PFC/JA-86-38

EXACT ANALYTICAL MODEL OF THE CLASSICAL WEIBEL INSTABILITY IN RELATIVISTIC ANISOTROPIC PLASMA

Peter H. Yoon

and

Ronald C. Davidson

June, 1986

Plasma Fusion Center
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139 USA

This research was supported in part by the Office of Naval Research and in part by the Department of Energy.

Submitted for publication in Physical Review A.

EXACT ANALYTICAL MODEL OF THE CLASSICAL WEIBEL INSTABILITY IN RELATIVISTIC ANISOTROPIC PLASMA

Peter H. Yoon and Ronald C. Davidson
Plasma Fusion Center
Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

ABSTRACT

Detailed properties of the Weibel instability in a relativistic unmagnetized plasma are investigated for a particular choice of anisotropic distribution function $F(p_\perp^2, p_z)$ that permits an exact analytical solution to the dispersion relation for arbitrary energy anisotropy. The particular equilibrium distribution function considered in the present analysis assumes that all particles move on a surface with perpendicular momentum $p_\perp = \hat{p}_\perp$ = const. and are uniformly distributed in parallel momentum from $p_z = -\hat{p}_z$ = const. to $p_z = +\hat{p}_z$ = const. (Here, the propagation direction is the z-direction.) The resulting dispersion relation is solved analytically, and detailed stability properties are determined for a wide range of energy anisotropy.

The classical Weibel instability[1,2] in a uniform plasma is a transverse electromagnetic instability driven by an anisotropy in the average kinetic energy of the constituent electrons and/or ions. This instability has a wide range of applicability to astrophysical plasmas,[3,4] and to laboratory plasmas[4] with intense microwave heating. For <u>nonrelativistic</u> anisotropic plasma, detailed properties of the Weibel instability are readily calculated[1,2] for a wide range of equilibrium distribution functions $F_j(p_\perp^2,p_z)$. For <u>relativistic</u> anisotropic plasma, however, because of the coupling of the perpendicular and parallel particle motions through the relativistic mass factor $\gamma = (1 + p_\perp^2/m_j^2c^2 + p_z^2/m_j^2c^2)^{\frac{1}{2}}$, stability properties are usually calculated in the limit of extreme energy anisotropy, or long perturbation wavelength,[4] which allow approximate analytical solutions to the electromagnetic dispersion relation. Here, "perpendicular" and "parallel" refer to directions relative to the propagation direction (the z-direction).

The purpose of the present <u>brief communication</u> is to investigate detailed properties of the Weibel instability in a relativistic unmagnetized plasma for a particular choice of anisotropic distribution function that permits an <u>exact</u> analytical solution to the dispersion relation for arbitrary energy anisotropy. This calculation is intended to provide qualitative insights regarding stability behavior for more general choices of equilibrium distribution function. The particular distribution function [Eq.(3)] considered in the present analysis assumes that all particles move on a surface with perpendicular momentum $p_\perp = \hat{p}_\perp$ = const. and are uniformly distributed in parallel momentum between $p_z = -\hat{p}_z$ = const. and $p_z = +\hat{p}_z$ = const. The resulting dispersion relation [Eq.(8)] can be solved analytically, and detailed stability properties are determined for a wide range of energy anisotropy. The dispersion relation [Eq.(15)] is also derived for the case where the particles have a thermal equilibrium distribution in parallel momentum p_z [Eq.(13)], and the corresponding range of unstable wavenumbers is determined in closed form [Eq.(18)]. Extension of the present analysis to include a uniform applied magnetic field $B_0\hat{e}_z$ will be the subject of a future investigation.

A. Theoretical Model

We investigate the electromagnetic stability properties of relativistic anisotropic plasma for wave perturbations propagating in the z-direction with wavevector $\underset{\sim}{k} = k_z\hat{e}_z$. Perturbations are about the class of uniform, field-free equilibria with distribution function

$$f_j^0(\underset{\sim}{p}) = \hat{n}_j F_j(p_\perp^2, p_z) \ , \tag{1}$$

where $\hat{n}_j$ = const. is the ambient density of the j'th plasma component, $p_\perp = (p_x^2 + p_y^2)^{1/2}$ is the particle momentum perpendicular to the propagation direction, and p_z is the parallel momentum. In the absence of applied magnetic field, the linear dispersion relation for transverse electromagnetic wave perturbations propagating in the z-direction is given by

$$\begin{aligned} 0 = D_T(k_z,\omega) = 1 - \frac{c^2k_z^2}{\omega^2} - \sum_j \frac{\omega_{pj}^2}{\omega^2}\int \frac{d^3p}{\gamma}\, \frac{(p_\perp/2)}{(\omega - k_z p_z/\gamma m_j)} \\ \times \left\{\left(\omega - \frac{k_z p_z}{\gamma m_j}\right)\frac{\partial}{\partial p_\perp} + \frac{k_z p_\perp}{\gamma m_j}\frac{\partial}{\partial p_z}\right\} F_j(p_\perp^2, p_z) \quad . \end{aligned} \tag{2}$$

