PFC/JA-84-29

AXISYMMETRIC, WALL-STABILIZED TANDEM MIRRORS

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August, 1984

ABSTRACT

We discuss the possibility of an axisymmetric tandem mirror in which stability accrues from wall stabilization. We find that the stability requirements are compatible with thermal barrier requirements so that the thermal barrier plug cell can also provide stabilization. Thus, a single axisymmetric end cell can plug and stabilize a high beta plasma solenoid. Self-stabilization of the central cell and other magnetic configurations are also discussed.

This work was supported by the U.S. Department of Energy Contract No. DE-AC02-78ET51013.

Submitted to Nuclear Fusion in September, 1984.

INTRODUCTION

A tandem mirror is a linear device in which confinement in a central solenoid results from electrostatic potential "plugs" located at either ends of the device. The efficiency of creating the potentials can be improved by the interposition of a "thermal barrier", between the central solenoid and plugs [1]. A thermal barrier is an along-the-field-line potential depression that serves to thermally insulate the central cell "thermal" electron population from a supra-thermal population in the end plug. The potential depression results from the interposition between the central cell and plug of a dense, localized hot electron population.

MHD stability in a tandem mirror usually derives from the presence of a quadrupole, "minimum-B" cell containing high plasma pressure. This "anchor" is required primarily for stabilizing low azimuthal mode number (m) modes, since higher order modes are stabilized by central cell Finite Larmor Radius (FLR) effects. However, the presence of a quadrupole cell necessarily leads to a more complicated coil set and a flux tube distortion away from axisymmetry. The non-axisymmetry of this system constrains equilibrium, can produce radial transport and enhances the drive for MHD and electrostatic (trapped-particle) ballooning instabilities.

One approach to obtaining enhanced axisymmetry is the axicell arrangement [2]. In this geometry, the thermal barrier and plug can be

produced in the same mirror cell, the so-called axicell, through the use of sloshing-ions. The quadrupole anchor can then be located outside of the confinement region. This arrangement provides axisymmetric ion confinement, but is more susceptible to trapped particle modes than other schemes [3].

Recent work by Berk et al. [4] suggests the possibility of the use of wall stabilization mechanism for m = 1 curvature driven modes. The source of stability is the image currents generated by placing the wall (or properly shaped conductors) in close proximity of a high-beta axially localized plasma. This work contained a low beta approximation and in the MHD limit, it has been generalized to arbitrary beta by Pearlstein and Kaiser [5].

In this paper, we will discuss schemes for obtaining a totally axisymmetric tandem mirror in which stability derives from the aforementioned wall-stabilization. We will show that the stabilization criteria are compatible with the requirements for the hot electron population of a thermal barrier. The hot electron population present in the axicell could both create the thermal barrier and provide MHD stability for m = 1 curvature driven instabilities. Higher m modes can be stabilized by Finite Larmor Radius stabilization deriving from both the axicell sloshing ions and the central cell thermal ions. Thus, the need for quadrupole anchors could be eliminated. The maximum beta that can be confined in the central cell would then be determined by MHD ballooning and trapped particle stability requirements.

Additionally, we consider the possibility of the central cell being self-stabilized by the same wall effect. The isotropic nature of the

central cell plasma will weaken the wall response and very high beta is required for stabilization. Nevertheless, this scenario raises the possibility of a high beta, linear confinement device containing simple axisymmetric plugs. Such a device would clearly produce a very desirable arrangement for a fusion reactor.

In Section II-A, we will discuss the axicell stabilization requirements. Section II-B will discuss central cell wall stabilization and in II-C other interesting anchor arrangements based on high beta ion population. Section III will summarize the conclusions.

II. WALL STABILIZED AXICELL REQUIREMENTS

We consider an axicell thermal barrier arrangement [2] in which the magnetic field and potential are as shown schematically in Fig. 1. The thermal barrier is formed by a disc-shaped high beta hot electron plasma that is mirror confined near the axicell field minimum (point b).

