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ANALYTICAL MODEL FOR THE PERPENDICULAR TEMPERATURE ENHANCEMENT IN LOWER-HYBRID CURRENT DRIVE

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Abstract

The enhancement of the perpendicular temperature inside the resonant region, observed in numerical studies of the two-dimensional Fokker-Planck equation, combined with unidirectional RF quasilinear diffusion, is modeled on the basis of the collisional relaxation equations. Strong RF diffusion is assumed and relativistic effects are taken into account. The resulting enhanced perpendicular temperature is a function of the position and the width of the applied RF spectrum. Good agreement with two-dimensional Fokker-Planck numerical results has been found.

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Experiments in current drive using lower-hybrid (LH) waves have shown that associated with the RF generation of current parallel to the magnetic field ($\overline{B}_0$), there is also a large increase in the electrons' energies perpendicular to $\overline{B}_0$ [1,2]. Numerical studies of the two-dimensional Fokker-Planck equation aimed at modeling these experiments also have shown a considerable enhancement of the perpendicular temperature (i.e., energy moment, $v_\perp^2/2$) of the electrons resonant with the wave[3,4]. Similar enhancement is found in a region which encompasses an interval in negative parallel momentum, symmetric to the resonant region. This latter enhancement is due to the pitch angle scattering process which connects the resonant region and its image part at negative $v_{||}$, mainly via the most energetic electrons. Recently there have also been analytical attempts at understanding these two-dimensional results[5,6]. Although they have provided satisfactory explanations for the observed current and figure of merit, they have been less successful in obtaining the enhanced perpendicular temperature.

In a recent, new analysis of LH current drive a model was developed for estimating the perpendicular temperature enhancement in the resonant region[7]. Here we incorporate relativistic effects, and detail the dynamics at the low-velocity interface of the resonant region. The model we use is based upon the premise that the high perpendicular energies in the distribution function are generated inside the resonant region in parallel velocity as a result of the interplay between strong parallel (RF) diffusion on a short time scale, combined with the collisional pitch angle scattering on a long time scale. This is also clear from the mentioned numerical Fokker-Planck studies; they show that for strong RF diffusion the role of the RF is to spread out the particle velocities in the parallel direction, while the large perpendicular temperatures are established by collisional processes. Furthermore, for strong RF diffusion the perpendicular temperature is essentially independent of RF power.[4]

We shall use the normalized (to the velocity of light c) velocity space $\beta_{\|}$, $\beta_{\perp}$ ($\beta = v/c$). The resonant region is defined by $\beta_{\|1} \leq \beta_{\|} \leq \beta_{\|2}$. The particles under consideration are mainly the particles in the low-velocity interface, i.e. the narrow layer that surrounds the boundary $\beta_{\|} = \beta_{\|1}$. These particles are acted upon by frictional drag and diffusion due to collisions and by the applied RF field for $\beta_{\|} \geq \beta_{\|1}$. The RF field force is considered as acting randomly in time on the basis of the random phase approximation of quasilinear theory and the randomizing flux-surface averaging of the electrons interacting with the LH fields. We assume that the RF-imposed parallel diffusion is strong in the sense that each resonant particle's parallel velocity is determined by the RF field force rather than collisions. Quantitatively speaking, we take the local (in $\beta_{\|}$) value of the parallel quasilinear diffusion coefficient to be much greater than the local collisional diffusivity. This assumption enables us to separate the collisional time scale (the longer one) from the time scale the individual particle diffuses in the parallel direction due to its interaction with the RF waves (which is a fast process).

Assuming a large and constant quasilinear diffusion coefficient within the resonant region, the probability of finding a particle anywhere in the resonant region is the same for times intermediate of the two aforementioned time scales. For times comparable to the collisional time scale the resonant particles are collisionally slowing down, until they reach the end of the resonant region ($\beta_{\|} = \beta_{\|1}$). The normalized momentum ($\gamma\beta_{\|}$) and energy (γ) slowing down equations are[6]:

$$\frac{d\gamma\beta_{\|}}{dt} = -(Z_i + 1 + \gamma)(\gamma^2 - 1)^{-3/2}\gamma^2\beta_{\|} \tag{1}$$

$$\frac{d\gamma}{dt} = -\gamma(\gamma^2 - 1)^{-1/2} \tag{2}$$

where Z_i is the ionic charge number and the time t is normalized to ν_c^{-1} ($\nu_c \equiv (4\pi n_e e^4 \ln \Lambda_{ee})/(m_e^2 c^3)$, Λ_{ee} = Coulomb logarithm). Eliminating the time from Eqs. (1) and (2) yields, after integration:

$$\beta_{\|} = K\beta^{Z_i+2}/(1+1/\gamma)^{Z_i+1} \tag{3}$$

where $\beta = \sqrt{\beta_{\|}^2 + \beta_{\perp}^2}$, and K is the constant of integration. This equation can now be transformed to an equation which relates parallel velocity ($\beta_{\|}$) to perpendicular one ($\beta_{\perp}$). By substituting $x = \beta_{\|}^2$, $y = \beta_{\perp}^2$ one obtains:

$$\frac{x}{x_0} = \left(\frac{x+y}{x_0+y_0}\right)^{Z_i+2}\left(\frac{1+\sqrt{1-x_0-y_0}}{1+\sqrt{1-x-y}}\right)^{2Z_i+2} \tag{4}$$

where x_0, y_0 are the initial values of x, y in the collisional slowing down process. Since $x_0^{1/2}$ can be anything between $\beta_{\|1}$ and $\beta_{\|2}$ with equal probability (due to the assumed strong RF diffusion), one can integrate Eq. (4) with respect to $x_0^{1/2}$ with a weighting function equal to unity. Setting $x = x_1 = \beta_{\|1}^2$, then y in Eq. (4) gives the final value of $\beta_{\perp}^2$ after the particle has been slowed down enough to exit the resonant region. Thus, averaging Eq. (4) with respect to $x_0^{1/2}$ yields (from here on we take $Z_i = 1$):

$$\begin{aligned}\left(1+\sqrt{1-x_1-y}\right)^2 (x_1+y)^{-3/2} x_1^{1/2}(x_2^{1/2}-x_1^{1/2}) \\ = \left(2+x_1+y_0+2\sqrt{1-x_1-y_0}\right)(y_0+x_1)^{-1/2} \\ -\left(2+x_2+y_0+2\sqrt{1-x_2-y_0}\right)(y_0+x_2)^{-1/2} \\ +\sin^{-1}(2y_0+2x_1-1) \\ -\sin^{-1}(2y_0+2x_2-1)\end{aligned} \tag{5}$$

This equation relates the initial perpendicular energy $y_0/2$ of the resonant particle to the final average $y/2$, after the particle has been slowed down to $x = x_1$. Since on the fast time scale the RF diffusion affects predominantly the parallel energy of the electrons entering the resonant region, assume first that a particle which enters the resonant region at the low parallel velocity end ($\beta_{\|1}$) has a perpendicular energy characteristic of the bulk temperature [$y_0^{(0)} = 2\beta_{th}^2$ ($\beta_{th} = v_{th}/c$)]. The pitch angle collisional scattering, which is the dominant collisional scattering process, provides a mechanism which transforms the parallel energy of the resonant particle into perpendicular energy. That is, the particles which now emerge from the $\beta_{\|1}$ boundary have $y^{(0)} > y_0^{(0)}$ [from Eq. (5)] and, therefore, constitute a population of electrons with high perpendicular velocities which are moving back and forth in pitch angle outside the resonant region. Some of the particles which have been released from the $\beta_{\|1}$ boundary, after being slightly slowed down by the thermal background, will return to that boundary with an intermediate perpendicular energy $y_0^{(1)}$, i.e. $y_0^{(0)} < y_0^{(1)} < y^{(0)}$. Then the same cycle is repeated again: fast parallel diffusion, slow pitch angle scattering, and so on.

The equilibrium state is characterized by a narrow layer near $\beta_{\|1}$, separating the resonant region (plateau) and the thermal bulk, populated by electrons having perpendicular energies intermediate between the bulk and plateau value. The resonant region is populated by electrons coming from this region which are characterized by an average entry perpendicular energy $y_0^{(s)} > 2\beta_{th}^2$. Since $y_0^{(s)}$ is in general unknown, we approach the problem with an iteration scheme. Equation (5) is first solved for $y^{(0)}$ with $y_0^{(0)} = 2\beta_{th}^2$. Then, in the next step we choose for $y_0^{(1)}$ the arithmetic mean of $y_0^{(0)}$ and $y^{(0)}$. In the n-th step one chooses for $y_0^{(n)}$ the arithmetic mean of $y_0^{(0)}$ and $y^{(n-1)}$), and so on. It is found that this scheme has extremely good convergence properties, that is, in five or so steps y does not change anymore. The final saturated value of y provides the plateau value of the perpendicular temperature $T_\perp \equiv y/2\beta_{th}^2$. The particles which are exiting from the $\beta_{\|1}$ boundary can also reach, in a few collisional times, the negative $\beta_\|$ side of velocity space and thus they imprint on the thermal background of those electrons the high perpendicular velocities they carry; this leads to a secondary small tail formation and perpendicular temperature enhancement in the negative $\beta_\|$ direction as is observed in numerical integrations of the 2-D Fokker-Planck equation.

