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Variational Quadratic Form for Low Frequency Electromagnetic Perturbations: (II) Application to Tandem Mirrors

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Abstract

In this paper the low frequency quadratic form governing the linear response of electromagnetic trapped particle modes is applied to a tandem mirror geometry and several problems are treated. It is shown that the long wavelength MHD wall stabilization mechanism persists even when the line bending energy is suppressed by allowing an electrostatic response. In another problem, the amount of charge uncovering needed to suppress electrostatic trapped particle modes is determined as the beta of the plasma rises to its critical MHD beta value, β_{cr}, and the required charge uncovering is shown to increase substantially as beta approaches β_{cr}. A third problem treats trapped particle instabilities driven by rotation and field line curvature in diffuse and steep boundary models. The steep boundary model offers the possibility of an enhanced "robust" FLR stabilization due to an amplification of the finite Larmor radius magnetic compressional term when steep pressure gradients are present. Otherwise, one finds relatively small regions of stability in parameter space when electric field drifts are comparable to diamagnetic drifts. Detailed

stability plots are presented for parameters applicable to the Tara experiment.

(1) Introduction:

In this work we apply the formalism developed in reference (1) to describe the low frequency linear response of a tandem mirror. We shall use an equilibrium model shown in figure (1) which is characterized by a three cell system. The relatively long solenoidal central cell of length L_c confines plasma having a uniform azimuthal rotational frequency. This region has an unfavorable field line curvature. At each end of the cell region are magnetic and electrostatic plugs which are characterized by good curvature. These plugs will be referred to as the magnetohydrodynamic (MHD) anchors. This region can in practice be quite complex being made up of quadrupole coil sets, choke coils and coils for thermal barriers. However, for our theoretical model, we will avoid these complexities, and consider the anchor regions as single cell mirrors that are nearly square well in shape as shown in figure (1). Besides having a favorable MHD curvature, these cells have a positive potential with respect to the central cell, and the plasma has a mean rotational frequency that can be different than the plasma rotational frequency in the central cell.

We analyze the linear stability of this system with respect to trapped particle modes taking into account the complete electromagnetic response of the system for non-eikonal perturbations assuming the bounce frequency of the plasma is arbitrary and that the plasma is collisionless. This analysis generalizes the work of Berk, et.al.[2] in that non-eikonal modes are treated and the bounce frequency is not assumed to be high. Kesner and Lane[3] developed a non-eikonal treatment of this problem under the assumptions that the perturbations were electrostatic and that the wave frequency was small compared to the bounce frequency. Liu and Horton[4] have also developed a non-eikonal analysis for electrostatic trapped particle modes for a plasma with a Gaussian radial pressure profile. In their analysis, electrons are treated in the high bounce frequency limit and the ions in the low bounce frequency limit. They also have studied the role of conducting walls on rotational modes. Byers and Cohen[5] recently pointed out a new axial rotational shear driven mode for a very special geometry. In the formalism presented here, this mode is described in a fairly general geometry.

In this work we analyze tandem mirror stability in a non-eikonal limit. We consider two types of radial pressure profiles: a flat pressure profile with a steep, but finite pressure gradient at the edge and an isothermal Gaussian density profile. We first of all consider whether the MHD wall stabilization condition found for displacement like modes [6,7] in regions of unfavorable curvature, which require line bending energy for stabilization, can be destabilized by allowing for excitations that are purely electrostatic. It is this type of excitation that produces trapped particle instabilities in the eikonal limit in systems that are MHD stable. We show that these "electrostatic" modes are in fact of larger magnetic energy than the MHD displacement-like modes in systems with somewhat steep pressure profiles. Thus, for long perpendicular wavelengths, the basic wall stabilization principle appears not to be altered by considering the electrostatic type oscillations that produce trapped particle instabilities at short wavelengths. Although we have not proved that this is true for a general diffuse pressure profile, we conjecture that it is true.

We then consider a system that has large MHD stabilization terms in the anchor but with a low ideal MHD critical beta such that the eigenfunction at marginal stability is nearly zero in the anchor region and nearly constant in the central cell. We then show how bending energy transforms the "electrostatic mode," to an ideal MHD mode, as the beta varies from very low values to values comparable to the critical beta value for MHD instability. This analysis shows that the marginal stability condition can be reduced significantly from the one obtained by combining the ideal MHD criterion with the zero beta charge uncovering stabilization[2] criterion of trapped particle modes.

Finally we consider the problem when equilibrium plasma rotation and finite Larmor radius effects compete with the MHD drive in a tandem mirror with strongly stabilized anchor regions so that the excitations are only in the central cell region when the plasma beta is well below the critical MHD beta ballooning limit. We then analyze stability with the above three effects. We point out that for the steep pressure profile model, a robust stabilization term can arise from the finite Larmor radius term at finite beta. This term is only of minor consequence if the profile is diffuse. We present detailed stability boundaries when this finite beta FLR term is unimportant. We emphasize parameters associated with the Tara experiment, which is designed to form an equilibrium for which the passing particles are all electrons and thereby produce a negative charge uncovering response. We show that if one neglects the resonant dissipation, stability is possible, but the bands of stability are rather narrow.

We will not consider the effect of resonant particle dissipation in this paper. This effect is considered elsewhere [8,9] and it generally leads to a further reduction in the instability threshold but at a much reduced growth rate.

In section II we present the basic equations that will be analyzed in this paper. The equations were derived in Ref. (1). In section III we analyze effects of the line bending on the trapped particle modes. In section IV we consider trapped particle modes driven by field line curvature and rotational drives. In section V we summarize our results.

II. Variational Quadratic Form

We begin with the variational quadratic form derived in reference (1).

$$W = \pi \int \frac{d\alpha ds}{B} \left\{ \frac{\sigma}{4\pi} \frac{\ell^2}{r^2} \left(\frac{\partial \chi}{\partial s} \right)^2 + \frac{B^2 \sigma r^2}{4\pi} \left[\frac{\partial}{\partial s} \left(\frac{\partial \chi}{\partial \alpha} + \frac{Q_L}{B} - \frac{\phi}{B} \frac{\partial B}{\partial \alpha} \right) \right]^2 \right.$$

$$+ \frac{\tau \ell^2}{4\pi} \left(Q_L - \frac{\sigma\kappa}{\tau} \frac{\phi}{r} \right)^2 - \frac{\ell^2 \kappa \phi^2}{Br} \left(\frac{\sigma}{\tau} \frac{\partial P_\perp}{\partial \alpha} + \frac{\sigma}{\tau} \frac{\partial P_\perp}{\partial \Phi} \frac{\partial \Phi}{\partial \alpha} + \frac{\partial P_{||}}{\partial \alpha} \right)$$

$$- \sum_j n_j m_j \omega^2 \left[r^2 \left(B \frac{\partial}{\partial \alpha} \left(\frac{\phi}{B} \right) + \frac{Q_L}{B} \right)^2 + \frac{\ell^2 \phi^2}{B^2 r^2} \right]$$

$$\left. - \sum_j n_j m_j A_j \left[r^4 B^2 \left(\frac{\partial}{\partial \alpha} \left(\frac{\phi}{rB} \right) \right)^2 + (\ell^2 - 1) \frac{\phi^2}{B^2 r^2} \right] \right\} + W_{kin}.$$

$$W_{kin} = i \sum_j \frac{2\pi^2}{(m_j c)^2} \int d\alpha d\epsilon d\mu \left(\omega \frac{\partial F_j}{\partial \epsilon} + \frac{\ell c}{q_j} \frac{\partial F_j}{\partial \alpha} \right)$$

$$\int_{-t_b}^{t_b} dt \int_{-\infty}^{+\infty} dt' \exp[i\Omega(t,t') \, sg \, (t-t')]$$

$$\bullet\left[-\, i\,sg\,(t-t')\, q_j \frac{d}{dt'}\left(\phi(t') - \chi(t')\right) + \ell c\mu Q_L(t') + q_j\phi(t')\,\omega_\kappa(t')\right]$$

$$\bullet\left[i\,sg\,(t-t')\, q_j \frac{d}{dt}\left(\phi(t) - \chi(t)\right) + \ell c\mu Q_L(t) + q_j\phi(t)\omega_\kappa(t)\right] . \qquad (1)$$

where

$$A_j \equiv \omega_E^2 - 2\omega_E\omega - (\omega-\omega_E)\ \frac{\ell c}{n_j B q_j}\ \frac{D}{D\alpha}\ (P_{\perp j}B) - \frac{\ell^2 c^2}{2 n_j q_j^2}\ \frac{1}{B}\ \frac{\partial B}{\partial \alpha}\ \frac{DL_j}{D\alpha} .$$

We list below the definitions of the various quantities appearing in Eq. (1).

j refers to species

$m_j \equiv$ mass $\quad q_j \equiv$ charge $\quad c =$ velocity of light

$\underset{\sim}{B} = \nabla\alpha \times \nabla\theta = B\ \underset{\sim}{b} =$ equilibrium magnetic field.

$B = |\underset{\sim}{B}|$

$\epsilon = m_j v^2/2 + q_j\Phi$

$\mu = m_j\ v_\perp{}^2/(2B)$

$v_{||}(\epsilon,\ \mu,\ s) = \pm\ (2/m_j)^{1/2}\ [\epsilon - \mu B - q_j\ \Phi]^{1/2}$

