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ABSTRACT

The effects of losses of the fast electron current carriers are included in a calculation of the efficiency of RF current drive. The analytical expressions obtained for the current drive efficiency J/P_A and bulk heating rate P_D/P_A are explicitly dependent on the fast electron confinement time τ_F. In present tokamak experiments, fast particle losses are predicted to reduce the current drive efficiency from 50% to more than an order of magnitude. However, in future fusion plasmas fast electron confinement appears to be sufficient to allow nearly ideal efficiencies to be achieved.

In tokamak experiments in which the plasma current is sustained by lower hybrid current drive, the current drive efficiency is generally found to be less than the efficiency predicted by Fisch and Boozer in Ref. 1. The cause or causes for this apparent discrepancy are not as yet understood, and in order to provide credible extrapolation of present experiments to fusion plasma conditions it appears that refinements in the theory of Ref. 1 are necessary. Here the effects of losses of the fast electron current carriers are considered. The current drive efficiency, heating rate, and power loss rate are calculated, including the effect of a finite fast particle confinement time τ_F, the Fisch-Boozer result being recovered in the limit $\tau_F \to \infty$.

The efficiencies are calculated from response functions following the method of Ref. 1. The response functions for the current χ_J, collisionally dissipated power χ_D, and power lost as a result of fast particle losses χ_L are given in Eqs. (1-3).

$$\chi_J = e \int_0^{\Delta t} \frac{dt}{\Delta t} v_{\parallel} \, \delta f \tag{1}$$

$$\chi_D = - \int_0^{\Delta t} \frac{dt}{\Delta t} \left[\frac{dE}{dt} \right]_{coll} \delta f \tag{2}$$

$$\chi_L = - \int_0^{\Delta t} \frac{dt}{\Delta t} E \left[\frac{d\delta f}{dt} \right]_{loss} \tag{3}$$

where $E = 1/2\, mv^2$, $v_{\parallel}$ is the velocity parallel to the magnetic field, $(dE/dt)_{coll}$ is the collisional energy loss rate, $(d\delta f/dt)_{loss}$ is the fast particle loss rate, and Δt is the averaging time which is taken as long compared to the collisional and particle loss time scales. The time integrals in Eqs. (1-3) are taken along the velocity space trajectory δf follows as a result of collisional slowing. Absorption of a quantum of RF wave energy by the electron population δf causes δf to undergo a velocity space displacement in the direction $\hat{s}$. In this process the time averaged power absorbed is given by Eq. (4):

$$P_A = \hat{s} \cdot \nabla_V \left[E \frac{\delta f_o}{\Delta t} \right] \tag{4}$$

where δf_o is the initial number of particles in the volume element d^3v affected by the wave absorption. In general $P_A \neq P_D$, where P_D is the power dissipated by collisions of the population δf with the bulk particles. The efficiency of interest gives the amount of current generated per unit of power absorbed; this efficiency is J/P_A. The response functions in Eqs. (1-3) yield the time averaged current J, the collisionally dissipated power P_D, and the power loss caused by fast particle losses in Eqs. (5-7).

$$J = \hat{s} \cdot \nabla_V \chi_J \tag{5}$$

$$P_D = \hat{s} \cdot \nabla_V \chi_D \tag{6}$$

$$P_L = \hat{s} \cdot \nabla_V \chi_L \tag{7}$$

and the efficiencies of current drive, bulk heating, and power loss are given by ratios of the quantities in Eqs. (5-7) to P_A. The expressions (1-3) can be evaluated by writing the response function integrals in the velocity representation. Here a dimensionless set of variables will be used with velocities normalized to $v_t = (T/m)^{1/2}$, and times to the bulk collision frequency $\nu_o = \omega_{pe}^4 \ln \Lambda/(2\pi n_o v_t^3)$; the variables w and u are defined as $w = v_\parallel/v_t$, $u^2 = (v_\parallel^2 + v_\perp^2)/v_t^2$. The change to the velocity representation is affected using the collisional slowing, momentum loss, and particle confinement equations. The collisional energy loss of the population δf is described by the equation[2] $du/dt = -\nu_E u$, and in the high velocity limit of interest $\nu_E = 1/2\ 1/u^3$. The momentum loss rate is $dw/dt = -\nu_m w$, where $\nu_m = (2 + Z_i)\nu_E$ and Z_i is the ion charge state, and the fast particle loss rate $(d\delta f/dt)_{loss}$ is given by Eq. (8):