Here, $\omega_{pj}^2 = 4\pi\hat{n}_j e_j^2/m_j$ is the nonrelativistic plasma frequency-squared; e_j and m_j are the charge and rest mass, respectively, of a j'th component particle; c is the speed of light <u>in vacuo</u>; $\gamma = (1 + p_\perp^2/m_j^2c^2 + p_z^2/m_j^2c^2)^{1/2}$ is the relativistic mass factor; the range of integration is $\int d^3p \cdots = 2\pi\int_0^\infty dp_\perp p_\perp \int_{-\infty}^\infty dp_z \cdots$;

and the normalization of F_j is $\int d^3pF_j(p_\perp^2,p_z) = 1$. In obtaining Eq.(2), the perturbations are assumed to have z- and t-dependence proportional to $\exp[i(k_z z - \omega t)]$, where k_z is the wavenumber and ω is the complex oscillation frequency with $\mathrm{Im}\omega > 0$, which corresponds to instability (temporal growth). For relativistic anisotropic plasma, we note that the perpendicular and parallel particle motions in Eq.(2) are inexorably coupled through the relativistic mass factor $\gamma = (1 + p_\perp^2/m_j^2c^2 + p_z^2/m_j^2c^2)^{\frac{1}{2}}$.

In the analysis that follows, we specialize to the case of stationary ions ($m_i \rightarrow \infty$) and consider a single active component of relativistic anisotropic electrons. Moreover, for simplicity of notation, the electron species labels are omitted from ω_{pe}^2, m_e, $F_e(p_\perp^2,p_z)$, etc. The resulting dispersion relations [Eqs.(8) and (15)] are readily generalized to the multi-component case.

B. Waterbag Distribution in Parallel Momentum

The dispersion relation (2) can be used to investigate detailed electromagnetic stability properties for a wide range of anisotropic distribution functions $F(p_\perp^2,p_z)$. For purposes of elucidating the essential features of the Weibel instability in relativistic anisotropic plasma, we make a particular choice of $F(p_\perp^2,p_z)$ for which the momentum integrals in Eq.(2) can be carried out in closed analytical form. In particular, it is assumed that the electrons move on a surface with perpendicular momentum $p_\perp = \hat{p}_\perp$ = const. and are uniformly distributed in parallel momentum between $p_z = -\hat{p}_z$ = const. and $p_z = +\hat{p}_z$ = const. That is, $F(p_\perp^2,p_z)$ is specified by

$$F(p_\perp^2,p_z) = \frac{1}{2\pi p_\perp}\,\delta(p_\perp - \hat{p}_\perp)\,\frac{1}{2\hat{p}_z}\,H(\hat{p}_z^2 - p_z^2)\ , \tag{3}$$

where $H(x)$ is the Heaviside step function defined by $H(x) = +1$ for $x > 0$, and $H(x) = 0$ for $x < 0$. Note from Eq.(3) that $\int d^3pF(p_\perp^2,p_z) = 1$. Because the electrons are uniformly distributed in parallel momentum for $|p_z| < \hat{p}_z$, we refer to the p_z-dependence of the distribution function in Eq.(3) as a "water-bag" distribution in p_z. For future reference, it is useful to introduce the maximum energy $\hat{\gamma}mc^2$, parallel speed $c\hat{\beta}_z$, and perpendicular speed $c\hat{\beta}_\perp$ defined by

$$\begin{aligned} \hat{\beta}_z &= \frac{\hat{p}_z}{\hat{\gamma}mc} \ , \quad \hat{\beta}_\perp = \frac{\hat{p}_\perp}{\hat{\gamma}mc} \ , \\ \hat{\gamma} &= \left(1 + \frac{\hat{p}_\perp^2}{m^2c^2} + \frac{\hat{p}_z^2}{m^2c^2}\right)^{\frac{1}{2}} \\ &= \left(1 - \hat{\beta}_\perp^2 - \hat{\beta}_z^2\right)^{-\frac{1}{2}} \ . \end{aligned} \tag{4}$$

We further introduce the effective perpendicular and parallel temperatures defined by

$$\begin{aligned} T_\perp &= \int d^3p \, \frac{p_\perp^2}{2\gamma m} F(p_\perp^2, p_z) \ , \\ \frac{1}{2} T_{||} &= \int d^3p \, \frac{p_z^2}{2\gamma m} F(p_\perp^2 \ p_z) \ . \end{aligned} \tag{5}$$

Substituting Eq.(3) into Eq.(5) and carrying out the required integrations over $p_\perp$ and p_z give

$$\begin{aligned} T_\perp &= \frac{1}{2} \hat{\gamma}mc^2\hat{\beta}_\perp^2 G(\hat{\beta}_z) \ , \\ T_{||} &= \frac{1}{2} \hat{\gamma}mc^2\left[1 - G(\hat{\beta}_z) + \hat{\beta}_z^2 G(\hat{\beta}_z)\right] \ , \end{aligned} \tag{6}$$

where $G(\hat{\beta}_z)$ is defined by

$$G(\hat{\beta}_z) = \frac{1}{2\hat{\beta}_z} \ln\left(\frac{1 + \hat{\beta}_z}{1 - \hat{\beta}_z}\right) . \tag{7}$$

From Eq.(7), we note that $G(\hat{\beta}_z)$ is a slowly increasing function of $\hat{\beta}_z$ with $G(\hat{\beta}_z) = 1 + \hat{\beta}_z^2/3 \cdots$ for $\hat{\beta}_z^2 \ll 1$. Moreover, in the limit of a nonrelativistic plasma with $\hat{\beta}_z^2 << 1$ and $\hat{\beta}_\perp^2 << 1$, Eq.(7) reduces to the expected results, $T_\perp \to (1/2)mc^2\hat{\beta}_\perp^2$ and $T_{||} \to (1/3)mc^2\hat{\beta}_z^2$. Depending on the relative values of $\hat{\beta}_\perp$ and $\hat{\beta}_z$, it is clear that the choice of distribution function in Eq.(3) can cover a wide range of energy anisotropy.