ℓ = azimuthal wave number

$\Phi\ (\alpha, s)$ = equilibrium electrostatic potential

$$\phi = \underset{\sim}{\xi} \bullet \nabla \alpha$$

$$Q_L = B_1 + \underset{\sim}{\xi} \bullet \nabla B$$

$$\underset{\sim}{B}_1 = \nabla \times \underset{\sim}{A}_1 = \underset{\sim}{b}\ B_1 + \nabla \times (\underset{\sim}{b}\ A_{||})$$

$$A_{||} = \frac{-i}{\ell} \frac{\partial \chi}{\partial s}$$

$$\underset{\sim}{\kappa} = (\underset{\sim}{b} \bullet \underset{\sim}{\nabla})\ \underset{\sim}{b} = \kappa \frac{\underset{\sim}{\nabla \alpha}}{|\nabla \alpha|}$$

$$r^2/2 = \int_0^{\alpha} \frac{d\alpha'}{B(\alpha', s)} + 0\ (\kappa r)$$

$$\tau = 1 + \frac{4\pi}{B} \frac{\partial P_{\perp}}{\partial B}$$

$$\sigma = 1 + \frac{4\pi \quad (P_{\perp} - P_{||})}{B^2}$$

$$\frac{DQ}{D\alpha}(\alpha,\ B,\ \Phi) = \frac{\partial Q}{\partial \alpha} + \frac{\partial B}{\partial \alpha}\frac{\partial Q}{\partial B} + \frac{\partial \Phi}{\partial \alpha}\frac{\partial Q}{\partial \Phi}$$

$$\omega_E = \ell c \frac{\partial \Phi}{\partial \alpha}$$

$$\omega_\kappa = \frac{v_{||}^2 \ \ell \nabla\theta \bullet \underset{\sim}{b} \times \underset{\sim}{\kappa}}{\omega_{cj}} = \frac{m_j v_{||}^2 \ \ell c \ \kappa}{q_j \ Br}$$

$$\omega_{\nabla B} \equiv \frac{\mu \ell \nabla\theta \bullet \underset{\sim}{b} \times \nabla B}{m_j \omega_{cj}} = \frac{\mu \ell c}{q_j} \frac{\partial B}{\partial \alpha}$$

$$\omega_{cj} = q_j B / m_j c$$

$$\Omega(t,t') = \int_{t'}^{t} dt'' \ (\omega - \omega_E - \omega_{\nabla B} - \omega_\kappa) dt''$$

$$sg\ (x) = \begin{cases} 1, & x > 0 \\ 0, & x < 0 \end{cases}$$

$$t\,(\epsilon,\mu,q_j,s) = \int_0^s \frac{ds}{|v_{||}|}, \quad t_b = \int_0^{s_b} \frac{ds}{|v_{||}|}, \quad v_{||}(s_b) = 0$$

$$n_j = \int d^3v\, F_j$$

$$\begin{Bmatrix} P_\perp \\ P_{||} \end{Bmatrix} = \sum_j \begin{Bmatrix} P_{\perp j} \\ P_{||j} \end{Bmatrix} = \sum_j m_j \int d^3v \begin{Bmatrix} v_\perp^2/2 \\ v_{||}^2 \end{Bmatrix} F_j$$

$$L_j = m_j^2 \int d^3v \, \frac{v_\perp^4}{4} F_j$$

The quantity $\underset{\sim}{\xi}$ refers to the conventional $\underset{\sim}{\tilde{E}}\times\underset{\sim}{B}$ displacement and all perturbed quantities are assumed to vary as exp $(i\ell\theta)$ where $\underset{\sim}{\tilde{E}}$ is the perturbed electric field. The equilibrium is taken here to be axisymmetric and the curvature small. Thus terms of order κr have been dropped in the calculation of the finite Larmor radius (FLR) terms. Equation (1) is strictly valid only for an axisymmetric tandem mirror as $\underset{\sim}{\kappa}$ is taken only in the $\nabla\alpha$ direction. However, the formalism is still valid for a situation in which strongly stabilizing non-axisymmetric minimum-B anchors force the mode to vanish in the anchors. In this case if the passing fraction is small and $\omega \gg \omega_\kappa$, $\omega_{\nabla B}$, the non-axisymmetry does not enter the problem in the order to which we work.

The further assumption has been made that there are no "hot" species whose drift frequency exceeds the drift frequency or MHD growth rates of the background core species. In the absence of such a hot species the compressional magnetic term Q_L is generally forced to be of order κr compared to ϕ/r^2 and the most unstable perturbations are incompressible. Terms proportional to Q_L in the calculation of the FLR contributions have been neglected. We note that the MHD limit is recovered when $\partial\phi/\partial s = \partial\chi/\partial s$, while the electrostatic limit is recovered when $\partial\chi/\partial s = 0$. If we treat $q_j\phi(t)\ \omega_\kappa + \ell c\mu\ Q_L(t) << q_j \frac{d}{dt}(\phi - \chi)$, so that the curvature terms can be neglected in the kinetic terms, the eigenfunctions will have either even or odd symmetry about the midplane. Specifically for even modes we can rewrite W_{kin} as,

$$W_{kin} = \sum_j \frac{8\pi^2 q_j^{\ 2}}{m_j^2 c^2} \int d\alpha d\epsilon d\mu \left(\omega \frac{\partial F_j}{\partial\epsilon} + \frac{\ell c}{q_j} \frac{\partial F_j}{\partial\alpha}\right)$$

$$\int_0^{t_b} dt \int_0^{t_b} dt' \ \frac{d}{dt} [\phi\ (t) - \chi\ (t)] \frac{d}{dt'} [\phi\ (t') - \chi\ (t')]$$

$$\Bigg\{ \cos\ [\Omega(t_b, t^<)]\ \ \sin\ [\Omega(t_b, t^>)]$$

$$- \cot [\Omega(t_b,0)] \sin [\Omega(t_b,t)] \sin [\Omega(t_b,t')] \Big\} \qquad (2)$$

where $t^{>} = \max(t,t')$, $t^{<} = \min(t,t')$.

We shall consider eigenfunctions ϕ and χ that are nearly constant in the central cell and anchor regions but change rapidly in the transition region from the central cell to anchor by amounts $\Delta\phi$ and $\Delta\chi$ respectively. Then for the evaluation of the kinetic term we have

$$\frac{d}{dt}(\phi - \chi) = v_{||}(\Delta\phi - \Delta\chi)\ \delta(s-s_0)$$

where s_0 is a point in the transition region. We consider the distribution functions F_j in the center cell (which consists of trapped and passing particles) to be Maxwellian, and for simplicity, without temperature gradients, so that

$$F_j = \frac{n_j(\alpha)}{(2\pi T_j/m_j)^{3/2}} \exp(-\epsilon/T) .$$

Then W_{kin} can be written as,

$$W_{kin} = -\sum_j \frac{8\pi^2 q_j^{\,2}(\Delta\phi - \Delta\chi)^2}{m_j^{\,2}c^2T_j} \int d\alpha\, (\omega - \omega_{Ec} - \omega_{*jc}) \int F_j d\epsilon d\mu$$

$$\left\{\cos\left[\Omega\left(t_b,\ t_0\right)\right]\ \sin\left[\Omega\left(t_b,\ t_0\right)\right] - \cot[\Omega(t_b,0)]\ \sin^2[\Omega\left(t_b,\ t_0\right)]\right\}\ . \quad (3)$$

where $t_0 = t(s_0)$, $\omega_{Ec} = \ell c\ \partial\Phi(s = 0,\alpha)/\partial\alpha \equiv$ electric field drift in central cell, $\omega_{*jc} = (\ell c T_j/q_j n_{jc})\ Dn_{jc}/D\alpha$ = diamagnetic drift in the central cell.

In this paper we neglect resonance contributions, and thus the term containing cot $[\Omega(t_b,0)]$ is considered small. This is the case if the anchor length is short compared to the length of the central cell. In this approximation we have,

$$W_{kin} = -2\pi \int d\alpha(\Delta\phi - \Delta\chi)^2 \sum_j \frac{q_j^2}{c^2 T_j} (\omega - \omega_{Ec} - \omega_{*j})\ (\omega - \omega_{Ea}) \int_{anchor} \frac{ds}{B_a} n_{pj} g_j(\omega), \quad (4)$$

where

$$g_j\ (\omega) = \frac{\frac{4\pi}{m_j^2} \int F_j\ \ d\epsilon d\mu\ \cos\ [\Omega(t_b,t_0)]\ \sin\ [\Omega(t_b,t_0)]}{(\ \omega - \omega_{Ea}) \int_{anchor} \frac{ds}{B_a} n_{pj}}\ . \quad (5)$$

B_a is the magnetic field in the anchor region and n_{pj} is the passing particle density in the anchor region. Note that $g_j(\omega) = 1$ if $\Omega\,(t_b, t_0) \ll 1$.

III. Finite Beta Coupling of Electrostatic and Electromagnetic Modes.

We consider in this section the transition from a low beta plasma where $\phi \sim O\,(1)$ and $\partial\chi/\partial s \sim O\,(\epsilon)$ to a plasma whose beta permits an electromagnetic balooning mode in which $\partial\chi/\partial s \sim \partial\phi/\partial s$. For simplicity we assume $\omega > \omega_{\nabla B}$ and we will neglect terms of $O\,(\epsilon^2)$ with $\epsilon = |\kappa r|$. Further, we order $r\partial^2/\partial s^2 \sim \kappa$, and $\omega^2 \simeq \kappa\, V_A{}^2/r \simeq \omega_i^{*2} \simeq \omega_E^2$ where $V_A^2 = B^2/(4\pi n_i m_i)$. In this limit the quadratic form to zeroth order in ϵ only involves Q_L and the constraint $Q_L = 0$ solves the lowest order variational equation. To next order in ϵ we obtain the variational form,

$$W = \pi \int \frac{ds d\alpha}{B} \left\{ \frac{\sigma}{4\pi} \quad \frac{\ell^2}{r^2} \left(\frac{\partial\chi}{\partial s}\right)^2 + \frac{B^2\sigma\, r^2}{4\pi} \left[\frac{\partial}{\partial s} \left(\frac{\partial\chi}{\partial\alpha} - \frac{\phi}{B} \; \frac{\partial B}{\partial\alpha} \right)\right]^2 \right.$$

$$- \frac{\kappa\ell^2\phi^2}{Br} \; \frac{D}{D\alpha} \; (P_\perp + P_\parallel) - n_i m_i \omega^2 \left[r^2 \left(B \frac{\partial}{\partial\alpha} \left(\frac{\phi}{B}\right)\right)^2 + \frac{\ell^2\,\phi^2}{B^2\, r^2} \right]$$

$$- n_i m_i \, A_i \left[r^4 \, B^2 \left(\frac{\partial}{\partial \alpha} \left(\frac{\phi}{rB} \right) \right)^2 + \frac{(\ell^2 - 1) \, \phi^2}{B^2 \, r^2} \right] \Bigg\} \tag{6}$$

$$+ W_{kin} + O\,(\epsilon^2) \, .$$

with W_{kin} given by Eq. (2).