$$\delta\dot{f} = -\delta f/\tau_F \tag{8}$$

with solution $\delta f = \delta f_o \exp\left(-\int_0^t dt'/\tau_F\right)$. In general, τ_F may be a function of u and w and represents an arbitrary fast particle transport process which causes current carriers to be lost from the plasma. Using the integrals of the slowing down equations to change to the velocity representation, the response functions are obtained, Eqs. (9-11):

$$\chi_J = \frac{2w}{\Delta t\ u^{2+Z_i}} \int_0^u du'\ u'^{4+Z_i} \exp\left[2\int_u^{u'} u''^2 \frac{du''}{\tau_F}\right] \tag{9}$$

$$\chi_D = \frac{1}{\Delta t} \int_0^u u'\ du' \exp\left[2\int_u^{u'} u''^2 \frac{du''}{\tau_F}\right] \tag{10}$$

$$\chi_L = \frac{1}{\Delta t} \int_0^u u'^4 \frac{du'}{\tau_F} \exp\left[2\int_u^{u'} u''^2 \frac{du''}{\tau_F}\right] \tag{11}$$

And using Eqs. (9-11) in expressions (5-7) yields the efficiencies of current generation J/P_A, heating efficiency P_D/P_A, and power loss rate P_L/P_A in Eqs. (12-14).

$$\frac{J}{P_A} = \frac{4}{\hat{s}\cdot\nabla u^2}\,\hat{s}\cdot\nabla\,\frac{w}{u^{2+Z_i}}\int_0^u du'\;u'^{4+Z_i}\,\exp\left[2\int_u^{u'} u''^2\,\frac{du''}{\tau_F}\right] \tag{12}$$

$$\frac{P_D}{P_A} = \frac{2}{\hat{s}\cdot\nabla u^2}\,\hat{s}\cdot\nabla\int_0^u u'\,du'\,\exp\left[2\int_u^{u'} u''^2\,\frac{du''}{\tau_F}\right] \tag{13}$$

$$\frac{P_L}{P_A} = \frac{2}{\hat{s}\cdot\nabla u^2}\,\hat{s}\cdot\nabla\int_0^u u'^4\,\frac{du'}{\tau_F}\,\exp\left[2\int_u^{u'} u''^2\,\frac{du''}{\tau_F}\right] \tag{14}$$

where in the most general case the fast particle confinement time τ_F is a function of w and u. Eqs. (12-14) contain the main result of this paper. The efficiencies calculated are explicitly dependent on the fast particle confinement time, and the loss free theory of Fisch and Boozer is recovered in the limit $\tau_F \to \infty$. Eqs. (12-14) can be applied to the cases of current drive by Landau damping or magnetic pumping, $\hat{s} = \vec{v}_{\parallel}/|v|$ or for current drive by cyclotron resonance, $\hat{s} = \vec{v}_{\perp}/|v|$.

To illustrate the content of Eqs. (12-14) the case of current drive by Landau damping is considered using a velocity independent model for the fast particle confinement time. In this case an analytical expression for J/P_A can be found for arbitrary values of τ_F. Taking $Z_i = 1$ and neglecting $v_\perp^2/v^2$ compared to $v_\parallel^2/v^2$, Eq. (12) reduces to:

$$\frac{J}{P_A} = \frac{\tau_F}{w}\left[1 + \frac{3\tau_F}{w^3}\left[1 - \left(1 + \frac{w^3}{\tau_F}\right)e^{-2w^3/3\tau_F}\right]\right] \tag{15}$$

In the limit $\tau_F \to \infty$ the Fisch-Boozer result $J/P_A = 4/3\ w^2$ is recovered. The current drive efficiency as a function of w is shown in Fig. 1 for various values of τ_F, where τ_F is the fast particle confinement time in units of the bulk collisional slowing time ν_0^{-1}. As shown in Fig. 1, at low velocities

J/P_A approaches the ideal Fisch-Boozer efficiency. When w is small the collisional slowing time is much faster than particle loss time, so the ideal efficiency is achieved. At higher velocities J/P_A first increases then reaches a maximum and begins to decrease slowly with $J/P_A \sim \tau_F/\omega$ at large ω, Fig. 1.

