constant-current straight ellipse in a confocal shell is peculiar in having zero surface current.

The linear stability properties are most easily discussed in terms of the second order energy perturbation δW caused by a small displacement ξ. We refer to quantities pertaining to the three types of perturbation: ideal shift, current shift and full energy-minimizing eigenmode using subscripts i, c, and m, respectively. Then the theorem is that

$$\delta W_i \geq \delta W_c \gtrsim \delta W_m \quad . \tag{2}$$

The stability properties of the current shift are intermediate between the ideal shift and the full eigenmode. This result means that, of the two simplified models for calculating stability, the constant current model is better than the ideal shift, in the sense that it is more conservative and closer to the exact MHD result.

The conditions under which the two inequalities hold are somewhat different. The first inequality, $\delta W_c \leq \delta W_i$ is shown in section 2 to hold under extremely general circumstances, requiring only that the external currents, flowing in any conductors around the plasma, be conservative (in the thermodynamic sense). The second inequality, $\delta W_m \lesssim \delta W_c$, requires more stringent conditions for its proof. A sufficient condition is that the system of plasma and conductors be mirror symmetric about a midplane and that kinetic pressure effects should be negligible. These pressure effects are shown to be first order small in the quantity $\epsilon\beta_p\xi_R/\xi$, where ϵ is the inverse aspect ratio, β_p is the poloidal beta and ξ is the flux surface displacement during the perturbation; for a purely vertical flux shift the component ξ_R in the major radial direction is zero. Section 3 discusses this condition and provides the proof.

2. Relative Stability of Current Shifts and Ideal Shifts

Statement:

The mechanical work done (and hence the free energy change) in translating a constant current distribution is not greater than that for translating an ideal conducting rigid body with the same initial current distribution, in a system of external conductors whose current response to flux changes is conservative (in the thermodynamic sense).

Remarks:

The mechanical work done is just δW, which when negative denotes instability. The conservativeness criterion means that the total electrical work done on any part of the external currents when traversing any closed loop in configuration space is zero. Any combination of fixed-flux or fixed-current conductors satisfies this requirement. So do conductors connected to arbitrary purely reactive external circuits (perfect inductors or capacitors). However, resistive circuits do not, because they are dissipative.

Proof:

Consider the following thought experiment. Start with an ideal shift, which can be thought of as a shift of a constant current distribution plus a perfectly conducting shell at the plasma surface. A certain amount of mechanical work, δW_i, is done in moving it, and in addition there is an electrical energy change in the system due to the flux and current changes in the external circuits. Now allow the plasma's surface shell to become resistive while the plasma is held in the perturbed position and its internal currents are kept fixed. The shell currents induced by the original motion will decay to zero and further changes will occur in the external fluxes and currents. The currents of the entire system will then be identical to what would have been obtained by a current shift (without the plasma's surface shell). Now return the plasma to its original equilibrium with constant currents and no shell. (Or so slowly that the shell currents are zero.) This requires an amount of mechanical work $-\delta W_c$, where δW_c is the work required to move a current shift plasma by the original amount. Again there will be some energy changes due to the external circuits.

Because of the conservative, reversible, nature of the external electrical circuits, the total sum of the electrical work done in these three stages is zero. Thus, if the energy dissipated in the shell resistance during the second (decay) phase is δW_d (which is always non negative and actually positive unless the shell currents are exactly zero) then

$$\delta W_i - \delta W_c = \delta W_d \geq 0 \ .$$

This completes the proof of the first inequality of our theorem.

3. Relative Stability of Current shifts and the Minimizing Ideal MHD Eigenmode

Statement:

The energy change, δW_c, in a current shift is greater than or equal to the energy change, δW_m, in the (similarly normed) minimizing ideal MHD perturbation, provided that

(i) one of the integral conditions around any flux surface

$$\oint (B_\phi \delta\psi/RB_p)\ d\ell = 0 \qquad \text{or} \qquad \oint (\delta\psi/R^2 B_p)\ d\ell = 0 \tag{3}$$

is satisfied, where $\delta\psi$ is the flux change due to the current shift and B_ϕ and B_p are the toroidal and poloidal fields, and

(ii) work done by plasma pressure is negligible.

A sufficient condition for (i) is that the system be mirror symmetric about a plane perpendicular to the direction of shift. The order of magnitude of the pressure terms relative to the remaining terms is $\epsilon\beta_p\xi_R/\xi$ where ϵ is the inverse aspect ratio, β_p is the usual ratio of kinetic pressure to poloidal magnetic field pressure and ξ_R is the radial component of the flux surface shift.