A. Bending Energy in Trapped Particle Mode

In the original analyses of the trapped particle it was shown that if the plasma beta is appreciably below the critical beta for MHD stability, an electrostatic trapped particle mode arises with $\chi = 0$ to allow for a mode to isolate itself in the unfavorable curvature regions without expending bending energy. The quadratic form of Eq. (6) shows that in a non-eikonal analysis bending energy cannot be entirely eliminated. If we choose $\chi = 0$, then there still persists a bending energy term

$$W_{bend} \equiv \int \frac{ds\,d\alpha}{4} \; B \, \sigma \, r^2 \left[\frac{\partial}{\partial s} \left(\frac{\phi}{B} \frac{\partial B}{\partial \alpha} \right) \right]^2 \, .$$

We note that this term contributes positive energy and can be important at low ℓ when compared to the destabilizing curvature drive term

$$W_{curv} = - \pi \ell^2 \int ds \; \frac{d\alpha}{B^2 r} \, \phi^2 \kappa \; \frac{D}{D\alpha} (P_\perp + P_{||}) \, .$$

Even if ϕ is constant we find that at moderate beta, where $\beta > \kappa \Delta$, W_{bend} is given by

$$W_{bend} = \int dsd\alpha\alpha \, \frac{\sigma\phi^2}{8} \left[\frac{\partial}{\partial s} \left(\frac{\partial \beta_\perp}{\partial \alpha} \right)^2 \right] + 0\left(\frac{\kappa\Delta}{\beta_\perp}\right)$$

where

$$\Delta^{-1} = \frac{rB}{P_\perp} \frac{\partial P}{\partial \alpha}, \; \beta_\perp = \frac{8\pi P_\perp}{B^2}, \; \alpha \simeq Br^2/2.$$

For a sharp profile where $\Delta/r_p \ll 1$, (r_p = r at the plasma edge), we observe that W_{bend} is greater than the $\ell = 1$ line bending stabilization terms recently calculated for a radial rigid displacement mode[6,7] by roughly a factor r_p/Δ. Thus, if the beta required for wall stabilization can be achieved for the $\ell = 1$ mode for a plasma with a relatively sharp pressure profile, the $\ell = 1$ "electrostatic" mode will also be stabilized. This argument suggests (though it is not proved) that for an arbitrary radial profile, the lowest energy state of a nearly rigid displacement $\ell = 1$ mode requires the condition $\phi = \chi$.

We also notice that since the additional bending energy is due to the term $\partial(\phi B^{-1} \partial B/\partial\alpha)/\partial s$, there is a limit on how sharply the electrostatic portion of the eigenfunction can vary in a region of finite beta. If we denote $\phi^{-1}\,\partial\phi/\partial s \equiv L_{eig}^{-1}$ and compare W_{bend} to W_{curv} we see that the mode will be stable when

$$\frac{r^2}{L_{eig}^2} > 2\ \frac{\iota^2}{\beta_\perp}\ |\kappa\Delta|\left(1 + \frac{\beta_{||}}{\beta_\perp}\right). \tag{7}$$

Unfortunately, the electrostatic portion of the perturbation varies rapidly in a region where the bulk of the particles reflect. This is typically near the magnetic mirror throat where the pressure is low and where the magnetic field is high. Thus the rapid variation in ϕ appears to occur naturally in a region of low beta and probably an accurate eigenfunction does not produce much field line bending energy.

B. Interaction of Electrostatic and Electromagnetic Ballooning Modes.

We now consider the interaction of MHD modes for which $\phi = \chi$ is normally assumed and electrostatic modes for which $\chi = 0$ is normally assumed. At finite beta these two polarizations interact. To demonstrate this interaction we consider a tandem configuration with the following properties: (1) a strongly stabilizing anchor of relatively high beta produced by magnetically trapped particles, (2) a relatively short destabilizing region in the central cell which could arise from a choke coil and (3) a well pumped transition region between choke coil and anchor that is relatively long compared to the choke coil length. The eigenfunction χ at $\beta = \beta_{cr}$, the MHD marginal stability point, is then small in the anchor to avoid the anchor stabilizing effect, rises through the transition region and is constant through the center cell. The critical beta is then of order $L^2_{choke}/L^2_{transition}$ where $L_{choke}(L_{transition})$ is the scale length of the choke coil (transition region). This type of eigenfunction resembles the shape of the electrostatic eigenfunction. We denote $\chi = \lambda\phi$ and we assume λ is a constant. We determine λ in the eikonal limit where $\nabla_\perp = i\ell/r\hat{\theta} + \kappa_\alpha \nabla\alpha$, with ℓ an integer and k_α constant along a field line and $k_\perp^2/B = \ell^2/2\alpha + 2\alpha k_\alpha^2$, then from the quadratic form given in Eq. (6) with $\omega_E = 0$, one finds,

$$(\omega - \omega_{*i})\ (\omega + \frac{\beta\omega_{*i}}{2}) + [Q\omega^2 - \Delta Q\omega_{*i}\omega]\ (1-\lambda)^2(k_\perp r)^{-2} \tag{8}$$

$$\gamma^2_{MHD}\left(1 - \lambda^2\ \frac{\beta_{cr}}{\beta}\right) = 0$$

where

$$\gamma^2_{MHD} = \frac{\dfrac{B\ell^2}{k_\perp^2} \displaystyle\int_c \frac{ds}{B^2 r}\ \kappa\ \frac{\partial}{\partial\alpha}\ (P_\perp + P_{||})\phi^2}{\displaystyle\int \frac{ds}{B^2}\ n_i m_i \phi^2}$$

$$\lambda^2 \gamma^2_{MHD} \frac{\beta_{cr}}{\beta} = 2 \frac{\int_{tr} ds \frac{a}{4\pi}\left(\frac{\partial\chi}{\partial s}\right)^2}{\int_c \frac{ds}{B^2} n_i m_i \phi^2}$$

with β_{cr} the critical beta for MHD stability and $\beta_\perp = 8\pi P_\perp/B_c^2$. We have used the fact that $k_\perp^2/B$ and Br^2 are constant along a field line. The "c" ("tr") under the integral sign refers to integration over the entire central cell (an end transition region). In the limit where $\Omega(t_b, t_0) << 1$ and $g_j(\omega) = 1$

$$Q = \sum_j 4\alpha\phi_c^2 \int_a \frac{ds}{B} \frac{n_{pj} q_j^2}{c^2 T_j} \Bigg/ \int_c \frac{ds n_i m_i}{B^2} \phi^2$$

with n_{pj} the density of passing particles, ϕ_c is the central cell value of ϕ, and the "a" symbol under the integral sign is for integration in the anchor.

$$\Delta Q\omega_{*i} = \sum_j 4\alpha\phi_c^2 \int_a \frac{ds}{B} \frac{n_{pj} q_j^2 \omega_{*cj}}{c^2 T_j} \Bigg/ \int_c \frac{ds n_i m_i}{B^2} \phi^2$$

with $\omega_{*i} \equiv [\ell c T_i \, Dn_c/D\alpha]/(q_i \, n_c)$, n_c is the central cell density.

We note that λ enters in two ways. First in the field line bending term in which finite λ acts to stabilize the mode and second in the charge separation term, $\omega \, \omega_{*i} \Delta Q \, (1-\lambda)^2$, in which finite λ destabilizes the mode.

If we now extremize with respect to λ, we find

$$\lambda = \frac{\beta\ (Q\omega^2 - \Delta\ Q\omega\ \omega_{*i})}{\beta\ (\omega^2 Q - \Delta Q\ \omega\omega_{*i}) - \beta_{cr}\gamma_{MHD}^2 k_\perp^2 r^2} \tag{9}$$

and the dispersion relation becomes,

$$\left(\Omega^2 - \frac{\Delta Q}{Q}\,\Omega_*\Omega - \frac{\beta_{cr}k_\perp^2 r^2}{\beta Q}\right)\left[(\Omega + \frac{\beta}{2}\,\Omega_*)(\Omega - \Omega_*) + 1\right] - \frac{\beta_{cr}}{\beta}\left(\Omega^2 - \frac{\Delta Q}{Q}\,\Omega\Omega_*\right) = 0, \tag{10}$$

with $\Omega = \omega/\gamma_{MHD}$ $\Omega_* = \omega_{*i}/\ \gamma_{MHD}$. In the analysis below we neglect the $\beta\omega_{*i}$ term. To solve this dispersion relation, we first neglect Ω_* terms. We then find for the unstable root,

$$\Omega^2 = \frac{1}{2}\left[\frac{\beta_{cr}}{\beta}\left(1 + \frac{k_\perp^2\ r^2}{Q}\right) - 1\right]$$

$$- \frac{1}{2}\left\{\frac{4\beta_{cr}k_\perp^2 r^2}{\beta Q} + \left[1 - \frac{\beta_{cr}}{\beta}\left(1 + k_\perp^2 r^2/_Q\right)\right]^2\right\}^{1/2}.$$

For conditions for which the inequality

$$\frac{4\beta k_\perp^2 r^2}{Q\beta_{cr}} \left(1 + \frac{k_\perp^2 r}{Q} - \beta/\beta_{cr}\right)^{-2} < 1 \ .$$

is satisfied, Ω^2 can be written approximately as

$$\Omega^2 \simeq \begin{cases} -\dfrac{1}{1 + Q(1-\beta/\beta_{cr})/k_\perp^2 r^2}, & \text{if } \dfrac{\beta}{\beta_{cr}} < 1 + \dfrac{k_\perp^2 r^2}{Q} \qquad \text{(11a)} \\[2em] -1 + \dfrac{\beta_{cr}}{\beta}\left(1 + \dfrac{k_\perp^2 r^2}{Q}\right), & \text{if } \dfrac{\beta}{\beta_{cr}} > 1 + \dfrac{k_\perp^2 r^2}{Q} \qquad \text{(11b)} \end{cases}$$

Eq. (11) demonstrates instability for β both greater than and less than β_{cr} in the absence of ion FLR. For β/β_{cr} of order one, instability in the absence of ion FLR is also present. We can explicitly evaluate Ω^2 when $\beta/\beta_{cr} = 1 + k_\perp^2 r^2/Q$ whereupon we have that

$$\Omega^2 \simeq -\frac{1}{(1 + Q/k_\perp^2 r^2)^{1/2}} \ . \qquad (12)$$

In summary, in the limit when Ω_* can be neglected, we always have instability and when $\beta/\beta_{cr} < 1$, the growth rate γ (in units of γ_{MHD}) is

$$\gamma = \left[\frac{k_\perp^2 r^2}{k_\perp^2 r^2 + Q\,(1 - \beta/\beta_{cr})} \right]^{1/2} \tag{13}$$

with the β dependence arising from the coupling of bending energy to the electrostatic modes.

To investigate stabilization we consider Ω_* finite and $|\Delta Q/Q| < 1$. We first consider $Q/k_\perp^2 r^2$ large, and if we first assume $\Omega\Omega_*$ and $\Omega^2 < 1$, Eq. (10) becomes,

$$\left(1 - \frac{\beta_{cr}}{\beta}\right) \Omega^2 - \frac{\Delta Q}{Q}\,\Omega_*\left(1 - \frac{\beta_{cr}}{\beta}\right)\Omega - \frac{\beta_{cr} k_\perp^2 r^2}{\beta Q} = 0 \tag{14}$$

The stability condition is then

$$\frac{(\Delta Q)^2}{Q}\,\Omega_*^2 > \frac{4 k_\perp^2 r^2}{1 - \beta/\beta_{cr}} \tag{15}$$

and as $\Omega \simeq \Delta Q\ \Omega_*/Q$, the assumptions $\Omega\Omega_*$ and $\Omega^2 < 1$ are valid if $\Delta Q/Q < 1$. We observe that the charge uncovering term ΔQ, needs to become larger as beta increases and the assumptions leading to Eq. (15) fail as $\beta \rightarrow \beta_{cr}$.