The heating efficiency P_D/P_A and the power loss fraction P_L/P_A are obtained in terms of the functions $\phi_n(x) = e^{-x^3} \int_0^x z^n e^{z^3} dz$. Evaluating the integrals in Eqs. (13) and (14) gives Eqs. (16) and (17):

$$\frac{P_D}{P_A} = 1 - 3\alpha u \; \phi_1(\alpha u) \tag{16}$$

$$\frac{P_L}{P_A} = \frac{3}{2}\alpha^3 u^3 - \frac{9}{2}\alpha u \; \phi_4(\alpha u) \tag{17}$$

where $\alpha = \left(2/(3\tau_F)\right)^{1/3}$. P_D/P_A and P_L/P_A obey the conservation law $P_D/P_A + P_L/P_A = 1$; P_D/P_A is plotted in Fig. 2. Surprisingly, P_D/P_A reached negative values at sufficiently large w, that is, an incremental increase in power absorption at large w can remove more energy because of increased particle losses than is supplied by the RF power absorbed. That the heating efficiency can be negative can be seen by considering a simple example. Suppose there is a velocity space loss process for which $\tau_F = \infty$ for $w \leq w_0$ and $\tau_F = 0$ for $w > w_0$. A population of electrons δf located at $w = w_0$ will transfer all of its energy E_0 to the bulk by collisions in the absence of RF power absorption since $\tau_F = \infty$ at $w = w_0$. If a quantum ΔE_0 of RF energy is absorbed by these particles the energy $(E_0 + \Delta E_0)\delta f_0$ is lost from the plasma since $\tau_F = 0$ for $w > w_0$, where the RF power absorption has pushed the population δf beyond w_0. The heating efficiency P_D/P_A of this process is therefore $(E_f^{coll} - E_i^{coll})/\Delta E_0 = -E_0/\Delta E_0 < 0$, where E_i^{coll} and E_f^{coll} are the energies dissipated via collisions respectively before and after absorption of the RF energy. If the velocity at which the power is absorbed is increased the result will be increased power flow to the bulk and higher current drive efficiency only if fast particle loss processes are negligible, but if τ_F is finite then at sufficiently high velocities the collisional slowing time becomes long and the energy loss due to fast particle losses can dominate.

Before Eq. (15) can be applied to experiments, a value for the fast particle confinement time τ_F is needed. Although τ_F has been measured in only a few cases it appears that for the electron energies of present experiments, $E \lesssim 1/2$ MeV, τ_F can be taken as approximately equal to the bulk electron energy confinement time τ_e.[3] In the case of a medium sized tokamak plasma with $T_e = 1$ keV, $n_e = 1 \times 10^{13}$ cm^{-3}, $\tau_e = 5$ msec, $\tau_F \cong \nu_0 \tau_e = 10^3$, and $w = 10$ for a typical fast electron velocity, Eq. (15) gives $J/P_A = 67$ as compared to $J/P_A = 133$ if electron confinement were ideal. In this example the energetic electron loss reduces the efficiency by approximately 50%. In smaller tokamaks τ_e is smaller and typical parameters are $T_e = 200$ eV, $n_e = 1 \times 10^{13}$ cm^{-3}, $\tau_e = 0.5$ msec, $\tau_F \cong \nu_0 \tau_e = 580$, and $w = 20$. Eq. (15) gives $J/P_A = 35$ as compared to the ideal confinement prediction of $J/P_A = 532$. In this case the current drive efficiency is reduced by a factor of ~15. In present tokamak experiments fast particle losses are predicted to seriously degrade the current drive efficiency; further, as evident in Figs. 1 and 2, the reduced values of J/P_A are correlated with reduced heating efficiency.

In future fusion plasmas, the normalized fast particle confinement time is expected to be much improved over the above examples, and w values will tend to be smaller because of the increased bulk temperatures. These effects will tend to improve the current drive efficiency. For example, with $T_e = 10$ keV, $n_e = 5 \times 10^{14}$ cm^{-3}, $\tau_e = 1$ sec, and $w = 6$, Eq. (15) predicts essentially the ideal efficiency. It appears that in moving to reactor plasma conditions significant improvements in the current drive efficiency can be anticipated in comaprison to those now achievable.