For the choice of distribution function $F(p_\perp^2,p_z)$ in Eq.(3), the $p_\perp$- and p_z- integrations required in Eq.(2) can be carried out in closed analytical form. Some straightforward algebra that makes use of Eqs.(2), (3), (4) and (7) gives the dispersion relation

$$0 = D_T(k_z,\omega) = 1 - \frac{c^2k_z^2}{\omega^2} - \frac{\omega_p^2/\hat{\gamma}}{\omega^2}\left[G(\hat{\beta}_z) + \frac{1}{2}\frac{\hat{\beta}_\perp^2}{(1-\hat{\beta}_z^2)}\left(\frac{c^2k_z^2-\omega^2}{\omega^2-c^2k_z^2\hat{\beta}_z^2}\right)\right] \quad (8)$$

Equation (8) is readily extended to the case of a multicomponent plasma by making the replacements $(\omega_p^2/\hat{\gamma})\cdots \to \sum_j(\omega_{pj}^2/\hat{\gamma}_j)\cdots$, $\hat{\beta}_z \to \hat{\beta}_{zj}$, $\hat{\beta}_\perp \to \hat{\beta}_{\perp j}$, etc. For a single active (electron) component, Eq.(8) can be expressed in the equivalent form

$$0 = \omega^4 - \omega^2\left\{c^2k_z^2(1+\hat{\beta}_z^2) + \frac{\omega_p^2}{\hat{\gamma}}\frac{\hat{\beta}_\perp^2}{2\hat{\beta}_z^2} - \frac{\omega_p^2}{\hat{\gamma}}\left[\frac{\hat{\beta}_\perp^2}{2\hat{\beta}_z^2(1-\hat{\beta}_z^2)} - G(\hat{\beta}_z)\right]\right\} + c^2k_z^2\hat{\beta}_z^2\left\{c^2k_z^2 - \frac{\omega_p^2}{\hat{\gamma}}\left[\frac{\hat{\beta}_\perp^2}{2\hat{\beta}_z^2(1-\hat{\beta}_z^2)} - G(\hat{\beta}_z)\right]\right\}, \quad (9)$$

which is a quadratic equation for ω^2. In Eq.(9), $\omega_p^2 = 4\pi\hat{n}_e e^2/m$ is the nonrelativistic electron plasma frequency-squared, and $G(\hat{\beta}_z)$ is defined in Eq.(7).

The dispersion relation (9) can be solved exactly for the complex oscillation frequency ω. In this regard, a careful examination of Eq.(9) shows that there are two classes of solutions for ω^2, namely, a fast-wave branch corresponding to stable oscillations with $\mathrm{Im}\omega = 0$ and $(\mathrm{Re}\omega)^2 > c^2k_z^2$, and a slow-wave branch which may or may not exhibit instability, depending on the degree of energy anisotropy. It is readily shown that the necessary and sufficient condition for the slow-wave branch to exhibit instability ($\mathrm{Im}\omega > 0$) is given by

$$\frac{\hat{\beta}_\perp^2}{2\hat{\beta}_z^2} > \left(1 - \hat{\beta}_z^2\right)G\left(\hat{\beta}_z\right) . \tag{10}$$

Moreover, when Eq.(10) is satisfied, the corresponding range of k_z^2 corresponding to instability is given by

$$0 < k_z^2 < k_0^2 \equiv \frac{\omega_p^2}{\hat{\gamma}c^2}\left[\frac{\hat{\beta}_\perp^2}{2\hat{\beta}_z^2(1 - \hat{\beta}_z^2)} - G(\hat{\beta}_z)\right] . \tag{11}$$

When Eq.(10) is satisfied, and k_z^2 is in the range specified by Eq.(11), the real oscillation frequency of the slow-wave branch satisfies $\mathrm{Re}\omega = 0$ and the growth rate of the unstable mode is given by

$$\mathrm{Im}\omega = \frac{1}{\sqrt{2}}\left(\left\{\left[c^2k_z^2\hat{\beta}_z^2 + \frac{\omega_p^2}{\hat{\gamma}}\frac{\hat{\beta}_\perp^2}{2\hat{\beta}_z^2} - c^2(k_0^2 - k_z^2)\right]^2 + 4c^4k_z^2\hat{\beta}_z^2(k_0^2 - k_z^2)\right\}^{\frac{1}{2}} - \left[c^2k_z^2\hat{\beta}_z^2 + \frac{\omega_p^2}{\hat{\gamma}}\frac{\hat{\beta}_\perp^2}{2\hat{\beta}_z^2} - c^2(k_0^2 - k_z^2)\right]\right)^{\frac{1}{2}} . \tag{12}$$

Note from Eq.(12) that $\mathrm{Im}\omega = 0$ for $k_z = 0$ and $k_z^2 = k_0^2$, and that $\mathrm{Im}\omega$ passes through a maximum for some value of k_z^2 intermediate between 0 and k_0^2.