For $\beta = \beta_{cr}$ we return to Eq. (10) and find that if $k_\perp^2 r^2/Q \ll 1$ and $\Delta Q/Q = 0$ we always get instability with

$$\gamma \simeq \left(\frac{k_\perp^2 r^2}{Q\Omega_*}\right)^{1/3} \frac{\sqrt{3}}{2}\ .$$

With $\Delta Q/Q < 1$ but finite and $\beta = \beta_{cr}$, Eq. (8) can be written approximately as

$$\Omega^3\Omega_* - \frac{\Delta Q}{Q}\,\Omega_*^2\Omega^2 + \frac{k_\perp^2 r^2}{Q} = 0.$$

Then the condition for stability at $\beta = \beta_{cr}$ is that

$$\frac{4}{27}\left(\frac{\Delta Q}{Q}\right)^3 \Omega_*^4 > \frac{k_\perp^2 r^2}{Q} \qquad (16)$$

which indicates the most pessimistic limit of Eq. (15) as β approaches β_{cr}. If we assume $Q \ll k_\perp^2 r^2$, Eq. (10) becomes

$$\Omega\,(\Omega-\Omega_*) + 1 = 0 \tag{17}$$

with the usual FLR stability condition for a flute mode in the central cell

$$\Omega_* > 2\ . \tag{18}$$

For $\beta/\beta_{cr} > 1$ (where we find $\Delta\phi = \Delta\chi$), Eq. (18) would appear to be the FLR stability criterion for a flute mode in the central cell for arbitrary $Q/k_\perp^2 r^2$. However, as shown in reference (9), for $\beta/\beta_{cr} > 1$, the mode tends to localize in the bad curvature region of the central cell and the stability condition is actually more pessimistic than Eq. (18).

IV. Non- Eikonal Analysis of Electrostatic Ballooning Modes

We now analyze the quadratic form assuming $\beta << \beta_{cr}$, so that we can consider modes with $\chi = 0$. We shall also assume the anchor length is small compared to the central cell length, and that the passing particles are Maxwellian in the anchor region (a reasonable model for electrons, and a marginal but simplifying model for ions). We consider parameters such that beta is sufficiently low that we can neglect the term proportional to $B^{-1}\partial B/\partial\alpha$ in the bending inertia and FLR terms and that the anchor region is strongly MHD stabilized so that the destabilizing eigenfunction is small there. We also use that $\alpha = Br^2/2$ which follows from the long thin and low beta approximations. The quadratic form given in Eqs. (1-4) then becomes

$$\delta W = \pi \int_c \frac{d\alpha ds}{B} \left[- \frac{\ell^2 \kappa \phi^2}{Br} \frac{D}{D\alpha} (P_\perp + P_{||}) \right.$$

$$- n_i m_i \omega^2 \left[r^2 \left(\frac{\partial\phi}{\partial\alpha}\right)^2 + \frac{\ell^2 \phi^2}{B^2 r^2} \right]$$

$$\left. - n_i m_i A_i \left\{ r^4 \left[\frac{\partial}{\partial\alpha} \left(\frac{\phi}{r}\right)\right]^2 + (\ell^2 - 1) \frac{\phi^2}{B^2 r^2} \right\} \right]$$

$$-2\pi \int_a \frac{d\alpha ds}{B} \phi^2 \sum_j \frac{q_j^2 n_{pj}}{c^2 T_j} (\omega - \omega_{Ec} - \omega_{*j}) (\omega - \omega_{Ea}) g_j(\omega) \qquad (19)$$

where the "c" and "a" under the integral signs refer to integration over the central cell and anchor respectively, and $g_j(\omega)$ is given in Eq. (5).

A. Equation for Sharp Boundary Model

To proceed further, we shall consider two radial profiles. The first is the sharp profile shown in fig. (2). Here the pressure and density profiles are taken as constant to a flux α_p, and then abruptly drop to zero. The equilibrium electric field is assumed to vary linearly with values

$$E = E_c \; r/(2\alpha_p/B_c)^{1/2}$$

in the central cell, and

$$E = E_a \; r/(2\alpha_p/B_c)^{1/2}$$

in the anchor. Hence ω_{Ec} and ω_{Ea} are independent of radius. We can then solve the variational equations of the quadratic form for ℓ-numbers satisfying the condition $\ell\Delta\alpha/\alpha_p < 1$ with $\Delta\alpha^{-1} = P_\perp^{-1}\partial P_\perp/\partial\alpha$.

In the region $\alpha < \alpha_p$, the Euler-Lagrange equation for the radial variation of Eq. (19), is

$$\frac{\partial}{\partial r} r \frac{\partial\phi}{\partial r} - \frac{\ell^2\phi}{r} - \frac{\tilde{Q}r\phi}{r_p^2} = 0 \tag{20}$$

where r refers now specifically to the radius at the mid-plane, $r \equiv (2\alpha/B_c^0)^{1/2}$, $r_p \equiv (2\alpha_p/B_c^0)^{1/2}$, $B_c^0 \equiv B(z = 0)$. The quantity $\tilde{Q}$ contains the effects of passing particles,

$$\tilde{Q} = \sum_j 2 \int_a \frac{ds}{B} \frac{r_p^2 n_{pj} q_j^2 \, g_j(\omega) \; (\omega - \omega_{Ea})/T_j c^2}{(\omega - \omega_{Ec}) \int_c \frac{ds}{B^2 B_c^0} n_i m_i} \tag{21}$$

$$= Q \frac{(\omega - \omega_{Ea})}{(\omega - \omega_{Ec})} .$$

We note that in specific limits that we shall study quantitatively, Q will be independent of ω.

The solution for ϕ in the region $r < r_p$ is then,

$$\phi = \frac{\phi_o I_\ell\left(\tilde{Q}^{1/2} \frac{r}{r_p}\right)}{I_\ell(\tilde{Q}^{1/2})} \tag{22}$$

where I_ℓ (x) is the Bessel function of imaginary argument of index ℓ and ϕ_o is a constant.

In the sharp gradient region the Euler-Lagrange equation becomes,

$$\frac{\partial}{\partial r} \frac{\omega^2}{V_A{}^2} r \frac{\partial}{\partial r} \phi - \frac{\ell^2}{rV_A^2} \omega^2 \phi$$

$$+ \frac{1}{r} \frac{\partial}{\partial r} \frac{A}{V_A^2} r^3 \frac{\partial}{\partial r} (\phi/r) - \frac{(\ell^2 - 1)}{r} \frac{A}{V_A^2} \phi$$

$$- \frac{r}{r_p^2}\left[\frac{\tilde{Q}}{V_A^2} (\omega-\omega_{Ec})^2 - \frac{\omega_{*i}}{V_A^2} \Delta\tilde{Q} (\omega-\omega_{Ec}) + \frac{\ell^2 \gamma^2_{MHD}}{V_A^2}\right] \phi = 0 \tag{23}$$

where

$$\widetilde{\Delta Q} = \sum_j 2\int_a \frac{ds}{B} \frac{n_{pj}}{T_i c^2} q_j q_i (\omega - \omega_{Ea}) r_p^2 g_j(\omega) \Bigg/ \int_c \frac{ds n_i m_i}{B_c^3} (\omega - \omega_{Ec}) \quad (24)$$

$$\gamma_{MHD}^2 = \int_c \frac{ds}{B^2 r_p} \kappa \frac{\partial}{\partial \alpha} (P_\perp + P_{||}) \Bigg/ \int_c \frac{ds n_i m_i}{B^3 r_p^2} \quad (25)$$

$$V_A^2 = \frac{B_c^2}{4\pi n_i m_i} .$$

and we assume no temperature gradients for the sake of algebraic simplicity. We also define

$$\Delta Q = \widetilde{\Delta Q} \quad \frac{(\omega - \omega_{Ec})}{(\omega - \omega_{Ea})}$$

and note that in the specific limits that we will analyze ΔQ will be independent of ω. In the layer $A >> \omega^2 \sim \omega_E^2$ (note, however, that $A \simeq \omega^2$ outside the layer). Then, to the lowest order we have the equation

$$\frac{1}{r} \frac{\partial}{\partial r} \frac{A}{V_A^2} r^3 \frac{\partial}{\partial r} (\phi/r) = 0 \quad (26)$$

with the solution $\phi = \phi_0 \, r/r_p$. If now we set $\phi = \phi_0 \, r/r_p + \phi_1$, and integrate over the layer, we obtain the relationship,

$$A \, \frac{r^3}{V_A^2} \frac{\partial}{\partial r} (\phi_1/r) \Bigg|_{r = r_p^+}^{r = r_p^-} = - \int_{r_p^+}^{r_p^-} dr \Bigg\{ r \, \frac{\partial}{\partial r}\left[\frac{\omega^2}{V_A^2} \, r\left(\frac{\phi_0}{r_p} + \frac{\partial \phi_1}{\partial r}\right)\right]$$

$$- \frac{\ell^2 \gamma^2{}_{MHD} \phi_0}{V_A^2} - (\ell^2 - 1) \frac{A}{V_A^2} \phi_0 + \frac{r^2}{r_p^2} \frac{\phi_0}{V_A^2} \omega_{*i} \tilde{\Delta Q} \; (\omega - \omega_{Ec}).$$

$$+ \, O\left(\frac{\ell \Delta \alpha_p}{r_p}\right)\Bigg\} \tag{27}$$

where we use $\ell \Delta \alpha_p / \alpha_p \sim \omega/\omega_{*i}$ and $\omega_E \sim \omega$. We assume the plasma density vanishes for $r > r_p$. B. Cohen, et.al.[10] have investigated the effect of density on the outside with walls for a similar model profile but without trapped particle effects.