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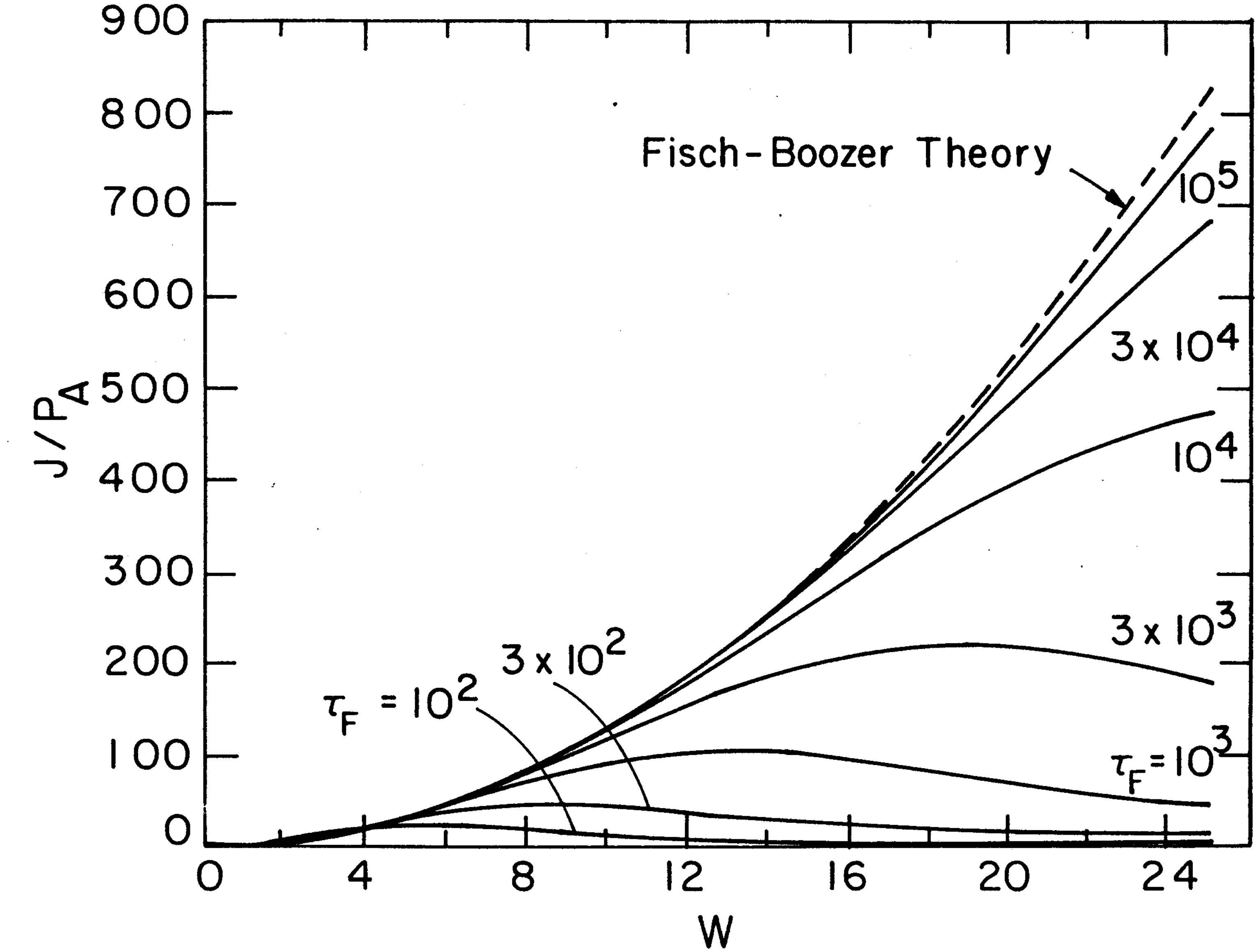
FIGURE CAPTIONS