In Fig. 1 we plot the ratio $T_\perp/T_B$ for a bulk temperature, T_B, of $1keV$ and $Z_i = 1$ as a function of the central position of the resonant region $v_0 = (v_{\|1} + v_{\|2})/2$ for various values of its semiwidth $\delta = (v_{\|2} - v_{\|1})/2$ (both normalized to the bulk thermal velocity $v_{th} = T_B^{1/2}/m_e^{1/2}$). The δ-labeled curves stop at the points indicated since relativity sets the limit $v_{2max}^2 + v_{\perp 0}^2 = (v_0 + \delta)_{max}^2 + v_{\perp 0}^2 = c^2/v_{th}^2$. It is clear that the presence of a maximum value for $T_\perp$ is a relativistic effect. We also observe that sometimes for a given v_2 there are two values of v_1 (corresponding to a narrow and a wide resonant region respectively) which give rise to the same perpendicular temperature. This has been verified by numerically solving the two-dimensional Fokker-Planck equation for both wide and narrow spectra. The portions of the curves where this double-valuedness occurs are dotted. There also exists a lower limit for v_1 below which strong Landau absorption would have prevented the wave from penetrating to the plasma core. In Fig. 1 we set $v_{1min} = 4$. The dashed line next to the $\delta = 1$ curve corresponds to the nonrelativistic limit for $\delta = 1$; the bending over of the curves, for each δ, is due to relativistic effects. The points in Fig. 1 correspond to numerical results from solving the two-dimensional Fokker-Planck equation.[4] The predictions of the models in Ref. 5 (KHB-curve) and in Ref. 6 (HB-curve) are also shown. We note that the KHB theory [Eq. (27) of Ref. 5] gives $T_\perp \rightarrow v_0\sqrt{2/\pi}$ as $\delta \rightarrow 0$, which is very close to the small δ curve of the present model; however the scaling with δ is completely different. The HB result shows correct scaling with the central location of the resonant region (v_0) but overestimates the plateau value for $T_\perp$ by a factor of almost 2.

It should be noted that Fig. 1 can be used to estimate the enhanced $T_\perp$ as a function of plasma radius. Thus, for example, if current is generated centrally in the plasma, where the temperature is highest, there $v_1 \approx 4$ and v_2 is whatever may be required (up to the accessibility limit for the particular plasma). But as we move out, away from the central core, both v_1 and v_2 increase as the electron temperature drops. Thus, although there may be little current generated away from the central plasma core (since $v_1 > 4$), the $T_\perp$ enhancement may persist, or even increase, albeit for fewer particles. This can give rise to energetic particles, at the plasma edge, which are poorly confined, and hence degrade the current drive efficiency.

As regards the enhanced $T_\perp$ for the core of the plasma, where the major current drive is usually desired, it is more useful to keep v_1 fixed at its lowest value. In Fig. 2 we plot the ratio $T_\perp/T_B$ for various values of the bulk temperature, with $Z_i = 1$ and $v_1 = 4$ (normalized to the thermal velocity) as a function of the ratio v_2/c (c being the velocity of light). The dotted lines correspond to the nonrelativistic result and the relativistic (solid) curves actually stop at the points indicated due to relativistic constraints ($v_2^2 + v_{\perp 0}^2 < c^2$). Here we note that as the bulk plasma temperature increases (from 1 keV to 5 keV) the maximum enhanced $T_\perp$ in the resonant region does not increase appreciably (from about 35 keV to 50 keV, respectively). This is encouraging for future current drive experiments in plasmas closer to ignition temperatures.

In conclusion we have presented a model for calculating the enhanced perpendicular temperature in the resonant region of lower-hybrid current drive, and shown that it satisfactorily predicts these temperatures as obtained from 2-D numerical integration of the Fokker-Planck equation. This approach in estimating the enhanced perpendicular temperature is used in a new analysis of lower-hybrid current drive presented elsewhere[7].