From the point of view of thermal barrier formation, the axicell midplane field should be low to minimize the trapping rate of central cell ions that traverse the barrier region. Furthermore, the temperature must be high to eliminate the expulsion of hot electrons from the barrier region and to reduce electron collisionality. Thus, high beta is desirable. Additionally, the hot electron power balance requires a minimization of the hot electron volume, which requires maximum hot electron anisotropy. For example, in the MARS tandem mirror reactor design study [6], the thermal barrier was formed by electrons with mean energy of 820 keV, beta of 50% and

an anisotropy (A = P_{\perp}/P_{\parallel} , the ratio of perpendicular to parallel pressures) of A = 4. The self consistent magnetic field at the thermal barrier was B_{b} = 1.2 T.

The stability requirement obtained from MHD for an isolated mirror cell obtains a simple form for a sharp boundary pressure model [5] and is given by

$$\omega^{2} = \int \left[\frac{p}{B_{vac}^{2}} \frac{r_{vac}}{r_{vac}} + \frac{1}{16} \left(\frac{\beta'_{vac}}{1 - \beta_{vac}} \right)^{2} (1 - p/B_{vac}^{2}) \right] ds$$

$$-0(1/\Lambda) > 0$$
 (1)

with $\beta_{vac} \equiv 8\pi P_{\perp}/B_{vac}^2$, and $p \equiv (P_{\perp} + P_{\parallel})/2$ and $\Lambda \equiv (R_w^2 + R_p^2)/(R_w^2 - R_p^2)$ with R_p and R_w the respective plasma and wall radii. Primes represent along-the-field-line derivations. This expression is valid at arbitrary beta and contains the long-thin approximation $(\partial/\partial z < \langle \partial/\partial r)$ and a sharp boundary pressure model. The first term in the integral will be recognized as the MHD drive due to the vacuum curvature and the second term represents the effect of wall stabilization. This expression is valid for large Λ , that is for the wall close to the plasma edge. The third term is small and can be shown to be destabilizing. For the wall right at the plasma edge $\Lambda + \infty$ and this term is zero and the first two terms of Eq. (1) can be viewed as

providing a necessary condition for stability. This condition can thus be written as S > 1 with S defined as

$$S = 1/16 \int ds(1 - p/2) (\beta''_{vac}(1 - \beta_{vac}))^2 / \int ds \frac{p}{B_{vac}^2} \frac{r''_{vac}}{r_{vac}}$$
(2)

Stabilization clearly requires a high beta axially localized disc. We note that, whereas at low beta, axial localization requires high plasma anisotropy, at high beta the plasma disc will dig a diamagnetic well and axial localization can be obtained with a more isotropic plasma.

In order to continue further, we must impose a dependence for P i.e. P_{\perp} (B). (The sharp boundary assumption eliminated any radial dependence.) The field line derivatives can then be evaluated using the long-thin equilibrium condition

$$8\pi P_{\perp} + B^2 = B_{\text{vac}}^2$$
(3)

given a vacuum field profile. Additionally, the dependence P (B) follows from the axial pressure balance:

$$\frac{\partial}{\partial B} (P_{\parallel}/B) = -P_{\perp}/B^2 .$$
 (4)

A simple pressure model that may be used to evaluate the stability requirements is

$$P_{\perp}(B) = P_{0} (1 - B^{2}/B_{1}^{2}) \qquad B < B_{1}$$
(5)

 $B > B_1$

with
$$P_{o} \equiv \frac{\beta_{o}}{8\pi} = \frac{B_{vo}^{2} B_{1}^{2}}{B_{1}^{2} - B_{vo}^{2} (1 - \beta_{o})}$$