In the nonrelativistic limit with $\hat{\beta}_\perp^2$, $\hat{\beta}_z^2 \ll 1$, the necessary and sufficient condition for instability in Eq.(10) becomes $\hat{\beta}_\perp^2/2\hat{\beta}_z^2 > 1$, and the range of instability is given by $0 < k_z^2 < k_0^2 \equiv (\omega_p^2/c^2)(\hat{\beta}_\perp^2/2\hat{\beta}_z^2 - 1)$. In the relativistic regime, however, the instability criterion in Eq.(10) is more complicated, which is illustrated in Fig. 1. In Fig. 1, the region of $(\hat{\beta}_\perp^2, 2\hat{\beta}_z^2)$ parameter space corresonding to instability is above the contour connecting the origin to $(\hat{\beta}_\perp^2, 2\hat{\beta}_z^2) = (0.580, 0.840)$, which corresponds to $\hat{\gamma} = \infty$. While the detailed form of the instability criterion in Eq.(10) differs from the nonrelativistic case, it is evident from Fig. 1 that the condition for instability in the relativistic regime is qualitatively the same, i.e., the Weibel instability exists for sufficiently large values of $\hat{\beta}_\perp^2/2\hat{\beta}_z^2$. Put another way, for the choice of distribution function in Eq.(3), the Weibel instability in a relativistic anisotropic plasma can be completely stabilized by increasing the thermal anisotropy $2\hat{\beta}_z^2/\hat{\beta}_\perp^2$ to sufficiently large values.

As a numerical example, shown in Fig. 2 are plots of normalized real frequency $\mathrm{Re}\omega/[\omega_p/\hat{\gamma}^{1/2}]$ [Fig. 2(a)] and growth rate $\mathrm{Im}\omega/[\omega_p/\hat{\gamma}^{1/2}]$ [Fig. 2(b)] versus normalized wavenumber $ck_z/[\omega_p/\hat{\gamma}^{1/2}]$ for $\hat{\gamma} = 9$ and several values of energy anisotropy $\hat{\beta}_\perp^2/2\hat{\beta}_z^2$. The real oscillation frequencies for both the fast-wave and slow-wave branches are presented in Fig. 2(a). Moreover, for $\hat{\gamma} = 9$, the slow-wave branch becomes completely stable ($\mathrm{Im}\omega = 0$ and $k_0^2 = 0$) for $\hat{\beta}_\perp^2/2\hat{\beta}_z^2 = 0.697$. It is evident from Fig. 2(b) that the strongest instability occurs for the largest energy anisotropy, i.e., $\hat{\beta}_z^2 = 0$ and $\hat{\beta}_\perp^2 = 80/81$ (for $\hat{\gamma} = 9$). Moreover, depending on the value of $\hat{\beta}_\perp^2/2\hat{\beta}_z^2$, the maximum growth rate in Fig. 2(b) can be a substantial fraction of $\omega_p/\hat{\gamma}^{1/2}$.

C. Thermal Equilibrium Distribution in Parallel Momentum

As a second example, we consider the case where the electron distribution function is specified by

$$F(p_\perp^2,p_z) = \frac{1}{2\pi p_\perp}\,\delta(p_\perp - \hat{p}_\perp)\,\frac{\exp(-\gamma mc^2/T_{||})}{2\hat{\gamma}_\perp mcK_1(\hat{\gamma}_\perp mc^2/T_{||})}\;. \tag{13}$$

Here, the ions are treated as a fixed background ($m_i \to \infty$), and a single component of relativistic, anisotropic electrons is assumed. In Eq.(13), $K_n(x)$ is the modified Bessel function of the second kind of order n; $\gamma = (1 + p_\perp^2/m^2c^2 + p_z^2/m^2c^2)^{1/2}$ is the relativistic mass factor; the constant $\hat{\gamma}_\perp$ is defined by $\hat{\gamma}_\perp = (1 + \hat{p}_\perp^2/m^2c^2)^{1/2}$; and $T_{||}$ is a positive constant. Note that the exponential factor in Eq.(13) corresponds to a thermal equilibrium distribution in parallel momentum p_z. Some straightforward algebra that makes use of Eqs.(5) and (13) shows that $T_{||}$ can be identified with the parallel temperature $\int d^3p(p_z^2/\gamma m) \times F(p_\perp^2,p_z)$, and that the effective perpendicular temperature $T_\perp = \int d^3p(p_\perp^2/2\gamma m)F(p_\perp^2,p_z)$ is given by

$$T_\perp = \frac{1}{2}\hat{\gamma}_\perp mc^2\left(\frac{\hat{p}_\perp}{\hat{\gamma}_\perp mc}\right)^2 \frac{K_0(\hat{\gamma}_\perp mc^2/T_{||})}{K_1(\hat{\gamma}_\perp mc^2/T_{||})} . \tag{14}$$

Substituting Eq.(13) into Eq.(2) and carrying out the integrations over $p_\perp$ and p_z give the dispersion relation

$$\begin{aligned} 0 = D_T(k_z,\omega) = 1 - \frac{c^2k_z^2}{\omega^2} - \frac{(\omega_p^2/\hat{\gamma}_\perp)}{\omega^2}\Bigg\{&\frac{K_0(\hat{\gamma}_\perp mc^2/T_{||})}{K_1(\hat{\gamma}_\perp mc^2/T_{||})} \\ &+ \frac{T_\perp}{\hat{\gamma}_\perp mc^2}\frac{(\omega^2 - c^2k_z^2)}{K_0(\hat{\gamma}_\perp mc^2/T_{||})}\int_0^\infty d\tau\tau K_0(\zeta)\Bigg\} . \end{aligned} \tag{15}$$