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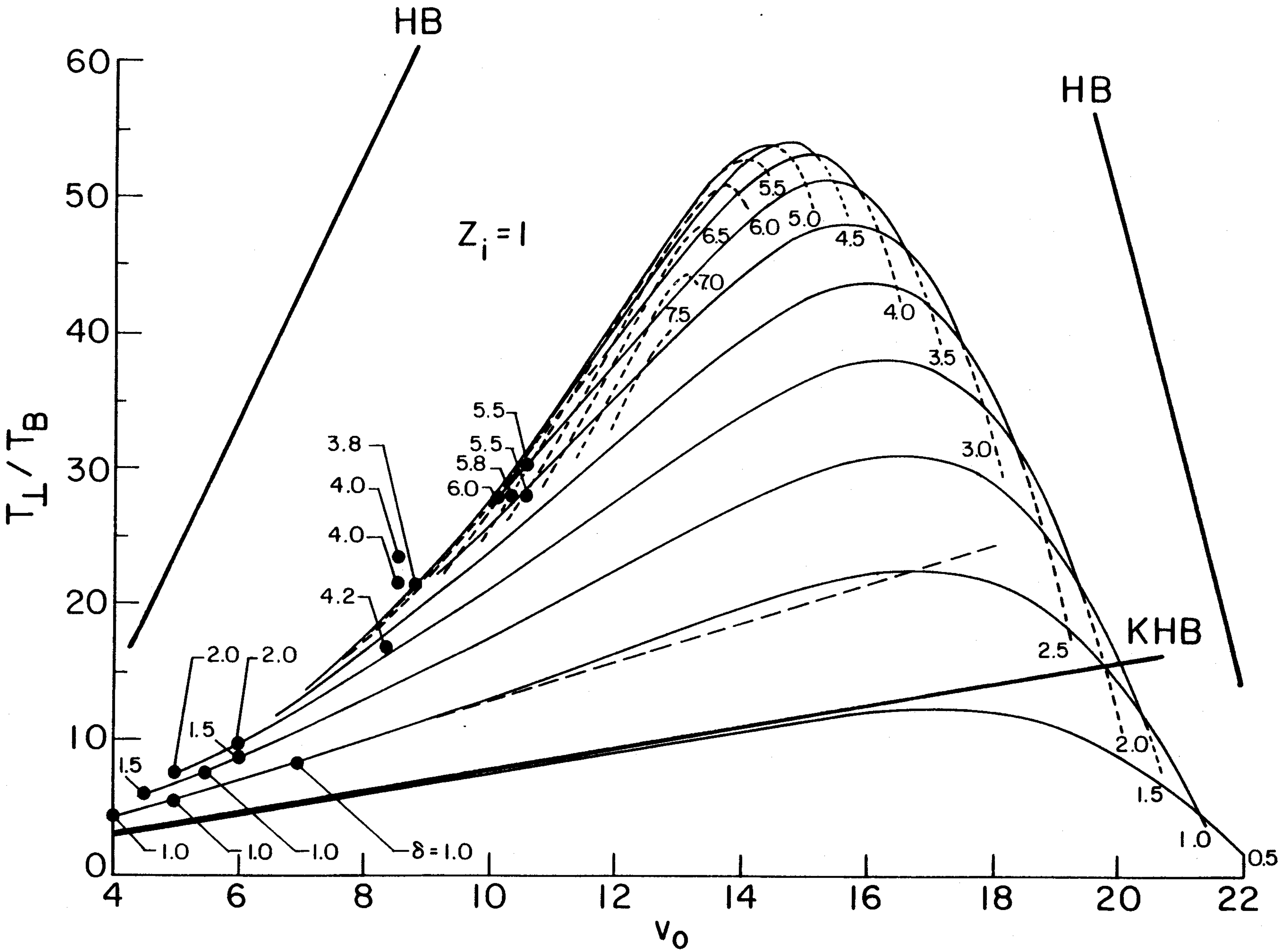
Figure Captions

1. Perpendicular temperature $T_\perp$ (normalized to the bulk temperature T_B) is plotted as a function of the central phase velocity position of the resonant region $v_0 = (v_{\|1} + v_{\|2})/2$ for various values of its semiwidth $\delta = (v_{\|2} - v_{\|1})/2$ (both normalized to the bulk thermal velocity v_{th}), and for $Z_i = 1$ and $v_1 \geq 4$. The dotted line corresponds to the nonrelativistic limit for $\delta = 1$. The points correspond to numerical results from solving the two-dimensional Fokker-Planck equation.[4] The KHB and HB-curves correspond to the predictions of the models of Ref. 5 and Ref. 6, respectively.

2. The ratio $T_\perp/T_B$ is plotted for various values of the bulk temperature, $Z_i = 1$ and $v_1 = 4$ (normalized to the bulk thermal velocity) as a function of the ratio v_2/c (c being the velocity of light). The dotted and solid curves correspond to the nonrelativistic and relativistic results, respectively.