Then using that $n_i \; (r = r_p^+) = 0$, $A \; (r = r_p^-) = \omega_{Ec}^2 - 2\omega_{Ec}\omega$,

the continuity of $\partial\phi/\partial r$ at $r = r_p^-$

$$\frac{\partial\phi}{\partial r}(r = r_p^-) = \frac{\phi_0}{r_p} + \frac{\partial\phi_1}{\partial r} = \frac{Q^{1/2}\,\phi_0}{r_p}\quad\frac{I_\ell'(\tilde{Q}^{1/2})}{I_\ell(\tilde{Q}^{1/2})}\,, \tag{28}$$

and that $\phi_1 \ll \phi_0$, we find,

$$\phi_0\,(\omega_{Ec}^2 - 2\omega\omega_{Ec})\left[\tilde{Q}^{1/2}\,\frac{I_\ell'(\tilde{Q}^{1/2})}{I_\ell(\tilde{Q}^{1/2})} - 1\right]$$

$$= -\,\phi_0\left[\omega^2\tilde{Q}^{1/2}\,\frac{I_\ell'(\tilde{Q}^{1/2})}{I_\ell(\tilde{Q}^{1/2})} - (\ell^2 - 1)\left[\bar{\omega}_{*i}(\omega - \omega_{Ec}) + \frac{\beta\bar{\omega}_{*i}^2 r_p}{2\Delta}\right] + \ell^2\bar{\gamma}_{MHD}^2\right.$$

$$\left. -\,\bar{\omega}_{*i}\Delta\,\tilde{Q}\,(\omega - \omega_{Ec}) + 0\left(\frac{\ell\Delta}{r_p}\right)\right] \tag{29}$$

where $\bar{\omega}_{*i} = c\ell P_{i\perp}/\,n_i q_i B_c r_p^2$, we assumed $\bar{\omega}_{*i} \simeq \omega$, and we have defined

$$\bar{\gamma}_{MHD}^2 = \frac{\displaystyle\int_c \frac{ds}{B^2 r}\kappa(P_\perp + P_{||})}{\displaystyle\int_c \frac{ds\,n_i m_i}{B^2}}\,.$$

$$\beta = \frac{8\pi P_\perp}{B_c^2}$$

and

$$\Delta^{-1} = \frac{-r_p B_c}{P_\perp} \frac{\partial P_\perp}{\partial \alpha}$$

Thus we obtain the dispersion relation,

$$q\Omega^2 + \Omega \left[2\omega_{Ec} - (\ell^2 - 1)\, \varpi_{*i} - \varpi_{*i}\, \Delta\tilde{Q}\right]$$

$$+ \ell^2\, \bar{\gamma}^{2}_{MHD} + \omega^2_{Ec} - (\ell^2 - 1)\, \frac{\beta}{2}\, \frac{r_p}{\Delta}\, \bar{\omega}^{2}_{*i} = 0 \qquad (30)$$

where

$$q = \tilde{Q}^{1/2} \frac{I'_\ell(\tilde{Q}^{1/2})}{I_\ell\, (\tilde{Q}^{1/2})}, \quad \Omega = \omega - \omega_{Ec}.$$

We also notice that it is a good approximation to use

$$\tilde{Q}^{1/2} \frac{I'_\ell(\tilde{Q}^{1/2})}{I_\ell\, (\tilde{Q}^{1/2})} \simeq |\ell| + \tilde{Q}^{1/2}$$

as this expression is valid for $\tilde{Q}$ small and $\tilde{Q}$ large.

B. Equation for Diffuse Profile

For the diffuse profile we take all pressures and densities to vary as $\exp(-r^2/2r_p^2)$ so that ω_{*i} is independent of r. We also take ω_E independent of r and assume that the equilibrium magnetic field is a constant through the center cell region. Then, variation with respect to ϕ of the quadratic form given in Eq. (19), after some manipulation yields,

$$\frac{\partial}{\partial r} \exp\left(-\frac{r^2}{2r_p^2}\right) r^3 \frac{\partial}{\partial r}\left(\frac{\phi}{r}\right) - (\ell^2 - 1)\exp\left(-\frac{r^2}{2r_p^2}\right)\phi$$

$$+ \exp\left(-\frac{r^2}{2r_p^2}\right)\frac{r^2}{r_p^2} G(\omega)\phi = 0 \qquad (31)$$

with

$$G(\omega) = -\frac{\omega^2 + \ell^2\gamma^2_{MHD} + \tilde{Q}(\omega-\omega_{Ec})^2 - \Delta\tilde{Q}(\omega-\omega_{Ec})\omega_{*i}}{(\omega-\omega_{Ec})(\omega-\omega_{Ec}-\omega_{*i})} \qquad (32)$$

where $\tilde{Q}$, $\Delta\tilde{Q}$ and γ_{MHD} are defined by Eq. (21), Eq. (24) and Eq. (25) respectively and $\omega_{*i} \equiv (\ell cT/q_i n_i)\partial n_i/\partial\alpha$.

Eq. (31) has been analyzed elsewhere[11,12], (using a simpler form for $G(\omega)$) and the eigenvalue condition is found to be,

$$G(\omega) = k_\perp^2 r_p^2 - 1$$

where $k_\perp^2 r_p^2 = |\ell| + 2\,|n|$ with $|n|$ an integer and $k_\perp$ can be interpreted as the perpendicular wave number in the eikonal approximation.

The dispersion relation for this profile can then be written as

$$q' \Omega^2 + \Omega\,[2\omega_{Ec} - (k_\perp^2 r_p^2 - 1)\,\omega_{*i} - \omega_{*i}\,\Delta Q - Q\Delta\omega_E]$$

$$+ \ell^2 \gamma_{MHD}^2 + \omega_{Ec}^2 + \Delta\omega_E \omega_{*i} \Delta Q = 0 \qquad (33)$$

where $q' = k_\perp^2 r_p^2 + Q$, and $\Delta\omega_E = \omega_{Ea} - \omega_{Ec}$.

C. Analysis of Dispersion Relations

If one compares the dispersion relation for the two models one observes similar features but with several significant modifications. In the steep pressure model, $q \simeq |\ell| + \tilde{Q}^{1/2}$, while $q = |\ell| + \tilde{Q}$ (for $n = 0$) for the diffuse profile. In the steep model a term linear in Ω (the FLR term) is proportional to $(\ell^2 - 1)\bar{\omega}_{*i}$ whereas the dispersion relation for the diffuse profile is proportional to $(|\ell| - 1)\omega_{*i}$. Finally, the sharp boundary profile has an additional stabilizing term proportional to $(\ell^2 - 1)\beta\omega_{*i}^2 r_p/(2\Delta)$ which can be significant if $\Delta/r_p << 1$. The corresponding term for the diffuse profile is always small if $\beta << 1$.

The dispersion relation for the sharp boundary model simplifies somewhat if we take $\Delta\omega_E = 0$ so that $\tilde{Q} = Q$ and $\Delta\tilde{Q} = \Delta Q$. There are two important limits that can be considered to evaluate $g_j(\omega)$ explicitly. One is where $\omega < \omega_{ba}$ for all species where ω_{ba} is the axial bounce frequency in the anchor region. In this case $g_j(\omega) = 1$ and ΔQ is proportional to the difference of the passing ion and electron populations in the anchor region. If most of the passing particles are ions, then $\Delta Q \simeq Q$, while if most of the passing particles are electrons then $\Delta Q \simeq - Q\, T_e/T_i$.

The other important limit is where $\omega_{bi} < \omega < \omega_{be}$. In this case $g_j(\omega) = 0$ for ions and unity for electrons. Above some moderate value of ℓ, this approximation can be expected to hold. For these ℓ numbers one can formally treat the equations as if there were no passing ions in the anchor region.

In addition, in these limits $g_j\ (\omega)$ has a small imaginary part, that is discussed in Ref. (1). In the applications studied here we neglect the imaginary contribution. However, these imaginary contributions can produce additional stabilizing and destabilizing mechanisms, and are discussed elsewhere [8,9].

Now, with the further approximations, $x\, I'_\ell\,(x)/I_\ell\,(x) \simeq |\ell| + x$ (this is an interpolation formula that is asymptotically valid if $x/|\ell| \ll 1$, $x/|\ell| \gg 1$), Eq. (30), the dispersion relation for the sharp profile, becomes,

$$(|\ell| + Q^{1/2})\ \Omega^2 + \Omega\ [2\omega_E - (\ell^2 - 1)\ \varpi_{*i} - \varpi_{*i}\ \Delta\ Q]$$

$$+\ \ell^2\ \gamma_{MHD}^{-2} + \omega_E^2 - (\ell^2 - 1)\ \frac{\beta r_p}{2\Delta}\ \varpi_{*i}^2 = 0 \qquad (34)$$

We then find that there is a band of stability in $\Omega_E = \omega_E/\varpi_{*i}$ which is given by

$$\text{Max}\ [\ -A_\ell - |B_\ell|] < \Omega_E < \text{Min}\ [-A_\ell + |B_\ell|] \qquad (35)$$

where,

$$A_\ell = \frac{1}{2}\,\frac{\Delta Q + \ell^2 - 1}{(|\ell| - 1 + Q^{1/2})}\,,$$

$$B_\ell^2 = (|\ell| + Q^{1/2})\left[\frac{(\ell^2 - 1 + \Delta Q)^2}{4(|\ell| - 1 + Q^{1/2})^2} - \frac{\Gamma - (\ell^2 - 1)\,\beta r_p/2\Delta}{|\ell| - 1 + Q^{1/2}}\right],$$

$$\Gamma = \frac{\ell^2\,\bar{\gamma}^{-2}_{MHD}}{\bar{\omega}^{-2}_{*i}} \qquad (36)$$

and we minimize and maximize with respect to ℓ. One observes that there is no stability band in Ω_E if

$$\Gamma > \mathrm{Min}\left[(\ell^2 - 1)\,\frac{\beta r_p}{2\Delta} + \frac{(\ell^2 - 1 + \Delta Q)^2}{4\,(|\ell| - 1 + Q^{1/2})}\right],$$

where the minimization is with respect to ℓ - number.

If ΔQ is positive, one achieves a wider range of stability with $\Omega_E < 0$ (radial equilibrium electric field inward) but with Ω_E not too large in magnitude. For Γ and β negligible, the stability in Ω_E is

$$\mathrm{Max}\left[-\frac{1}{2}\;\frac{(\Delta Q+\ell^2-1)\;[1+(|\ell|+Q^{1/2})^{1/2}sg(x_\ell)]}{|\ell|-1+Q^{1/2}}\right]<\Omega_E<$$

$$\mathrm{Min}\left\{\frac{1}{2}\cdot\frac{(\Delta Q+\ell^2-1)}{(|\ell|-1+Q^{1/2})}\left[(|\ell|+Q^{1/2})^{1/2}sg(x_\ell)-1)\right]\right\} \tag{37}$$

where $x_\ell = \Delta Q + \ell^2 - 1$ and

$$sg\ (x) = \begin{cases} 1, & x > 0 \\ -1, & x < 0 \end{cases} .$$

Note that with $\Omega_E > 0$ stability cannot be achieved with $Q = 0$, but at finite Q and ω_E sufficiently small, stability is also possible. For moderate values of Q and ΔQ, the $\ell = 1$ condition tends to be the most stringent stability condition.

If ΔQ is negative, there is a range of values where stability is particularly difficult to achieve, viz. $\Delta Q \simeq -\ell^2+1$ where the negative charge uncovering from the anchor region balances the positive charge uncovering finite Larmor radius terms. For $-\Delta Q -\ell^2+1 > 0$, $\Omega_E > 0$ is most favorable to stability (an outward electric field) while if $-\Delta Q -\ell^2+ 1 < 0$, $\Omega_E < 0$ is most favorable for stability (an inward electric field); apparently conflicting requirements.