Remarks:

The idea behind this proof is that the result is trivial provided that the current shift can be shown to be equivalent to some ideal MHD perturbation. Since the work done by the plasma pressure is not (normally) included in the current shift energy balance, the equivalence requires that this work be negligible. Similarly, the current shift model includes no changes in the toroidal field energy, while the MHD energy minimization process does. Thus, the proof is successful only if the ideal MHD perturbation to which the current shift corresponds is one in which the toroidal field energy is not changed. Perturbations of this type have been called "slip motions" by Rebhan and Salat[11]. Jensen and Thompson [19] have also used almost equivalent conditions. The integral condition guarantees this toroidal field invariance. It acts as an additional constraint on the energy minimization and is satisfied automatically only in certain special situations, such as the mirror symmetric one.

Proof:

Consider a current shift. We wish to find an ideal MHD perturbation ξ to which it corresponds. The current shift causes a change in the poloidal flux $\delta\psi(R,z)$ arising from the current shift itself and from the changes in the external currents induced by the shift. Then since any ideal perturbation convects poloidal flux, the ideal perturbation must satisfy

$$\xi \cdot \nabla\psi = \delta\psi \quad . \tag{4}$$

Thus the component of the perturbation perpendicular to the flux surfaces is defined by the current shift. Other components are free to be chosen as desired in order to satisfy other constraints.

Now consider the second order energy change (the energy principle [20]). Assuming there to be no surface current in the equilibrium state, it may be written:

$$\delta W = \delta W_f + \delta W_v \quad , \text{ with}$$

$$\delta W_f = \frac{1}{2}\int_f \frac{1}{\mu_0}|\nabla\wedge(\xi\wedge B)|^2 + (j\wedge\xi).\nabla\wedge(\xi\wedge B) + (\xi.\nabla p)\nabla.\xi + \gamma p(\nabla.\xi)^2 \quad d^3x$$

$$\delta W_v = \frac{1}{2\mu_0}\int_v(\delta B_v)^2 \; d^3x \; - \; \frac{1}{2\mu_0}\int_S(B_v.\delta B_v)\xi.ds \qquad . \tag{5}$$

where δW_f is the contribution from the fluid (plasma) region, δW_v is the contribution from the vacuum region and B_v refers to the field in the vacuum region. There may be additional contributions from external circuit energy changes. However, if we can demonstrate the equivalence inside the plasma of the current shift and the ideal pertubation, such changes will be identical for either.

The simplified model effectively eliminates from consideration the terms in the energy principle that arise from p and the toroidal field, B_ϕ. In an MHD equilibrium the total field may be written

$$B = B_p + B_\phi = \nabla\phi\wedge\nabla\psi + F\nabla\phi \quad , \tag{6}$$

where ϕ is the toroidal angle and F is constant on flux surfaces. If we use the symmetry properties of the equilibrium and perturbation, which, amongst other things, imply that $\nabla\wedge(\xi\wedge B_\phi)$ and $\nabla\wedge B_p$ have no poloidal component, then after considerable algebra, we can write:

$$\begin{aligned}\delta W_f = \frac{1}{2\mu_0}\int \quad d^3x \quad \{ \; & \nabla\wedge(\xi\wedge F\nabla\phi) \;.\; F\nabla\wedge(\xi\wedge\nabla\phi) \\ & - [\nabla(\xi.\nabla\phi)\wedge\nabla\psi] \;.\; [\; 2\nabla\wedge(\xi\wedge F\nabla\phi) + (\nabla F\wedge\nabla\phi)\wedge\xi + \frac{dF}{d\psi}(\xi.\nabla\psi)\nabla\phi \;] \\ & + |\nabla(\xi.\nabla\phi)\wedge\nabla\psi|^2 + |\nabla\phi|^2|\nabla(\xi.\nabla\psi)|^2 + \nabla\phi.[\nabla\wedge(\nabla\phi\wedge\nabla\psi)]\xi.\nabla(\xi.\nabla\psi) \\ & + \mu_0[\; (\xi.\nabla p)\nabla.\xi + \gamma p(\nabla.\xi)^2 \;] \quad \}. \end{aligned} \tag{7}$$

To reach this form, an integration by parts has been performed on the $dF/d\psi$ term.

Now the second line of terms in this equation, which are all dependent on B_ϕ, will be zero if

$$\nabla(\xi.\nabla\phi)\wedge\nabla\phi = 0. \tag{8}$$

This is a requirement that any motion in the toroidal direction consist of a rigid rotation of each flux surface about the major axis. It is plain from the third line of terms that this requirement on ξ_ϕ minimises the energy perturbation. Once this criterion is satisfied ξ_ϕ no longer enters into δW_f so we take ξ_ϕ to be zero from now on. Then, providing the first term vanishes, i.e. either

$$\nabla\wedge(\xi\wedge F\nabla\phi) = 0 \qquad \text{or} \qquad \nabla\wedge(\xi\wedge\nabla\phi) = 0, \tag{9}$$

the terms involving B_ϕ will all vanish. These alternative conditions generalize the "slip motion" condition of Rebhan and Salat, who used only the second because they were concerned with the large aspect ratio tokamak limit, in which the conditions become identical. The two conditions, while not identical in general, are so similar that the following treatment applies essentially unchanged to either. We will concentrate on the first, which is the more general. The forms for the second can then be obtained by putting F=1.