In Eq.(15), the complex argument of $K_0(\zeta)$ is defined by

$$\zeta^2 = \left(\frac{\hat{\gamma}_\perp mc^2}{T_{||}} - i\omega\tau\right)^2 + c^2k_z^2\tau^2 , \tag{16}$$

where $\mathrm{Im}\omega > 0$ corresponds to instability, and $T_\perp$ is defined in Eq.(14). The τ-integral in Eq.(15) must generally be evaluated numerically,[5,6] or in the context of asymptotic expansions[7] for large or small values of $|\zeta|$. Unlike Eq.(8) [or Eq.(9)], the dispersion relation (15) generally incorporates the effects of collisionless dissipation (Landau damping) by the p_z-distribution in Eq.(13). For the slow-wave branch, it can be shown from Eq.(15) that the necessary and sufficient condition for instability is given by

$$\frac{T_\perp}{T_{||}} > \left[\frac{K_0(\hat{\gamma}_\perp mc^2/T_{||})}{K_1(\hat{\gamma}_\perp mc^2/T_{||})}\right]^2 , \tag{17}$$

where $T_\perp$ is defined in Eq.(14). Moreover, when Eq.(17) is satisfied, it is found that $\mathrm{Re}\omega = 0$ (for the slow-wave branch) over the range of unstable wavenumbers specified by

$$0 < k_z^2 < k_0^2 \equiv \frac{\omega_p^2}{\hat{\gamma}_\perp c^2}\left[\frac{T_\perp}{T_{\parallel}}\,\frac{K_1(\hat{\gamma}_\perp mc^2/T_{\parallel})}{K_0(\hat{\gamma}_\perp mc^2/T_{\parallel})} - \frac{K_0(\hat{\gamma}_\perp mc^2/T_{\parallel})}{K_1(\hat{\gamma}_\perp mc^2/T_{\parallel})}\right] . \tag{18}$$

Note from Eq.(18) that the marginal stability point k_0^2 (where $\mathrm{Im}\omega = 0 = \mathrm{Re}\omega$) can be calculated in closed analytical form. This follows from the identity

$$\lim_{\mathrm{Im}\omega \to 0_+}\left[c^2k_z^2\int_0^\infty d\tau\tau K_0(\zeta)\right]_{\mathrm{Re}\omega = 0} = \frac{\hat{\gamma}_\perp mc^2}{T_{\parallel}}\,K_1(\hat{\gamma}_\perp mc^2/T_{\parallel}) . \tag{19}$$

Finally, shown in Fig. 3 is a plot of the stability boundary in the parameter space $(T_\perp/\hat{\gamma}_\perp mc^2,\ T_{\parallel}/\hat{\gamma}_\perp mc^2)$ calculated numerically from Eq.(17). The region above the curve in Fig. 3 corresponds to instability, which requires sufficiently large thermal anisotropy $T_\perp/T_{\parallel}$.

ACKNOWLEDGMENTS

This research was supported in part by the Office of Naval Research and in part by the Department of Energy.

REFERENCES

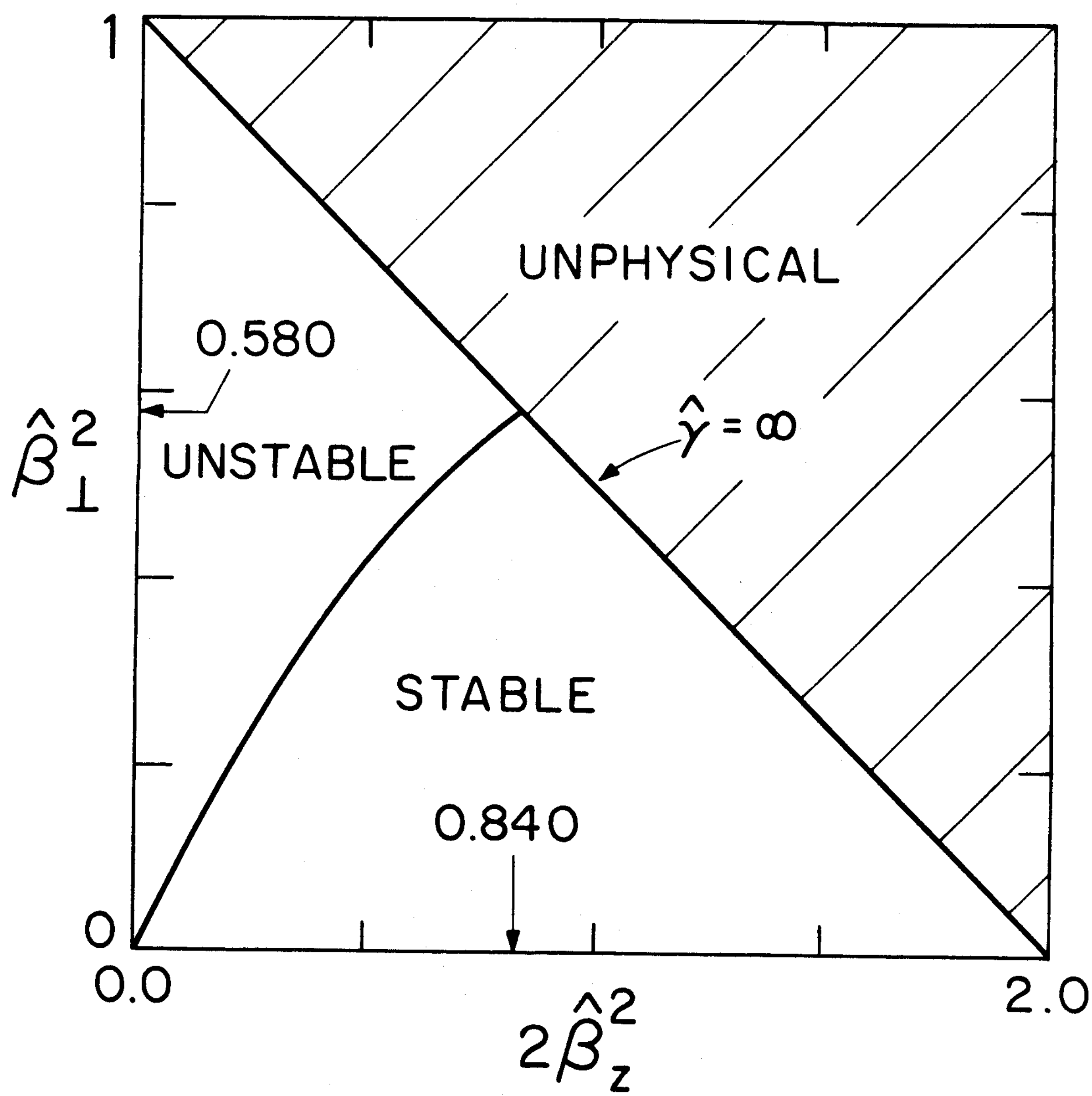
1. E.S. Weibel, Phys. Rev. Lett. 2, 83 (1959).
2. R.C. Davidson, Handbook of Plasma Physics, Volume 1 (A.A. Galeev and R.N. Sudan, eds., North-Holland Publishing Co., New York, 1983) pp. 521-585; cf., pp. 552-573, and references therein.
3. C.S. Wu and L.C. Lee, Astrophys. J. 230, 621 (1979).
4. N.T. Gladd, Phys. Fluids 26, 974 (1983), and references therein.
5. K.T. Tsang, Phys. Fluids 27, 1659 (1984).
6. P.L. Pritchett, Phys. Fluids 27, 2393 (1984).
7. K. Imre, Phys. Fluids 5, 459 (1961).