0

 β_{0} is the midplane vacuum beta ($\beta_{0} \equiv 8\pi P_{\perp}(s=0)/B_{vac}^{2}$ (s = 0)) and B_{vo} , B_{1} are respectively the vacuum midplane field and field at which the pressure goes to zero. This distribution function has been called an ideal distribution since it does not contain a mirror mode limit. In this model, the pressure goes to zero at $B = B_{1}$ which is less than the peak mirror field, B_{m} . (We define a vacuum mirror ratio $R_{L} \equiv B_{1}/B_{vo}$ which is less than the axicell mirror ratio $R_{T} \equiv B_{m}/B_{vo}$.) Thus R_{L} determines the localization of the hot electron disc. From Eq. (3), we can obtain P_{\parallel} (B) and the midplane anisotropy

$$P_{\perp o}/P_{\parallel o} = \frac{B_{\perp} + B_{\perp}}{B_{\perp} - B_{o}}$$
 (6)

with B_0 the midplane (beta depressed) field. Notice that as beta increases B_0 will decrease and the anisotropy will approach 1.

Assuming a parabolic vacuum field, we can now evaluate Eq. (2). Fig. 1 displays the stabilization factor, S, as a function of midplane beta. Distributions with a small mirror ratio, R_L , have a larger anisotropy and exhibit stronger stabilization. Since these profiles can have the same spatial extent, it is clearly anisotropy (not axial localization) that determines stability.

A second model pressure distribution that allows pressure to extend out to the mirror peak is given by

$$P_{a,b}(B) = nP_{o}(B/B_{m})^{2} (1 - B/B_{m})^{n-1}$$
(7)

with
$$P_o = \frac{\beta_o B_m^2}{8\pi n (1 - \beta_o) (1 - \sqrt{1 - \beta_o}/R_{vac})^{n-1}}$$

and R = B/B = B/(B $\sqrt{1-\beta}$). This distribution has an anisotropy given by

 \mathbf{i}_{i}

$$P_{\perp o} / P_{\parallel o} = n/(B_{m o} - 1)$$
 (8)

Again, assuming a parabolic vacuum field, we evaluate the stability factor. Curves for S = 1 and S = 2 are shown in Fig. 3 as a function of midplane anisotropy. The circles represent the results for the ideal distribution (shown in Fig. 2). The very close agreement between these two pressure models implies that the stabilization depends on the midplane anisotropy and is not sensitive to the exact $P_{\perp}(B)$ dependence. (Some deviation between the pressure models is seen for S = 2.)

The pressure model of Eq. (7) contains a mirror mode limitation on β given by

$$\frac{\beta_{o}}{1-\beta_{o}} < \frac{8\pi (B/B-1)}{(n+1-2B/B)}$$

The mirror mode limit which is a boundary imposed by equilibrium is indicated by the dashed curve in Fig. 2. Thus, we observe a stability window between the wall stabilization requirements and the requirements of equilibrium at high beta.

The MARS thermal barrier operating point, $\beta_0 = 50\%$, A = 4 is also indicated on Fig. 3. We see that this operating point falls at S \circ 2.5.

A last point of interest that can be gleaned from this distribution is the effect of a "sloshing" hot population. From Eq. (7), we can determine that the pressure will peak at $B_{m} = \frac{2B}{m}/(1 + n)$. For large n and mirror ratio, we then find

$$R_{slosh} \equiv \frac{B_{slosh}}{B_{o}} \circ 2 P / P$$

For $P_{\perp}/P_{\parallel} < 2$ the pressure peak moves off the mid-plane and Fig. 2 indicates an increase in the required midplane beta for stabilization. Thus, if stability of anisotropy driven modes requires a sloshing character to the high beta component, we can still obtain a stable regime.

In summary, we observe the existence of a high beta stability window which may be bounded by equilibrium requirements. We note that there is a decreased stabilization as the wall moves back from the plasma. Berk, et al. [3] showed that at low beta the stabilization is proportional to $(r_p/r_w)^2$ so this effect is not expected to be strong when the plasma edge is near to the wall. Additionally, in a tandem mirror the added instability drive that comes from the central cell would further increase the desired S value. An appropriate value of the stabilization factor, S, might therefore be S ~ 2 .