The Tara experiment[13] is designed to have $\Delta Q < 0$. However, stability can still be achieved over limited parameter regimes, if $|\Delta Q|<3$. In this case, for typical parameters, $\ell = 2$ determines the most stringent inequality of the right side of Eq. (36) and $\ell = 1$ determines the most stringent inequality of the left hand side of Eq. (36). The stability condition for $\beta = 0$ is then found to be,

$$- (1 + Q^{1/2})^{1/2} \left[\frac{\Delta Q^2}{4Q} - \frac{\Gamma}{Q^{1/2}} \right]^{1/2} - \frac{\Delta Q}{2Q^{1/2}} < \Omega_E$$

$$< (2 + Q^{1/2})^{1/2} \left[\frac{(3 + \Delta Q)^2}{4 (1 + Q^{1/2})^2} - \frac{\Gamma}{1 + Q^{1/2}} \right]^{1/2} - \frac{(3 + \Delta Q)}{2 (1 + Q^{1/2})} \qquad (38)$$

For example if Γ is negligible we find the stability range of Ω_E is

$$-[sg(\Delta Q)(1 + Q^{1/2})^{1/2} + 1] \frac{\Delta Q}{2Q^{1/2}} < \Omega_E < [sg(3+\Delta Q)(2 + Q^{1/2})^{1/2} - 1] \frac{(3+ \Delta Q)}{2(1+ Q^{1/2})}$$

(39)

In Fig. (2) we present the numerical evaluation of the stability band in Ω_E given by Eq. (35) as a function of Q for $\Delta Q/Q = -.5$. This choice is appropriate for the Tara geometry as we discuss in Section V. The cross-hatched regions in the figure indicate for which values of Ω_E as a function of Q the model predicts stability for various values of the stability parameter $\Gamma \equiv (\ell\gamma_{MHD}/\omega_{*i})^2$. The sides of the stable regions are labeled by the mode number ℓ which imposes the most stringent constraint on stability in that region of parameter space. Note that when $Q = 2(\ell^2-1)$ the negative charge uncovering due to electrons cancels the positive charge uncovering due to the ion finite Larmor orbit and the system is unstable for all values of Ω_E. As Γ increases, the islands shrink and vanish altogether for a critical value of Γ. The stability island corresponding to the lowest values of Q completely disappears for $\Gamma \simeq .18$. The last stable point occurs at $\Omega_E = 0$ and $Q \simeq 2.7$. The width of the stability band for this value of Q at $\Gamma = 0$ is $-.25 < \Omega_E < .28$.

We now analyze the dispersion relation of the diffuse profile, Eq.(33). We note that if $k_\perp^2 r_p^2 \gg 1$ this dispersion relation is identical with the dispersion relation one would derive in the eikonal approximation. First we consider $\Delta\omega_E = 0$, so that $\Delta\tilde{Q} = \Delta Q$ and $\tilde{Q} = Q$. We then find that the band of stability in $\Omega_E \equiv \omega_E/\omega_{*i}$ is given by

$$\mathrm{Max}\left[-C_{\ell,n} - |D_{\ell,n}|\right] < \Omega_E < \mathrm{Min}\left[-C_{\ell,n} + |D_{\ell,n}|\right] \tag{40}$$

$$C_{\ell,n} = \frac{1}{2}\frac{(|\ell| + 2|n| - 1 + \Delta Q)}{(|\ell| + |2n| - 1 + Q)}$$

$$D^2_{\ell,n} = (|\ell| + 2|n| + Q)\left[\frac{(|\ell| + 2|n| - 1 + \Delta Q)^2}{4\,(|\ell| + 2|n| - 1 + Q)^2}2 - \frac{\Gamma_d}{|\ell| + 2|n| - 1 + Q}\right]$$

$\Gamma_d = \ell^2\gamma^2_{MHD}/\ \omega^2_{*i}$, and we minimize and maximize with respect to ℓ and n.

If $\Delta Q > 0$, one achieves the best stability with $\Omega_E < 0$, but with Ω_E not too large in magnitude. For Γ negligible the stability in Ω_E is,

$$-\frac{1}{2}\frac{(2|n| + \Delta Q + |\ell| - 1)[1 + (|\ell| + 2|n| + Q)^{1/2}\mathrm{sg}(x_{n,\ell})]}{(|\ell| + 2|n| - 1 + Q)} < \Omega_E < \tag{41}$$

$$\frac{1}{2}\frac{(\Delta Q + |\ell| + 2|n| - 1)}{(|\ell| + 2|n| - 1 + Q)}\left[(|\ell| + 2|n| + Q)^{1/2}\mathrm{sg}(x_{n,\ell}) - 1\right]$$

where $x_{n,\ell} = 2|n| + \Delta Q + |\ell| - 1$. The stability band disappears if

$$\Gamma_d > \frac{(|\ell| + 2|n| - 1 + \Delta Q)^2}{4\,(|\ell| + 2|n| - 1 + Q)} \tag{42}$$

If ΔQ is negative, there is a range of ℓ and n values where stability is particularly difficult to achieve, viz. $\Delta Q = |\ell| + 2|n| - 1$ where the negative charge and positive charge uncovering mechanisms cancel. If Q and $-\Delta Q \simeq 1$, there is a limited range of parameters where stability can be achieved. If in Eq. (41) we take $\ell = 1$, $n = 0$ for the left hand side and $\ell = 2$ for the right hand side we obtain the following stability band,

$$- (1 + Q)^{1/2} \left[\frac{\Delta Q^2}{4Q^2} - \frac{\Gamma_d}{Q} \right]^{1/2} - \frac{\Delta Q}{2Q} < \Omega_E < - \frac{(1 + \Delta Q)}{2\,(1 + Q)} +$$

$$\frac{(2 + Q)^{1/2}}{2(1 + Q)} \left[(1 + \Delta Q)^2 - 4\Gamma_d\ (1+Q) \right]^{1/2} . \qquad (43)$$

If Γ_d is negligible, the stability band is

$$- \left[(1+Q)^{1/2} sg(\Delta Q) + 1 \right] \frac{\Delta Q}{2Q} < \Omega_E < \frac{1}{2} \frac{(1 + \Delta Q)}{(1 + Q)} \left[(2+Q)^{1/2}\ sg(1+\Delta Q) - 1 \right] \qquad (44)$$

In Fig. (3) we present a numerical evaluation of the stability bands in Ω_E for the diffuse profile given by Eq. (40), again for $\Delta Q/Q = -.5$. The stability criteria given by the diffuse profile are more stringent than the criteria of the steep profile. Note the change in scale in Ω_E from the corresponding scale and the relatively low values of Γ which cause the islands to shrink to a point. The stability island at $\Gamma = 0$ between $Q = 0$

and Q = 2 vanishes altogether for $\Gamma \simeq .027$ at $Q \simeq .92$. For this value of Q the width of the stability band at $\Gamma = 0$ is $-.1 < \Omega_E < .1$.

We now consider the case when $\Delta\omega_E = \omega_{Ea} - \omega_{Ec} \neq 0$. The stability condition found from Eq. (33) is,

$$[Q\Delta\Omega_E + (k_\perp^2 r_p^2 - 1) + \Delta Q - 2\Omega_{Ec}]^2$$

$$> 4\ [k_\perp^2 r_p^2 + Q]\ [\ \Gamma_d + \Omega_{Ec}^2 + \Delta\Omega_E \Delta Q] \tag{45}$$

where $\Delta\Omega_E = \Delta\omega_E/\omega_{*i}$ and $\Omega_{Ec} = \omega_{Ec}/\omega_{*i}$

If we set $\Gamma_d = 0$ and $\Omega_{Ec} = 0$ the only instability drive present is due to the axial shear in the electric field rotation. This drive for the instability of a non-flute mode was first treated by Byers and Cohen(5) for a somewhat special equilibrium with zero ion temperature.

The present dispersion relation is for a more general equilibrium, albeit in the small anchor-length approximation. For $\Delta Q < 0$, which is negative charge uncovering, instability is possible if $\Delta\Omega_E < 0$, which corresponds to an inward electric field in the anchor. We assume $k_\perp^2 r_p^2 \gg 1$, and that the frequency will be high enough so that ions do not contribute to Q or ΔQ (therefore $Q = - T_i/T_e \, \Delta Q \equiv - \tau\Delta Q$). It then follows from Eq. (45) that the instability band in $k_\perp^2 r_p^2$ is determined from the condition,

$$k_\perp^4 r_p^4 + 2k_\perp^2 r_p^2 \Delta Q \, [1 - \Delta\Omega_E(2 + \tau)] + \Delta Q^2(1 + \tau\Delta\Omega_E)^2 < 0 \qquad (46)$$

The k-spectrum of the instability is then

$$- Y < \frac{k_\perp^2 r_p^2}{|\Delta Q|} - 1 + \Delta\Omega_E(2 + \tau) < Y \qquad (47)$$

with

$$Y = \left[(1 - \Delta\Omega_E(2 + \tau))^2 - (1 + \tau\Delta\Omega_E)^2 \right]^{1/2}$$

Real values for Y exist for $-\Omega_E > 1$, which defines the threshold for instability. Note that the $k_\perp$ values for instability must satisfy

$$\frac{k_\perp^2 r_p^2}{|\Delta Q|} > 3 + \tau,$$

so that the assumption $k_\perp^2 r_p^2 \gg 1$ is valid if $|\Delta Q| > 1$.

The importance of this instability is that the drive is due to electric fields in the anchor which can be large compared to central cell electric fields, since the temperature of the plasma in the anchor can be much larger than the plasma in the central cell. If the electric field is inward, the stabilizing effect of the MHD anchor is subverted by the formation of an eigenfunction with a negligible amplitude in the anchors.

V. Summary

We have applied the variational form derived in Ref. 1 to several stability problems relevant to tandem mirrors; especially to tandem mirror trapped particle modes. The analysis in this paper is primarily applied to long wavelength modes. The problems studied and their results are summarized here.

(a) Wall Stabilization

Previous calculations of wall stabilization[6,7] assumed $\phi = \chi$, or equivalently, when there is no rotation, zero electric field along the field line. Wall stabilization of the $\ell = 1$ displacement mode then causes an induced line-bending term. The initial work on trapped particle modes[2] observed that if $\chi = 0$, the stabilization due to line bending is removed for eikonal modes. However, in the non-eikonal theory, additional line-bending energy exists even if $\chi = 0$. Further, if $\ell\Delta/r_p < 1$, where Δ^{-1} is the pressure gradient, r_p the plasma radius and ℓ the mode number, this bending energy is larger than the bending energy arising from the MHD wall-stabilization for displacement-like modes. Hence, if a system is stable to an MHD displacement-like mode, it will be stable to the "electrostatic" ($\chi = 0$) displacement-like mode. Our proof depends on the assumption $\Delta \ll r_p$. We have not as yet proved that $\chi = \phi$ is the lowest energy perturbation for the $\ell = 1$ displacement mode in the case $\Delta \sim r_p$. However, we conjecture that when there is sufficient beta to achieve wall stabilization of the $\ell = 1$ displacement mode, $\chi = \phi$ will be close to the minimizing energy perturbation when an arbitrary pressure profile is considered.