FIGURE CAPTIONS

Fig. 1. Region of $(\hat{\beta}_\perp^2, 2\hat{\beta}_z^2)$ parameter space corresponding to instability [Eq.(10)].

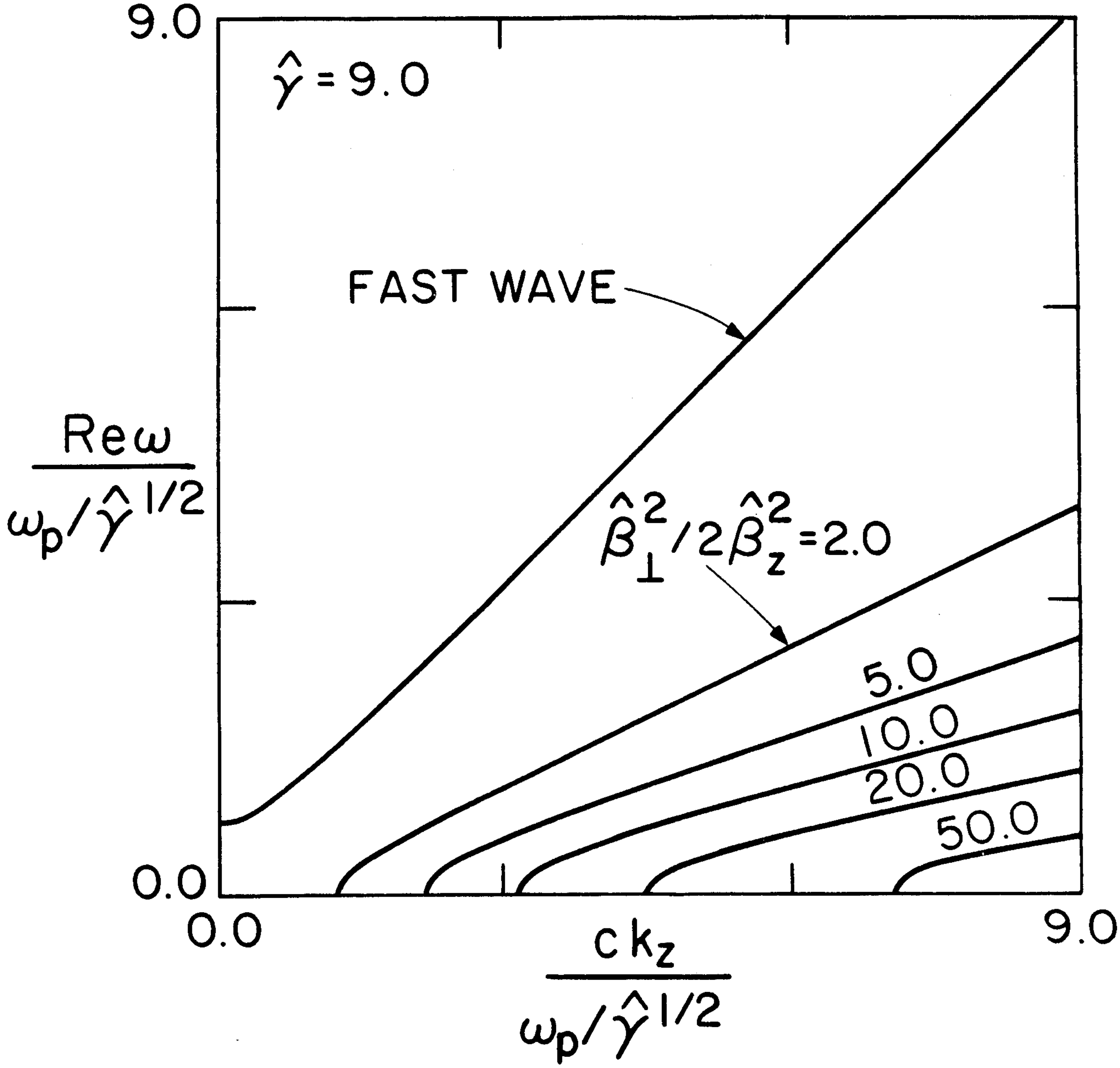
Fig. 2 Plots of (a) normalized real frequency $\mathrm{Re}\omega/[\omega_p/\hat{\gamma}^{\frac{1}{2}}]$ and (b) normalized growth rate $\mathrm{Im}\omega/[\omega_p/\hat{\gamma}^{\frac{1}{2}}]$ versus $ck_z/[\omega_p/\hat{\gamma}^{\frac{1}{2}}]$, for $\hat{\gamma} = 9$ and several values of $\hat{\beta}_\perp^2/2\hat{\beta}_z^2$ [Eq.(9)].