HB
HB
KHB
$Z_i = 1$
$T_\perp / T_B$
v_o
0
10
20
30
40
50
60
4
6
8
10
12
14
16
18
20
22
5.5
5.5
5.8
6.0
3.8
4.0
4.0
4.2
2.0
2.0
1.5
1.5
1.0
1.0
1.0
δ=1.0
5.5
6.5
6.0
5.0
4.5
7.0
7.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5

Figure 1

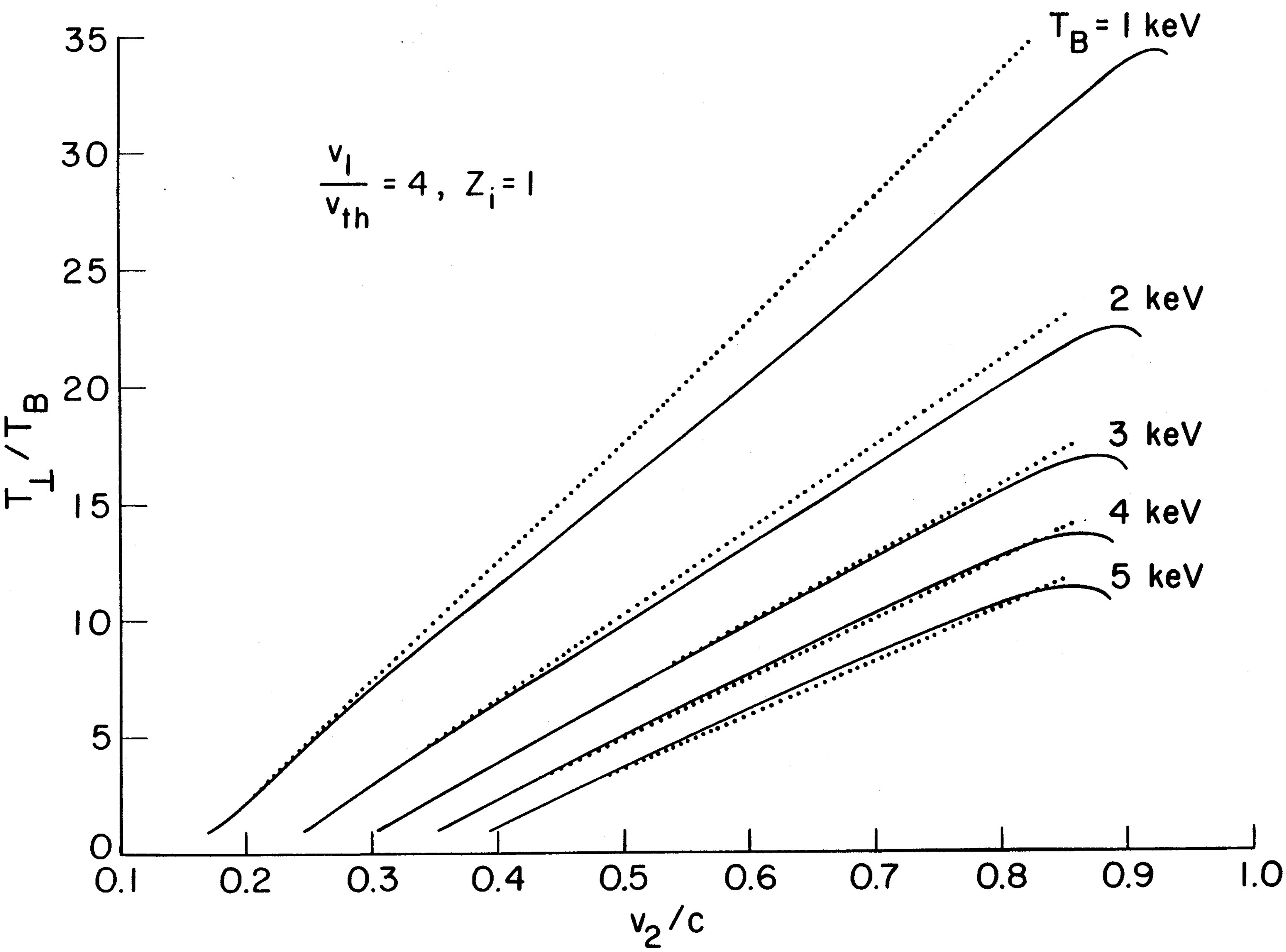

$T_B = 1$ keV
2 keV
3 keV
4 keV
5 keV
$\frac{v_1}{v_{th}} = 4, \; Z_i = 1$
$T_\perp / T_B$
35
30
25
20
15
10
5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
v_2/c

Figure 2