(b) Charge-Separation Stabilization of Trapped Particle Mode at Finite Beta

The original theory of trapped particle modes obtained a charge separation stability condition in a low-beta limit. Tandem mirror designs are based on satisfying the trapped particle stabilization condition for beta values approaching, and even exceeding the MHD critical beta limit, β_{cr}. We have treated the stability of a finite beta trapped particle mode for the following type of geometry: an anchor region that strongly stabilizes flute modes with the lengths in the anchor and transition regions much shorter than the overall central cell length and with the length of the transition region larger than the bad curvature region of the central cell. We then find that the charge separation stability criterion becomes more severe as the beta aproaches the critical beta for marginal stability. If β is not too close to β_{cr}, we find in the eikonal limit that the stability condition is,

$$\left(\frac{\Delta Q}{Q}\right)^2 \Omega_*^2 > \frac{4k_\perp^2 r^2}{(1 - \beta/\beta_{cr})Q} .$$

which is identical to that given in Eq. (15). The reader is referred to the text for definitions. We note here that $\Delta Q/Q$ is the relative charge separation of the passing particles. The formula was derived with the assumption, $\Delta Q/Q; k_{\perp}^2 r^2/Q \ll 1$. The factor, $1/(1-\beta/\beta_{cr})$, indicates the enhanced difficulty of satisfying the stability condition as beta aproaches the MHD beta condition. The most stringent condition on the charge separation parameter, $(\Omega_* \Delta Q/Q)$, occurs as β approaches β_{cr}. At this value of β the stability condition is

$$\left(\frac{\Delta Q}{Q}\right)^2 \Omega_*^2 > \left(\frac{27}{4} \frac{k_{\perp}^2 r^2}{Q} \frac{Q}{\Delta Q}\right)^{1/2} .$$

If the goal of a tandem mirror design is to operate near or above the critical beta for MHD stability, the above criterion should be satisfied during build-up to eliminate the non-resonant curvature-driven trapped particle mode.

In the analysis used to obtain the above results, it was assumed that the anchor region strongly stabilizes flute modes and that the anchor length is much smaller than the central cell length.

(c) Rotation and Curvature Driven Modes

We have analyzed trapped particle-rotational driven modes in the presence of curvature for two pressure profiles: (1) a steep pressure profile at a radius $r = r_p$ where we performed the analysis when $\ell\Delta/r_p < 1$;

(2) a diffuse pressure profile where exact analytic solutions can be obtained. We found that the steep pressure profile is considerably more stable than the diffuse profile. One reason is the possibility of a robust stabilization mechanism, for $\ell \geq 2$, that arises from the finite beta FLR term. This term becomes extremely important if $\beta r_p/\Delta \gtrsim 1$, in which case a robust stability arises for electric field rotational rates comparable to the diamagnetic drift frequency. As pointed out in Ref. 6, this robust stabilization term, together with wall stabilization of the $\ell = 1$ term, can provide for a strongly stabilized system which has only positive energy excitations.

At low beta, when $\beta r_p/\Delta \ll 1$, the sharp boundary model still gives a stability condition that is nearly an order of magnitude more favorable than the diffuse profile. The reason for the improvement is not clear, and seems to be the result of detailed quantitative calculations. Nevertheless, even with a steep pressure profile, stringent stability constraints arise for the trapped particle mode driven by the combination of rotational and curvature drives.

We can apply our results to the Tara experiment, which is designed to operate with only electrons passing from the center cell into the anchor region.[13] We note that our analysis is restricted to conditions in which the plasma can be treated as collisionless which requires $\omega \sim \omega_{*i} > \nu_e$.

where ν_e is the electron collision frequency. Such a collisionality condition has not yet been fulfilled for $\ell = 1$ in the current unplugged mode of operation of Tara, but the collisionless regime is expected to be achieved in upcoming experiments. Analysis of the Tara experiment in terms of our model is complicated by the addition of axisymmetric plug cells between the solenoidal central cell and the stabilizing minimum B anchor. Ions with an average energy of 12 keV are injected into these plugs which provide the main MHD destabilizing drive. The longer center cell plasma with an ion temperature of 400 eV contributes to the inertia and to the FLR terms. To apply our analysis we treat the plug region as part of the center cell. This is appropriate to the extent that the mode is flute-like through the two regions. From Eq. (19) we note that the drive and inertia terms in each region are weighted by the factor $\int d\ell/B^2$. Using the steep pressure gradient model and the long-thin expression for the curvature, $\kappa \simeq r/L_{cv}^2$, where L_{cv} is the characteristic length of the curvature region we obtain

$$\gamma_{MHD}^2 \simeq \frac{\dfrac{L_{cv}\,P_c}{B_c^2\,L_{cv}^2} + \dfrac{L_p\,P_p}{B_p^2\,L_p^2}}{n_i m_i\left(\dfrac{L_c}{B_c^2} + \dfrac{L_p}{B_p^2}\right)}$$

where we assume n_i is constant axially. Using $L_c = 5\text{m}$, $L_p = 1\text{m}$, $L_{cv} = 1\text{m}$, $B_c = 2\text{kG}$ and $B_p = 5\text{kG}$, we obtain

$$\gamma_{MHD} \simeq 2 \times 10^5 \text{ sec}^{-1}$$

In evaluating the expression for ω_{*i}, the contributions from the pressure on the center cell and plug must be weighted by the appropriate factor of $\int d\ell/B^2$. We account for this by introducing an effective temperature, T_{eff}

$$T_{eff} \equiv \frac{\frac{L_c}{B_c^2} T_{i,c} + \frac{L_p}{B_p^2} T_{i,p}}{\frac{L_c}{B_c^2} + \frac{L_p}{B_p^2}} \simeq 760 \text{ eV}$$

The relevant value of ω_{*i} is thus

$$\omega_{*i} = \frac{\ell c}{eB_c r_p^2} T_{eff} \simeq 4 \times 10^5 \ell \text{ sec}^{-1}$$

for $r_p \simeq 10$ cm. We note that $\Delta\tilde{Q}$ defined by Eq. (24) must be evaluated for $T_i = T_{eff}$, while $\tilde{Q}$ defined by Eq. (21) is evaluated only for electrons and hence involves the center cell electron temperature, approximately 400 eV. Thus $\Delta Q/Q \simeq -.5$ assuming that $\Delta\omega_E = 0$.

We thus conclude that the stability parameter, $\Gamma \equiv \left(\ell\gamma_{MHD}/\omega_{*i}\right)^2$, is approximately equal to .25. We note that this is greater than the value of .18 at which the stability island in the steep profile model disappears for Q = 2.7. The prediction of the Gaussian profile theory is yet more pessimistic. We note, however, that the numbers are sensitive to the values chosen for r_p and $T_{i,c}$. The value of the stability parameter shows that the excitation of a trapped particle mode in a configuration of the Tara design is probable.

(b) Axial Shear Driven Mode

We have generalized the description used in Ref. 5 to describe the trapped particle mode obtained by axial shear in the equilibrium electric fields when the electric field is inward in the MHD anchor region. In our description, the geometry is more realistic than given in Ref. 5, and the analysis is valid even in the non-eikonal limit. The onset for stability is roughly the same condition as in Ref. 5. However, the large $k_\perp$ stabilization condition differs. We do not obtain their condition as we have considered a limit of small anchor length. Our stabilization mechanism depends on finite T_i/T_e, which was not predicted in Ref. 5, which treated $T_i/T_e = 0$.

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Figure Captions

Fig. 1. Equilibrium model of tandem mirror. The quantities B_c, L_c, Φ_c, ω_{Ec} denote the magnetic field, length, equilibrium electrostatic potential and equilibrium ExB drift frequency in the center cell. The corresponding quantities subscripted by the letter "a" refer to the anchor region. The quantity B_m denotes the maximum magnetic field, s labels distance along the field line from the origin at the mid-plane.

Fig. 2. Stability windows in Ω_E as a function of Q for the steep pressure profile model for $\Delta Q/Q = -.5$ and $(\ell\gamma_{MHD}/\omega_{*i})^2 = 0., .1, .18$. The boundaries of the stable region are labelled with the mode numbers ℓ, which give the most stringent stability criteria at that point in the (Ω_E, Q) parameter space.

Fig. 3. Stability windows in Ω_E as a function of Q for the diffuse pressure profile for $\Delta Q/Q = -.5$ and $(\ell\gamma_{MHD}/\omega_{*i})^2 = 0., .01, .027$.