Fig. 3. Regions of $(T_\perp/\hat{\gamma}_\perp mc^2, T_{||}/\hat{\gamma}_\perp mc^2)$ parameter space corresponding to instability [Eq.(17)].



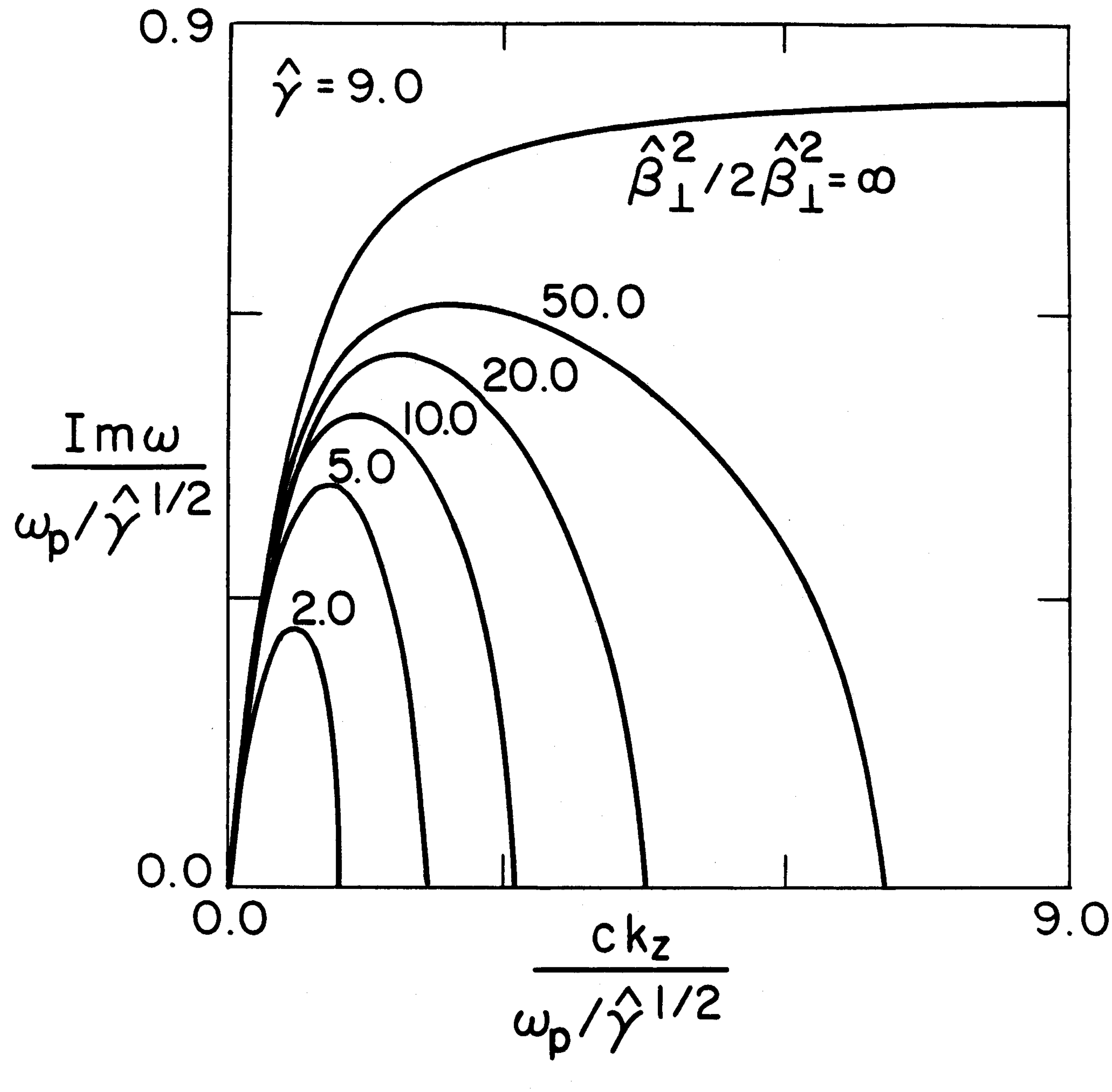
1
0
0.0
2.0
$\hat{\beta}_{\perp}^{2}$
$2\hat{\beta}_{z}^{2}$
UNPHYSICAL
0.580
UNSTABLE
$\hat{\gamma} = \infty$
STABLE
0.840