Up to now, we have considered the MHD result of Ref. [5] and one could question the applicability of the MHD formalism. The energy of the hot electron forming disc is expected to be in the 500 to 900 keV range and their high beta drift frequency $\omega_D \sim \beta_h \omega_{\star h}$, with $\omega_{\star h}$ the hot species diamagnetic drift frequency will greatly exceed the central cell diamagnetic drift frequency, $\omega_{\star c}$, and the MHD growth rate that characterizes MHD modes. Thus, the electrons should be considered a "hot" species in the accepted EBT terminology.

One can show, however, that the failure to satisfy a "decoupling" condition will lead to an MHD like response of the hot ions (Ref. 7, Eq. (73) and Fig. 1) even if the core beta is negligible. This decoupling condition sets the requirement

$$\gamma_{\rm MHD}^{\prime}/\omega_{\rm kh}^{\prime} > 0.5 \tag{10}$$

with
$$\gamma_{MHD}^2 = \gamma_a^2 (1 - S) + \gamma_c^2$$
.

and γ_a , γ_c are respectively the axicell and central cell MHD growth rates. For stability S > 1 and $|\gamma_a^2(1-S)| > \gamma_c^2$ so that $\gamma_{MHD}^2 < 0$. We can estimate

$$\gamma_{MHD}^{2}/\omega_{kh}^{2} \simeq \frac{T_{c}}{T_{a}} \left(\frac{L_{a}}{r_{a}}\right)^{2} \frac{V_{n}}{V_{c}} \frac{n_{eh}}{n_{c}} \frac{1}{\left(k_{\perp} \rho_{i}\right)_{c}^{2}}$$
(11)

with the subscripts c and a representing central cell and axicell quantities. V is volume, k_{\perp} wave number (m/r_c) , L_a the axicell half-length and ρ_i ion gyroradius. We have approximated the curvature as $\kappa_a \circ r_a/L_a^2$. Estimating $T_a/T_c \circ 25$, $L_a/r_a \circ 5$, $V_c/V_a \circ 20$, $n_c/n_{eh} \circ 10$, $k_{\perp} \rho_{ic} \circ 0.013$, $B_c/B_a \circ 2$ we obtain $\gamma_{MHD}/\omega_{kh} \circ 6$. A critical issue in the above scheme is startup. The hot electrons in the axicell thermal barrier may be started up in the fashion of the E.B.T. experiment [8]. That is, the electrons could be heated up to sufficient energies to decouple from the core plasma while maintaining a sufficient density in the low beta core (below the Van Dam-Lee limit) to prevent high frequency interchange mode (the Berk-Dominguez mode). Once the hot electron beta exceeds the required beta for wall stabilization, the sloshing ion beams could turn on to since they will be stabilized by the electrons for the m = 1 mode.

Higher azimuthal mode numbers $(m \ge 2)$ can be stabilized by ion Finite Larmor Radius effects (FLR). Berk, et al. [4] suggest that the high beta FLR stabilization term should enter the dispersion relation in the following manner:

$$\omega^{2} = \gamma_{\text{MHD}}^{2} \left(1 - \text{S/m} - \frac{\beta_{i} \frac{k_{\perp}^{2} \rho_{i}}{2\kappa\Delta}}{2\kappa\Delta} \right)$$

with β_i the ion beta, k_{\perp} the perpendicular wave vector $(k_{\perp} = (m^2 - 1)/r_p^2)$, κ the curvature, Δ the pressure gradient scale length and g is a geometric factor representing the fact that the ions which supply the FLR are sloshing and fill a larger volume than the electrons (which dominate the drive and wall term). This then yields the approximate requirement for FLR stabilization:

$$(\rho_i/r_a)^2 > \frac{(1-S/m)}{(m^2-1)} \frac{2\kappa\Delta}{g\beta_i}$$

We will take $\kappa \circ r_a/L_a^2$ with L_a the axicell half length. Then, for m = 2 and typical parameters; S = 1.5, $r_a/L_a = 0.1$, $\Delta \circ r_a/3$, g = 2, $\beta_1 = 0.1$, we find $\rho_1/r_a > .05$ which would require for B = 1.5 T, $r_a = 1$ m and tritium ions a temperature of $T_i > 180$ keV. In the MARS study the upper limit on energy based on adiabaticity was about 700 keV [6]. We note that the energetic "sloshing-ions" form a second "hot" species and must satisfy the same criterion, Eq. (10), as the electrons.

With the thermal barrier electrons providing stabilization for m = 1modes and the sloshing-ions providing stability for m > 1, the axicell would be stable to all instability. The central cell could then be started up which would add some MHD instability drive as well as additional FLR stabilization. Importantly, when the axicell dominates (and stabilizes the MHD drive) the central cell enters the decoupling condition as a result of inertia. From Eq. 11, we observe that the central cell temperature does not enter the decoupling condition. This means that during startup the axicell can stabilize the relatively cool central cell (when Eq. 10 is satisfied). Of course, if the central cell is self-stabilized, as discussed below, we need not consider the decoupling condition.

II-B. Central Cell Wall Stabilization

Tandem mirror economics favors very high beta central cell operation [6,9]. High beta permits designs to lower the field up to a limit set by alpha adiabaticity and to raise the plasma pressure, up to a limit set by neutron wall loading [8]. Additionally, at high beta with a relatively sharp boundary pressure profile as is desired for wall stabilization [4] classical conduction within the edge region can limit the desired edge gradient. These effects are examined in Ref. 9 and a central cell beta in the range of 80 to 90% is found to be favorable.

The strength of the stabilization depends on the spatial localization of beta within the mirror. This localization is weak for the isotropic central cell plasma but can become significant at high beta. The only nonisotropic component in the central cell is the hot alpha particles produced by D-T fusions and they are only mildly anisotropic and account for less than 20% beta. (The stabilization caused by alphas can be enhanced through the use of polarized nucleii but this possibility will not be analyzed here.)

Since the central cell plasma is isotropic, the pressure becomes independent of B and axial position, which will greatly simplify the alongthe-field-line integrals in Eq. (1). In this limit, $P_{\perp}(B) = P_{\parallel}(B) = P_{o}$.

To evaluate Eq. (1), we choose a parabolic field shape $B(z) = B_{vo} + (B_m - B_{vo})(s^2/L^2)$ with L the central cell ramp length. Since the central cell mirror ratio is large, typically greater than 8, we can allow the limit on

integration to extend to infinity to obtain an analytic estimate of S. Additionally, in this estimate, we will set the factor $(1 - p/B_v^2(s)) + (1 - \beta_0/2)$ and thus remove this factor from the integral.

The integral along the axis is performed with the aid of the residue theorem and yields the stabilization factor S.

$$S = \frac{16}{3} \frac{(1 - \beta/2)}{\beta_{o}} \left[1 + \frac{6 \sqrt{\beta'} - 5}{4(1 - \sqrt{\beta_{o}})^{1/2}} - \frac{6 \sqrt{\beta} + 5}{4(1 + \sqrt{\beta_{o}})^{1/2}} \right]$$
(12)

and D = $3\pi/32$ β_0/L is the vacuum curvature drive consistent with Eq. 12.

Figure 3 shows S vs. β_0 from Eq. (10) as well as the exact result obtained from a numerical integration. The S = 1 boundary is seen to occur at $\beta_0 = 83\%$ and above this beta value the curve rises steeply. Notice that the parabolic field scale length has dropped out of the ratio indicating an insensitivity to the axial field profile.