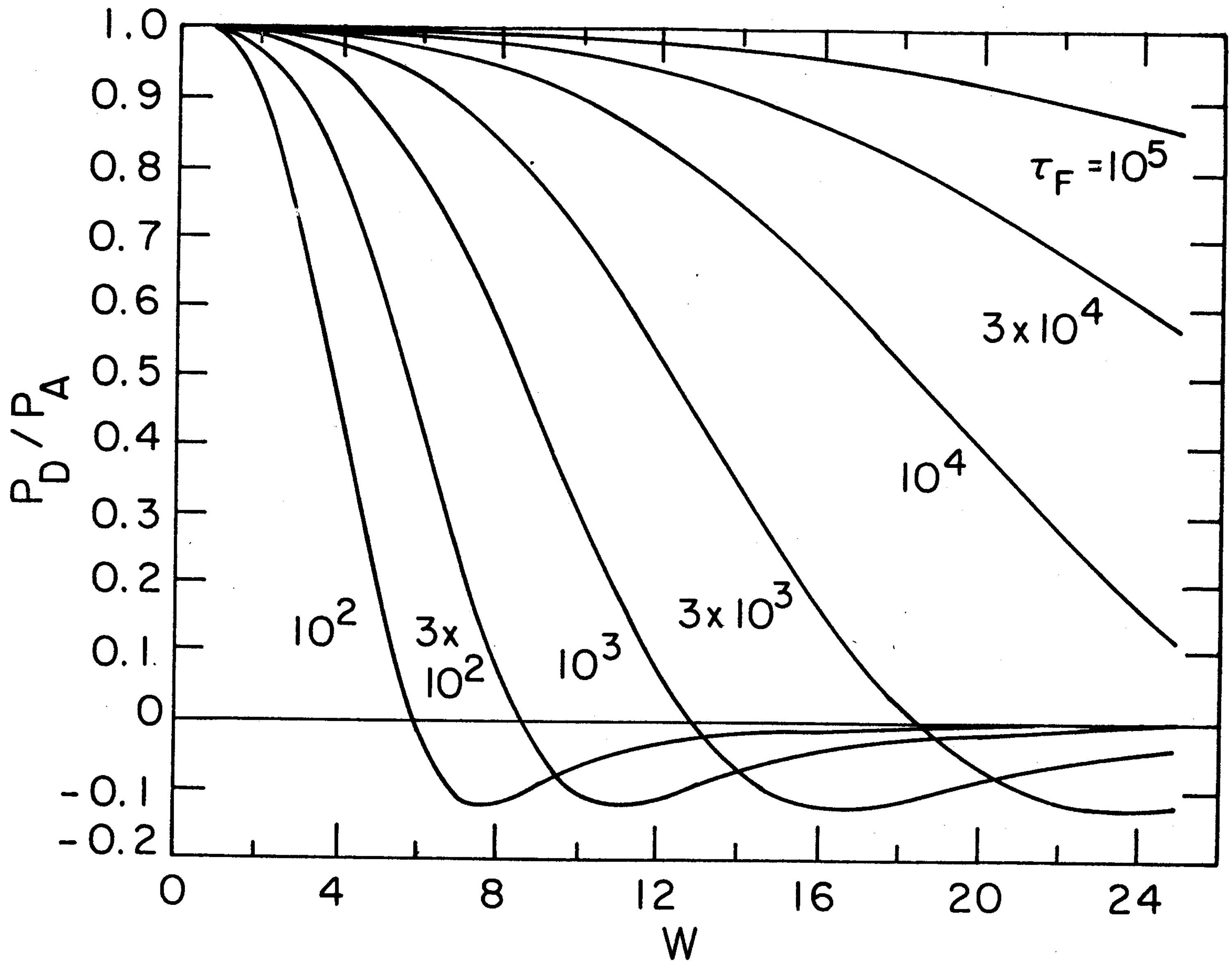
Fig. 1 Current drive efficiency J/P_A vs. normalized parallel velocity w for various values of τ_F, the fast electron confinement time (in units of the bulk collision time ν_0^{-1}).

Fig. 2 Bulk heating efficiency P_D/P_A vs. w for various τ_F.



Fisch-Boozer Theory
10^5
3×10^4
10^4
3×10^3
$\tau_F = 10^3$
3×10^2
$\tau_F = 10^2$
J/P_A
W
900
800
700
600
500
400
300
200
100
0
0
4
8
12
16
20
24

Fig. 1



1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
P_D / P_A
0
4
8
12
16
20
24
W
$\tau_F = 10^5$
3×10^4
10^4
3×10^3
10^3
3×10^2
10^2

Fig. 2