Fig. 1



9.0
0.0
$\hat{\gamma} = 9.0$
FAST WAVE
$\frac{Re\omega}{\omega_p/\hat{\gamma}^{1/2}}$
$\hat{\beta}_\perp^2/2\hat{\beta}_z^2 = 2.0$
5.0
10.0
20.0
50.0
0.0
9.0
$\frac{ck_z}{\omega_p/\hat{\gamma}^{1/2}}$

Fig. 2(a)



0.9
0.0
0.0
9.0
$\hat{\gamma} = 9.0$
$\hat{\beta}_\perp^2/2\hat{\beta}_\perp^2 = \infty$
50.0
20.0
10.0
5.0
2.0
$\frac{\mathrm{Im}\,\omega}{\omega_p/\hat{\gamma}^{1/2}}$
$\frac{ck_z}{\omega_p/\hat{\gamma}^{1/2}}$

Fig. 2(b)

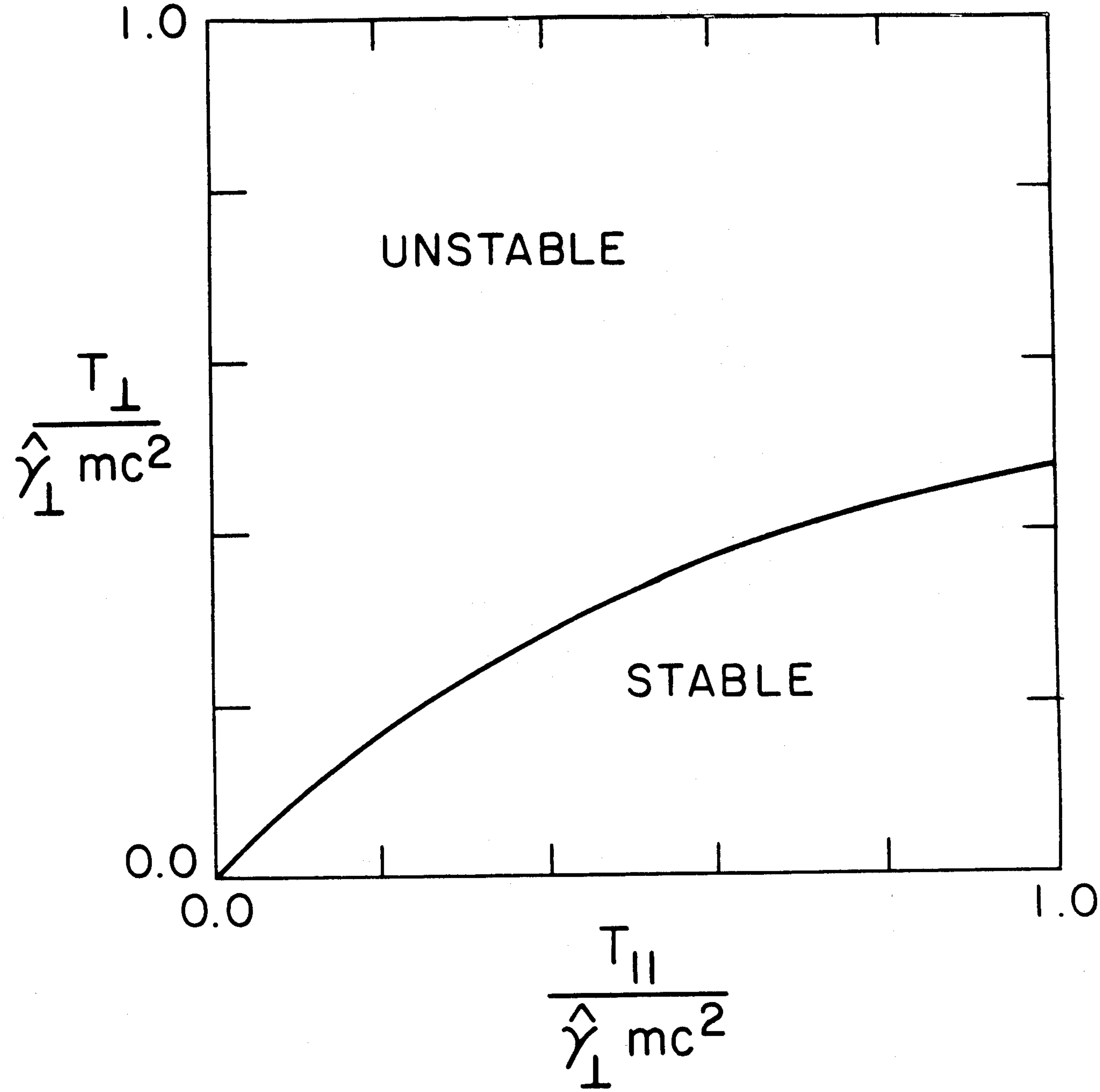

1.0
UNSTABLE
$\frac{T_\perp}{\hat{\gamma}_\perp mc^2}$
STABLE
0.0
0.0
1.0
$\frac{T_{\parallel}}{\hat{\gamma}_\perp mc^2}$

Fig. 3