The vacuum contribution, δW_v, may be evaluated as the surface integral using the internal fields B and $\delta B = \nabla\wedge(\xi\wedge B)$ (rather than the vacuum fields) provided that the equilibrium has no surface currents (i.e. if p and ∇p are zero at the plasma surface). Substitution readily demonstrates that the condition $\nabla\wedge(\xi\wedge F\nabla\phi) = 0$ is sufficient to make the B_ϕ contribution to δW_v zero.

The equation $\nabla\wedge(\xi\wedge F\nabla\phi)=0$ requires ξ to be expressible as

$$\xi = \frac{1}{F|\nabla\phi|^2}(\nabla\phi\wedge\nabla\omega) \tag{10}$$

where ω is a scalar potential, a kind of stream function of the poloidal flow.

Now we demonstrate that a perturbation equivalent to the current shift can indeed be found, satisfying the constraints, provided an integral condition is satisfied. First note that any perturbation satisfying the slip constraints will give

$$\xi.\nabla\psi = \frac{(\nabla\phi\wedge\nabla\omega).\nabla\psi}{F|\nabla\phi|^2} = \frac{(\nabla\psi\wedge\nabla\phi).\nabla\omega}{F|\nabla\phi|^2} \quad . \tag{11}$$

Thus when $\xi.\nabla\psi$ is given, the variation of ω round any flux surface is determined by simply integrating this equation. For if ℓ denotes arc length around the flux surface in the poloidal direction, we can write

$$\omega(\ell,\psi)-\omega(0,\psi) = \int_0^{\ell}\frac{(\nabla\psi\wedge\nabla\phi).\nabla\omega}{|\nabla\psi\wedge\nabla\phi|}\,d\ell = \int_0^{\ell}\frac{F|\nabla\phi|^2\delta\psi}{|\nabla\psi\wedge\nabla\phi|}\,d\ell \quad , \tag{12}$$

where integrals are along the flux surface, and we have incorporated the current shift form $\xi.\nabla\psi = \delta\psi$. We must require ω to be single valued, however. This requires the complete integral once around the flux surface to be zero. Noting that $B_\phi = F|\nabla\phi|$ and $B_p = |\nabla\psi\wedge\nabla\phi|$, this readily reduces to Eq(3) as was to be proved. The alternate form of the slip criterion, $\nabla\wedge(\xi\wedge\nabla\phi)=0$, gives rise to the alternate condition in Eq(3), by setting F=constant. Inspection of Eq(12) shows that if we are dealing with a mirror symmetric case, where F, $|\nabla\psi\wedge\nabla\phi|$, $\nabla\phi$, and $d\ell$ are even and $\delta\psi$ is odd, then the integral is indeed zero by symmetry. Otherwise there is no guarantee that the integral condition is satisfied.

Although there is still the integration constant $\omega(0,\psi)$ free to be chosen for each flux surface, Eq(12) essentially determines the ξ that we are seeking. This ξ is not in general compressionless and choice of $\omega(0,\psi)$ cannot make $\nabla.\xi = 0$ in general. Therefore the pressure terms in δW cannot necessarily be ignored. Their magnitude can readily be established by noting that

$$\nabla\wedge(\xi\wedge F\nabla\phi) = -F\left[\frac{1}{F}\xi.\nabla F - \frac{2\xi_R}{R} + \nabla.\xi\right]\nabla\phi \quad . \tag{13}$$

Therefore the perturbation which satisfies the slip condition has

$$\nabla.\xi = \frac{2\xi_R}{R} - \frac{\xi.\nabla F}{F} \quad . \tag{14}$$

Using the zeroth order equilibrium pressure balance, the order of magnitude of $\nabla F/F$ can be seen to be $[(B_p/B_\phi)^2 + \beta_t]$. Therefore the order of magnitude of the term $\mu_0(\xi.\nabla p)\nabla.\xi$ relative to the driving term of the axisymmetric mode, $\nabla\phi.[\nabla\wedge(\nabla\phi\wedge\nabla\psi)]\xi.\nabla(\xi.\nabla\psi)$, is

$$\frac{\mu_0 \xi \frac{p}{a}[(2\xi_R/R) + \frac{\xi}{a}(B_p^2/B_\phi^2 + \beta_t)]}{B_p^2\ \xi^2/a^2} = O(\beta_p \frac{a}{R}\frac{\xi_R}{\xi}) + O(\beta_t) + O(\beta_p\beta_t), \tag{15}$$

where a is the minor radius. For the alternate form of the slip condition, $\nabla\wedge(\xi\wedge\nabla\phi)=0$, the last two terms are absent. They may therefore be safely ignored because if the plasma pressure were high enough for them to be significant we could always take the alternate slip condition as our choice and thus eliminate them.