Recently, studies have been performed by Potok [9] to model a tandem mirror with a one central cell beta near 1. It is shown in this work that a beta of 80 - 90% is optimum. As indicated in this work, at high beta one is limited in the ability to lower the field by alpha adiabaticity and to raise plasma pressure by neutron wall loading. In addition, classical confinement (of particles and energy) serves to limit the edge pressure gradient (recall we desire $R_p/R_w \leq 1$). For β_{cc} of 90%, $n_c = 1.2 \times 10^{14} \text{ cm}^{-3}$, plasma radius

= 1.7 m, electron temperature = 30 keV and solenoid vacuum field = 2 T Potok finds a resulting wall loading is 3.5 MW/m^2 . Alternatively, he finds for a vacuum field of 2.5 T and a plasma radius of 1.3 m a wall loading of 6.9 MW/m^2 at the same electron temperature. The MARS design [7] has a central cell field of 4.7 T and a neutral wall loading of 4.2 MW/m^2 . Thus, high beta can significantly reduce central cell field requirements and raise neutron wall loading.

II-C. Other Anchor Configurations

A high-beta hot electron or ion disc is subject to high beta anisotropy driven instability such as the relativistic Whistler [10] and cyclotron maser instability for electrons [10, 11]. At high beta relativistic effects will tend to stabilize the Whistler modes [10]. For ions the Alfven ioncyclotron instability [12] will present a limitation an beta. AIC modes can be stabilized by axial localization [12] although this process is limited by the requirements on ion adiabaticity. Importantly, the large gyroradius of ions can lead to strong FLR stabilization.

Two possible ion disc configurations are shown schematically in Fig. 4. In these configurations, the ion disc forms an inside anchor, (preferable to an outside anchor due to superior trapped particle mode stabilization properties).

In Fig. 4a, the anchor cell is located beyond the central cell choke coil. The high beta anchor ion population would require an axial extent of

10 to 20 ion gyroradii for adiabaticity. AIC stability would also require an axial extent of not more than this [13]. Power requirements can be minimized by pumping the passing ions to prevent local trapping. For example, for $B_{vac} = 2T$, $n_a = 7 \times 10^{13} \text{ cm}^{-3}$, $T_i = 500 \text{ keV}$ ($\beta = 0.6$), the power required (drag and pitch angle scatter) is about 3 MW/m³. The pumping requirements are increased in proportion to the effective anchor volume which is about 5 m³. This arrangement will increase the length of the end cell region, increasing the trapping current of the passing ions and thereby the minimum central cell length required for ignition.

Pumping requirements can be diminished by placing the anchor on the inside of the choke coil (Fig. 4b). A small throttle coil creates a mirror (mirror ratio ~ 2 to 3) between the central cell and anchor so as to confine most of the energetic central cell alphas, which would otherwise suffer from loss of adiabaticity. In this case, however, the density rises to the central cell level which would increase power requirements for the hot ions. For a hot ion density of 7×10^{13} cm⁻³ and a core ion density of 2×10^{14} cm⁻³, the required power for the anchor is about 9 MW/m³ of volume.

III. Conclusions

Use of wall stabilization for m = 1 curvature driven modes is seen to present the possibility of a completely axisymmetric tandem mirror reactor that can operate at high betas and have compact and relatively simple end plugs. Requirements are seen to be compatible with a thermal barrier. Furthermore, there is a possibility that the central cell be self-stabilized which would permit beta near one and eliminate the possibility of trapped particle modes.

Finally, it is important to note that the wall effect could be obtained through the use of properly shaped and positioned conductors. This possibility would cut down on sputtering, wall loading and increase the access to the plasma for neutral beams.

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ACKNOWLEDGEMENTS

I would like to thank B. Lane for many important discussions. Additionally, I would like to acknowledge important contributions from H. Berk (U. Texas), L. D. Pearlstein, G. Smith, and B. G. Logan (LLNL) and J. Shuy (SAI). This work was supported by U. S. DOE Contract No. DE-ACO2-78ET51013.







FIGURE 2

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FIGURE 3

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FIGURE 4