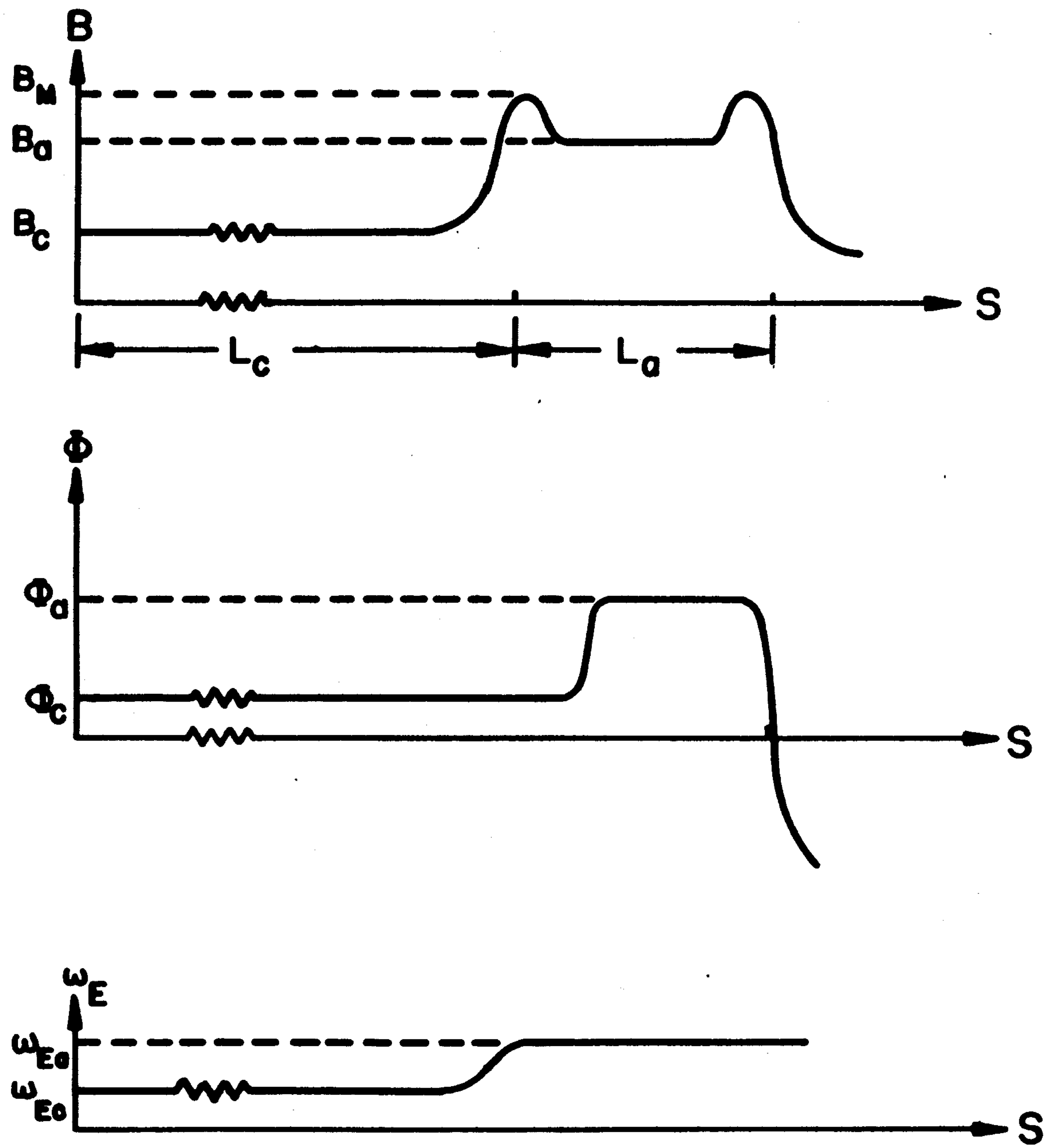
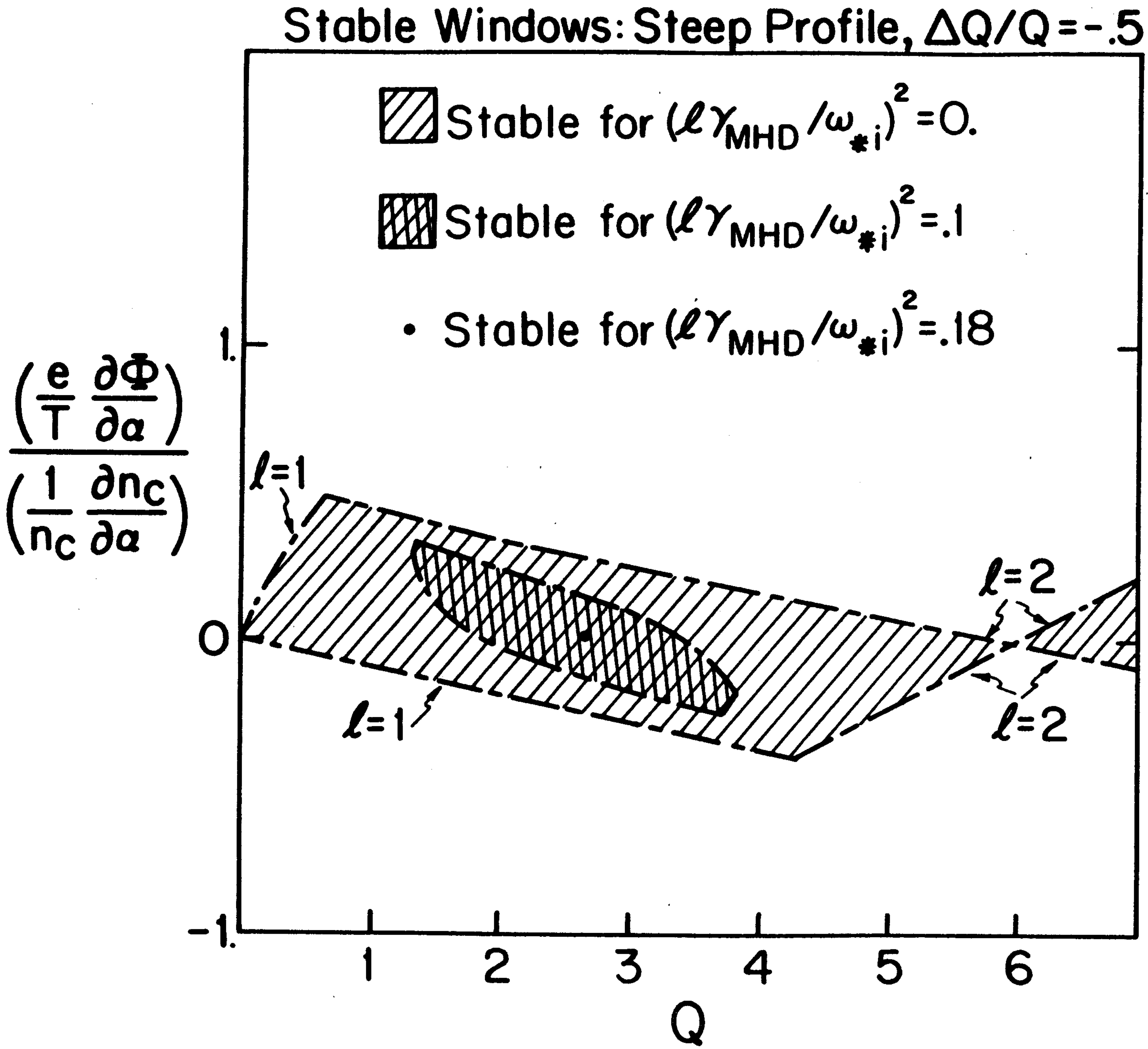


FIG. 1



Stable Windows: Steep Profile, ΔQ/Q=-.5
Stable for $(\ell\gamma_{MHD}/\omega_{*i})^2$=0.
Stable for $(\ell\gamma_{MHD}/\omega_{*i})^2$=.1
• Stable for $(\ell\gamma_{MHD}/\omega_{*i})^2$=.18
$\left(\frac{e}{T}\frac{\partial\Phi}{\partial\alpha}\right) \Big/ \left(\frac{1}{n_c}\frac{\partial n_c}{\partial\alpha}\right)$
1.
0
-1.
ℓ=1
ℓ=1
ℓ=2
ℓ=2
1
2
3
4
5
6
Q

FIG. 2

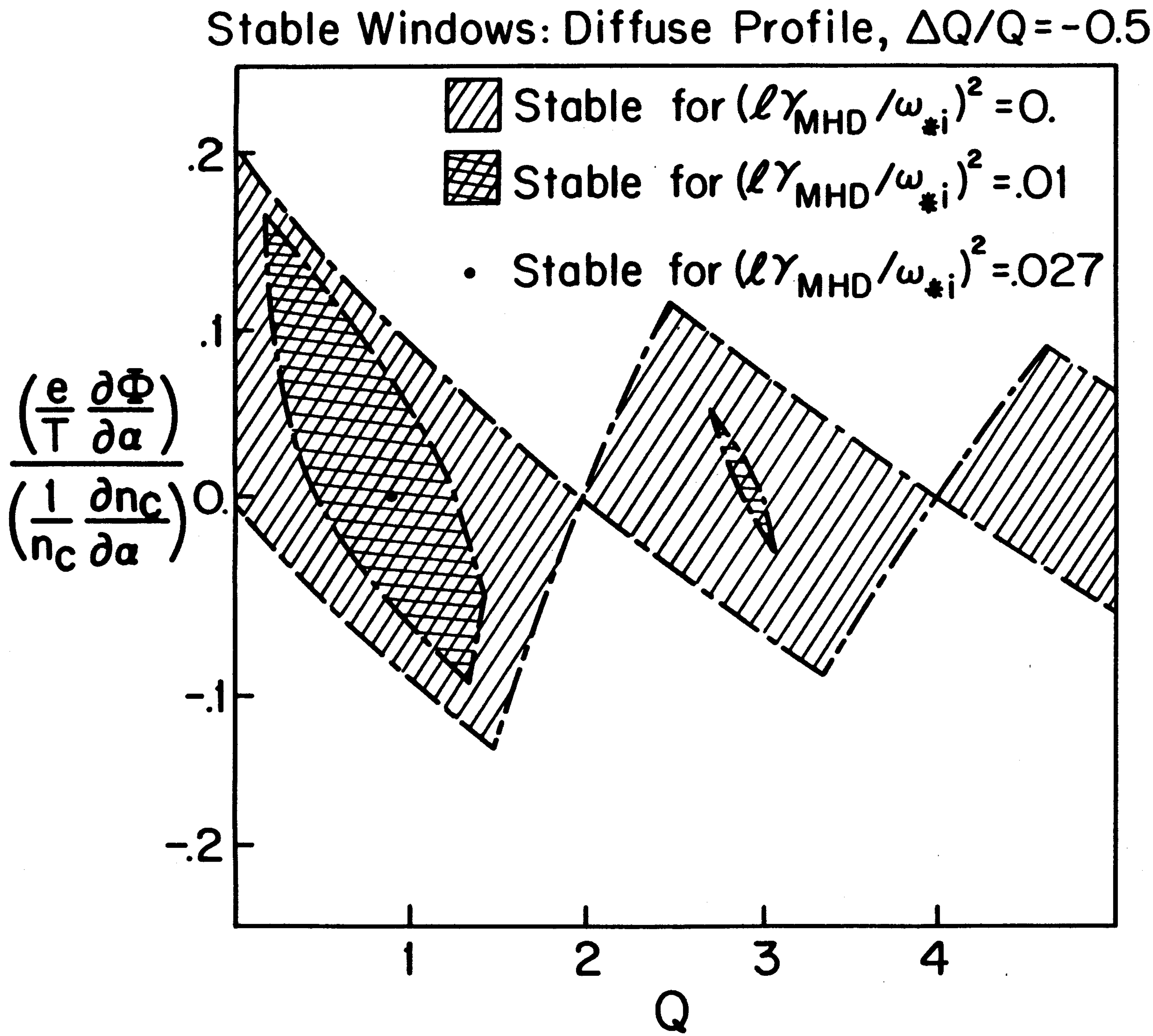

Stable Windows: Diffuse Profile, $\Delta Q/Q=-0.5$
Stable for $(\ell\gamma_{MHD}/\omega_{*i})^2=0.$
Stable for $(\ell\gamma_{MHD}/\omega_{*i})^2=.01$
Stable for $(\ell\gamma_{MHD}/\omega_{*i})^2=.027$
$\dfrac{\left(\frac{e}{T}\frac{\partial\Phi}{\partial\alpha}\right)}{\left(\frac{1}{n_c}\frac{\partial n_c}{\partial\alpha}\right)}$
.2
.1
0.
-.1
-.2
1
2
3
4
Q

FIG. 3