That completes our proof.

4. Discussion

Despite the generality of the inequality $\delta W_c \leq \delta W_i$, it might seem that its relevance is sharply reduced by the requirement that the external circuit currents be conservative. Perhaps the most important situation of interest is when the plasma is surrounded by resistive walls. This appears to be ruled out by the conservative restriction. It turns out, however that the inequality can be made relevant to the resistive wall case using approximate results of Haney and Freidberg [6]. They showed that a variational estimate of the growth rate, γ, of a mode within a resistive stabilizing wall whose time constant is τ is

$$\gamma\tau = -\frac{\delta W_\infty}{\delta W_b} , \tag{16}$$

where δW_∞ is the energy change for a perturbation in the absence of the wall and δW_b is that in the presence of a perfectly conducting wall. What this equation shows is that the magnitude of the energy for a perfectly conducting wall, δW_b, (assumed positive, for otherwise the plasma is unstable on the ideal timescale) and for no wall, δW_∞, determine the growth rate of the resistive mode. Thus our relative ordering of δW_c and δW_i for conservative external circuits determines also the relative ordering of the resistive growth rates for their respective perturbations, at least to the accuracy of the variational approximation.

One might wonder at the restrictiveness of the conservative condition and whether it ought to be necessary. That there must be some restriction placed on the external current response may be understood by considering a situation in which active current control is allowed. It seems clear that if there is any difference between the field perturbations for the current shift and ideal shift cases, then a linear feedback law can be devised that reverses the ordering and makes the ideal shift more unstable than the current shift. This could be done by applying a strong positive feedback to the difference between the perturbations. Thus a restriction is necessary that excludes such a situation. The conservative condition does this. Whether, however, there is some direct demonstration that for a passive wall (i.e. allowing only dissipation of energy) the growth rate estimate for the ideal shift is less than for the current shift, in a more rigorous sense than the variational demonstration, is unknown to this author.

The relative stability of current shifts and the minimizing eigenmode is less dependent on such additional arguments to demonstrate its relevance to the resistive wall case. On the other hand, the restriction to mirror-symmetric cases is a fairly serious one since there is considerable interest currently in "single null" divertor configurations which are not mirror-symmmetric. It is clear, again, that some form of restriction of the configuration under consideration is inevitable. As an obvious example, horizontal rather than vertical shifts are axisymmetric modes. But it is known that they are not adequately modelled by constant current shifts. Instead, for circular plasmas, adiabatic scaling expressions are known [21], including changes in total current and minor radius with R. The mirror symmetry requirement excludes horizontal shifts from consideration for the current shift model. However it appears that the present formalism may be valuable in developing more general shift models that are appropriate for horizontal shifts.

Another limitation concerning the comparison between current shifts and the minimizing eigenmode is that the inequality is itself only approximate because of the compression term that is left over. This term, which has been shown to be of relative order $\epsilon\beta_p\xi_R/\xi$, will in general be stabilizing. Thus it tends to increase δW_m relative to δW_c. In many cases, particularly for moderate beta ($\beta_p \lesssim 1$) and triangularity ($\xi_R/\xi << 1$), it will be negligible. In extreme cases it may lead to a significant violation of the approximate inequality. This weakens the impact of the present theorem but since it leads to a relatively more conservative prediction by the use of the current shift model this may not be regarded as a disqualifying fault. It may in fact mean that the current shift model is an even better estimate.

It should be noted that the eigenmode that we have shown to be more unstable than the current shift is the full minimizing eigenmode, including the possibility of radial (ξ_R) motions. Bobbio et al [7] found numerically that for INTOR shaped plasmas the current shift model gave more unstable perturbations than a general incompressible ideal MHD perturbation. This result does not violate our theorem because of their restriction to incompressible perturbations, of the form $\boldsymbol{\xi} = \xi_z(R)\,\hat{\mathbf{z}}$. What it does show is that, for this plasma, the current shift model estimate is better than even a generalized vertical flux shift model of this type. A similar result has also been observed during numerical studies of other plasmas [22].

In summary, then, the main force of the present investigation is to show that for essentially all practical purposes the model consisting of a constant shift of the current profile gives a superior estimate of axisymmetric stability than the constant shift of ideal MHD flux surfaces.

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